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Abstract: In this paper, we propose a queueing model to analyze the performance of an opportunistic spectrum access (OSA) system with service interruptions operating over heterogeneous channels in which the service transmission rate and the service interruption rate after the transmission is resumed are generally different than their value prior to the interruption. We first propose Markov chain models to analyze this system under memoryless service time and availability periods. Based on simplification assumptions, we also provide an analytical z-Transform analysis of the Markov models. The Markov model and approximations are validated with accurate system simulations. We also provide numerical results illustrating the non-convex relations between the traffic metrics and system parameters and that the proposed models are essential for optimal OSA network planning and operation. We further analyze and discuss the OSA queuing model for general distribution of service time and availability periods. The analytical and simulation results indicate that for usual system parameters, the queue average occupancy is similar for different distributions of service time and availability periods and that the memoryless Markov models can be used to accurately predict the heterogeneous OSA system traffic performance.

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1 Introduction

Opportunistic spectrum access (OSA) is expected to be widely deployed in next generation wireless networks to address the fast traffic growth in wireless networks [1–4]. In OSA networks, the packet transmissions are frequently interrupted because the cognitive radio (CR) users must stop using their operating channel when the channel's primary users appear or if the channel quality drops below a minimum threshold. During the interruption, depending on the OSA medium access control (MAC) algorithm, the CR users might stay on the channel until it becomes available again or switch to a new channel [5]. In both cases, the operating channel after the interruption might have different parameters (channel bandwidth, probability of primary user arrivals, propagation conditions, level of interference, etc.) than the operating channel prior to the interruption.

Our objective in this work is to obtain a queueing model, which is is an important tool to analyze CR traffic performance [6], in an OSA network with heterogeneous operating channels. In the queueing model, the operating channel is the server of the queue. We therefore have a system with a server with a time-variant service rate subject to frequent service interruptions occurring at a time-variant rate.

1.1 Related work

Queueing models for servers with interruptions have been previously studied [7–11]. However, all those works have considered a server with a time-invariant service rate and server interruption rate. Two-class preemptive queuing models have been proposed for OSA networks where the CR server interruptions are modeled as the busy periods of the preemptive primary traffic [6, 12–14]. With this approach, the server is considered time-invariant with a constant service rate and interruption rate. Furthermore, it is not straightforward to use those models to study OSA networks with generally distributed operating and interruption periods.

In [15] we proposed a new queueing model for OSA networks for a single class of CR traffic for general operating and interruption period lengths. However, a time-invariant service rate and server interruption rate is also assumed in this model. In [16] an OSA queuing model with variable service rate is studied. But the work did not address the recovery periods and variable interruption rate. The queueing model is also specific to the MAC protocol studied in the paper. The applicability of queueing theory to analyze multiuser multichannel cognitive radio networks has been discussed in [17,18] where channels are assumed to be similar. In [19], the authors discuss a queueing model for a multiuser cognitive radio network with a variable service rate, but the notion of recovery periods is not considered. Variable service rate has also been addressed in [20] but for an underlay model; the notion of interruption is therefore not applicable.

1.2 Contributions

Our major contribution in this paper is, to the best of our knowledge, the first accurate Markov chain (MC) queuing model for an OSA system using heterogeneous channels with different transmission and interruption rates in the presence of interruptions with random periods. To obtain analytically tractable results, some simplifying assumptions, such as memoryless packet and interruption arrival process, have naturally been made to the cognitive network model. We also introduce another MC model for generally distributed interruptions where the arrival process during interruptions is approximated as a bunch arrival. Those two models can be numerically solved to find the distribution of the number of packets and therefore analyze the OSA network traffic metrics for different network parameters' values.

In some cases, such as for large number of channels and queue lengths, or for real-time control algorithm implementations such as the MAC protocol, channel sensing order selection, and user admission, the MC models can be computationally too intensive to solve. Another important contribution of this paper is therefore a novel analytical z-transform analysis to approximate the traffic metrics for the heterogeneous OSA network queue model. The z-transform analysis is also extended to general distributions of service time and availability periods.

The queueing models developed in this paper can be used to analyze the traffic metrics of different OSA networks using a set of heterogeneous channels. Although the performance study of an OSA network for specific MAC protocols is outside the scope of this paper, we present numerical results in this paper illustrating the complex non-convex relationships between the traffic metrics and demonstrating that tools, such as the ones we introduce in this paper, are essential for OSA network planning. Even though the analysis is made in the presence of some commonly used simplification assumptions, we further provide some simulation results demonstrating that the simplified model is a good analytical approximation for more complicated and realistic scenarios.

The remainder of this paper is organized as follows. Section 2 presents the OSA system and queueing models considered in this paper. In Section 3, we present the accurate two-dimensional MC model and the approximate z-transform analysis for exponentially distributed recovery periods. The case of general interruption periods is studied in Section 4. The accuracy of those different models and approximations is verified with Monte-Carlo simulations of the system in Section 5. We further analyze and discuss in Section 6 the OSA queuing model for general distributions of service time and availability periods. Finally, Section 7 concludes the paper.

2 System model

As illustrated in Figure 1, the CR node alternates between operating and recovery periods. The operating periods begin from the time that the CR node switches to a new channel and last until a recovery or spectrum search has to be performed. Without loss of generality, we will use the term recovery to designate the period during which the CR node must stop its transmission and search over channels according to an arbitrary channel search algorithm depending on the protocol used by the CR network and, possibly, compete with other users to reserve a channel according to a medium access control (MAC) protocol. At the end of the recovery period, the CR node will select a channel of type i, $i \in \{1, \dots, B\}$, and resume its packet transmission. We assume in this paper that the B types of channels are heterogeneous with different parameters such as channel bandwidth, link signal to noise plus interference ratio (SINR) and primary user occupancy distribution. Therefore, the system parameters are different between the operating periods before and after a recovery period. We also use, without loss of generality, the term failure event to designate the event triggering the start of a recovery period or equivalently the end of an operating period. A failure event can be due to several factors such as the appearance of primary users, a bad link quality in the operating channel or a periodic trigger.

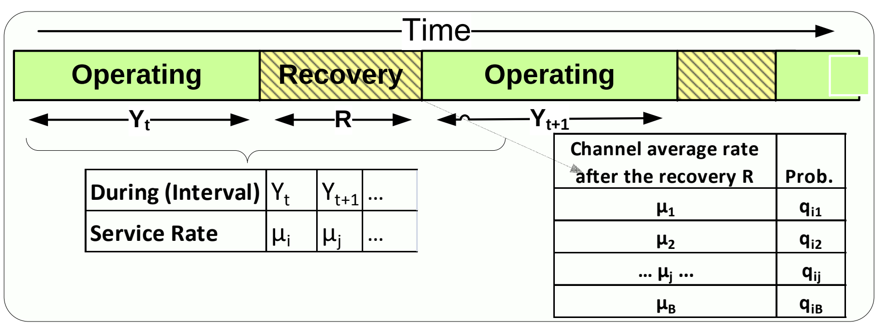

Figure 1: Cognitive radio node operation in opportunistic spectrum access.

The operating period has a random length Y with a general distribution. We assume that the distribution type is the same for the B types of channel (for example, they are all exponentially distributed) but the parameter (for example, the average) is different for each channel. The general distribution case is studied in Section 6.1 but, as discussed there, it is very difficult to obtain analytical queue performance results for non-memoryless operating period distributions. For this reason, the exponential distribution has widely been used in the literature to model the operating periods' length [16, 21–23]. Results presented in this paper also show that the analytical results with the exponential distribution are good approximations for other distributions. For the memoryless case analyzed in Sections 3 and 4, the operating period over channel type

i, $i \in \{1, \ldots, B\}$, is modeled with an exponential distribution with parameter α_i. Thus, when the CR node is operating on channel i, failure events occur at a rate α_i.

The service time depends on both the packet length and the channel transmission rate. In this model, we consider a time-invariant Poisson packet arrival process with rate λ and time-invariant packet length distribution and parameters. During an operating period, it is also assumed that the channel transmission rate is constant. However, the transmission rate is channel dependent (for example, due to the different bandwidth or channel link quality) and changes for each operating period. We therefore have a system where the service time probability distribution is the same for all operating periods, but its parameters change for each operating period (for example, the service time is exponentially distributed with a different mean for each operating period). The general service time distribution case is studied in Section 6.2 and the memoryless packet length (and thus memoryless service time) is analyzed in Sections 3 and 4. For the later case, the service rate for channel type i, $i \in \{1, \ldots, B\}$, is modeled with an exponential distribution with parameter μ_i.

The recovery period had a random length R. Since the recovery period length depends on the system parameters, we model it with a time-invariant distribution which does not depend on the channel used during the previous operating period. In Section 3, we study the case of a recovery period with an exponential distribution with the parameter β (i.e., $E[R] = 1/\beta$) and in Section 4 we consider a generally distributed recovery period. The probability that channel j will be selected at the end of the recovery period given that channel i was used before the recovery period is assumed to be known and it is given by q_{ij}.

Several factors affect the distribution of the recovery period R and the channel selection probabilities q_{ij}. In multiuser scenarios, the MAC and opportunistic scheduling of users influence the distribution of R and the probabilities q_{ij}. For instance, in probabilistic MAC models, based on Aloha or CSMA, the recovery time to find a new channel includes not only the search time, but also the competition time (including backoff periods) with other nodes [5, 24]. Even if the user is blocked due to other users transmitting on all channels, the total time of blocking until a successful channel reservation is included in the recovery time, which can be found for instance using a renewal model (i.e., if the user is blocked, the competition process to reserve a channel is renewed). In a network where a spectrum server assigns the channels, the recovery time is a deterministic short period of negotiation with the spectrum server until a new channel is assigned. Other factors, such as the channel search algorithm, channel sensing strategy, and number of channels should also be taken into account to determine the distribution of R. Note that the exponential distribution is a good approximation for the commonly used random channel search where channels are sensed one by one until the first available channel is found [25] and for multiuser Aloha competition for channel reservation [24]. A methodology to find the recovery period distribution for two baseline multichannel opportunistic spectrum access MAC protocols is also provided in [24].

q_{ij}, the probability of finding a channel with a specific service rate after the period R, also depends on several factors such as the channel availability, channel models (fading and interference), multiuser channel assignment, and, in some cases, on the service rate of the channel used before the recovery. For example, we are using the common assumption in the literature [5, 16] that each channel is used by one user. However, given that multiple users may share the channels, we can assume that when a channel is multiplexed between multiple users, its service rate is also divided equally between the users. So, in the proposed queueing model, we can have a row for each possible service rate which would depend on the number of users assigned to the channel. The q_{ij} then depends on the number of users assigned to a channel and the channel scheduling strategy. Naturally, finding the q_{ij} values with channel sharing can be more involved [26].

A combination of prior statistical knowledge about the channels (availability probability, channel fading model, etc.) in addition to system parameters (MAC protocol, channel search and sensing algorithms, channel assignment algorithm, number of users, number of channels, etc.) is therefore required to determine the distribution of R and the q_{ij}. The approach to find those is thus case-dependent and is out of the scope of this paper. However, the objective of this paper is to propose and analyze a general queueing model which can be used to find the packet level performance once those distribution are found. As discussed, the proposed model in this section is general and may be used for per-node performance analysis in different heterogeneous multichannel opportunistic spectrum access network scenarios with multiple homogenous users.

3 Queue model for exponentially distributed recovery periods

In this section, we consider exponential distributions for the operating periods, service time, and recovery periods, which enable the development of an accurate queue model. In Section 3.1, we present a Markov chain model for this CR queue and in Section 3.2, we derive approximate analytical queue performance results.

3.1 Markov chain model

Figure 2 shows the exact bi-dimensional Markov chain model for the case of exponentially distributed recovery periods. State (k, i), $k = 0, 1, \ldots,$ and $i = 1, \ldots, B$, indicates that there are k packets in the system operating over a channel of type i. State $(k, 0i)$, $k = 0, 1, \ldots,$ and $i = 1, \ldots, B$, indicates that there are k packets in the system which is in a recovery period, and the last operating channel before the start of the current recovery period has been channel i. When a failure occurs, the state makes a transition from (k, i) to state $(k, 0i)$, while at the end of a recovery period, the state makes a transition from $(k', 0i)$ to state (k', j) with a probability q_{ij}. The states $(k, 0i)$ enable us to accurately model the packet arrivals during the recovery period, but the recovery periods must be exponentially distributed to be able to introduce them into the Markov chain. Note that B separate lines are needed for the recovery periods because of the memory between consecutive operating channels.

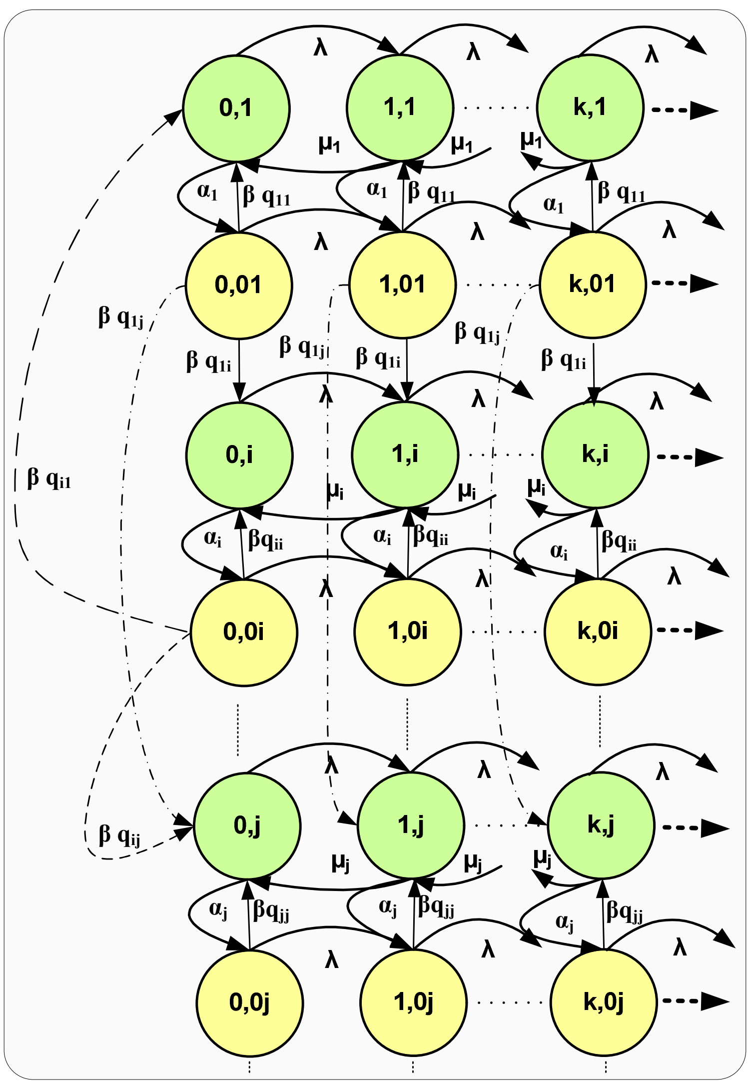

Figure 2: Markov chain for the queue with variable service rate and exponential recovery periods (M1).

For convenience, let us define $\mu_{0i} = 0, \forall i > 0$. The transition rates are then as follows:

$$p_{(k,i)(l,j)} = \begin{cases} \lambda & i = j \text{ and } l = k+1 \\ \mu_i & i = j, i \in \{1, \ldots, B\} \text{ , } l = k-1 \text{ and } k > 0 \\ \alpha_i & i \in \{1, \ldots, B\} \text{ , } j = 0i \text{ and } l = k \\ \beta q_{mj} & i = 0m \text{ , } m \in \{1, \ldots, B\} \text{ , } j \in \{1, \ldots, B\} \text{ and } l = k \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

The balance equations are then given by

$$\pi(0,i)(\alpha_i + \lambda) = \pi(1,i)\mu_i + \beta \sum_{j=1}^{B} \pi(0,0j)q_{ji} \qquad i = 1, \ldots, B \quad (2)$$

$$\pi(k,i)(\alpha_i + \lambda + \mu_i) = \pi(k+1,i)\mu_i + \pi(k-1,i)\lambda + \beta \sum_{j=1}^{B} \pi(k,0j)q_{ji} \qquad k > 0, \; i = 1, \ldots, B \quad (3)$$

and

$$\pi(k,0i)(\beta + \lambda) = \pi(k-1,0i)(\lambda) + \pi(k,i)\alpha_i \qquad k > 0, \; i = 1, \ldots, B. \quad (4)$$

$$\pi(0,0i)(\beta + \lambda) = \pi(0,i)\alpha_i \qquad i = 1, \ldots, B. \quad (5)$$

The Markov chain above can then be solved numerically and used to obtain the exact system performance metrics.

3.2 Analytical approximations

The previous Markov chain can be numerically solved, however it could be computationally intensive for large number of channels and large queues. We thus propose to use two different approximations based on the symmetric structure of this Markov chain to obtain analytical results for this model which could be used, for instance, in real-time CR network scheduling and configuration algorithms. Let us first define Q_i as the steady-state probability of operating over channel i which implies either the channel being used now is i (being in a row with an i index) or i was the last channel before a recovery (being in a row with a $0i$ index). The Q_i values can be directly found from a Markov chain, illustrated in Figure 3, where each state represents a whole line in the bi-dimensional Markov chain and the transition probabilities (rates) are given by $\alpha_i' s$ and q_{ij} values. Note that as β is unique and thus the same for all rows, in finding the steady-state probabilities of operation over channel type i, the recovery periods are neglected.

In the first approximation approach, we neglect the dependence between the rows for each service rate to approximate the z-transform of the number of packets in the queue as

$$\pi_z(z) \approx \sum_{i=1}^{B} Q_i \pi_z(z,i), \quad (6)$$

where $\pi_z(z,i)$ is the z-transform of the distribution of the number of packets in the system for a queue with a fixed service capacity μ_i. That is, we approximate $\pi_z(z)$ as the weighted sum of independent Markov chains for different service rates, where the weights are the probabilities of having each service rate.

In this Markov chain, each service rate is itself modeled by a two-dimensional Markov chain with two lines: one line for the operating state and the second for the recovery period. The z-transform of each line cannot be obtained because it does not correspond to a complete probability distribution (i.e., the probabilities of a line do not sum to 1). To solve this problem, we introduce $P_{(k,i)}$ and $P_{(k,0i)}$, which are the conditional probabilities that the system is in states (k,i) and $(k,0i)$, respectively, given that the system is in an operating period or a recovery period, respectively. From the symmetric structure, it can be immediately seen that $\pi(k,i) = \frac{\beta}{\alpha_i+\beta} P_{(k,i)}$ and $\pi(k,0i) = \frac{\alpha_i}{\alpha_i+\beta} P_{(k,0i)}$.

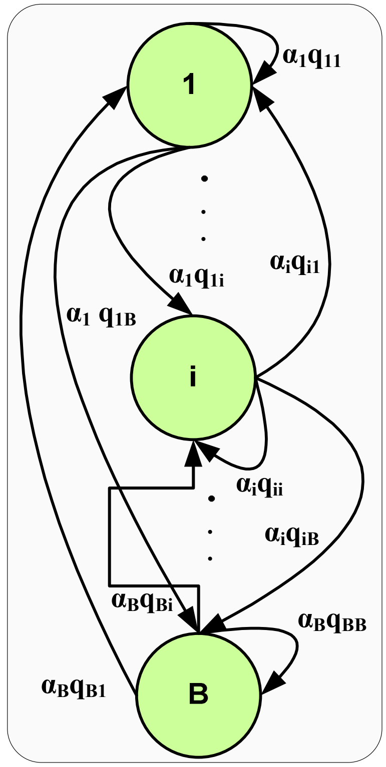

Figure 3: The steady-state probabilities of working on a channel of type i (Q_i) are the steady state probabilities of the Markov chain above.

Let $q_{ii} = 1$ (and thus $q_{ij} = 0$, $\forall j \neq i$) to decouple the service rates. We then obtain the following balance equations as a function of the conditional probabilities $P_{(k,i)}$ and $P_{(k,0i)}$:

$$P_{(0,i)}\beta(\alpha_i + \lambda) = P_{(1,i)}\beta\mu + P_{(0,0i)}\beta\alpha_i, \tag{7}$$

$$P_{(0,0i)}\alpha_i(\beta + \lambda) = P_{(0,i)}\beta\alpha_i, \tag{8}$$

$$(\lambda + \alpha_i + \mu)\beta P_{(k,i)} = \mu\beta P_{(k+1,i)} + \lambda\beta P_{(k-1,i)} + \beta\alpha_i P_{(k,0i)} \qquad k > 0, \tag{9}$$

and

$$(\lambda + \beta)\alpha_i P_{(k,0i)} = \lambda\alpha_i P_{(k-1,0i)} + \beta\alpha_i P_{(k,i)}, \qquad k > 0. \tag{10}$$

Note that in all of the above equations, $(\alpha_i+\beta)$ is common in the denominator and was thus dropped. Because $P_{(k,i)}$ and $P_{(k,0i)}$ are complete probability distributions, we can then obtain their z-transforms $P_{z\,i}(z)$ and $P_{z\,0i}(z)$, respectively, by computing $\sum_{k=1}^{\infty} z^k$ on both sides of the last two equations. We can then find that $P_{z\,0i}(z)$ is equal to:

$$P_{z\,0i}(z) = \frac{\beta P_{z\,i}(z) - \beta P_{(0,i)} + (\lambda+\beta)P_{(0,0i)}}{(\lambda+\beta) - \lambda z} = \frac{\beta P_{z\,i}(z)}{(\lambda+\beta) - \lambda z} = P_{z\,i}(z)A_z(z). \tag{11}$$

$A_z(z)$ represents the z-transform of the number of arrivals during exponential recovery periods. $P_{z\,i}(z)$ can then be determined as:

$$P_{z\,i}(z) = \frac{\mu_i(1-\rho)(1-z)}{\mu_i(1-z) - \lambda z(1-z) - \alpha_i z(1 - \frac{\beta}{\lambda+\beta-\lambda z})}. \tag{12}$$

$P_{(0,i)}$ is equal to $1 - \rho_i = 1 - \lambda E[X_{b,i}]$ [27]. We can now find $P_{(0,0i)}$ from (8), and from there, we can find that $\pi(0,i)$ and $\pi(0,0i)$. Finally, the $\mathbb{P}_0$ of the system can be found to equal:

$$\mathbb{P}_0 = \pi(0,i) + \pi(0,0i) = \frac{1-\rho_i}{1 + \frac{\lambda\alpha_i}{\beta(\alpha_i+\beta+\lambda)}}. \tag{13}$$

$\pi_z(z,i)$, the z-transform of the number of packets in an unconditioned queue with the rate μ_i, can be found equal to:

$$\pi_z(z,i) = \frac{\beta}{\alpha_i+\beta} P_{z\,i}(z) + \frac{\alpha_i}{\alpha_i+\beta} P_{z\,0i}(z) = P_{z\,i}(z)(\frac{\beta+\alpha_i A_z(z)}{\alpha_i+\beta}). \tag{14}$$

The average number of packets in the system can be given by:

$$\begin{aligned}\pi_z^{'}(1,i) = \bar{N} &= P_{z\,i}{}'(1) + \frac{\alpha_i}{\alpha_i+\beta} A_z^{'}(1)\\ &= \frac{\rho_i}{1-\rho_i} + \frac{\alpha_i \lambda^2 E[R^2]}{2\mu_i(1-\rho_i)} + \frac{\alpha_i}{\alpha_i+\beta}\lambda E[R],\end{aligned} \tag{15}$$

where $A_z^{'}(z)$ is the first derivative of A. Combining those results with (6), the overall queue performance can be analytically approximated.

For the second approximation approach, we define the average service rate ($\bar{\mu}$) and the average failure rate ($\bar{\alpha}$) of the bi-dimensional queue as:

$$\bar{\mu} = \sum_{i=1}^{B} Q_i \mu_i, \tag{16}$$

$$\bar{\alpha} = \sum_{i=1}^{B} Q_i \alpha_i. \tag{17}$$

We then approximate the model with a system with homogeneous channels where the service rate is $\bar{\mu}$ and the operating periods are exponentially distributed with the parameter $\bar{\alpha}$. That is, the z-transform of the number of packets in the queue is approximated as:

$$\pi_z(z) \approx \pi_z(z,\bar{\mu},\bar{\alpha}), \tag{18}$$

where $\pi_z(z,\bar{\mu},\bar{\alpha})$ is the z-transform of the distribution of the number of packets in the system for a queue with a fixed service capacity $\bar{\mu}$. $\pi_z(z,\bar{\mu},\bar{\alpha})$ can be found using the same approach as outlined previously to find $\pi_z(z,i)$.

4 Queue model for general interruptions

In this queue model, we simplify the system by assuming that all of the packets arriving during the recovery period actually arrive at the end of the recovery period, or equivalently, at the beginning of the next operating period. However, this model is a simplification and indeed a lower bound, as it neglects a small part of the waiting time for the packets that arrive during a recovery period. On the other hand, this assumption provides a lower bound analysis for generally distributed recovery periods. Exponential distributions for the operating periods and service time are still considered.

The evolution of the CR queue can be modeled with a continuous time Markov model, as illustrated in Figure 4. As the service rate may change between different operating periods, we use a two-dimensional Markov chain model where line i, $i = 1,\dots,B$, corresponds to an operating period with channel i. State (k,i), $k = 0,1,\dots,$ and $i = 1,\dots,B$ indicates that k packets are accumulated in the queue and that channel i with service rate μ_i and failure rate α_i is currently used. In each state of the Markov chain, in addition to transitions caused by the arrival and service processes, there are also transitions due to failure events. Those transitions depend on the channel selected after the recovery period and on the number of packet arrivals during this recovery period. For example, suppose that the current state is (k,i). If a failure occurs, the CR node performs a recovery and selects channel j, $j = 1,\dots,B$ with probability q_{ij}. The next state will then be $\{(k,j), (k+1,j),\dots\}$, which depends on the number of arrivals (probability of n arrival, a_n, in R). The number of arrivals during the random recovery period R is denoted by the random variable A_r and its distribution is given by:

$$a_n = Pr(A_r = n) = \int_0^{\infty} \frac{e^{-\lambda t}(\lambda t)^n}{n!} f_R(t)dt, \tag{19}$$

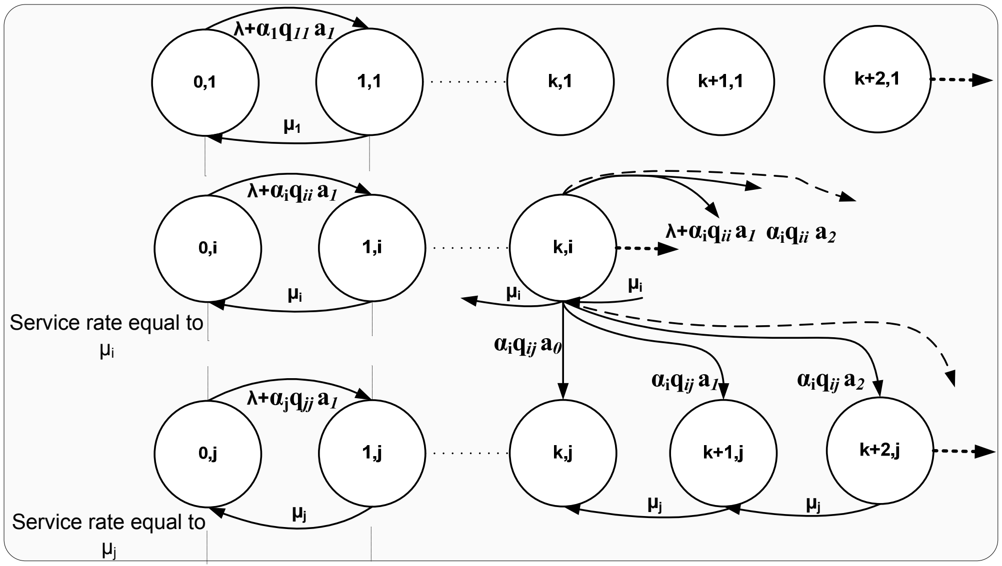

Figure 4: Approximate Markov chain for the queue with a variable service rate (M2).

where $f_R(.)$ is the probability density function of R.

In the proposed Markov chain, the *transition rate* from state (k,i) to state (l,j) can be expressed by:

$$p_{(k,i)(l,j)} = \begin{cases} \lambda + \alpha_i a_1 q_{ii} & i = j \text{ and } l = k+1 \\ \mu_i & i = j \text{ , } l = k-1 \text{ and } k > 0 \\ \alpha_i a_{(l-k)} q_{ij} & i \neq j \text{ and } l \geq k \text{ or } i = j \text{ and } l > k+1 \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

For the state $(0,i)$, the boundary steady-state balance equation can be written as:

$$(\lambda + \alpha_i)\pi(0,i) = \mu_i \pi(1,i) + a_0 \sum_{j=1}^{B} \alpha_j q_{ji} \pi(0,j). \tag{21}$$

For the states where $k > 0$, we have:

$$(\lambda + \alpha + \mu_i)\pi(k,i) = \mu_i \pi(k+1,i) + \lambda\pi(k-1,i) + \sum_{j=1}^{B}\sum_{n=0}^{k} a_n \alpha_j q_{ji} \pi(k-n,j), \tag{22}$$

and the last equation:

$$\sum_{i=1}^{B}\sum_{k=0}^{\infty} \pi(k,i) = 1. \tag{23}$$

Numerical techniques can then be used to solve these balance equations to find the steady-state probabilities $\pi(k,i)$ of this Markov chain. From this distribution, different performance parameters of interest can be obtained such as the queue occupancy, the system time and the waiting time. It is also straightforward to develop analytical approximations for this model using an approach similar to Section 3.2.

5 Simulation and numerical results

To investigate the accuracy of derived results, the first moment of the average number of packets in the system ($E[N]$) has been evaluated using the numerical solutions for the two proposed Markov models and the analytical approximations, and the results are compared with accurate Monte-Carlo simulations of the system. We investigated the CR performance for exponentially distributed operating periods and recovery periods with averages of 75 ms ($E[Y] = E[Y_i] = 1/\alpha_i = 75$ ms $\forall i$) and 25 ms ($E[R] = 1/\beta = 25$ ms), respectively. We also present results for a high system availability case where $E[Y] = E[Y_i] = 750$ ms ($\forall i$). We consider a system with $B = 6$ and rates $\mu_i = [0.27, 0.4, 0.47, 0.8, 0.87, 1]$ (packets/ms). The transition probabilities are independent of the original states and given by $q_{ij} = [0.3, 0.25, 0.15, 0.15, 0.1, 0.05]$ $\forall i$. In the figures, M2 refers to the lower-bound model presented in Section 4 and M1 to the accurate model for the exponential recovery period presented in Section 3, 'Num' refers to the Markov chain numerical solution, and 'An-Apx1' and 'An-Apx2' refer to the two approximate analytical solutions. Note that using simulation results, we find an appropriate buffer size to be able to truncate the Markov chains and solve them numerically.

As expected and illustrated in Figures 5, 6 and 7, the simulated $E[N]$ and $E[N^2]$ and their numerical evaluation using the first Markov chain are identical. As discussed, the second model provides a lower bound for the exact simulated performance. Regarding the analytical approximations, the first approximation is built on a weighted sum of B fixed-rate queues (Eq. (6)) and is therefore sensitive to instability in any one of the B queues. This approximation is therefore accurate as long as the weakest queue with the lowest service rate has a queue utilization, ρ_i, lower than one. For example, for $E[Y] = 75$ ms (Figure 5), we can easily find that $\rho_1 = 1$ for $\lambda = 0.2025$. The first approximation thus starts to diverge when λ approaches 0.2025. The second approximation based on Eq. (6) underestimates $E[N]$, and the gap between the exact result and the approximation increases as a function of the arrival rate. This can be explained by considering that increasing the arrival rate makes the heterogeneity of the channels more important. This approximation thus underestimates the impact of the weakest queue on the overall performance by assuming homogeneous channels with a single average service rate. Figure 7 depicts the queue performance when availability periods are much longer than recovery periods (for this case $E[Y] = 30E[R]$). Longer availability periods imply that the system operates as a fixed-rate queue for long periods of time and the dependency between the different lines of the Markov chain thus becomes less important. The first approximation, which is based on this assumption, therefore provides a very tight bound for the exact performance of the queue.

The previous results have confirmed the exactitude of the Markov model for heterogeneous spectrum access and the first approximation tightness in the regime where all the channels are stable. In the following, we will use the accurate Markov model to gain some insights on the queue performance for an heterogeneous OSA network with two channel types. We assume a packet arrival rate of $\lambda = 0.12$ and a recovery period with an average length $E[R] = \frac{1}{\beta} = 10$. The parameters of the type 2 channel are an average availability period length of $E[Y_2] = 150$ and an average packet service time of $E[Y_2] = 7.5$. Figure 8 shows $E[N]$, the

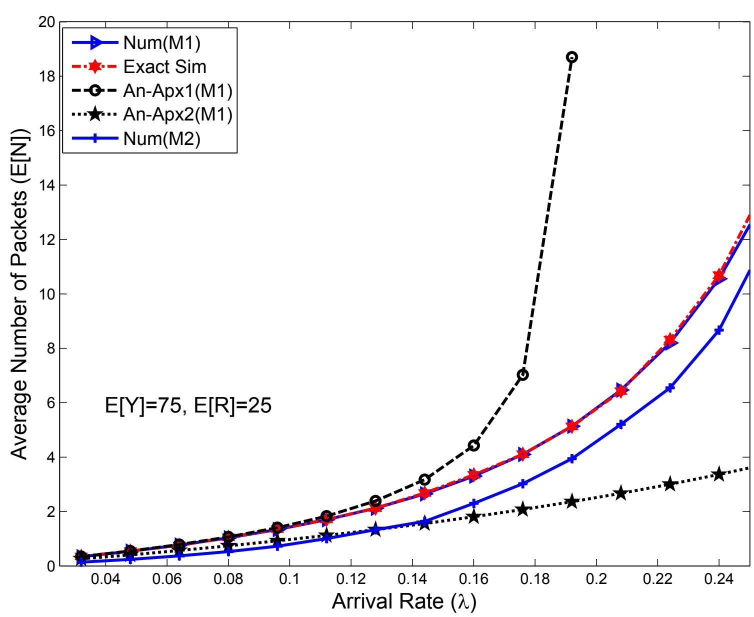

Figure 5: $E[N]$ versus the arrival rate λ.

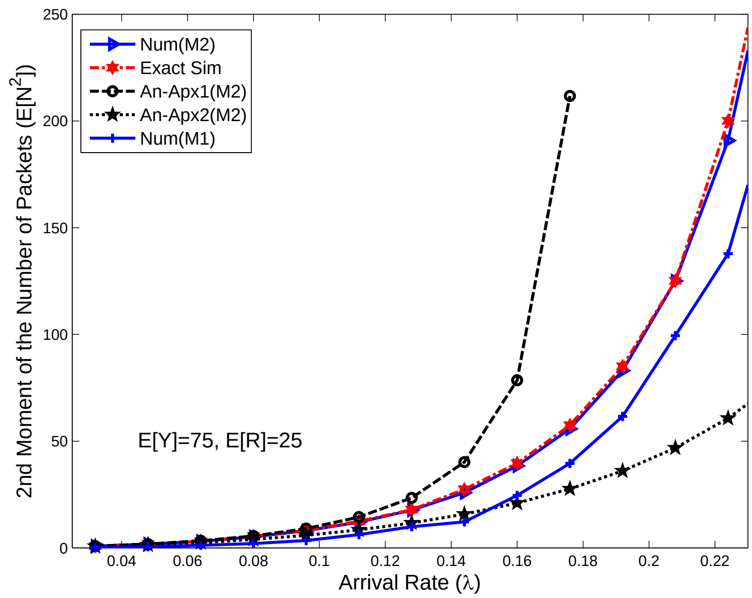

Figure 6: $E[N^2]$ versus the arrival rate λ.

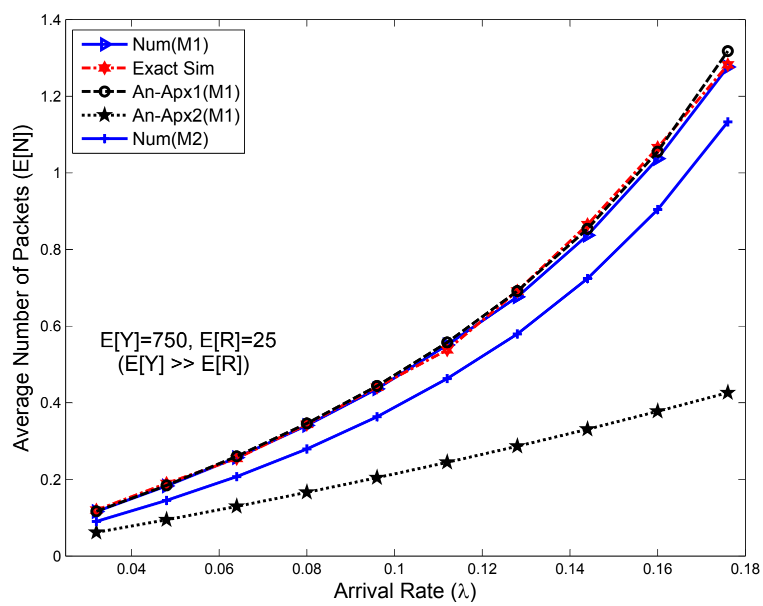

Figure 7: $E[N]$ when availability periods are much longer than recovery periods.

average number of packets in the system, as a function of q_1 ($q_2 = 1 - q_1$), the probability of selecting a type 1 channel after an interruption, for different parameters of the type 1 channel. $E[Y_1]$ varies between 20 and 70 while $E[T_1] = 0.1E[Y_1]$ and therefore varies between 2 and 7. That is, the more reliable channels with longer availability periods have slower service rates.

First, it can be observed from this figure that $E[N]$ is a non-convex function of q_1 and that the lowest value of $E[N]$ is always achieved when only one channel type is used ($q_1 = 0$ or 1). We observed the same behavior for the minimal value of $E[N]$ for all system parameters that we studied. It is straightforward to see that this behavior will not be affected if we had different recovery distributions for each channel type, and if the average recovery time for a transition toward a channel type is longer than the time for staying on that channel type (which is normally the case since there is an additional switching time due to the transition between channels). We therefore conjecture that the optimal $E[N]$ in an heterogeneous OSA system, where

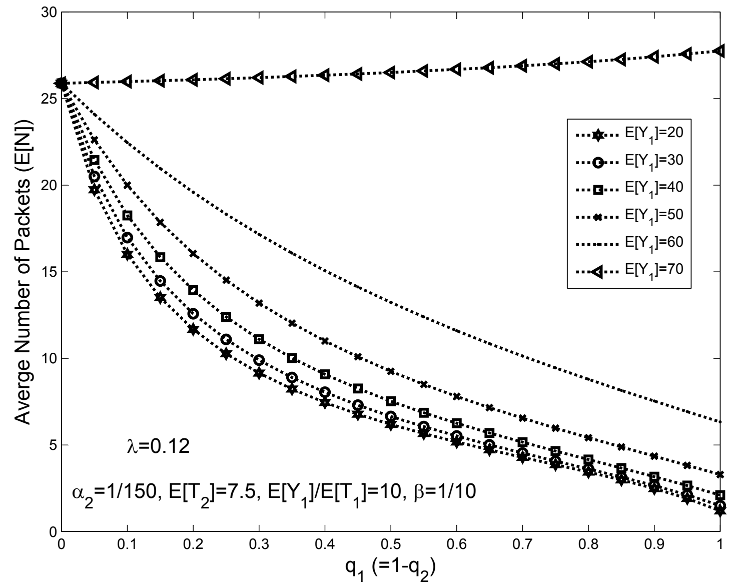

Figure 8: $E[N]$ versus q_1 for two types of channel ($B = 2$).

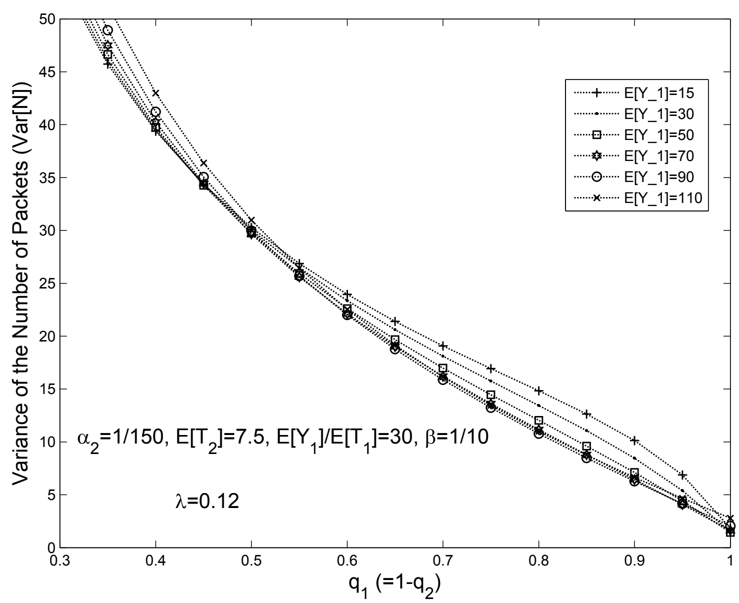

Figure 9: $Var[N]$ versus q_1 for two types of channel ($B = 2$).

the transition probabilities between channel sets are fixed, is achieved when the system always stays on the channel type with the minimum (single-row calculated) $E[N]$. However, due to the system complexity and non-convexity of $E[N]$, proving this conjecture remains an open problem.

Figure 9 shows the variance of N, which is related to the jitter and has an important impact on the buffer size design, as a function of the arrival rate for $E[T_2] = E[Y_2]/30$. Figures 8 and 9 demonstrate how the Markov chain analysis can be used for OSA network planning. For example, assume that the type 2 channels are the basic channels available to the OSA network and that the network designer needs to select the type 1 channel set. q_1 depends on this selection since it is a function of, without limitations, the average channel availability, the channel service rate, the number of available channels in the set and the number of users. It can be observed that depending on the value of q_1 for the different channel types, the optimal type 1 channel varies. For example, assuming that $q_1 = 0.2$ for $E[Y_1] = 20$, then in order to minimize $E[N]$, the type 1 channel set with $E[Y_1] = 40$ should be selected if for those channels we have $q_1 > 0.28$. Figure 9 also shows that there is multiple crossover points where the best channel set changes, further illustrating the non-convexity of the system performance and the importance of carefully selecting the channel sets as a function of the system parameters.

6 General operating periods and service times

Memoryless operating periods and service times were assumed in Sections 3 and 4 to develop Markov chain models. Although the memoryless model has been shown to be a good assumption [21], we explore in this section the analysis and the performance of the CR queues with general distributions for the operating periods and service times.

6.1 Operating periods

The operating period distribution depends on several factors such as the remaining availability part of the selected channel during the recovery period. Since the system has no knowledge of how long this channel has been available so far, we should use renewal theory results [28, 29]. Furthermore, the distribution of the remaining parts of the operating period will be different after each packet transmission, which further complicates the analysis. As discussed in [7, 15], even for a fixed service rate queue with non-exponentially distributed operating periods, only analytical approximations can be obtained for the queue performance. An approximation for the heterogeneous system can be obtained by using the approximate analytical results for a queue with fixed service rate provided in [15] for each variable service rate queue into the two approximations (6) and (18). Results obtained by this approach are called 'Apx1-AnaApx' and 'Apx2-AnaApx' in the figure.

In [21] it was discussed that one of the best models for the channels availability periods in cognitive radio networks is to assume a constant part in addition to an exponential tail. We therefore modeled the operating periods with a Pareto distribution with the same average as the exponential distribution used before ($E[Y]$ =75 ms) but a larger variance (shape parameter = 0.43, scale parameter = 18.3825 and threshold parameter = 42.75).[1] The constant part with this distribution is thus 42.75 ms. We also investigated the case where the CR system has constant operating periods (with the same average) to model a protocol where the system remains on a channel for a fixed given time and not until the moment where the channel becomes unavailable. Simulation and analytical results are presented in Figure 10 for both cases as well as for the exponential distribution. We can observe that the impact of the higher moments of the operating periods on the average number of packets is small so that the queue performance has very close results for the different distributions. This can be explained by the fact that the packet service time is much smaller than the average operating periods. We can also observe that the average number of packets in the system for both Pareto and constant distributions is slightly lower than for the exponential distribution. This is due to the guaranteed constant part where no failure event can occur. We also see that the analytical approximations quality is similar for the general distributions as for the the exponential distribution. From those results, we can conclude that the performance results (either analytical or numerical with the Markov chain model) obtained with the memoryless model can provide acceptable approximations of the queue performance with generally distributed operating periods.

6.2 Service time

For a generally distributed service time, we essentially have an M/G/1 queue with interruptions. For the average queue occupancy, from [27, Eq. 37] and for a queue with a fixed service rate we have:

$$E[N] = \lambda E[X_b] + \frac{\lambda^2 E[X_b^2]}{2(1 - \lambda E[X_b])} + \frac{\lambda E[R^2]}{2(E[Y] + E[R])}. \tag{24}$$

where $E[X_b]$ is the average completion time [15] and is given by $E[T](1 + \alpha E[R])$. $E[T]$ is the real service time of the packets (packet length divided by the queue service rate). Using (24) for each service rate, we can

[1] Generalized Pareto distribution is used to generate Pareto instances.

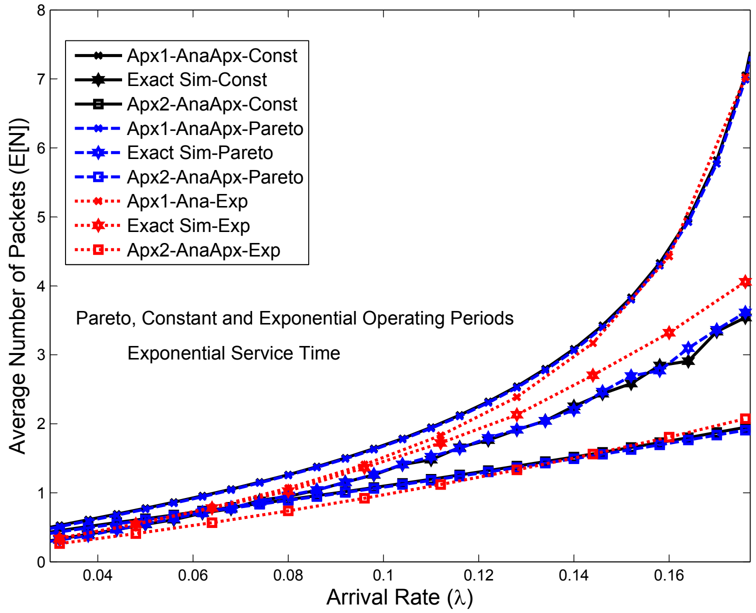

Figure 10: $E[N]$ when operating periods are distributed with Pareto and constant distribution ($E[Y] = 75$), compared to exponential distribution. All other parameters are the same as in Section 5.

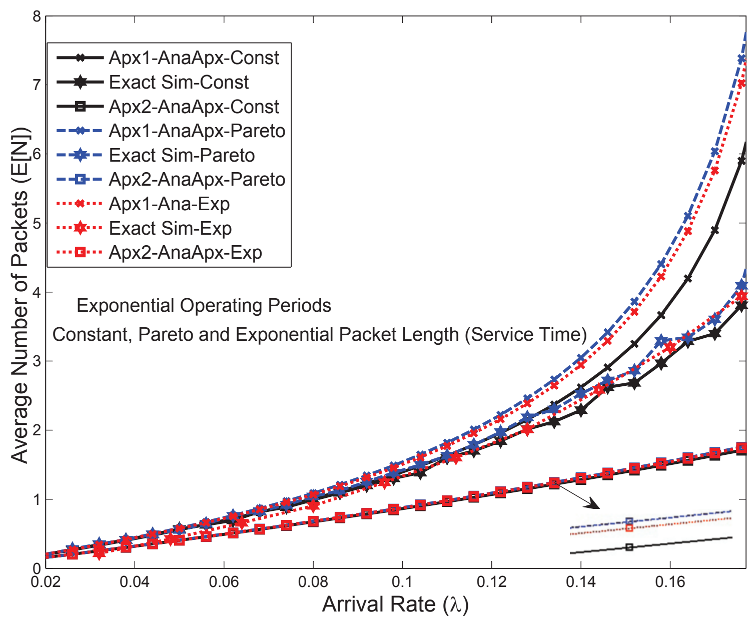

Figure 11: $E[N]$ when packet length is distributed with Pareto and constant distribution. The packet length average and all other parameters are the same as in Section 5.

obtain the same analytical approximations with Eqs. (6) and (18) for the heterogeneous CR system. The two bounds will thus be given by $\sum_{i=1}^{B} Q_i E[N(\mu_i)]$ and $E[N(\bar{\mu})]$ (called 'Apx1-AnaApx' and 'Apx2-AnaApx' in the figures). We have performed simulations for two different distributions: Pareto distribution with a large variance (shape parameter = 0.43, scale parameter = 3.6765 and threshold parameter = 8.55) and constant packet size, both with the same average as for the exponentially distributed packet length in Section 5. As illustrated in Figure 11, we can observe similar results for different packet length distributions and the two analytical approximations can still be used for performance analysis of a cognitive radio system with generally distributed service times.

7 Conclusion

A queueing model with frequent interruptions and heterogeneous channels with variable service rates and failure rates was studied in paper for OSA in CR networks where the new channel after spectrum handover does not necessarily provide the same service rate and availability as the last channel. Modeling the queue as a two-dimensional Markov chain, we established numerical evaluations and analytical approximations for the general case in which recovery periods can have any distribution and for the specific case when they are exponentially distributed. Simulation results are presented to validate our analysis. We also investigated the performance for general distributions of the service time and operating periods and showed that in realistic scenarios, the performance is similar as for memoryless distribution.

This work constitutes a first step for the traffic level study of OSA networks. Future research includes the modeling of MAC protocols for heterogeneous OSA networks to study their performance using the analytical tools provided in this paper. Another interesting research area is to use dynamic transition probabilities between channel sets, for example as a function of the instantaneous number of packets in the system, to obtain a better performance than with the fixed transition probability scheme studied in this paper.

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