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Coalition Formation for Sourcing Contract Design with Cooperative Replenishment in Supply Networks

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Abstract. This paper studies a coalition formation problem for cooperative replenishment with a single supplier and multiple firms. We consider that a vertical cooperation between the supplier and the set of firms is already set using periodic contracts, and investigate the profitability of a horizontal cooperation between firms. Three business situations are inspected, focusing either on the collaboration opportunities in the ordering, the inventory holding or the transportation process. In each collaborative situation, a decision-making approach is applied to assess the expected profitability of each firm and to determine if firms should form coalitions in order to maximize their profits. An exact solution method based on a game-theoretic approach is developed, named cooperative replenishment algorithm (CRA), which generates core-stable coalitions for cost savings regarding all partners' standpoints. An analytical study of stability conditions regarding the ordering, holding and transportation cost structures was performed to enhance the CRA resolution procedure. Extensive experiments based on realistic instances are provided to validate the performance of the CRA proposed in terms of solution stability and the convergence in terms of running time. The computational results confirmed also the potential benefit of horizontal collaboration between firms in terms of profit maximization compared to the stand alone situation. They provided encouraging findings on the profit enhancements that sharing inventories and transportation pooling practices could produce within the supply chain.

Keywords: Supply networks, game theory, coalition formation, quantity discounts, contracts, joint replenishment, collaborative transportation.

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I. Introduction

Supply chains (SC) have been characterized as organizational networks that are linked through upstream and downstream processes and activities that produce value in the form of products and services delivered to the hands of the ultimate customer (Christopher, 1998). A Supply Network (SN) is a configuration of facilities geographically deployed in order to serve a predetermined customer base. To respond to customers' demands with the adequate service level, products are procured, stored and distributed to demand zones using a SN involving several facilities owned or rented by the firm. In such network, day-to-day procurement, warehousing, transportation and demand management decisions generate product flows in the network, with associated costs and revenues. For a given product-market, where a set of firms are operating, their SNs can be viewed as a complex network encompassing a large number of decision making units. Each unit may have its own objectives and makes its decisions so as to maximize self-profits. Unfortunately, by acting solely, these firms could dismiss collaboration opportunities enhancing their profits. At the strategic level, each firm decides on the number, the location and the mission of the facilities to operate and selects the portfolio of suppliers to employ in order to design a value-creating SN (Klibi et al., 2010a). When the SN designed by the firm becomes operational, it is managed to respond on a daily basis to customers' demands through planning and control processes. At the tactical level, firms have to anticipate future demand and decide on inventory management, ordering and replenishment policies, which organize the concrete relationship between the firms and their suppliers. This is generally done by selecting appropriate forecasting methods, quantities to order, transportation options, and coordination/collaboration mechanisms such as contracts and information sharing (Tsay et al., 1999; Cachon and Zipkin, 1999). **Figure 1** illustrates the business context considered, where a two-stage SN is depicted: the firms' facilities stage where products are stored and delivered to customers when orders are received; and the suppliers' stage dedicated to the replenishment of the firms' facilities from pre-assigned suppliers' sites using different transportation means.

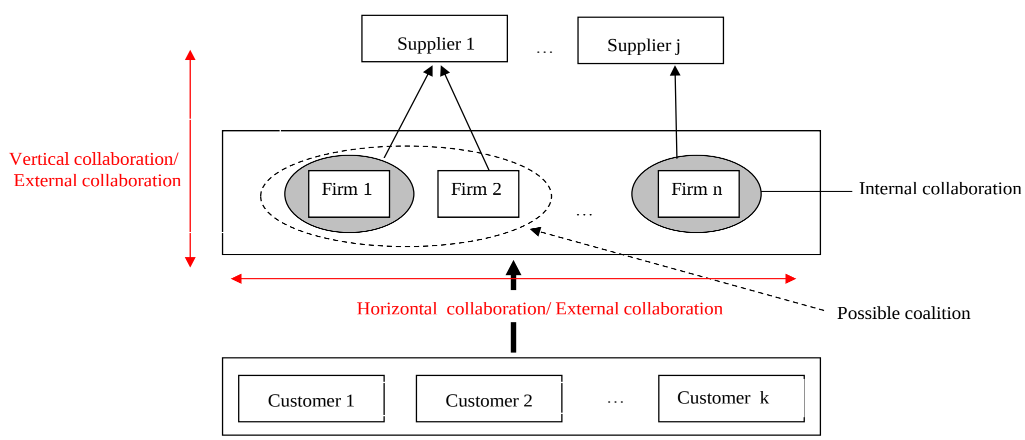


Figure 1- The Business Context under Collaboration Opportunities

Nowadays, major preoccupations of firms are to deal with the growing volatility in demand (Christopher and Holweg, 2011), the inherent scarcity of raw materials and non-renewable energies and their increasing costs (Shell, 2013), and the uncertainty and disruptions in global sourcing (Klibi and Martel, 2013). In these circumstances, more efficiency, sustainability and flexibility in the SNs are necessary for firms in order to remain competitive. In this line of thinking, collaboration is foreseen as one of mechanisms to help mitigating these preoccupations and to contribute to the achievement of contemporary business objectives. As illustrated in **Figure 1**, two main categories of collaboration exist: vertical collaboration (Chen et al., 2001) and horizontal collaboration (Krichen et al., 2011).

In counterpart of their concentration on core competencies to be effective, current firms must rely on activities' externalization, subcontracting and on vertical collaboration in the SC. The visibility offered by the latter in the SNs is nowadays widely accepted, but seems not sufficient to overcome the issues underlined above. New actions such as dual sourcing, assets sharing, resources pooling, mutualisation or joint replenishment have been recently proposed to offer more flexibility and sustainability to SNs (Christopher and Holweg, 2011). These practices are usually associated to horizontal cooperation in the SC and their implementation necessitates efficient coordination strategies, in order to find the best compromise between procurement, inventory holding and transportation operations optimization within the SC. The horizontal collaboration matches competitors or non-competitors SC entities that have built relationships and integrated processes by joining their orders/goals (Mangan et al., 2008) or sharing their resources (Simatupang and Sridharan, 2002). Horizontal collaboration is among the contemporary practices as one could see in recent large scale projects such as CO3 (www.co3-project.eu/), Moduluscha (www.modulushca.eu/) and the Physical Internet Manifesto (www.physicalinternetinitiative.org/).

The interest of such horizontal cooperative practices can be attributed to the increased appeal of alliance/coalition in different industries where a group of firms initiated collaborative replenishment, warehousing, ordering and/or transportation activities with a positive impact on their SNs performance. However, there is still a lack of models that can address these operations jointly in a collaborative process. In practice, firms could be interested to collaborate in the ordering, the inventory holding, and/or the transportation activity in a supply context. Accordingly, in the context of SNs, a coalition/alliance characterizes the group of firms that decides to work jointly by grouping their orders for a given product-market and synchronizing their replenishment policies. The aim of the approach is to find the most profitable coalition structure for each firm under collaboration opportunities. To address these questions, we consider in this paper a coalition formation mechanism that may allow firms according to the incentives of various collaborative situations to maximize their own profit. This tactical decision-making process tends to find the most profitable coalition/alliance for all firms while designing the best sourcing contract.

In order to study extensively the benefit of horizontal collaborations between a set of firms, this paper proposes a collaboration-based modeling approach of the coalition formation problem for the design of sourcing contracts. The contribution of this paper is threefold. First, while most papers in the literature on horizontal collaboration address the joint replenishment problem assuming the coalition already formed, we propose here to tackle the coalition formation problem, when the sourcing contract must be designed, in order to better anticipate the expected profits from collaboration under ordering, inventory holding and transportation costs. Second, to account for the particularities of realistic supply networks, three collaborative contexts are studied in order to investigate the collaboration opportunities offered alternatively by the ordering, the inventory and the transportation activities. Accordingly, three typical cooperative replenishment situations are investigated, namely: (a) Cooperative Ordering with Direct Shipment (CODS), (b) Joint Replenishment with Shared Warehousing (JRSW) and (c) Joint Replenishment with Synchronized Shipment (JRSS). Third, to build the most profitable coalitions, we propose a game theoretic modeling approach based on core-stability concept. Game theory is viewed as a powerful tool to cope with such decisions since it reaches to generate possible coalitions and to identify an equilibrium situations that satisfy all players in the game (Shenoy, 1979). We develop in this paper a cooperative replenishment algorithm (CRA) that manages the coalition formation process using the core-stability condition. Several problem instances are addressing to assess the efficiency of the CRA in generating appropriate coalition structures that fulfills all firms' requirements. We show through an empirical investigation the saving in computational time and convergence of our approach. Furthermore, an experimental study, of several cooperative replenishment situations, underlines the importance of the collaboration in the cost saving and illustrates how firms tend to further collaborate when joint ordering, inventory holding and transportation costs are proposed.

The rest of the paper is organized as follows. Section 2 consists in an overview on vertical and horizontal collaboration in the context of SNs. Section 3 states the description and the formulation of the coalition formation problem. Section 4 presents the three replenishment contexts investigated. Section 5 discusses the stability condition of the coalition structures and proposes a cooperative replenishment algorithm to solve the coalition formation problem. Numerical results are presented and discussed in Section 6 in the objective to show the performance of the proposed algorithm in generating stable coalitions. Finally, Section 7 summarizes our findings and proposes future research avenues.

II. Vertical and Horizontal Collaboration in the Supply Chain

This section attempts to briefly overview the existing works in the vertical coordination and collaboration in the supply chain, based on contracting mechanisms. In addition, it reviews current state of the art on the horizontal collaboration in the supply networks. It presents also typical collaboration cases between firms reported from practice in the recent years. Comprehensive reviews on the coordination mechanisms in the supply chain are found in Arshinder et al., (2011) and Cachon, (2003).

On the one hand, vertical collaboration between suppliers and firms is generally governed by a contract or an information sharing mechanism that improves the performance of the SC. Such protocols offer the potential to prevent and simulate the level of the efficiency when acting in collaborative SCs (Chen et al., 2001; Cachon and Fisher, 2000). The focus of this paper is dedicated to the contracting coordination mechanisms based on its positive impacts for the better management of entities' relationships, for cost saving and for risk reduction. As shown in **Figure 1,** the vertical coordination takes place on the upstream of the SC between suppliers and firms that agree with a specific contract structure as with the vendor managed inventory (VMI) approach; and on downstream side managing relationship between firms and customers as with the customer relationship management (CRM). State of the art contracting coordination is characterized by the object of the contract, the number of periods and the involved entities. The object of the contract can be either the quantity discounts (Weng, 2004), the buy backs (Wei et al., 2013), the quantity flexibility (Tsay, 1999), the revenues sharing (Yao et al., 2008, Zhou and Wang, 2009) and the subsidies/penalties (Cachon, 2003). Note that regarding the number of periods in contracts, most of the existing papers considered single period models. Generally, such contracts are designed as two-echelon in the SC including one supplier and one firm (Jemai and Karaesmen, 2007; Cachon, 2003; Cachon, 2005); or multiple suppliers and one firm (Hu et al., 2013). Zaho et al, (2010) addressed a coordination issue for manufacturer-retailer supply chains using option contracts. They demonstrated that the contract option can coordinate the supply chain with risk sharing. One should mention that these former works and also Hu et al., (2013) studied the contracting problem under the assumption of a deterministic demand process. Finally, Cachon and Lariviere (2005) modeled a stochastic version of the demand process when considering the contract-based coordination mechanism.

Among the managerial insights underlined in the literature cited above is that the quantity and pricing decisions in the SC are the most influent components that can encourage suppliers and firms to cooperate. Regarding the quantity discount contract, the main incentive is to encourage suppliers and firms to produce and purchase profitable quantities for the whole SC. Comprehensive literature reviews on quantity discounts are given by Dolan and Frey (1987) and Viswanthan and Wang (2003). The latter classified the existing papers according to whether there are one or multiple firms involved in the coordination mechanism. Corbett and De Groote, (2000) and Corbett et al., (2004) demonstrated that the optimal contracts for the supplier turn out to be quantity discount contracts. Chen et al. (2001) focused on the transaction and channel efficiency using the discount quantity contract for a SC including one supplier and one firm. Alternatively, many researchers studied the role of quantity discounts as an efficient channel coordination mechanism (Cachon, 2003; Jaber et al., 2006).

On the other hand, the horizontal collaboration makes firms cooperate in order to share private information and resources, with the ultimate goal of increasing their individual profits. Such options are generally announced in a supplier’s contract (Cachon, 2002; Krichen et al., 2011). As a result, there exists a strong incentive to decrease firms’ costs by coordinating activities, such as ordering, warehousing and transportation. In the recent years, we observed a high interest to the study of the main structure of firms’ alliances for risk and cost reductions (Mollenkopf et al., 2010). Elomri et al., (2012) focused on the transportation activity as a horizontal collaboration mechanism between multiple firms with one supplier. They used a coalition formation model for cost allocation in which firms interact on the transportation level. They formulated the problem as a cooperative game that aims to determine efficient coalitions and proved numerically that core-stable coalition structures are profitable for the whole system. Moreover, horizontal coordination mechanism on the ordering activity between multiple firms and one supplier was addressed in Meca et al., (2005). The objective was to determine the Economic Order Quantity (EOQ) considering the ordering and holding costs of the collaborating firms. In this case, when gathering their orders, firms share ordering costs based on their mean number of orders. The model assumes that firms share only the information of their order frequencies and shows that the problem converges to a non-empty core of the game. In order to minimize firms’ costs for a single supplier and multiple firms, Krichen et al., (2011) modeled an EOQ with quantity discounts and delays in payment. They proposed a game theoretic approach that generates coalition structures for firms with delays in payments and quantity discounts. Similarly, Ozen et al. (2008) considered a SC involving multiple firms, multiple warehouses and a single supplier. They proposed a cooperative game for the storage of items in common warehouses where orders are supplied via warehouses available within a predefined lead time. A two-level centralized model is proposed with permissible delay in payment and profit sharing scenarios to encourage firms gather their orders and jointly supply their orders using common warehouses. The literature review reveals that despite the increased interest in horizontal collaboration, to the best of our knowledge all existing models don't integrate simultaneously, ordering, and warehousing and transportation cooperation. Their inclusion at the sourcing contract design level is crucial to adequately calibrate the revenues and operational costs of coalition formation. This paper is aimed at overcoming this drawback.

Although SC contracts are used to be an efficient tool for making entities coordinate independently, game theoretic approach (Ozen et al., 2008, Nagarajan and Sosic, 2008) play an important role to help entities either to agree or not with the proposed contract and to answer how the profit/cost sharing mechanism is employed. Indeed the game theory provides a mathematical background for modeling the SC decision-making system and generating solutions in competitive or conflicting situations. One should notice that the existing literature on game theory is classified into cooperative and non-cooperative games. In the cooperative game, considered in this paper, participants are called to deal with a specific contract in such way all members are satisfied with the decided contract, and thus there is no room for conflict or competition when each entity is called to tempt his optimal coalition of partners. The core concept in game theory approach can be an efficient way to form profitable coalitions and has been successfully applied in Meca et al, (2005). Various cooperative situations are presented in **Table 1** based on typical cases collected from the literature. These cases concern as well the ordering, the warehousing and the transportation activities in the SN. **Table 1** promotes for several large firms their recent collaboration initiatives and which activities they tend to cooperate in. It shows the portfolio of activities involved in the case of horizontal collaboration and underlines some innovative collaborative practices such as pooling and sharing resources.

		Activities		
Case (Sector, Reference)	***Actions and benefits***	***Ordering***	***Warehousing***	***Transportation***
Mars and Nestle (Food, Logistics Manager, 2009)	• Shared truck to deliver combined loads (over 60 loads shared during 11 weeks) • Elimination of 7.5 miles of duplicate trips	✓		
Kelloggs and Kimberly Clark (Food, Logistics Manager, 2008)	• Shared warehousing to consolidate small orders • Reduction of empty running vehicles and of replenishment cycles		✓	
Nestle Waters and Danone (Food, Nestle, 2010)	• Shared warehousing and distribution of goods • Full utilization of warehouses involved and improved truck utilization • Reduction of warehousing and transport costs		✓	✓
Henkel Colgate, Palmolive and Sara-lee (Personal Care, Henkel, 2010)	• Manufacturers store their products on the same warehouse • Synchronization of flows using common VMI and distribution • Less inventory on retailer warehouse while increasing the service level • Better optimization of the trucks, reduction of 241 tons of CO^2 • Saving generated 50% less deliveries, 292.300 Km, 97 liters of fuel in 2008	✓	✓	✓
Spar Belgium (Retail, CO3 project, 2014)	• Consideration of a neutral Trustee company to manage the collaboration • Designate 3PLs to provides joint replenishment from 2 to 4 suppliers • Reduction of the transport kilometers with 66% and the number of trucks of 2/3			✓

Table 2- Practical Examples of Horizontal Collaboration initiatives

III. Description and formulation of the coalition formation Problem

In this section, we start with a description of the business context of the replenishment problem and the definition and the modelling of the stand-alone situation experienced by a given firm. Then, the concept of coalition formation is introduced and the problem modelling for a set of firms is proposed. This section ends with the presentation of various cooperative replenishment situations, based on the coalition formation, and their modelling features.

III.1. The Business Context

The business context studied in this paper is the following. A set of firms (retailers or wholesalers) operating in the same geographical area purchase the same product (it could refer to a family of similar products) from a unique supply source (supplier or manufacturer) that is positioned locally or abroad (i.e. global source). This product is sold by each firm to its own customers by shipping the daily orders received to a number of demand zones using contract carriers. In order to provide a good service, each firm operates its own distribution center (DC) located in the product-market coverage area so that it is able to provide next day delivery to the predetermined set of demand zones. In addition, to satisfy the varying quantity of product requested by its customers on a daily basis, each firm have to keep its own inventory level within the located DC (assumed to be large enough to meet the requirements of the inventory policy of the firm). For each firm, during its business period, the inventory level must be filled periodically, based on its replenishment plan, and using individual shipping. This context describes the standalone case incurred by each firm when acting solely. When looking to the entire product-market zone, the replication of such business operations gives rise to the cluster of independent SNs illustrated in **Figure 1** where a set of firms operate in a standalone setting.

We assume that strategic decisions on the SN structure, involving the selection of the supply source and the location of the DC are already fixed for each firm and will not be revoked for a long term (Klibi et al., 2010a). In order to maximize its return on investments during its lifespan, each firm must insure that the costs and revenues generated when using the SN designed are optimized periodically (i.e. on a tactical rolling horizon). More specifically, each firm have to minimize for the tactical horizon (typically a year) the replenishment (ordering and transportation) costs, the inventory holding costs and the customers' delivery costs. In addition each firm must maximize its revenues by satisfying the requested customers' demand with the adequate service and by decreasing the purchasing price contracted with the supplier for that period.

Figure 2 illustrates a realization of the tactical plan over the planning horizon T for a set of DCs, each one representing a distinct firm that is replenishing solely. The planning horizon T is partitioned into a set of discrete periods $t \in T$ representing days or weeks depending on the granularity of the demand process. Let I denotes the set of firms, $i \in I$ corresponds to the index of each firm in this set, and j denotes the unique supply sourcing (supplier) of the SNs. For each firm $i \in I$, along the planning horizon, its customers order varying quantities of the product on a periodic basis. Let d_{it} denotes the sum of orders received by the DC of firm i from its customers at period t. As depicted by the **Figure 2**, we assume that the cumulative demand received for period t at a given DC from a subset of its assigned customers is characterised by a *Normal* process[1]. When considering that d_{it} is a random variable fitting a *Normal* distribution, this provides explicitly that the total demand of firm i (i.e. $\sum_{t\in T} d_{it}$) is also a random variable characterized by a Normal process. However, in the beginning of the planning horizon, when the tactical decisions have to be fixed, the realisation of the demand process d_{it} is not known with certainty. Thus, only an estimate of the future demand could be anticipated in order to optimize the tactical plan. This anticipation is usually based on the historical path of the firm and is used to provide its future replenishment plan. Accordingly, at the decision moment, let $\hat{D}_i$ denotes the estimated total demand of firm i in its DC for the planning horizon T (the hat "^" indicates that this value represents an approximation of the future demand).

[1] This assumption is reasonable since the number of customers assigned to each depot is sufficiently large to benefit from the Central Limit Theorem, even if the customers demand process is modeled as a compound Poisson process.

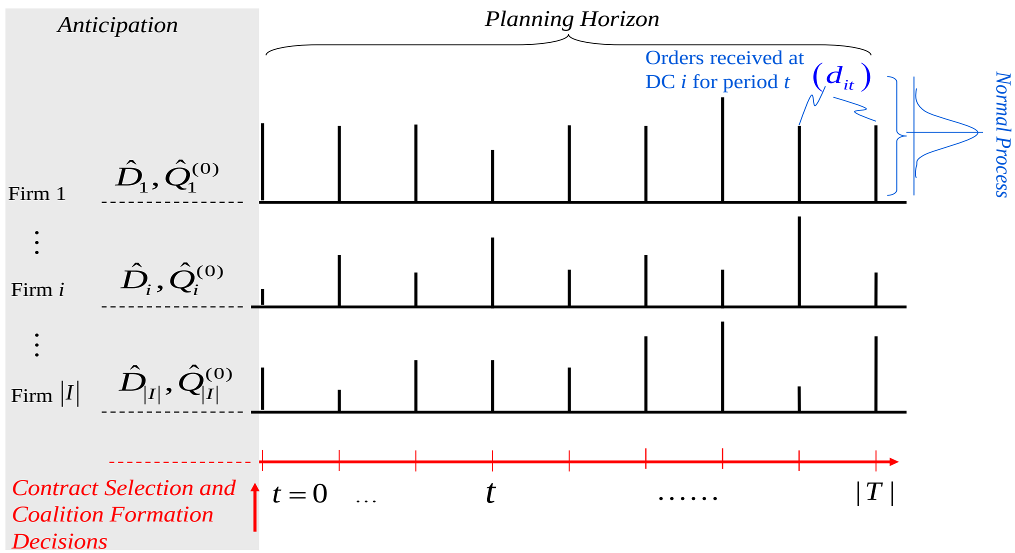


Figure 2- Firms Demand Process at DCs for the Planning Horizon

When future demand is anticipated, each firm can optimize solely its own revenues and costs for the planning horizon considered by finding the best compromise between all the cost components and the purchasing price, i.e. by optimizing the ordering quantity. This latter is crucial since it depends on the replenishment costs values incurred by the firm, on the demand behavior from its customers, and on the purchasing costs structure offered by the contracted supplier. For instance, it is well known that when demand is characterised by a *Normal* process the optimal ordering quantity can be determined by the so-called economic ordering quantity (EOQ) formula. Let $\hat{Q}_i$ denotes the individual ordering quantity defined in the replenishment plan of firm $i \in I$ based on the demand $\hat{D}_i$, $i \in I$. For a given firm *i*, an ordering sequence of $\hat{Q}_i$ must be defined for a subset of periods $t \in T$ in order to satisfy its demand $\hat{D}_i$ during the planning horizon. Under normality assumption on $\hat{D}_i$, the replenishment becomes a sequence of equal and individual order sizes $\hat{Q}_i$ for regular periods along the planning horizon. Let N_i be the number of replenishments (i.e. orders frequency) along the planning horizon, corresponding implicitly to the repartition of the demand $\hat{D}_i$ into lots $\hat{Q}_i$ for firm $i \in I$. When the firm is operating solely, the anticipated values correspond to $\hat{D}_i$ and $\hat{Q}_i^{(0)}$ underlining the standalone setting (referred by subscript 0) of the firm's decisions as shown in **Figure 2**. Once the ordering quantity and frequency are determined by the firm, an agreement on the contract terms for the tactical horizon with the supply entity is pinpointed. The specification of the contract features and the characteristics of the relationship between the two SC entities, determines the degree of the vertical collaboration between them. Flexible contracts, VMI or CPFR approaches discussed in Section 2 pertain to this type of collaboration.

At the operational level, when the replenishment plan is executed and deliveries are made, the customer orders are fulfilled from the assigned DCs. It is through this order fulfilments process that sales revenues and operational costs are generated. Let $\hat{P}_i(.)$, $\hat{R}_i(.)$ and $\hat{C}_i(.)$ denote, the profit, the sales revenues and the total operational costs anticipated for firm $i \in I$, respectively. One should mention that, at its replenishment periods, each firm i (based on its orders size $\hat{Q}_i^{(0)}$) requests from its contracted carrier the trucks required to supply the products from the source location[2] to the DC using Single Truckload (STL) transportation. The STL refers to the case where a not full trailer is delivered to a single destination (illustrated later on in **Figure 4**).

The context described in this subsection corresponds to the stand-alone replenishment situation, where each firm operates individually. Note that the customers' delivery costs are assumed to be sunk costs in what follows since the modeling approach employed is not optimizing the DCs to demand zones transportation stage. The next subsection provides the modeling features of this problem. Next, the standalone replenishment planning problem described here will be contrasted with various collaborative business situations.

III.2. **The Standalone Replenishment Problem Modelling**

Initially, the supplier *j*, must develop the purchasing cost structure in terms of the quantity discount to propose on the product-market, independently from the number of firms and their requirements. More specifically, the supplier must fix for each ordering level the associated unitary purchasing cost. In practice, the discount offer proposed encourages the maximization of the sales volume for the supplier that attempts to benefit from economies of scale in production, in inbound transportation and warehousing savings. Let c_j denotes the unitary purchasing cost incurred by the firms for each unit of product sourced from supplier *j*. As mentioned, the c_j value depends on the ordered quantity required, and thus the purchasing costs incurred by the firm *i* are function of $\hat{Q}_i$. Let $\hat{G}_i\left(\hat{Q}_i\right)$ denotes the purchasing costs function applied by the supplier based on quantity $\hat{Q}_i$ of a given firm *i*. As the discount is a potential option to encourage ordering larger quantities, we propose a digressive purchasing cost function that is controlled by a minimal prefixed threshold $Q_{\min}$ and a maximal threshold $Q_{\max}$. The purchasing costs function $\hat{G}_i\left(\hat{Q}_i\right)$ relies on a maximal value *c* that defines the unitary cost for purchasing products and on a quantity discount rate *e* reflecting the purchasing costs decrease when the ordered quantities increase. These values are defined so that for any ordering level beyond the maximal quantity we have $c > c - eQ_{\max}$ and when the quantity exceeds the maximal level $Q_{\max}$ we have $c_{\min} < c - eQ_{\max}$, where $c_{\min}$ is the fixed minimal cost. Accordingly, the unitary purchasing costs for a given firm $i \in I$ could be anticipated using the estimated quantity to order $\hat{Q}_i$ and the supplier cost structure given by equation (1), adapted from Fazel et al. (1998):

$$c_j = \begin{cases} c & if \quad \hat{Q}_i < Q_{\min} \\ c - (e\hat{Q}_i) & if \quad Q_{\min} \leq \hat{Q}_i < Q_{\max} \\ c_{\min} & if \quad \hat{Q}_i \geq Q_{\max} \end{cases} \tag{1}$$

The two thresholds $Q_{\max}$ and $Q_{\min}$ are obtained as follows:

$$Q_{\max} = \frac{c - c_{\min}}{e}, \; Q_{\min} = \frac{Q_{\max}}{2}$$

Afterward, the anticipated costs and revenues of each firm $i \in I$ are derived based on the anticipated demand $\hat{D}_i$ and the decision on quantities to order $\hat{Q}_i$. Let $\hat{R}_i^0$, $\hat{C}_i^0(\hat{Q}_i)$ and $\hat{G}_i^0(\hat{Q}_i)$ denote the sales revenues, the total operational costs, and the purchasing costs of the firm $i \in I$ in a standalone replenishment context, respectively. In this case, firm *i* aims to maximize its profit, denoted by $\hat{P}_i^0\left(\hat{Q}_i\right)$, by minimizing its operational costs $\hat{C}_i^0(\hat{Q}_i)$, and increasing its net revenues

[2] In the case of a global supplier, the local location could be associated to an entry port or to an inland cargo hub.

(i.e. sales revenues R_i^0 minus purchasing costs $G_i^0(\hat{Q}_i)$) using the following ordering quantity-based equation :

$$\hat{P}_i^0(\hat{Q}_i) = \hat{R}_i^0 - \hat{C}_i^0(\hat{Q}_i) - \hat{G}_i^0(\hat{Q}_i) \tag{2}$$

where the purchasing costs are computed by $\hat{G}_i^0(\hat{Q}_i) = c_j \hat{D}_i$ using relation (1). By integrating the purchasing costs presented in (1), the profit function could be written as follow:

$$\hat{P}_i^0(\hat{Q}_i) = \begin{cases} \hat{R}_i^0 - \hat{C}_i^0(\hat{Q}_i) - c\hat{D}_i & if \quad 0 < \hat{Q}_i < Q_{\min} \\ \hat{R}_i^0 - \hat{C}_i^0(\hat{Q}_i) - (c - (e\hat{Q}_i))\hat{D}_i & if \quad Q_{\min} \le \hat{Q}_i < \\ \hat{R}_i^0 - \hat{C}_i^0(\hat{Q}_i) - c_{\min}\hat{D}_i & if \quad \hat{Q}_i \ge Q_{\max} \end{cases} \tag{3}$$

Moreover, anticipated sales revenues $\hat{R}_i^0$, for a given firm i, are obtained by multiplying the estimated total demand $\hat{D}_i$ by the unitary sales price, denoted by p_i, which gives the following relation: $\hat{R}_i^0 = p_i \hat{D}_i$. Regarding the operational costs $\hat{C}_i^0(\hat{Q}_i)$, recall that it takes into account the entire replenishment process by computing the ordering costs, the inventory holding costs, and the inbound transportation costs. Accordingly, we propose in this paper a typical modeling of the ordering and holding costs functions, similarly to Elomri et al. (2012), but a more elaborated transportation costs function is provided here. This latter proposes to capture variable and fixed parts of the transportation fees using, a distance and weight-based, linear function that estimates the unit arc-flow costs. The variable costs part is related to the shipment weight that depends on the anticipated quantity to order by shipment and it is given by $c_v \hat{Q}_i$, where c_v is the unitary variable cost by product unit associated to the shipment. The fixed costs part is associated to the traveled distance to ship from the source location (local site or an entry port/hub) to the firms' DCs. Since the distance to travel is known in advance, the fixed costs is computed a priori by $c_a + c_d d_{ji}$ where c_a is a fixed charge for the truck usage, c_d is the unitary distance-rate to ship one unit of product, and d_{ji} is the traveling distance from location *j* to *i*. Hence the anticipated transportation costs incurred by a given firm *i* could be computed as follows:

$$TC_i^0\left(\hat{Q}_i\right) = c_a + c_d d_{ji} + c_v \hat{Q}_i \quad , \forall i \in I \tag{4}$$

The ordering costs for firm *i*, denoted by OC_i^0, depend mainly on a fixed unitary ordering cost A and the number of placed orders $\hat{N}_i$ (recall that $\hat{N}_i = \hat{D}_i / \hat{Q}_i$). In addition, the inventory holding costs, denoted by HC_i^0, involves a linear relation between the unitary cost of capital to hold one unit of product (denoted by h_i) and the average size of inventories kept at DCs between two replenishments, calibrated to be $\hat{Q}_i/2$. The ordering and the holding cost functions are given by (5) and (6), respectively.

$$OC_i^0(\hat{Q}_i) = A\hat{N}_i, \ \forall i \in I \tag{5}$$

$$HC_i^0(\hat{Q}_i) = h_i \frac{\hat{Q}_i}{2}, \ \forall i \in I \tag{6}$$

The anticipated total operational costs incurred by a firm i are thus computed using the following summation:

$$C_i^0(\ddot{Q}_i) = OC_i^0(\ddot{Q}_i) + HC_i^0(\ddot{Q}_i) + TC_i^0(\ddot{Q}_i)$$

Based on the set of operational costs functions and revenues formula introduced above, the profit structure for a given firm i presented in (3) can be rewritten as follows:

$$\hat{P}_i^0\left(\hat{Q}_i\right) = \begin{cases} p_i\hat{D}_i - \left(A\dfrac{\hat{D}_i}{\hat{Q}_i} + h_i\dfrac{\hat{Q}_i}{2} + c_a + c_d d_{ji} + c_v\hat{Q}_i\right) - c\hat{D}_i & if \quad 0 \le \hat{Q}_i < Q_{\min} \\ p_i\hat{D}_i - \left(A\dfrac{\hat{D}_i}{\hat{Q}_i} + h_i\dfrac{\hat{Q}_i}{2} + c_a + c_d d_{ji} + c_v\hat{Q}_i\right) - \left(c - e\hat{Q}_i\right)\hat{D}_i & if \quad Q_{\min} \le \hat{Q}_i < Q_{\max} \\ p_i\hat{D}_i - \left(A\dfrac{\hat{D}_i}{\hat{Q}_i} + h_i\dfrac{\hat{Q}_i}{2} + c_a + c_d d_{ji} + c_v\hat{Q}_i\right) - c_{\min}\hat{D}_i & if \quad \hat{Q}_i \ge Q_{\max} \end{cases} \quad (7)$$

According to the formulation given in (7), the optimal ordered quantity $Q_i^{0,*}$ that maximizes the profit for firm i is obtained by solving $\frac{d\hat{P}_i^0(\hat{Q}_i)}{d\hat{Q}_i}$ which can be written according to the purchasing cost structure in (1) as follows:

$$Q_i^{0,*} = \begin{cases} \sqrt{\dfrac{2\hat{D}_i(A + c_a + c_d\hat{D}_i)}{h_i}} & if \quad 0 < \hat{Q}_i < Q_{\min} \\ \sqrt{\dfrac{2\hat{D}_i(A + c_a + c_d\hat{D}_i)}{h_i - 2e\hat{D}_i}} & if \quad Q_{\min} \le \hat{Q}_i < Q_{\max} \\ \sqrt{\dfrac{2\hat{D}_i(A + c_a + c_d\hat{D}_i)}{h_i}} & if \quad \hat{Q}_i \ge Q_{\max} \end{cases} \quad (8)$$

In what follows, given the standalone setting described above, the opportunity for a given firm i to form a coalition is discussed.

III.3. **Coalition Formation Problem Description and Modelling**

When looking to the entire structure of the SNs pertaining to the cluster of operating firms I in the product-market considered here, the collection of individual firm-based decisions corresponds to a set of a decentralized decision making processes. In this situation several questions may arise: Is there any benefit for these firms to improve their individual economic performance when they collaborate throughout their SNs? If so, what is the best coalition formation for each firm in order to maximize its profits? How will the firms react toward collaboration according to the profit sharing mechanism proposed? Finally, when various joint replenishment situations are investigated, one expects the same coalitions to be formed? More specifically, the main issue is to inspect if the firms $i \in I$ could increase their individual profit $P_i(.)$, when they collaborate? Depending on the replenishment situation, which is the best compromise for each firm i between the purchasing costs $G_i(.)$ and the operational costs $C_i(.)$? and thus in each replenishment situation how the best ordering quantity Q_i could be determined ? To answer to these questions we must define the coalition formation problem for a joint replenishment process.

Aissaoui et al. (2007) resume the purchasing decision process into six major components: make/buy, supplier selection, contract negotiation, design collaboration, procurement, and sourcing analysis. Here, we are clearly focusing to the contract negotiation and the design collaboration components by deciding for each firm on the coalition to form in order to design the sourcing contract taking into account the decision to collaborate or not with other firms. As mentioned, the sourcing contract was usually designed by the firms solely or jointly based only on the quantity discount incentives, but not by anticipating the benefits of joint replenishment in transportation and/or in warehousing. Accordingly, the sourcing contract design can be viewed as a distributed decision-making process (Schneeweiss, 2003) aiming to identify profitable coalitions to form by anticipating the revenues and costs generated by the firms collaboration. **Figure 3** describes the four steps encountered in the collaborative contract design decisional process when one supplier and a set of independent firms are involved. In step (1), the supplier provides for the set of firms requiring its product a purchasing cost structure encompassing the quantity-based unitary costs feature as given in equation (1). In such contract proposition, the supplier has to fix the main contract parameters: the discount value, the unitary cost incurred and the minimum and maximum quantities.

In step (2), given the supplier offer, firms are tempted to evaluate the opportunity to operate individually or in cooperation with other firms in order to maximize their own profits. In the cooperative situation, firms will try to form profitable coalitions regarding all the partners within. The anticipation of a profitable coalition essentially imply to share among the partners a number of information about their estimated annual demands $\hat{D}_i$, as well as their estimated orders quantities $\hat{Q}_i$. In some cases this could be piloted by an external logistics service provider (i.e. bidding 3PLs, currently contracted carriers) to guarantee the neutrality and confidentiality of the process. For that reason the design process illustrated here incorporates the intervention of an external entity in step (2). Recall that we assume here that in the standalone situation each firm rely solely on a contracted carrier to replenish its products. Such entity could be responsible for the whole coalition formation project or simply be mandated for the execution of ordering, holding, and/or transportation activities for the formed coalition. Typically, in step (2) the external entity considered provided the necessary information in terms of bids and charges, and in terms of transportation capacities. See for instance the case of Nestle Waters and Danone in (2010) reported in **Table 1**. The idea of Trustee entity was also introduced in the case of retail inbound horizontal collaboration in Belgium as a neutral actor that facilitates the collaboration among a large number of suppliers (www.co3-project.eu/).

Step (3) reflects the decision made by the firms participating in the collaboration opportunity to join or not the coalition based on the anticipated profitability. Remind that a coalition/alliance characterizes the group of firms that decides to work jointly while grouping their orders and synchronizing their policies. All joining firms will operate as a single decision making unit as showed in **Figure 3** to decide about the contract structure proposed by the supplier j. Finally, in step (4) when coalition formation decision is acted, a joint replenishment plan is optimised and submitted by the cooperating firms to the supplier. More specifically, the coalition decides on the joint ordering level and ordering frequency contracted with the supplier.

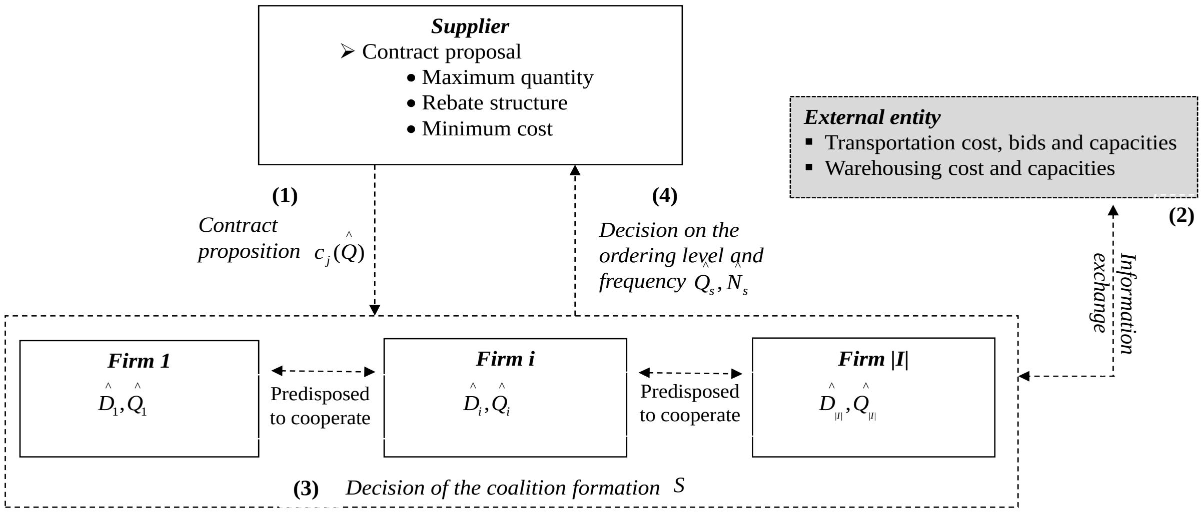


Figure 3- Decisions Process of the Contract Design Problem with collaboration

Based on the decision-making process described above, the following modeling approach for the coalition formation problem is provided. Let S denotes a given coalition composed by a collection of firms $i \in I$ such as $S \subseteq I$. For a given coalition S , let I_S be the subset of firms assigned to S , such as $I_S = \arg\{i | i \in I \cap S\}$ and $I_s \subseteq I$. Conversely, let $S(i)$ refers to the coalition of firm $i \in S$. Note that the set of all possible coalitions is denoted by Ψ so that each coalition is indexed by $S \in \Psi$. When a coalition S is formed, the set of firms I_S pertaining to the alliance must pool their ordered quantity and must be aligned on the number of orders. Let $\hat{Q}_S$ denotes the ordering quantity for coalition $S \in \Psi$ and let $\hat{N}_S$ denotes the orders frequency for coalition $S \in \Psi$ along the planning horizon. We assume also the existence of a collective ordering cost A_S associated to the characteristics of a given coalition S . First, one should determine a unique number of orders $\hat{N}_S$, for all firms $i \in S$ so that they are invited to adopt the same replenishment cycle $\hat{N}_S$ as proposed in Meca et al., (2005) and in Fiestras-Janeiro et al. (2011). Consequently, an agreement should be adopted relatively to the individual $\hat{N}_i$ for all $i \in S$, which are generally not necessarily aligned when firms are beforehand acting solely. As the profit function is inversely proportional to cooperative ordering number, $\hat{N}_S$ can be expressed as follows (Krichen et al., 2011):

$$\hat{N}_s = \min(\hat{N}_i), \forall i \in I_s \tag{9}$$

Based on the alignment of the ordering cycles, the ordered quantity of each firm in the coalition S must be adjusted. Let $\hat{Q}_{i,s}$ denotes the quantity ordered by the firm $i \in S$ which is defined in accordance to the single frequency $\hat{N}_i$ and based on the new cooperative $\hat{N}_s$ as follows: according to the cooperative order number as:

$$\hat{Q}_{i,S} = \begin{cases} \dfrac{\hat{D}_i}{\hat{N}_S} & if \quad \hat{N}_i > \hat{N}_S \\ \hat{Q}_i & if \quad \hat{N}_i = \hat{N}_S \end{cases} \tag{10}$$

Finally, by forming a coalition, firms pertaining to the alliance must coordinate their orders and thus share a unique ordering cost A_S . Hence, a communication cost will be needed to manage business operations on day to day basis between coalition members. Furthermore, that cost can be also used to support the design of a distributed database among all coalitions' members. Such mechanism is called in practice as Electronic Data Interchange (EDI) which is a good example of standards developed in order to share documents between organizations in a standardized electronic form and in an automated manner (Bhatt, 2001; Audy et al., 2012). In a generic way, one should expect that the coalition ordering cost be function of the firms' individual ordering cost and the size of the coalition: $A_S = f(A, |S|)$. Consequently, we propose to include a communication cost to be incurred by firms belonging to the coalition. This cost increases proportionally to the number of firms in the coalition. Given a coalition S, its supplementary ordering cost is equal to $c_c |S|$ where c_c is the unit communication cost. Then, the coalition's ordering cost to be placed with any order can be stated as:

$$A_s = A + c_c |S| \tag{11}$$

In addition, when firms collaborate, they share information in a timely manner and use different approaches to synchronize their activities efficiently. In such cases, a cumulative holding cost is assigned once a coalition is formed. Elomri et al., 2012 proposed that cost where an EOQ is used as a reordered policy. In this way, firms forming a coalition are pooling their orders to maximize profit by reducing the unit purchasing cost (see equation (1)). Furthermore, once a coalition is formed, one should ask how much profit each firm could gain? This depends on the sharing opportunities offered by the replenishment situation. To this end, the next section describes a number of replenishment situations that offer various opportunities for alliances to share not only orders but also transportation and/or storage activities. By forming coalitions, firms group their orders and decide about the most profitable coalition to be adopted. The decision of joining coalition is taken according to the realized profit of each firm in a given coalition. Let $\hat{P}_S(.)$, denotes the cooperative profit evaluated for a coalition $S \in \Psi$. We define by $\hat{P}_{i,S}(.)$ the profit allocated to a firm $i \in S$ that suppose a sharing mechanism that the coalition profit will be distributed among all the members proportionally to the quantity $\hat{Q}_{i,S}$. This volume-based gain sharing mechanism has been successfully applied recently in a retailing context (www.co3-project.eu). The individual profit $\hat{P}_{i,S}$ (.) is calculated by:

$$\hat{P}_{i,S}(.) = \frac{\hat{Q}_{i,S} \times \hat{P}_S(.)}{\hat{Q}_S}, \ \forall i \in S \tag{12}$$

For each firm i, various coalitions in ψ can be profitable with regard to the cost saving realized by the cooperation. Let ψ' the set of all profitable coalitions belonging to ψ, $\psi' \subset \psi$. Equation (12) assumes also that each firm savings on purchasing, transportation and storage costs would be proportional to the ordered quantity within the coalition. Cooperative game theory offers a natural paradigm to deal with cost saving problems (Meca el al., 2005). To define a cooperative game, two ingredients are needed: a set of players (here the firms) and a characteristic function which assigns to each possible coalition of players a numerical value (here the profit value) to be interpreted as a measure of its payoff. Let $\hat{P}_s^l = \{\hat{P}_{1,s}^l, \hat{P}_{2,s}^l, ..., \hat{P}_{|s|,s}^l\}$ be the set of profit realized by firms in a given coalition S. Indeed, the members of a coalition S may be interested in forming a coalition S, if $\hat{P}_s^l > \sum_{i \in I_s} \hat{P}_i^0$. The main objective is to find among all profitable coalitions the stable ones. We denote by ψ^* the set of stable coalitions; $\psi^* \subset \psi'$. Coalition structures are stable in the sense that no firm has an incentive to deviate (e.g stability condition is discussed in details in

Section V). Stable coalition in the game theory is in the sense of the Coalition Structure Core (Aumann and Drze, 1974). Core concept is defined as follows:

$$\hat{P}_{i,s}(.) > \hat{P}_{i,s'}(.) \quad \forall i \in I_s, i \in I_{s'}, s \in \psi, s' \in \psi' \tag{13}$$

Once stable coalitions are obtained, let $P^*_{i,s}(.)$ and $Q^*_{i,s}$ be respectively the stable profit and ordered quantity for a firm $i \in S$. Consequently, suitable replenishment decisions to join stable coalitions are to be taken from firms to procure products at the lowest possible price. Three replenishment situations are to be defined and discussed in the next section.

IV. The Investigation of Three Cooperative Replenishment Situations

As described in the business context and illustrated by decision-making process in **Figure 3**, in response to the supplier offer, each firm can contract individually or collaborate with other firms. Typically, firms are interested by new purchasing contracts based on discount structures offered by the supplier which could be reached only when they decide to join their ordering process. Moreover, as reported in **Table 1**, several practical examples showed that firms could also benefit from collaboration in the transportation and warehousing operations by pooling their orders and the entire replenishment process. In this paper, we propose to study three alternative replenishment situations that could favor horizontal collaboration on these operations. These three situations are namely: Collaborative Ordering with Direct Shipments (CODS), Joint Replenishment with Shared Warehousing (JRSW), and Joint Replenishment with Shared Shipments (JRSS). These situations are explained hereafter and are illustrated in **Figure 4** in order to mimic the replenishment process in each case. To inspect separately the outcomes of these situations, let $\hat{P}^l_S(.)$ be the cooperative profit where an index l=1, 2, 3, is used to denote CODS, JRSW and JRSS situations, respectively (recall that l=0 corresponds to the standalone situation). Recall that the objective of the study of these three alternative situations is first to inspect in each case the impact of the replenishment incentives on the coalitions to form, and secondly to analyse the insights toward horizontal collaboration based on the drivers offered in the ordering, transportation and warehousing operations. As illustrated in **Figure 4**, these replenishment situations rely on two transportation options: Single truckload (STL) or Multi-drop truckload routes (MTL) (Klibi et al., 2010b). Based on transportation costs function (4), the transportation fees charged with each of these two modes were computed differently depending on the replenishment situation (i.e. subscripts 1 for STL and 2 for MTL were added to all the parameters of the transportation costs function (4)).

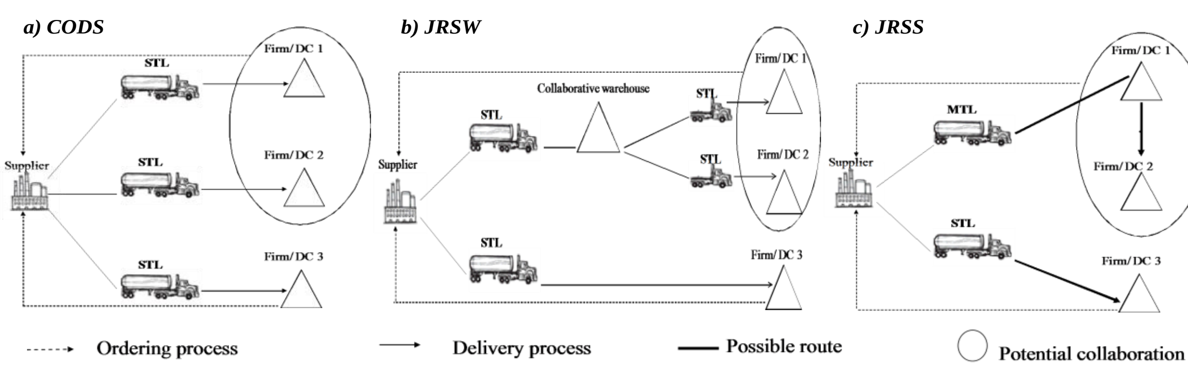


Figure 4- The Replenishment Process of Each of the Three Situations Investigated

The first cooperative situation consists in collaborating solely in the ordering process. In this case, firms group their quantities to order for each scheduled time period. Based on the ordering policy in (9), joint orders are computed with (10) for each time period. Once orders are placed, each firm could plan with its own transportation service provider (3PL) for the delivery of its individual part of the joint order using a single truckload shipment (STL). *CODS* situation is illustrated in **Figure 4.a),** and corresponds to classical joint ordering process as discussed in (Krichen et al., 2011) and in several practical case in Table 1. For a firm $i \in I_s$, the optimisation of the transportation costs is not always possible because the replenishment level $\hat{Q}_{i,S}$ depends on the coalition $S(i)$ parameters $\hat{Q}_S$, $\hat{N}_S$ and thus, are not always ideally calibrated to have a full truckload as in the individual ordering case. For this first replenishment situation, the coalition profit $\hat{P}_S^1(.)$ is given by expression (14). It includes the four cost components: ordering, holding, transportation and purchasing, and consider the joint ordering cost expressed in (11). For the set of the collaborating firms *S*, using $\hat{Q}_S$, the sum of individual shipment transportation cost in STL is computed based on expression (4) and the joint purchasing cost ($\hat{G}_S^1(\hat{Q}_S)$) are given by equation (3). The collaborative profit $\hat{P}_S^1(.)$ according to the CODS replenishment situation is as follows:

$$\hat{P}_s^1 = \begin{cases} \sum_{i\in I_s} p_i \hat{D}_i - (A_s \frac{\hat{D}_s}{\hat{Q}_s} + \sum_{i\in I_s} h_i \frac{\hat{Q}_i}{2} + \left[\sum_{i\in I_s}(c_a^1 + c_d^1 d_{ij}) + c_v^1 \hat{Q}_s\right]\frac{\hat{D}_s}{\hat{Q}_s}) - c\hat{D}_s & 0 < \hat{Q}_s < \hat{Q}_{\min} \\ \sum_{i\in I_s} p_i \hat{D}_i - (A_s \frac{\hat{D}_s}{\hat{Q}_s} + \sum_{i\in I_s} h_i \frac{\hat{Q}_i}{2} + \left[\sum_{i\in I_s}(c_a^1 + c_d^1 d_{ij}) + c_v^1 \hat{Q}_s\right]\frac{\hat{D}_s}{\hat{Q}_s}) - (c - e\hat{Q}_s)\hat{D}_s & \hat{Q}_{\min} \le \hat{Q}_s < \hat{Q}_{\max} \\ \sum_{i\in I_s} p_i \hat{D}_i - (A_s \frac{\hat{D}_s}{\hat{Q}_s} + \sum_{i\in I_s} h_i \frac{\hat{Q}_i}{2} + \left[\sum_{i\in I_s}(c_a^1 + c_d^1 d_{ij}) + c_v^1 \hat{Q}_s\right]\frac{\hat{D}_s}{\hat{Q}_s}) - c_{\min}\hat{D}_s & \hat{Q}_s \ge \hat{Q}_{\max} \end{cases} \quad (14)$$

In the second cooperative situation JRSW, firms gather their individual orders to be delivered in joint shipments to a common warehouse or hub. The incentive behind proposing such business case arises from the consideration of long distances between geographical locations. In fact, when suppliers are located in foreign countries, intermediate warehouses are required for temporary storages (see **Figure 4.b**). As reported in (Ozen et al., 2008), this issue was observed in several manufacturing facilities in Asia to markets in Europe and North America, which suffer from long supplier lead times. The authors proposed that firms collaborate through inventory pooling and make allocations after demand realization in order to enhance their profit. In the same way, we considered in this paper that in the *JRSW* situation, a set of collaborative warehouses, denoted W^S, would be available for coalition S. These latter are assumed to be part of the private network of hubs managed by the 3PLs and thus in-between connections transportation fees are not considered in the firms charges. Subsequently, the transportation lane take into account of the inbound transportation (supplier - 3PLs origin warehouse $w \in W^S$) and outbound transportation (3PLs destination warehouse $w' \in W^S$ – DC $i, i \in S$). The inbound transportation lane is accomplished jointly in STL which could be in full truckload if Q_s corresponds to the 3PL truck capacity. A final shipment step consists in delivering to firms via single STL shipments. Here, the total travelled distance from the supplier to the $|S|$ ship-to points is given by $\delta_s = d_{jw} + \sum_{i\in S} d_{w'i}$ be the travelled distance to the $|S|$ destinations, where $|S|$ is the number of firms in the coalition S. Hence, for a given coalition S, the total cost in the JRSW situation is based on shared ordering and inventory holding cost components, and thus, the total profit $\hat{P}_S^2(.)$ is written as follow:

$$\hat{P}_s^2 = \begin{cases} \sum_{i\in I_s} p_i \hat{D}_i - (A_s \frac{\hat{D}_s}{\hat{Q}_s} + \sum_{i\in I_s} h_i \frac{\hat{Q}_i}{2} + \left[c_a^1\left(|S|+1\right) + c_d^1 d_{ij} + c_v^1 \hat{Q}_s\right]\frac{\hat{D}_s}{\hat{Q}_s}) - c\hat{D}_s & 0 < \hat{Q}_s < \hat{Q}_{\min} \\ \sum_{i\in I_s} p_i \hat{D}_i - (A_s \frac{\hat{D}_s}{\hat{Q}_s} + \sum_{i\in I_s} h_i \frac{\hat{Q}_i}{2} + \left[c_a^1\left(|S|+1\right) + c_d^1 d_{ij} + c_v^1 \hat{Q}_s\right]\frac{\hat{D}_s}{\hat{Q}_s}) - (c - e\hat{Q}_s)\hat{D}_s & \hat{Q}_{\min} \le \hat{Q}_s < \hat{Q}_{\max} \\ \sum_{i\in I_s} p_i \hat{D}_i - (A_s \frac{\hat{D}_s}{\hat{Q}_s} + \sum_{i\in I_s} h_i \frac{\hat{Q}_i}{2} + \left[c_a^1\left(|S|+1\right) + c_d^1 d_{ij} + c_v^1 \hat{Q}_s\right]\frac{\hat{D}_s}{\hat{Q}_s}) - c_{\min}\hat{D}_s & \hat{Q}_s \ge \hat{Q}_{\max} \end{cases} \tag{15}$$

Finally, for the third replenishment situation *JRSS*, the delivery process from the supplier's warehouse to the firm's local warehouses is synchronized in a multi-drop truck load (MTL) through pooling product transportation. A MTL refers to the case where the truck delivery route involves more than one destination (Klibi et al., (2010b)) and thus serves multiple DCs within one trip as presented in **Figure 4.c**). The idea is inspired from the transportation pooling and its benefits as discussed in Pan et al., (2012). The authors proposed a pooling strategy at the transportation level by sharing the freight network at a national level in order to reduce CO2 emissions. This opportunity offers lower transportation costs since the travelled distance is shared by the involved firms. One way to evaluate the transportation costs here is by solving the joint inventory-routing problem (Coelho et al., 2014). Another approach, used in this paper, is to build an evaluation function that is based on a linear regression route length estimator (introduced by Daganzo (1984) and extended in Klibi et al., (2010b)) to account for STL and MTL transportation. In this case, let ∂_s be the total distance travelled by the firms of the coalition from the supplier to their respective DCs. This distance is approximated by the following expression: $\partial_s = |S|(\frac{\rho}{|S|+1} + 0.57\sigma^2)$.

Hence, in the *JRSS* situation firms forming a coalition are sharing ordering, inventory holding and transportation costs and their cooperative profit can be written as follow:

$$\hat{P}_s^3 = \begin{cases} \sum_{i\in I_s} p_i \hat{D}_i - (A_s \frac{\hat{D}_s}{\hat{Q}_s} + \sum_{i\in I_s} h_i \frac{\hat{Q}_i}{2} + \left[c_a^2 |S| + c_d^2 \partial_s + c_v^2 \hat{Q}_s\right]\frac{\hat{D}_s}{\hat{Q}_s}) - c\hat{D}_s & 0 < \hat{Q}_s < \hat{Q}_{\min} \\ \sum_{i\in I_s} p_i \hat{D}_i - (A_s \frac{\hat{D}_s}{\hat{Q}_s} + \sum_{i\in I_s} h_i \frac{\hat{Q}_i}{2} + \left[c_a^2 |S| + c_d^2 \partial_s + c_v^2 \hat{Q}_s\right]\frac{\hat{D}_s}{\hat{Q}_s}) - (c - e\hat{Q}_s)\hat{D}_s & \hat{Q}_{\min} \le \hat{Q}_s < \hat{Q}_{\max} \\ \sum_{i\in I_s} p_i \hat{D}_i - (A_s \frac{\hat{D}_s}{\hat{Q}_s} + \sum_{i\in I_s} h_i \frac{\hat{Q}_i}{2} + \left[c_a^2 |S| + c_d^2 \partial_s + c_v^2 \hat{Q}_s\right]\frac{\hat{D}_s}{\hat{Q}_s}) - c_{\min}\hat{D}_s & \hat{Q}_s \ge \hat{Q}_{\max} \end{cases} \tag{16}$$

The shared profit of firm $i \in I_s$ regarding situation $l \in \{1,2,3\}$ is denoted by $P_{i,s}^l$. So, the sharing mechanism reported in equation (12) becomes:

$$\hat{P}_{i,S}^l = \hat{Q}_{i,S}^l \hat{P}_S^l \big/ \hat{Q}_S \quad \forall i \in S \tag{17}$$

Similarly, the profit incurred by the stable coalition for each firm, is greater than the profit generated by any other coalition. Consequently, suitable replenishment decisions to join stable coalitions are to be taken from firms to procure products at the lowest possible price. The three cooperative situations presented above will be evaluated in various industrial contexts and compared to the standalone situation in Section V.

V. Solution Approach

V.1. Stability Conditions of the Coalition Structure

This section proposes a game theoretical solution approach for generating profitable coalition structures and selecting stable ones based on decision rules relying on the core (Aumann and Drze, 1974). The core stability concept used in cooperative games takes into account the individual and coalitional rationalities needed for reaching best compromises between all firms in the supply chain (Krichen et al, 2011). The core stability concept is determined with the following two features:

- *Coalitional rationality condition:* A profit P_s^l is said to be coalitionally rational if it satisfies: $P_s^l > \sum_{i\in s} P_i^0$, $l = 1,2,3$
- *Stability condition*: A coalition s *is said to be stable if it satisfies*: $P_{i,s} > P_{i,s'}, \forall i \in I_s, i \in I_{s'}, s \in \Psi, s' \in \Psi'$.

For the coalition formation problem, enumerating all the possible coalitions and testing their stability is too time consuming, if not impossible. For that reason, we first developed in this section inequality conditions that eliminate beforehand some non-profitable coalitions, and thus enhance considerably the solution approach efficiency. Next to that, the cooperative replenishment Algorithm (CRA) steps will be presented in details. To start with, only profitable coalitions are interesting to accept with respect to profits equations (3) and (14)-(17), and thus must be computed and compared efficiently. To this end, a set of inequalities was developed and their validity was proved analytically. Depending on the replenishment situation, these latter are based on the trade-offs produced on the ordering costs, the inventory holding costs and the transportation costs. Accordingly, for each replenishment situation $l = 1,2$ or 3, two coalition-based expressions, denoted by $\alpha_{i,s}^l$ and $\theta_{i,s}^l$, must be calculated and compared to check if it is beneficial or not for a given firm i to join a coalition S. Considering a given replenishment situation l, the decision to accept or to discard the coalition S for a given firm i is based on the following decision rule:

- Accept the coalition if $\alpha_{i,s}^l < \theta_{i,s}^l$
- Refuse it otherwise

Considering the CODS replenishment situation, each firm i has interest to accept to join coalition S if the inequality above holds where $\alpha_{i,s}^1$ and $\theta_{i,s}^1$ are given by expression (18) :

$$\alpha_{i,s}^1 = c_c |S|, \theta_{i,s}^1 = A\left(\frac{\hat{D}_i (\hat{Q}_s)^2}{\hat{D}_s \hat{Q}_{i,s} \hat{Q}_i} - 1\right) - \chi_{i,s}^1 \frac{(\hat{Q}_s)^2}{\hat{D}_s \hat{Q}_{i,s}}; if\ Q_{\min} \le \hat{Q}_i < Q_{\max} \tag{18}$$

For the JRSW replenishment situation (l=2) we prove that the inventory holding cost of the common local warehouse h_s may affect the stability of the core structure. Then, to improve the stability condition for the JRSW, the following condition (19) is required:

$$\alpha_{i,s}^2 = h_s, \theta_{i,s}^2 = \frac{2\chi_{i,s}^2 + h_i \hat{Q}_i}{\hat{Q}_{i,s}}; if\ Q_{\min} \le \hat{Q}_i < Q_{\max} \tag{19}$$

Finally, for the JRSS replenishment situation (l=3), we proved that the 3PL travelled distance can perturb the transportation costs and thus impact on the coalition. Consequently, a given coalition is expected to be stable if:

$$\alpha_{i,s}^{3} = \partial_s \, , \theta_{i,s}^{3} = \frac{d_{ji} \hat{D}_i \hat{Q}_s}{\hat{Q}_s \hat{Q}_{i,s}} + \frac{\hat{Q}_s \chi_{i,s}^{3}}{c_d^k \hat{Q}_{i,s}} ; if \; Q_{\min} \leq \hat{Q}_i < Q_{\max} \tag{20}$$

The mathematical proof of conditions (18)-(20) is given in **Appendix A.** These three conditions are encompassed in the resolution algorithm hereafter to enhance its efficiency.

V.2. **The Cooperative Replenishment Algorithm**

We present now the schema of the CRA which is composed by three main steps. The first determines the individual profit for all firms in the standalone context. The second step is a filtering process that keeps only cost saving coalitions based on the profit structure given by (3) and (14)-(17). It proceeds by enumerating for each firm the subset of firms that can be beneficial to cooperate with. Subsequently, all non-profitable coalitions are discarded initially when performing the preliminary calculations of thresholds $\alpha_{i,s}^{l}$ and $\theta_{i,s}^{l}$ in order to compare coalition-based and standalone profits. The third step of the process consists in applying the core-stability test in order to find, among the current set of profitable coalitions, the stable ones. With these steps, the CRA produces the most profitable repartition of the set of firms into a set of stable coalitions that are mutually exclusives and collectively exhaustive. Subsequently, the CRA is tested and validated on extensive numerical instances.

***CRA** (Inputs:* $\hat{Q}_i$, Ψ , $s \in \Psi$, I ; $\hat{Q}_{i,s}$, $P^l_{i,s}$, Ψ' , $s' \in \Psi'$, $s'' \in \Psi$; ***Outputs**:* $Q^{*,l}_{i,s}$, $P^{*,l}_{i,s}$, Ψ^* , $s^* \in \Psi^*$

1. For all $i \in I$ **do**
Compute the SA profit $\hat{P}^0_i(.)$ using equation **(3)**
End for
$\psi' = \emptyset, \psi^* = \emptyset$
2. For all $S \in \psi$ **do**
Stop=true
2.1 While $i \in I_s$ **and** *NotStop*
Compute $\alpha^1_{i,s}$ and $\theta^1_{i,s}$ based on $\hat{P}^1_{i,s}$ and $\hat{Q}^1_{i,s}$ with equations[3] **(10)** , **(14)** and **(18)**
If ($\alpha^1_{i,s} > \theta^1_{i,s}$) **then**
Stop= true
End if
End while
2.2 If *NotStop* **then**
$S' = S$
$\psi' = \psi' \cup \{S'\}$
End if
End for
3. For all $S' \in \psi'$ **do**
Stable=true
3.1 While $S'' \in \psi'$ and $i \in I_s$ **and** *Stable=True*
If ($\hat{P}^1_{i,s''} > \hat{P}^1_{i,s'}$) **then**
Stable = false
End if
End while
3.2 If *Stable=true* **do**
$S^* = S'$
$\psi^* = \psi^* \cup \{S^*\}$
$P^{*,1}_{i,s} = \hat{P}^1_{i,s}, \forall i \in I_s$
$Q^{*,1}_{i,s} = \hat{Q}^1_{i,s}, \forall i \in I_s$
End if
End for

Figure 5- The Cooperative Replenishment Algorithm

VI. Numerical Results and Discussion

VI.1. Plan of Experiments

In order to test the modeling and solution approaches proposed to solve the coalition formation problem, several problem instances were generated based on the following dimensions: market size, network configuration and cost structure (high / low costs). All these instances will be tested for the set of replenishment situations proposed (CODS, JRSW or JRSS) and compared to the SA. The horizon length is fixed to one year and one product family is

[3] **(18)** should be replaced by **(19)** if $l = 2$ (JRSW) and by **(20)** if $l = 3$ (JRSS)

considered in this study. We considered that the number of firms should not exceed 30, which is congruent with the industrial examples reported in **Table 1** and sufficient to validate the solution approach efficiency. Thus, the market size n in our tests ranges between 5 and 30 firms. The firms' market-size for the product-market considered and total demands received are calibrated based a North American distribution context studied in Klibi et al. (2010b). Using the customers' characterization (large, medium, or small) and their associated demand size, a repartition within the set of firms is provided to calibrate their market share. Accordingly, an aggregated yearly demand for each firm is derived and two main classes of firms are generated:

i. *Small-sized Firms / product-demand (SF)* with a yearly total demand level within the interval of [120, 220] pallets.
ii. *Large-sized Firms / product-demand (LF)* with a yearly total demand level within the interval of [480, 580] pallets.

With the firms' classes defined above, three configuration of firms' networks will be investigated: (1) a network containing only large sized firms (LN), (2) network including only small sized firms (SN) and network comprising both large and small sized firms (HN). Furthermore, in order to capture different cost structures, two levels of fixed and variable costs were considered. As provided in **Table 2**, for each level, ordering, inventory holding and purchasing costs are varied. Note that the values for low costs attribute are fine-tuned based on the work of Elomri et al. (2012) for the inventory holding component, and inspired from the work of Krichen et al. (2011) for the purchasing structure. These values are augmented by factor 2 to obtain the corresponding values for high costs attribute. Note also that the ordering costs component was calibrated based on an average trucks fulfilment in pallets such as the shipments frequency on a yearly basis is about $N \in [6, 10[$ for SF and $N \in [12, 20[$ for LF. Finally, the product family average price is fixed to 60$ for all firms.

Regarding the transportation costs, the fixed and variables charges are estimated in **Table 3** for the STL and MTL transportation modes. These are estimated by regression with historical data in Klibi et al. (2010b) under the assumption that a 400-miles distance limit is defined between supplier's and firms' depots.

	Low costs	*High costs*
Ordering cost	$A = 10,$; $c_c = 10$	$A = 100$; $c_c = 20$
Holding cost	$h \in [15, 25]$; $h_s = 35$	$h \in [30, 50]$; $h_s = 70$
Purchasing cost	$e = 0.01$; $c = 15$; c_{min}=12	$e = 0.02$; $c = 30$; c_{min}=24

Table 2- Input Problems Cost Structure

	c_a^k	c_d^k	c_v^k
k=1 (STL)	60	0.15	0.375
k=2 (MTL)	60	0.3	0.75

Table 3- Parameters for the Transportation Structure

Each problem instance is denoted by the triplet $(\boldsymbol{n}, \boldsymbol{x}, \boldsymbol{y})$ where $\boldsymbol{n}$ refers to the market size, $\boldsymbol{x}$ to the network configuration and $\boldsymbol{y}$ to the costs structure. It is defined as follows:

$$(n, x, y): n \in \{5,10,15,20,25,30\}; x \in \{SN, LN, HN\}; y \in \{LC, HC\}$$

Recall that each of these 36 instances $(\boldsymbol{n}, \boldsymbol{x}, \boldsymbol{y})$ must be solved for the SA situation and threefold for the cooperative situations (CODS, JRSW, JRSS). This combination yields 144 problems to solve with the CRA. All the computational experiments are performed in Java language on a 32 bits computer with Pentium (R) CPU, 2.13 GHz, and 4GB of RAM. The next section presents the numerical results of the tested instances and discussed their managerial insights.

VI.2. **Numerical Results**

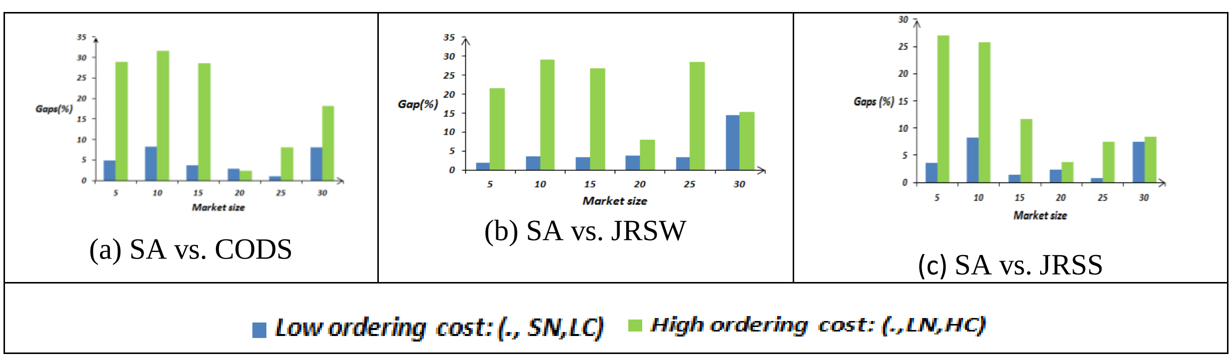
Given the 36 problem instances specified previously, this section inspects for each cooperative situation if the collaboration is a promising support to further increase the profits of firms involved. In addition, based on the profits realised by the firms in the coalitions formed, we compare the individual profits before and after cooperating in order to discuss the efficiency of the incentives offered by a given cooperative situation. Note that this section is focusing on the analyses of all the instances with similar companies' networks either with large sized firms (*LN*) or with small sized firms (*SN*). The instances considering hybrid network attribute are discussed in the subsequent section.

Analysis of the profit allocation

First, to measure the degree of cooperation between firms, we compare the gap in terms of profit realized between the stand-alone case and the cooperative one for each replenishment situation $l = 1, 2, 3$ for high. This latter is based on the sum of the profit of all the coalitions formed in that market. For a given replenishment situation l, Gap^l is computed as follows:

$$Gap^l = \sum_{s \in \Psi^*} \left[\left(\hat{P}_s^l(.) - \sum_{i \in I_s} \hat{P}_i^0(.) \right) \Big/ \sum_{i \in I_s} \hat{P}_i^0(.) \right]$$

Figure 6 illustrates such gaps for the two following instances with opposite attributes: $(., SN, LC)$ and $(., LN, HC)$ where high and low ordering costs values are compared on each market size. It shows separately in graphics a), b) and c) the difference between the SA and CODS, JRSW and JRSS, respectively. One can observe from **Figure 6** that all gap values are positive and that in general they are higher when the ordering fixed cost is high with the three cooperative situations (except with instance of size 20 for the JRSW). This underlines that when the fixed ordering costs are high, firms are more encouraged to collaborate in order to reduce this cost component. Especially under JRSW and JRSS situations, the gap with the SA is much more marked. We clearly deduce that the degree of collaboration is increasingly proportional to the fixed ordering costs A. The detailed gap values are given in **Appendix B** for various instances with ordering costs A values ranging between 10 and 100.



(a) SA vs. CODS

(b) SA vs. JRSW

(c) SA vs. JRSS

Figure 6- Gaps Comparison between SA and Cooperative Situations

Analysis of the Cardinality of the stable coalition structure

The impact of collaboration is also measured in terms of the number of coalitions formed within the network: the more this number decreases the more the degree of collaboration is high. Ideally the grand coalition is formed with all the firms present in the network. **Table 4** reports the number of stable coalitions within the structure belonging to set ψ^* when the market size increases, and using instances with attributes $(.SN, LC)$ and $(., LN, HC)$. This table expresses that the cardinality of the coalition structure for the three situations is, for all instances, less than the number of firms. This well explains the cooperative tendency of the proposed scenarios as they reflect various behavioral cooperative protocols. Indeed, it is noticeable that for 34 out of 36 of the instances the number of firms within the coalition increases with the problem size (see **Table 4, Figure 1** and **Figure2** in **Appendix B**). However, the cardinality of the coalition structure increases proportionally to the market size, which probably underlines the limits of collaboration opportunities in this context. This behavior appears to be quite similar in all the replenishment situations. In this table, are highlighted the lowest number of entities observed in the stable coalition structure. For instance, the last raw of **Table 4** provides the average proportion of distinct entities in the stable coalition structure for the CODS, JRSW and JRSS situations in both instances inspected. These results confirm the good incentives toward collaboration, regarding inventory pooling, proposed by JRSW situation. Hence, we notice that the proportion of the stable coalition's size is better with the high costs attributes which explain the efficiency of collaboration in terms of cost savings. Besides, the obtained results show that collaboration has a significant performance for the three cooperative situations.

	Cardinality of the coalition structure					
	$(., SN, LC)$			$(., LN, HC)$		
n	*CODS*	*JRSW*	*JRSS*	*CODS*	*JRSW*	*JRSS*
5	1	1	1	1	1	1
10	3	1	5	3	1	3
15	9	6	6	8	5	6
20	14	10	11	12	9	9
25	18	14	15	16	11	11
30	20	18	19	18	15	17
Average coalition formation (in %)	53.1%	**39.3%**	48.1%	47.9%	**33.7%**	39.3%

Table 4- Stable Coalition's Size in SA and in Cooperative Situations

In complement, **Table 5** reports the number of firms involved in each coalition according to each replenishment situation. For instance, when $n = 10$, the CODS situation generates a coalition structure composed of three coalitions: the first coalition involves 8 firms, and the remaining firms operate individually. This gives the following structure:

Coalition 1: 8 firms /*Coalition 2*: 1 firm /*Coalition 3*: 1 firm.

n	*CODS*	*JRSW*	*JRSS*
5	(5)	(5)	(5)
10	(8,1,1)	(10)	(6,1,1,1,1)
15	(7,1,1,1,1,1,1,1,1)	(10,1,1,1,1,1)	(10,1,1,1,1,1)
20	(7,1,1,1,1 ... 1)	(11,1,1,1,1 ... 1)	(10,1,1,1,1 ... 1)
25	(8,1,1,1, ...1)	(12,1,1,1, ...1)	(11,1,1,1, ...1)
30	(11, 1,1, ... ,1)	(13, 1,1, ... ,1,1)	(12, 1,1, ... ,1)

Table 5- Number of Firms in SA and Cooperative Situations for $(.,SN,LC)$ Instances

As reported in **Table 5**, for each instance, the CRA always generates a stable coalition structure that splits the whole set of firms into: one coalition that encompasses all cooperative firms and a set of singletons indicating the stand-alone behavior of the remaining ones. We also notice that, in spite of the existence of collaboration, the stand-alone position is dominant in the generated coalitions with respect to the number of stand-alone firms. The grand coalition is obtained only when $n = 5$ for all replenishment situations which is probably due to the cost of coalition impacted. This is congruent with real world cases reported in **Table 1** that point out the managerial and costs difficulties to go beyond larger number of firms within a coalition. A unique case of grand coalition was observed when $n = 10$ with the JRSW situation which could be explained by the importance of the inventory pooling incentives when small firms are involved. Under the CODS situation, the number of firms being in stand-alone is more important in comparison to JRSW and JRSS, since the former offer only cooperation on the ordering costs. These results underline the importance of incentives to cooperate in warehousing and transportation within the replenishment process.

Performance Analysis in terms of degree of collaboration

All collaborative situations incur an ordering cost A_s expressed in terms of the fixed ordering cost A and the communication cost c_c, as reported in equation (11). Hence, for a fixed value of A, the coalition formation is proportional to c_c. We conducted a sensitivity analysis on the effect of the c_c by solving the CRA with various c_c values, in order to point out key threshold values $Th(c_c)$ that enhance firms cooperation. **Figure 7** illustrates such analysis for the problem instance with attributes $(10, LN, HC)$ and regarding all replenishment situations. In this case, starting with a c_c=0 and solving iteratively the CRA, a coalition formation is observed until $Th(c_c)$ reaches the values of $30A$, $35A$ and $43A$ for CODS, JRSS and JRSW situations, respectively. These values are very useful to determine whether a coalition formation is possible or not, and could be of key importance for firms in order to establish the adequate communication cost c_c without altering the cooperation opportunity. In addition, this figure underlines also that the threshold value for the CODS is lower than the other situations. This is explained by the dependency of this collaborative situation to the ordering costs whereas the two others beneficiates from addition incentives in the transportation and the warehousing. Similar analysis for problem instance with low ordering costs attributes (i.e. (10, LN, LC)) provides $Th(c_c)$ values of $15A$, $17A$ and $21A$, for CODS, JRSS and JRSW situations, respectively. This is congruent with the previous results due to the huge difference between low costs and high costs structure used in these instances.

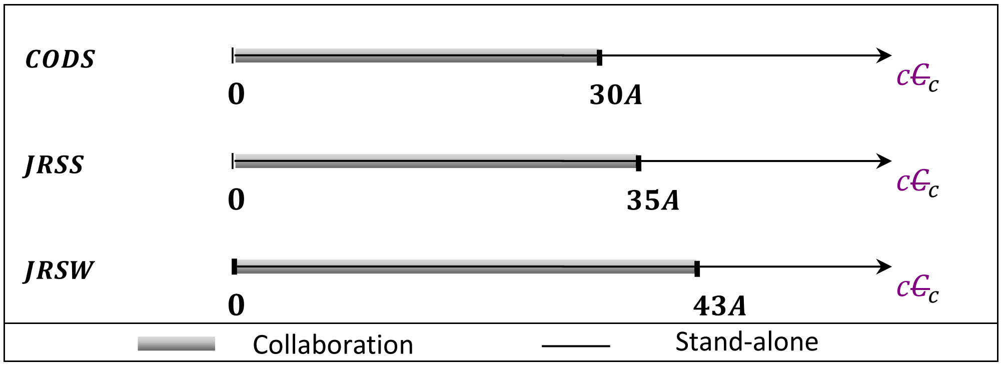


Figure 7- Extreme Values for Cooperation for instance (10,LN,HC)

Performance analysis in terms of the cost savings

As mentioned, the anticipated total operational costs incurred by a firm are composed by the ordering costs (eq. 5), the inventory holding costs (eq. 6) and the transportation costs (eq. 4). In the subsequent analysis we computed the proportion of each of these cost components in the total operational costs in order to inspect their relevance in the coalition formation decision-making process.

As CODS situation is committed with the design of a joint ordering between firms, we drive an analysis on the improvement of the ordering cost in the stand alone situation and this collaborative one. We report in **Table 6**, the proportion of the ordering cost regarding the total operational cost based on the results of $(.,SN,,LC)$ instances attribute. The empirical results show that the ordering costs proportion when firms collaborate ranges ascendingly between 62.32% to 75.43% for n=5 to n=30, respectively. This give rise to an average gap of 11% improvement compared to the stand-alone situation. This results well explains the incentive of firms to collaborate under ordering cost saving. Joint ordering computations with a high market costs is given in **Table 5** in **Appendix B**.

$\boldsymbol{n}$	$\boldsymbol{Stand-alone}$	$\boldsymbol{Collaboration}$
5	68.94%	62.32%
10	85.39%	72.13%
15	85.73%	73.21%
20	86.24%	73.58%
25	87.12%	74.47%
30	87.61%	75.43%
$\boldsymbol{Average}$	**83.50%**	**71.85%**

Table 6- Proportion of the Ordering Cost in the Total Cost Function

Regarding the purchasing cost, the selection of the unitary purchasing landings (see eq. (1)), for each firm is a key decision in the cost savings pursuit. One can note from **Figure 8** the benefit from collaboration where all the firms move from the first landing in the SA to the second or the third landing in CODS. With $(.,SN,,LC)$ instances attribute, all firms in collaboration beneficiate from the second landing in 4 out of 6 instances and from the third landing for the 2 remaining ones. Alternatively, with $(.,LN,,HC)$ instances attribute, the unitary purchasing cost for the collaborative firms moves in all the cases to the third landing.

From the JRSW standpoint, the inventory holding cost is the most relevant component of the total operational costs. We explore in what follows its impact on the collaboration degree. **Table 7** shows that the proportion of the inventory holding cost when each firm operates individually (second column of **Table 7**) is, for all problem instances, greater than its equivalent in the collaborative framework (third column of **Table 7**). In average, such proportion moves from 15.39% for stand-alone position to 11.33% for the collaborative situation. Such decrease is due to the sharing of warehouses once the cooperation is launched which well explains the efficiency of using collaborative warehouses in the cost saving. In addition, we notice that the proportion of the holding cost increases with the market size which indicates that the number of collaborative firms increases. Finally, we should mention that the same behavior is observed for instances $(.,\boldsymbol{LN},\boldsymbol{HC})$ (see **Table 6** in **Appendix B**).

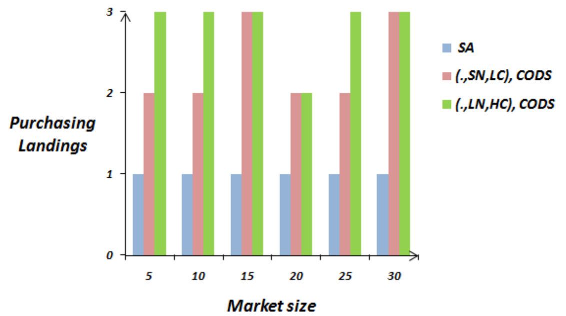


Figure 8- Optimal Purchasing Cost Landing in SA and CODS

n	$Stand - alone$	$Collaboration$
5	14.21%	10.11%
10	15.4%	10.42%
15	15.7%	11.34%
20	15.07%	11.26%
25	15.87%	12.33%
30	16.13%	12.39%
$\boldsymbol{Average}$	**15.39%**	**11.33%**

Table 7- Proportion of the Holding Cost in the Total Cost Function

Since for the JRSS situation the transportation cost is the most dominant component, we compute, in **Table 8**, its influence on the firms' collaboration. We also report the proportion of the transportation cost when each firm takes the stand-alone position versus the cooperation under a low market assumption. We can underline that the transportation cost is proportional to the number of firms. It is also noticeable that the transportation cost decreases from 19.23% to 11.03% in average. This cost drop is due to the reduction of the number of used STL, as collaborative firms group their orders in common MTL. Hence, the collaboration concerns firms' distribution activities by sharing logistics resources. Besides, the traveled distance can be minimized regarding the ship to points in the stand-alone situation. The impact of the joint replenishment in the reduction of the transportation costs is also observed with the high market costs (see **Table 6** in **Appendix B**).

n	$\boldsymbol{Stand - alone}$	$\boldsymbol{Collaboration}$
5	16.85%	9.24%
10	19.72%	10.82%
15	19.74%	10.99%
20	19.01%	11.56%
25	19.96%	11.76%
30	20.15%	11.83%
$\boldsymbol{Average}$	**19.23%**	**11.03%**

Table 8- Proportion of the Transportation Cost in the Total Cost Function

Performance Analysis in terms of the convergence of the CRA

Due to its NP-hardness, proven in Dung (2004), we proposed in section V.2 an efficient CRA exact method that converges to core-stable coalition structures in a reasonable computation time for realistic size instances (see **Table 4**). **Table 9** reports the performance of the stability condition (discussed in section V.1) in terms of the number of addressed coalitions that enhances considerably the solution time of the CRA. For instance, with $n = 10$, we notice that thinks to the stability conditions introduced 84% of non-profitable coalitions that are discarded by the CRA beforehand. Similarly, for the JRSW and JRSS the percentages of removed coalitions are respectively 75% and 80%. As the CRA reduces considerably the number of checks, we notice that the CPU time, for all 36 instances, is subsequently minimized regarding their resolution when a complete enumeration is applied. **Figure 9** illustrates this huge gap in terms of solution time between our CRA method and a classical enumeration-based algorithm (denoted EA). Hence, despite the huge number of alternatives which is about $2^n - 1$ (Dung, 2004), we observed that CRA is able to solve the whole benchmarking set in a computation time close to 630 seconds. Supplementary results are provided in **Appendix B**.

	Discarded coalitions (in %)		
n	$CODS$	$JRSW$	$JRSS$
5	25%	25%	50%
10	84%	75%	80%
15	80.8%	80.3%	93.4%
20	85.02%	85.02%	95.52%
25	87.04%	98.5%	98.5%
30	87.3%	90.17%	90.17%

Table 9- Performance Results of the CRA

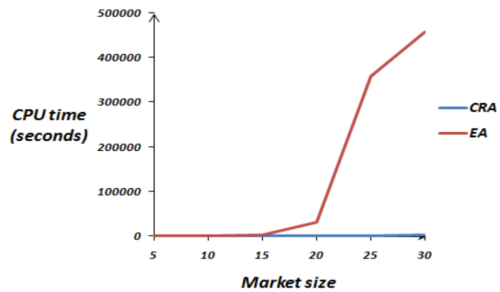


Figure 9- CPU Time per Market Size

VI.3. Analysis of the Case of Hybrid Networks

In this subsection, the analysis is focusing on the results obtained for instances where hybrid network is considered (i.e. composed by SF and LF simultaneously). Here, we compared the gap stand-alone versus cooperation and the cardinality of the stable coalitions generated when considering low and high market costs, as reported in **Table 10**. When making small and large firms collaborate, we observe that all gap values are positive which underlines the profitability of the collaboration also in case of hybrid network (see **Table 10**). We also notice that the gap values are more much important when the ordering fixed cost is high which reflect the efficiency of collaboration in term of cost saving. This is also deduced when observing the coalition size and comparing instances with attributes $(.,.,LC)$ and$(.,.,HC)$. For the six instances of the market size, it is noticeable that for 3 out of 6 instances the gap values within the small network configuration exceeds there equivalent in the hybrid network. In addition, we note that the gap value within the large network is dominant regarding the hybrid network (except for $n = 20$ and$n = 25$). This well explains that the collaboration is in general more pronounced when firms with the same demand profile are involved, rather than the matching of hybrid profile.

n	Gap		(number SF ; number LF)	(.,.,LC), CODS		(.,.,HC), CODS	
	SN	LN		Gap	$\|Coal.\|$	Gap	$\|Coal.\|$
5	4.82%	**28.92**%	(2; 3)	**6.78**%	4	26.02%	4
10	**8.18**%	**31.55**%	(5; 5)	7.96%	6	30.64%	6
15	**8.63**%	**28.65**%	(7; 8)	6.57%	6	24.07%	10
20	2.87%	6.37%	(10; 10)	**5.33**%	5	**16.33**%	6
25	0.96%	7.99%	(12; 13)	**4.60**%	7	**19.77**%	9
30	**8.10**%	**18.17**%	(15; 15)	2.96%	7	15.09%	8

Table 10- Gap Results in Hybrid Network under CODS

VII. Conclusions

We developed in this paper a game theoretic approach for a single supplier-multiple collaborative firms that act jointly to maximize their profits. Three collaborative situations, corresponding to realistic contexts, are considered in order to illustrate the importance of incentives offered in the replenishment process: CODS focusing on the joint ordering process, JRSW proposing the idea of sharing warehousing activities, and JRSS adding the collaborative transportation opportunity. The coalition formation problem investigated in this paper is difficult to solve, thus an efficient CRA exact method is developed. To reach a cooperative decision that fulfills all firms' requirements, the core-stability concept is used in the computation of coalition structures within a cooperative replenishment algorithm that outputs appropriate profits stability. The effectiveness of the CRA in generating stable coalitions is validated empirically by addressing several problem sizes.

The experimental study highlighted the importance of horizontal collaboration in the firms' pursuit of profit growth. It also underlined the broad range of cooperation opportunities in the supply chain, not only on the ordering process as always suggested. These experiments confirmed that it presence of quantity discount contract, joint ordering decisions are clearly profitable, mainly for firms with small sized demand level. Also, the obtained results showed that, essentially in a global context, using shared warehousing in the replenishment process, enhances the firms profit compared to the stand-alone situation. This emerging collaboration with shared warehouses could be one of the solutions to hedge lead-time and demand uncertainties in case of global sourcing. The investigation of pooling transportation as a collaborative situation revealed also that important costs savings could be realized thanks to the traveled routes reduction. This could also be interpreted that horizontal collaboration enhances the ecologic performance of firms in such coalition. Furthermore, our investigation illustrated that firms tend to further collaborate when high ordering, inventory holding and transportation costs are characterizing the business context and thus, the size of the coalitions formed are larger. Another behavior observed is that the collaboration is in general more pronounced when firms with the same demand profile are involved, rather than the matching of hybrid profile (i.e. firms with small sized demand and large sized demand). Finally, using a volume-based profit sharing mechanism, we observed that the coalition stability produced in all the instances: one coalition as large as possible; and several singletons of firms working in a standalone setting.

This paper assumed the business case of one supplier based sourcing policy and stationary customers demand process. Further works should address the multiple supplier-multiple firms' business context where multiple sourcing strategies are imposed. Various stationary and nonstationary demand processes should also be investigated in order to study their impact on the firm's collaboration decisions. In addition, a more detailed anticipation of the inventory-routing problems characteristics could be integrated in the collaborative decisional process. Finally, when all these features are added, the solvability of the coalition formation problem will be challenged, and thus, the development of meta-heuristics could be a promising research avenue in order to generate near-stable coalition structures in a reasonable running time.

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Appendix A: Proofs of the stability conditions

This appendix details the stability conditions discussed in **Section V** for all replenishment situations:

- ***CODS Case :***

When $Q_{\min} \leq \hat{Q}_s < Q_{\max}$, the decision of a firm i to join or not a coalition depends on the following inequality:

$$\hat{P}_i^0 < \hat{P}_{i,s}^1$$

$$\hat{P}_i^0 < \frac{\hat{P}_S^1 \hat{Q}_{i,S}}{\hat{Q}_S}$$

$$R_i^0 - OC_i^0 - HC_i^0 - TC_i^0 - (c - e\hat{Q}_i)\hat{D}_i < (\sum_{i \in I_S} p_i \hat{D}_i - OC_s^1 + HC_s^1 + C_s^1 + (c - e\hat{Q}_s)\hat{D}_s)\frac{\hat{Q}_{i,s}}{\hat{Q}_s}$$

$$p_i \hat{D}_i - A\frac{\hat{D}_i}{\hat{Q}_i} - h_i \frac{\hat{Q}_i}{2} + (c_a + c_d d_{ji} + c_v \hat{Q}_i)\frac{\hat{D}_i}{\hat{Q}_i} - (c - e\hat{Q}_i)\hat{D}_i < (\sum_{i \in I_S} p_i \hat{D}_i - A_s \frac{\hat{D}_s}{\hat{Q}_s} - \sum_{i \in I_S} h_i \frac{\hat{Q}_i}{2} + ((\sum_{i \in I_S}(c_a + c_d d_{ji})) + c_v \hat{Q}_s)\frac{\hat{D}_s}{\hat{Q}_s} - (c - e\hat{Q}_s)\hat{D}_s))\frac{\hat{Q}_{i,s}}{\hat{Q}_s}$$

$$A\frac{\hat{D}_i}{\hat{Q}_i} - A_s \frac{\hat{D}_s \hat{Q}_{i,s}}{(\hat{Q}_S)^2} > p_i \hat{D}_i - h_i \frac{\hat{Q}_i}{2} - (c_a + c_d d_{ji} + c_v \hat{Q}_i)\frac{\hat{D}_i}{\hat{Q}_i} - (c - e\hat{Q}_i)\hat{D}_i - (\sum_{i \in I_S} p_i \hat{D}_i)\frac{\hat{Q}_{i,S}}{\hat{Q}_S} + (\sum_{i \in I_S} h_i \frac{\hat{Q}_i}{2})\frac{\hat{Q}_{i,S}}{\hat{Q}_S} + ((\sum_{i \in I_S}(c_a + c_d d_{ji})) + c_v \hat{Q}_s)\frac{\hat{D}_s \hat{Q}_{i,s}}{(\hat{Q}_S)^2} + (c - e\hat{Q}_s)\frac{\hat{D}_s \hat{Q}_{i,s}}{\hat{Q}_s}$$

$$A\frac{\hat{D}_i}{\hat{Q}_i} - A_s \frac{\hat{D}_s \hat{Q}_{i,s}}{(\hat{Q}_S)^2} > \chi^1$$

$$A\frac{\hat{D}_i}{\hat{Q}_i} - (A + c_c |s|)\frac{\hat{D}_s \hat{Q}_{i,s}}{(\hat{Q}_S)^2} > \chi^1$$

$$c_c |s| < A(\frac{(\hat{Q}_S)^2 \hat{D}_i}{\hat{D}_s \hat{Q}_{i,s} \hat{Q}_i} - 1) - \chi^1 \frac{(\hat{Q}_S)^2}{\hat{D}_s \hat{Q}_{i,s}}$$

If $\hat{Q}_s \leq Q_{\max}$ the parameter χ^1 is given by:

$$\chi^1 = p_i \hat{D}_i - h_i \frac{\hat{Q}_i}{2} - (c_a + c_d d_{ji} + c_v \hat{Q}_i)\frac{\hat{D}_i}{\hat{Q}_i} - (c - e\hat{Q}_i)\hat{D}_i - (\sum_{i\in I_S} p_i \hat{D}_i)\frac{\hat{Q}_{i,S}}{\hat{Q}_S} + (\sum_{i\in I_S} h_i \frac{\hat{Q}_i}{2})\frac{\hat{Q}_{i,S}}{\hat{Q}_S} + ((\sum_{i\in I_S}(c_a + c_d d_{ji})) + c_v \hat{Q}_s)\frac{\hat{D}_s \hat{Q}_{i,s}}{(\hat{Q}_S)^2} + c_{\min}\frac{\hat{D}_s \hat{Q}_{i,s}}{\hat{Q}_s}$$

With $Q_{\min} < \hat{Q}_s$, the parameter χ^1 will be:

$$\chi^1 = p_i \hat{D}_i - h_i \frac{\hat{Q}_i}{2} - (c_a + c_d d_{ji} + c_v \hat{Q}_i)\frac{\hat{D}_i}{\hat{Q}_i} - (c - e\hat{Q}_i)\hat{D}_i - (\sum_{i\in I_S} p_i \hat{D}_i)\frac{\hat{Q}_{i,S}}{\hat{Q}_S} + (\sum_{i\in I_S} h_i \frac{\hat{Q}_i}{2})\frac{\hat{Q}_{i,S}}{\hat{Q}_S} + ((\sum_{i\in I_S}(c_a + c_d d_{ji})) + c_v \hat{Q}_s)\frac{\hat{D}_s \hat{Q}_{i,s}}{(\hat{Q}_S)^2} + c\frac{\hat{D}_s \hat{Q}_{i,s}}{\hat{Q}_s}$$

- ***JRSW Case :***

When $Q_{\min} \leq \hat{Q}_s < Q_{\max}$, the condition stability for each firm i is given according to the following parameter χ^2 :

$$\chi^2 = A\frac{\hat{D}_i}{\hat{Q}_i} + h_i \frac{\hat{Q}_i}{2} + (c_a + c_d d_{ji} + c_v \hat{Q}_i)\frac{\hat{D}_i}{\hat{Q}_i} + (c - e\hat{Q}_i)\hat{D}_i - A_s \frac{\hat{D}_s \hat{Q}_{i,s}}{(\hat{Q}_s)^2} - ((\sum_{i\in I_S}(c_a + c_d d_{ji})) + c_v \hat{Q}_s)\frac{\hat{D}_s \hat{Q}_{i,s}}{(\hat{Q}_s)^2}$$

If $\hat{Q}_s \leq Q_{\max}$, the parameter χ^2 adopted to determine the core stability is expressed as:

$$\chi^2 = A\frac{\hat{D}_i}{\hat{Q}_i} + h_i \frac{\hat{Q}_i}{2} + (c_a + c_d d_{ji} + c_v \hat{Q}_i)\frac{\hat{D}_i}{\hat{Q}_i} + c_{\min}\hat{D}_i - A_s \frac{\hat{D}_s \hat{Q}_{i,s}}{(\hat{Q}_s)^2} - ((\sum_{i\in I_S}(c_a + c_d d_{ji})) + c_v \hat{Q}_s)\frac{\hat{D}_s \hat{Q}_{i,s}}{(\hat{Q}_s)^2} - c_{\min}\frac{\hat{D}_s \hat{Q}_{i,s}}{\hat{Q}_s}$$

With $Q_{\min} < \hat{Q}_s$, the parameter χ^2 will be:

$$\chi^2 = A\frac{\hat{D}_i}{\hat{Q}_i} + h_i\frac{\hat{Q}_i}{2} + (c_a + c_d + d_{ji} + c_v\hat{Q}_i)\frac{\hat{D}_i}{\hat{Q}_i} + c\hat{D}_i - A_s\frac{\hat{D}_s\hat{Q}_{i,s}}{(\hat{Q}_s)^2} - ((\sum_{i\in I_S}(c_a + c_d d_{ji})) + c_v\hat{Q}_s)\frac{\hat{D}_s\hat{Q}_{i,s}}{(\hat{Q}_s)^2} - c\frac{\hat{D}_s\hat{Q}_{i,s}}{\hat{Q}_s}$$

- ***JRSW Case :***

When $Q_{\min} \le \hat{Q}_s < Q_{\max}$, the condition stability for each firm i is given according to the following parameter χ^3 :

$$\chi^3 = A\frac{\hat{D}_i}{\hat{Q}_i} + h_i\frac{\hat{Q}_i}{2} + (c_a + c_v\hat{Q}_i)\frac{\hat{D}_i}{\hat{Q}_i} + (c - e\hat{Q}_i)\hat{D}_i - A_s\frac{\hat{D}_s\hat{Q}_{i,s}}{(\hat{Q}_s)^2} - (\sum_{i\in I_S}h_i)\frac{\hat{Q}_{i,S}}{2} - (c_a\beta + c_v\hat{Q}_s)\frac{\hat{D}_s\hat{Q}_{i,s}}{(\hat{Q}_s)^2} \quad -(c - e\hat{Q}_s)\frac{\hat{D}_s\hat{Q}_{i,s}}{\hat{Q}_s}$$

If $Q_s \le Q_{\max}$, the expression of the parameter χ^3 will be:

$$\chi^3 = A\frac{\hat{D}_i}{\hat{Q}_i} + h_i\frac{\hat{Q}_i}{2} + (c_a + c_v\hat{Q}_i)\frac{\hat{D}_i}{\hat{Q}_i} + c_{\min}\hat{D}_i - A_s\frac{\hat{D}_s\hat{Q}_{i,s}}{(\hat{Q}_s)^2} - (\sum_{i\in I_S}h_i)\frac{\hat{Q}_{i,S}}{2} - (c_a\beta + c_v\hat{Q}_s)\frac{\hat{D}_s\hat{Q}_{i,s}}{(\hat{Q}_s)^2} - c_{\min}\frac{\hat{D}_s\hat{Q}_{i,s}}{\hat{Q}_s}$$

When $Q_{\min} < Q_s$, χ^3 becomes:

$$\chi^3 = A\frac{\hat{D}_i}{\hat{Q}_i} + h_i\frac{\hat{Q}_i}{2} + (c_a + c_v\hat{Q}_i)\frac{\hat{D}_i}{\hat{Q}_i} + c\hat{D}_i - A_s\frac{\hat{D}_s\hat{Q}_{i,s}}{(\hat{Q}_s)^2} - (\sum_{i\in I_S}h_i)\frac{\hat{Q}_{i,S}}{2} - (c_a\beta + c_v\hat{Q}_s)\frac{\hat{D}_s\hat{Q}_{i,s}}{(\hat{Q}_s)^2} - c\frac{\hat{D}_s\hat{Q}_{i,s}}{\hat{Q}_s}$$

Appendix B: Supplementary Results

We present in what follow complementary results of the 36 problem instances considered in this paper to assess the performance of our proposed approach in solving the coalition formation problem. The gap comparison between stand-alone and collaborative profits, designed in Figure 6 of Section VI.2, is based on the three following tables, respectively according to the CODS, JRSW and JRSS situations. Here, further ordering cost values in addition to 10 and 100 were tested to appreciate the impact of this latter of the profit.

	$(.,SN,LC)$						$(.,LN,HC)$					
	$A=10$		$A=30$		$A=50$		$A=60$		$A=80$		$A=100$	
n	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$
5	54185.90	**56800.07**	53421.425	**60058.41**	56812.33	**61489.82**	67806.10	**81990.23**	64055.04	**78319.95**	62296.81	**80314.82**
10	61487.90	**66522.06**	70314.15	**77403.68**	70181.05	**78865.34**	130443.79	**157542.18**	126273.82	**155533.76**	124493.12	**163779.3**
15	165892.35	**171929.97**	163340.95	**170368.09**	161896.74	**169512.7**	197795.62	**220481.19**	188996.48	**213033.83**	191454.76	**246310.11**
20	217433.71	**223675.58**	227994.09	**235266.36**	211269.45	**222724.37**	261135.85	**292858.06**	251412.13	**282551.81**	248390.75	**264223.41**
25	284244.92	**286982.98**	265166.03	**272761.2**	281092.35	**289288.19**	327866.72	**350355.97**	324141.23	**357296.4**	319335.77	**344875.78**
30	290080.10	**313582.42**	278471.68	**306060.25**	282481.82	**352012.41**	390080.10	**413582.42**	378471.68	**406060.25**	382481.82	**452012.41**

Table 1- Results of the SA profit and CODS cooperative profit (*Million* $)

	$(.,SN,LC)$						$(.,LN,HC)$					
	$A=10$		$A=30$		$A=50$		$A=60$		$A=80$		$A=100$	
n	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$
5	54185.90	**55234.6**	53421.425	**57510.08**	56812.33	**59051.39**	67806.10	**77339.93**	64055.04	**72974.97**	62296.81	**75613.59**
10	61487.9	**63630.13**	70314.15	**74881.73**	70181.05	**77084.71**	130443.79	**161002.65**	126273.82	**156880.08**	124493.12	**160484.37**
15	165892.35	**171387.46**	163340.95	**168639.41**	161896.74	**172518.91**	197795.62	**231115.22**	188996.48	**236792.44**	191454.76	**242522.66**
20	217433.71	**225806.41**	227994.09	**238792.07**	211269.45	**224729.86**	261135.85	**301001.16**	251412.13	**314808.29**	248390.75	**267970.18**
25	284244.92	**293577.98**	265166.03	**279042.68**	281092.35	**296612.85**	327866.72	**396533.58**	324141.23	**371447.14**	319335.77	**409974.49**
30	290080.10	**332018.73**	278471.68	**328485.88**	282481.82	**340708.45**	390080.10	**432018.73**	378471.68	**428485.88**	382481.82	**440708.45**

Table 2- Results of the SA profit and JRSW cooperative profit (*Million* $)

	$., SN, LC)$						$(., LN, HC)$					
	$A = 10$		$A = 30$		$A = 50$		$A = 60$		$A = 80$		$A = 100$	
n	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$	$SA.$	$COOP.$
5	54185.90	**56135.00**	53421.425	**59388.18**	56812.33	**60738.14**	67806.10	**80617.14**	64055.04	**77027.04**	62296.81	**79131.52**
10	61487.9	**66564.97**	70314.15	**77444.61**	70181.05	**77660.24**	130443.79	**155729.1**	126273.82	**153601.72**	124493.12	**156670.52**
15	165892.35	**168102.86**	163340.95	**169427.24**	161896.74	**168494.16**	197795.62	**218908.23**	188996.48	**211742.32**	191454.76	**213736.41**
20	217433.71	**222672.88**	227994.09	**234209.86**	211269.45	**221280.36**	261135.85	**290776.62**	251412.13	**280398.29**	248390.75	**257489.33**
25	284244.92	**286578.05**	265166.03	**271089.67**	281092.35	**288295.72**	327866.72	**348822.18**	324141.23	**346983.48**	319335.77	**343326.03**
30	290080.10	**311807.17**	278471.68	**394088.52**	282481.82	**314613.18**	390080.10	**411807.17**	378471.68	**404088.52**	382481.82	**414613.18**

Table 3- Results of the SA profit and JRSS Cooperative Profit (*Million* $)

In order to analyze the cardinality of the stable coalition structures, **Table 4** below reports, for all problem instances, the number of cooperative firms within the generated coalition. Based on these results we shaped in subsequent **Figures 1** and **2**, the coalition' size behavior regarding the market's size and costs: low and high costs.

	$(., SN, LC)$			$(., LN, HC)$		
n	*CODS*	*JRSW*	*JRSS*	*CODS*	*JRSW*	*JRSS*
5	5	5	5	5	5	5
10	7	9	4	7	10	7
15	6	9	9	7	10	9
20	6	10	9	8	11	11
25	7	11	10	9	14	14
30	10	12	11	12	15	13

Table 4- Number of Firms within the Coalition

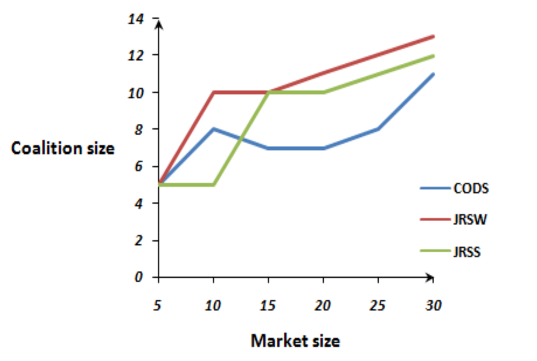

Figure 1- Coalition Size' Behavior With the Market Size for $(.,SN,LC)$ Instances

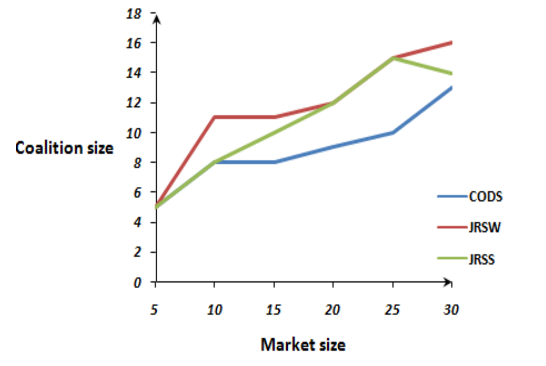

Figure 2- Coalition Size' Behavior With the Market Size for $(.,LN,HC)$ Instances

To detail the analysis of the collaboration performance in terms of cost saving, discussed in **Tables 5, 6** and **7** of **Section VI.2**, the complementary outputs on the proportion of the ordering, holding and transportation costs in the total operational cost under high market costs are given.

n	$Stand-alone$	$Collaboration$
5	68.94%	67.43%
10	85.39%	84.24%
15	85.73%	84.86%
20	86.24%	85.39%
25	87.12%	86.72%
30	87.61%	86.96%
Average	**83.50%**	**82.60%**

Table 5- Proportion of the Ordering Cost in the Total Cost Function with High Market size

n	$Stand-alone$	$Collaboration$
5	14.21%	10.51%
10	15.4%	10.72%
15	15.7%	12.56%
20	15.07%	12.38%
25	15.87%	13.23%
30	16.13%	13.67%
Average	**15.39**%	**12.17**%

Table 6- Proportion of the Holding Cost in the Total Cost Function with High Market Costs

n	$Stand-alone$	$Collaboration$
5	16.85%	9.84%
10	19.72%	11.22%
15	19.74%	11.89%
20	19.01%	11.72%
25	19.96%	12.14%
30	20.15%	12.61%
Average	**19.23**%	**11.57**%

Table 7- Proportion of the Transportation Cost in the Total Cost Function with High Market Costs

Table 8 detail the computation of the CPU time used to analyze the performance of the proposed CRA in terms of resolution efficiency. Such computations are illustrated by **Figure 9** in **Section VI.2**.

n	CRA	EA
5	17.1	77.2
10	92.03	832.32
15	20.81	2673.29
20	479.02	30712.42
25	422.32	357344.22
30	2752.24	457121.27
Average	**630.58**	**141460.12**

Table 8- The Impact of CRA on the CPU Time per Market Size (in seconds)