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Joint effect of commodity price and geological uncertainty over the life of mine and ultimate pit limit

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Abstract: Mining operations are highly affected by risk, commodity price and geology being acknowledged as the most relevant risk factors. Considering these uncertainties at an early stage is a key element of project success, and doing so by including annual re-evaluations of a project allows for a more realistic approach. A real-options based evaluation is presented herein to assess the effects of uncertainty over the life of an open pit mining operation and the consequent ultimate pit limit modifications. A case study at a gold mine shows the value of having flexibility to expand or stop mining early when dealing with stochastic scenarios of both the deposit being assessed and related commodity prices. Stochastic scenarios provide useful probabilistic data that traditional methods ignore, inspite the fact that they have a major effect on optimal pit limits and mine design. A geometric Brownian motion with Poisson jumps model is used to forecast price, and direct block simulation is used to model geological uncertainty. The proposed model is optimized using the commercial software ILOG-Cplex; results show that including the option to stop mining or expand increases the operation's value, allowing management to assess mine designs and prepare for changes that may have substantial ramifications.

1 Introduction

Mine reserves are function of both internal and external uncertainties, making mining projects one of the riskiest investment subjects in the market and at the same time, one of the most profitable. The complex combination of uncertainties makes it challenging for mine planners to determine, with some reasonable certainty, strategic design variables such as production capacity and the life-of-mine at the planning stage of the project. In response, later adjustments to mine designs and plans must be done according to the changing technical and economic context of the project. These adjustments are typically not considered in the initial project evaluation stages and this can undervalue projects while prevents preparing in advance as well as taking advantage of opportunities or hedging from risk.

Traditional evaluation methods deal with uncertainty by assuming an average case scenario, usually chosen conservatively, and then optimizing the project as if all inputs used are known. This has been shown to lead to deviations from targets and suboptimal mine plans, as well as creates misleading designs (e.g. Albor and Dimitrakopoulos, 2009, 2010; Dowd, 1994). In other words, traditional evaluation is a heavy static response to a continuously changing reality. An alternative approach considers the 'stochastic nature' of the variables involved, which facilitates designing flexible operations, allowing the planning process to include decision-maker's reactions to future conditions and simultaneously optimizing the timing of the reaction's costs. These probabilistic approaches (Dimitrakopoulos, 2011) provide a tree of possible outcomes and potential project values which constitute a better representation of the project's actual performance.

Goal of open pit mine planning is to first determine an optimal extraction schedule or life-of-mine (LOM) production schedule that maximizes project value and, subsequently, decides where/when to stop mining. The latter defines the ultimate pit limit of the deposit and the infrastructure location for the operation. The current study focuses on the second stage of the mine planning process, assuming that the initial stage has already been optimized, by creating a model that considers metal price and geological uncertainties to determine the life-of-mine of a mining operation and with this, a stochastically-defined optimal ultimate pit limit. A key topic to stress is that if a mining operation is not prepared for pit limit variations, there are various potential problems that may arise and prevent the operation from benefiting from opportunities. A typical example of this is placing infrastructure in strategic locations where an open pit could expand. Similarly, having leases and contracts that do not allow for flexibility, make it hard for decision makers to hedge from unfavorable scenarios.

Because of the high costs associated with exploration and the limited information obtained from drilling composites, geology and metal content within a mineral deposit is highly uncertain. Godoy and Dimitrakopoulos (2004) state that geological uncertainty is a major contributor to not meeting project expectations, and demonstrate that accounting for it significantly reduces the deviation from production targets and increases project value. Dowd (1994) refers to geological uncertainty as the most significant factor in mine planning, and uses conditional or stochastic simulation to assess technical risk. Conditional simulation is a methodology that, amongst other uses, can be utilized to quantify the risk associated to grade and tonnage assessment from sparse data by generating equally probable representations of the orebody. These representations respect the spatial correlation and variability of the deposit, and provide a probabilistic assessment over a group of blocks (Ravenscroft, 1992; Dowd 1994, 1997; Goovaerts, 1997). This stochastic approach uses a set of equally probable orebody realizations as an input for developing optimal long-term open pit mine plans (Asad and Dimitrakopoulos, 2013). To date, this method has been successfully implemented in various projects, and will be used in the study described in Section 3. Further applications of the method in comparison with deterministic assessments can be found in Ramazan and Dimitrakopoulos (2013), Albor and Dimitrakopoulos (2010) and Dimitrakopoulos and Jewbali (2013).

Among the different stochastic simulation techniques, an efficient and straightforward method to generate multiple equiprobable representations of a mineral deposit is direct block simulation (DBSim), thoroughly described in Godoy (2002). DBSim is a step forward from the generalized sequential Gaussian simulation (GSGS) in Dimitrakopoulos and Luo (2004), which mixes the upside characteristics of the covariance matrix decomposition method (LU) with the qualities of the well-known sequential Gaussian simulation (SGS) (Goovaerts, 1997). LU is capable of simulating rapidly a group of nodes but is computationally expensive method, while SGS is easily implemented but becomes very slow as the number of nodes to simulate increases. The DBSim method divides the volume to be simulated into groups of nodes representing the selective mining unit (SMU) defined by the operation. Subsequently, each of the groups is visited following a random path, as in SGS, and inside each group, the internal nodes are simulated by LU decomposition, which in these conditions is a fast and feasible method given the reduced size of the groups.

The main difference between DBSim and the previous GSGS method, is that in this case, once the internal nodes of a group are simulated, they are averaged and only this value is stored, liberating the memory of storing each individual node, which at the end, would be averaged up anyway once the re-blocking process takes place. Because of this memory release, DBSim results are computationally inexpensive and simple to implement. It's important to note that, once the nodes of a group are averaged, one is left with a block value which must be used to condition subsequent blocks being simulated, so the covariance of block to block and block to node support are needed. This is a straightforward step; however, it is an important difference between this method and its predecessors. Because of the mentioned computational efficiency and practicality of DBSim, this method is used in the case study described herein to generate multiple simulations of the studied orebody.

In addition to geological uncertainty, despite the fact that market risk is widely acknowledged, for simplification purposes, projects are traditionally evaluated assuming certainty in the price trend. Price shifts have a decisive effect over a project, and although they can't be controlled, it is possible to increase the flexibility of the project in order to prepare it to react timely to these changes. Generally, a fall in commodity price causes the ore content of the final pit to be less valuable and, in addition, that less material is profitable to extract which triggers the final pit limit to shrink and the LOM to decrease (as the operation stops mining before expected). Similarly, if commodity price rises, the LOM will likely increase, as new material is now profitable to extract and, in consequence, the pit limit expands.

This ore/waste relation is mainly represented by the cut-off grade, which, together with the scheduling process, links geological and market uncertainty, as it is calculated as a function of price, operational capacities and costs. Many efforts have been made to obtain an optimized cut-off grade strategy that provides the desired production targets and maximizes project's value. Further details on cut-off grade selection may be found in Lane (1988), Rendu (2008), Asad and Dimitrakopoulos (2012), and others.

McCarthy and Monkhouse (2003) state that not considering the commodity price's uncertainty and the managerial flexibility in the evaluation process results in underestimations of the optimal LOM, which leads to plants with extra capacity, higher initial investments and, in general, a loss of capital. The same authors also mention that this can be handled by using real options valuation (RO) approach, which has shown to provide alternative results to account for the effects of market uncertainty over the project's value. In a standard RO model, the underlying state variable (i.e. the commodity price) is formulated as a stochastic process, enabling the examination of the uncertain behavior of the variable (Shibata, 2006); with this, the model is capable of valuing flexibility and integrating it into the project by considering decision-making. In mining, Moel and Tufano (2002) use RO to explain the operational state dynamics of a group of 285 mines, defining when to exercise and when to hold on to options of closing, subject to price uncertainty. This valuation method has been implemented to consider, mainly, commodity price and exchange rate uncertainties. Similarly, Sabour and Wood (2009) and Dimitrakopoulos and Sabour (2007) consider metal price uncertainty and show that RO method incorporates the ability of project managers to react to change based on new information. Other examples of RO over market uncertainties can be found in Lemelin et al. (2007), Cardin et al. (2008), Sabour and Poulin (2010).

To formulate price behavior as a stochastic process, price forecasting models are used which examine and include the price's variability in the evaluation model. To do this, forecasts traditionally recur to random walks, with different corrections according to the asset being evaluated. Dixit and Pindyck (1994) state that precious metals' behavior are better represented by geometric Brownian motions, dependent on a drift and a volatility whereas base metals follow a mean reverting process, i.e. they have a cyclical behavior tending to a long term price. Even though these methods may seem simplistic compared to newer models, they allow for a basic representation with a small number of parameters, and thus are easy to interpret and calibrate from market data, which reduces the chance of model errors. For these reasons, they have been used for decades, and are the pillars of newer models (Blanco et al., 2001). An extensive description of econometrics and price forecasting models based on random walks is given in Dixit and Pindick (1994) and Campbell et al. (1996). Newer and more sophisticated models are also available (Mingming and Jinliang, 2012; Jammazi and Aloui, 2012; Meade, 2010; Tan et al. 2010); within them, the integration of random walks with diffusion jump has proven to show better price behavior results when applied to energy and commodity prices, without the need of extensive assumptions as input (Shafie and Topal, 2010; Mendez and Lamothe, 2009; Blanco and Soronow, 2001). Because of this, the forecasts done for the case study presented will include a Poisson exponential jump diffusion model to the classic Geometric Brownian motion. Details on how to add the jump process are found in Appendix A.

Some authors have already attempted to create a joint model that considers price and geological uncertainties simultaneously. Sabour et al. (2008) refer to price, exchange rate and geological uncertainty, and develop a process to rank the simulated mine designs, in order to select the most favorable one. Meagher et al. (2009) include price and exchange rate variability, as well as geological simulations for pushback design by using minimum cut algorithm. Similarly, Asad and Dimitrakopoulos (2013) account for both uncertainties and represent the orebody as a directed graph to optimize pushback design; the authors use the maximum flow algorithm to formulate a maximum closure problem that provides operationally feasible pushbacks by minimizing their size difference, and at the same time, provides higher value and bigger pit limits when compared to deterministic designs. Other examples of joint analysis for price and geological uncertainty can be seen in Sabour and Poulin (2006), Musingwini et al. (2007) and Dimitrakopoulos and Sabour (2007). This paper will consider commodity price and geological uncertainty simultaneously in order to assess the optimal ultimate pit limits of a mine and obtain a quantitative method to calculate the potential value of further pit expansions.

The next section starts by describing the existing methodologies to model the different types of uncertainty, and continues by explaining the proposed method to assess the potential of pit expansion options. Subsequently, a case study shows the benefits of the method, and further explains its implementation. Finally, conclusions and implications for future research are presented.

2 Proposed uncertainty-modeling methodology

To allow for the joint consideration of market and geological uncertainties in the mine design and planning process, a three-step methodology is proposed. The first step consists of creating an analytical review of the project, such as its costing structure, financing requirements, flexibility opportunities, etc., thus generating a base case scenario. The second step looks to both create flexibility in the engineering design of the project at hand, and model the uncertainties acting over the project by generating stochastic price paths and orebody models. Finally, the third step consists of developing a flexible mine planning evaluation model considering an annual re-evaluation of the mine's operational state by integrating managerial flexibility and the option of re-deciding the destination of extracted blocks at each period; this is done according to the 'current' circumstances, both technical and economic. This last step can be also represented as the value of keeping the 'option of closing', instead of exercising it on a previous period.

Step 1: Project review and operational assumptions

To create the base case model, it is first necessary to specify the costing model (CAPEX and OPEX), the initial mining and processing capacities of the operation, the metal price (assumed constant), processing recovery and more, in order to design an optimal schedule for the estimated orebody model. Additionally, other financial data is required such as the operation's required continuing expenses, the depreciation method, taxes, etc. With this information, the mining schedule is generated, which provides the 'base case' ultimate pit limit, as well as initial project value. This corresponds to the conventional pit design and project evaluation, and will be used as point of comparison for the subsequent stochastic analysis.

As an initial step in this complex research direction, the schedule generated here is kept constant over the case study, fixing the material that is extracted in each period (from the beginning of the extraction, to the end of the initial life of mine).

Step 2: Stochastic modeling and creation of flexibility

The second step is divided in two sections: firstly, with the creation and inclusion of flexibility options into the project's design in order to make the operation more responsive in case the expected context changes; and secondly, with the generation of multiple equiprobable orebody simulations to account for geological uncertainty, and the forecasting of metal price to account for market variability. The last is done by using a price model chosen depending on the commodities involved and the historical information available (as mentioned in Section 3).

Flexibility is included in the mine design by modifying the pit design and ultimate pit limit defined by the schedule generated in the base case. As the study looks to assess the possibility of the operation expanding or contracting from its original limits, there must be a mining sequence available in case the operation decides to expand. To do this, a three step process is followed: first, the rock contained in the initial ultimate pit is removed from the orebody model. Second, the commodity price is increased over its regular value, in order to generate subsequent nested pits that extend further than the initial pit limit (Whittle, 1999), and a continuing schedule is done for the new material. Finally, as there is no common rock between the initial base-case pit and the expanded schedule, both models are combined to create an 'expanded schedule and ultimate pit limit'. This keeps the base case's schedule, pushback design and ultimate pit limit unchanged, and at the same time, defines a continuing mining sequence in case the operation decides to expand.

Just as in the base-case schedule, expansions are considered fixed over all the analysis. It must be noted that keeping the schedule fixed no matter what scenario is being evaluated is a limitation of the model presented, as the designed schedule is only optimal for the context in which it was created, and a change in geology or price would lead to scheduling changes. However, this is done here only as an initial step and further studies on this subject are proposed as future research.

Step 3: Flexible expansion evaluation

To evaluate a mine plan and assess the possibility of expansion, the 'expanded schedule' generated in the previous step is run over multiple equiprobable scenarios, where a scenario consists on one of the orebody simulations and a price path forecasted, which account for ore grade and commodity price uncertainties respectively. This way, for each period (simulation and current metal price) it is possible to obtain the annual revenue of the base case, as well as for each scenario. Together with this, managerial flexibility is considered by re-defining the destination of each block in every period: mill, if rock grade is above the cut-off, and waste dump, if it is not. This decision is taken by maximizing the project value based on a changing cut-off grade, depending on the current selling price as well as the grade-tonnage curve of the orebody model. All the optimization work is done with the optimization tool CPLEX from IBM (ILOG CPLEX v12.1 User's Manual for CPLEX, 2010).

For this study, the project evaluation is performed on an annual basis, along with the standard yearly long-term plan of an open pit mine, so that the decision to extract or stop mining is revised at the end of each year, while considering respectively the capital and operational expenses on the first case, and the closing costs on the second. The actual project evaluation is done as a financial American option by considering the flexibility of being able to close (or exercise the option to stop mining) at any time, incurring in the corresponding costs (Hull, 1997). This means that for a given scenario the project is evaluated backwards, from the ultimate (expanded) pit limit, when the operation stops mining and the LOM is reached, up to the starting year, along the whole forecasted price path. The evaluation is done sequentially for consecutively decreasing closing years and, by optimizing with the objective of maximizing the value of the mining operation, it is possible to obtain the probability of being operational or closed at time t, what provides a reliable range of feasible project values.

The previous methodology is represented in Fig. 1, where Mine 'X' is evaluated from its ultimate pit limit, achieved by year 'N'(where is the maximum LOM, or $\max LOM$ including all the expansions available), until 'today', for decreasing values of N. Every year, the operation has the probability of expanding ($\mathbf{p}_{\mathrm{E}}$), or stopping mining ($\mathrm{p}_{\mathrm{C}} = 1 - \mathbf{p}_{\mathrm{E}}$), and the project value is calculated as the maximum between the sunk costs, and the total project value of closing on year 't' ($TPV_{C(t)}$), for $t = \{0, \ldots, N\}$. This way, the evaluation

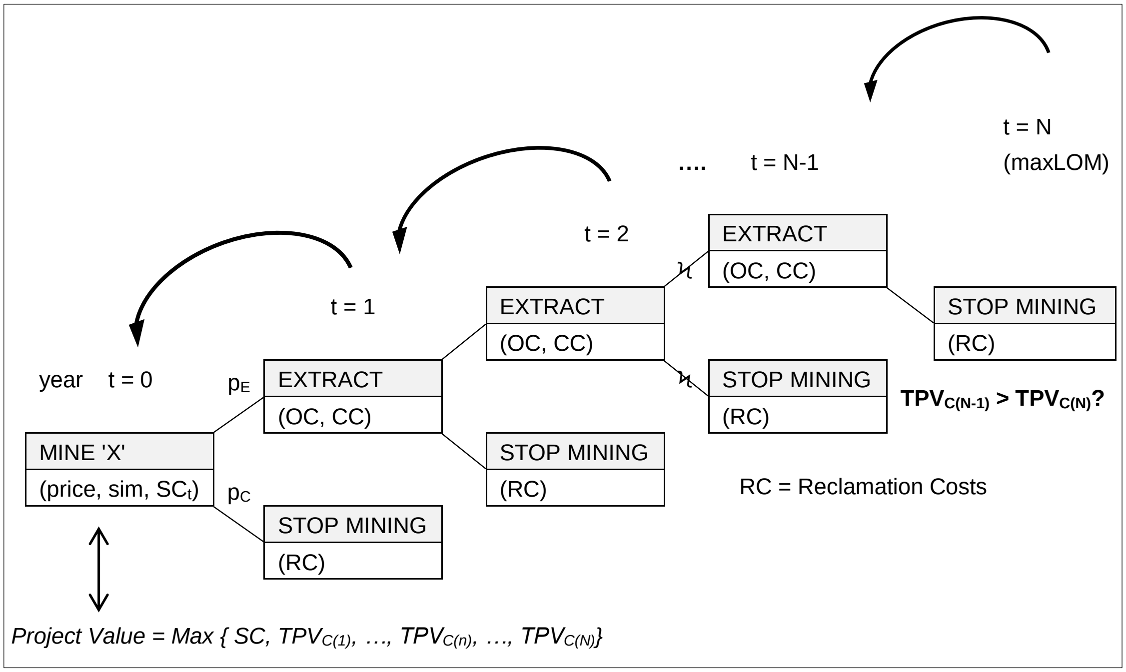


Figure 1: Evolution of the proposed stochastic mine planning evaluation model

process looks backwards, one year at a time, checking if the project's value would increase if the operation closed earlier (i.e. if $TPV_{C(t-1)} > TPV_{C(t)}$), considering the ore available and the time value of money.

With this analysis, the LOM distribution is obtained by taking the argument of the project's maximum value, which, assuming that the current time is 't^*', the orebody is simulation 'S' and the price path is forecast 'P', is represented in Eq. (1).

$$LOM(t^*, S, p) = \underset{t^* \leq T \leq \max LOM}{\arg\max} \left(\sum_{t=t^*}^{T-1} V_t + V_{C(T)} \right) = \underset{t^* \leq T \leq \max LOM}{\arg\max} (TPV_{C(T)}) \tag{1}$$

where:

$$V_{E(t)} = f\left(S, p, Ore_{t,S}, CAPEX_{t,S}, OPEX_{t,S}\right) = \text{Value of extracting in year '}t\text{'}$$

$$V_{C(T)} = f\left(\sum_{t=0}^{T} Ore_{t,S} | S, p\right) = \text{Value of stopping mining at period '}T\text{'}$$

$$TPV_{C(t)} = \text{Total project value if mining stops in period '}t\text{'}$$

For example, if the operation decides to stop mining at period 'T'($LOM = T$, with $T > t^*$), this means that (i) there is an overall positive value for continuing extracting, and (ii) that the maximum profit (considering the costs) is obtained by operating until the end of year 'T' (i.e. $TPV_{C(T)} > TPV_{C(t)}, \forall t = t^*, \ldots, N$), even if at any given time 't' extracting is temporarily not profitable (with $t^* < t < T$). This means that even if the costs of extracting on a given year are higher than the revenues obtained, the operation should continue if there is a subsequent period that presents a '*minimum profit*', which is high enough to pay for its extraction costs and the previous year's losses. In other words, to continue the operation past the current moment 't^*', the following condition must be met:

$$\left(\sum_{t=t^*+1}^{T-1} V_t \right) + V_{C(T)} \geq \text{ Minimum Profit}, \quad \forall T = \{t^* + 1, \ldots, \max LOM\} \tag{2}$$

If this condition is satisfied for any 'T' ($T = \{t^* + 1, \ldots, \max LOM\}$), then the operation 'expands' to the next period ('$t^* + 1$'), if not, it means that no combination of future extractions will increase the value of the project and so, the operation should stop mining in that period. In any case, the reclamation costs are considered to be incurred in time 'T', and are calculated depending on the cumulative ore production up to that year.

To obtain the ultimate pit limit and LOM probability distribution (i.e. the probability to stop mining at each year, together with the range of project values), this process is repeated for all simulations and for thousands of price paths. This practice provides useful information by periodically allowing new external information to be included and used in the planning process. For example, if the mine is in operation on a given year, it is useful to know what is the probability of obtaining a higher project value if the production continues (and for how long it should continue), in order to plan for infrastructure and equipment arrangements ahead of time.

It is assumed that the decision to stop mining is irreversible, and that if the operation decides to continue in production, this decision is maintained until the next year's re-evaluation (i.e. if production continues, the whole tonnage considered in that period must be extracted). Metal price is also assumed constant throughout one period. This way, an optimum LOM which maximizes the project value is obtained for each simulated price path and simulated orebody.

To assess the influence of each individual variable in the performance of the operation and the ultimate pit limit, the case study presented next performs separate analysis for each variable. More specifically, firstly, considers multiple orebody realizations over a constant price path; then, evaluates the performance of the basic estimated orebody model over multiple stochastic price paths, and finally, integrates both uncertainties through joint evaluation.

3 Case study

The proposed methodology is demonstrated at an open pit gold mine to show how these concepts are applied in a real world problem, and the advantages that the information obtained can provide in the strategic decision-making processes. The gold mine being evaluated has an extraction capacity of 15Mtpa, and a processing capacity of 5Mtpa. The operation doesn't consider stockpiling, so at each period the extracted material is either taken to the mill to be processed at a cost of 13 US\$/t, or to the waste dump, at only a transportation cost; in any case, material is extracted and transported at a cost of 1.8 US\$/t. Additionally, it is assumed that the mining targets of each period are met, and that the mill will process rock as long as it has a grade higher than the cut-off grade selected for that period. For this case study, the block destination is defined by using Lane's optimal cut-off grade selection to maximize NPV according to the mine and mine-mill capacity relation (and, respectively, below in Eqs. (3) and (4)), where the maximum cut-off value between them is selected, and corresponds to the minimum grade that a block must have to be transported to the mill to be processed, instead of the waste dump. The equations used to define these cut-offs are shown next.

$$g_m = \frac{\text{Processing Cost}}{(\text{Selling Price} - \text{Refining Cost}) \times \text{Recovery}} \tag{3}$$

$$g_m = \left(\frac{\text{Processing Capacity}}{\text{Mining Capacity}} - \frac{Ore(k^*)}{\text{Total Rock}}\right) \cdot \frac{g(k^*+1) - g(k^*)}{\left(\frac{Ore(k^*+1) - Ore(k^*)}{\text{Total Rock}}\right)} + g(k^*) \tag{4}$$

Calculating g_m for each period from Eq. (3) is straightforward. However, to obtain g_{cm}, the grade-tonnage curve is used (see Fig. 4 as reference). Let $k^* \in K$, $K = \{1, \dots, k\}$ be a certain section defined by two consecutive cut-offs ('x' axis in Fig. 4); in the previous equation, $g(k^*)$ and $g(k^*+1)$ correspond to the lowest grades (cut-off grades) of two consecutive sections, and $Ore(k^*)$ and $Ore(k^*+1)$ correspond to the total amount of ore available with those corresponding cut-offs (Asad, 2007; Asad and Dimitrakopoulos, 2012).

From the previous equations, it can be seen that g_m provides the minimum cut-off grade for the operation to profit from the processing of the rock (to cover all the corresponding expenses), and g_{cm} ensures that, given the amount of ore available for a given cut-off grade, the capacities are met but not exceeded. This way, if for example, the price is high, g_{cm} will probably define the cut-off, as the tonnage of profitable rock may exceed the mill's capacity, and if the price is low, g_m will define the cut-off, as it may be not profitable to process most of the rock, even if there is mill capacity left unused.

The operational expenses, together with the initial investment and the present value of the continuing capital costs (equipment, infrastructure and closing costs) are shown in Table 1. For simplicity, it is assumed that the project is fully founded by its owners, and the depreciation is done linearly over 5 year period.

Table 1: Case study's cost structure

Cost Structure			
OPEX	Mining Processing Selling	US$/t rock US$/t ore US$/oz.	1.8 13.0 5.0
CAPEX	Initial Investment	MUS$	350
	Infrastructure Maintenance Equipment Closure	MUS$ (r = 8%)	70
Tax over Revenue		%	18

3.1 Base case

Initially and to build a base case, no uncertainty is taken into account and the project is evaluated in the conventional way of static discounted cash flow. An initial gold price of 700 US$/oz. is considered, which increases linearly to 900 US$/oz. with an annual growth of 50 US$/year. The deposit is discretized in eleven thousand blocks of 15×15×10 meters. The base-case orebody model is assumed perfectly known (referred to as "E-type") and is obtained by averaging the grades of 20 conditional simulations of the orebody. This averaging generates a smoothed representation of the data that generally presents errors related to an overestimation of the amount of ore and an underestimation of the ore's grade, or vice-versa (for further details on effects of estimated vs. simulated orebody models see Albor and Dimitrakopoulos, 2009).

To define the destination of each block (waste dump or mill), the cut-off grade is calculated for each period, as explained in Eqs. (3) and (4), according to the gold's current selling price and the mine's global grade-tonnage curve. This curve is obtained from the schedule designed using Milawa NPV algorithm included in the Whittle Optimization Software (Whittle, 2009), generated by optimizing over the mentioned price trend. In this case, the average cut-off grade is of 0.39ppm, which results in a total of 53.2 million tons of ore. This and other base case assumptions are presented in Table 2. With this information, a base-case cash flow is conducted, presenting a net present value (NPV) of 89.2M USD$ for the initial pit limit and resulting in an 11 year LOM operation according to the base case schedule.

To create and include flexibility in the mine design, a revenue factor from 0.3 to 3.0 is used to increase the metal price and schedule further extractions past the ultimate pit limit. This is done by exporting the blocks from the initial pit, and re-scheduling the remaining 'ore' using again the Milawa NPV algorithm. Figure 2, shows a cross-section of the deposit with the initial ultimate pit limit, and the flexibility of possible expansions, presented as two further stages. This scheduling increases the operation's LOM up to 15 and 16 years respectively; however, according to this static net present value evaluation, the mining extensions are not profitable, reducing the project's value by 2.4% and 2.5% respectively (please see Table 3). Even though these value reductions may seem marginal, they do cause the rejection of the expansions with the traditional scheduling process, and leave the ultimate pit limit as an 11-year design.

3.2 Stochastic case

The previous discounted cash flow (DCF) results show that there is a 100% probability of the mine operating until year 11 and a 0% chance of expanding any further. If the variable's evolution was perfectly known,

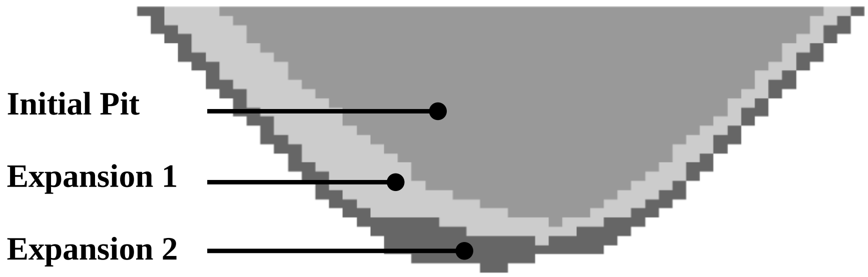


Figure 2: Cross section of the deposit for the initial pit and the available expansions

Table 2: Base Case data and operational assumptions

Base Case		
Price of Au	US$/oz.	700–900
Reserves	Mt	53.3
Extraction Rate	Mtpa	15.0
Processing Rate	Mtpa	5.0
Discount Rate	%	8
Recovery	%	90
Average grade of ore	ppm	1.20
Average cut-off grade	ppm	0.39
NPV	**MUS$**	**89.2**

Table 3: Economic evaluation results for the base case analysis

Base Case (DCF)	
Initial Ultimate Pit (LOM=11)	$ 89,238,244
Value Expansion 1 (LOM=15)	–$ 2,120,700
Value Expansion 2 (LOM=16)	–$ 2,250,100

this would be the case; however, the mine's context will certainly change and, by acquiring new information, decision-makers will adapt the operation accordingly. This is why the main evaluation advantage of real options (RO) is that, in comparison to the static DCF valuation, RO do consider the dynamic quality of the decision-making process, quantify it and include it in the project's value. This is in comparison to the DCF evaluation, which assumes that nothing will change from the moment the evaluation is done until the end of the project's life.

3.2.1 Case of stochastic geology

To quantify and account for the stochasticity of the grade in the orebody, the direct block simulation method mentioned earlier is used to create 20 equally probable representations of the orebody. It has been shown that after a certain number of simulated orebodies, usually around 15, risk analysis over different schedules and mine planning parameters converge to stable results (Albor and Dimitrakopoulos, 2009). This is not surprising and reflects well-understood support-scale or volume-variance effects. Accordingly, life-of-mine planning seeks the total material or group of blocks to be extracted each production year (some thousands of mining blocks) and not each mining block individually, the related volumes and variability of grades and material types become orders of magnitude lower, leading to stable schedules and converging production forecasts based on a relatively low number of simulated orebodies. An additional obvious positive aspect of this is that the computational intensity of both generating and using orebody simulations becomes substantially more efficient and practical. Given the above, the 20 simulated models of the orebody, as used in this study, is a sufficiently large number to quantify the pertinent uncertainty. This means that adding more realizations does not affect the results presented herein.

Just as in the base case, the cut-offs are calculated for each of the orebody simulations by using Lane's formulas and are re-estimated in every period. This means that the destination of each block is re-defined subject to the geology that is encountered in the orebody simulation, and managerial flexibility is considered by having the option to expand the operation past its initial limits, if the conditions are favorable (this will be referred to as the flexible case 'FC').

The difference between the initial base case and the simulations is presented in Fig. 3a for ore quantity and Fig. 3b for NPV. The simulated models present in average 14% less ore than the base case, but at the same time, more than a 21% increase in the NPV. This happens because the smoothness of the estimated model (E-type in graphs) causes the deposit to contain more medium grade blocks, and in this case, it increases the total ore tonnage (as the block's grade distribution is mostly over the selected cut-off grade), which causes extra processing costs without the benefit of more metal being produced. Simulations present higher grade dispersion with extreme values, making ore blocks more profitable to process, as there are fewer blocks with higher grade (that is, more revenue with fewer costs). This fact if further presented in Fig. 4, where it is possible to see the grade tonnage curve for each of the 20 simulations, as well as for the estimated model. This graph shows, for different cut-off grades, the amount of ore available (over that given cut-off), as well as the average grade of the ore considered. Thus, for higher cut-offs, there is less tonnage available, but the average grade increases. The 20 simulations tend to have a similar behavior, with a maximum variability of about 25 thousand tons, and a maximum grade variability of 0.5ppm. However, there is a high difference between the simulations and the estimated model, with almost a 75 thousand ounce difference of gold and 1ppm difference in average grade.it can be seen that the e-type model tends to overestimate the tonnage for low to medium cut-off grades (between 0.6 and 1.3ppm), but strongly underestimates the tonnage of high grade material (over a cut-off grade of 1.5ppm). In summary, as explained earlier, the estimated model overestimates medium grade tonnage, but at the same time, underestimates the average grade of the ore available.

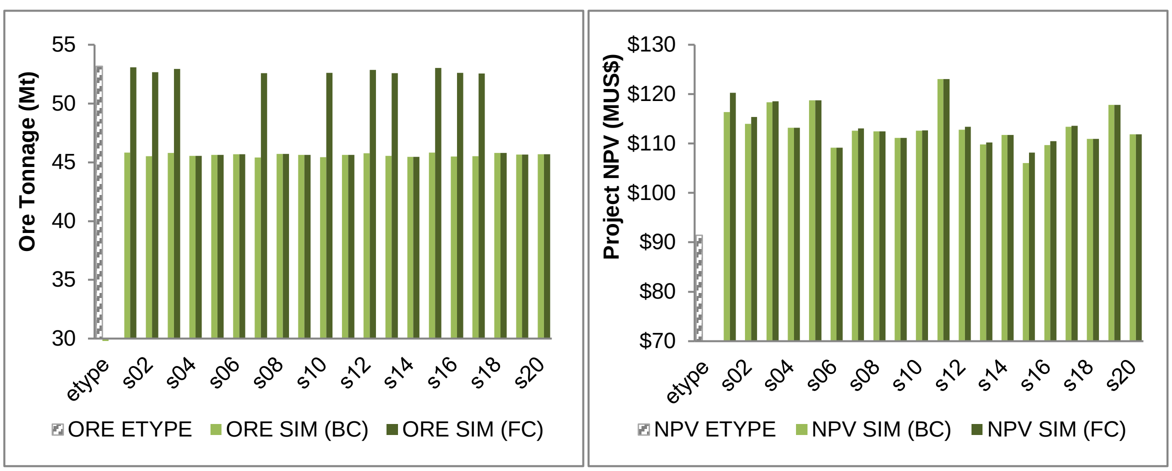


Figure 3: Results of base case and stochastic orebody simulations over the (a) Ore tonnage and (b) NPV

The evaluation is done considering the initial ultimate pit, or base case (BC) design of 11 years of LOM, and the expanded schedule of up to 16 years. Results show that, if the flexible option is considered, 50% of the time the operation will decide to expand up to the 15-year LOM pit limit. As mentioned earlier, just by considering geological simulations, project value is on average, increased by 21%. If together with this the option to expand is included, this difference increases to a 22% compared to the initial evaluation (as presented in Table 4). Even though the increased value that is purely due to the expansion may seem marginal, longer projects allow for new opportunities, and usually have a better social acceptance. These results are presented in the right-most columns of Figs. 3a and 3b, under the label of 'FC'. In this case, the second stage expansion is not profitable for any of the related simulations.

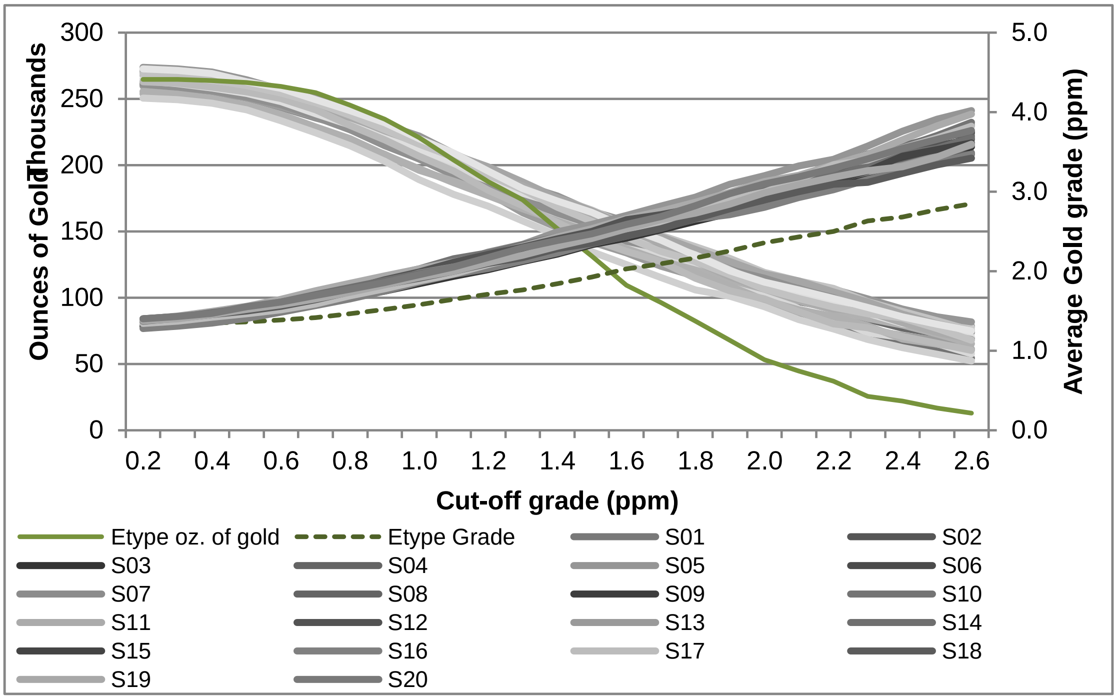


Figure 4: Grade tonnage curve for the average etype model, and for the 20 orebody simulations for different cut-off grades

Table 4: Economic evaluation results for the uncertain geology case

ROV - GEOLOGY CASE	
Initial Ultimate Pit (LOM=11)	$ 113,281,947
Value Expansion 1 (LOM=15)	$ 506,450
Value Expansion 2 (LOM=16)	$ –

For clarification purposes, the ROV of the geological uncertainty case is calculated by considering the optimal state of each simulation, considering that decision-makers have free will to close the operation early or keep extracting past the initial limits. Table 5 details the annual operational and capital costs, the ore production tonnage and average grade for the initial pit limit, the two possible expansions for the estimated model (‘ETYPE’), and the 50% percentile value of the simulated orebodies (‘SIM P50’), which means that half of the simulated scenarios exceed the presented value. The reclamation costs are specified in the final column of Table 5, and are calculated proportionally to the cumulative ore tonnage extracted, while they correspond to the costs that the project must incur only on the year that they decide to stop mining and close the operation. This table also shows that the first year of any of the two expansions requires extra capital expenses, mainly for mining works, accesses, scaling and support.

A summary of the project’s cash flow for each of the cases studied is presented in Table 6. The first section shows the base case’s results, and the second case presents the results of the stochastic geology case just described. Even though the NPV case presents higher revenues, the costs are also higher. This is caused because of two reasons: first, the overestimation of ore tonnage in the estimated model depicted in Fig. 3a which results in the processing costs to peak (OPEX), and second, because this static evaluation does not consider the possibility that the project’s value may actually increase if the operation closes later or earlier than expected, depending on the turn of events. This last possibility is also considered in cases 3 and 4 in Table 6, described in the following sections.

Table 5: Operational data of the initial pit and available expansions for the estimated model and simulated models

	YEAR	ORE (Mt)		GRADE (ppm)		OPEX (US$/t)		CAPEX (MUS$)		RECLAMATION (MUS$)	
		ETYPE	SIM (P50)	ETYPE	SIM (P50)	ETYPE	SIM (P50)	ETYPE	SIM (P50)	ETYPE	SIM (P50)
Initial Ultimate Pit	**0**							350.00	350.00		
	1	4.83	4.17	1.34	1.51	90.79	82.23	15.60	14.55	0.80	0.70
	2	4.78	4.23	1.44	1.64	90.20	83.04	7.80	7.30	1.60	1.40
	3	4.95	3.90	0.97	1.14	83.13	69.33	3.73	3.31	2.43	2.05
	4	4.99	4.05	1.04	1.18	76.67	64.41	9.09	8.72	3.26	2.73
	5	4.99	4.40	1.27	1.41	90.39	82.62	5.96	5.72	4.09	3.46
	6	4.97	4.17	1.30	1.50	92.64	82.13	5.49	5.16	4.92	4.15
	7	4.96	4.22	1.01	1.14	80.13	70.35	3.42	3.13	5.75	4.85
	8	4.87	4.51	1.51	1.60	91.50	86.83	9.45	9.31	6.56	5.61
	9	4.96	4.03	0.96	1.11	92.29	80.09	6.36	5.99	7.39	6.28
	10	3.85	3.63	1.46	1.53	77.92	74.97	5.04	4.95	8.03	6.88
	11	4.99	4.43	1.18	1.29	91.03	83.67	4.44	4.22	8.86	7.62
Exp. 1	**12**	1.20	1.07	1.20	1.31	41.88	40.21	**17.37**	**17.32**	9.06	7.80
	13	2.92	2.58	1.08	1.18	65.47	61.04	5.54	5.41	9.55	8.23
	14	1.24	1.03	0.93	1.08	43.33	40.52	4.00	3.91	9.75	8.40
	15	2.94	2.46	0.96	1.11	65.64	59.41	3.21	3.12	10.24	8.81
Exp. 2	**16**	0.15	0.14	0.94	1.06	9.68	9.45	**3.21**	**3.09**	10.27	8.83

Table 6: Cash flow summary for the four stages of the analysis

Cash Flow (MUS$)	(1) NPV - BASE CASE		
	Init Pit	Exp 1	Exp 2
Revenue	1,790.08	2,006.40	2,016.08
OPEX	956.68	1,173.00	1,182.68
CAPEX	432.88	463.01	466.22
NPV	**89.24**	87.12	84.87
Cash Flow (MUS$)	**(2) ROV - GEOLOGY**		
	Init Pit	**Exp 1**	**Exp 2**
Revenue	1,750.59	1,851.19	1,960.86
OPEX	858.47	959.07	1,068.74
CAPEX	428.74	458.48	461.60
NPV	113.28	**113.78**	111.21
Cash Flow (MUS$)	**(3) ROV - PRICE**		
	Init Pit	**Exp 1**	**Exp 2**
Revenue	1,342.58	1,819.96	1,898.21
OPEX	854.35	967.77	968.36
CAPEX	421.51	447.83	448.56
NPV	123.92	140.22	**145.95**
Cash Flow (MUS$)	**(4) ROV - GEO & PRICE**		
	Init Pit	**Exp 1**	**Exp 2**
Revenue	1,265.09	1,741.89	1,820.30
OPEX	774.96	887.80	888.43
CAPEX	424.60	444.89	445.62
NPV	135.64	158.39	**158.58**

3.2.2 Case of stochastic market

The following case looks at the effect of gold price variability over the project's evaluation, and the influence this has over the operation's size and potential expansions. In this case, the orebody is represented by the estimated model, so the only difference from the base case is the inclusion of a stochastic price forecasting model.

Historical data can be used to obtain the required parameters of the formulation. Figure 5 shows the monthly gold price from January 1990 to December 2012, where the positive drift as well as the occasional jumps can be clearly perceived. For this case study, gold prices where simulated using a geometric Brownian motion with a Poisson exponential jump diffusion model, so as to include the sudden extreme changes in price that have been seen recently in gold's price behavior. The parameters for the model where obtained by maximum likelihood over the past 15 years of price data, presenting a volatility of 13.8%, and a drift of 2.8%. In this case, a jump is defined as a change of price in two consecutive periods of more than 3 standard deviations along the 15 year period. If a jump is found, its value is removed from the time series and the standard deviation is re-calculated to study the presence of more jumps. In this case, this process is done three times, which gives a frequency of 0.1 jumps per year, with a jump size of 10% (difference in size between the moment after the jump and its previous period), and a jump volatility of 15% (difference in size between different jumps). In this case, the direction of the jump used for the simulation is 50% probability upwards and 50% probability downwards. For further details on implementing the jumps and the parameters required, please refer to Appendix A.

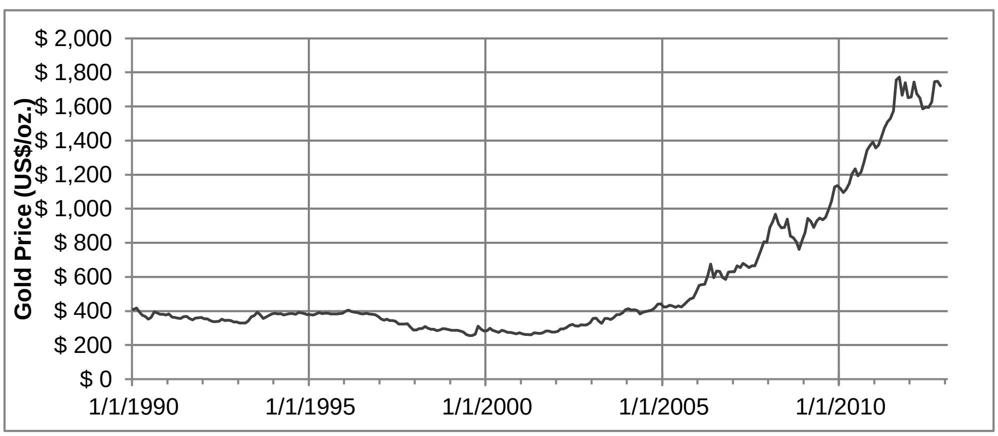


Figure 5: Historical monthly gold price data from January 1990 to December 2012

For clarification purposes, one price simulation is a price path or vector of annual gold prices, which starts form 'today' at the current initial price of 700 US$/oz. (as in the base case), and evolves stochastically until time 'T', which corresponds to the maximum life of mine considering all expansions available. This means that all simulations share the same starting point of 700 US$/oz. and spread out proportionally to time, according to the model and its parameters. Figure 6 presents 10 price paths that show the price evolution along the periods, where it is possible to see the effect of the added "jumps" into the model. Price forecast 1 (PF 1) presents an upwards jump along year 10, and has a favorable trend overall. PF 2 shows an upwards jump along year 1, and a downwards jump during year 8, which reduces the overall variability of that particular path. Finally, PF 3 shows a downwards jump along year 2, and steady variability from there on, providing a negative scenario to evaluate. From this, it can be seen that the forecasting model developed will provide an overall analysis of the possible price behaviors along the evaluation period.

The previous stochastic price model is applied over the operation, and the project is evaluated as explained in Section 3, assuming that the schedule is fixed for the initial pit limit as well as for the expansions. In this case, 20,000 price simulations are generated; as the computational cost of generating each forecast is marginal, and given the high variability of commodity prices per year, and unlike the case of stochastically and spatially

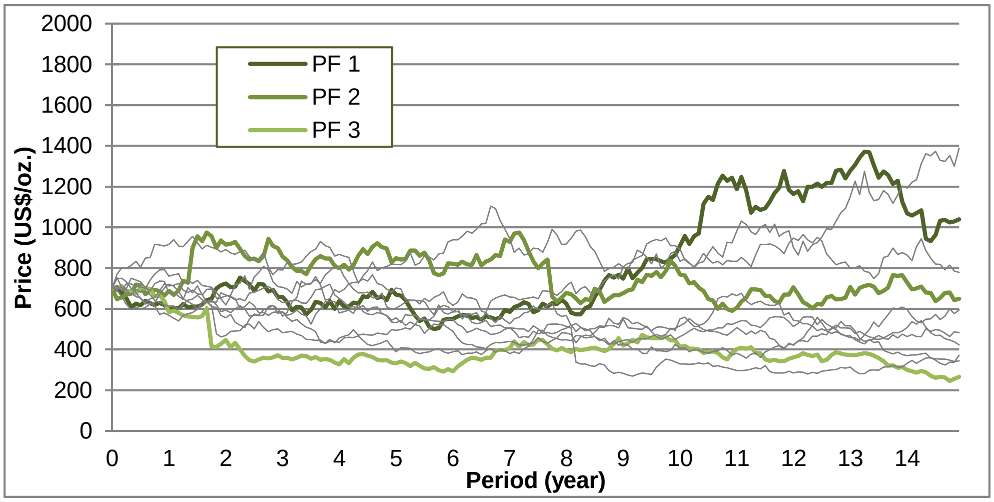


Figure 6: Example of 10 price forecasts generated with parameters obtained from historical data

simulating orebodies, more commodity price simulations provide a better range representing the commodity price's distribution over time, which over 16 years (time span of the forecasts) varies considerably. However, it is seen that the results already converge to a similar distribution with half the amount of simulations generated. The summary of the obtained cash flow for this case is presented in the third case of Table 6, showing that having the flexibility to expand until year 15 can increase the project's value by more than 13% and almost 18%, if the second expansion is considered. This suggests that according to the described conditions, both expansions would be feasible and profitable for the project.

It must be noted that these values consider having the flexibility to expand until year 15 or 16, which doesn't mean that the mine will actually be operating invariably until years 15 or 16. It does mean that management can decide to stop mining whenever is considered optimal, and also expand past the initial pit's limit. This is clearly presented in Fig. 7, where the left axis shows the mine's probability of being in operation from year to year (with curves labelled 'Prob. Open'), and the right axis shows the frequency of optimal life of mine for each of the 20,000 price simulations (labelled 'LOM Freq.'). This is shown for the current case of price uncertainty ('ETYPE'), and for the global model which includes price uncertainty as well as geological uncertainty ('SIM'), which is described in the next section. For comparison, Fig. 7 also includes the base case's annual probability of being operational, labelled 'Prob. Open (BC)'. This figure shows that there is a 10% chance that the mine closes beforetime by year 8 due to the price fluctuations, a 45% probability of the mine expanding to 15 years and a 5% to 16 years, and only a 30% chance of closing as expected by year 11. In contrast, the base case shows an absolute 100% - 0% probability shift of being open by year 11, where none of the expansion stages available are of any interest.

An advantage of ROV is that it allows for obtaining distributions for results, such as the one presented in Fig. 7, and cannot be obtained by the traditional evaluation methods. The information provided here can be highly valuable for strategic decision-making processes, allowing for informed solutions, ensuring that the operation has sufficient flexibility and contractual freedom to review the option of closing by year 8, if the price scenario is unfavorable; similarly, ensuring that the plant and any other fixed infrastructure is located away from the 16-year LOM pit limit, to avoid relocations in case gold price increases and the operation chooses to expand. In both cases, there may be some additional costs involved, such as extra transportation if the plant is further, or less-favorable contract negotiations if the lease terms are shorter, however these costs are marginal compared to the probable profits obtained or the losses prevented. All these may be considered as the cost of having (and maintaining) the 'option of closing', which is quantified and included in the evaluation process.

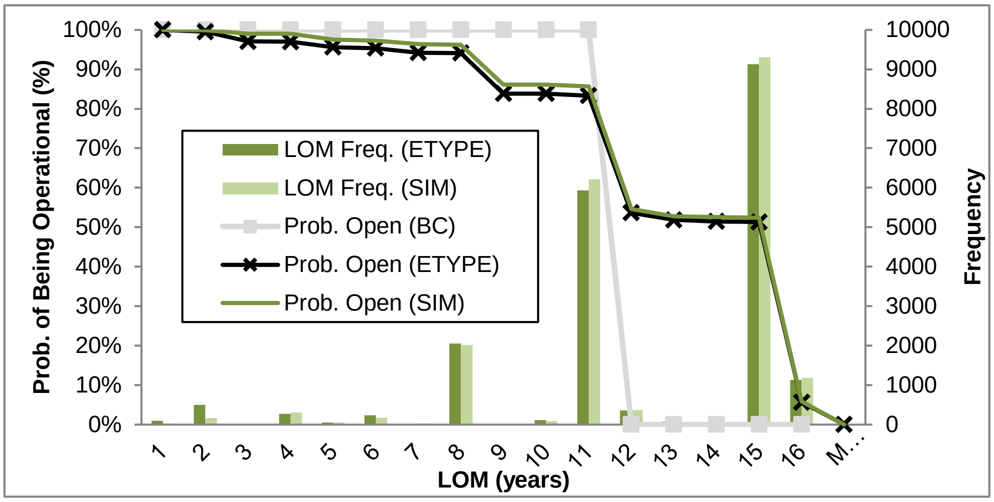


Figure 7: Probability of mine being open and frequency of LOM considering (i) uncertain price (dark green) and (ii) price and geology (light green) for the estimated and simulated models

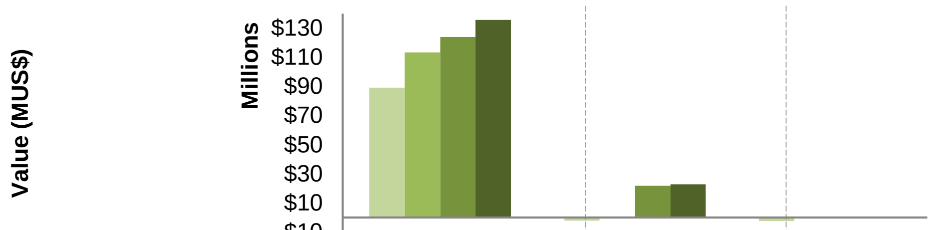
3.2.3 Joint stochastic model

This final step combines geological uncertainty with the stochastic price model in order to create a global stochastic evaluation model. In this case, the procedure is the same as the one described in Section 2, where the expanded schedule is applied over each of the orebody realizations and price paths to obtain different project values and expected LOMs. To obtain comparable results with the previous case of market uncertainty, the 20 simulated orebody models are evaluated over the same 20,000 price paths generated in step 3.2.2, that is, for each estimated model evaluation, there are 20 other evaluations done over the same price path - one for each of the 20 orebody simulations.

This case's cash flow is summarized in the final case of Table 6, where the first and second expansions have the potential of increasing the project's value by 16% and 17% respectively over the initial ultimate pit's value. The actual LOM frequency and the operation's probability of being open are presented in the right-most column of Fig. 7, where it is shown that the probability of expanding to 15 and 16 years is even higher than in the case presented in Section 3.2.2 (46% and 6% respectively); however, the results in this section are consistent. Figure 8 presents the independent value of each of the individual stages, and for each of the evaluation cases, showing that there is an important value difference between considering or not considering the different uncertainty. However, the ROV of the three stochastic cases do agree on the same thing: that there is an important profit to be made if the option of delaying or advancing the mine closure is considered given the underlying uncertainties, a fact that goes unnoticed by the traditional DCF method presented in the leftmost column of Fig. 8. Thus, accounting for uncertainty leads to higher rewards for a project that presents less risk.

It is also noted from Fig. 8 that, if the price is considered stochastic (cases 3 and 4), the value difference between taking or not taking into account the geological uncertainty reduces from the initial pit to the 1st and 2nd expansions. This is expected, given that if the mine decides to expand, there has been a favorable economic and technical development of the uncertainties, with probably high prices, which causes more material to be classified as potential ore in both the estimated model and the simulated models, making the 'smoothness' of the estimated model less damaging to the overall project value. However, this only occurs if the prices are 'high enough' to make the low grade blocks of the simulations profitable to process.

For a more detailed evolution of the project's performance, Fig. 9 shows a risk analysis of the project's annual cash flow for the initial pit and the two following expansions, for cases 3 and 4, considering only price uncertainty in the first case ('ETYPE'), and geology as well as price in the second ('SIM'). In the figure, 'Stopped Mining' represents the annual probability of the operation having decided to stop the operation,



	Initial Ultimate Pit (LOM=11)	Value Expansion 1 (LOM=15)	Value Expansion 2 (LOM=16)
(1) BASE CASE	$89,238,168	-$2,120,700	-$2,250,100
(2) ROV - GEOLOGY CASE	$113,281,871	$506,456	$-
(3) ROV - PRICE CASE	$123,919,763	$21,876,089	$156,221
(4) ROV - JOINT CASE	$135,639,184	$22,751,901	$191,568

Figure 8: Value of individual expansions for each of the four stage analysis

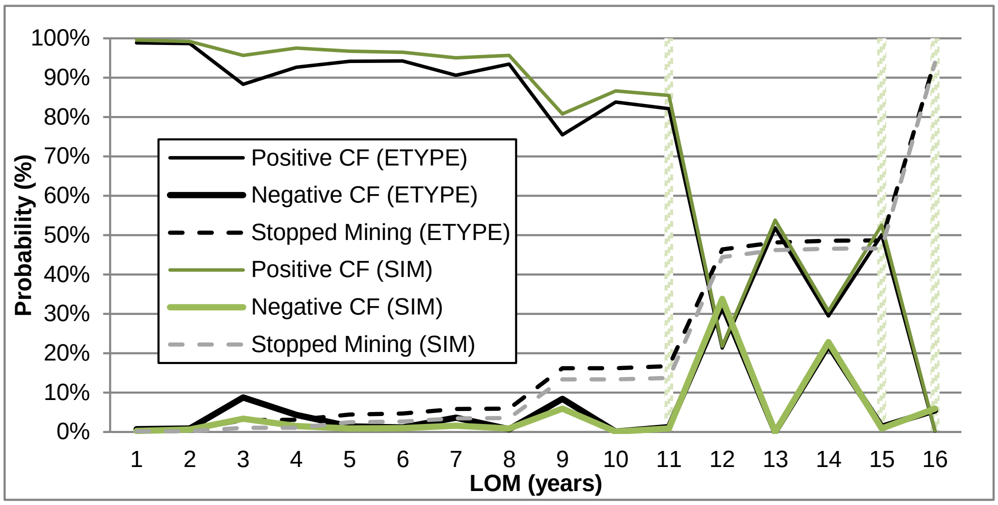


Figure 9: Risk analysis of annual cash flows for the initial pit and two expansions, considering uncertain price for estimated and stochastically simulated orebodies

which after that it is irreversibly closed. 'Negative CF' shows the probability of having a negative net cash flow on a particular period, which implies that even though there is negative net cash flow, the overall project value is higher if the mine is operating during that period, which suggests that extracting may cover some of the closing costs, or that subsequent years will pay for that year's losses. For example, years 13 and 15 show that if the operation is open, extracting those years will generate positive net cash flows in almost every case, which would pay for the losses incurred on the previous years (12 and 14) where the probability of having negative cash flow is almost the same as the probability of having a profit. This risk analysis helps managers to have a broader and better understanding of the possible behavior of the operation being studies, and to detect the crucial periods where strategic decisions (such as expanding or stopping mining) will have to be made.

From Fig. 9 it is possible to see that even though the estimated model with price uncertainty case described in Section 3.2.2 behaves similarly to the simulations analyzed in this section, there is a consistent gap between the two cases, where the estimated model has a constantly lower probability of annual positive cash flow and a higher probability of a negative one. Together with this, the estimated model presents a higher probability of closing in comparison to the simulated orebodies. The difference, however, diminishes past the initial pit limit as the favorable price scenario shadows the consequences of smoothing over the project's evaluation.

4 Conclusions and future work

The present study analyzes the efficiency of traditional evaluation methods in assessing the potential performance of a mining operation under uncertain geological (grade) and commodity price scenarios, and provides an alternative real option-based method that includes the option of expanding or contracting the initial ultimate pit limit, subject to these uncertainties.

A case study in an open pit gold mine was used to describe the method proposed and shows its benefits. Experimental results indicate that traditional methods tend to underestimate the size of the final pit, ignoring possibly profitable expansions, as documented in the past only for geological uncertainty (Albor and Dimitrakopoulos, 2010), and that considering uncertainties in the evaluation model as well as allowing for the operation to react to these uncertainties significantly increases the project value.

The proposed method accounts for the value that management and decision-makers generate when taking advantage of opportunities and hedging from unfavorable scenarios, which are unknown at initial stages of the project. It is shown that real options are able to include this flexibility value in the global evaluation model. Together with this, it is also shown that uncertainty-based analysis provides probabilistic results, which in turn helps decision-makers be prepared to react to the continuously changing context of a mine project.

This analysis can be considered as a comprehensible way of including different sources of uncertainty in project evaluation and mine design, allowing to assess potential pit limit modifications at the early planning stage of the project, defining infrastructure-free zones and providing highly valuable information to decision makers so that slight adjustments can be done to the design at no or very low cost, so as to facilitate the execution of these flexibilities, and prevent potential unnecessary large losses in the future.

Even though the actual metal price, or the exact ore grade can't be known with certainty, accounting for their uncertainty helps analyze the possible range of outcomes that the project might have when faced to the certain changes in context. To do this, direct block simulation proved to be an effective method to represent the geological uncertainty of the deposit, and the geometric Brownian motion with Poisson jumps provided a reasonable range of market values to evaluate the effect of price uncertainty in the initial mine design. However, there are some important limitations to the method presented herein: Mainly assuming that the schedule is fixed over all the evaluation stages, as, subject to price and geological changes, the schedule ceases to be optimal; this limits the applicability of the study to only assessing the performance of one given schedule. Another limitation is the relevance that the inputs and parameters fed to the model have over the final value, which characterize the different uncertainties considered in the analysis.

Further studies should focus on creating a stochastic integer programing model to generate an uncertainty-based schedule that includes price and geological uncertainty, so as to expand the applicability of the methodology presented herein. Together with this, the future developments should include the different complexities that mine operations contain, such as multi-pits, multiple products and/or multiple processing methods, as well as stockpiling options.

A Appendix

The inclusion of a 'jump' component to complement the variable's modeling has been proven to generate a better representation of price behaviors when applied to energy and commodities such as gold and copper, without the requirement of extensive assumptions as input (Shafie and Topal, 2010; Blanco and Soronow, 2001). This was first noticed by Merton (1976), who derives an option pricing formula that continues the work of Black and Scholes (1973), and considers stock returns as a mixture of a continuous behavior with a jump-Poisson process, both of which only depend on the current price, respecting the Markov properties of the models. Oldfield et al. (1977) introduced empirical data to support the idea of modeling stock returns as a combination of a continuous process with discrete jumps. Even though these studies are focused on stock returns, they can as well be applied for commodity price forecasting, as they also behave as market derivatives. This was done by Mendez and Lamothe (2009), who modeled copper price by incorporating Gaussian Poisson exponential stochastic processes (or 'jumps') to the usual mean reverting process, and by Blanco and Soronow (2001), who incorporated a jump-diffusion process to a geometric mean reverting process to forecast energy prices.

The mathematical formulation to incorporate Poisson jumps into the forecasting model is done by adding a Poisson diffusion process (dq) into the random walk (geometric Brownian motion in this case, but the addition is the same for a mean reverting process), and is presented in Eq. (A1). Mathematically, as presented in Eq. (A1), the jump is integrated into the model by simply adding an extra diffusion term (dq) in the original random walk model.

$$dx = \mu x dt + \sigma x dz + dq, \tag{A.1}$$

where, to define the stochastic process x, $\mu =$ is the percentage drift, $\sigma =$ the percentage volatility, $dt =$ the differential time step, and $dz =$ the Wiener process or Brownian motion.

The regressive solution of the stochastic differential equation model is given in Eq. (A2).

$$x_t - x_{t-1} = \mu \cdot \Delta t \cdot x_{t-1} + (\sigma^2 \cdot \sqrt{\Delta t}) \cdot x_{t-1} \cdot \varepsilon_{t1} + \eta \cdot (\kappa + \delta \varepsilon_{t2}) \cdot x_{t-1} \tag{A.2}$$

In the regressive form shown in Eq. (A2), $\kappa =$ stands for the expected jump size, $\delta =$ is the standard deviation of the jump, $\varepsilon_{t1}, \varepsilon_{t2} =$ a normally distributed random variables for jump and volatility respectively, and η is a binary variable that takes the value of 0 with probability $1 - \lambda dt$ if there is no jump, and a value of 1 with probability λdt if there is one, where $\lambda = \lambda_u + \lambda_d$ corresponds to the total frequency of a jump, and λ_u, $\lambda_d =$ frequency of upwards and downward jumps respectively. Here, λ represents the total frequency of jumps per year.

All these values can be obtained by inspection of the historical data. To simulate these jumps, if $\eta = 1$, a uniform random variable $u \sim U[0, 1]$ is used to identify if the jump is upwards (a sudden increase in price) or downwards (a sudden fall in price), where if $u < (\lambda_u/\lambda)$, the price jump is upwards, and if $u > (1 - \lambda_u/\lambda) = \lambda_d/\lambda$, the jump is considered to go downwards.

In the current study, a jump is considered a change in returns between two consecutive periods (from historical data) of more than three standard deviations. For a more detailed explanation in the implementation process and examples see Mendez and Lamothe (2009) and Blanco and Soronow (2001).

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