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Wrong-way risk of interest-rate instruments

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Abstract: Wrong-way risk arises when the value of a financial transaction is adversely correlated with the creditworthiness of the counterparty. This paper investigates wrong-way-risk effects on the pricing of counterparty credit risk for interest-rate instruments. These effects are captured via the correlations between the default of the counterparty and the two relevant market risk factors, namely, the level and the volatility of the instantaneous spot interest rate. Our empirical findings show that the wrong-way effect induced by the dependence between the interest-rate volatility and the default intensity is generally small, even for volatility-sensitive derivatives. However, a dependence between the interest-rate level and the default intensity has a sizeable impact on counterparty risk and gap risk.

Keywords: Counterparty credit risk, wrong-way risk, gap risk, interest-rate volatility, dependence, interest-rate derivatives

Résumé : Dans le cadre du calcul du risque de contrepartie, le risque de corrélation réfère à une situation où la valeur d'une transaction financière est corrélée avec la solvabilité de la contrepartie. Cet article analyse l'impact du risque de corrélation sur le prix du risque de contrepartie des instruments dérivés de taux d'intérêt. Cet impact est mesuré en supposant une corrélation entre l'intensité de défaut de la contrepartie et deux facteurs de risque pertinents, soit le niveau et la volatilité du taux spot instantané. Nos résultats empiriques montrent que l'impact d'une dépendence entre la volatilité du taux d'intérêt et l'intensité de défaut est en général mineur, même pour les dérivés dont la valeur dépend de la volatilité. Par ailleurs, une dépendence entre le niveau du taux d'intérêt et l'intensité de défaut a un impact significatif sur le risque de contrepartie.

Mots clés : Risque de contrepartie, intensité de défaut, risque de corrélation, volatilité du taux d'intérêt, dépendance, instruments dérivés des taux d'intérêt

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1 Introduction

A *credit valuation adjustment* (CVA hereafter) is the price of counterparty default risk. It is a positive adjustment that is deducted from the default-free value of a derivative position in order to account for the possibility that the counterparty fails to meet its contractual payments. This pricing adjustment has been prescribed by regulators since the second (2006) installment of the Basel Committee on Banking Supervision Accords [2]. The CVA depends both on the market risk factors driving the price of a financial instrument and on the counterparty's default probability. This makes it a complex hybrid product that is generally difficult to price. An additional issue arises when the creditworthiness of the counterparty and the value of the transaction are correlated. The risk resulting from a positive (*resp. negative*) correlation between the credit exposure and the counterparty default event is known as *wrong-way risk* (WWR) (*resp. right-way risk (RWR)*). Wrong-way risk can be particularly significant during periods of systemic credit risk characterized by default contagion. As WWR is the most hazardous manifestation of counterparty credit risk, the third (2011) Basel Accords [3] raised the standards of counterparty risk management by incorporating this additional source of risk in CVA pricing. However, when WWR is included, the additional complexity pertaining to modeling the dependence between the market risk factors and the default probability of the counterparty renders the CVA pricing problem a more challenging task.[1]

Counterparty risk is mainly inherent to the over-the-counter (OTC) derivatives markets. According to the *Bank for International Settlements Statistical Bulletin* [1], the global notional amount outstanding for OTC interest-rate contracts at the end of 2016 was $368 trillion, accounting for 76% of the OTC derivatives markets. Interest-rate instruments, such as swaps, forward agreements and options, thus make up the largest portion of market-counterparty risk worldwide.

While there is a considerable amount of literature dealing with counterparty-credit-risk pricing, this important field of research remains relatively limited when it comes to interest-rate products, despite their very sizeable market. Empirical evidence of the existence of WWR in OTC fixed-income markets is provided in Duffie (1996) [16], where WWR is related to the level of the short-term interest rate since defaults tend to be observed more frequently when interest rates are falling. As far as WWR modeling is concerned, several approaches have been proposed in the context of interest-rate instruments to capture the dependency between market-risk factors and the counterparty's default intensity. A first category of approaches introduces WWR into CVA pricing by assuming a functional relationship between the counterparty's hazard rate and the level of the stochastic interest rate (see, e.g., Gargouri et al. 2017 [19], who test four different functional forms to model this dependency for pricing the CVA of interest-rate swaps in the Hull & White 2012 [26] framework). A second approach relies on the use of copulas (see, e.g., Cherubini 2013 [13] and Černý & Witzany 2015 [12] in the context of interest-rate swaps). In a third approach, dependency between the default intensity and the short-term interest rate is created by instantaneously correlating the Brownian motions driving their respective stochastic processes (see, e.g., Brigo & Pallavacini 2008 [8], Brigo et al. 2009 [9] and Brigo et al. 2013 [6]).

It is worth noting that, although the assumptions concerning the dynamics of the market and default factors, as well as how their dependence is modeled, differ among these three categories of approaches, in all cases correlation is established via the level of the interest rate. This choice of correlation modeling is the standard framework to account for right-way/wrong-way risk in the fixed-income market. In the case of interest-rate swaps, however, such a modeling choice results in a wrong-way effect for payer positions and a right-way effect for receiver positions when the correlation is positive, so that a joint wrong-way behavior for interest-rate-swap positions, which may be relevant during stress periods, is precluded. As first advocated in Gregory (2012) [21], a joint and simultaneous WWR effect for both interest-rate-swap counterparties can be captured by correlating the default intensity of the counterparty with the volatility (rather than the level) of the interest rates. More recently, Harris et al. (2015) [22] examine the effect of correlation between the interest-rate volatility and the default intensity of the counterparty on the expected loss of collateralized interest-rate swaps. Note that, even when an interest-rate swap is fully collateralized, there is the additional

[1] See, e.g., Gatarek & Jablecki (2016), Hofer (2016) and Carr & Ghamami (2017) for recent contributions on CVA pricing under WWR.

risk that the value of the position will move significantly after the counterparty's default but before it can be unwound or replaced, resulting in a potentially significant loss (the so-called *gap risk*), particularly during turbulent periods.[2] Harris et al. (2015) [22] find that the correlation between the interest-rate volatility factor and the default rate has a non-negligible wrong-way effect on gap risk, but do not assess the magnitude of this effect relative to that of the correlation between the interest-rate level and the default intensity.

This paper contributes to the previously described literature by proposing a general setting for pricing the counterparty risk of interest-rate instruments that simultaneously takes into account the presence of two possible WWR effects. These WWR effects are captured by correlating two risk factors—namely, the level and the volatility of the interest rate—with the counterparty's default intensity. In addition to measuring the joint influence of these two factors, such a setting allows us to disentangle their individual effects and assess their relative importance. Our paper provides a broad analysis by covering different types of standard interest-rate instruments, which can be either volatility insensitive (such as swaps) or volatility sensitive (such as caps and floors).

We find that, when considered individually, the wrong-way effect induced by the correlation between the interest-rate volatility and the default intensity is generally small for all interest-rate derivatives examined in this paper—including the volatility-sensitive ones—as well as for gap risk. However, when it comes to the dependence between the interest-rate level and the default intensity, we find that its impact on the CVA is particularly significant, largely dominating the impact of volatility. When both correlations are nonzero, we find that they contribute positively to the resulting combined wrong-way effect, especially when both these effects are wrong-way and the counterparty's credit risk is high. A sensitivity analysis shows the robustness of our findings.

The remainder of this paper proceeds as follows. Section 2 describes the stochastic dynamics chosen for the short-term interest-rate and for the default intensity and describes the CVA pricing model. Section 3 presents the calibration methodology, and Section 4 reports our empirical results. Section 5 covers to the sensitivity analysis of the CVA price with respect to the models' parameters. Section 6 concludes.

2 The counterparty-credit-risk pricing framework

In this section, we present our CVA pricing framework, that is, the description of the interest-rate and the default-intensity dynamics considered in this study, as well as the CVA pricing procedure. We consider a defaultable contract between an investor and a counterparty with an inception date $t = 0$ and maturity T, where τ is the counterparty's default time. We use a multifactor interest-rate model and a reduced-form interpretation of credit risk, where market-risk factors and default intensity are Markovian. In what follows, the notation $\mathbb{E}_t[\cdot]$ represents the expectation, under the risk-neutral measure, conditional to the information available at t.

The approach we take for modeling the dependency between credit exposure and counterparty default consists of instantaneously correlating the Brownian motions driving the market-risk factors and the counterparty's default intensity. This approach for modeling WWR, combined with a multifactor interest-rate model, allows us to capture a wide range of future-exposure scenarios.

2.1 The interest-rate model

We consider the CCG model of Casassus et al. (2005) [11] for the instantaneous spot interest rate, which is a stochastic-volatility extension of the well-known one-factor Vasicek (1977) [32] interest-rate model and of its time-inhomogeneous version proposed by Hull & White (1990) [27]. This model is also a particular case of the general Heath-Jarrow-Morton (Heath et al. 1992 [23]) stochastic-volatility framework proposed in Trolle & Schwartz (2009) [31]. The CCG model features Unspanned Stochastic Volatility (USV) behavior, i.e., it is designed in such a way that the stochastic-volatility factor does not affect bond prices but does affect bond-option prices. In the earlier versions of stochastic-volatility interest-rate models, such as

[2] The effect of gap risk on the collateralized bilateral CVA of interest-rate swaps has also been examined in Brigo et al. (2013) [6] in the case where the hazard rate is independent of market factors.

Fong & Vasicek (1991) [18] and Longstaff & Schwartz (1992) [28], bond prices depend on both the level and the volatility variables, meaning that bonds can serve as building blocks for hedging strategies that span the universe of interest-rate derivatives. However, empirical evidence of USV has been documented in a number of papers; for instance, Collin-Dufresne & Goldstein (2002) [14] show that there are volatility factors that influence interest-rate derivatives but do not affect the bond yield curve. We choose a model with this fundamental characteristic in order to analyze the effects of the correlation between the counterparty default and the interest-rate volatility on the CVA of volatility-insensitive instruments (e.g., interest-rate swaps) and volatility-sensitive products (e.g., caps and floors). In the case of interest-rate swaps, volatility only plays a role in the uncertainty of the future exposure, whereas in the case of bond options, the exposure itself is directly affected by the volatility factor.

The CCG model in its short-rate form is written under the risk-neutral measure as follows:

$$\begin{aligned} dr_t &= \xi_r(\theta_{rt} - r_t)dt + \sqrt{V_t}dZ_{1t}, && (1)\\ d\theta_{rt} &= \left(\gamma(t) - 2\xi_r\theta_{rt} + \frac{V_t}{\xi_r}\right)dt, && (2)\\ dV_t &= \xi_V(\theta_V - V_t)dt + \nu_V\sqrt{V_t}\left(\rho_{rV}dZ_{1t} + \sqrt{1-\rho_{rV}^2}dZ_{2t}\right), && (3) \end{aligned}$$

where Z_{1t} and Z_{2t} are two independent Brownian motions and $\gamma(t)$ is a deterministic function of time. The instantaneous spot interest rate r_t follows a mean-reverting process, where the parameter ξ_r is the speed of adjustment, and the stochastic process θ_{rt} models the long-run interest-rate level. V_t is the variance process of the instantaneous spot rate, which is a square-root process where the parameter ξ_V is the speed of adjustment, θ_V is the long-run variance level and ν_V is the volatility of volatility. The parameter ρ_{rV} represents the correlation between the Brownian components of r_t and V_t. Note that, while there are two stochastic factors (the two Brownian processes), the model is Markovian in three state variables (r_t, θ_{rt}, V_t).[3]

Denote respectively by

$$\begin{aligned} \Gamma_t(u) &\equiv \exp\left(-\int_t^u r_s ds\right)\\ P(t,u) &\equiv \mathbb{E}_t\left[\Gamma_t(u)\right] \end{aligned}$$

the discount factor for $u > t$ and the price at t of a zero-coupon bond with a principal of 1, maturing at u, under the CCG model. Casassus et al. (2005) [11] show that $P(t,u)$ is given by the following expression:

$$P(t,u) = \exp(-B_r(u-t)r_t - B_\theta(u-t)\theta_{rt} - A_r(t,u)), \tag{4}$$

where

$$\begin{aligned} B_r(\delta) &= \frac{\left(1-e^{-\xi_r\delta}\right)}{\xi_r},\\ B_\theta(\delta) &= \frac{\left(1-e^{-\xi_r\delta}\right)^2}{2\xi_r},\\ A_r(t,u) &= \int_t^u \gamma(s)B_\theta(u-s)ds, \end{aligned}$$

showing that the specification of θ_{rt} is designed so that the model features USV behavior, that is, bond prices do not depend on the volatility process.

[3] Collin-Dufresne & Goldstein (2002) [14] show that at least three state variables are needed in order to exhibit USV in an affine framework.

2.2 The default-intensity model

To model the default intensity, we consider the following square-root Cox-Ingersoll-Ross (CIR) representation (Cox et al. 1985 [15]) under the risk-neutral measure:

$$d\lambda_t = \xi_\lambda(\theta_\lambda - \lambda_t)dt + \nu_\lambda\sqrt{\lambda_t}\left(\rho_{r\lambda}dZ_{1t} + \frac{\rho_{V\lambda} - \rho_{rV}\rho_{r\lambda}}{\sqrt{1-\rho_{rV}^2}}dZ_{2t} + \sqrt{1-\rho_{r\lambda}^2 - \frac{(\rho_{V\lambda} - \rho_{rV}\rho_{r\lambda})^2}{1-\rho_{rV}^2}}dZ_{3t}\right), \quad (5)$$

where Z_{3t} is a third Brownian motion independent from Z_{1t} and Z_{2t}, and $\rho_{r\lambda}$ (respectively $\rho_{V\lambda}$) is the correlation between the Brownian components of r_t and λ_t (respectively V_t and λ_t). The CIR model is the standard choice for modeling the default intensity because it ensures the positivity of the process and has desirable mean-reverting properties.

Denote by

$$D(t,u) \equiv \mathbb{E}_t\left[\exp\left(-\int_t^u \lambda_s ds\right)\right]$$

the survival probability between t and u under the CIR model. This probability has a closed-form expression, recalled in Equation (7) in the Appendix.

2.3 The general model for pricing CVA

We price the unilateral CVA under the risk-neutral measure using the following general formula (see Brigo & Masetti 2006 [7]):

$$\mathrm{CVA}_t = \mathbb{E}_t\left[\mathrm{LGD}\mathbf{1}_{t<\tau\leq T}\Gamma_t(\tau)\max\{W_\tau;0\}\right],$$

where $\mathbf{1}_{\mathcal{C}}$ is the indicator function of condition $\mathcal{C}$, LGD is the loss given default (computed as one minus the recovery rate), τ is the default time, T is the maturity of the contract, and W_t is the residual (default-free) value of the contract at t.

To price the CVA using Monte Carlo simulation, we apply the following procedure:

- Simulate the default time and the trajectories of the risk factors that affect W_t;
- Calculate $\mathrm{LGD}\mathbf{1}_{t<\tau\leq T}\Gamma_t(\tau)\max\{W_\tau;0\}$ for each scenario; if $\tau > T$, this would simply equal zero; and
- Average over all scenarios to get an estimate of the CVA.

3 Calibration methodology and results

We conduct our empirical analysis using the market conditions observed on December 5, 2008. This date falls in the period of significant systemic credit risk that immediately followed Lehman's bankruptcy. It was also chosen by Harris et al. (2015) [22] as the date for assessing the impact of WWR driven by the correlation between the interest-rate volatility and the hazard rate on the gap risk of collateralized interest-rate swaps. Note that it is hard to find an instrument that critically depends on the correlation parameters $\rho_{r\lambda}$, $\rho_{V\lambda}$ and ρ_{rV}, so that these parameters have to either be estimated historically from the joint evolution of interest rates and CDS spreads, or set on an expert-judgement basis. To allow us to compare our results to those of Harris et al. (2015) [22], we use the same correlation parameters, based on the joint historical evolution of interest rates and default intensities during the financial crisis.

3.1 Calibration of the interest-rate model

The inputs for the interest-rate model are r_0, θ_0, $\gamma(t)$, ξ_r, the variance process parameters V_0, ξ_V, θ_V, ν_V, and the correlation coefficient ρ_{rV}.

First, we calibrate the initial term structure of interest rates, using the Nelson-Siegel-Svensson (NSS) scheme (Nelson & Siegel 1987 [29] and Svensson 1994 [30]). Under the NSS scheme, the curve of instantaneous forward rates is assumed to have the following parametric form:

$$f(t) = \beta_0 + \beta_1 \exp\left(-\frac{t}{\zeta_1}\right) + \beta_2 \frac{t}{\zeta_1} \exp\left(-\frac{t}{\zeta_1}\right) + \beta_3 \frac{t}{\zeta_2} \exp\left(-\frac{t}{\zeta_2}\right).$$

The parameters $(\beta_0, \beta_1, \beta_2, \beta_3, \zeta_1, \zeta_2)$ are estimated by minimizing the sum of squared errors between the NSS model-implied rates and the rates actually observed. We use Libor rates with maturities of 3, 6 and 9 months, and swap rates with maturities of 1, 2, 3, 4, 5, 7, 10, 15, 20 and 30 years. The data on interest rates is obtained from Bloomberg. The estimated NSS parameters are reported in Table 1.

Table 1: Estimated Nelson-Siegel-Svensson parameters on December 5, 2008.

β_0	β_1	β_2	β_3	ζ_1	ζ_2
0.0254	-0.0023	-0.0453	0.0483	1.6366	3.1673

For a given ξ_r, we then obtain the deterministic function of time $\gamma(t)$ of the CCG model and the initial values r_0 and θ_0, using

$$\begin{aligned} \gamma(t) &= \frac{1}{\xi_r}\frac{d^2 f(t)}{dt^2} + 3\frac{df(t)}{dt} + 2\xi_r f(t) \qquad (6) \\ r_0 &= f(0) \\ \theta_0 &= r_0 + \frac{1}{\xi_r}\frac{df(0)}{dt}. \end{aligned}$$

As shown in Casassus et al. (2005) [11], this ensures that the model-implied initial forward curve exactly fits the market forward curve.

Note that, because of the USV property of the CCG model, the volatility process parameters need to be extracted from derivative prices. Casassus et al. (2005) [11] show that the correlation parameter ρ_{rV} drives the skewness of implied volatility smiles, and that a good fit is obtained from the restriction to $\rho_{rV} = 0$ for at-the-money (ATM) options. Accordingly, we calibrate the interest-rate model parameters $(V_0, \xi_r, \xi_V, \theta_V, \nu_V)$ by minimizing the sum of squared errors between the model-implied interest-rate option prices under the restricted CCG model and the prices actually observed on December 5, 2008, for ATM options. The parameters are calibrated to ATM cap prices with maturities of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 25 and 30 years, computed from Bloomberg's Black log-normal implied-volatility matrix (Black 1976 [4]). Cap prices under the restricted CCG model are obtained using Equation (8) given in the Appendix. The calibrated parameters of the interest-rate model are reported in Table 2.

Table 2: Calibrated parameters of the interest-rate model on December 5, 2008.

V_0	ξ_r	ξ_V	θ_V	ν_V
0.0007	0.09	0.11	0.0006	0.008

For OTM options, we use the historical estimation in Harris et al. (2015) [22] for the correlation parameter $\rho_{rV} = -0.7045$, along with the parameter values in Table 2.

3.2 Calibration of the default-intensity model

The inputs for the default-intensity model are $\lambda_0, \theta_\lambda, \xi_\lambda, \nu_\lambda$, and the correlations $\rho_{r\lambda}$ and $\rho_{V\lambda}$. The information on the default intensity is embedded in the counterparty's credit default swap (CDS) quotes. As in Harris et al. (2015) [22], our analysis is conducted for eight counterparties representing the major American banks and financial companies, that is, Bank of America, Citi Group Inc., Goldman Sachs Group Inc., JP Morgan Chase & Co., American Express, American International Group Inc., Morgan Stanley, and Merrill Lynch.

We calibrate the default-intensity vector of parameters $(\lambda_0, \theta_\lambda, \xi_\lambda, \nu_\lambda)$ to the observed term structure of CDS premia by minimizing the sum of squared errors between the model-implied CDS spreads (see

Equation (11) in the Appendix) and the market-quoted ones. We consider CDS spreads with maturities of 1, 2, 3, 4, 5, 7 and 10 years. Data on CDS spreads is obtained from Bloomberg. Figure 1 reports the CDS-spread curves observed on December 5, 2008, for the different counterparties considered in this study, while Table 3 indicates the calibrated values of the parameters of the default-intensity model.

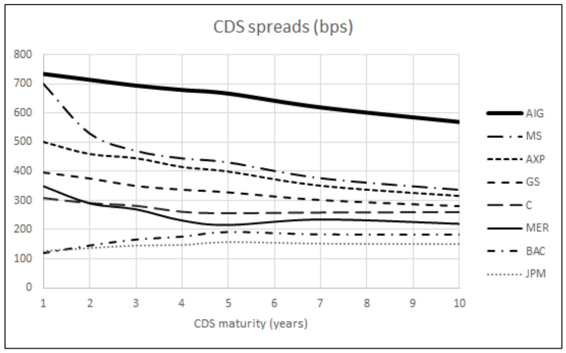

Figure 1: CDS-spread curves for the different counterparties on December 5, 2008.

Table 3: Calibrated parameters of the default-intensity model for the different counterparties on December 5, 2008.

Counterparty	Ticker	λ_0	θ_λ	ξ_λ	ν_λ
American International Group Inc.	AIG	0.1213	0.0933	0.0506	0.1780
Morgan Stanley	MS	0.1712	0.0464	1.4014	0.1023
American Express	AXP	0.0876	0.0133	0.1517	0.1071
Goldman Sachs Group Inc.	GS	0.0696	0.0330	0.2495	0.1023
Citi Group Inc.	C	0.0535	0.0337	0.2596	0.1973
Merrill Lynch	MER	0.0839	0.0316	1.5976	0.1989
Bank of America	BAC	0.0070	0.0341	1.4851	0.1249
JP Morgan Chase & Co.	JPM	0.0152	0.0269	1.6085	0.1799

As mentioned before, the two remaining parameters, $\rho_{r\lambda}$ and $\rho_{V\lambda}$, must be estimated statistically based on historical data. We use the values estimated by Harris et al. (2015) [22] based on the joint historical evolution of interest rates and default intensities during the financial crisis; these estimated values are reported in Table 4 for the different counterparties. Note that $\rho_{r\lambda}$ is negative and $\rho_{V\lambda}$ is positive for all counterparties. This is in line with the conditions observed during the financial crisis, where the increase in counterparty risk was accompanied by a decrease in the interest-rate level and an increase in interest-rate volatility, due to monetary policy and high uncertainty.

Table 4: Values of the correlations for the different counterparties (Harris et al. 2015).

Counterparty	$\rho_{r\lambda}$	$\rho_{V\lambda}$
American International Group Inc.	−0.5352	0.6271
Morgan Stanley	−0.2751	0.3149
American Express	−0.4284	0.5769
Goldman Sachs Group Inc.	−0.6758	0.6491
Citi Group Inc.	−0.5485	0.3247
Merrill Lynch	−0.4851	0.6677
Bank of America	−0.4619	0.3859
JP Morgan Chase & Co.	−0.4044	0.5576

Note that, since the chosen firms are major players in the financial market, default of any of them may have effects that are not taken into account by the default intensity model (5) (e.g. contagion or other systemic effect). These counterparties are chosen for illustration purposes, in order to use a range of plausible hazard rate parameters inferred from market data, and to allow a comparison with the results reported in Harris et al. (2015). A robustness analysis is performed in Section 5 using hypothetical parameter values and a range of correlation coefficients.

4 Impact of correlations on counterparty credit risk

To investigate the relative impact of the correlations $\rho_{r\lambda}$ and $\rho_{V\lambda}$ on the price of counterparty risk, we consider four distinct cases. Case 1 ($\rho_{V\lambda} = \rho_{r\lambda} = 0$) corresponds to a hazard rate that is independent from all sources of market risk. Case 2 ($\rho_{V\lambda} \neq 0$ and $\rho_{r\lambda} = 0$) assumes a correlation between the hazard rate and the level of the interest rate, while Case 3 ($\rho_{V\lambda} = 0$ and $\rho_{r\lambda} \neq 0$) assumes a correlation between the hazard rate and the volatility of the interest rate. Finally, Case 4 ($\rho_{V\lambda} \neq 0$ and $\rho_{r\lambda} \neq 0$) corresponds to a situation where the hazard rate is correlated with both the level and the volatility of the interest rate. Throughout this study, the recovery factor is assumed to be nil.

We first analyze the CVA of interest-rate swaps, where the CVA is driven by an exposure corresponding to the full default-free value of the swap. In this case, due to the USV nature of the interest-rate model, the volatility only plays a role in the uncertainty of the future exposure. The same exercise is then performed for caps and floors. Since caps and floors are interest-rate options and are thus contingent on the volatility state, one may expect that $\rho_{V\lambda}$ will have an important effect for this kind of instrument. Finally, we study the impact of correlations on gap risk by computing the CVA of collateralized interest-rate swaps. For this analysis, we assume a two-week replacement period upon default, during which the swap value may vary, so that the role played by the interest-rate volatility may be significant.

The simulation results are based on $100,000$ scenarios and 260 discretization steps for the underlying processes. We use the same sample of random shocks for all situations in order to ensure that comparative results are not affected by sampling errors.

4.1 Interest-rate swaps

Here we compare the CVA of interest-rate swaps implied by the four previously defined correlation schemes. We consider both payer and receiver positions with exposure to each of the eight counterparties considered in this study. We compute the CVA (reported in basis points) for each correlation scheme. Our results are provided in Appendix A.4 (Tables 7–11) and are illustrated in Figures 2 to 6 for swap maturities ranging from one to five years.

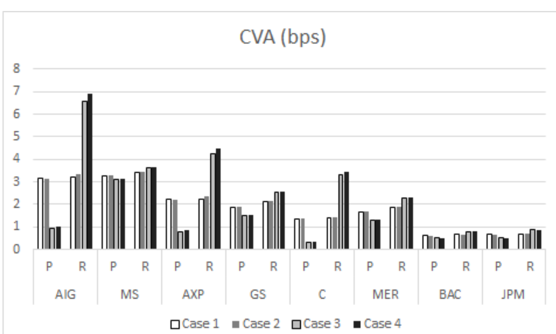

Figure 2: CVA for 1-year swaps.

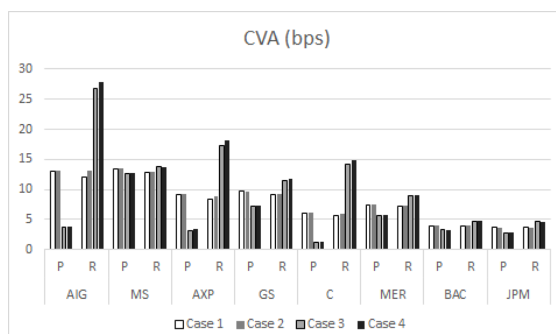

Figure 3: CVA for 2-year swaps.

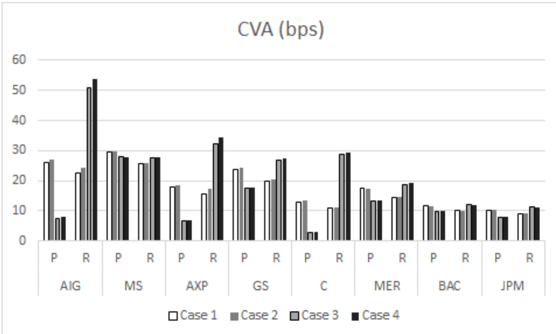

Figure 4: CVA for 3-year swaps.

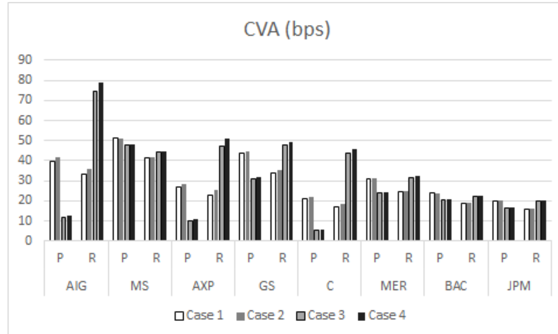

Figure 5: CVA for 4-year swaps.

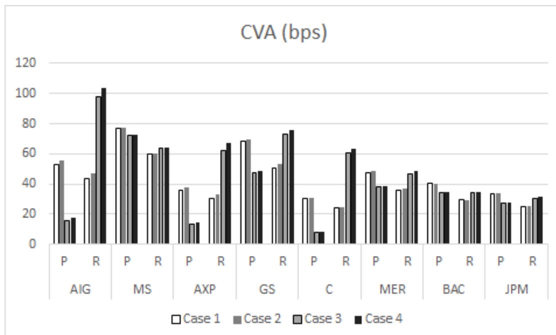

Figure 6: CVA for 5-year swaps.

Figures 2 to 6 show qualitatively similar results across all swap maturities, up to a scaling factor for the CVA values (the maximum CVA is 7 bps for one-year maturity swaps, while it is 100bps when the maturity is five years). We observe that the impact of correlation $\rho_{r\lambda}$ (Case 3) has a right-way effect (a decrease in the CVA with respect to Case 1) on the payer position and a wrong-way effect (an increase in the CVA with respect to Case 1) on the receiver position. These are the expected effects since $\rho_{r\lambda}$ is negative for all counterparties. Note that this effect is particularly significant, yielding a large variation in the CVA for most cases. Indeed, the CVA more than doubles for many counterparties, regardless of the maturity of the swap, when the WWR induced by the correlation $\rho_{r\lambda}$ for the swap receiver position is taken into account (the largest relative effect being an increase of 164.20% (an increase of 17.81 with respec to a CVA of 10.85 bps) for the three-year swap receiver position with exposure to counterparty Citi Group Inc.). On the other hand, except for very few cases, the correlation $\rho_{V\lambda}$ (Case 2) has the expected wrong-way effect on both the payer and the receiver positions. This is intuitive since a positive correlation between the uncertainty on the interest-rate movements and the default probability would generally result in a CVA increase, whatever the position. However, it is worth noting that the magnitude of this wrong-way effect is minor compared to the effect of $\rho_{r\lambda}$, as measured in Case 3. The maximum computed percentage variation of the CVA due to $\rho_{V\lambda}$ with respect to the no-correlation case is 12.41% (an increase of 1.92 with respect to a CVA of 15.48 bps) for the receiver position engaged in a three-year swap with AXP. When a correlation with both the level and the volatility is present (Case 4), it is always the substantially larger effect of $\rho_{r\lambda}$ that predominates, driving the overall percentage change in the CVA. A possible explanation for the relatively non-critical role of $\rho_{V\lambda}$ observed for interest-rate swaps may be the fact that, under a USV framework, swap exposures themselves are not affected by the volatility variable. On the other hand, it is interesting to notice from Figures 2 to 6 that right-way and wrong-way effects are far from being symmetric in nonzero correlation cases, with an exposure to the counterparty that is generally higher and can be substantially more sizeable from the swap receiver's point of view—the most hazardous position exposure when $\rho_{r\lambda}$ is negative.

4.2 Interest-rate caps and floors

We now examine the effect of $\rho_{V\lambda}$ and $\rho_{r\lambda}$ on the CVA of interest-rate derivatives whose exposures depend on the volatility factor, that is, interest-rate caps and floors. Our CVA results are reported in the Appendix (Tables 12–17) and are illustrated in Figures 7 to 11 for ATM caps and floors with maturities ranging from one to five years and in Figure 12 for OTM five-year caps. For ATM options, CVA prices are obtained under the restricted CCG model with $\rho_{rV} = 0$, while OTM options prices are obtained with $\rho_{rV} = -0.7045$.

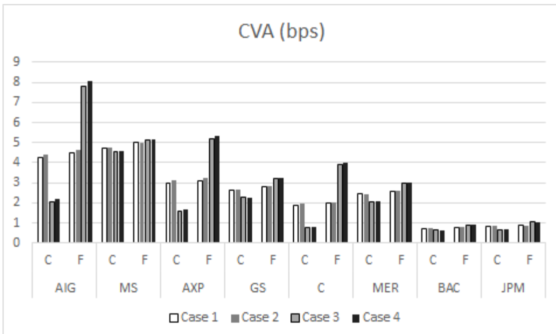

Figure 7: CVA for 1-year caps and floors.

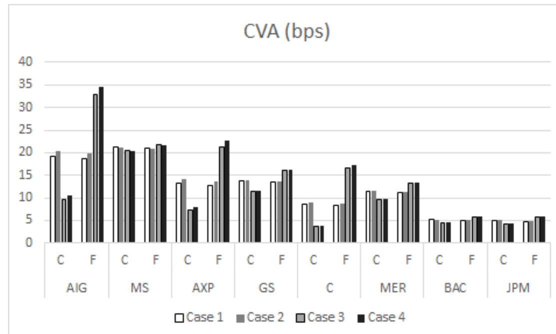

Figure 8: CVA for 2-year caps and floors.

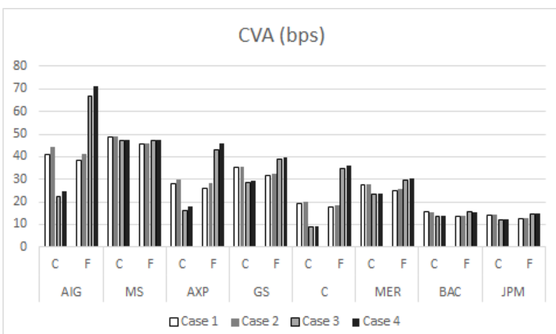

Figure 9: CVA for 3-year caps and floors.

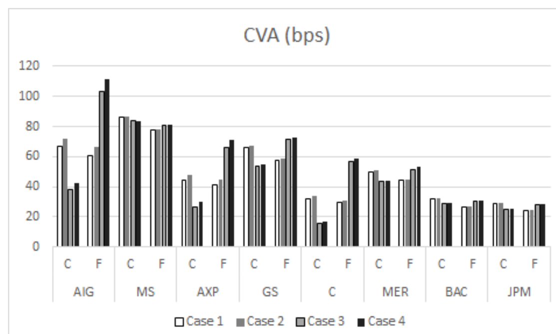

Figure 10: CVA for 4-year caps and floors.

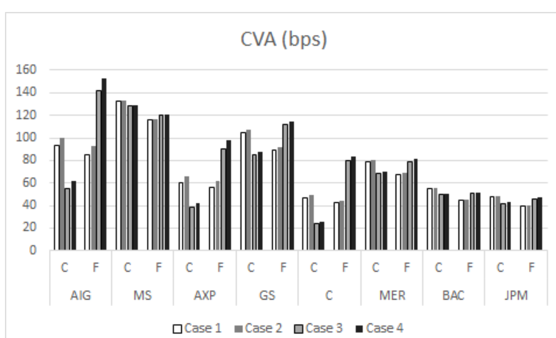

Figure 11: CVA for 5-year caps and floors.

The examination of Figures 7 to 11 shows that the correlation $\rho_{V\lambda}$ always has a wrong-way effect on both instruments (Case 2), while $\rho_{r\lambda}$ has a right-way effect on caps and a wrong-way effect on floors (Case 3). In the same way as for swaps, the effect of the correlation $\rho_{r\lambda}$ on the CVA dominates that of $\rho_{V\lambda}$ for interest-rate options. Interestingly, the materiality of the $\rho_{V\lambda}$ effect is generally negligible, especially for shorter-maturity options, with the highest being an increase of 10% (8.52 bps) with respect to the no-correlation CVA (84.7 bps) for a five-year floor option with AIG. This may seem surprising since option exposures are affected by volatility. However, while it is true that the volatility factor affects exposure, which is an expectation of discounted future cash flows, the realized cash flows are not explicitly related to volatility. We contend that for a given factor to generate significant right-way/wrong-way risk, it should have a direct influence on the cash flows of the derivative product. Our results suggest that the materiality of right-way/wrong-way risk on the price of counterparty risk is located neither at the exposure level (expectation of cash flows) nor at the uncertainty level (volatility of cash flows), but rather at the cash-flow level itself.

Finally, Figure 12 provides a comparison of CVA prices of ATM and OTM five-year caps, where the strike price of OTM caps is fixed at 0.02 above the ATM strike price. We observe that the impacts of the correlation scenarios on the CVA of OTM options are not qualitatively different from the results obtained for ATM options.

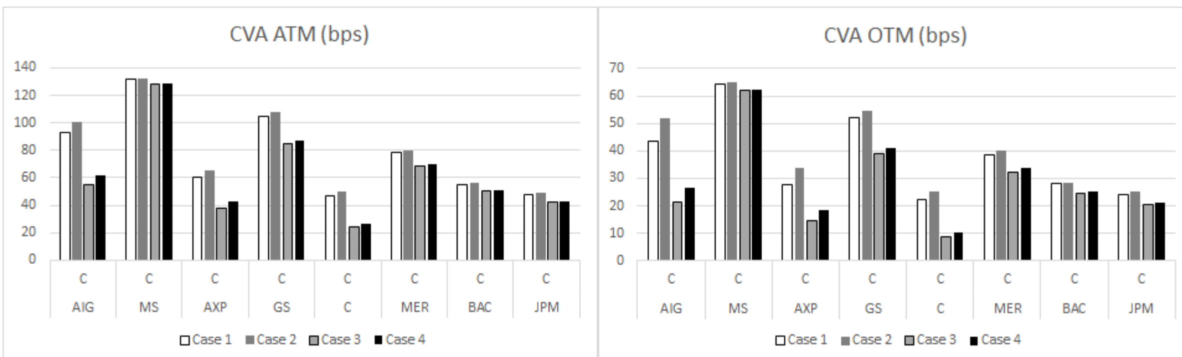

Figure 12: CVA for 5-year ATM and OTM caps.

4.3 Impact of correlations on the CVA of collateralized interest-rate swaps

We now examine the impact of the correlations $\rho_{V\lambda}$ and $\rho_{r\lambda}$ on gap risk by computing the CVA of collateralized interest-rate swaps. We assume that the swaps are fully collateralized and that a two-week period elapses between the default event and the date where positions are replaced or unwound. Gap risk arises from a potential significant movement of the interest rates during that two-week period where collateral is not posted; if the value of the trade increases during this period, the available collateral is then insufficient to cover the swap value, and a loss corresponding to the difference between the two values is suffered. In this context, an increase in interest-rate volatility may play a key role, since a positive $\rho_{V\lambda}$ can in turn lead to a CVA increase. On the other hand, the role of $\rho_{r\lambda}$ in this situation is not expected to be significant. Figure 13 and Table 18 report on the CVA values for a five-year collateralized swap according to the four correlation schemes and for the eight counterparties considered in this paper.

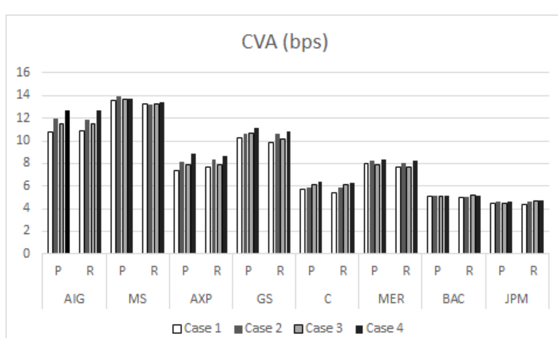

Figure 13: CVA for a 5-year collaterallized swap with a 2-week gap period.

Case 2 in Figure 13 corroborates the expected individual wrong-way effect of $\rho_{V\lambda}$. However, the magnitude of this effect is small, reaching at the maximum only a 10.51% change (an increase of 1.14 bps) with respect to the no-correlation case (10.8 bps) for a payer swap with AIG. On the other hand, the correlation $\rho_{r\lambda}$ (Case 3) does not exhibit a clear individual effect in terms of sign, and the corresponding percentage variation is also small (the maximum value is 12.42%, that is, an increase of 0.67 bps for a receiver swap with Citi Group). In sum, although the effect of $\rho_{V\lambda}$ dominates in certain cases (generally for riskier counterparties), it remains small, resulting in a variation of the expected loss of between 0.02 and 1.14 basis points. Note that the numbers we obtain are close to the figures reported in Harris et al. (2015) [22], who analyze the single effect of $\rho_{V\lambda}$ on the expected loss of collateralized swaps under the same assumptions. However, instead of basing their interpretations on the unconditional expected loss, they consider the conditional expected loss, which is defined as the average loss in defaulted scenarios. Under this last metric, they find the effect of $\rho_{V\lambda}$ to be relevant, without comparing it to the effect of $\rho_{r\lambda}$.

To conclude this section, we find that incorporating the correlation $\rho_{V\lambda}$ has a minor effect on the price of counterparty risk, regardless of whether or not the instrument is collateralized, and, more interestingly, of whether or not the instrument is volatility sensitive. The individual impact of the correlation $\rho_{r\lambda}$ on the CVA is however significant for the non-collateralized instruments considered in this paper. The impact of WWR is amplified when both correlations are included, especially for credit-riskier counterparties. While Harris et al. (2015) [22] report a non-negligible wrong-way effect of $\rho_{V\lambda}$ on the CVA of collateralized interest-rate swaps, and Gregory (2012) [21] suggests incorporating right-way/wrong-way risk for interest-rate derivatives by correlating the default intensity with the interest-rate volatility, our results show that the WWR induced by $\rho_{V\lambda}$ makes a relatively small contribution to the overall CVA variation when compared with the effect of $\rho_{r\lambda}$. From a practitioner's perspective, the additional effect of $\rho_{V\lambda}$ may seem too small to be accounted for. Nevertheless, given that the interest-rate level and its volatility represent the two distinct underlying risk factors that drive WWR in the interest-rate market by clearly impacting the credit exposure through their respective correlations with the hazard rate, these two sources of risk may need to be taken into account simultaneously (as in Case 4) to fairly price and hedge the CVA of an interest-rate derivative position.

5 Robustness analysis

In this section, we perform a sensitivity analysis to validate the robustness of the interpretations made in the previous section. To this end, we stress the parameters of the interest-rate and default-intensity models and we vary the correlation parameters in order to determine the extent to which the CVA is impacted by $\rho_{r\lambda}$ and $\rho_{V\lambda}$ for the (non-collateralized) instruments considered in this paper. The key parameters to be stressed are the volatility of the interest-rate volatility process ν_V and the volatility of the default intensity ν_λ, since these are the parameters that amplify the joint random shocks caused by the correlations. We put ν_V equal to 0.03 and set $\xi_V = 0.8$ in order to ensure that the interest-rate-volatility variable remains strictly positive. We set $\rho_{rV} = 0$ and keep all the other parameters of the interest-rate model at their original values. For the default-intensity parameters, we consider a hypothetical company having a high credit-risk profile, for which the default-intensity volatility ν_λ is set equal to 0.4. The stressed parameters of the interest-rate and the default-intensity models are reported in Tables 5 and 6.

Table 5: Stressed parameters of the interest-rate model.

V_0	ξ_r	ξ_V	θ_V	ν_V
0.0007	0.09	0.8	0.0006	0.03

Table 6: Stressed parameters of the default-intensity model.

λ_0	θ_λ	ξ_λ	ν_λ
0.17	0.17	0.5	0.4

Figure 14 depicts the incremental effect of the correlations $\rho_{V\lambda}$ and $\rho_{r\lambda}$ on the CVA of non-collateralized instruments with a five-year maturity. Results are expressed in terms of percentage variation of the CVA price in Case 2 (respectively Case 3) with respect to that of Case 1, as a function of $\rho_{V\lambda}$ (respectively $\rho_{r\lambda}$). Correlation values range from -0.9 to 0.9, making it possible to assess the effects of potential extreme correlation scenarios. Figure 14 clearly shows that the effect of $\rho_{V\lambda}$ is minor compared to that of $\rho_{r\lambda}$, for all instruments. The contribution of $\rho_{V\lambda}$ reaches a maximum of around 11% for extreme values, while the effect of $\rho_{r\lambda}$ is much more substantial. This validates the conclusion that, for non-collateralized instruments, the contribution of $\rho_{V\lambda}$ is generally small and always largely dominated by the contribution of $\rho_{r\lambda}$ to the overall additional counterparty credit risk driven by both correlations.

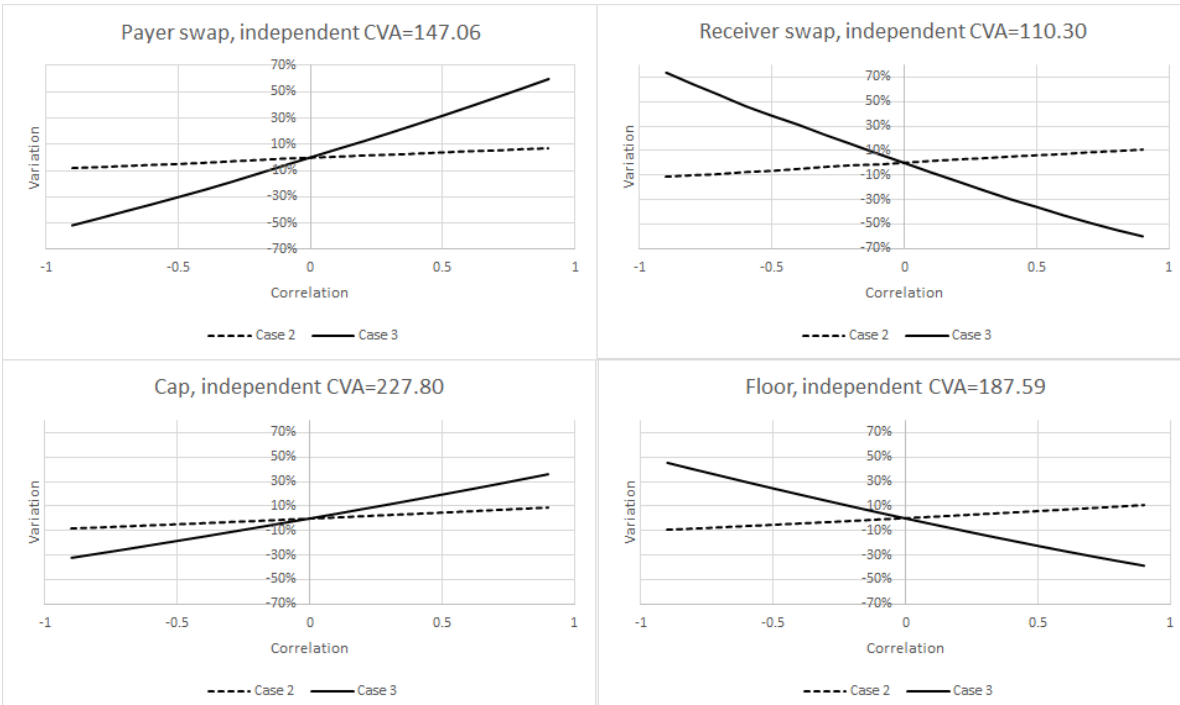

Figure 14: Percentage variation of the CVA with respect to Case 1 (independence) for non-collateralized swaps, caps and floors as a function of the correlation between the interest-rate level and the default intensity ($\rho_{r\lambda}$), and the correlation between the interest-rate volatility and the default intensity ($\rho_{V\lambda}$).

6 Concluding remarks

In this paper, we investigate the impact of two sources of WWR on the pricing of counterparty credit risk in the interest-rate market. The first source is induced by the dependence between the interest-rate level and the counterparty's default intensity, while the second stems from the dependence between this default intensity and the interest-rate volatility. We consider an interest-rate model featuring USV in order to analyze the effects of correlations on volatility-insensitive products, such as interest-rate swaps, and volatility-sensitive instruments, such as interest-rate caps and floors. We also investigate the effects of these correlations on collateralized instruments for which the gap risk is relevant.

We find that a positive correlation between the volatility and the default intensity generally has the expected wrong-way effect. However, this effect is small, barely contributing to the gap risk for collateralized instruments on the one hand, and largely dominated by that of the correlation between the interest-rate level and the default intensity for non-collateralized instruments, on the other hand. This is true for both interest-rate swaps and options. Although one would have expected to find a more significant impact of volatility on the CVA, especially for interest-rate options whose exposures depend on the volatility state, our results suggest that this right-way/wrong-way risk would be material only at the very bottom level of realized cash flows. Our conclusions are backed by the numerical results of the robustness analysis that we conduct in this paper.

An interesting avenue of research would be to investigate the effects of the correlations of market-risk factors with the default intensity of counterparties in other OTC financial markets (e.g., equity options, FX, etc.) and to consider a variety of other market models. In particular, it would be interesting to investigate the impact of the volatility correlation on the CVA of timer options, where the cash flows are directly affected by the volatility factor.

A Appendix

A.1 Survival probability in the CIR model

$$D(t,u) = A_\lambda(u-t)\exp(-\lambda_t B_\lambda(u-t)) \tag{7}$$

where

$$\begin{aligned} A_\lambda(\delta) &= \left(\frac{2h\exp\left(\frac{(h+\xi_\lambda)\delta}{2}\right)}{2h+(h+\xi_\lambda)\exp(h\delta)-1}\right)^{\frac{2h\theta_\lambda}{\nu_\lambda^2}} \\ B_\lambda(\delta) &= \left(\frac{2(\exp(h\delta)-1)}{2h+(h+\xi_\lambda)\exp(h\delta)-1}\right) \\ h &= \sqrt{\xi_\lambda^2+2\nu_\lambda^2}. \end{aligned}$$

A.2 Cap prices under the CCG model

Consider an interest-rate cap with a strike rate K and m reset dates $t_1, t_2, ..., t_m = T$, where $t_j = t_{j-1} + \delta$, $j = 1, ...m$. The price of the cap at date $t < t_1$ is the sum of the prices at t of m underlying caplets expiring at the same dates, with a strike rate K and term δ:

$$Cap_t\left(m,\delta,T,K\right) = \sum_{j=1}^{m} Cpl_t\left(\delta,t_j,K\right). \tag{8}$$

Let $Put_t(T_0,T,K)$ represent the price at date t of a put option with strike K and maturity T_0 written on the value of a zero-coupon bond maturing at T:

$$Put_t(T_0,T,K) = \mathbb{E}_t\left[\exp\left(-\int_t^{T_0} r_s ds\right)\left(K-P(T_0,T)\right)\mathbf{1}_{P(T_0,T)<K}\right].$$

We then have

$$Cpl_t\left(\delta,t_j,K\right) = (1+\delta K)\, Put_t\left(t_j-\delta, t_j, \frac{1}{1+\delta K}\right). \tag{9}$$

Casassus et al. 2005 [11] show that the price of a put option under the restricted CCG model ($\rho_{rV} = 0$) is given by

$$\begin{aligned} Put_t(T_0,T,K) &= K\left(\frac{\psi_{t,T_0,T}(0)}{2} - \frac{1}{\pi}\int_0^\infty \frac{Im\left(\psi_{t,T_0,T}(ix)e^{-ix\log(K)}\right)}{x}dx\right) \\ &\quad - \left(\frac{\psi_{t,T_0,T}(1)}{2} - \frac{1}{\pi}\int_0^\infty \frac{Im\left(\psi_{t,T_0,T}(1+ix)e^{-ix\log(K)}\right)}{x}dx\right), \end{aligned} \tag{10}$$

where the Fourier transform $\psi_{t,T_0,T}(w) = \mathbb{E}_t\left[\exp\left(-\int_t^{T_0} r_s ds\right)e^{w\log P(T_0,T)}\right]$ has the following expression:

$$\psi_{t,T_0,T}(w) = \exp\left(M(T_0-t) + N(T_0-t)V_t + wP(t,T) + (1-w)P(t,T_0)\right),$$

and where M and N are two deterministic functions that satisfy the system of ODEs

$$\begin{aligned}\frac{dN(\delta)}{d\delta} &= \frac{\nu_V^2}{2}N(\delta)^2 - \xi_V N(\delta)\\ &\quad +\frac{1}{2}\left((wB_r(\delta+T-T_0)+(1-w)B_r(\delta))^2 - wB_r(\delta+T-T_0)^2-(1-w)B_r(\delta)^2\right),\\ \frac{dM(\delta)}{d\delta} &= \theta_V\xi_V N(\delta),\end{aligned}$$

subject to the initial conditions $M(0) = N(0) = 0$. This system can be solved numerically using efficient standard techniques, and Formula (10) can be evaluated using direct numerical integration or efficient Fast Fourier Transform (FFT) techniques.

A.3 Credit default swap evaluation

In a CDS contract, the protection buyer pays fixed periodic fees (at discrete dates $0 < t_1 < t_2 < ... < t_m = T$) to the protection seller until the maturity of the contract or the default event. In return, if default occurs, the contract seller agrees to pay the buyer the amount of money lost due to the default of the reference entity. The standard price of a CDS with maturity T at date 0 (as seen from the protection buyer's point of view) is

$$\mathrm{CDS} = \mathbb{E}_0\left[\mathrm{LGD}\mathbf{1}_{0<\tau\le T}\Gamma_0(\tau) - \sum_{j=1}^{m}\Gamma_0(t_j)\delta_j R\mathbf{1}_{\tau>t_j} - \Gamma_0(\tau)(\tau - t_{k(\tau)})R\mathbf{1}_{0<\tau<T}\right],$$

where R is the premium rate, and $\delta_j = t_j - t_{j-1}$ and $k(\tau)$ is the index of the payment date that immediately precedes the default event. The fair premium rate is chosen so as to make the CDS value nil at $t = 0$. Under the default-intensity and the interest-rate models considered in this study, the value of the CDS does not allow for a closed-form solution because of the correlation between the interest rate and the default intensity. However, it has been shown in the financial literature that the impact of this correlation on the CDS price is minor (see, e.g., Brigo & Alfonsi 2005 [5]), so that the interdependence between interest-rate and credit risks has become a common assumption in CDS pricing models. Under independence, the CDS value is then written as follows:

$$\begin{aligned}\mathrm{CDS} &= \mathrm{LGD}\int_0^T P(0,s)\left(-\frac{d}{ds}D(0,s)\right)ds\\ &\quad -\sum_{j=1}^{m}\delta_j R P(0,t_j)D(0,t_j)\\ &\quad -R\int_0^T P(0,s)\left(-\frac{d}{ds}D(0,s)\right)(s-t_{k(s)})ds,\end{aligned}$$

where both $P(0,t)$ and $D(0,t)$ are known in closed form—see Equations (4) and (7) respectively. The fair premium rate is then given by

$$R = \frac{\mathrm{LGD}\int_0^T P(0,s)\left(-\frac{d}{ds}D(0,s)\right)ds}{\sum_{j=1}^m \delta_j P(0,t_j)D(0,t_j) + \int_0^T P(0,s)\left(-\frac{d}{ds}D(0,s)\right)(s-t_{k(s)})ds}. \tag{11}$$

A.4 Numerical results

Table 7: CVA (in basis points) of 1-year swaps for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda} = \rho_{r\lambda} = 0$	Case 2 $\rho_{V\lambda} \neq 0$, $\rho_{r\lambda} = 0$		Case 3 $\rho_{V\lambda} = 0$, $\rho_{r\lambda} \neq 0$		Case 4 $\rho_{V\lambda} \neq 0$, $\rho_{r\lambda} \neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Payer	3.18	3.14	−0.03	0.95	−2.23	1.00	−2.18
	Receiver	3.19	3.36	0.17	6.59	3.40	6.88	3.69
MS	Payer	3.28	3.28	0.01	3.12	−0.15	3.11	−0.16
	Receiver	3.44	3.47	0.02	3.61	0.16	3.63	0.18
AXP	Payer	2.21	2.22	0.01	0.77	−1.44	0.84	−1.37
	Receiver	2.24	2.35	0.11	4.26	2.02	4.47	2.23
GS	Payer	1.86	1.88	0.01	1.52	−0.35	1.52	−0.35
	Receiver	2.13	2.17	0.04	2.54	0.41	2.58	0.45
C	Payer	1.34	1.38	0.03	0.30	−1.04	0.33	−1.01
	Receiver	1.39	1.43	0.04	3.33	1.94	3.44	2.06
MER	Payer	1.65	1.67	0.02	1.30	−0.35	1.31	−0.33
	Receiver	1.89	1.89	0.00	2.27	0.38	2.31	0.41
BAC	Payer	0.62	0.62	0.00	0.50	−0.11	0.51	−0.11
	Receiver	0.66	0.67	0.01	0.80	0.14	0.80	0.14
JPM	Payer	0.66	0.64	−0.02	0.52	−0.14	0.50	−0.16
	Receiver	0.68	0.70	0.01	0.87	0.18	0.88	0.20

Table 8: CVA (in basis points) of 2-year swaps for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda} = \rho_{r\lambda} = 0$	Case 2 $\rho_{V\lambda} \neq 0$, $\rho_{r\lambda} = 0$		Case 3 $\rho_{V\lambda} = 0$, $\rho_{r\lambda} \neq 0$		Case 4 $\rho_{V\lambda} \neq 0$, $\rho_{r\lambda} \neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Payer	13.04	13.23	0.19	3.69	−9.35	3.84	−9.20
	Receiver	12.11	13.09	0.98	26.75	14.64	27.95	15.84
MS	Payer	13.42	13.47	0.05	12.66	−0.76	12.68	−0.74
	Receiver	12.85	12.91	0.06	13.73	0.88	13.73	0.88
AXP	Payer	9.13	9.34	0.21	3.14	−5.99	3.42	−5.71
	Receiver	8.31	8.95	0.64	17.24	8.92	18.10	9.79
GS	Payer	9.67	9.69	0.02	7.20	−2.47	7.26	−2.41
	Receiver	9.12	9.20	0.08	11.52	2.41	11.77	2.65
C	Payer	6.11	6.24	0.14	1.22	−4.89	1.28	−4.83
	Receiver	5.63	5.89	0.25	14.25	8.62	14.83	9.20
MER	Payer	7.35	7.44	0.09	5.65	−1.71	5.71	−1.65
	Receiver	7.18	7.31	0.13	9.01	1.83	9.16	1.97
BAC	Payer	4.00	3.99	−0.01	3.32	−0.67	3.32	−0.68
	Receiver	3.89	3.96	0.07	4.70	0.80	4.71	0.82
JPM	Payer	3.69	3.72	0.03	2.85	−0.83	2.91	−0.78
	Receiver	3.68	3.70	0.03	4.63	0.95	4.70	1.02

Table 9: CVA (in basis points) of 3-year swaps for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda} = \rho_{r\lambda} = 0$	Case 2 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} = 0$		Case 3 $\rho_{V\lambda} = 0,\ \rho_{r\lambda} \neq 0$		Case 4 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} \neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Payer	26.06	27.11	1.05	7.56	−18.50	8.09	−17.97
	Receiver	22.78	24.21	1.43	50.91	28.13	53.66	30.88
MS	Payer	29.59	29.57	−0.02	27.85	−1.74	27.97	−1.62
	Receiver	25.82	25.88	0.06	27.54	1.72	27.75	1.93
AXP	Payer	18.00	18.65	0.64	6.54	−11.46	7.08	−10.93
	Receiver	15.48	17.41	1.92	32.24	16.75	34.39	18.90
GS	Payer	23.85	24.44	0.59	17.37	−6.48	17.58	−6.27
	Receiver	19.86	20.38	0.53	26.86	7.00	27.42	7.57
C	Payer	12.91	13.35	0.44	2.83	−10.08	2.95	−9.96
	Receiver	10.85	11.32	0.47	28.66	17.81	29.37	18.52
MER	Payer	17.37	17.51	0.14	13.26	−4.12	13.34	−4.03
	Receiver	14.57	14.78	0.21	18.78	4.21	19.13	4.56
BAC	Payer	11.62	11.70	0.08	9.77	−1.85	9.91	−1.71
	Receiver	10.14	10.15	0.01	12.02	1.88	12.04	1.90
JPM	Payer	10.03	10.32	0.29	7.99	−2.04	8.16	−1.87
	Receiver	9.04	9.14	0.09	11.22	2.17	11.33	2.28

Table 10: CVA (in basis points) of 4-year swaps for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda} = \rho_{r\lambda} = 0$	Case 2 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} = 0$		Case 3 $\rho_{V\lambda} = 0,\ \rho_{r\lambda} \neq 0$		Case 4 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} \neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Payer	39.42	41.63	2.22	11.63	−27.79	12.74	−26.68
	Receiver	33.15	36.06	2.92	74.73	41.58	78.82	45.67
MS	Payer	51.12	51.26	0.14	48.05	−3.06	48.37	−2.75
	Receiver	41.37	41.67	0.29	44.36	2.99	44.42	3.05
AXP	Payer	26.84	28.45	1.61	10.00	−16.83	10.86	−15.98
	Receiver	23.12	25.44	2.32	46.97	23.85	50.79	27.67
GS	Payer	43.63	44.54	0.92	31.15	−12.48	31.60	−12.03
	Receiver	34.07	35.22	1.14	47.96	13.89	49.22	15.15
C	Payer	21.25	21.84	0.59	5.38	−15.87	5.55	−15.70
	Receiver	17.26	18.21	0.95	44.03	26.77	45.74	28.48
MER	Payer	31.09	31.10	0.01	24.20	−6.88	24.38	−6.70
	Receiver	24.38	24.82	0.44	31.48	7.10	32.69	8.30
BAC	Payer	23.84	23.87	0.02	20.42	−3.42	20.59	−3.26
	Receiver	18.72	18.89	0.16	22.02	3.30	22.27	3.54
JPM	Payer	20.17	20.12	−0.05	16.43	−3.75	16.47	−3.70
	Receiver	16.00	16.16	0.16	19.82	3.82	20.09	4.09

Table 11: CVA (in basis points) of 5-year swaps for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda} = \rho_{r\lambda} = 0$	Case 2 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} = 0$		Case 3 $\rho_{V\lambda} = 0,\ \rho_{r\lambda} \neq 0$		Case 4 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} \neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Payer	52.97	55.32	2.35	16.09	−36.88	17.65	−35.32
	Receiver	43.71	46.80	3.10	98.06	54.35	103.40	59.70
MS	Payer	76.79	77.06	0.26	72.68	−4.11	72.89	−3.90
	Receiver	59.59	59.92	0.33	63.59	4.00	63.83	4.24
AXP	Payer	35.53	38.05	2.52	13.46	−22.07	14.74	−20.79
	Receiver	30.09	33.11	3.02	62.33	32.24	67.08	36.99
GS	Payer	68.11	69.44	1.34	47.29	−20.81	48.77	−19.34
	Receiver	50.97	53.52	2.55	73.19	22.22	75.42	24.45
C	Payer	30.54	30.93	0.39	8.12	−22.42	8.39	−22.15
	Receiver	24.11	24.78	0.67	60.77	36.66	63.76	39.65
MER	Payer	47.75	48.70	0.94	37.85	−9.90	38.31	−9.44
	Receiver	36.26	36.99	0.73	46.96	10.69	48.58	12.31
BAC	Payer	40.35	40.34	−0.01	34.61	−5.74	34.93	−5.42
	Receiver	29.35	29.62	0.27	34.60	5.25	34.94	5.58
JPM	Payer	33.47	33.86	0.39	27.60	−5.87	27.67	−5.80
	Receiver	24.83	25.24	0.41	30.64	5.81	31.28	6.45

Table 12: CVA (in basis points) of 1-year caps and floors for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda} = \rho_{r\lambda} = 0$	Case 2 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} = 0$		Case 3 $\rho_{V\lambda} = 0,\ \rho_{r\lambda} \neq 0$		Case 4 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} \neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Cap	4.27	4.42	0.16	2.07	−2.19	2.20	−2.07
	Floor	4.47	4.65	0.18	7.81	3.34	8.09	3.63
MS	Cap	4.73	4.73	0.01	4.57	−0.15	4.58	−0.15
	Floor	4.99	5.00	0.01	5.16	0.17	5.17	0.18
AXP	Cap	2.99	3.11	0.12	1.60	−1.39	1.70	−1.30
	Floor	3.12	3.25	0.13	5.17	2.04	5.36	2.24
GS	Cap	2.63	2.65	0.02	2.26	−0.37	2.27	−0.36
	Floor	2.79	2.81	0.02	3.22	0.43	3.24	0.45
C	Cap	1.88	1.94	0.06	0.76	−1.13	0.79	−1.10
	Floor	1.97	2.02	0.05	3.92	1.95	4.01	2.04
MER	Cap	2.43	2.46	0.02	2.08	−0.36	2.10	−0.34
	Floor	2.57	2.60	0.03	2.98	0.42	3.01	0.45
BAC	Cap	0.74	0.74	0.00	0.63	−0.11	0.64	−0.10
	Floor	0.79	0.80	0.00	0.92	0.12	0.92	0.13
JPM	Cap	0.82	0.83	0.01	0.68	−0.14	0.69	−0.14
	Floor	0.87	0.88	0.01	1.05	0.17	1.06	0.18

Table 13: CVA (in basis points) of 2-year caps and floors for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda} = \rho_{r\lambda} = 0$	Case 2 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} = 0$		Case 3 $\rho_{V\lambda} = 0,\ \rho_{r\lambda} \neq 0$		Case 4 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} \neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Cap	19.17	20.34	1.17	9.64	−9.53	10.51	−8.66
	Floor	18.62	19.96	1.33	33.01	14.39	34.67	16.04
MS	Cap	21.18	21.24	0.06	20.42	−0.75	20.48	−0.69
	Floor	20.88	20.95	0.07	21.68	0.80	21.75	0.87
AXP	Cap	13.19	14.08	0.89	7.29	−5.90	7.95	−5.24
	Floor	12.83	13.67	0.84	21.34	8.50	22.66	9.82
GS	Cap	13.84	14.02	0.18	11.46	−2.38	11.63	−2.21
	Floor	13.47	13.67	0.19	16.12	2.64	16.33	2.86
C	Cap	8.69	9.07	0.38	3.67	−5.02	3.87	−4.82
	Floor	8.44	8.71	0.27	16.71	8.27	17.37	8.93
MER	Cap	11.40	11.58	0.18	9.65	−1.75	9.82	−1.58
	Floor	11.19	11.38	0.20	13.17	1.99	13.40	2.21
BAC	Cap	5.19	5.23	0.04	4.50	−0.69	4.54	−0.66
	Floor	4.96	5.00	0.04	5.72	0.76	5.77	0.81
JPM	Cap	5.03	5.12	0.08	4.20	−0.83	4.27	−0.76
	Floor	4.84	4.93	0.09	5.78	0.95	5.89	1.05

Table 14: CVA (in basis points) of 3-year caps and floors for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda} = \rho_{r\lambda} = 0$	Case 2 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} = 0$		Case 3 $\rho_{V\lambda} = 0,\ \rho_{r\lambda} \neq 0$		Case 4 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} \neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Cap	41.18	44.20	3.03	22.14	−19.03	24.51	−16.67
	Floor	38.38	41.48	3.11	66.60	28.22	71.00	32.62
MS	Cap	48.71	48.89	0.18	47.03	−1.68	47.21	−1.51
	Floor	45.45	45.63	0.18	47.22	1.77	47.41	1.96
AXP	Cap	27.87	29.92	2.05	16.18	−11.69	17.88	−9.99
	Floor	26.06	28.20	2.14	42.88	16.83	45.81	19.75
GS	Cap	35.14	35.77	0.64	28.68	−6.46	29.25	−5.89
	Floor	31.89	32.54	0.66	39.00	7.11	39.74	7.86
C	Cap	19.27	20.19	0.92	8.84	−10.43	9.43	−9.84
	Floor	17.95	18.65	0.70	35.02	17.07	36.09	18.14
MER	Cap	27.29	27.81	0.52	23.34	−3.95	23.80	−3.49
	Floor	25.16	25.70	0.54	29.63	4.46	30.25	5.09
BAC	Cap	15.48	15.61	0.13	13.65	−1.83	13.77	−1.71
	Floor	13.53	13.66	0.14	15.49	1.97	15.65	2.12
JPM	Cap	14.10	14.36	0.26	12.00	−2.10	12.23	−1.87
	Floor	12.48	12.75	0.27	14.81	2.33	15.12	2.64

Table 15: CVA (in basis points) of 4-year caps and floors for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda} = \rho_{r\lambda} = 0$	Case 2 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} = 0$		Case 3 $\rho_{V\lambda} = 0,\ \rho_{r\lambda} \neq 0$		Case 4 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} \neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Cap	66.67	71.73	5.07	38.02	−28.65	42.29	−24.38
	Floor	61.07	66.70	5.63	103.39	42.33	111.17	50.11
MS	Cap	86.35	86.69	0.34	83.55	−2.80	83.89	−2.46
	Floor	77.55	77.90	0.35	80.53	2.98	80.90	3.35
AXP	Cap	44.14	47.81	3.68	26.89	−17.25	29.87	−14.27
	Floor	40.92	44.68	3.76	66.23	25.30	71.26	30.34
GS	Cap	66.10	67.54	1.44	53.67	−12.43	54.97	−11.13
	Floor	57.34	58.83	1.49	71.24	13.90	72.94	15.61
C	Cap	32.39	34.08	1.69	16.05	−16.34	17.17	−15.22
	Floor	29.51	30.96	1.44	56.54	27.03	58.90	29.38
MER	Cap	49.97	50.97	1.01	43.26	−6.70	44.18	−5.79
	Floor	44.12	45.17	1.05	51.76	7.64	52.97	8.85
BAC	Cap	32.37	32.66	0.28	28.94	−3.43	29.20	−3.17
	Floor	26.80	27.09	0.29	30.48	3.68	30.80	4.01
JPM	Cap	28.53	29.06	0.54	24.69	−3.83	25.18	−3.35
	Floor	23.95	24.51	0.56	28.19	4.24	28.84	4.89

Table 16: CVA (in basis points) of 5-year caps and floors for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda} = \rho_{r\lambda} = 0$	Case 2 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} = 0$		Case 3 $\rho_{V\lambda} = 0,\ \rho_{r\lambda} \neq 0$		Case 4 $\rho_{V\lambda} \neq 0,\ \rho_{r\lambda} \neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Cap	92.80	100.31	7.51	55.42	−37.38	61.97	−30.83
	Floor	84.70	93.22	8.52	141.68	56.98	152.29	67.59
MS	Cap	132.15	132.71	0.56	128.16	−3.99	128.70	−3.45
	Floor	116.10	116.67	0.57	120.47	4.37	121.06	4.96
AXP	Cap	60.12	65.79	5.67	38.18	−21.94	42.63	−17.49
	Floor	55.96	61.32	5.36	89.83	33.87	97.77	41.81
GS	Cap	104.81	107.45	2.65	85.07	−19.74	87.46	−17.35
	Floor	88.86	91.58	2.72	111.70	22.84	114.81	25.95
C	Cap	46.89	49.68	2.79	24.32	−22.57	26.06	−20.82
	Floor	42.27	44.15	1.88	79.93	37.66	83.08	40.81
MER	Cap	78.45	80.10	1.65	68.69	−9.76	70.20	−8.25
	Floor	67.59	69.30	1.71	78.94	11.35	80.91	13.32
BAC	Cap	55.39	55.88	0.49	50.06	−5.33	50.52	−4.87
	Floor	44.72	45.23	0.51	50.53	5.81	51.09	6.38
JPM	Cap	47.85	48.78	0.93	41.99	−5.86	42.84	−5.01
	Floor	39.15	40.11	0.96	45.73	6.59	46.84	7.70

Table 17: CVA (in basis points) of OTM 5-year caps for different correlation schemes. The OTM strike price is fixed at 0.02 above the ATM strike price. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda}=\rho_{r\lambda}=0$	Case 2 $\rho_{V\lambda}\neq 0,\ \rho_{r\lambda}=0$		Case 3 $\rho_{V\lambda}=0,\ \rho_{r\lambda}\neq 0$		Case 4 $\rho_{V\lambda}\neq 0,\ \rho_{r\lambda}\neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Cap	43.49	51.70	8.21	21.45	−22.04	26.76	−16.73
MS	Cap	64.55	65.11	0.56	61.96	−2.59	62.50	−2.04
AXP	Cap	27.89	33.97	6.08	14.49	−13.40	18.65	−9.24
GS	Cap	51.93	54.66	2.73	38.86	−13.07	41.20	−10.74
C	Cap	22.28	25.02	2.74	8.91	−13.37	10.11	−12.16
MER	Cap	38.60	40.31	1.71	32.42	−6.19	33.87	−4.74
BAC	Cap	28.06	28.58	0.51	24.61	−3.46	25.06	−3.00
JPM	Cap	24.08	25.04	0.97	20.32	−3.76	21.14	−2.93

Table 18: CVA (in basis points) of 5-year collateralized swaps for different correlation schemes. Δ is the variation in basis points with respect to Case 1.

		Case 1 $\rho_{V\lambda}=\rho_{r\lambda}=0$	Case 2 $\rho_{V\lambda}\neq 0,\ \rho_{r\lambda}=0$		Case 3 $\rho_{V\lambda}=0,\ \rho_{r\lambda}\neq 0$		Case 4 $\rho_{V\lambda}\neq 0,\ \rho_{r\lambda}\neq 0$	
Counterparty	Position	CVA	CVA	Δ	CVA	Δ	CVA	Δ
AIG	Payer	10.80	11.93	1.14	11.48	0.69	12.65	1.86
	Receiver	10.91	11.87	0.96	11.48	0.57	12.65	1.74
MS	Payer	13.57	13.91	0.34	13.66	0.09	13.70	0.14
	Receiver	13.23	13.24	0.02	13.24	0.01	13.42	0.20
AXP	Payer	7.41	8.13	0.72	7.92	0.51	8.83	1.42
	Receiver	7.68	8.35	0.67	7.91	0.23	8.71	1.03
GS	Payer	10.28	10.66	0.38	10.67	0.39	11.12	0.84
	Receiver	9.88	10.57	0.69	10.20	0.32	10.79	0.91
C	Payer	5.71	5.88	0.17	6.09	0.38	6.34	0.64
	Receiver	5.43	5.84	0.41	6.11	0.67	6.28	0.85
MER	Payer	7.97	8.25	0.28	7.94	−0.04	8.32	0.35
	Receiver	7.71	8.00	0.29	7.68	−0.02	8.22	0.51
BAC	Payer	5.09	5.17	0.09	5.11	0.02	5.16	0.07
	Receiver	4.94	5.08	0.14	5.15	0.20	5.12	0.17
JPM	Payer	4.51	4.60	0.09	4.52	0.01	4.65	0.14
	Receiver	4.37	4.58	0.21	4.63	0.26	4.74	0.37

References

[1] Bank for International Settlements. (2017). BIS Statistical Bulletin. June 2017.

[2] Basel Committee on Banking Supervision. (2006). Basel II: International Convergence of Capital Measurement and Capital Standards: A Revised Framework - Comprehensive Version. Bank for International Settlements.

[3] Basel Committee on Banking Supervision. (2011). Basel III: A Global Regulatory Framework for More Resilient Banks and Banking Systems. Bank for International Settlements.

[4] Black, F. (1976). The pricing of commodity contracts. Journal of Financial Economics, 3(1–2), 167–179.

[5] Brigo, D., and Alfonsi, A. (2005). Credit Default Swap Calibration and Derivatives Pricing with the SSRD Stochastic Intensity Model. Finance and Stochastics, 9(1), 29–42.

[6] Brigo, D., Capponi, A., Pallavicini, A., and Papatheodorou, V. (2013). Pricing Counterparty Risk Including Collateralization, Netting Rules, Re-Hypothecation and Wrong-Way Risk. International Journal of Theoretical and Applied Finance. 6(2).

[7] Brigo, D., and Masetti, M. (2006). Risk Neutral Pricing of Counterparty Risk. In: Pykhtin, M. (Editor), Counterparty Credit Risk Modelling: Risk Management, Pricing and Regulation. Risk Books, London.

[8] Brigo, D., and Pallavicini, A. (2008). Counterparty Risk and Contingent CDS under correlation. Risk Magazine, February issue.

[9] Brigo, D., Pallavicini, A., and Papatheodorou, V. (2009). Bilateral counterparty risk valuation for interest-rate products: impact of volatilities and correlations. Working Paper, arXiv:0911.3331v3.

[10] Carr, P., and Ghamami, S. (2017). Derivatives pricing under bilateral counterparty risk. Journal of Risk, 20(1), 77–107.

[11] Casassus, J., Collin-Dufresne, P., and Goldstein, R. (2005). Unspanned stochastic volatility and fixed income derivatives pricing. Journal of Banking and Finance, 29(11), 2723–2749.

[12] Černý, J., and Witzany, J. (2015). Interest Rate Swap Credit Valuation Adjustment. The Journal of Derivatives, 23(2), 24–35.

[13] Cherubini, U. (2013). Credit Valuation Adjustment and Wrong Way Risk. Quantitative Finance Letters, 1(1), 9–15.

[14] Collin-Dufresne, P., and Goldstein, R. (2002). Do Bonds Span the Fixed Income Markets? Theory and Evidence for Unspanned Stochastic Volatility. Journal of Finance, 57(4), 1685–1730.

[15] Cox, J. C., Ingersoll, J. E., and Ross, S. A. (1985). A Theory of the Term Structure of Interest Rates. Econometrica, 53(2), 385–407.

[16] Duffee, G. R. (1996). On Measuring Credit Risks of Derivative Instruments. Journal of Banking and Finance, 20(5), 805–833.

[17] Duffie, D., Pan, J., and Singleton, K. (2000). Transform analysis and option pricing for affine jump-diffusions. Econometrica, 68(6), 1343–1376.

[18] Fong, H. G., and Vasicek, O. A. (1991). Fixed-income volatility management. Journal of Portfolio Management, 17(4), 41–46.

[19] Gargouri, A., Lai, V.S., and Soumaré, I. (2017). Revisiting Interest Rate Swap Valuation with Counterparty Risk, Wrong-Way Risk, and OIS Discounting. The Journal of Fixed Income, 26(3), 63–80.

[20] Gatarek, D., and Jablecki, J. (2016). Modeling joint defaults in correlation-sensitive instruments. Journal of Credit Risk, 12(3), 15–42.

[21] Gregory, J. (2012). Counterparty Credit Risk and Credit Value Adjustment: A Continuing Challenge for Global Financial Markets. John Wiley and Sons.

[22] Harris, G. R., Wu, T. L., and Yang, J. (2015). The relationship between counterparty default and interest rate volatility and its impact on the credit risk of interest rate derivatives. Journal of Credit Risk, 11(1), 93–127.

[23] Heath, D., Jarrow, R. and Morton, A. (1992). Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation. Econometrica, 60(1), 77–105.

[24] Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. The Review of Financial Studies, 6(2), 327–343.

[25] Hofer, M. (2016). Path-consistent wrong-way risk: a structural model approach. Journal of Risk, 19(1), 25–42.

[26] Hull, J. and White, A. (2012). CVA and Wrong-Way Risk. Financial Analysts Journal, 68(5), 58–69.

[27] Hull, J. and White, A. (1990). Pricing Interest Rate Derivative Securities. The Review of Financial Studies, 3(4), 573–592.

[28] Longstaff, F. A., and Schwartz, E. S. (1992). Interest Rate Volatility and the Term Structure: A Two-Factor General Equilibrium Model. The Journal of Finance, 47(4), 1259–1282.

[29] Nelson, C. R., and Siegel, A. F. (1987). Parsimonious Modeling of Yield Curves. The Journal of Business, 60(4), 473–489.

[30] Svensson, L. E. O. (1994). Estimating and Interpreting Forward Interest Rates: Sweden 1992–1994. IMF Working Paper, WP/94/114.

[31] Trolle, A., B., and Schwartz, E. S. (2009). A General Stochastic Volatility Model for the Pricing of Interest Rate Derivatives. Review of Financial Studies, 22(5), 2007–2057.

[32] Vasicek, O. A. (1977). An equilibrium characterization of the term structure. Journal of Financial Economics, 5(2), 177–188.