SUPPLY CHAIN FLOW PLANNING METHODS:
A REVIEW OF THE LOT-SIZING LITERATURE

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Summary

The objective of this paper is to present, in a unified body, the most important research results dealing with material flow planning in a supply chain, and in particular with deterministic lot-sizing methods. After identifying the most relevant characteristics of material flow planning problems, we present clears and simple formulations of the different lot-sizing problems treated in the literature and we discuss their solution methods. Special attention is given to the multi-facility material flow coordination problems and a new formulation of the general problem is proposed.

Résumé

L'objectif de ce document est de présenter, d'une manière unifiée, les plus importants travaux de recherche qui portent sur les problèmes de pilotage des flux de matières dans les réseaux logistiques et, en particulier, les méthodes de lotissement déterministes. Après avoir identifié les caractéristiques fondamentales de ces problèmes, nous présentons des formulations des principaux cas traités dans la littérature et nous discutions les méthodes de solutions disponibles. Un intérêt spécial est porté aux problèmes de lotissement dans les réseaux logistiques qui incluent plusieurs installations et une nouvelle formulation du problème général est proposée.

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I. Introduction

Strong foreign competition in an increasingly global marketplace has forced firms to turn their attention towards streamlining operations in order to generate savings from a slimmer and more reactive Supply Chain. A.T. Kearney, a management consulting firm (Sengupta and Turnbull [1996]) estimated that supply chain costs represent more than 80% of the cost structure in a typical manufacturing company. For retailers the supply chain costs represent between 70% and 80% of their total costs. From these numbers we can deduce that even a slight improvement in the process of managing the supply chain can be translated into millions of dollars on the bottom line. Before going further, let's define the "Supply Chain".

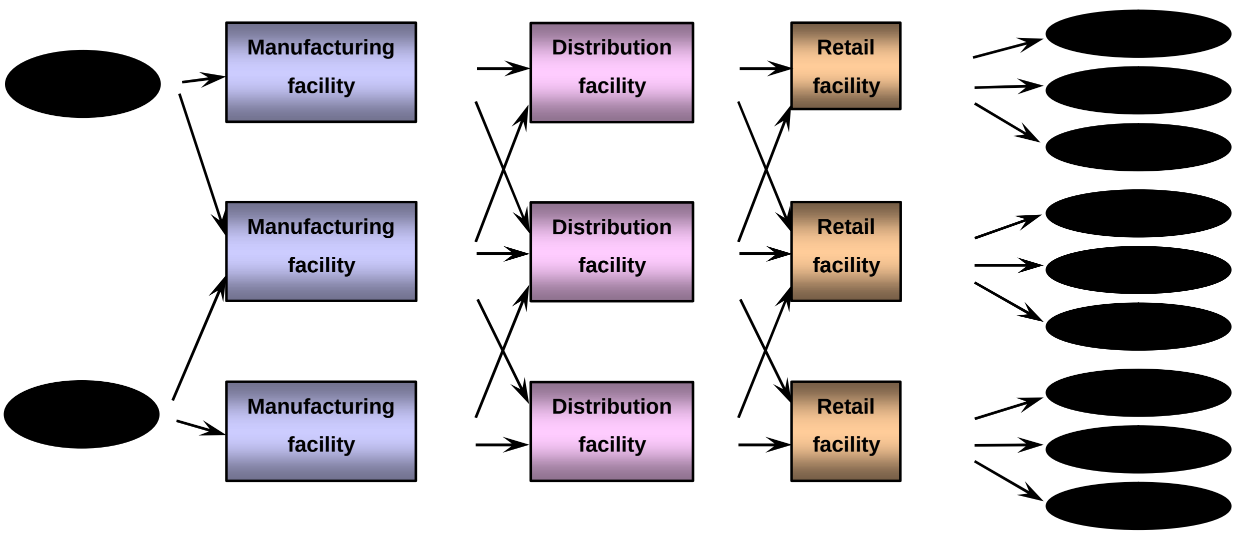


Figure 1: Supply Chain Network

Lee and Billigton [1993] define the Supply Chain (SC) as a network of facilities that performs the functions of procurement of material, transformation of material to intermediate and finished products and distribution of finished products to customers. Thus, as viewed in Figure 1, a SC is a multi-level multi-echelon production-distribution system. Each facility in the network is represented by a node. Arcs indicate how the items flow from one facility to another. Assuming that no item is consumed in its own production, the general supply chain is thus represented by an acyclic directed network.

Once strategic planning has specified the shape and structure of the supply chain, as well as its production, transportation and storage capacity, the question to answer is how to plan and control the flow of materials in the network, or how to mange the supply chain? Supply chain management (SCM), as defined by Sengupta and Turnbull [1996], is the process of effectively managing the flow of materials and finished goods from vendors to customers using manufacturing facilities and warehouses as potential intermediate stops. This problem is one of the most important challenges, especially in a competitive environment that every enterprise seeks to resolve successfully.

The performance of a SC, as observed by Cohen and Lee [1988], can be measured with respect to: (1) the cost of the products delivered to markets, (2) the level of service provided to customers and (3) the responsiveness and flexibility of the production/distribution system. Managing the supply chain effectively can improve customer service levels dramatically, reduce excess inventory in the system, and cut excess costs from the logistic network.

The scientific literature on SCM problems is directly linked to the solution of lot-sizing problems in production-distribution networks, and it can be classified into categories based on the number of locations considered, the number of stages (or echelons) in the system, the number of items considered, the presence or absence of capacity constraints, and the characteristics of demand (see Figure 2). Note that the models found in the literature for each class of problems identified above, may also include different cost structures and allow for backorders or not.

Since one can deal with the stochastic nature of real manufacturing-distribution networks through the use of safety buffers, rolling planning horizons and planning time fences (Graves [1988], Martel *et al.* [1995], Martel [1995]**….**); and since most decision systems are implemented in practice as if the supply chain environment was deterministic over a specified finite horizon, we restrict our review in this text to deterministic models. Note, however, that stochastic supply chain models have also attracted the attention of researchers (see for example, Federgruen [1993] and Axsäter [1993]).

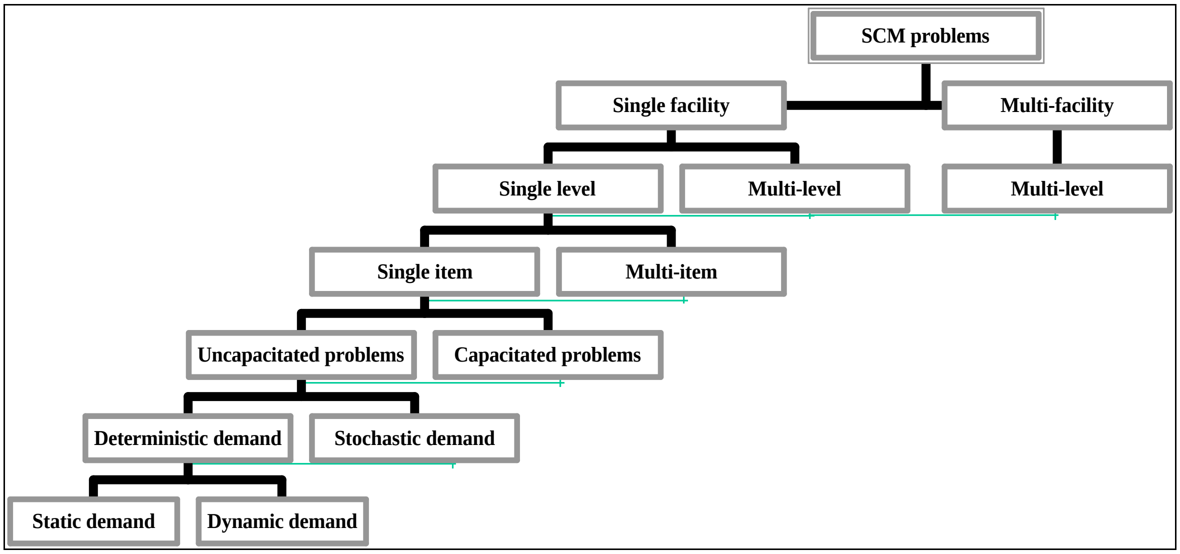


Figure 2: A Classification of SCM Problems

Since there is no unified body, to our knowledge, of literature that deals with the issue of production/distribution planning coordination among multiple plants/warehouses in a vertically integrated firm, we present a detailed review of the major contributions for different important particular cases of this problem. The organization of the text is based on the problem classification presented in Figure 2. A review of the major results related to the lot-sizing problems in a single facility is presented in the next section. The third section reviews the major contributions related to the solution of lot-sizing problems in multi-facility networks.

II. Single Facility Problems

Single facility systems are the most studied in the literature. Even if some researchers' claim that they deal with multi-facility situations, most of them don't consider transportation costs and thus the system they consider is often a multi-machine or multi-stage single facility system. Due to the predominant assumption that the general multi-facilities problem can be solved by optimizing the costs of each facility independently, and due to the difficulties of solving such big size problems in real time, the interdependency between the different facilities in the system are often not considered. In this section, we review the most important contributions to the solution of single facility problems in both single stage and multi-stage situations.

1. Single Level Problems (SLP)

Single stage (echelon) systems are common in procurement and distribution contexts. Even if single stage systems don't describe truly the real situation of the majority of production systems, they have attracted the attention of a large proportion of researchers. This can be explained by the fact that single stage methods may be generalized to deal effectively with multi-stage situations, they may be used as routines for multi-stage methods, and they may also give good insights and ideas to deal effectively with more complex problems.

In single-stage production lot-sizing problems, the manufacturing process is characterized by a single level product structure in which products are directly produced from raw materials without intermediate stocking points or subassemblies. Product demands are derived from customer orders and/or market forecasts. In a procurement/distribution context, single-echelon lot-sizing problems generally consider only the procurement and the inventory costs of a single stocking point and do not take transportation costs into account.

Before going further in our SLP review, we present a generic formulation of the problem. Using the notation in Figure 3 the SLP can be formulated as follows:

$$\text{Min} \sum_{t=1}^{T} \left[S_t \delta(\sum_{i=1}^{n} Q_{it}) + \sum_{i=1}^{n} \left(C_{it}(Q_{it}) + H_{it}(I_{it}) \right) \right] \qquad \textbf{(SLP)}$$

subject to:

$$Q_{it} + I_{i(t-1)} - I_{it} = d_{it} \qquad i = 1,\dots,n;\ t = 1,\dots,T \qquad \text{(SLP1)}$$

$$\ell_{jt} d(\sum_{i=1}^{n} Q_{it}) \le \sum_{i=1}^{n} R_{ij}(Q_{it}) \le u_{jt} d(\sum_{i=1}^{n} Q_{it}) \qquad j = 1,\dots,J;\ t = 1,\dots,T \qquad \text{(SLP2)}$$

$$Q_{it} \in \Omega_{it} \qquad i = 1,\dots,n;\ t = 1,\dots,T \qquad \text{(SLP3)}$$

The objective function includes production/procurement costs, inventory costs and set-up costs. For each period and for each item, constraints (SLP1) describe the relationship between the inventory level at the beginning and the end of the period, the demand and the production/procurement quantity of an item. Constraints (SLP2) enforce resource (capacity) restrictions. Constraints (SLP3) make sure that the production quantities respect the norms imposed (ex: minimum lot size, standard packs, etc). Unless otherwise stated, we assume in what follows that $\Omega_{it} = \{Q_{it} \mid Q_{it} \ge 0\}$. Note that this formulation allows for negative inventory levels I_{it} (backorders).

1.1. Uncapacitated Single Item Problems

The SLP when there is a single item (n = 1) and when there is no capacity constraints was at the origin of early developments in the field of Lot Sizing and Scheduling. Those developments are namely the Economic Order Quantity (EOQ) model (Harris [1913]) (in the case of static demand) and the Wagner-Whitin (WW) model [1958] (in the case of dynamic demand). In the following subsections, we present the major contributions to solve this problem in the case of static and dynamic demand.

Indexes

t: Planning periods ($t = 1, \dots, T$)
i: Items ($i = 1, \dots, n$).
j: Resources ($j = 1, \dots, J$).

Decision variables

Q_{it}: Lot size of item i at period t.
I_{it}: Inventory of item i at the end of period t.

Parameters

u_{jt}: Capacity of resource j available at period t.
ℓ_{jt}: Minimum amount of resource j which can be used at period t.
d_{it}: Effective demand for item i at period t.
c_{it}: Unit production/procurement cost for item i at period t.
s_{it}: Set-up cost for item i at period t.
S_t: Family set-up cost at period t.
h_{it}: Unit holding cost for item i at period t.

Functions

$H_{it}(.)$: Holding cost function for item i at period t.
$C_{it}(.)$: Production/procurement cost function for item i at period t.
$R_{ij}(.)$: Resource j consumption function for item i.
$\delta(.)$: Defined by $\delta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases} \quad \forall\, x \geq 0$

Sets

Ω_{it}: Set of the possible values for Q_{it}.

Figure 3: Basic Notation for the SLP

1.1.1. Static Demand

The first lot sizing and scheduling model is the well-known EOQ-model developed by Harris [1913]. The EOQ-model assumes an infinite planning horizon ($T = +\infty$), a continuous demand at a constant rate (d), an infinite production rate (no capacity constraints), as well as no backlogging possibilities ($I \geq 0$). The production and the holding cost functions in the EOQ-model do not vary in time and are given respectively by: $C(Q) = s\delta(Q)$ and $H(I) = hI$. Using elementary calculus, it can be shown that under these assumptions the optimal production quantity is given by $\sqrt{2sd/h}$. Several extensions to the basic EOQ model are discussed in Hax and Candea [1984]. They cover models which allow for backlogging, lost sales and quantity discounts. Tersine and Price [1981] discuss the temporary price discounts case. Solutions to finite horizon cases where costs are time-dependent are presented by Lev and Weiss [1990] and Gascon [1995]. Several other variants of this basic problem are also found in the literature.

1.1.2. Dynamic Demand

Wagner and Whitin [1958] proposed a dynamic programming algorithm to solve the problem when demand is time varying, there is no backlogging possibilities, no capacity constraints, and when the production/procurement and the holding cost functions are given respectively by $C_t(Q_t) = s_t\delta(Q_t)$ and $H_t(I_t) = h_tI_t$. The WW-model has an $O(T^2)$ running time. In subsequent work, researchers focused on generalizing the WW-procedure to solve more complicated versions of the original problem. Veinott [1963] showed that even if production and inventory costs are general concave functions, the problem is still solvable by an $O(T^2)$ dynamic programming algorithm. Zangwill [1966, 1969], Gupta and Brennan [1992] (among others) generalized the WW-procedure to solve the problem when backlogging is allowed. Martel and Gascon [1998] proposed an algorithm to solve the problem when inventory holding cost is a percentage of the product cost. Extensive research has been done in the case of a perishable, deteriorating or obsolete product. The main references in this case are Veinott [1960], Van Zyl [1964], Ghare and Schrader [1963], Shah [1977], Cohen [1977], Tadikamalla [1978], Nahmias and Wang [1979], Freidman and Hoch [1978] and Jain and Silver [1994]. Since the WW-algorithm is widely used as a routine in solving more complicated problems, recent work on this problem focused on bringing down its theoretical running time by exploiting the special (cost) properties of the problem and by an appropriate choice of data structure (see Federgruen and Tzur [1991], Wagelmans *et al*. [1992] and Aggarwal and Park [1993]). Also, a large number of heuristics have been proposed to solve the WW-model and its variants. The major contributions are De Matteis and Mendoze [1968] (The Part Period Balancing heuristic), Gorham [1968] (The Least Unit Cost Heuristic), Berry [1972] (The EOQ Based Period Order Quantity heuristic), Silver and Meal [1973], Groff [1979], Zoller and Robrade [1988], Gupta and Brennan [1992], and Jain and Silver [1994].

1.2. Capacitated Single Item Problems

The aim of this class of problems is to determine the optimal or near optimal production plan of a single product while known demands are satisfied and capacity restrictions are respected.

1.2.1. Static Demand

The basic EOQ model in the case of a finite production rate is discussed in Hax and Candea [1984], among others.

1.2.2. Dynamic Demand

In the case of dynamic demand the well-known lot-sizing models that deal explicitly with capacity restrictions are the *Capacitated Lot-Sizing Problem* (CLSP), the *Continuous Setup Lot-sizing Problem* (CSLP) and the *Discrete Lot-sizing and Scheduling Problem* (DLSP). In general, these problems ignore setup times. The definition, the complexity and the solution procedures of the single item version of each of the above mentioned problems is discussed in what follows.

1.2.2.1. The Capacitated Lot-sizing Problem (CLSP)

The CLSP aims to determine a production/procurement plan that minimizes production/procurement and inventory holding costs while known demand are satisfied without backlogging ($I_{it} \geq 0$) and capacity restrictions are respected. In the single level CLSP, only one resource is considered (J=1) and no lower bound on resource usage is imposed ($\ell_{jt} = 0 \ \forall$ j, t). The single item CLSP is shown to be NP-Hard for arbitrary cost functions or time varying capacity by Florian *et al*. [1980] and by Bitran and Yanasse [1982]. Even if it is NP-Hard, the basic problem with production cost, holding cost and resource consumption functions given respectively by $C_t(Q_t) = s_t\delta(Q_t) + c_tQ_t$, $H_t(I_t) = h_tI_t$, and $R_t(Q_t) = Q_t$, can be solved by a successful dynamic programming approach proposed by Chen *et al*. [1994]. Many authors proposed polynomial algorithms to solve the constant capacity version of the problem. Florian and Klein [1971] presented an $O(T^4)$ dynamic programming algorithm based on the shortest path method to solve the constant capacity case with concave costs. Their algorithm can deal with the backlogging situation. Jagannathan and Rao [1973] extended Florian and Klein's results to a more general production cost function which is neither concave nor convex. Van Hoesel and Wagelmans [1996] proposed a more efficient $O(T^3)$ dynamic programming algorithm to solve the constant capacity, concave production costs and linear holding costs case. Hill [1997] reduced the constant capacity problem where the production/procurement and the inventory cost are time-invariant and are given respectively by $C(Q_t) = s\delta(Q_t)$ and $H(I_t) = hI_t$ to a typical WW model that can be solved efficiently. The case with time-dependent capacity was also treated in the literature. An $O(2^T)$ dynamic programming algorithm, proposed by Baker *et al.* [1978], solve this problem when costs are constant and when the capacity varies from period to period. Florian *et al.* [1980] extended Florian and Klein's [1971] dynamic programming algorithm to the problem with arbitrary capacities. However, the required computation time becomes substantially larger. Kirca [1990] offered improvements to their algorithm. Lambert and Luss [1982] studied the problem in which the capacity limits are integer multiples of a common divisor and devised an efficient algorithm. In the case of a general cost function, Pochet [1988] proposed a procedure based on polyhedral techniques in combination with a branch and bound procedure. Chen *et al.* [1992.b] proposed a dynamic algorithm for the case of a piecewise linear cost function with no assumption of convexity or concavity, where arbitrary capacity restrictions on inventory and backlogging are allowed. Other contributions for restricted versions of the problem are found in Bitran and Matsuo [1986], Chen *et al.*[1992.a], Chung and Lin [1988] and Chung *et al*. [1994].

1.2.2.2. The Continuous Setup Lot-sizing Problem (CSLP)

The CSLP is closely related to CLSP. However, two important differences exist. First, CSLP allows for at most one product to be produced per period ("small" time bucket model), while in CLSP no limitations exist with respect to the maximum number of products to be produced per period ("Large" time bucket model). Second, CSLP assumes that if a product is produced in two successive periods, then there is no need to setup the machine for the second period batch.

The single item CSLP is showed to be NP-Hard by Florian *et al* [1980]. An exact algorithm for this problem is discussed by Karmarkar *et al*. [1987].

1.2.2.3. The Discrete Lot-Sizing and Scheduling Problem (DLSP)

The DLSP has a large similarity with the CSLP in that it also assumes at most one item to be produced per period ("small" time bucket model), as well as batch setup costs. However, in DLSP the quantity produced in each period is assumed to be zero or equal to the full production capacity. Models of this type are called "all or nothing" models in the literature.

The DLSP has attracted the attention of several researchers mainly because of its importance when developing decomposition algorithms for multi-item problems. Van Wassenhove and Vanderhenst [1983] discuss a hierarchical production planning problem in which the single item DLSP with general cost structures and zero setup times appeared as a subproblem. To solve this problem they use a straightforward dynamic programming algorithm. Other contributions related to this problem are Magnanti and Vachani [1990], Lasdon and Terjung [1971], Cattrysse *et al*. [1993] and Solomon [1991] among others.

1.3. Uncapacitated Multi-item Problems

The main concern of this class of problems is to determine production/procurement lots for multiple products over a finite (in the case of dynamic demand) or infinite (in the case of static demand) planning horizon so as to minimize the total cost, while known demand is satisfied. The total relevant cost generally consists of setup costs, inventory holding costs, production/procurement costs and backlogging costs. Note, however, that when there is no joint setup cost, there is no interdependency between products because there are neither capacity constraints nor parent-component relationships. Therefore, decisions can be made for each product separately.

1.3.1. Static Demand

When no capacity restrictions are imposed, the multi-item problem is relevant when joint setup/order costs exist. In the constant demand case, this is known as the *Economic Order Quantity with Joint Replenishment* (EOQJR) problem. This problem has the same assumptions as those of the classical Economic Order Quantity (EOQ), except for the major setup/order cost. The objective is to determine the joint frequency of production/order cycles and the frequency of producing/procuring individual items so as to minimize the total cost per unit of time. The EOQJR problem occurs, for example, when several items are purchased from the same supplier. In this case, the fixed order cost can be shared by replenishing two or more items jointly. EOQJR may also be attractive if a group of items uses the same vehicle or the same machine. Van Eijs *et al.* [1992] distinguished between two types of strategies used by the algorithms proposed to solve this problem: the "indirect grouping strategy" and the "direct grouping strategy". Both strategies assume a constant replenishment cycle (the time between two subsequent replenishments of an individual item). The items that have the same replenishment frequency form a "group" (set of items that are jointly replenished).

The algorithms that use the "indirect grouping strategy" assume a constant family replenishment cycle (basic cycle). The replenishment cycle of each item is an integer multiple of this basic cycle time. The problem is then to determine the basic cycle time and the replenishment frequencies of all items simultaneously. A group is (indirectly) formed by those items that have the same replenishment frequency. An optimal enumeration procedure to solve this problem is found in Goyal [1974 (a)] and Van Eijs [1993]. Unfortunately, the running time of those procedures grows exponentially with the number of items. Recently, Wildeman *et al.* [1997] proposed an efficient optimal solution method based on global optimization theory (Lipschitz optimization). The running time of this procedure grows linearly in the number of items. On the other hand, heuristic methods for the problem are discussed by Brown [1971], Shu [1971], Goyal [1973, 1974 (b)], Silver [1976], Kaspi and Rosenblatt [1983, 1985, 1991], Goyal and Deshmukh [1993] and Hariga [1994].

The replenishment cycles of individual items in "the direct grouping strategy" are not imposed to be an integer multiple of a basic cycle. The problem is to form (directly) a predetermined number of groups that minimizes the total cost. Heuristics that use this strategy can be found in Page and Paul [1976], Chakravarty [1981] and Bastian [1986].

Based on a simulation study, Van Eijs *et al.* [1992] showed that the "indirect grouping strategy" slightly outperforms the "direct grouping strategy" and that it requires less computer time.

1.3.2 Dynamic Demand

In the dynamic demand case the problem is known as the *multi-item dynamic lot-sizing problem with joint set-up costs* (LPJS). If we eliminate the resource constraints (SLP2) from the SLP model (page 9), the LPJS formulation is obtained. The basic LPJS does not allow for backorders ($I_{it} \geq 0$). In addition, the production/procurement and the inventory costs functions in the basic LPSJ are given respectively by $C_{it}(Q_{it}) = s_{it}\delta(Q_{it}) + c_{it}Q_{it}$, $H_{it}(I_{it}) = h_{it}I_{it}$. This problem generally involves two types of fixed ordering costs: major (S_t) and minor (s_{it}) set-up costs. In any period a major fixed cost is charged when at least one item is ordered. It has been shown that the LPJS is NP-hard (Afentakis and Gavish [1986]). Accordingly, most of the research on this problem has concentrated its efforts on the exploitation of its special structure to develop optimal or heuristic procedures. Zangwill [1966] showed that there exists an optimal policy in which the schedule of each item is of Wagner and Whitin type. All the existing approaches for the LPJS in the literature make use of this property to generate solutions for the problem. The algorithms suggested by Zangwill [1966], Kao [1979], Veinott [1969] and Silver [1979] are based on different dynamic programming formulations of the problem. However, all theses procedures fail to solve problems with practical dimensions due to high memory and extensive computational effort requirements. Branch and Bound procedures are proposed by Erenguc [1988], Afentakis and Gavish [1986], Kirca [1995], Robinson and Gao [1996]. The lower bounds in Erenguc [1988] are computed by ignoring the major set-up costs and solving independent uncapacitated single item lot-sizing problems. In Afentakis and Gavish [1986], lower bounds are obtained by applying the Lagrangean relaxation method. By solving the linear relaxation dual of a new problem formulation, Kirca [1995] proposed an efficient way to obtain tight lower bounds. The same idea was exploited also by Robinson and Gao [1996] to obtain the lower bounds, but instead of solving the linear relaxation to optimality, the authors use a heuristic dual ascent method to solve the "condensed dual" of the relaxed problem. Different kind of heuristic methods were also proposed to solve the LPJS. See Atkins and Iyogun [1988], Chung and Mercan [1992], Federgrum and Tzur [1994], Joneja [1990] (who proposed a bounded worst case heuristic) among others. Some optimality conditions were proposed by Haseborg [1982].

1.4. Capacitated Multi-item Problems

Problems belonging to this class are concerned with determining production/procurement lots for multiple products over a finite or infinite planning horizon so as to minimize total costs, while known demands are satisfied and capacity restrictions are respected. The total relevant cost generally consists of setup costs, inventory holding costs, production/procurement costs, and backlogging costs. The limited availability of production resources introduces some interdependency between products, which leads to complex coordination problems where decisions can no longer be made for each product separately.

1.4.1. Static Demand

When demand is constant, researchers have been interested in determining cyclical production schedules for multiple products over an infinite planning horizon to minimize the sum of setup and inventory holding costs, while demand must be fulfilled without backlogging. Setup and inventory costs are assumed to be constant over time and are given respectively by $C_i(Q_i) = s_i\delta(Q_i)$ and $H_i(I_i) = h_iI_i$. This problem is known in the literature as ELSP (Economic Lot Sizing Problem). If each product i is treated independently and is produced (at a production rate r_i) in cycles of length T_i (the time between two successive production runs for the same product i), the cost per unit time is give by:

$$c_i = \frac{s_i}{T_i} + \frac{h_i d_i (1 - \frac{d_i}{r_i}) T_i}{2}$$

ELSP can then be formulated mathematically by the following constrained objective function:

ELSP:

$$\min_{(T_i, i=1,\ldots,n) \in \Gamma} \sum_{i=1}^{n} \left(\frac{s_i}{T_i} + \frac{h_i d_i (1 - \frac{d_i}{r_i}) T_i}{2} \right)$$

A solution (T_i, i = 1,…, n) is feasible only if in the corresponding production schedules no two items are produced at the same time. Γ is the set of all feasible solutions.

Nevertheless ELSP has been proved to be NP hard (Hsu [1983]). In his review on ELSP, Elmaghraby [1978] differentiates between two types of solution procedures: (i) analytical approaches that find an optimal solution to a restricted version of the problem, and (ii) heuristic procedures that search for acceptable solutions to the original problem.

1.4.1.1 ELSP analytical approaches

Analytical approaches to solve ELSP restrict Γ to a particular set before solving the problem. The well known analytical approaches are the common cycle approach due to Hanssmann [1962] which restrict Γ to $\{(T_i, i = 1,\ldots, n) | T_i = T\}$ where T is the length of the common cycle and the basic period approach due to Bomberger [1966] which restrict Γ to $\{(T_i, i = 1,\ldots, n) | T_i = N_i T\}$ where N_i is an integer multiplier of the basic period length T. Once a feasible basic period length T has been determined (by some search method), the corresponding multipliers N_j can be determined easily by a dynamic programming algorithm. Extensions and improvements of the basic period approach can be found in Elmaghraby [1978], Axsäter [1983], Hendriks and Wessels [1978] and Boctor [1985].

1.4.1.2. ELSP heuristic approaches

Most proposed heuristics to solve ELSP require that cycle times be integer multiple of a basic period. This condition was shown to be a necessary feasibility condition by Boctor [1982]. Salomon [1991] in his description of “basic period” heuristics showed that they include three (3) main components: (i) a procedure to compute the parameters N_i and T, and (ii) a procedure to detect whether a given choice of parameters is infeasible, and (iii) a rule to modify the multipliers in case of unfeasibility. Since (ii) is NP-Hard (see Hsu [1983]), the procedures that deal with the feasibility problem are frequently of a heuristic nature. “Basic period” heuristics were proposed by Madigan [1968], Stankard and Gupta [1969], Doll and Whybark [1973], Goyal [1973], Saipe [1977], Haessler [1979], Park and Yun [1984] and Boctor [1985, 1987].

Some interesting variants of the ELSP problem were also studied. Fujita [1978] and Dobson [1987] worked on the non-zero setup times sequence dependent problem. Carreno [1990] studied the parallel machine ELSP where m identical machines are available to process the set of products. The family setup costs, and discounts cases were studied by Silver and Peterson [1985].

1.4.2 Dynamic demand

In the case of dynamic demands the well-known lot-sizing models that deal explicitly with capacity restrictions are the Capacitated Lot-Sizing Problem (CLSP), the Continuos Setup Lot-sizing Problem (CSLP) and the Discrete Lot-sizing and Scheduling Problem (DLSP). In general, these problems ignore setup times. The definition of these problems is given in section 1.2.2. The complexity and the solution procedures of the multi-item version of each of the above mentioned problems are discussed in what follows.

1.4.2.1 The Capacitated Lot-Sizing Problem (CLSP)

Since the multi-item CLSP is NP-Hard (see Chen and Thizy [1987]), little work has been done to solve the problem optimally. Exact solution procedures have been proposed by a number of authors. When set-up times are considered, Gelders *et al.* [1986] and Diaby *et* *al.* [1992a] propose a Lagrangean relaxation based Branch and Bound procedure. Barany *et al*. [1984] and Leung *et al*. [1989] use cut-generation techniques within a Branch and Bound procedure to reach an optimal solution. Eppen and Martin [1987] and Pochet and Wolsey [1991] propose a stronger formulation of the problem (a formulation with tighter lower bounds from its linear relaxation problem).

On the other hand, heuristic methods for CLSP were proposed by many authors. Maes and Van Wassenhouve [1988] classifies these methods in two (2) categories: single resource heuristics and mathematical programming based heuristics.

The single resource heuristics are of greedy type and can also be divided into two subsets (as suggested by Salomon): the "period by period" heuristics and the "improvement heuristics". The production/procurement plan in the "period by period" heuristics are determined by finding a plan for period 1 and proceeding up to period T while ensuring feasibility during the whole process. Such heuristics have been suggested by Eisenhut [1975], Lambrecht and Vanderveken [1979], Dixon and Silver [1981] and Maes and Van Wassenhove [1986].

The "improvement heuristics" start with a solution for the complete horizon. This solution may be infeasible. From this starting solution, feasible production schedules are generated by simple shifting routines. Such heuristics were suggested by Dogramaci *et al*. [1981], Karani and Roll [1982] and Van Nunen and Wessels [1978].

Salomon [1991] divides the mathematical programming heuristics into three categories: relaxation heuristics, linear programming heuristics and column generation heuristics. Relaxation heuristics are based on relaxing the "difficult" constraints of the problem. This relaxation leads to an easier problem that can be solved efficiently. A perturbation method, in combination with a search procedure, is then used to reach a "good" solution. Lagrangean relaxation heuristics are proposed by Thizy and Van Wassenhove [1985], Millar and Yang [1993] (relaxation of the capacity constraints), Chen and Thizy [1987] (relaxation of demand constraints), among others. Column generation heuristics are based on set-covering or set-partitioning approaches and can be found in Chen and Thizy [1987] and Cattrysse *et al.* [1990]. The last, but not the least, are the linear programming heuristics based on alternative formulations of the problem that have tight linear relaxation solutions or better structures. Those solutions are then perturbed (if they are not feasible for the original problem) in different ways to find feasible plans. Linear programming heuristics can be found in Maes *et al.* [1989] and Gilbert and Madan [1991].

A different approach based on an item-by-item strategy, which cannot be included in any of the heuristic categories described above, was proposed by Kirca and Kökten [1994]. Every iteration, a set of items is scheduled over the planning horizon and the procedure terminates when all items are scheduled. The behavior of some heuristics is examined in Maes [1987]. In general, single resource heuristics are faster in terms of computational times than mathematical programming procedures and they are more transparent (easier to understand). Mathematical programming procedures, on the other hand, tend to yield solutions of better quality and to be more general.

When setup times are considered, the problem of finding a feasible solution has been proved to be NP-Hard (Maes *et al.* [1993]). To deal with this additional complexity, heuristics that consider setup times allow over-time work options at some costs. Most of the heuristics available are of the relaxation type and can be found in Billington *et al.* [1986], Lozano [1989], Trigeiro *et al.* [1989], Diaby *et al.* [1992b], Kirka [1990] (ignores setup costs) and Mercan and Ereguc [1993] (ignore setup costs and consider a family set-up time). Backlogging possibilities are allowed by Pochet and Wolsey [1989] and by Millar and Yang [1993, 1994]. Dixon and Poh [1990] consider a related problem where capacity constraints are not on production but on storage. Inventories may be constrained due to limited space or financial resources. Dixon and Poh [1990] proposed a relaxation heuristic to solve this problem.

1.4.2.2 The Continuous Setup Lot-Sizing Problem (CSLP)

The multi-item CSLP has been studied by Karmarkar and Scharge [1985] who presented a Branch and Bound procedure to solve it. A Lagrangean relaxation of the capacity constraints lead to easier subproblems that are solved efficiently within a sub-gradient optimization technique to reach tight lower bounds. An extension to the generic CSLP that considers parallel machines was studied by De Matta and Guignard [1989] who proposed a heuristic method based upon a Lagrangean relaxation combined with a greedy algorithm.

1.4.2.3. The Discrete Lot-Sizing and Scheduling Problem (DLSP)

The multi-item DLSP has been treated by Solomon [1991]. Besides studying the complexity of some extensions of the generic multi-item DLSP, Solomon [1991] proposed dynamic programming approaches and heuristics to solve the generic DLSP. He also proposed two column generation based heuristics for the DLSP with nonzero setup times. Fleischmann [1990] contributed to solve the problem by a Branch and Bound algorithm. Lagrangean relaxation and dynamic programming approaches are used to obtain the lower bounds while the upper bounds are obtained by successive approximation techniques.

2. Multi-Level Problems

The *Multi-Level Lot-sizing Problem* (MLLP) considers products that are manufactured or procured through several levels possibly involving several production/stocking points. The objective in these problems is to determine a multi-stage production/procurement schedule which minimizes the total cost while known demand is fulfilled. Relevant costs consist generally of inventory and production/procurement costs.

There are two types of demand in the MLLP: independent demand and dependent demand. Independent demand is triggered from outside the firm. Dependant demand is triggered by the production/supply required to fulfil the independent demand. Dependant demands create interdependencies between the different items considered in the system and hence between their planning decisions which make the MLLP a lot more difficult to solve than the SLP.

The structure of a multi-level production system can be represented by a directed acyclic graph, as shown in Figure 4, where nodes represent items and/or stocking points and where arcs describe the relationship between the different nodes. Although a single graph can be used to represent production and distribution activities, it should be noted that the nodes-arc structure does not have the same interpretation in both cases. In a production context, nodes typically represent components which are part of some finished product (also a node) and the arcs describe the bill of materials with the associated positive gozinto factors a_{ik} (the amount of item i required to produce one unit of item k) and lead-times t_i. In a distribution structure, the nodes correspond to the stocking points where the finished products are held and the arcs represent the possible flows between these locations with their associated lead-times. In a distribution context, all gozinto factors are equal to one. In what follows, the set of all the nodes in the network is denoted by N, while the set of immediate successors of an item/stocking point i is represented by F_i, the number of constrained resources considered in the systems by J and the set of items that use resource j by M_j.

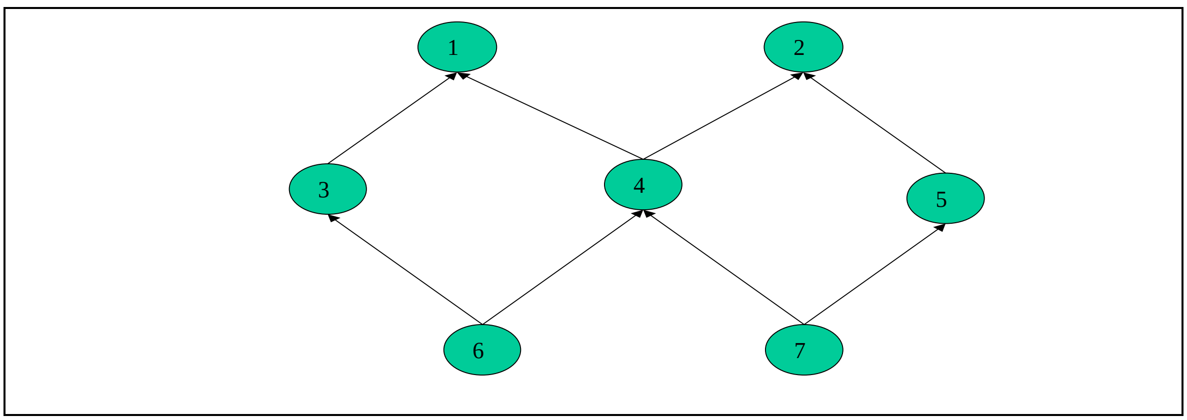
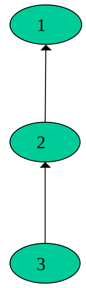


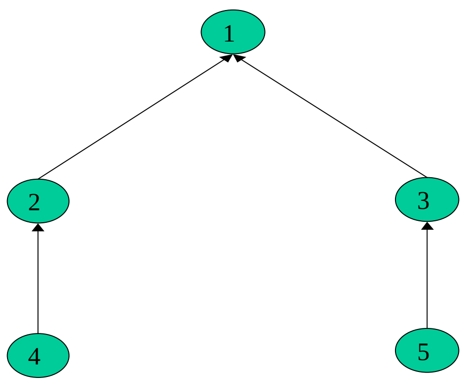
Figure 4: General product structure

Four types of system structures are studied in the literature:

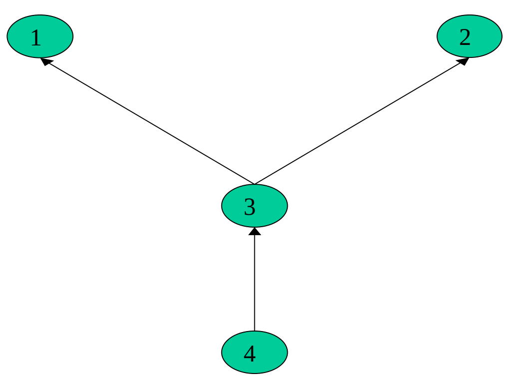
- The *serial* system structure: each item has at most one successor and one predecessor (only one end-item can be considered).



- The *assembly* product structure: each item has at most one successor (only one end-item can be considered).



- The *arborescent* system structure: each item/location has at most one predecessor



- The *general assembly* structure: the number of successors and predecessors are not restricted (Figure 4).

Adapting the notation introduced in Figure 3 to this multi-level context, the MLLP can be formulated mathematically as a mixed integer program:

$$\text{Min} \sum_{i \in N} \left(\sum_{t=1}^{T-\tau_i} C_{it}(Q_{it}) + \sum_{t=1}^{T} H_{it}(I_{it}) \right) \qquad \text{(MLLP)}$$

subject to:

$$Q_{i(t-t_i)} + I_{i(t-1)} - I_{it} - \sum_{k \in F_i} a_{ik} Q_{kt} = d_{it} \qquad i \in N;\ t = t_i + 1, ..., T \qquad \text{(MLLP1)}$$

$$I_{i(t-1)} - I_{it} - \sum_{k \in F_i} a_{ik} Q_{kt} = d_{it} \qquad i \in N;\ t = 1, ..., t_i \qquad \text{(MLLP2)}$$

$$\sum_{i \in M_j} R_{ij}(Q_{it}) \leq u_{jt} \qquad j = 1, ..., J;\ t = 1, ..., T \qquad \text{(MLLP3)}$$

$$Q_{it} \geq 0 \qquad i \in N;\ t = 1, ..., T \qquad \text{(MLLP4)}$$

The objective function includes production costs and inventory holding/shortage costs. Restrictions imposed on the solutions of the model are given by constraints (MLLP1) to (MLLP4). Constraints (MLLP1) state the well-known inventory balance condition which must exist between production, independent demand, dependent demand and inventory for each node-period combination. Constraints (MLLP2) simply gives the inventory level for each item at the end of periods 1,…, t_i. Constraints (MLLP3) enforce resource (capacity) restrictions at each level. Constraints (MLLP4) make sure that the production quantities are non-negative. Note that this formulation allows for negative inventory levels I_{it} (backorders).

Since 1960, tens of thousands of manufacturers have installed Manufacturing and Distribution Resource Planning (MRP II/DRP) systems to help make production/supply management decisions in a multi-level system. Unfortunately these systems focus mainly on materials requirements planning which is only a small part of the manufacturing process and do not consider capacity constraints (MLLP3) explicitly. Although MRP II and DRP use derived demand instead of forecasted demand for levels preceding the final stage of production/distribution, the problem of lot-sizing under resource constraints remains ill-solved. Lot-sizing in those systems is done on a level-by-level basis only. Major drawbacks of this approach are now well known (Billington *et al.* [1983]).

A review of some of the most important papers published on the different special cases of the MLLP, depending on demand characteristic (static or dynamic), on capacity constraints (capacitated or uncapacitated) and on the number of end-items considered (single or multiple) is presented in what follows.

2.1. Uncapacitated single item problems

In this section we consider MLLP when capacity restriction on resources are not present and when only one final product is considered. It is usually unrealistic to assume that resources are unrestricted and available in abundance. However, many approaches to capacitated multi-level lot sizing and scheduling problems begin by finding lot sizes without considering capacity constraints. The solution is then modified to eliminate unfeasibilities. A review of both the static demand and the dynamic demand versions of the uncapacitated MLLP is provided in the following.

2.1.1. Static demand

Power-of-two policies were proposed to solve single end-item uncapacitated MLLP when demand is static and an infinite planning horizon is considered. Under a power-of-two policy, all items are replenished at constant intervals and only when their inventory drops to zero; moreover the replenishment intervals are all power-of-two multiples of a common base planning period (lot sizes are allowed to vary from one level to another). With power-of-two policies it is possible to derive solutions that are very close to optimum (Roundy [1986]). Such methods can be found in Maxwell and Muckstadt [1985] for general product structures, in Atkins and Sun [1995] for serial systems (backlogging is allowed at the final level) and in Sun and Atkins [1997] (backlogging is allowed at the final level) for assembly systems.

2.1.2. Dynamic demand

The dynamic version of the uncapacitated MLLP is shown to be NP-hard for general product structures (Arkin *et al.* [1989]). The production/procurement and inventory holding costs in the basic version of the problem are time independent and are given respectively by $C_i(Q_{it}) = s_i\delta(Q_{it})$ and $H_i(I_{it}) = h_iI_{it}$. Backlogging is not allowed and independent demands occur for the final product only. The problem is also difficult to handle from a computational point of view in the sense that straight forward relaxation of the integrality constraints on setup variables lead to poor quality of lower bounds (Salomon [1991]). Research on this problem was initiated by Zangwill [1966], Veinott [1969] and Love [1972]. Most of the work published deals with serial and assembly systems because of their interesting structure (notice that only one final product can be considered in the serial and the assembly systems). Those contributions are based on the "nested" property discovered by Veinott [1969]: if independent demands occur for end items only, and if production/procurement costs are constant over time, then there exist an optimal solution to the uncapcitated MLLP for which $\delta(Q_{it}) = 0$ whenever $\sum_{j \in F_i} \delta(Q_{jt}) = 0$ (production/procurement cannot occur at one operation unless it also occurs at all of its immediate successor operations).

When there is no independent demand for components, the serial system is known to be polynomially solvable by an $O(NT^4)$ dynamic programming algorithm (Love [1972]). In practice the algorithm is of little use, because it requires a large amount of memory and because the running time grows rapidly with the problem size. The complexity of the assembly system is still open (Chopra *et al.* [1997]). Under some assumptions the assembly system problem is polynomially solvable (see Arkin *et al*. [1989]).

Exact procedures to solve dynamic uncapacitated MLLP when only one final product is considered use either dynamic programming, relaxation or reformulation approaches. The computational effort of the dynamic programming methods increases exponentially with the problem size, therefore these methods are inefficient for large problems. Dynamic programming algorithms can be found in Crowston and Wagner [1973] and Zangwill [1966] for assembly systems**.** On the other hand, relaxation based procedures generally use the Lagrangian relaxation technique to obtain an easier problem that can be solved efficiently. Generally the coupling constraints (MLLP1) are relaxed which leads to easily solvable shortest path problems. The solution of the relaxed problem is used as a lower bound in a specialized B&B procedure. Upper bounds are generally obtained by simple heuristics. Relaxation procedures can be found in Afentakis *et al.* [1984] and Rosling [1985] for assembly systems and in Afentakis and Gavish [1986] for general product structures**.** Reformulation approaches propose more efficient formulations of the problem, often by using the echelon stock concept. The resulting models can be solved by specialized cutting plan algorithms exploiting standard optimization software. Reformulated models are found in Mcknew *et al***.** [1991] and in Clark and Armentano [1993] (production costs are time-dependent) for assembly systems (independent demand for components are allowed). Steinberg and Napier [1980], Pochet and Wolsey [1991] and Clark and Armentano [1993] proposed reformulations for general systems.

Since the computation time and memory required by exact procedures make them useless for "real-world" cases, several heuristics have been proposed for the problem with general and assembly product structures. The heuristics proposed in the literature may be divided into level-by-level, period-by-period and local search methods. The level-by-level algorithms are based on a decomposition of the problem into single-level subproblems in combination with cost adaptation procedures that account for interactions between adjacent levels. Level-by-level heuristics are found in Blackburn and Millen [1982], McLaren [1976], Graves [1981] and in Coleman and McKnew, [1991] for assembly systems**.** In period-by-period heuristics, the solution of the t-time period problem is used to find the solution of the (t+1)-time period problem. This is repeated till a solution for the T-time period problem is obtained. This strategy gives to those algorithms the advantage of controlling the nervousness of the generated policy (Joneja [1991]). Such heuristics can be found in Afentakis [1987] for assembly systems. Local search techniques were proposed in Salomon [1991] who used simulated annealing and tabu search to solve the problem.

2.2. Capacitated single item problems

When capacity restrictions are taken into consideration, the main concern of the MLLP is to find a production/procurement schedule that minimizes the total cost when demand are fulfilled and capacity restrictions are respected. Capacity restrictions are generally considered at one level of the product structure defined as the "bottleneck". In what follows, we discuss the single item capacitated MLLP in both the static demand and the dynamic demand cases. The resource consumption function for item i at resource j is generally assumed to be linear.

2.2.1. Static demand

Several approaches have been proposed to solve a relaxed version (items reorder intervals are restricted to be integer multipliers of a basic cycle) of the single product capacitated MLLP when demand is static over an infinite horizon, and when production/procurement and inventory holdings costs are constant and given respectively by $C_i(Q_{it}) = s_i\delta(Q_{it})$ and $H_i(I_i) = h_iI_{it}$. Those approaches may be divided into two classes. The first class consists of approaches which allow different lot sizes (reorder intervals) for the different production/procurement levels (Jensen and Khan [1972], Crowston *et al*. [1973], Schwarz and Scharge [1975], Williams [1982], Blackburn and Millen [1984], Moily [1986], Jackson *et al*. [1988], Karimi [1989], Atkins *et al.* [1992]). On the other hand, Szendrovits [1975, 1976] and Goyal [1976], among others assume the same lot size for all the stages.

2.2.2. Dynamic demand

When demand is time-dependent, an exact procedure to solve the single end-item MLLP when capacity constraints are considered, and when production/procurement and inventory holding costs are given respectively by $C_{it}(Q_{it}) = s_{it}\delta(Q_{it})$ and $H_{it}(I_{it}) = h_{it}I_{it}$, can be found in Billington *et al*. [1986]. The paper proposes a B&B algorithm using Lagrangean relaxation to generate lower bounds, while upper bounds are obtained using simple smoothing heuristics. Lozano *et al.* [1989] proposed a primal-dual solution procedure (includes set-up times).

Heuristic algorithms have attracted more attention than exact procedures. The heuristics proposed may be divided into three categories: level-by-level, relaxation and cost adjustment methods. Level-by-level heuristics can be found in Gabbay [1979] when there are multiple constrained resources and in Zahorik *et al.* [1984] when there is a single capacitated resource for serial systems with linear production/procurement costs. Relaxation heuristics are based on the LP-relaxation of the so-called “facility location” formulation of the MLLP. Such algorithms are proposed by Maes *et al.* [1991] (include set-up times), for serial systems, and by Salomon [1991], for assembly systems, who used the LP relaxation to generate solutions for the problem. Cost adjustment heuristics aim to adapt some single level heuristics (like the Dixon-Silver procedure) to handle capacitated multi-level systems. Cost adjustment heuristics can be found in Billington *et al.* [1989], in Maes and Van Wassenhove [1991] and in Harrison and Lewis [1996] (minimize inventory and backorders costs) for serial production environment systems with multiple constrained resources.

2.3. Uncapacitated multi-item problems

In this section we discuss the uncapacitated MLLP when multiple end-items are considered in both the static demand and the dynamic demand cases. This problem occurs generally when general product structures are considered. The production/procurement and inventory holding costs in the basic version of the problem are given respectively by $C_i(Q_{it}) = s_i\delta(Q_{it})$ and $H_i(I_{it}) = h_iI_{it}$. A literature review of the problem is presented in what follows.

2.3.1. Static demand

In the static demand case, approaches for solving the uncapacitated multiple end-items MLLP may be divided into two classes. Approaches of the first class allow different reorder intervals (lot sizes) for different levels and are generally based on the power-of-two policy (reoreder intervals are restricted to be a power-of-two multiplier of a basic cycle). Such methods can be found in Maxwell and Muckstadt [1985], Roundy [1986] and in Federguen and Zheng [1995] for general product structures where external demands may occur at any of the network's nodes and orders are delivered instantaneously. Power-of-two methods are showed to be within 2% of a lower bound for the minimum cost even in the case where external demands are allowed for components and where joint set-up cost is a general monotone submodular function (Roundy [1986] and Federguen *et al.* [1992]). Approaches of the second class assume a common reorder interval (lot-size) for all products (Hsu and El-Najdawi [1990]), El-Najdawi and Kleindorfer (1993)).

2.3.2 Dynamic demand

In the dynamic demand case, we can classify the literature of the uncapcitated MLLP into two classes. Approaches that solve a restrictive version of the problem optimally and approaches that propose heuristics procedures to solve the original problem. Approaches for restrictive versions of the problem can further be divided into two categories: those that allow different lot sizes for different levels (Afentakis and Gavish [1986]) and those that impose the same lot-size for the different levels (Karmarkar *et al.* [1985]).

Few heuristics have been proposed to solve the multiple end-items case. Local search and relaxation heuristics can be found respectively in Kuik and Salomon [1990] and in Salomon [1991] for general product structures (independent demand may occur for all items).

2.4. Capacitated multi-item problems

The capacitated MLLP occurs when production/procurement schedules have to be determined for multi-level production-inventory systems in the presence of capacity constraints on resources. In the basic version of the MLLP production/procurement and inventory holding costs are given respectively by $C_i(Q_{it})= s_i\delta(Q_{it})$ and $H_i(I_{it})= h_iI_{it}$, only one bottleneck exists at a given level in the product structure and backlogging is not allowed. A review of the literature for the static demand and the dynamic demand versions of this problem are discussed in the following.

2.4.1. Static demand

Research to solve the MLLP when capacity restrictions and multiple end-products are considered is not very abundant. One of the few contributions to solve this problem, when demand is static, is Federgruen and Zheng [1993] who generalized the methods proposed by Maxwell and Muckstadt [1985] and Roundy [1986] by proposing an optimal power-of-two policy for general systems when there is upper and lower bounds on capacity and bounds on the frequency with which individual items need to be replenished at each level, and where external demands may occur for components. Hill *et al.* [1997] proposed greedy heuristics and an optimal algorithm to solve the problem in the case of a general system with non-instantaneous lead times and multiple capacity constraints (a capacity constraint at each level).

2.4.2. Dynamic demand

The MLLP when capacity restrictions and multiple final products are considered is very hard to solve optimally. When multiple constrained resources are considered, Maes *et al.* [1991] proved that simply finding a feasible solution to the multiple end-items case is NP-Complete. This explains the absence, to our knowledge, of exact procedures to solve this problem. Some attempts to find a tighter formulation of the problem were made by Stadtler [1996] (includes setup and overtime). On the other hand, heuristic approaches have attracted some attention from researchers. The heuristic approaches available can be divided into relaxation, cost adjustment and local search procedures. Relaxation heuristics are based on the Lagrangean relaxation techniques. They are found in Billington *et al*. [1983] for the single capacity-constrained resource case and in Tempelmeir and Derstroff [1996] for general product structures and multiple capacity constraints. Cost adjustment heuristics aim to adapt a single-level procedure (Dixon-Silver [1981]) to solve the multi-level problem. Such heuristics are found in Tempelmeir and Helber [1994], Helber [1995] and Katok *et al.* [1998] who generalized the Harrison and Lewis [1996] work (single item) to the multi-item case. Local search methods to solve the problem have been proposed by Salomon *et al.* [1993] who used simulated annealing and taboo search techniques.

III. Multi-facility Problems

Multi-facility systems are complex procurement-production-distribution networks covering a large part of the supply chain. Each plant in the network represents a multistage system in which the flow of products may be serial, parallel, assembly or general (Billington *et al.* [1983]). Lot sizing problems, in this case, are complicated by the interdependence of different plants.

To reduce the size of the problem, Billington *et al*. [1983] suggest that in most production systems there are only a few 'constrained facilities' i.e. bottleneck work-centers where capacity is likely to be a binding constraint so as to cause scheduling difficulties. Thus, the authors state that lot-sizing is only critical for the constrained work-centers and other work-centers can often be scheduled on a lot for lot basis. According to this point of view Karmarkar *et al.* [1992] reduce a complex manufacturing system into its most critical 'constrained facilities' by representing it by what they call an "approximate composite model". This representation must be able to capture the salient features of the original system. Once this is achieved, the very difficult problem described above can be formulated as a multi-stage lot-sizing problem where each stage is a site. Such formulations differ from models for multi-stage lot-sizing in a single site since they must capture the effect of transportation costs between the different sites. Clearly this is a very difficult problem to solve optimally. However, good heuristics would help quantify the benefits of coordination as compared to the current practice of optimizing the material flows plant by plant, which ignores the supply chain sites interdependencies (Bhatnagar *et al.* [1993]).

Since the problem is very difficult to solve optimally, only restricted versions of it were considered by some researchers that attempted to at least evaluate the benefits of coordination. The constant demand case attracted the attention of most of those researchers. In particular, three network structures were considered: (1) one origin with multiple destinations network, (2) multiple origins with one consolidation point and multiple destinations network, and (3) multiple origins with multiple consolidation points and multiple destinations network. Since most of the particular cases considered in the literature are treated from a distribution point of view, production capacity constraints are usually not considered. In the case of dynamic demand, to our knowledge, no work has been done to solve the coordination problem except the simulation study of Chandra and Fisher [1994]. In what follows, we present the different multi-facility lot-sizing coordination problems that were discussed in the literature.

1. Uncapacitated single item problems

In this section we discuss the lot sizing coordination problem when there is no production capacity constraint and when only a single product is considered in the system.

1.1. Static demand

In the case of a single item and static demand, the only structure that was considered is the one origin and multiple destinations network. Figure 5 summarizes the different contributions. The one origin multi-destination networks were studied by several authors who aimed to coordinate production, distribution and transportation decisions. The products are delivered from the origin by vehicles that combine deliveries to several retailers into efficient vehicle routes. The objective is to determine replenishment policies that specify the delivery quantities and the vehicle routes so as to minimize long-run average production, distribution and transportation costs. Such schedules should specify a list of routes, the frequency of deliveries on the routes, as well as the lot size for each of the retailers on the route. Optimal strategies are difficult to find and often very complex since the problem is related to the classic Vehicles Routing Problem, which is notoriously NP-Hard (Anily [1986]). Thus, studies on this problem restrict their search to a class of easily solvable and effective strategies.

Anily and Federgruen [1990] studied the distribution problem of a single product from one warehouse to geographically dispersed retailers by a fleet of capacitated vehicles. The warehouse acts as a break-bulk center and does not keep any inventory. Deterministic constant demand of the products occurs at the retailers. Trucks have a limited capacity, the holding costs at all the retailers are identical, and the transportation costs consist of a cost per mile and a fixed cost of hiring a truck. All demands must be met on time. To solve this problem, the authors considered only policies that belong to a class of family-based replenishment strategies. The retailers are first split into a family of demand points with identical demand rates and the resulting problem is then solved. The replenishment strategy is restricted to a class of strategies in which whenever a demand point in a route is replenished; all other demand points along the route are also replenished. In other words, they restrict their research of policies to the ones where demand is split into regions, and whenever any retailer in a region orders, all retailers in that region also order. The overall region is partitioned into sub-regions. A retailer can be assigned to several sub-regions. Each sub-region is responsible for a certain fraction of the sales of each of its retailers.

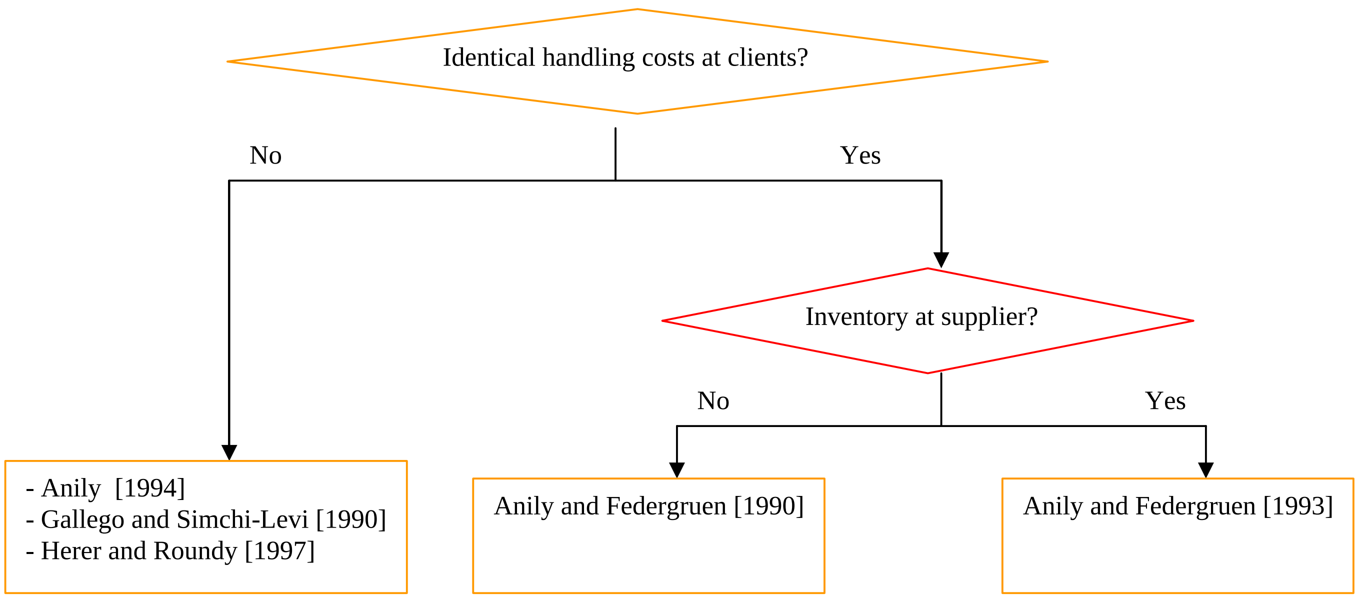


Figure 5: Literature Review of the SCMP:
The Uncapacitated Single Item & Constant Demand Case

Using this restricted class of strategies, Anily and Federgruen reduce the original problem to an Euclidean Vehicle Routing Problem with a cost function that depends on the length of the route and the number of points visited. They propose two heuristics to solve this problem which they show to be asymptotically optimal within the given class of policies, as the number of retailers tends to infinity. Both heuristics include two stages. The first stage involves the derivation of a lower bound, which is obtained by partitioning the set of demand points into groups. In the second stage, all groups obtained by solving the partitioning problem are combined into larger families of demand points. In each of these families, efficient vehicle routes are developed using regional partitioning heuristics. The two heuristics differ in the way the partitioning problem is solved in the first stage. While the first heuristic uses the average distance from the warehouse to the demand points in a group as the lower bound for the Traveling Salesman Problem solution for these demand points, the second heuristic uses twice the maximum distance from the warehouse to a demand point in the group as the lower bound. For any group of demand points that are replenished together, an EOQ type formula is derived for the replenishment cost. In their model, Anily and Federgruen also impose a constraint on the number of demand points that can be replenished together. They show that both heuristics are asymptotically convergent. However, asymptotic convergence of the heuristic is guaranteed only when bounds are imposed on the total demand (number of demand points) that a particular route can serve.

Anily and Federgruen [1993] extended this work to the case where inventory is kept at the warehouse as well as at the retailers. Anily [1994] generalized the results to the case where retailers may have different holding costs and where the warehouse is allowed to keep inventory. Gallego and Simchi-Levi [1990] treated a similar problem where retailers are allowed to have different holding costs and a different fixed ordering/transportation cost. The authors obtain a simple lower bound on the average total cost. Moreover they show that if the EOQ of each of the retailers is at least 71% of truck's capacity, then a simple heuristic using only "direct shipments" (i.e. each route consists of single retailer) comes within 6% of the lower bound. Herer and Roundy [1997] analyzed the same problem as Gallego and Levi but with an uncapacitated truck. Herer and Roundy [1997] used a different approach to solve the problem. The cost function in their model includes fixed ordering costs and linear inventory costs. The fixed ordering costs are assumed to be the sum of two cost functions. The first cost function is a given monotone non-negative submodular function of the set of retailers placing orders at a given point in time. The second part of the order cost function (i.e. transportation cost) is a per-mile charge times the length of the traveling salesman tour through the central warehouse and the retailers that are visited. Inventory costs depend only on the stock at hand, as in the standard EOQ model. At each point in time when one or more of the retailers place an order, the authors assume that the delivery vehicle follows a traveling salesman tour through the warehouse and the retailers that are currently placing orders. Under these assumptions, the authors solve the problem by calculating power-of-two reorder intervals. The main contribution of Herer and Roundy [1997] is the generalization of Roundy's [1986] work. They showed that the cost of the best power-of-two policy is within $2\sqrt{\alpha}$ percent of the cost of an optimal policy where the order cost function is α-submodular.

1.2. Dynamic demand

In the case of a single product and a dynamic demand, Diaby and Martel [1993] proposed a modified Lagrangian relaxation algorithm to solve an arborescent multi-echelon distribution system. The costs in the multi-echelon system are linear inventory holding costs and general piece wise linear transportation costs. Using the lagrangian relaxation, the authors decomposed the problem to two easily solvable sub-problems. The lower obtained by solving these sub-problems is used in a Branch and Bound algorithm to find an optimal solution.

Though the paper restricts itself to a specific network structure, and doesn't consider production, it is one of the firsts to consider more general network structures with general piece-wise linear functions.

2. Capacitated single item problems

When the problem is capacitated, the shape of its feasible region is more complex. Thus, it is more difficult to identify the characteristics of an optimal solution and find a short cut to reach that ultimate solution as was done by Wagner and Whitin [1958]. When computer speed and memory was very limited, it was not possible to deal with the complexity obtained when considering the different facilities interactions at the same time. Even for the case of a single product and static demand, the capacitated multi-facility problem remained untackled by the research community. When demand is time dependent, the problem is more complex. The research community has not yet examined the flows planning problem in a single item capacitated multi-facility network.

3. Uncapacitated multi-item problems

In this section, we present the multi-product version of the lot-sizing coordination problem for the case where production/distribution capacity is not considered.

3.1. Static demand

For systems where static demand for multiple products is assumed and where resource capacities in the facilities are not restricted, three categories of supply chain networks were considered by the research community. The one-origin one-transit one-destination network, the one-origin multi-destination network and the multi-origin multi-destination network with one or several transshipment points. A literature review of the flow coordination problem in these networks when demand is static and multiple products are considered is presented in what follows.

3.1.1. One-origin One-destination Network

Speranza and Ukovich [1994] dealt with the problem of determining the frequencies at which several products must be shipped on a common link to minimize the sum of transportation and inventory costs. Several products must be shipped on the same link using finite capacity vehicles that may leave only at some given discrete times. The products are produced with known production rates and are required at a different location (i.e. warehouse, customer) with a consumption rate equal to their production rate. The inventory cost is linear and is the same in the plant, in the break-bulk (transshipment point) and in the warehouse. The transportation cost is proportional to the number of vehicles used for each shipment (constant per truck per trip). Different types of vehicles can be considered with different capacities and costs. The problem to solve is when to ship each product and in what quantities in a way that minimizes the sum of transportation and inventory costs. In addressing this issue, integer and mixed integer linear programming formulation models of different particular cases of the problem are proposed.

3.1.2. One-origin Multi-destination Network

For the one-origin multi-destination case, Viswanathan and Mathur [1997] studied a multi-product generalization of the distribution system considered by Anily and Federgruen [1990]. The authors developed a heuristic that generates a stationary nested joint replenishment policy. A policy is defined to be stationary if the replenishment of each item is made at equally spaced points in time, i.e. with constant replenishment intervals. A policy is said to be nested if whenever the replenishment interval of a given item is larger than that of another item, the former is an integer multiple of the second. A product at a specific retailer is referred to as an item, and the same product at different retailers is considered as different items. Inventories are kept at the retailers, but not at the warehouse, which acts as a break-bulk center. The problem is formulated as a joint replenishment problem (JRP) and a simple heuristic is proposed. Since the vehicles are capacitated, the heuristic groups items in clusters where power-of-two reorder policies are followed. However, if all the items in a cluster are replenished at the same frequency, then the common replenishment interval may be a general integer, rather than a power-of-two multiple of a basic planning period.

3.1.3. Multi-origin Multi-destination Network

The coordination problem when there are multiple origins, multiple destinations and one or several consolidation points, is to determine the pattern of direct (i.e. directly from the origin to the destinations) and indirect (i.e. via a consolidation point) shipments that minimizes the long run average cost. Figure 6 summarizes the major contributions to this case. Blumenfeld *et al.* [1985] aim to determine optimal shipping strategies between origins and destinations by analyzing the trade-offs that exist between transportation, inventory and production set-up costs for networks with one consolidation terminal. They assume that the inventory cost is linear, and that it is the same for the whole system (i.e.in the origins/destinations and in transit). Transportation cost is a fixed function. Inbound and outbound shipments at the consolidation terminal are independent and the mix of parts in each shipment is in proportion to the demands. For an origin-destination, only two options are considered: shipping all parts directly or shipping all parts via the terminal. The authors proposed a simple heuristic to determine routes (direct or via a terminal) and shipment sizes ca simultaneously for shipments on networks with concave cost functions and a consolidation terminal. The heuristic fixes shipment sizes from the origins to the consolidation terminal and determines the optimal cost for each scenario by solving a set of independent sub-networks, each with one origin, one consolidation terminal, and many destinations. For a small number of origins, this strategy reduces the routing (direct or indirect shipping) enumeration to a manageable number**.** Bulmenfeld *et al.* [1987] reports a successful implementation of this research at GM's Delco electronics division that resulted in a 26% ($2.9 million per year) reduction in logistics costs. The same problem with many origins to few destinations has been analyzed by Hall [1984, 1987] who developed a systematic method to determine if an origin should ship directly to a destination or through a consolidation point. Hall [1987] also suggests that the developed systematic method combined with Bulmenfeld *et al.* can be used to solve many origins to many destinations problems.

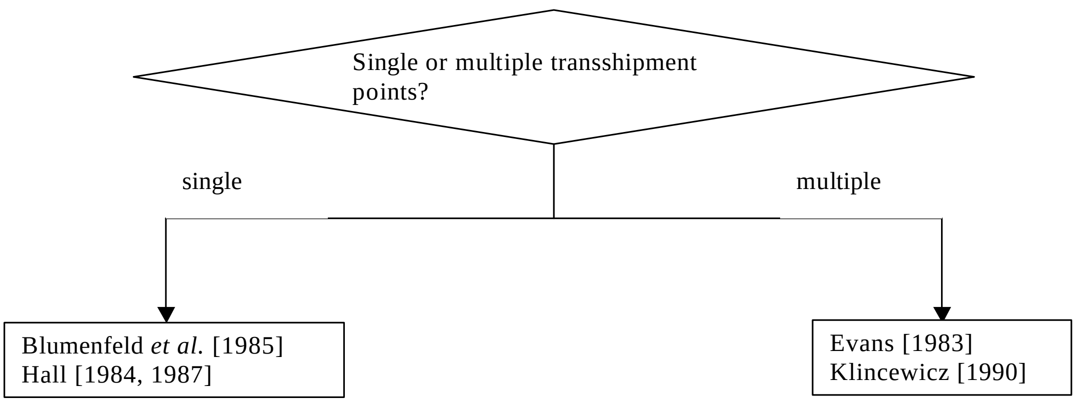


Figure 6: Literature Review of the SCMP.
The Uncapacitated Multi-origin Multi-destination & Constant Demand Case

Evans [1983] treated the multiple origins, multiple destinations and multiple consolidation terminals case. Each source produces only one product and this product must be sent to each destination. Shipping costs are linear functions of the volume shipped. No inventory costs are considered. For each origin-destination, the problem is whether to ship direct or via a consolidation terminal. Klincewicz [1990] also studied this problem but under the assumption that shipping costs are piecewise linear concave functions of the volume shipped. Klincewicz [1990] solved this problem optimally for two special cases. In the first case, the sources-to-terminal shipping costs are linear functions of the volume; in the second case, the terminal-to-destinations shipping costs are linear functions of the volume. In both cases, the problem may be reduced to a set of concave cost facility location problems. For more general cases, the author proposed several heuristics based on solving sequences of the special case with linear costs.

3.2. Dynamic demand

When demand is dynamic and multiple products are considered, the problem gets more complicated. The lot-sizing process must now take products interaction into account. To the best of our knowledge, no work has dealt with this problem.

4. Capacitated multi-item problems

The multi-facility problem with capacity constraints, multi-product, and dynamic demand didn't attract much interest from the research community. This is mainly due to its complexity. However, some work has been done to further explain the benefits of coordinating production, distribution, and routing decisions. Chandra and Fisher [1994] designed a computational study to investigate the value of coordinating production, distribution, and routing. Their study involves a plant that produces a number of products over time and at the same time can hold inventory. The demand for each product at each retail outlet is known for each period of the planning horizon (dynamic demand) and must be satisfied without any backlogging. Only one production resource is considered at the facility and all vehicles have the same capacity; transportation costs is the sum of a fixed vehicle cost per route in each period t and a cost of direct travel from location to location (fixed).

The problem is to schedule production and distribution so as to minimize production set-up, transportation and inventory costs. Production lead times are considered explicitly. They assume that the operations of the plant can be modeled as a capacitated lot sizing problem and that the operation of the fleet of vehicles can be modeled as a standard multi-period local delivery routing problem.

The authors' computational study compares two different ways of managing this system. The first approach solves production scheduling and vehicle routing problems separately while the other one coordinates them within a single model. In the decoupled strategy, the authors first determine a production schedule that minimizes the cost of setups and inventory subject to meeting total demand per period. This is solved by a cutting plane algorithm based on Pochet and Wolsey [1991]. They then schedule vehicle deliveries of products to customers, by the application of some heuristics followed by a local improvement procedure that attempts to determine whether changing the timing of customer delivery can reduce costs subject to inventory availability according to the production schedule. For the coupled production and distribution strategy, a local improvement heuristic that starts from the decoupled solution and search for cost-reducing changes is used. The production/distribution schedule with the lowest total cost is taken to be the new schedule. The process is repeated until no improving changes can be found. The reduction in total operating costs ranged from 3% to 20%.

The authors indicate that the benefits of coordination increase as the length of the planning horizon, the number of products and retailers outlet, and vehicle capacity increases. It is also found beneficial to coordinate these functional activities when production capacity at the plant (the origin) is less binding, and distribution costs are higher relatively to the production costs.

Recently, in a Ph.D. thesis at Drexel University, Mallaya [1999] combined supply chain decisions regarding production plans, delivery quantities and vehicle routing in one integer-programming model. The objective is to minimize a total cost that includes production setup costs, linear inventory costs, and linear transportation costs over a finite planning horizon. Dynamic demand of multiple products needs to be satisfied for each period of time. In addition, production, inventory and transportation capacities are considered explicitly in the model. The author proposes a modified Lagrangian approach to solve the large-size integer-programming problem obtained. Although the author has spent effort explaining the Lagrangian relaxation approach proposed, no computational study is included to show the practicability of the approach.

IV. A General Supply Chain Planning Model

To provide a basis for further research, in this section, we propose a general formulation of the *Supply Chain Planning Problem* (SCPP) under deterministic dynamic demands. The notation used to formulate the problem is summarized in Figure 7. Let N be the set of the SC facilities considered in the system, P_n the set of products that can be produced/assembled/stocked in facility n, C_n the set of facility n clients, F_n the set of facility n suppliers. To model resource limitations, let's use R_n as the set of resources or processes in facility n, K_{nw} as the set of products in P_n that consume resource w ($w \in R_n$), b_{wnt} as the capacity of resource w available in facility n at period t. The production process (bill of materials) is modeled using U_k as the set of product k immediate successors (products that require item k as a component in their production process), and a_{ki}, the number of units of product k required to produce one unit of product i.

The model outputs are the quantity of product k to produce in facility n at period t (Q_{knt}), the inventory of product k in facility n at the end of period t (I_{knt}), and the quantity of product k to ship from facility n to facility j in period t (R_{knjt}).

These model outputs have to be determined to minimise the supply chain total cost, which includes transportation, production and inventory holding costs. The transportation cost between facility n and facility j ($j \in C_n$) in period t is given by the function $Z_{njt}(S_{njt})$ where S_{njt} is the amount, in some standard transportation unit (pallet, cases,…), of products transported from facility n to facility j in period t. The production cost of product k in facility n for period t is represented by the function $C_{knt}(Q_{knt})$. Finally, the inventory holding cost of product k in facility n for period t is given by $H_{knt}(I_{knt})$.

We assume that the flow planning problem has to be solved over a finite horizon. Without loss of generality the production and the transportation lead times are assumed to be zero. This assumption is reasonable since in real planning problems, the production and the transportation lead times are often less than the length of a planning period. The external demands are assumed to be dynamic (time-varying) and deterministic.

Sets:

N:	Set of the supply chain facilities considered in the model.
P_n:	Set of products that can be fabricated/assembled/stocked in facility n.
C_n :	Set of facility n clients ($n \in N$).
F_n :	Set of facility n suppliers ($n \in N$).
R_n :	Set of resources in facility n ($n \in N$).
K_{nw} :	Set of products in P_n that use resource w ($K_{wn} \subset P_n$).
U_k :	Set of product k immediate successors (in the bill of materials structure).

Variables:

Q_{knt} :	Production lot size of product k in facility n ($k \in P_n$) at period t.
R_{knjt} :	Quantity of product k transported from facility n to facility j at period t.
I_{knt}:	Inventory of product k in facility n at the end of period t.
S_{njt} :	Amount, in some standard transportation unit (pallet, …), of products transported from facility n to facility j ($j \in C_n$) during period t.

Parameters:

a_{ki} :	Number of units of product k required to produce one unit of product i.
b_{wnt} :	Capacity of resource w available in facility n at period t.
d_{knt} :	Product k external demand in facility n ($k \in P_n$) at period t.
r_k :	Product k conversion coefficient to the standard transportation unit used (pallet, …)
T:	Number of periods in the planning horizon (t = 1, … , T).
K:	Number of products (k = 1, … , K).

Functions:

$Z_{njt}(\ )$:	Shipping cost function between facility n and facility j ($j \in C_n$) at period t
$H_{knt}(\ )$:	Holding cost function of product k in facility n at period t ($k \in P_n$).
$C_{knt}(\ )$:	Production cost of product k in facility n at period t ($k \in P_n$).
$CAP_{wn}(\ )$:	Capacity consumption function of resource w in facility n

Figure 7: General Notation

Under the conditions described above, the SCPP can be formulated as follows:

$$\text{Min } \sum_{t=1}^{T}\left[\sum_{n \in N}\left\langle \sum_{k \in P_n}\left(C_{knt}(Q_{knt}) + H_{knt}(I_{knt})\right)\right\rangle + \sum_{j \in C_n} Z_{njt}(S_{njt})\right] \qquad \textbf{(SCPP)}$$

subject to:

$$\sum_{j\in F_n / k\in P_j} R_{kjnt} + Q_{knt} + I_{kn(t-1)} - I_{knt} - \sum_{i\in(U_k \cap P_n)} a_{ki} Q_{int} - \sum_{j\in C_n / k\in P_j} R_{knjt} = d_{knt} \qquad \forall n\in N, k\in P_n, t=1,...,T \qquad (1)$$

$$S_{njt} - \sum_{k\in P_n \cap P_j} r_k R_{knjt} = 0 \qquad \forall n\in N, j\in C_n, t=1,...,T \qquad (2)$$

$$\sum_{k\in K_{wn}} CAP_{wn}(Q_{knt}) \le b_{wnt} \qquad \forall n\in N, w\in R_n, t=1,...,T \qquad (3)$$

$$Q_{knt} \ge 0 \qquad \forall n\in N, k\in P_n, t=1,...,T \qquad (4)$$

$$R_{knjt} \ge 0 \qquad \forall n\in N, j\in C_n, k\in P_n \cap P_j, t=1,...,T \qquad (5)$$

In this model, the objective function is the sum of inventory, production and transportation costs. For each planning period, for each facility and for each product that can be produced/assembled/stocked in this facility, constraints (1) assure that the flow going into this facility is equal to the flow going out. Constraints (2) define the amount of items transported from facility n to its client facility j for each period t. Constraints (3) enforce the capacity limitation of each resource at each facility.

V. Conclusion

The objective of this paper was to present a unified framework for the research literature on deterministic flow planning problems. A description and categorization of the most relevant literature was also presented. Special consideration was given to the multi-facility flow coordination problem. Important issues for this problem were identified and a novel formulation for the general problem under dynamic demand was proposed.

It is obvious from the multi-facility section of the paper that several important flow coordination issues are not studied in the research literature. First, most studies assume static (deterministic) demand. Little work has been done on the multi-facility problem when demand is time-dependent, which is the case generally encountered in real life. Second, the treatment of the production process, if considered, is a gross simplification of the actual situation. The few models dealing with multi-facility coordination consider, only single stage manufacturing systems. However, when the product being manufactured is complex, for example in the case of computers, telecommunication equipment, etc., production is often divided between a number of "focused factories", which cannot be represented by a single production stage. Third, important savings coming from economies of scale are not considered in these models. Larger production/purchasing and/or shipping quantities can lead to major discounts as it has been shown in many realistic situations. Fourth, only very simple network structures were considered by these studies, which limit their application.

These shortcomings of the lot-sizing literature are explained mainly by the fact that, in the past, sales data and information on inventory status at the downstream nodes of the supply chain were not available for the upstream members, which made it unrealistic for researchers to include the different actors of the supply chain in their models. Nowadays, improvements in information and communication technology (e.g. bar coding/scanning, EDI, Internet) provide many opportunities to increase the efficiency of the total supply chain. Taking a systems view of the supply chain and integrating its several partners and actors in the decision making process can lead to a sustainable competitive advantage. In order to capture the potential of modern information technologies to make better decisions on an enterprise-wide basis and develop win-win strategies, a set of planning and decision support tools with models that are able to capture the complexity of real production processes, the saving from economies of scale (specially the ones related to the transportation costs) and the dynamic nature of demand, is required. Since these models are NP-hard, the search for tight lower bounds and efficient heuristics is the route to take from both a practical and a theoretical point view. A generalization of the methods available to solve single facility lot-sizing problems to procedures capable of tackling multi-facility problems may yield high payoffs.

REFERENCES

1) AGGARWAL, A. and J.K. PARK, "Improved Algorithms for Economic Lot Size Probelms", *O.R.*, 41 (1993), 549-571.

2) AFENTAKIS P, B. GAVISH and U. KARMARKAR, "Computationally Efficient Optimal Solutions to the Lotsizing Problem in Multistage Assembly Systems", *Management Science*, 30 (1984), 222-239.

3) AFENTAKIS, P. and B. GAVISH, "Optimal Lot-Sizing Algorithms for Complex Product Structures", *O.R.*, 34 (1986), 237-249.

4) AFENTAKIS, P., "A Parallel Heuristic Algorithm for Lot-Sizing in Muli-Stage Production Systems. *IIE Transactions*, (1987), 34-42.

5) ANILY, S., "Integrating Inventory Control and Transportation Planning", Ph.D dissertation, Columbia University, New York, (1986).

6) ANILY, S. and A. FEDERGRUEN, "One Warehouse Multiple Retailers Systems with Vehicle Routing Costs", *Management Science*, 36 (1990), 92-114.

7) ANILY, S. and A. FEDERGRUEN, "Two-Echelon Distribution Systems with Vehicle Routing Costs and Central Inventories", *Operations Research*, 41 (1993), 37-47.

8) ANILY, S., "The General Multi-retailer EOQ Problem with Vehicle Routing Costs", *E.J.O.R.*, 79 (1994), 451-473.

9) ARKIN, E., D. JONEJA and R. ROUNDY, "Computational Complexity of Uncapacitated Multi-echelon Production Planning Problems", *Operations Research Letters*, 8 (1989), 61-66.

10) ATKINS, D.R. and P.O. IYOGUN, "A Heuristic with Lower Bound Performance Guarantee for the Multi-Product Dynamic Lot-Sizing Problem", *IIE Transactions*, December (1988), 369-373.

11) ATKINS, D., M. QUEYRANNE and D. SUN, "Lot Sizing Policies for Finite Production Rate Assembly Systems", *Operations Research*, 40 (1992), 126-140.

12) ATKINS, D.and D. SUN, “98%-Effective Lot Sizing for Series Inventory Systems with Backlogging”, *Operations Research*, 43 (1995), 335-345.

13) AXSÄTER, A., “An Extension of The Extended Basic Period Approach for Economic Lot Scheduling Problems”, *Technical Report*, Department of Production Economics, Link∅ping Institute of Technology, Sweden, 1983.

14) AXSÄTER, A., “Continuous Review Policies for Multi-Level Inventory Systems with Stochastic Demand”, in Graves *et al.* Eds., *Handbooks in OR & MS*, 4 (1993), 175-197.

15) BAKER K.R., P. DIXON, M.J. MAGAZINE and E.A. SILVER, “An Algorithm for the Dynamic Lotsize Problem with Time-varying Production Capacity Constraints”, *Management Science*, 24 (1978), 1710-1720.

16) BARANY, I., T.J. VAN ROY, and L.A. WOLSEY, “Strong Formulations for Multi-items Capacitated Lotsizing”, *Management Science*, 30 (1984), 1255-1261.

17) BASTIAN, M., “Joint Replenishment in Multi-item inventory Systems”, Journal of the Operational Research Society, 37 (1986), 1113-1120.

18) BERRY W.L., “Lot sizing Procedures for Requirements Planning Systems: a Framework for Analysis”, *Production and Inventory Management*, 13 (1972), 19-34.

19) BHATNAGAR, R., P. CHANDRA and S.K. GOYAL, “Models for Multi-plant Coordination”, *European Journal of Operational Research*, 67 (1993), 141-160.

20) BILLINGTON, P.J., J.O. MC CLAIN, and L.J. THOMAS, “Mathematical Programming approaches to Capacity Constrained MRP Systems: Review, Formulation and Problem Reduction”, *Management Science*, 29 (1983), 1126-1141.

21) BILLINGTON, P.J., J.O. MC CLAIN, and L.J. THOMAS, “Heuristics for Multilevel Lot-sizing with a Bottleneck”, *Management Science*, 32 (1986), 989-1006.

22) BILLINGTON, P.J., J. BLACKBURN, J. MAES, R. MILLEN and L.N. VAN WASSENHOVE, “Multi-Product Scheduling in Capacitated Multi-Stage Serial Systems”, *Technical*

Report 8908/A, Econometric Institute, Erasmus Universiteit Rotterdam, The Netherlands, 1989.

23) BITRAN, G. and H.H. YANASSE, “Computational Complexity of the Capacitated Lot Size Problem”, *Management Science*, 28 (1982), 1174-1185.

24) BITRAN G. and H. MATSUO, “Approximation Formulations for the Single-Product Capacitated Lot Size Problem”, *O.R.*, 34 (1986), 63-74.

25) BLACKBURN J.D. and R.A MILLEN, “Improved Heuristics for Multi-Stage Requirements Planning Systems”, *Management Science*, 28 (1982), 44-56.

26) BLACKBURN, J.D. and R.A MILLEN, “Simultaneous Lot-sizing and Capacity Planning in Multi-stage Assembly Processes”, *E.J.O.R.*, 16 (1984), 84-93.

27) BLUMENFELD, D.E, BURNS, L.D., J.D. DILTZ and C.F. DAGANZO, “Analyzing Tradeoffs between Transportation, Inventory and Production Costs on Freight Networks, *Transportation Research*, 19B (1985), 361-380.

28) BLUMENFELD, D.E, BURNS, L.D., C.F. DAGANZO, M.C. FRICK and R.W. HALL, “Reducing Logistics Costs at General Motors”, *Interfaces*, 17 (1987), 26-47.

29) BOCTOR, F.F., “Single Machine Lot Scheduling: A Comparison of Some Solution Procedures”, *R. A. I. R. O.*, 19 (1985), 389-402.

30) BOCTOR, F.F. (1987), “The G-group Heuristic for Single Machine Lot Scheduling”, *International Journal of Production Research*, 25 (1987), 363-379.

31) BOMBERGER, E., “A Dynamic Programming Approach to a Lot Size Scheduling Problem”, *Management science*, 12 (1966), 778-784.

32) BROWN, R.J., “Decision Rules for Inventory Management”, Holt, Reinhart and Winston, New York, (1967), 50-55.

33) CARRENO, J.J., “Economic Lot Scheduling for Multiple Products on Parallel Identical Processors”, *Management science*, 36 (1990), 348-358.

34) CATTRYSSE, D., M. SALOMON, R. KUIK, and L.N. VAN WASSENHOVE, “A Dual Ascent and Column Generation Heuristic for the Discrete Lotsizing and Scheduling Problem with Setup Times”, *Management Science*, 39 (1993), 477-486.

35) CHANDRA, P. and M.L. FISHER, “Coordination of production and distribution planning”, *Journal of the Operational Research Society*, 72 (1994), 503-517.

36) CHAKRAVARTY, A.K., “Multi-item Inventory Aggregation into Groups”, *Journal of the Operational Research Society,* 32 (1981), 19-26.

37) CHEN, W.H. and J.M. THIZY, “Analysis of Relaxation for the Multi-item Capacitated Lotsizing Problem”, *Technical Report* 86-103, School of Management & Department of Industrial Engineering, State University at Buffalo, N.Y., (1987).

38) CHEN, H., D.W. HEARN, and C. LEE, “A New Dynamic Programming Algorithm for the Single Item Capacitated Dynamic Lot Size Model”, Research Report 92-7 (1992.a), Department of Industrial and Systems Engineering University of Florida.

39) CHEN, H., D.W. HEARN, and C. LEE, “A Dynamic Programming Algorithm for Dynamic Lot Size Models with Piecewise Linear Costs”, Research Report 92-27 (1992.b), Department of Industrial and Systems Engineering University of Florida.

40) CHEN, H., D.W. HEARN, and C. LEE, “A New Dynamic Programming Algorithm for the Single Item Capacitated Dynamic Lot Size Model”, *J. of Global Optimization*, 4 (1994), 285-300.

41) Chopra, S., M.R. Rao an C.Y. Tsai, “Computational Study of the Multiechelon Production Planning Problem”, *Naval Research Logistics,* 44 (1997), 1-19.

42) CHUNG, C. and C.M. LIN, “An O(T) Algorithm for the NI/G/NI/ND Capacitated Lot Size Problem”, *Management Science*, 21 (1988), 420-426.

43) CHUNG, C. and M.H. MERCAN, “A Heuristic Procedure for a Multi-product Dynamic Lot-Sizing Problem With Coordinated Replenishments”, **Working Paper Series (), March** 4 (1992),

44) CHUNG, C., J. FLYNN and C.M. LIN, “An Effective Algorithm for the Capacitated Single Item Lot Size Problem”, *E.J.O.R,*, 75 (1994), 427-440.

45) COHEN, M., "Joint Pricing and Ordering Policy for Exponentially Decaying Inventory with Known Demand", *Naval Research Quarterly*, 24 (1977), 257-268.

46) COHEN, M. and H. LEE, "Strategic Analysis of Integrated Production – Distribution Systems: Models and Methods", *O.R.*, 36 (1988), 216-228.

47) COLEMAN, B.J. and M.A. MCKNEW, "An Improved Heuristic for Multilevel Lot Sizing Material Requirements Planning", Decision Sciences, 22 (1991), 136-156.

48) CROWSTON, W. B., M. WAGNER and J.F WILLIAMS, "Economic Lot Size Determination in Multi-Stage Assembly Systems", *Management Science*, 19 (1973), 517-527.

49) CLARK, A.R. and V.A. ARMENTANO, "Echelon Stock Formulation for Multi-stage Lot-sizing with Component Lead Times", *Int. J. Systems Sci*., 24 (1993), 1759-1775.

50) CROWSTON, W. B., M. WAGNER, "Dynamic Lot Size Models for Muti-stage Assembly Systems", *Management Science,* 20 (1973), 14-21.

51) DE MATTA, R. and M. GUIGNARD, "Production Scheduling with Sequence-Independent Changeover Cost", *Technical Report*, Wharton School, University of Pennsylvania, (1989).

52) DE MATTEIS J.J. and A.G. MENDOZE, "An economic lot sizing technique", *IBM System Journal*, 7 (1968), 30-46.

53) DIABY, M., H.C. BAHL, M.H. KARWAN, and S. ZIONTS, "Capacitated Lot-sizing and Scheduling by Lagrangean Relaxation", *E.J.O.R.*, 59 (1992a), 444-458.

54) DIABY, M., H.C. BAHL, M.H. KARWAN, and S. ZIONTS, "A Lagrangean Relaxation Approach for Very-Large-Capacitated-Scale Capacitated Lot-Sizing", *Management Science*, 38 (1992b), 1329-1340.

55) DIABY, M. and MARTEL, A., “Dynamic Lot Sizing for Multi-Echelon Distribution Systems with Purchasing and Transportation Price Discounts” *Operations Research*, 41 (1993), 48-59

56) DIXON, P.S. and E.A. SILVER, “A Heuristic Solution Procedure for the Multi-item, Single Level, Limited Capacity, Lotsizing Problem”, *Journal of Operations Management*, 2 (1981), 23-39.

57) DIXON, P.S. and C.L. POH, “Heuristic Procedures for Multi-item Inventory Planning with Limited Storage”, *IIE Transactions*, 22 (1990), 112-123.

58) DOBSON, G., “The Economic Lot-Scheduling Problem: Achieving Feasibility Using Time-Varying Lot Sizes”, *Operations Research*, 15 (1987), 764-771.

59) DOLL, C.L. and D.C. WHYBARK, “An iterative Procedure for the Single-Machine Multi-Product Lot Scheduling Problem”, *Management Science*, 20 (1973), 50-55.

60) EISENHUT, P.S., “A Dynamic Lotsizing Algorithm with Capacity Constraints”, *AIIE Transactions*, 7 (1975), 170-176.

61) ELMAGHRABY, S., “The Economic Lot Scheduling Problem (ELSP): Review and Extensions”, *Management Science*, 24 (1978), 587-598.

62) El-NAJDAWI, M.K. and P.R. KLEINDORFER, “Common Cycle Lot-size Scheduling for Multi-product, Multi-stage Production”, *Management Science*, 39 (1993), 872-885.

63) EPPEN, G.D. and R.K. MARTIN, “Solving Multi-item Capacitated Lot-sizing Problems Using Variable Redefinition”, *Operations Research*, 35 (1987), 268-277.

64) ERENGUC, S.S., “Multiproduct Dynamic Lot-Sizing Model with Coordinated Replenishments”, *Naval Research Logistics*, 35 (1988), 1-22.

65) EVANS, J.R., “A Network Decomposition Aggregation Procedure for a Class of Multicommodity Transportation Problems”, *Networks*, 13 (1983), 197-205.

66) FEDERGRUEN, A., M. QUEYRANNE and Y.S. ZHENG, “Simple Power-Of-Two Policies are Close to Optimal In a General Class of Production/Distribution Networks with General Joint Setup Costs”, *Mathematics of Operations Research*, 17 (1992), 951-963.

67) FEDERGRUEN, A., “Centralized Planning Models for Multi-Echelons Inventory Systems under Uncertainty”, in Graves et al. Eds., *Handbooks in OR & MS*, 4 (1993), 133-173.

68) FEDERGRUEN, A. and Y.S. ZHENG “Optimal Power-of-Two Replenishment Strategies in Capacitated General Production/Distribution Networks”, *Management Science*, 39 (1993), 710-727.

69) FEDERGRUEN, A. and M. TZUR, “The joint Replenishment Problem with Time-Varying Costs and Demands: Efficient, Asymptotic and ε-Optimal Solutions”, *O.R.*, November-December (1994), 1067-1086.

70) FEDERGRUEN, A. and Y.S. ZHENG “Efficient Algorithms for Finding Optimal Power-of-two Policies for Production/Distribution Systems with General Joint Setup Costs”, *Operations Research*, 43 (1995), 458-470.

71) FLEISCHMANN, B., “The Discrete Lot-Sizing and Scheduling Problem”, *E.J.O.R.*, 44 (1990), 337-348.

72) FLORIAN, M. and M. KLEIN, “Deterministic Production Planning with Concave Costs and Capacity Constraints”, *Management science*, 18 (1971), 12-20

73) FLORIAN, M., J.K. LENSTRA, and A.H.G. RINNOOY KAN, “Deterministic Production Planning: Algorithms and Complexity”, *Management science*, 26 (1980), 12-20.

74) FRIEDMAN, Y. and Y. Hoch, “A Dynamical Lot-size Model with Inventory Deterioration”, *INFOR*, 16 (1978), 183-188.

75) FUJITA, S., “The Application of Marginal Analysis to the Economic Lot Scheduling Problem”, *AIIE Transactions*, 10 (1978), 354-361.

76) GABBAY, H., “Multi-Stage Production Planning”, *Management Science*, 25 (1979), 1138-1148.

77) GALLEGO, G. and D. SIMCHI-LEVI, “On the Effectiveness of Direct Shipping Strategy for the one Warehouse Multi-retailer R-systems”, *Management Science*, 36 (1990), 240-243.

78) GASCON, A., “On the finite horizon EOQ model with cost changes”, *Operations Research.*, 43 (1995), 716-717.

79) GELDERS, L.F., MAES, J., and VAN WASSENHOVE, L.N., “A Branch and Bound Algorithm for the Multi-item Single Level Capacitated Dynamic Lotsizing Problem”, in S. Axsaster et al. (eds.), ¨Multi-stage Production Planning and Inventory Control¨, *Lectures Notes in Economics and Mathematical Systems* 266, Springer-Verlag, Berlin, 1986, 92-108.

80) GHARE, P., and G. SCHRADER, “A Model for Exponentially Decaying Inventories”, *Journal of Industrial Engineering*, 14 (1963), 238-243.

81) GILBERT, K.C., and M.S. MADAN, “A Heuristic for a Class of Production Planning and Scheduling Problems”, *IIE Transactions*, 23 (1991), 282-289.

82) GORHAM T., “Dynamic order quantities”, *Production and Inventory Management*, 20 (1968), 75-81.

83) GOYAL, S.K., “Scheduling a Multi-product Single-machine System”, *Operational Research Quarterly*, 24 (1973 **(a)**), 261-266.

84) GOYAL, S.K., “Determination of Economic Packaging Frequency of Items Jointly Replenished”, ”, *Management Science*, 20 (1973 (b)), 232-235.

85) GOYAL, S.K., “Determination of Optimum Packaging Frequency of Jointly Replenished Items”, *Management Science*, 21 (1974 (a)), 436-443.

86) GOYAL, S.K., “Optimum Ordering Policy for a Multi-item Single Supplier System”, ”, *Operational Research Quarterly*, 25 (1974 (b)), 293-298.

87) GOYAL, S.K., “Note on Manufacturing Cycle Time Determination for A Multi-Stage Economic Production Quantity Model”, *Management Science*, 23 (1976), 332-333

88) GOYAL, S.K. and S.G. DESHMUKH, “A Note on the Economic Ordering Quantity of Jointly Replenished Items”, *International Journal of Production Research*, 31 (1993), 2959-2962.

89) GRAVES, S.C., “Multi-stage Lot-sizing: An Iterative Procedure”, *in L.B. Schwartz (Ed.), Multi-Level Production/ Inventory Control Systems: Theory and Practice*, North Holland, New York, (1981).

90) GRAVES, S.C., “Safety Stocks in Manufacturing Systems”, *Journal of Manufacturing Operations Management*, 1 (1988), 67-101.

91) GROFF, K.G., “A Lot sizing rule for time-phases component demand”, *Production and Inventory Management*, 20 (1979), 47-53.

92) GUPTA, S.M. and L. BRENNAN, “Heuristic and Optimal Approaches to Lot-sizing Incorporating Backorders: An Empirical Evaluation”, *INT. J. PROD. RES.*, 30 (1992), 2813-2824.

93) HALL, R.W., “Principles for Routing Freight Through Transportation Terminals”, Report GMR-5579, General Motors Research Laboratories, Warren, Mich.

94) HALL, R.W., “Direct Versus Terminal Freight Routing on a Network with Concave Costs”, *Transportation Research*, B21 (1987), 287-298.

95) HANSMANN, F., “Operation Research in Production and Inventory”, *John Wiley & Sons*, New York, (1962).

96) HARIGA, M., “Two New Heuristic Procedures for the Joint Replenishment Problem”, *Journal of Operation Research Society*, 45 (1994), 463-471.

97) HARISSON, T.P. and H.S. LEWIS, “Lot Sizing in Serial Assembly Systems with Multiple Constrained Resources”, Management Science, 42 (1996), 19-36.

98) HASEBORG, F., “On the Optimality of Joint Ordering Policies in a Multi-product Dynamic Lot Size Model with Individual and Joint Set-up Costs”, *E.J.O.R.*, 9 (1982), 47-55.

99) HAX A. C. and D. CANDEA, “Production and Inventory Management”, Printice-Hall, Inc, Englewood Cliffs, New Jersey, (1984).

100) HELBER, S., “Lot Sizing in Capacitated Production and Control Systems”, *Operations Research Spektrum*, 17 (1995), 5-18.

101) HENDRIKS, T.H.B. and J. WESSELS, “Repetitive Schemes for the Single Machine, Multiproduct Lot-size Problem”, *In Proceedings of Operations Research Verfahren XXVI* (1978), 571-581.

102) HERER, Y. and R. ROUNDY, “Heuristics for a One-Warehouse Multiretailer Distribution Problem with Performance Bounds”, *Operations Research*, 45 (1997), 102-115.

103) HILL, R.M., “Note: Dynamic Lot Sizing for a Finite Rate Input Process”, *Naval Research Logistics*, 44 (1997), 221-228.

104) HILL, A.V., A.S. RATURI and C.C. SUM, “Capacity-Constrained Reorder Intervals for Materials Requirement Planning Systems”, *IIE Transactions*, 29 (1997), 951-963.

105) HSU, W., “On the General Feasibility test of Scheduling Lot Sizes for Several Products on One Machine”, *Management Science*, 29 (1983), 93-105.

106) HSU, J.I.S. and M. El-NAJDAWI, “Common Cycle Scheduling in a Multistage Production Process”, *Engineering Costs and Production Economics*, 20 (1990), 73-80.

107) IYOGUN, P. and D. ATKINS, “A Lower Bound and an Efficient Heuristic for Multistage Multiproduct Distribution Systems”, *Management Science*, 39 (1993), 204-217.

108) JACKSON, P.L., W.L. MAXWELL and J.A. MUCKSTADT, “Determining Optimal Reorder Intervals in Capacitated Production-Distribution Systems”, *Management Science*, 34 (1988), 938-958.

109) JAGANNATHAN, R. and M.R. RAO, “A Class of Deterministic Planning Problems”, *Management science*, 19 (1973), 1295-1300.

110) JAIN, K. and E.A. SILVER, “Lot Sizing for a Product Subject to Obsolescence or Perishability”, *E.J.O.R.*, 75 (1994), 287-295.

111) JENSEN P. A. and KHAN H. A., “Scheduling in a Multistage Production System with Set-up and Inventory Costs”, *AIIE Transactions*, 4 (1972), 126-133.

112) JONEJA, D., “The Joint Replenishment Problem: New Heuristics and Case Performance Bounds”, *O.R.*, 38 (1990), 711-723.

113) JONEJA, D., “Multi-echelon Assembly Systems with Nonstationary Demands: Heuristics and Worst Case Performance Bounds”, *Operations Research*, 39 (1991), 512-517.

114) KAO, E.P.C., “A Multi-product Dynamic Lot-Sizing Model with Individual and Joint Set-Up Costs”, O.R., 27 (1979), 279-284.

115) KARIMI, I.A. (1989), “Optimal Cycle Times in a Two-Stage Serial System with Set-up and Inventory Costs”, *IIE Transactions*, 21 (1989), 324-332.

116) KARMARKAR, U.S. and L. SCHRAGE, “The Deterministic Dynamic Product Cycling Problem”, Operation Research, 33 (1985), 326-345.

117) KARMARKAR, U.S., S. KEKRE, and S. KEKRE, "Lotsizing in Multi-Item Multi-Machine Job Shops", *IIE Transactions*, 17 (1985), 290-298.

118) KARMARKAR, U.S., S. KEKRE, and S. KEKRE, "The Deterministic Lot Sizing Problem with Startup and Reservation Costs", *O.R.*, 35 (1987), 389-398.

119) KARMARKAR, U.S., S. KEKRE, and S. KEKRE, "Multi-item batching heuristics for minimization of queuing delays", *European Journal of Operational Research*, 58 (1992), 99-111.

120) KARNI, R. and Y. ROLLI, "A Heuristic Algorithm for the Multi-item Lotsizing Problem with Capacity Constraints", *AIIE Transactions*, 14 (1982), 249-256.

121) KASPI, M, and M.J. ROSENBLATT, "An improvement of Silver's Algorithm for the Joint Replenishment Problem", *IIE Transactions*, 17 (1983), 264-267.

122) KASPI, M, and M.J. ROSENBLATT, "The Effectiveness of Heuristic Algorithms for Multi-item Inventory Systems with Joint Replenishment Costs", *International Journal of Production Research*, 23 (1985), 109-116.

123) KASPI, M, and M.J. ROSENBLATT, "On the Economic Ordering Quantity for Jointly Replenished Items", *International Journal of production Research*, 29 (1991), 107-114.

124) KIRCA, Ö., "A Heuristic Procedure for the Dynamic Lot-Size Problem with Set-up Time", *Technical Report*, Middle East Technical University, Ankara, Turkey, (1990).

125) KIRCA, Ö., "An Efficient Algorithm for the Capacitated Single Item Dynamic Lot Size Problem", *E.J.O.R.*, 45 (1990), 15-24.

126) KIRCA, Ö. and M. KÖKTEN, "A New Heuristic Approach for the Multi-Item Dynamic Lot Sizing Problem", *E.J.O.R.*, 75 (1994), 332-341.

127) KIRCA, Ö., "A Primal-Dual Algorithm for the Dynamic Lotsizing Problem with Joint Set-Up Costs", *Naval Research Logistics*, 42 (1995), 791-806.

128) KLINCEWICZ, J.G, "Solving A Freight Transport Problem Using Facility Location Techniques", *Operations Research*, 38 (1990), 99-109.

129) KUIK R. and M. SALOMON, "Multilevel Lotsizing Problem: Evaluation of a Simulated-Annealing Heuristic", *European Journal of Operational research*, 45 (1990), 25-37.

130) LAMBERT, A. and H. LUSS, “Production Planning with Time-Dependent Capacity Bounds”, *E.J.O.R.*, 9 (1982), 275-280.

131) LAMBRECHT, M. and H. VANDERVEKEN, “Heuristic Procedures for the Single Operation Multi Item Loading Problem”, *AIIE Transactions*, 11 (1979), 319-326.

132) LASDON L.S. and R.C. TERJUNG, “An efficient algorithm for multi-item scheduling”, *O.R.*, 19 (1971), 946-969

133) LEE, H. and C. BILLIGTON, “Material Management in Decentralized Supply Chain”, *O.R.*, 41 (1993), 837-845.

134) LEE, H., PADMANABAHAN V. and WHANG S., “Information Distortion in a Supply Chain: The Bullwhip Effect”, *Management Science*, 43 (1997), 546-558.

135) LEV, B. and H.J. WEISS, “Inventory models with cost changes”, *O.R.*, 38 (1990), 53-63.

136) LEUNG, J.M.Y., T.L. MAGNANTI, and R. VACHANI, “Facets and Algorithms for the Capacitated Lotsizing”, *Mathematical Programming*, 45 (1989), 331-359.

137) LOVE, S., “A Facilities in Series Model with Nested Schedules”, *Management Science*, 18 (1972), 327-338.

138) LOZANO, S., “Multilevel Lot-sizing with a Bottleneck Work Center”, *Master’s thesis*, Katholieke Universiteit Leuven, Belguim, (1989).

139) MADIGAN, J.C., “Scheduling a Multi-product Single-machine System for an infinite Planning Period”, *Management Science*, 14 (1968), 713-719.

140) MAES, J., and L.N. Van WASSENHOVE, “A simple Heuristic for the Multi-Item Single Level Capacitated Lot Sizing Problem”, Letters of the Operational Research Society, 4 (1986), 265-274.

141) MAES, J., “Capacitated Lotsizing Techniques in Manufacturing Resource Planning”, Ph.D. thesis, Katholieke Universiteit Leuven, Belguim, (1987).

142) MAES, J., and L.N. Van WASSENHOVE, “Multi-item Single-level Capacitated Dynamic Lot-sizing Heuristics: a General Review”, *Journal of the Operation Research Society*, 39 (1988), 991-1004.

143) MAES, J., and L.N. Van WASSENHOVE, “Capacitated Dynamic Lotsizing Heuristic for Serial Systems”, *International Journal of Production Research*, 29 (1991), 1235-1249.

144) MAES, J., J.O. McClain and L.N. Van WASSENHOVE, “Multilevel Capacitated Lotsizing Complexity and LP-Based Heuristics”, E.J.O.R., 53 (1991), 131-148.

145) MAGNANTI, T.L., and R. VACHANI, “A Strong Cutting-Plane Algorithm for Production Scheduling with Changeover Costs”, *O.R.*, 38 (1990), 456-473.

146) Mallya, S.N., “Demand Driven Production and Distribution: Seamless Integration of Downstream Supply Chain Decisions Using Electronic Data Interchange”, Ph.D dissertation, Drexel University, (1999)

147) MARTEL, A., “Planning Policies for Multi-Echelons Supply Systems with Probabilistic Time-Varying Demands”, *Publication de Recherche du GREGOR* (*Groupe de Recherche en Gestion des Organisations*), Institut d’Administration des Entreprises de Paris-Université Paris 1- Panthéon-Sorbonee. 1995.08.

148) MARTEL, A., M. DIABY, F. BOCTOR, “Multiple Items Procurement Under Stochastic Nonstationary Demands”, *E.J.O.R.*, 87 (1995), 74-92.

149) MARTEL, A. and A. GASCON, “Dynamic lot-sizing with price changes and price-dependent holding costs”, to appear in *E.J.O.R.* (1998).

150) MAXWELL, W.L. and J.A. MUCKSTADT, “Establishing Consistent and Realistic Reorder Intervals in Production-Distribution Systems”, *Operations Research*, 33 (1985), 1316-1341.

151) McLaren, B.J., “Multi-level Lot Sizing Procedures in a Material Requirements Planning Environnemnt”, *Discussion Paper 64, Division of research/School of Business*, Indiana University, (1976).

152) MCKNEW, M.A., C. SAYDAM and B.J. COLEMAN, “An Efficient Zero-One Formulation of the Multilevel Lot-Sizing Problem”, *Decision Sciences*, 22 (1991), 280-294.

153) MERCAN, H.M. and S.S. ERENGUC, “A Mult-family Dynamic Lot Sizing Problem with Coordinated Replenishments: A Heuristic Procedure”, *Int. J. Prod. Res.*, 31 (1993), 173-189.

154) MILLAR, H.H. and M. YANG, “An Application of Lagrangean Decomposition to the Capacitated Multi-item Lot Sizing Problem”, *Computers Ops Res.*, 20 (1993), 409-420.

155) MILLAR, H.H. and M. YANG, “Lagrangean Heuristics for the Capacitated Multi-item Lot-Sizing Problem with Backordering”, *International Journal of Production Economics*, 34 (1994), 1-15.

156) MOILY, J.P., “Optimal and Heuristic Procedures for Component Lot-Splitting in Multi-Stage Manufacturing Systems”, *Management Science*, 32 (1986), 113-125.

157) NAHMIAS, S., and S. WANG, “A Heuristic Lot-size Reorder Point Model For Decaying Inventories”, *Management science*, 25 (1979), 90-97.

158) PAGE, E., and R.J. PAUL, “Multi-product Inventory Situations with One Restriction”, *Operational Research Quarterly*, 27 (1976), 815-834.

159) PARK, K.S. and D.K. YAN, “A Stepwise Partial Enumeration Algorithm for the Economic Lot Scheduling Problem”, *IIE Transactions*, 16 (1984), 363-370.

160) POCHET, Y., “Valid Inequalities and Separation for Capacitated Economic Lot Sizing”, *Operations Research Letters*, 7 (1988), 109-116.

161) POCHET, Y. and L.A. WOLSEY, “Solving the Multi-item Lot-sizing Problems Using Strong Cutting Planes”, Management Science, 37 (1991), 53-67.

162) ROBINSON, E.P. and L. GAO, “A Dual Ascent Procedure for Multiprodut Dynamic Demand Coordinated Replenishment with Backlogging”, *Management science*, 42 (1996), 1556-1564.

163) ROSLING, K., “Optimal Lot-Sizing for Dynamic Assembly Systems”, *Technical Report 152*, Link∅ping Institute of Technology, Sweden, 1985.

164) ROUNDY, R., “A 98%-effective Lot-sizing Rule for a Multi-product, Multi-stage Production/inventory System”, *Mathematics of Operations Research*, 11 (1986), 699-727.

165) SALOMON, M. (eds.), “Multi-stage Production Planning and Inventory Control”, *Lectures Notes in Economics and Mathematical Systems* 355, Springer-Verlag, Berlin, 1991, 92-108.

166) SCHWARZ, L.B. and L. SCHRAGE, “Optimal and System Myopic Policies for Multi-Echelon Production/Inventory Assembly Systems”, *Management Science*, 21 (1975), 1285-1294.

167) SENGUPTA, S. and J. TURNBULL, “Seamless Optimization”, *IIE Sol*., October (1996), 369-373.

168) SHAH, Y., “An Order Level Lot-size Inventory Model for Deteriorating Items”, *AIIE Transactions*., 9 (1977), 190-197.

169) SHU, F., “Economic Ordering Frequency for two items Jointly Replenished”, *Management Science*, 17 (1971), 406-410.

170) SILVER E.A. and H.C. MEAL, “A heuristic for selecting lot size quantities for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment”, *Production and Inventory Management*, 14 (1973), 64-74.

171) SILVER E.A., “A Simple Method of determining Order Quantities in Joint Replenishments Under Deterministic Demand”, *Management science*, 22 (1976), 1351-1361.

172) SILVER E.A., “Coordinated Replenishment of Items Under Time-Varying Demand: A Dynamic Programming Formulation”, *Naval Research Logistics Quarterly*, 26 (1979), 141-151.

173) SILVER E.A. and PETERSON, “Decision sysytems for Inventory Management and Production Planning”, *Jhon Wiley & Sons*, New York, (1985).

174) SPERANZA, M.G. and W. UKOVICH, “Minimizing Transportation and Inventory Costs for Several Products on a Single Link”, Operations Research, 42 (1994), 879-894.

175) SUN, D. and D. ATKINS, “98%-Effective Lot-sizing for Assembly Inventory Systems with Backlogging”, *Operations Research*, 45 (1997), 940-951.

176) STANGARD, M.F. AND S.K. GUPTA, “A Note on Bomberger’s Approach to Lot Size Scheduling: Heuristic Proposed”, *Management science*, 15 (1969), 449-452.

177) STADTLER, H., “Mixed Integer Programming Model Formulations for Dynamic Multi-item Multi-level Capacitated Lotsizing”, *E.J.O.R.*, 94 (1996), 561-581.

178) STEINBERG, E. and H.A. NAPIER, “Optimal Multi-level Lot Sizing for Requirements Planning Systems”, *Management Science*, 26 (1980), 1258-1271.

179) SZENDROVITS, A.Z., “Manufacturing Cycle Time Determination for A Multi-Stage Economic Production Quantity Model”, *Management Science*, 22 (1975), 298-307

180) SZENDROVITS, A.Z., “On the Optimality of Sub-Batch Sizes for A Multi-Stage EPQ Model-A Rejoinder”, *Management Science*, 23 (1976), 334-338.

181) TADIKAMALLA, P.R., “An EOQ Model for Items with Gamma Distributed Deterioration”, *AIIE Transactions*, 10 (1978), 100-103.

182) TEMPELMEIER, H. and S. HELBER, “A Heuristic for Dynamic Multi-item Multi-level Capacitated Lotsizing for General Product Structures”, *E.J.O.R.*, 75 (1994), 296-311.

183) TEMPELMEIER, H. and M. DERSTROFF, “A Lagrangean-based Heuristic for Dynamic Multilevel Multiitem Constrained Lotsizing with Setup Times”, *Management Science*, 42 (1996), 738-758.

184) TERSINE R.J. and R.L. PRICE, “Temporary Price Discounts and EOQ”, *Journal of Purchasing and Materials Management*, 2nd Quarter (1981), 23-27.

185) THIZY, J.M. and L.N. VAN WASSENHOVE, “Lagrangean Relaxation for the Multi-Item Capacitated Lotsizing Problem: a Heuristic Implementation”, IIE Transactions, 17 (1985), 308-313.

186) TRIGEIRO, W.W., L.J. THOMAS, and J.O. Mc Clain, “Capacitated Lot Sizing with Setup Times”, *Management science*, 35 (1989), 353-366.

187) VAN EIJS, M.J.G., R.M.J. HEUTS and J.P.C. KLEIJNEN, “Analysis and Comparison of two Strategies for Multi-item Inventory Systems with Joint Replenishment Costs”, *E.J.O.R.*, 59 (1992), 405-412.

188) VAN EIJS, M.J.G., "A Note on the Joint Replenishment Problem under Constant Demand", *Journal of the Operational Research Society*, 44 (1993), 185-191.

189) VAN HOESEL, C.P.M. and A.P.M. WAGELMANS, "An O(T^3) Algorithm for the Economic Lot-sizing Problem with Constant Capacities", *Management science*, 42 (1996), 142-150.

190) VAN NUNEN, J.A.E.E. and J. WESSELS, "Multi Item Lot Size Determination and Scheduling under Capacity Constraints", *E.J.O.R.*, 2 (1978), 36-41.

191) VAN ZYL, G.J.J., "Inventory Control for Perishable Commodities", Unpublished Ph.D. Dissertation, University of North Carolina.

192) VAN WASSENHOVE, L.N. and P. Vanderhenst, "Planning Production in a Bottleneck Department", *E.J.O.R.*, 12 (1983), 127-137.

193) VEINOTT, A. F., "Optimal Ordering, Issuing and Disposal of Inventory with Known Demand", Unpublished Ph.D. Dissertation, Columbia University, (1960).

194) VEINOTT, A.F., Uunpublished class notes for the Program in Operations Research at Stanford University, (1963).

195) VEINOTT, A.F., "Minimum Concave Cost Solution of Leontieff Substitution Models of Multi-Facility Inventory Systems", *O.R.*, 17 (1969), 267-291.

196) VISWANATHAN, S. and K. MATHUR, "Integrating Routing and Inventory Decisions in One-Warehouse Multiretailer Multiproduct Distribution Systems", Management Science, 43 (1997), 294-312.

197) WAGELMANS, A.P.M., S.V. HOSSEL AND A. KOLEN, "Economic Lot-sizing: An O(n logn) Algorithm that Runs in Linear Time in the Wagner-Whitin Case", *O.R.*, 40 (1992), (145-156).

198) WAGNER, H. AND T.M. WHITIN, "Dynamic Version of the Economic Lot Size Model", *Man. Sci.*, 5 (1958), 89-96

199) WILDEMAN, R.E., J.B.G. FRENK, R. DEKKER, "An efficient Optimal Solution Method for the Joint Replenishment Problem", *E.J.O.R.*, 99 (1997), 433-444.

200) WILLIAMS, J.F., “Heuristic Techniques for Simultaneous Scheduling of Production and Distribution in Multi-Echelon Structures: Theory and Empirical Comparisons”, *Management Science*, 27 (1981), 336-352.

201) WILLIAMS, J.F., “On the Optimality of Integer Lot Size Ratios in Economic Lot Size Determination in Multi-Stage Assembly Systems”, *Management Science*, 28 (1982), 1341-1349.

202) ZAHORIK, A., L.J. THOMAS and W.W. TRIGERIO, “Network Programming Models for Production Scheduling in Multi-Stage, Multi-Item Capacitated Systems”, Management Science, 30 (1984), 308-325.

203) ZANGWILL, W.I., “A Deterministic Multiproduct, Multifacility Production and Inventory Model”, *Ops. Res.*, 14 (1966), 486-507.

204) ZANGWILL, W.I., “A Backlogging Model and a Multi-Echelon Model of a Dynamic Economic Lot Size Production System”, *Man. Sci.*, 15 (1969), 506-527.

205) ZOLLER, K. and A. ROBRADE, “Efficient Heuristics for Dynamic Lot Sizing”, *INT. J. PROD. RES.*, 26 (1988), 249-265.