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Maryam DARVISH
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Flexible Two-Echelon Location Routing

Maryam Darvish[1], **Claudia Archetti**[2], **Leandro C. Coelho**[1*], **Maria Grazia Speranza**[2]

[1] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT) and Department of Operations and Decision Systems, 2325, rue de la Terrasse, Université Laval, Québec, Canada, G1V 0A6

[2] Department of Economics and Management, University of Brescia, Via S. Faustino 74/b-25122 Brescia, Italy

**Corresponding author: leandro.coelho@cirrelt.ca*

ABSTRACT

This paper deals with an integrated routing problem in which a supplier delivers a commodity to its customers through a two-echelon supply network. The commodity is first sent from a single depot to a set of distribution centers (DCs). Then, from the DCs, it is delivered to customers on the basis of their requests. A limited planning horizon is considered and the objective is to minimize the total cost consisting of the sum of the shipping costs from the depot to the DCs, the traveling costs from DCs to customers, the location costs, and the penalty costs for any unmet demand. On top of this basic setting, we study two sources of flexibility: flexibility in due dates and flexibility in the network design. The former establishes an interval within which the customer requests can be satisfied while the latter is related to the possibility of deciding which DCs are convenient to be rented at each period of the planning horizon. We present a mathematical formulation of the problem together with different classes of valid inequalities. Extensive computational tests are made on randomly generated instances to show the value of the two kinds of flexibility. Computational and business insights are discussed. The results show that the combined effect of the two kinds of flexibility leads to total average savings of up to almost 35%.

Keywords: Multi-depot vehicle routing, location routing, integrated logistics, flexibility.

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1. Introduction

The recent literature on routing problems is evolving to the study of more and more complex problems. This complexity stems from different sources, among which integration and flexibility are the most investigated. By integration, we mean to include broader parts of the decision systems, and not only the one focused on the pure stand-alone routing. One of the most classical examples of integrated routing problem is the Inventory Routing Problem (IRP) where routing is integrated with inventory management (see Bertazzi and Speranza (2012, 2013); Coelho et al. (2013) for surveys and tutorials and Archetti and Speranza (2016) for a study on the value of integration in IRPs). Other important examples of integrated routing problems are the ones in which network design issues are integrated in routing decisions, an example being location routing problems (see Prodhon and Prins (2014) for a recent survey) and two-echelon vehicle routing problems (see Cuda et al. (2015) for a recent survey and Guastaroba et al. (2016) for a more general survey on transportation problems with intermediate facilities).

Flexibility is related to the possibility of relaxing some constraints in order to save costs. For example, in a distribution problem where customers requests have to be satisfied within a planning horizon, one may achieve cost savings if flexibility in the due dates is allowed, as shown in Archetti et al. (2015). Another study related to the advantage of flexibility in routing problems is provided in Archetti et al. (2017) where the authors study the flexible periodic vehicle routing problem, that is a generalization of the periodic vehicle routing problem in which no visiting schedule is considered.

In this paper we study a routing problem where both integration and flexibility are considered. In particular, we study a problem coming from a real application where a supplier has to build a distribution plan to serve the customers through a two-layer distribution network. A single commodity is produced at a production plant, or stocked at the depot, and is distributed from there to a set of distribution centers (DCs). Then, the commodity is delivered to customers from the DCs. A planning horizon is considered which is discretized in periods, typically days. The supplier has the possibility to choose among the available DCs on a daily basis. In fact, we consider the DCs as the rented space in physical facilities shared with other companies and managed by a third party. Daily customers requests are known and dynamic. Moreover, each order has a due date, which represents the latest delivery date. Each order has to be entirely fulfilled in one delivery. A penalty is related to an unmet demand, i.e., to orders which are not satisfied by the delivery due date. Products are shipped from the depot to the selected DCs via full truckloads, and from DCs to customers via milk runs. The supplier has to take four simultaneous decisions: which DCs to use in each period, when to satisfy the orders of customers, from which of the selected DCs to ship to the customers, and how to create vehicle routes from the selected DCs to the customers.

We call this problem the Flexible Two-Echelon location routing Problem (F-2E-LRP), in which the objective is to minimize the total costs consisting of the sum of the shipping costs from the depot to the DCs, the delivery cost from the DCs to the customers, the renting cost of DCs, and the penalty cost for the unmet demand. The F-2E-LRP merges integration issues related to the decision of which DCs to rent, and flexibility issues coming from two sources:

- the possibility of selecting amongst the available DCs on a daily basis;
- the possibility of selecting the day when customer orders are satisfied, provided that either the due date is respected or a penalty is paid.

The F-2E-LRP is motivated by recent interest in collaborative business. As an important variant of the sharing economy, in collaborative business, companies gain by sharing their assets, capacities or in general their infrastructure with others (Savelsbergh and Van Woensel, 2016). While Savelsbergh and Van Woensel (2016) name some advantages for sharing assets, in this paper we highlight the potential advantages of optimizing a supply chain over different kinds of flexibility. From the academic point of view, as mentioned earlier, the F-2E-LRP is also related to several well known problems, including location routing, inventory routing, and multi-depot vehicle routing problems. Location routing is the problem of determining the location of facilities that are then used to distribute goods to customers (see Prodhon and Prins (2014) and Drexl and Schneider (2015) for recent surveys). The F-2E-LRP extends the location routing problem as it considers a two-echelon network and a planning horizon where the location decision is taken on a daily basis. The link with inventory routing is due to the fact that DCs may be used to store goods from one day to another before being delivered to customers. Finally, the F-2E-LRP is related to the multi-depot vehicle routing problem (Renaud et al., 1996; Cordeau et al., 1997; Lahyani et al., 2015) as routes serving customers depart from different DCs.

In addition to the above mentioned problems, the F-2E-LRP is an extension of the works presented in Archetti et al. (2015) and Darvish et al. (2016). Archetti et al. (2015) study the multi-period vehicle routing problem with due dates. They propose several formulations, solve them through branch-and-cut and compare their performance. This work extends the work of Archetti et al. (2015) by first adding intermediate facilities (DCs) where goods are stored and second, by considering the possibility of choosing among several DCs on a daily basis. Darvish et al. (2016) study a multi-echelon integrated lot sizing-distribution problem considering both delivery time windows and facility location decisions. A key difference between the F-2E-LRP and that problem is the use of vehicle routes to manage the distribution to customers instead of direct shipments, which significantly enriches the problem setting investigated by Darvish et al. (2016).

The contributions of this paper are summarized as follows. We introduce the F-2E-LRP and propose a mathematical formulation along with different classes of valid inequalities. We run a large set of experiments on randomly generated instances to show the value of flexibility, both in terms of due dates, in terms of network design, and on their combined effect. The results highlight the cost saving advantages of both types of flexibility. In particular, we show that the combined effect of the two kinds of flexibility leads to a saving in total cost of up to almost 35%. We also provide computational and business insights based on this analysis.

The remainder of the paper is organized as follows. In Section 2 we formally describe the problem while in Section 3 we present a mathematical formulation together with different classes of valid inequalities. We present the results of the computational experiments in Section 4, followed by our conclusions in Section 5.

2. Problem description

In the F-2E-LRP a supplier delivers a single commodity to its customers through a two echelon supply chain which consists of the supplier plant, referred to as the depot, and a set of DCs. The supplier decides on a daily basis which subset of DCs to use in order to distribute the goods to its final customers. DCs are replenished by shipping full truckloads from the depot. Goods are consolidated at the DCs and distributed to final customers via milk runs. Without loss of generality, each day all DCs are available to be rented for a fee. The paid fee covers the fixed cost to use a vehicle, for which only routing costs are due. Dynamic customer orders are known in each period, and a due date is associated with them, i.e., each order must be satisfied within its due date, otherwise it is subject to a penalty per period of delay per unit.

Let $\mathcal{T}$ indicate the discretized planning horizon, typically days, of length T. Let $\mathcal{C}$ represent the set of customers and $\mathcal{D}$ the set of potential DCs, each with a single vehicle available for the distribution (if the DC is selected). Each customer $c \in \mathcal{C}$ has a known demand d_c^t for each period $t \in \mathcal{T}$. Once the customer places an order, the demand could be fulfilled from any of the selected DCs within a due date r, which is fixed for all customers in all periods. Late orders are not lost but any demand fulfilled after the due date is subject to a penalty cost p per period. Although the demand is known a priori, no demand can be satisfied in advance. Let f_i be the daily fee for DC i. Each selected DC is rented for one day. If the same DC is selected for two or more consecutive days, it can hold inventory from one day to another up to a capacity C_i, $i \in \mathcal{D}$. When a DC is not rented in a given day, any remaining previous inventory is lost. We assume that the fee f_i covers all the handling costs of products kept in the DCs, hence, no inventory holding cost is due.

All products are stored in DCs before being sent to the customers. Each DC possesses a vehicle with capacity Q. The vehicle may visit several customers per day in a single trip, starting and ending at the same DC. No partial shipment of an order is possible. Different orders from the same customer in different periods may be either bundled together or shipped separately from the same or different DCs, and/or in different periods, but one order from a customer in a period cannot be split in different periods.

Transportation costs are accounted as follows. Each shipment from the depot to DC i costs s_i and has a transportation capacity W. Vehicle routes from each DC to any of the customers incur a cost which is based on the distance traveled. A distance matrix c_{ij} is known, $i, j \in \mathcal{C} \cup \mathcal{D}$, where c_{ij} is the cost of traveling from location i to location j. No transshipment between DCs is allowed, i.e., goods stored at a DC are distributed to customers only.

The objective of the F-2E-LRP is to minimize the total cost of distribution, including the DC rental fees, the transportation costs from the depot to the DCs and from DCs to the final customers as well as the late delivery penalty.

3. Problem formulation

In this section we propose the mathematical programming formulation for the F-2E-LRP. It extends a commodity-flow formulation initially proposed by Garvin et al. (1957) and extensively used in Koç et al. (2016), Lahyani et al. (2015), and Salhi et al. (2014).

The commodity flow formulation for the F-2E-LRP makes use of the following variables:

- binary variables x_{ij}^{dt} indicate whether a vehicle from DC d traverses arc (i, j) in period t;
- binary variables y_i^{dt} take value one if and only if a vehicle from DC d visits node i in period t;
- continuous variables z_{ij}^t represent the remaining load on the vehicle when traversing arc (i, j) in period t, i.e., after visiting node i and before visiting node j;
- continuous variables q_{id}^t indicate the quantity delivered to customer i from DC d in period t;
- continuous variables S_i^t represent the amount of goods backlogged for customer i in period t;
- binary variable w_d^t take value one if DC d is rented in period t;
- continuous variables I_d^t represent the amount of inventory in DC d in period t;
- continuous variable g_d^t represent the quantity shipped to DC d in period t;
- binary variables α_i^{tp} indicate whether the demand of customer i in period t is satisfied in period p. These will be used to ensure that the delivery of a demand will not be split over several periods.

The F-2E-LRP is formulated as follows:

$$\text{minimize} \sum_{t\in\mathcal{T}} \left(\sum_{i\in\mathcal{D}} f_i w_i^t + \sum_{i\in\mathcal{D}} s_i g_i^t + \sum_{d\in\mathcal{D}} \sum_{i\in\mathcal{D}\cup\mathcal{C}} \sum_{j\in\mathcal{D}\cup\mathcal{C}} c_{ij} x_{ij}^{dt} \right) + \sum_{t=1}^{T+1} \sum_{i\in\mathcal{C}} p S_i^t \tag{1}$$

subject to

$$\sum_{d\in\mathcal{D}} y_i^{dt} \leq 1 \quad i \in \mathcal{C}, t \in \mathcal{T} \tag{2}$$

$$y_i^{dt} \leq y_d^{dt} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T} \tag{3}$$

$$\sum_{j\in\mathcal{D}\cup\mathcal{C}} x_{ij}^{dt} + \sum_{j\in\mathcal{D}\cup\mathcal{C}} x_{ji}^{dt} = 2y_i^{dt} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T} \tag{4}$$

$$\sum_{j \in \mathcal{D} \cup \mathcal{C}} x_{ij}^{dt} = \sum_{j \in \mathcal{D} \cup \mathcal{C}} x_{ji}^{dt} \quad i \in \mathcal{C}, i \neq j, d \in \mathcal{D}, t \in \mathcal{T} \tag{5}$$

$$x_{ij}^{dt} = 0 \quad i \in \mathcal{C}, j \in \mathcal{D}, d \in \mathcal{D}, j \neq d, t \in \mathcal{T} \tag{6}$$

$$x_{ij}^{dt} = 0 \quad i \in \mathcal{D}, j \in \mathcal{C}, d \in \mathcal{D}, i \neq d, t \in \mathcal{T} \tag{7}$$

$$\sum_{i \in \mathcal{C} \cup \mathcal{D}} z_{ij}^{t} - \sum_{i \in \mathcal{C} \cup \mathcal{D}} z_{ji}^{t} = \sum_{d \in \mathcal{D}} q_{jd}^{t} \quad j \in \mathcal{C}, t \in \mathcal{T} \tag{8}$$

$$\sum_{i \in \mathcal{D}} \sum_{j \in \mathcal{C}} z_{ij}^{t} = \sum_{j \in \mathcal{C}} \sum_{d \in \mathcal{D}} q_{jd}^{t} \quad t \in \mathcal{T} \tag{9}$$

$$z_{ij}^{t} \leq \sum_{d \in \mathcal{D}} Q x_{ij}^{dt} \quad i, j \in \mathcal{C} \cup \mathcal{D}, t \in \mathcal{T} \tag{10}$$

$$q_{id}^{t} \leq Q y_{id}^{t} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T} \tag{11}$$

$$\sum_{d \in \mathcal{D}} \sum_{t' \leq t} q_{id}^{t'} \leq \sum_{t' \leq t} d_{i}^{t'} \quad i \in \mathcal{C}, t \in \mathcal{T} \tag{12}$$

$$S_{i}^{t+1} \geq \sum_{t' \leq t} d_{i}^{t'} - \sum_{d \in \mathcal{D}} \sum_{t' \leq t+r} q_{id}^{t'} \quad i \in \mathcal{C}, t \in \mathcal{T} \tag{13}$$

$$\sum_{i \in \mathcal{C}} S_{i}^{0} = 0 \tag{14}$$

$$\sum_{p \geq t}^{T+1} \alpha_{i}^{tp} = 1 \quad i \in \mathcal{C}, t \in \mathcal{T} \tag{15}$$

$$\sum_{d \in \mathcal{D}} q_{id}^{p} = \sum_{t \in \mathcal{T}, t \leq p} \alpha_{i}^{tp} d_{i}^{t} \quad i \in \mathcal{C}, p \in \mathcal{T} \tag{16}$$

$$y_{i}^{dt} \leq w_{d}^{t} \quad i \in \mathcal{C} \cup \mathcal{D}, d \in \mathcal{D}, t \in \mathcal{T} \tag{17}$$

$$I_{d}^{t} \leq C_{d} w_{d}^{t} \quad d \in \mathcal{D}, t \in \mathcal{T} \tag{18}$$

$$I_{d}^{t} = I_{d}^{t-1} + g_{d}^{t} - \sum_{i \in \mathcal{C}} q_{id}^{t} \quad d \in \mathcal{D}, \quad t \in \mathcal{T} \setminus \{0\} \tag{19}$$

$$I_{d}^{1} = I_{d}^{0} + g_{d}^{0} - \sum_{i \in \mathcal{C}} q_{id}^{0} \quad d \in \mathcal{D} \tag{20}$$

$$g_{d}^{t} \leq W w_{d}^{t} \quad d \in \mathcal{D}, t \in \mathcal{T} \tag{21}$$

$$z_{ij}^{t} = 0 \quad i, j \in \mathcal{D}, t \in \mathcal{T} \tag{22}$$

$$x_{ij}^{dt} = 0 \quad i, j, d \in \mathcal{D}, t \in \mathcal{T} \tag{23}$$

$$y_{i}^{dt}, \alpha_{i}^{tp}, x_{ij}^{dt} \in \{0, 1\} \tag{24}$$

$$S_{i}^{t}, z_{ij}^{t}, I_{d}^{t}, w_{d}^{t}, g_{d}^{t}, q_{id}^{t} \in \mathbb{Z}^{*}. \tag{25}$$

The objective function (1) minimizes the total cost composed of the fixed renting costs of the DCs, transportation costs to the DCs, distribution costs to the customers, and the late delivery penalties. Constraints (2) impose that a customer is visited at most once per period, and constraints (3) ensure that customers are visited only from the rented DCs. Constraints (4) and (5) are degree constraints. Constraints (6) and (7) forbid a vehicle to start a route from a DC and finish at another. Constraints (8) ensure the connectivity of a route, while constraints (9) ensure that the quantity loaded on vehicles from all DCs is delivered to customers in the same period. Constraints (10) impose a bound on the z variables and ensure that vehicle capacities are respected. Constraints (11) link the delivery quantities with the DC used for delivery to that customer. Constraints (12) impose that no demand can be satisfied in advance. Constraints (13) and (14) determine the amount of stockout. Constraints (15) and (16) ensure that each demand of each customer is delivered exactly once. Constraints (17) allow routes to start only from rented DCs, while constraints (18) impose capacity constraints on the selected DCs. Constraints (19) set the inventory level at each DC and (20) indicate that the initial inventory is equal to zero. Constraints (21) guarantee that only rented DCs receive deliveries from the depot and the delivery respects the transportation capacity. Constraints (22) and (23) forbid vehicles to travel between DCs. Constraints (24)–(25) define the nature and bounds of the variables.

We also propose the following valid inequalities to strengthen formulation (1)–(25):

$$z_{ij}^t = 0 \quad i \in \mathcal{C}, j \in \mathcal{D}, t \in \mathcal{T} \tag{26}$$

$$z_{ii}^t = 0 \quad i \in \mathcal{C} \cup \mathcal{D}, t \in \mathcal{T} \tag{27}$$

$$x_{ii}^{dt} = 0 \quad i \in \mathcal{C} \cup \mathcal{D}, d \in \mathcal{D}, t \in \mathcal{T} \tag{28}$$

$$x_{id}^{dt} \leq y_i^{dt} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T} \tag{29}$$

$$x_{di}^{dt} \leq y_i^{dt} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T} \tag{30}$$

$$x_{ij}^{dt} + y_i^{dt} + \sum_{h \in \mathcal{D}, h \neq d} y_j^{ht} \leq 2 \quad i, j \in \mathcal{C}, i \neq j, d \in \mathcal{D}, t \in \mathcal{T} \tag{31}$$

$$x_{ij}^{dt} + x_{ji}^{dt} \leq 1 \quad i, j \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T} \tag{32}$$

$$\sum_{i \in \mathcal{C}} \sum_{t' \in \mathcal{T}, t' \leq t} \sum_{d \in \mathcal{D}} q_{id}^{t'} \leq \sum_{d \in \mathcal{D}} \sum_{t' \in \mathcal{T}, t' \leq t} Q w_d^{t'} \quad t \in \mathcal{T} \tag{33}$$

$$\sum_{i \in \mathcal{C}} \sum_{t'=t}^{T+1} S_i^{t'} \geq \sum_{i \in \mathcal{C}} \sum_{t'=t}^{T} d_i^{t'} - \sum_{d \in \mathcal{D}} \sum_{t'=t}^{T} Q w_d^{t'}, \quad t \in \mathcal{T} \tag{34}$$

$$\sum_{i\in\mathcal{C}}\sum_{t'=t_1}^{t_2} q_{id}^{t'} \leq \sum_{d\in\mathcal{D}}\sum_{t'=t_1}^{t_2} Qw_d^{t'}, \quad t_1, t_2 \in \mathcal{T}, t_1 \geq t_2 \tag{35}$$

$$2y_d^{dt} \leq \sum_{j\in\mathcal{D}\cup\mathcal{C}} x_{dj}^{dt} + \sum_{j\in\mathcal{D}\cup\mathcal{C}} x_{jd}^{dt} \quad d \in \mathcal{D}, t \in \mathcal{T}. \tag{36}$$

Constraints (26) impose that the vehicles return empty to the DCs, breaking symmetries in the solutions that differ only in the quantity loaded. Constraints (27) and (28) forbid links between a node and itself. Constraints (29) and (30) strengthen the link between routing and visiting variables. Inequalities (31) exclude infeasible vehicle routes that visit customers assigned to two different DCs, and (32) are two-cycle elimination constraints. Inequalities (33) state that total deliveries to all customers from all DCs up to period t' should not exceed the total capacities of all vehicles used during the t' periods. Constraints (34) establish that the backlog has to be at least equal to the exceeding demand with respect to the capacity of the vehicles used, while (35) state that the quantity delivered from a DC is bounded by the vehicle capacity multiplied by the number of days in which the DC is used. Finally, (36) impose that at most one route can start and end at a DC in each day, in case the DC is rented.

4. Computational Experiments

The formulation presented in Section 3, together with its valid inequalities, has been solved through CPLEX 12.7.0 and IBM Concert Technology. No separation of constraints or valid inequalities is needed as they are all in polynomial number. All computations are conducted on Intel Core i7 processor running at 3.4 GHz with 64 GB of RAM installed, with the Ubuntu Linux operating system. The maximum execution time is 10,800 seconds.

The goal of our experiments is to assess the value of two types of flexibility: the one arising from the possibility of modifying the design of the supply chain network, and the one gained by relaxing due dates on customers requests. In order to highlight the values of these two types of flexibility we have conducted experiments on randomly generated instances. In Section 4.1 we explain how the instances are generated. In Section 4.2 we assess the value of the network design flexibility, and in Section 4.3 we show the value of the flexibility obtained by relaxing the due date. Finally, in Section 4.4 we combine and analyze the effects of both type of flexibility.

4.1. Instance generation

We randomly generated instances for the F-2E-LRP according to the parameter values specified in Table 1. The instance generation is done as follows. For each combination of number of customers and number of days (10 combinations), we first generate instances with only one DC. Then, instances with two and three DCs are created by using instances with one DC and adding DC locations at random. This way, the advantage of having multiple DCs, if any, is completely imputable to their availability and not to different customer and DC locations. In addition, we considere three different values for the capacity of the vehicle performing deliveries from DCs to customers: tight, normal or loose. The capacity value of the truck shipping the goods from the depot to the DCs (W) is set to a sufficiently large value so that flexibility in network design and/or due dates is fully exploited. Penalty cost p is also set to a high value in order to force deliveries within due dates if possible. Finally, concerning due dates, we considere three cases: no due date ($r = 0$, i.e., the customer request has to be satisfied when it is released), next day delivery ($r = 1$) or delivery within two days ($r = 2$). For each combination of the above mentioned parameters we generate five instances by randomly choosing the values of C_i, d_i^t, f_i, s_i, X_i and Y_i, as specified in Table 1, for a total of 1,350 instances.

4.2. Flexibility from changing the network design

In order to assess the value gained from the flexibility in supply chain network design, we solve each instance under two different scenarios allowing the model to choose among the available DCs. In the first scenario, called *fixed network design*, we impose the DCs selected in the first period to remain unchanged throughout the planning horizon, whereas in the second scenario, called *flexible network design*, the model re-evaluates the decision on which of the DCs should be rented in each period. Thus, in the first scenario we have $w_d^t = w_d^1$ for each DC, i.e., if we decide to rent (not rent) a DC, it remains rented (not rented) for the entire planning horizon, while in the second scenario w_d^t remains flexible per period as defined in Section 3. As the goal in this section is to compare the two network designs, we set $r = 0$ in order to exclude any effect of due dates.

Table 2 presents, for the first scenario, the average costs and optimality gaps over all five instances with the capacity specified in the first column and the number of customers and days specified in the second and third columns, respectively. The table compares the results for

Table 1: Input parameter values

Name	Parameter	Values
Periods	T	$\{3,6\}$
DCs	D	$\{1,2,3\}$
Customers	C	$\{5,10,15,20,25\}$
Vehicles	K	One per facility
Due dates	r	$\{0,1,2\}$
Demands	d_i^t	[0, 5]
DC rental fees	f_i	[100,150]
Shipping costs (plant-DC)	s_i	[1,5]
X coordinates	X_i	[0,100]
Y coordinates	Y_i	[0,100]
Shipping costs (DC-customers)	c_{ij}	$\left\lfloor \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2 + 0.5} \right\rfloor$
Penalty cost	p	1000
Inventory capacities	C_i	$[2,3] \times D_{max}$
Full truckload capacity	W	D_{max}
Tight vehicle capacity	Q_T	D_{min}
Normal vehicle capacity	Q_N	$\left\lceil \frac{D_{min} + D_{max}}{2} \right\rceil$
Loose vehicle capacity	Q_L	D_{max}

where D_{max} equals the total demand of the peak day ($\max_t \sum_{i \in \mathcal{C}} d_i^t$) and D_{min} equals the total demand of the day with the lowest demand ($\min_t \sum_{i \in \mathcal{C}} d_i^t$)

different number of DCs. For each number of DCs we report the cost of the best solution found and the optimality gap. In addition, for the case where the number of DCs is equal to 2 and 3, we report, in column 'guaranteed savings', the gap between the cost of the solution with the corresponding number of DCs and the lower bound of the solution with one DC. This way, we provide an upper bound on the savings that are achieved by introducing new DCs. Note that, when the instance with one DC is solved to optimality, this upper bound corresponds to the exact value of the savings achieved. The guaranteed savings are calculated as $100 \times \frac{Cost - LB}{Cost}$, where $Cost$ is the value reported in column 'Cost' and LB is the lower bound of the solution of the same instance with one DC.

Table 2 shows that the cost savings become more relevant when the vehicle capacity tightens. While the global average cost for all three vehicle capacity scenarios has a decreasing trend as the number of available DCs increases, the biggest influence of adding extra DCs is observed under the tight capacity scenario. Moreover, the savings are more substantial when moving from 1 to 2 DCs then moving from 2 to 3 DCs. In fact, the average savings achieved with 2 DCs are 59% while in the case of 3 DCs they are 62%. Concerning solution time, we see that, depending on the size of the instance and the capacity scenario, the average CPU time over all five instances varies, but in general when more DCs are available, the problem becomes more difficult to solve. Small size instances such as the ones with three periods, five customers and three DCs are solved in less than a second for all capacity scenarios. However, even after three hours of computation, instances with six periods, 25 customers, and two or three DCs under tight capacity could only be solved with almost 1.40% optimality gap. Moreover, when the vehicle capacity decreases, the problem takes on average 60% more time to be solved.

Table 3 compares the best solutions obtained by the fixed network design with the ones obtained by the flexible network. The guaranteed savings are calculated as in Table 2 with LB being equal to the lower bound of the solution related to the fixed network design. The last column of the table reports the average reduction in the number of rented DCs. In particular, for each solution, the number of rented DCs is calculated as the sum of the number of DCs rented in each day of the planning horizon. Then, the reduction in the number of rented DCs is calculated as $100 \times \frac{Number\ of\ rented\ DC_{Fixed} - Number\ of\ rented\ DC_{Flexible}}{Number\ of\ rented\ DC_{Fixed}}$. Note that, when calculating the reduction in the number of DCs, we considered the best solution found in both the fixed and the flexible network design case, which corresponds to the optimal solution when the optimality gap is equal to 0. As the table indicates, the flexible network design always yields lower costs. However, this difference is more evident for the cases with normal and tight vehicle capacities. In these cases, the model not only reduces the cost but also decreases the number of DCs rented throughout the planning horizon. On average the flexible network design reduces costs by almost 6% and, at the same time, it uses 15% fewer DCs. The difference between the two designs is more significant under the normal vehicle capacity scenario, for which we observe 11% savings in total cost and 28% reduction in the number of rented DCs.

Table 2: DC availability in the fixed network design with $r = 0$

	Instances		# of $DC = 1$		# of $DC = 2$			# of $DC = 3$		
	Periods	Customers	Cost	Gap (%)	Cost	Gap (%)	Guaranteed savings (%)	Cost	Gap (%)	Guaranteed savings (%)
Loose	3	5	1005.60	0.00	1001.20	0.00	0.44	986.40	0.00	1.91
	3	10	1469.60	0.00	1436.20	0.00	2.27	1390.60	0.00	5.38
	3	15	1591.40	0.01	1500.40	0.00	5.71	1469.20	0.00	7.67
	3	20	1613.60	0.01	1561.00	0.00	3.25	1535.60	0.00	4.83
	3	25	2131.40	0.01	1762.40	0.18	17.31	1757.80	0.09	17.52
	6	5	2221.40	0.01	2108.00	0.00	5.10	2085.40	0.00	6.12
	6	10	3023.00	0.01	2953.00	0.00	2.31	2813.40	0.00	6.92
	6	15	3232.20	0.01	3150.60	0.00	2.52	3079.60	0.00	4.71
	6	20	3468.40	0.01	3385.60	0.47	2.28	3372.60	0.00	2.75
	6	25	4200.60	0.01	3978.00	0.52	5.29	3788.60	0.23	9.80
Average			**2395.72**	**0.01**	**2283.64**	**0.12**	**4.66**	**2227.92**	**0.03**	**6.76**
Normal	3	5	4965.00	0.00	3385.60	0.00	31.81	1334.60	0.00	73.12
	3	10	5621.40	0.00	1819.20	0.00	67.64	1744.40	0.00	68.97
	3	15	12691.60	0.01	1896.00	0.00	85.06	1863.80	0.00	85.31
	3	20	13697.00	0.01	1950.40	0.00	85.76	1903.20	0.00	86.10
	3	25	18598.20	0.01	2205.40	0.18	88.14	2174.80	0.82	88.31
	6	5	23216.60	0.01	2938.80	0.00	87.34	2807.20	0.00	87.91
	6	10	49408.40	0.01	3799.80	0.00	92.31	3603.60	0.00	92.71
	6	15	27217.60	0.01	3917.20	0.00	85.16	3838.00	0.00	85.90
	6	20	46887.40	0.01	4220.80	0.47	91.00	4172.20	0.55	91.10
	6	25	46174.00	0.01	4804.80	0.52	89.59	4565.00	0.82	90.11
Average			**24847.72**	**0.01**	**3093.80**	**0.12**	**80.43**	**2800.68**	**0.22**	**84.95**
Tight	3	5	17669.00	0.00	2757.40	0.00	84.39	1456.40	0.00	91.76
	3	10	22485.20	0.00	2242.40	0.00	90.030	1841.00	0.00	91.81
	3	15	35300.60	0.01	1938.20	0.00	94.51	1894.40	0.00	94.63
	3	20	43081.60	0.01	1981.00	0.39	95.40	1941.20	0.29	95.49
	3	25	71617.60	0.01	2269.80	0.21	96.83	2215.80	1.04	96.91
	6	5	95632.60	0.01	14302.40	0.00	85.04	4953.60	0.00	94.82
	6	10	184902.80	0.01	11712.00	0.03	93.67	3889.80	0.00	97.90
	6	15	191139.40	0.01	5531.20	0.55	97.11	4159.40	1.08	97.82
	6	20	205257.60	0.01	4333.00	1.15	97.89	4269.20	1.32	97.92
	6	25	197439.60	0.02	4922.80	1.44	95.51	4645.20	1.41	97.65
Average			**106452.60**	**0.01**	**5199.02**	**0.38**	**93.24**	**3126.60**	**0.51**	**95.67**
Global average			**44565.35**	**0.01**	**3525.49**	**0.20**	**59.44**	**2718.40**	**0.25**	**62.46**

Table 3: Fixed vs. flexible network designs with $r = 0$

Instances			Fixed		Flexible			
	Periods	Customers	Cost	Gap (%)	Cost	Gap (%)	Guaranteed saving (%)	Reduction in # DCs
Loose	3	5	986.40	0.00	984.20	0.00	0.22	0.00
	3	10	1390.60	0.00	1389.00	0.00	0.12	0.00
	3	15	1469.20	0.00	1469.20	0.00	0.00	0.00
	3	20	1535.60	0.00	1533.40	0.00	0.14	0.00
	3	25	1757.80	0.09	1756.60	0.12	**0.00**	0.00
	6	5	2085.40	0.00	2071.80	0.00	0.65	0.00
	6	10	2813.40	0.00	2809.20	0.00	0.15	0.00
	6	15	3079.60	0.00	3079.20	0.00	0.01	0.00
	6	20	3372.60	0.00	3372.60	0.01	0.00	0.00
	6	25	3788.60	0.23	3785.00	0.28	**0.00**	0.00
Average			**2227.92**	**0.03**	**2225.02**	**0.04**	**0.13**	**0.00**
Normal	3	5	1334.60	0.00	1142.00	0.00	14.43	30.00
	3	10	1744.40	0.00	1544.00	0.00	11.49	30.00
	3	15	1863.80	0.00	1664.20	0.00	10.71	26.67
	3	20	1903.20	0.00	1725.60	0.00	9.33	26.67
	3	25	2174.80	0.82	1946.60	0.93	9.75	30.00
	6	5	2807.20	0.00	2405.60	0.00	14.31	30.00
	6	10	3603.60	0.00	3277.20	0.00	9.06	23.33
	6	15	3838.00	0.00	3354.00	0.00	12.61	35.00
	6	20	4172.20	0.55	3828.20	0.53	7.74	23.33
	6	25	4565.00	0.82	4154.60	0.29	8.23	26.67
Average			**2800.68**	**0.22**	**2504.20**	**0.18**	**10.77**	**28.17**
Tight	3	5	1456.40	0.00	1306.60	0.00	10.29	21.21
	3	10	1841.00	0.00	1643.60	0.00	10.72	27.27
	3	15	1894.40	0.00	1767.20	0.00	6.71	16.67
	3	20	1941.20	0.29	1802.60	0.27	6.87	20.00
	3	25	2215.80	1.04	2084.60	0.87	4.93	16.67
	6	5	4953.60	0.00	4592.60	0.00	7.29	19.44
	6	10	3889.80	0.00	3648.80	0.09	6.20	15.15
	6	15	4159.40	1.08	3956.20	1.38	3.85	13.64
	6	20	4269.20	1.32	4068.60	1.52	3.43	13.33
	6	25	4645.20	1.41	4527.00	1.84	1.16	8.33
Average			**3126.60**	**0.51**	**2939.78**	**0.6**	**6.14**	**17.17**
Global average			**2718.40**	**0.25**	**2556.33**	**0.27**	**5.68**	**15.11**

4.3. Flexibility from due dates

In this section we evaluate the value of the flexibility obtained by relaxing due dates. We compare the costs and difficulty in solving the problems when no due dates are considered, i.e., $r = 0$, and when due dates are $r = 1$ and $r = 2$. We separate our analysis for fixed and flexible network designs in order to interpret the benefits coming from due dates only. Results are presented in Table 4 for the fixed network design and in Table 5 for the flexible network design. In both cases we consider three DCs. In both tables, the guaranteed savings are calculated by comparing the best solution value for the cases with $r = 1$ and $r = 2$ with the lower bound obtained with $r = 0$.

In general, delivery with larger due dates reduces the costs but makes the problem more difficult to solve. For both network designs, larger savings are achieved when changing from the case with no due dates ($r = 0$) to next day delivery ($r = 1$), rather than from the next day delivery to a two-day delivery ($r = 2$). However, comparing Tables 4 and 5, this difference is more significant in the flexible network design. In other words, when the location of DCs is fixed, offering larger due dates does not have as significant cost saving effect as it has in flexible networks. Overall, while serving the demand the next day rather than on the same day reduces the cost by 16% on fixed design, on a flexible network design the savings go up to 22%. Changing from next day delivery to delivery within two days leads to 4% additional savings in fixed networks and 10% in flexible ones.

Table 4: Value of flexibility from due dates for 3 DCs and fixed network design

Instances			$r=0$		$r=1$			$r=2$		
	Periods	Customers	Cost	Gap (%)	Cost	Gap (%)	% guaranteed savings over $r=0$	Cost	Gap (%)	% guaranteed savings over $r=0$
Loose	3	5	986.40	0.00	904.20	0.00	8.33	897.20	0.00	9.04
	3	10	1390.60	0.00	1277.60	0.00	8.13	1248.00	0.00	10.25
	3	15	1469.20	0.00	1344.40	0.00	8.49	1320.20	0.00	10.14
	3	20	1535.60	0.00	1378.20	0.34	10.25	1337.40	0.82	12.91
	3	25	1757.80	0.09	1541.00	3.04	12.26	1523.00	2.87	13.28
	6	5	2085.40	0.00	1714.40	0.00	17.79	1609.40	0.00	22.83
	6	10	2813.40	0.00	2244.40	0.00	20.22	2065.20	0.00	26.59
	6	15	3079.60	0.00	2408.40	0.59	21.80	2200.60	0.52	28.54
	6	20	3372.60	0.00	2820.20	2.91	16.38	2678.80	2.67	20.57
	6	25	3788.60	0.23	3203.40	6.25	15.25	3169.40	12.21	16.15
Average			**2227.92**	**0.03**	**1883.62**	**1.31**	**13.89**	**1804.92**	**1.91**	**17.03**
Normal	3	5	1334.60	0.00	1096.60	0.00	17.83	1071.60	0.00	19.71
	3	10	1744.40	0.00	1428.40	0.00	18.12	1364.80	0.00	21.76
	3	15	1863.80	0.00	1553.00	0.00	16.68	1439.60	0.00	22.76
	3	20	1903.20	0.00	1539.20	1.64	19.13	1452.80	2.13	23.67
	3	25	2174.80	0.82	1726.60	2.43	19.95	1681.40	4.02	22.05
	6	5	2807.20	0.00	2423.80	0.00	13.66	2273.60	0.00	19.01
	6	10	3603.60	0.00	2818.40	0.00	21.79	2649.40	0.24	26.48
	6	15	3838.00	0.00	2854.40	1.26	25.63	2777.20	2.65	27.64
	6	20	4172.20	0.55	3453.40	4.96	16.77	3255.40	6.74	21.54
	6	25	4565.00	0.82	3917.40	11.17	13.47	3556.00	9.79	21.46
Average			**2800.68**	**0.22**	**2281.12**	**2.15**	**18.30**	**2152.18**	**2.56**	**22.61**
Tight	3	5	1456.40	0.00	1319.20	0.00	9.42	1267.80	0.00	12.95
	3	10	1841.00	0.00	1564.80	0.00	15.00	1499.80	0.00	18.53
	3	15	1894.40	0.00	1630.60	0.41	13.93	1507.80	0.35	20.41
	3	20	1941.20	0.29	1672.80	2.59	13.58	1543.80	3.14	20.24
	3	25	2215.80	1.04	1958.40	7.58	10.69	1891.00	10.12	13.76
	6	5	4953.60	0.00	2795.60	0.00	43.56	2685.60	0.00	45.78
	6	10	3889.80	0.00	3227.80	0.16	17.02	3046.80	0.56	21.67
	6	15	4159.40	1.08	3495.80	4.50	15.04	3282.40	4.02	20.22
	6	20	4269.20	1.32	3645.80	7.53	13.46	3382.40	6.73	19.71
	6	25	4645.20	1.41	3991.00	10.18	12.86	3659.60	8.55	20.09
Average			**3126.60**	**0.51**	**2530.18**	**3.29**	**16.46**	**2376.70**	**3.35**	**21.34**
Global average			**2718.40**	**0.25**	**2231.64**	**2.25**	**16.22**	**2111.27**	**2.60**	**20.33**

Table 5: Value of flexibility from due dates for 3 DCs and flexible network design

	Instances		$r = 0$		$r = 1$			$r = 2$		
	Periods	Customers	Cost	Gap (%)	Cost	Gap (%)	% guaranteed savings over $r = 0$	Cost	Gap (%)	% guaranteed savings over $r = 0$
Loose	3	5	984.20	0.00	835.40	0.00	15.12	733.80	0.00	25.44
	3	10	1389.00	0.00	1162.20	0.00	16.33	991.80	0.00	28.60
	3	15	1469.20	0.00	1227.40	0.00	16.46	1032.60	0.00	29.72
	3	20	1533.40	0.00	1254.60	0.68	18.18	1040.60	0.85	32.14
	3	25	1756.60	0.12	1454.40	1.65	17.10	1280.80	4.81	27.00
	6	5	2071.80	0.00	1554.40	0.00	24.97	1336.40	0.00	35.50
	6	10	2809.20	0.00	2090.00	0.00	25.60	1807.60	0.73	35.65
	6	15	3079.20	0.00	2255.00	1.67	26.77	1925.20	2.71	37.48
	6	20	3372.60	0.01	2620.00	2.71	22.31	2359.40	5.33	30.04
	6	25	3785.00	0.28	2872.40	6.54	23.90	2601.40	9.85	31.08
Average			**2225.02**	**0.04**	**1732.58**	**1.33**	**20.67**	**1510.96**	**2.43**	**31.26**
Normal	3	5	1142.00	0.00	945.60	0.00	17.20	874.40	0.00	23.43
	3	10	1544.00	0.00	1234.40	0.00	20.05	1056.00	0.00	31.61
	3	15	1664.20	0.00	1312.00	0.00	21.16	1140.00	0.00	31.50
	3	20	1725.60	0.00	1327.80	1.02	23.05	1117.20	1.77	35.26
	3	25	1946.60	0.93	1537.60	2.82	20.27	1370.60	7.28	28.93
	6	5	2405.60	0.00	1821.40	0.00	24.29	1654.20	0.00	31.24
	6	10	3277.20	0.00	2329.00	1.48	28.93	2050.60	1.71	37.43
	6	15	3354.00	0.00	2474.80	3.80	26.21	2216.60	7.63	33.91
	6	20	3828.20	0.53	2897.80	7.69	23.90	2663.20	10.97	30.06
	6	25	4154.60	0.29	3145.60	9.31	24.06	3144.20	20.27	24.10
Average			**2504.20**	**0.18**	**1902.60**	**2.61**	**22.91**	**1728.70**	**4.96**	**30.75**
Tight	3	5	1306.60	0.00	1066.60	0.00	18.37	1006.20	0.00	22.99
	3	10	1643.60	0.00	1337.80	0.00	18.61	1210.60	0.00	26.34
	3	15	1767.20	0.00	1394.40	0.95	21.10	1233.40	1.13	30.21
	3	20	1802.60	0.27	1434.60	3.08	20.20	1249.80	3.44	30.48
	3	25	2084.60	0.87	1782.60	11.03	13.74	1558.20	9.82	24.60
	6	5	4592.60	0.00	2475.80	0.00	46.09	2276.80	0.00	50.42
	6	10	3648.80	0.09	2835.40	3.98	22.22	2565.20	3.11	29.63
	6	15	3956.20	1.38	3026.80	9.14	22.42	2756.00	8.68	29.36
	6	20	4068.60	1.52	3204.60	10.44	20.02	2879.40	10.01	28.14
	6	25	4527.00	1.84	3503.00	12.99	21.17	3414.40	19.86	23.17
Average			**2939.78**	**0.60**	**2206.16**	**5.16**	**22.39**	**2015.00**	**5.60**	**29.53**
Global average			**2556.33**	**0.27**	**1947.11**	**3.03**	**21.99**	**1751.55**	**4.33**	**30.51**

4.4. Fixed versus flexible network design with due dates

In this section we analyze the combined effect of both types of flexibility. Table 6 provides a general overview on the flexibility gained from the network design and the due dates. We compare the solutions obtained from both fixed and flexible network designs by assuming different due dates. As shown in the table, the cost for both designs decreases as the due date increases. The cost of the flexible design is always lower than the one of the fixed design and the difference between the costs increases when the value of r increases. This is consistent with the results shown in the previous section. Moreover, the gap between the costs of fixed vs. flexible network are more relevant for the cases of normal and loose vehicle capacity. Concerning optimality gaps, we see that they increase with the value of r and with a tight vehicle capacity. In all cases, the flexible case is always more difficult to solve and presents larger optimality gaps.

In order to have a better estimation of the savings achieved by combining the two types of flexibility, in Table 7 we compare the most inflexible case, i.e., with $r = 0$ and a fixed network design, against the most flexible one which has two-day delivery due date and a flexible network design. Although the most flexible problem is harder to solve to optimality, we could observe cost savings of 35% with 27% reduction in the number of used DCs, on average. While in cases with tight or normal capacity this reduction in the total cost is related to a remarkable fewer number of DCs rented (32% and 43% less, respectively), for the cases with loose vehicle capacity, an average of 31% less cost is gained with a reduction in the number of rented DCs of only 5%.

Table 6: Cost of fixed and flexible designs for 3 DCs with different due dates

Instances			$r = 0$				$r = 1$				$r = 2$			
	Periods	Customers	Fixed		Flexible		Fixed		Flexible		Fixed		Flexible	
			Cost	Gap (%)	Cost	Gap (%)	Cost	Gap (%)	Cost	Gap (%)	Cost	Gap(%)	Cost	Gap (%)
Loose	3	5	986.40	0.00	984.20	0.00	904.20	0.00	835.40	0.00	897.20	0.00	733.80	0.00
	3	10	1390.60	0.00	1389.00	0.00	1277.60	0.00	1162.20	0.00	1248.00	0.00	991.80	0.00
	3	15	1469.20	0.00	1469.20	0.00	1344.40	0.00	1227.40	0.00	1320.20	0.00	1032.60	0.00
	3	20	1535.60	0.00	1533.40	0.00	1378.20	0.34	1254.60	0.68	1337.40	0.82	1040.60	0.85
	3	25	1757.80	0.09	1756.60	0.12	1541.00	3.04	1454.40	1.65	1523.00	2.87	1280.80	4.81
	6	5	2085.40	0.00	2071.80	0.00	1714.40	0.00	1554.40	0.00	1609.40	0.00	1336.40	0.00
	6	10	2813.40	0.00	2809.20	0.00	2244.40	0.00	2090.00	0.00	2065.20	0.00	1807.60	0.73
	6	15	3079.60	0.00	3079.20	0.00	2408.40	0.59	2255.00	1.67	2200.60	0.52	1925.20	2.71
	6	20	3372.60	0.00	3372.60	0.01	2820.20	2.91	2620.00	2.71	2678.80	2.67	2359.40	5.33
	6	25	3788.60	0.23	3785.00	0.28	3203.40	6.25	2872.40	6.54	3169.40	12.21	2601.40	9.85
Average			**2227.92**	**0.03**	**2225.02**	**0.04**	**1883.62**	**1.31**	**1732.58**	**1.33**	**1804.92**	**1.91**	**1510.96**	**2.43**
Normal	3	5	1334.60	0.00	1142.00	0.00	1096.60	0.00	945.60	0.00	1071.60	0.00	874.40	0.00
	3	10	1744.40	0.00	1544.00	0.00	1428.40	0.00	1234.40	0.00	1364.80	0.00	1056.00	0.00
	3	15	1863.80	0.00	1664.20	0.00	1553.00	0.00	1312.00	0.00	1439.60	0.00	1140.00	0.00
	3	20	1903.20	0.00	1725.60	0.00	1539.20	1.64	1327.80	1.02	1452.80	2.13	1117.20	1.77
	3	25	2174.80	0.82	1946.60	0.93	1726.60	2.43	1537.60	2.82	1681.40	4.02	1370.60	7.28
	6	5	2807.20	0.00	2405.60	0.00	2423.80	0.00	1821.40	0.00	2273.60	0.00	1654.20	0.00
	6	10	3603.60	0.00	3277.20	0.00	2818.40	0.00	2329.00	1.48	2649.40	0.24	2050.60	1.71
	6	15	3838.00	0.00	3354.00	0.00	2854.40	1.26	2474.80	3.80	2777.20	2.65	2216.60	7.63
	6	20	4172.20	0.55	3828.20	0.53	3453.40	4.96	2897.80	7.69	3255.40	6.74	2663.20	10.97
	6	25	4565.00	0.82	4154.60	0.29	3917.40	11.17	3145.60	9.31	3556.00	9.79	3144.20	20.27
Average			**2800.68**	**0.22**	**2504.20**	**0.18**	**2281.12**	**2.15**	**1902.60**	**2.61**	**2152.18**	**2.56**	**1728.70**	**4.96**
Tight	3	5	1456.40	0.00	1306.60	0.00	1319.20	0.00	1066.60	0.00	1267.80	0.00	1006.20	0.00
	3	10	1841.00	0.00	1643.60	0.00	1564.80	0.00	1337.80	0.00	1499.80	0.00	1210.60	0.00
	3	15	1894.40	0.00	1767.20	0.00	1630.60	0.41	1394.40	0.95	1507.80	0.35	1233.40	1.13
	3	20	1941.20	0.29	1802.60	0.27	1672.80	2.59	1434.60	3.08	1543.80	3.14	1249.80	3.44
	3	25	2215.80	1.04	2084.60	0.87	1958.40	7.58	1782.60	11.03	1891.00	10.12	1558.20	9.82
	6	5	4953.60	0.00	4592.60	0.00	2795.60	0.00	2475.80	0.00	2685.60	0.00	2276.80	0.00
	6	10	3889.80	0.00	3648.80	0.09	3227.80	0.16	2835.40	3.98	3046.80	0.56	2565.20	3.11
	6	15	4159.40	1.08	3956.20	1.38	3495.80	4.50	3026.80	9.14	3282.40	4.02	2756.00	8.68
	6	20	4269.20	1.32	4068.60	1.52	3645.80	7.53	3204.60	10.44	3382.40	6.73	2879.40	10.01
	6	25	4645.20	1.41	4527.00	1.84	3991.00	10.18	3503.00	12.99	3659.60	8.55	3414.40	19.86
Average			**3126.60**	**0.51**	**2939.78**	**0.60**	**2530.18**	**3.29**	**2206.16**	**5.16**	**2376.70**	**3.35**	**2015.00**	**5.60**
Global average			**2718.40**	**0.25**	**2553.77**	**0.27**	**2231.64**	**2.25**	**1947.11**	**3.00**	**2111.27**	**2.60**	**1751.55**	**4.33**

Table 7: Comparison between the most inflexible and the most flexible scenarios

	Instances		Most inflexible		Most flexible			
	Periods	Customers	Cost	Gap (%)	Cost	Gap (%)	Guaranteed savings (%)	% reduction in # DCs
Loose	3	5	986.40	0.00	733.80	0.00	25.61	0.00
	3	10	1390.60	0.00	991.80	0.00	28.68	0.00
	3	15	1469.20	0.00	1032.60	0.00	29.72	0.00
	3	20	1535.60	0.00	1040.60	0.85	32.23	0.00
	3	25	1757.80	0.09	1280.80	4.81	27.07	0.00
	6	5	2085.40	0.00	1336.40	0.00	35.92	20.00
	6	10	2813.40	0.00	1807.60	0.73	35.75	10.00
	6	15	3079.60	0.00	1925.20	2.71	37.49	16.67
	6	20	3372.60	0.00	2359.40	5.33	30.04	3.33
	6	25	3788.60	0.23	2601.40	9.85	31.18	3.33
Average			**2227.92**	**0.03**	**1510.96**	**2.43**	**31.37**	**5.33**
Normal	3	5	1334.60	0.00	874.40	0.00	34.48	40.00
	3	10	1744.40	0.00	1056.00	0.00	39.46	43.33
	3	15	1863.80	0.00	1140.00	0.00	38.83	40.00
	3	20	1903.20	0.00	1117.20	1.77	41.30	43.33
	3	25	2174.80	0.82	1370.60	7.28	36.46	46.67
	6	5	2807.20	0.00	1654.20	0.00	41.07	45.00
	6	10	3603.60	0.00	2050.60	1.71	43.10	45.00
	6	15	3838.00	0.00	2216.60	7.63	42.25	48.33
	6	20	4172.20	0.55	2663.20	10.97	35.81	41.67
	6	25	4565.00	0.82	3144.20	20.27	30.55	38.33
Average			**2800.68**	**0.22**	**1728.70**	**4.96**	**38.33**	**43.17**
Tight	3	5	1456.4	0.00	1006.2	0.00	30.91	33.33
	3	10	1841.0	0.	1210.6	0.00	34.24	36.36
	3	15	1894.4	0.00	1233.4	1.13	34.89	33.33
	3	20	1941.2	0.29	1249.8	3.44	35.43	33.33
	3	25	2215.8	1.04	1558.2	9.82	28.94	30.00
	6	5	4953.6	0.00	2276.8	0.00	54.04	30.56
	6	10	3889.8	0.00	2565.2	3.11	34.05	27.27
	6	15	4159.4	1.08	2756.0	8.68	33.02	34.85
	6	20	4269.2	1.32	2879.4	10.01	31.65	33.33
	6	25	4645.2	1.41	3414.4	19.86	25.45	30.00
Average			**3126.60**	**0.51**	**2015.00**	**5.60**	**34.26**	**32.24**
Global average			**2718.40**	**0.25**	**1751.55**	**4.33**	**34.65**	**26.91**

Finally, Table 8 provides the computation times. We can see that they increase for both network designs as the due date increases and, in general, the flexible network design takes slightly longer.

Table 8: Computation time of fixed and flexible network designs

	Instances		Fixed			Flexible		
	Periods	Customers	$r = 0$	$r = 1$	$r = 2$	$r = 0$	$r = 1$	$r = 2$
Loose	3	5	0	1	1	0	1	1
	3	10	14	82	37	6	41	66
	3	15	8	1726	1372	7	285	401
	3	20	53	4176	6998	26	3753	5332
	3	25	2436	10566	10051	2717	9122	10801
	6	5	3	11	9	2	25	29
	6	10	19	331	1342	16	2992	5075
	6	15	153	7177	7911	54	10800	9564
	6	20	647	10800	10800	2552	8751	10800
	6	25	6066	10800	10801	10800	10800	10800
Average			**940**	**4567**	**4932**	**1618**	**4657**	**5287**
Normal	3	5	0	1	1	1	4	3
	3	10	9	116	166	10	128	111
	3	15	34	2417	3682	27	902	5215
	3	20	192	6599	7085	233	6901	7613
	3	25	5679	9036	8767	4938	10485	10801
	6	5	3	26	15	5	48	106
	6	10	361	949	3384	74	7530	8594
	6	15	571	10541	10800	937	10800	10800
	6	20	6706	10801	10801	7072	10800	10800
	6	25	10377	10800	10800	7746	10800	10800
Average			**2393**	**5129**	**5550**	**2104**	**5840**	**6484**
Tight	3	5	0	2	1	0	4	3
	3	10	13	104	168	10	217	500
	3	15	102	5091	5250	159	5222	8138
	3	20	3541	10380	10800	4058	10786	9034
	3	25	8949	10801	10801	7221	10801	10801
	6	5	9	130	201	8	1040	4009
	6	10	9559	5637	6659	2445	10800	10800
	6	15	1413	10800	10800	10801	10800	10800
	6	20	10378	10801	10800	10458	10800	10800
	6	25	10801	10801	10800	10801	10800	10801
Average			**4476**	**6455**	**6628**	**4596**	**7127**	**7569**
Global average			**2603**	**5383**	**5703**	**2773**	**5875**	**6447**

5. Conclusions

In this paper we have introduced the F-2E-LRP and proposed a mathematical formulation along with different classes of valid inequalities for it. Inspired by recent works in the sharing economy and extending several classes of the routing problems, the F-2E-LRP combines integration issues related to the decision of which facilities to rent, and flexibility issues coming from two sources: network design and delivery due dates. The results obtained from the experiments on randomly generated instances show the value of flexibility, both in terms of due date and network design. The results highlight the cost saving advantages of both types of flexibility.

This study opens different avenues for future research. In particular, being the F-2E-LRP a highly complex problem which can find applications in real distribution settings, it would be worthwhile to propose heuristic algorithms that can handle large size instances.

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