CURRICULUM

Mathematics 416

Secondary School

Québec

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Direction de la formation générale des jeunes

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Minister of Education

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Introduction

Mathematics 416 is a program designed for Secondary IV students attending schools in Québec.

To prepare young Quebecers for the demanding world of the twenty-first century, schools must focus on the students' cognitive growth and the development of basic skills (i.e. communication and problem-solving skills as well as the ability to work with technology).

As an important component of a solid basic education, this mathematics program provides fertile ground for the development of the skills that students will require in the future. As Resnick and Kloffer have noted, "Graduates must not only be literate; they must also be competent thinkers."[1]

Mathematics 416 is part of the basic secondary school curriculum and is designed for students who have successfully completed the Secondary III mathematics course. In this program, the students not only build on what they learned in the first cycle of secondary school, but also develop the skills required by every citizen to function productively in society.

Because society is changing so rapidly, and owing to developments in the field of mathematics education, it is important to stress the interconnection of knowledge, skills and attitudes in the teaching of this program.

Three Major Guiding Principles

Current knowledge of the learning process and the focus of student learning have led to an emphasis on three principles intended to guide teachers in their work with students. These principles are as follows: to encourage the students to participate actively in the learning process, to encourage them to use a problem-solving approach at each stage of the learning process and to encourage them to use the appropriate technology for each task.

Encouraging Students to Take an Active Part in Their Own Learning Process

A great many research studies have shown that students should play a central role in their own learning process. In short, they should be ultimately responsible for their education:

1. L. B. Resnick and L. E. Klopfer, "Toward the Thinking Curriculum: An Overview," in *Toward the Thinking Curriculum: Current Cognitive Research, 1989 Yearbook of the Association for Supervision and Curriculum Development*, ed. Lauren B. Resnick and Leopold E. Klopfer (Alexandria, Va.: Association for Supervision and Curriculum Development, 1989), 1.

> The construction of a given concept is a complex process that depends first and foremost on the student. Concepts are not directly transmitted from a knowledgeable person to a student who supposedly knows nothing in a given field. Before they tackle new subject matter, students have already developed their own conceptions, which are well organized, practical and sometimes fairly resistant to the changes targeted in a course of study.
>
> Thus, teaching involves creating situations in which students draw on their own knowledge. Teaching involves structuring the learning process around their strategies and thinking in order to try to get them to make progress in the construction of a given concept.[2]

To help students acquire the knowledge and skills targeted by this program, it is important to design learning situations that call upon their powers of observation and dexterity and that involve manipulations, exploration, construction and simulations. Through these activities, the students analyze hypotheses, actively look for solutions, discuss their approaches, analyze concepts or theories from their own point of view while taking into account other points of view, actively question the meaning and consequences of the procedures they use and relate the knowledge they have acquired to their own experience. These situations encourage the students to reflect, act, react and establish connections with what they have already learned.

Another way teachers can encourage students to participate in their learning process is by developing a suitable teaching approach. Teachers will do more to help young people build their knowledge by asking them questions than by giving them the answers.

Any question that helps students get on the right track or find the answers to their questions by themselves encourages them to participate in their own learning.

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2. Nadine Bednarz, "L'enseignement des mathématiques et le Québec de l'an 2000," excerpted from Richard Pallascio, ed., *Mathématiquement vôtre! Défis et perspectives pour l'enseignement des mathématiques* (Montréal: Les éditions Agence d'ARC inc., 1990), 69 (Free translation).

Encouraging Students to Use a Problem-Solving Approach at Every Stage in the Learning Process

Problem solving is an essential teaching and learning tool in several general education programs (e.g. pure sciences, social studies) and is an integral part of any mathematical activity. Problem solving is not a separate theme, but rather a process that should be applied throughout the program and that provides a suitable context for learning concepts and acquiring skills.

> Problem solving is both a basic skill that students should develop and an effective teaching approach that promotes the development of mathematical knowledge, thinking skills, socio-affective attitudes and problem-solving strategies.[3]

Learning through problem-solving calls for the active involvement of the students and the use of questions. It is important that the teacher ask the students questions and that they in turn ask questions of each other and the teacher.

Some problem-solving tasks may be more difficult than others and the problems themselves can be quite varied. For instance, the students may encounter the following:

> ... problems with solutions requiring students to choose an appropriate combination of knowledge and skills from among several possible combinations seen in the past.[4]

They may even encounter the following:

> ... problems requiring students to create a new combination of knowledge and skills, exercise a great deal of intellectual independence and use plausible reasoning in order to solve them.[5]

Problem solving is a very effective means of developing knowledge and skills. The quality of learning depends on the variety of the problems assigned and on their level of difficulty. In a learning context, the students can even be presented with very challenging problems. By solving these problems, the students can discover such things as properties, relationships and strategies by themselves. A wide variety of problems allows students to conceptualize their knowledge and develop numerous problem-solving strategies. Problem solving is a way of learning and a way of teaching.

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3. Québec, ministère de l'Éducation, *Mathematics Curriculum Guide, Elementary School, Booklet K, Problem Solving,* Code 16-2300-11A (Québec: ministère de l'Éducation, 1989), 47-51.

4. Ibid, p. 15.

5. Ibid, p. 15.

The problems can be related to the students' environment and used at various stages in the learning process. Problem solving can help students learn new concepts and develop skills or help them expand their knowledge and reinforce what they have learned.

Thus, problems provide an opportunity to:

- apply and integrate mathematical knowledge (e.g. concepts, properties, algorithms, techniques, procedures);

- develop intellectual skills (e.g. organizing, structuring, abstracting, analyzing, synthesizing, estimating, generalizing, deducing, justifying);

- develop positive attitudes (e.g. becoming aware of one's potential, respecting the opinions of others, and being imaginative and creative, rigorous and precise);

- use different problem-solving strategies (e.g. looking for patterns, representing a problem by means of a figure or a graph, constructing a table, referring to a known model, using a formula, formulating an equation, working backwards).

The emphasis on problem solving does not mean that exercises have no part in the teaching or in the learning of mathematics. Exercises play a different role, but one that is complementary to that of problem solving. For instance, exercises can help students consolidate skills and habits that they have already begun to develop. They can also help the students practise applying definitions and properties that they have learned in class. Exercises can neither replace nor be replaced by problems.

By using a problem-solving approach, the students become accustomed to referring to a known mathematical model and are thereby more likely to attain the terminal objectives. The teacher should also assist the students in using a procedure that will enable them to acquire more knowledge and generate other models. This will help them attain the global objectives in accordance with the first guiding principle, namely, to encourage the students' active participation.

Students must have the opportunity to analyze their work methods and organize their thinking. In short, they must be able to learn how to learn.

Encouraging Students to Use the Appropriate Technology for Each Task

New technology is being used in all human activities and is bringing about a veritable revolution. It has had a noticeable impact on employment and has often changed the nature of work itself. The ability to work with electronic devices has become an essential skill because they will form an integral part of the students' everyday life.

In school, this technology can influence the teaching of mathematics and the way students learn. In addition to facilitating calculations, graphing, and data management, it enables students to deal with more complex problems. Technology complements the problem-solving tools that students already have. Students will have to learn how and when to use these different tools.

It is important to take the guiding principles into account when integrating new technology into the learning process and the evaluation of learning.

Connection with Previous Programs

With continuity in learning, students can review topics they have already studied and further develop their conceptions and representations. This mathematics program enables students to build on the knowledge acquired in elementary school and in the first three years of secondary school.

This learning process will be dynamic if the learning activities allow the students to use their previously acquired knowledge and skills in new situations and help them to become more proficient at applying what they have learned.

As they acquire new knowledge, the students will review the following skills and concepts acquired in previous programs:

- number sense and facility with operations;
- the habit of estimating;
- proportionality;
- the concept of a variable;
- translation from one mode of representation to another;
- the types of dependence characterizing the relationship between variables;
- definitions, properties, theorems or corollaries related to different geometric concepts;
- spatial relationships;
- the ability to organize and process statistical data;
- simulation of random events and the concept of probability.

Evaluation of Learning

Orientations and Practices Relating to the Evaluation of Learning

> The evaluation of student learning has come in for a great deal of discussion in the Québec education system over the last decade and it is surely no exaggeration to say that this field has been and to some extent remains a subject of scrutiny. Teachers today are more knowledgeable about the evaluation of student learning than they were in the past...[6].

It is important to draw on all the available expertise in evaluation and ensure that evaluation practices increasingly tie in with the essential learning pursued in the programs of study. Thus, the aim should be to establish greater consistency between the spirit of these programs and evaluation practices.

Procedures for Evaluating Learning

When evaluating student learning, teachers should keep in mind the purpose of evaluation. Whether the goal is to give immediate educational feedback (formative evaluation) or to determine whether one or more terminal objectives have been attained (summative evaluation), evaluation provides individual students with useful information about their learning progress. It also helps teachers to assess the organization of program content and the effectiveness of teaching methods. Since the program is aimed at helping students acquire a solid basic education and the skills that will enable them to adapt to a constantly changing society,

> ...the evaluation of learning should take into account the various components of human development and the complex nature of education, [and] be consistent with the learning activities carried out in the classroom...[7].

In this program, the students not only acquire knowledge, but also learn how to investigate, communicate, represent, reason and use a variety of approaches in order to solve problems. They also acquire other skills and attitudes.

Because the students' knowledge, skills and attitudes are constantly evolving, it is necessary to create situations which will yield information that, after criterion- or norm-referenced interpretation, is likely to provide a reliable indication of each student's or group's knowledge.

6. Conseil supérieur de l'éducation, *Évaluer les apprentissages au primaire : un équilibre à trouver* (Québec: Direction des communications du CSE, 1992), 1 (Free translation).

7. Ibid., 2 (Free translation).

Since "paper-and-pencil" evaluation may not be appropriate to every aspect of this program, a certain amount of adaptation will be necessary. Depending on the specific goals and in keeping with a spirit of diversification, the following means of evaluation could be appropriate:

- Log
- Oral presentation of a solution or a mathematical subject
- Quiz
- Class discussion
- Group project
- Interview
- Comprehensive examination comprising a number of sections
- Evaluation during computer-assisted learning activities
- Observation checklist
- Self-evaluation
- and so on

The different types of evaluation must also take into account the variety of learning activities:

- Manipulation activity
- Communication activity (oral or written, individual or group)
- Estimation activity
- Activity using a calculator
- Activity using a computer
- and so on

When planning education evaluation, it is important to vary the means of evaluation. However, this does not mean that only one type of evaluation (i.e. diagnostic, formative and summative) should be used. Choices must be made in this regard.

The evaluation of learning, be it formative or summative, is essentially aimed at improving both learning and teaching.

As Esther Paradis notes in *L'évaluation des apprentissages : valoriser sa mission pédagogique*, isn't it essentially a matter of rediscovering the educational merit of evaluation?[8]

8. Esther Paradis, *L'évaluation des apprentissages : valoriser sa mission pédagogique* (Québec: Fédération des enseignantes et des enseignants de commissions scolaires, Centrale de l'enseignement du Québec, 1992), 26 (Free translation).

Relative Importance of the General Objectives

The following table shows the relative importance of each general objective.

General Objectives	%
1. To help students learn to apply their knowledge of algebra.	38
2. To enable students to analyze geometric situations.	38
3. To help students develop the ability to analyze statistical data.	24

PROGRAM CONTENT

Program Structure

This program is made up of global, general, terminal and intermediate objectives. These objectives should reflect the aims of mathematics education and the guiding principles mentioned previously.

Global Objectives

Objectives that summarize the *role* that mathematics plays in providing students with the basic education they need to integrate into our changing society. These global objectives remain the same throughout the five years of secondary school and form the nucleus around which the objectives for each level are structured.

General Objectives

Objectives that specify the context in which the global objectives will be pursued and that describe in general terms the *expected educational outcomes* associated with each program theme. General objectives can be broken down into a set of terminal objectives.

Terminal Objectives

Objectives that clarify the general objectives and describe the *anticipated results.* Each objective is described in three paragraphs:

- The first paragraph indicates what the students have learned from their previous studies.

- The second paragraph provides criteria for determining whether the students have attained the terminal objective.

- The third paragraph outlines activities that are consistent with the general objective, the global objectives and the guiding principles. In this way, it reflects the spirit of the program.

The terminal objective is attained when the students are able to establish a link between a situation and acquired knowledge. This ability is directly related to attainment of the terminal objective and not to attainment of each of the underlying intermediate objectives, a complex object of knowledge being more than the sum of its parts. Hence, the primary goal is to have the students achieve the terminal objectives of the program. The degree to which the terminal objectives of the program are attained is directly related to the appropriateness of the measurement instruments, which must take into account the scope of the intermediate objectives and the context outlined by the general objective and the global objectives.

Intermediate Objectives

Objectives that specify the scope of a terminal objective, intermediate objectives might also be described as "reference objectives." They are not intended as a series of steps to be completed one after the other. Such a process would give a very fragmented picture of teaching and learning. Rather, intermediate objectives are:

- aspects of a theme that have been chosen for the program;
- clarifications to ensure that the terminal objective is clearly understood;
- guidelines that indicate the connection between the terminal objective and student learning;
- prerequisites for attaining a terminal objective.

Program Objectives

Global Objectives

Establishing Connections

Increasing the students' ability to establish connections between the knowledge they are acquiring and the knowledge they already have in mathematics and other disciplines, and encouraging them to view their knowledge as a tool that can be useful to them in everyday life.

Problem Solving

Increasing the students' ability to analyze the data associated with a problem and use appropriate strategies to arrive at a solution that they will be able to verify, interpret and generalize.

Communicating

Increasing the students' ability to grasp and transmit information and to express their thoughts clearly, using mathematical language.

Reasoning

Increasing the students' ability to formulate hypotheses and verify them using an inductive or a deductive method.

GENERAL OBJECTIVE 1

To help students learn to apply their knowledge of algebra

Our world is not static but rather one of constantly changing and often interdependent elements. The study of functions makes it possible to describe different types of dependence.

A function is one of the most important mathematical concepts and should therefore be incorporated into every aspect of the curriculum. In Secondary I, students were introduced to the basic idea that two quantities are related to each other in a certain way. In this course, they continue examining this principle by studying exponential or step functions.

Students must develop an intuitive understanding of the way variables affect one another. They can do this by graphing a wide range of phenomena and using the different modes of representation to view a situation in other ways. Drawing on what they learned in previous courses, students can determine if a phenomenon is best represented by a continuous or discontinuous line or curve.

Students should use functions to create models of familiar situations. The students should be able to identify and compare families of functions as well as understand, analyze and use systems of functions, including the functions studied in Secondary III.

By using technology, students do not have to master algebraic manipulations, which may sometimes obscure relatively simple concepts.

Terminal Objective 1.1

To analyze variations using different modes of representation

In Secondary III, students illustrated the type of dependence characterizing the relationship between variables. They studied situations in which the variables are directly or inversely proportional, as well as other situations in which one of the variables is proportional to the square of the other.

Students who have attained Terminal Objective 1.1 of this program will be able to use different modes of representation to analyze a situation and thereby differentiate between families of functions. Through exploration activities, the students learn to identify the different families of functions intuitively. The students will also study situations where the relationship between variables is represented by exponential or step functions. The students are not expected to be able to write an equation for a given situation or name the type of relation involved. The following table indicates the different types of translations from one mode of representation to another; the shaded boxes indicate the translations covered in this objective. Note that the students have used this approach to study different types of relations since Secondary II. The Roman numerals indicate the level(s) at which these translations were studied.

TRANSLATIONS FROM ONE MODE OF REPRESENTATION TO ANOTHER

from \ to	words or drawing	table of values	graph	rule or equation
words or drawing	II	II	II	II and III
table of values	II		III	III
graph	II			III
rule or equation	II and III	III		II

The use of a wide variety of situations that help the students learn the basic properties of each family of functions is consistent with the global objectives, General Objective 1 and the guiding principles. It is advisable to use graphic display calculators or computers to enable students to explore, investigate, describe or explain the relationships between variables.

1.1

Intermediate Objectives

- To determine the dependent variable and the independent variable in a given situation.
- To make a table of values for a given situation.
- To determine the most appropriate scale for the graph of a given situation.
- To draw a graph representing a particular situation, given a table of values.
- To compare different situations expressed by means of the same mode of representation.

Terminal Objective 1.2

To solve problems dealing with systems of linear relations

In Secondary III, the students solved problems involving direct or partial variation.

Students who have attained Terminal Objective 1.2 of this program will be able to use different modes of representation to solve problems involving systems of linear relations. The concept of a function studied up to now can be applied to more complex situations in which several functions are considered simultaneously. At this point, however, the students will simply study situations that can be represented by straight lines. The following table indicates the different types of translations from one mode of representation to another; the shaded boxes indicate the translations covered in this objective. Note that the students have used this approach to study different types of relations since Secondary II. The Roman numerals indicate the level(s) at which these translations were studied.

TRANSLATIONS FROM ONE MODE OF REPRESENTATION TO ANOTHER

to / from	words or drawing	table of values	graph	rule or equation
words or drawing	II	II	II	II and III
table of values	II		III	III
graph	II			III
rule or equation	II and III	III		II

Activities in which the students learn that one mode of representation is more appropriate than another in a given situation are consistent with the global objectives, General Objective 1 and the guiding principles. Technology makes it possible to examine more realistic situations because students are no longer obliged to use complex arithmetic or algebraic manipulations that could actually make problems difficult to solve.

1.2

Intermediate Objectives

- To represent a situation by a system of linear relations.
- To describe a real-life situation expressed as a system of linear relations.
- To make a table of values for a system of linear relations.
- To determine the most appropriate scale for the graph of a system of linear relations.
- To draw a graph representing a system of linear relations.
- To justify the interpretation of a system of linear relations by using one or more modes of representation.
- To determine specific values of a system of linear relations with the degree of precision required for that situation.

GENERAL OBJECTIVE 2

To enable students to analyze geometric situations

The study of geometry provides an ideal opportunity to introduce the students to the deductive method and help them understand it so they can use it to solve problems. From the beginning of secondary school, the students progress through a hierarchy of levels in developing their geometric thinking skills. They first learned to recognize shapes and then analyzed the properties of these shapes before making deductions by establishing relationships between these properties. They must now discover that the reasoning used to solve a problem is similar to the sound, structured argumentation needed to present a proof. Formal reasoning as such should be emphasized so that students can learn to present more organized proofs. While requiring the students to work out relatively simple proofs, it is important to use a dynamic approach that fosters their imagination and creativity.

In the course of their studies, the students have established a system of geometric relationships pertaining to angles, triangles, quadrilaterals, circles, polygons and solids. After studying transformations and their characteristics in the first cycle of secondary school, the students are now prepared to use the concepts of isometry and similarity to solve problems in Secondary IV.

After applying the concept of similarity to different figures, the students will discover that certain trigonometric ratios are derived from ratios of similitude involving similar right triangles. With the help of these tools and proportional reasoning, the students can determine measures and solve problems pertaining to measurements that cannot be found directly.

Terminal Objective 2.1

To solve problems using the concept of similarity

Through numerous exploration and observation activities in the first cycle of secondary school, students built up a body of knowledge about various geometric figures. In addition, they constructed figures[1] resulting from isometries or dilatations and were able to state the principal properties of each type of transformation.

Students who have attained Terminal Objective 2.1 will be able to solve problems involving the concepts of similarity and isometry by structuring their solutions and, if necessary, justifying the steps in their reasoning by referring to relevant definitions, theorems or corollaries. A close connection should be established with Terminal Objective 2.1 of the Secondary III mathematics program to ensure that students understand that the concept of similarity is directly derived from the characteristics of geometric transformations. By defining the concept of similarity in this way, we can apply it to any two- or three-dimensional figure. As a result, cases involving similar or isometric triangles, which were examined as theorems in Euclidean geometry, become properties of similarity transformations in transformational geometry. Given similar or isometric figures, the students will discover that there is always at least one similarity transformation or isometry which maps one figure onto another. The proofs assigned to the students should be within their capability. For both solids and similar polygons, the students will be asked to deduce certain measures or ratios required to solve problems.

Activities in which the students must organize the work involved in solving problems are consistent with the global objectives, General Objective 2 and the guiding principles. The students will learn to distinguish a probable or possible explanation from a fact or from the information required to make an argument convincing. By analyzing geometric situations and solving problems, the students learn to appreciate the importance of developing their reasoning abilities.

1. For all the objectives pertaining to geometry, the term figure refers to a polygon or a solid.

2.1

Intermediate Objectives

- To distinguish similar or isometric figures from those that are not.
- To describe a similarity transformation or an isometry involving two polygons.
- To support an assertion[2] used in presenting a proof involving the concepts of similarity or isometry.
- To deduce certain measures in similar figures from an appropriate geometric principle.[3]
- To justify an assertion[4] used to solve a problem involving the concept of similarity.

2. See Appendix.

3. See Appendix.

4. See Appendix.

Terminal Objective 2.2

To solve problems using trigonometric ratios

The students developed the skills required to work with trigonometric ratios by studying the concepts of ratio and proportion in the first cycle of secondary school as well as the material covered in Terminal Objective 2.1 of this course.

Students who have attained Terminal Objective 2.2 will be able to use trigonometric ratios to find the measures needed to solve a variety of problems. Problems should not be limited to determining the measure of a side or an angle in a right triangle or in another type of triangle. They should also involve using such information to deduce other data needed to solve a problem. To help students establish connections between mathematical concepts they already know, it is important to show them that trigonometric ratios are derived from ratios of corresponding sides in similar right triangles. By using a calculator, the students can concentrate on geometric reasoning rather than on calculations. Geometric properties of right triangles with an acute angle of 30°, 45° or 60° can be used to deduce certain measures and easily establish the trigonometric ratios for these angles. Using certain trigonometric principles, students can determine distances, lengths and heights that would be more difficult to measure directly.

Activities in which the students learn to use different modes of representation of a problem, estimate results and evaluate ratios mentally are consistent with the global objectives, General Objective 2 and the guiding principles. Through these activities, the students will discover that if they are given two measurements in a right triangle, they have enough information to find a third measurement in that triangle. By solving a variety of problems related to different fields of activity in the real world, the students can establish many connections between the different mathematical skills and concepts they have learned.

2.2

Intermediate Objectives

- To deduce the measures of a right triangle using trigonometric ratios.
- To deduce the measures of triangles from various geometric principles.[5]
- To justify an assertion used to solve a problem.[6]

5. See Appendix.

6. See Appendix.

GENERAL OBJECTIVE 3

To help students develop the ability to analyze statistical data

To be informed and productive, a person should be able to handle data and make intelligent decisions based on quantitative or qualitative arguments. The emphasis should therefore be on analyzing situations rather than just finding a single numerical answer. Students will learn to ask pertinent questions and present an analysis while developing their critical sense.

In the first cycle of secondary school, the students organized and presented data in tables and graphs. They also saw that they could use certain descriptive measures (mean, median, mode, range) to synthesize data and thus provide information on various phenomena. In Secondary IV, the students will begin examining measures of position and will be prepared to study the concept of dispersion.

It would also be advisable to examine the source of the data to see how it was obtained. The students will learn to assess the strengths and weaknesses of the data-gathering process and then acquire some tools for analyzing situations.

With this approach, the students will learn to use data rather than produce it. They should be given the opportunity to investigate and discuss such things as public opinion polls, media ratings and census data.

Terminal Objective 3.1

To solve problems that involve gathering data

In the first cycle of secondary school, the students organized data in the form of tables or graphs, usually working with given information. They continued to study phenomena involving chance and also used certain measures to summarize data (mean, median, mode and range).

Students who have attained Terminal Objective 3.1 will be able to assess the reliability of a sample and the relevance of the data used when solving problems involving predictions about a population. If the initial hypothesis is valid, the sample should provide an accurate picture of the population under study. The students should check the size of the sample and the data-gathering methods to ensure that a study is as unbiased and error-free as possible. The students already know several ways of summarizing data graphically or numerically. They must learn to follow certain principles in processing data to ensure that they draw appropriate conclusions. When presenting their results or conclusions, the students can use everyday language to support their arguments.

Activities in which the students learn to develop a critical attitude towards survey data are consistent with the global objectives, General Objective 3 and the guiding principles. They should become aware that a survey consists of several elements, all of which can affect the accuracy of the results. During their discussions and investigations, the students should watch for biases in the selection of data, for errors in measurement and for distortions in graphic or numerical representations of data, both in the media and in their own work.

3.1

Intermediate Objectives

- To distinguish between a sample and a population.
- To justify the decision to prepare a census, a poll or a study to obtain information.
- To describe the characteristics of a representative sample of a given population.
- To choose an appropriate sampling method when gathering data.
- To determine the possible sources of bias when gathering data.
- To compare two samples from the same population.

Terminal Objective 3.2

To solve problems using measures of position

In the first cycle of secondary school, the students learned to use certain tools (measures of central tendency and range) to analyze information and presented data in the form of tables or graphs (bar, broken-line and circle graphs as well as histograms).

Students who have attained Terminal Objective 3.2 will be able to solve problems using the graphic or numerical tools they have to analyze information. They are to use these tools to study the variability of a distribution. The students will use measures of position to determine the rank of a data value in relation to other values in a distribution, or to identify the possible variations among various data values in the distribution. In continuing to explore methods of analyzing data, the students will increase their knowledge of mathematical models by studying the box-and-whisker plot. This graph not only highlights certain characteristics of a distribution, but also gives the students an idea about the dispersion of the data.

Activities in which the students can present information about a set of data are consistent with the global objectives, General Objective 3 and the guiding principles. Technology should be used to facilitate the analysis and interpretation of the situation. Emphasis should be placed on analyzing and presenting the situation. In this way, the students will learn to interpret graphs and understand the connections between graphic and numerical representations of the same situation.

3.2

Intermediate Objectives

- To distinguish between measures of central tendency, measures of position and measures of dispersion.
- To assign a quintile, a quartile or a percentile rank to a data value in a distribution.
- To determine the data value(s) that are assigned a given rank.
- To use measures of position to compare data.
- To construct a box-and-whisker plot.
- To interpret a box-and-whisker plot.
- To find qualitative information about the dispersion of the data in a one-variable distribution, using measures of position and measures of central tendency.

Appendix

Principles Related to Themes Introduced in *Mathematics 416*

Through their activities in geometry, the students increase their understanding of concepts and perfect several skills. Using definitions, properties, theorems and corollaries related to similarity or certain relations dealing with measurements in a triangle, they can deduce measures and justify an assertion used to present a proof or solve a problem.

1- If two corresponding (or alternate interior or alternate exterior) angles are congruent, then they are formed by two parallel lines and a transversal.

2- If a transversal intersects two parallel lines then:

- the alternate interior angles are congruent;
- the alternate exterior angles are congruent;
- the corresponding angles are congruent.

3- The angles and sides of isometric figures[1] are equal in measure.

4- Figures are isometric if and only if there is an isometry or a composite of isometries that makes one figure coincide with the other.

5- Two triangles whose corresponding sides are congruent must be congruent.

6- If two sides and the contained angle of one triangle are congruent to two sides and the contained angle of another triangle, then the triangles must be congruent.

7- If two angles and the contained side of one triangle are congruent to two angles and the contained side of another triangle, then the triangles must be congruent.

8- Transversals intersected by parallel lines are divided into segments of proportional lengths.

9- Any straight line that intersects two sides of a triangle and is parallel to a third side forms a smaller triangle similar to the larger triangle.

10- The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.

11- Similar figures have congruent corresponding angles and proportional corresponding sides.

12- Two figures are similar if and only if there is a dilatation or a composite of transformations that preserves the order of points, the measures of the corresponding angles and the ratio of the corresponding sides.

13- If two angles of one triangle are congruent to two angles of another triangle, then the triangles must be similar.

14- If the lengths of the corresponding sides of two triangles are in proportion, then the triangles must be similar.

1. The term figure designates a plane figure or a solid.

15- If the lengths of two sides of one triangle are proportional to the lengths of two sides of another triangle and the contained angles are congruent, then the triangles must be similar.

16- In similar polygons:

- the ratio between the measures of corresponding angles is 1;
- the ratio between the lengths of corresponding elements is equal to the ratio between the lengths of the corresponding sides;
- the ratio of the areas is equal to the square of the ratio between the lengths of the corresponding sides.

17- Figures with a similarity ratio of 1 or -1 are isometric.

18- In similar solids, the ratio of the volumes is equal to the cube of the ratio between the lengths of the corresponding sides.

19- The length of the side opposite the 30° angle in a right triangle is half the length of the hypotenuse.

20- **Hero's (or Heron's) Formula**

The area S of a triangle whose sides measure *a, b and c* is:

$$S = \sqrt{p(p - a)(p - b)(p - c)}$$

where $p = \frac{1}{2}(a + b + c)$ (i.e. half the perimeter of the triangle).

21- **The Law of Sines**

The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides. This can be expressed as follows:

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

In the Secondary I and Secondary II programs, the students began to build up a system of axioms. In order to deduce certain measurements and justify certain steps involved in solving problems, the students must apply the following principles as well as those studied in Secondary III.

Secondary I Program

1. Adjacent angles whose external sides are in a straight line are supplementary.
2. Vertically opposite angles are congruent.
3. The sum of the measures of the interior angles of a triangle is 180°.
4. In any triangle, the length of any side is less than the sum of the lengths of the other two sides.
5. In any triangle, the length of any side is greater than the difference of the lengths of the other two sides.
6. In any triangle, the longest side is opposite the largest angle.
7. In any isosceles triangle, the angles opposite the congruent sides are congruent.
8. In any equilateral triangle, each of the angles measures 60°.
9. In any right triangle, the acute angles are complementary.
10. In any isosceles right triangle, each of the acute angles measures 45°.
11. The axis of symmetry of an isosceles triangle contains a median, a perpendicular bisector, an angle bisector and an altitude of the triangle.
12. The axes of symmetry of an equilateral triangle contain the medians, perpendicular bisectors, angle bisectors and the altitudes of the triangle.
13. The opposite angles of a parallelogram are congruent.
14. The opposite sides of a parallelogram are congruent.
15. The diagonals of a parallelogram bisect each other.
16. The diagonals of a rectangle are congruent.
17. The diagonals of a rhombus are perpendicular to each other.

1. The diagonals from one vertex of a convex polygon form **n — 2** triangles, where n is the number of sides in that polygon.

2. In a convex polygon, the sum of the measures of the exterior angles, one at each vertex, is 360°.

3. The sum of the measures of the interior angles of a polygon is 180° (n — 2), where n is the number of sides in the polygon.

4. Three non-collinear points determine one and only one circle.

5. All the perpendicular bisectors of the chords of a circle meet at the centre of that circle.

6. All the diameters of a circle are congruent.

7. In a circle, the measure of the radius is half the measure of the diameter.

8. The axes of symmetry of a circle contain its centre.

9. The ratio of the circumference of a circle to its diameter is a constant known as π.

10. In a circle, the measure of the central angle is equal to the measure of its intercepted arc.

11. In a circle, the ratio of the measures of two central angles is equal to the ratio of the measures of their intercepted arcs.

Secondary III Program

1. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

2. A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other two sides.

3. In any convex polyhedron, the sum of the number of vertices and the number of faces is equal to the number of edges plus two.

4. Any translation and any dilatation will transform a straight line into another line parallel to it.

5. Isometries or dilatations have one or more of the following properties:

   - they preserve collinearity;
   - they preserve parallelism;
   - they preserve the order of points;
   - they preserve the orientation of the plane;
   - they preserve distances and measures of angles.

Bibliography

Baracs, Janos, and Richard Pallascio. "Le développement de la perception spatiale." In *Bulletin de l'AMQ*, December, 1981, 5-11.

Bergeron, Anne, and Jacques Bordier. *Enseignement des probabilités et des statistiques au secondaire*, PPM 5029, Télé-université, 1982.

Bednarz, Nadine. "L'enseignement des mathématiques et le Québec de l'an 2000." In *Mathématiquement vôtre! Défis et perspectives pour l'enseignement des mathématiques*, edited by Richard Pallascio. Montréal: Les éditions Agence d'ARC inc., 1990.

Bertrand, Richard, in collaboration with Claude Valiquette. *Pratique de l'analyse statistique des données.* Sillery: Presses de l'Université du Québec, 1986, 23-140.

Bordier, Jacques, et al. *La mathématique et l'activité humaine. Rencontre avec Pascal C.*, Télé-université, 1979, 57-75 and 200-238.

Clements, Douglas H., and Michael T. Battista. "Geometry and Spatial Reasoning." In *Handbook of Research on Mathematics Teaching and Learning,* edited by Douglas A. Grouws. New York: National Council of Teachers of Mathematics, Research Interpretation Project, Macmillan Publishing Company, 1992, 420-464.

Conseil supérieur de l'éducation. *Évaluer les apprentissages au primaire : un équilibre à trouver*. Québec: Direction des communications, 1992.

Gaulin, Claude. "The Need for Emphasizing Various Graphical Representations of 3-dimensional Shapes and Relations." In *Proceedings of the 9th PME Conference,* Noordwijkerhout, Netherlands, 1985, vol. II, 53-71.

Gaulin Claude, and Eva Puchalska. "Coded Graphical Representations: A Valuable but Neglected Means of Communicating Spatial Information in Geometry, Developments in School Mathematics Education Around the World." In *Proceedings of the UCSMP International Conference of Mathematics Education.* Reston, Va.: National Council of Teachers of Mathematics, 1987, 514-539.

Janvier, Claude. "Le volume comme instrument de conceptualisation de l'espace." In *Topologie structurale*, vol. 18, CIRADE, UQAM, 1992, 63-75.

Janvier, Claude, Catherine Girardon, and Jean-Charles Morand. "Mathematical Symbols and Representations." In *Research Ideas for the Classroom: High School Mathematics*, edited by P.S. Wilson. New York: Macmillan Publishing Company, 1993, 79-102.

Kieran, Carolyn. "The Learning and Teaching of School Algebra." In *Handbook of Research on Mathematics Teaching and Learning,* edited by Douglas A. Grouws. New York: National Council of Teachers of Mathematics, Research Interpretation Project, Macmillan Publishing Company, 1992, 390-419.

Laquerre, Justin, and Alain Taurisson. *Manipulations géométriques et exemples de démonstrations. Activités géométriques,* PMM 5017, Télé-université, 1978, 35-56 and 173-192.

Pallascio, Richard, Richard Allaire, and Pierre Mongeau. "Représentation de l'espace et enseignement de la géométrie." In *Topologie structurale,* vol. 19, CIRADE, UQAM, 1992, 71-82.

Papillon, Vincent, Dominique Dion, and Richard Pallascio. "Activités d'entraînement à la perception structurale." In *Bulletin de l'AMQ,* December, 1985, 30-35.

———. "Activités d'entraînement à la perception structurale." In *Bulletin de l'AMQ*, March 1986, 29-33.

———. "Activités d'entraînement à la perception structurale." In *Bulletin de l'AMQ*, May 1986, 28-32.

Paradis, Esther. *L'évaluation des apprentissages : valoriser sa mission pédagogique.* Québec: Fédération des enseignantes et des enseignants de commissions scolaires, Centrale de l'enseignement du Québec, 1992.

Québec. Ministère de l'Éducation. *Mathematics Curriculum Guide, Elementary School, Booklet K, Problem Solving.* Code 16-2300-11A. Québec: ministère de l'Éducation, 1989.

Resnick, L. B., and L. E. Kloffer. "Toward the Thinking Curriculum: An Overview." In *Toward the Thinking Curriculum: Current Cognitive Research, 1989 Yearbook of the Association for Supervision and Curriculum Development.* Alexandria, VA: ASCD, 1989.

Wagner, Sigrid, and Sheila Parker. "Advancing Algebra." In *Research Ideas for the Classroom: High School Mathematics*, edited by P.S. Wilson. New York: Macmillan Publishing Company, 1993, 119-139.

Yakimanskaya, I. S. *The Development of Spatial Thinking in Schoolchildren, Soviet Studies in Mathematics Education*, vol. 3. Translated by Robert H. Silverman, Reston, Va.: National Council of Teachers of Mathematics, 1991.

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