

# Analytical Solution for Probabilistic Logic with Conditional Probabilities

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### **Abstract**

Given a set of logical sentences and probabilities that these sentences are true, the probabilistic logic problem consists in determining whether these probabilities are consistent. When so, the probabilistic entailment problem is to determine the range of values of the probability associated with an additional logical sentence such that the resulting system of sentences and probabilities remains consistent.

These problems are well-solved, both analytically and numerically. In this note, we propose an analytical solution method for their extension where conditional probabilities are given instead of or in addition to probabilities. This method is illustrated by automated deduction, and improvement in one case, of inference rules with conditional probabilities gathered by Frisch and Haddawy (1994).

### **Résumé**

Étant donné un ensemble de phrases logiques et de probabilités que ces phrases soient vraies, le problème de la logique probabiliste consiste à déterminer si ces probabilités sont cohérentes. Si c'est le cas, le problème de l'inférence probabiliste est de déterminer l'étendue des valeurs de la probabilité d'une phrase logique additionnelle telle que le système de phrases logiques et de probabilités résultant demeure cohérent.

Ces problèmes sont bien résolus, tant analytiquement que numériquement. Dans cette note nous proposons une méthode de résolution analytique pour leur extension où des probabilités conditionnelles sont données au lieu de ou en plus de probabilités. La méthode est illustrée par la déduction automatique, et l'amélioration dans un cas, de règles d'inférence avec des probabilités conditionnelles réunies par Frisch et Haddawy (1994).

## 1 Introduction

Probabilistic logic (Nilsson [1]) is an approach to reasoning under uncertainty which agrees both with standard logic and probability theory. It has been initiated by Boole [2] and developed by Hailperin [3; 4; 5], and others, using tools from linear programming. Since the appearance of [1] theory and applications of probabilistic logic have been the subject matter of over 100 papers, see Nilsson [6], Chandru and Hooker [7], Hansen and Jaumard [8] for recent surveys.

The Probabilistic Logic problem [1; 9; 10] also called **Probabilistic Satisfiability** (or PSAT for short), is defined as follows. Let  $S = \{S_1, S_2, \dots, S_m\}$  be a set of  $m$  logical sentences defined on a set of  $n$  boolean variables  $X = \{x_1, x_2, \dots, x_n\}$  with the usual operators  $\vee$  (disjunction),  $\wedge$  (conjunction) and  $\neg$  (negation or complementation). Let  $\pi = (\pi_1, \pi_2, \dots, \pi_m)$  be a probability vector such that  $\pi_i$  defines the probability that sentence  $S_i$  is true, for all  $i = 1, 2, \dots, m$ .

Let  $w = (w_1, w_2, \dots, w_m)$  be a truth assignment for  $S$ , where  $w_i$  is equal to 1 if  $S_i$  has value true, and to 0 otherwise. A vector  $w \in \{0, 1\}^m$  is a *possible world* if there exists a truth assignment over  $X$  which leads to  $w$  over  $S$  and it is an *impossible world* otherwise. Let  $W$  denote the set of possible worlds and set  $k$  equal to  $|W|$  (note that  $k \leq 2^n$ ).

Let  $p = (p_1, p_2, \dots, p_k)$  be a probability distribution on  $W$ , with  $0 \leq p_j \leq 1$  ( $j = 1, \dots, k$ ) and  $\sum_j p_j = 1$ . The probability distribution  $p$  is *consistent* if it satisfies the set of logical sentences together with their probabilities, i.e., if for each  $S_i$ , the sum of  $p_j$ 's over all truth assignments  $w^j$  that satisfy  $S_i$  (i.e.,  $w_i^j = 1$ ) equals  $\pi_i$ .

For a given set  $S$  of sentences, let  $A = (a_{ij})$  be an  $m \times k$  matrix such that  $a_{ij}$  is equal to 1 if  $S_i$  is true for  $w^j$ , and equal to 0 otherwise. The linear programming formulation of the probabilistic logic problem is to determine whether or not there exists a vector  $p$  such that:

$$(P) \quad \begin{cases} \mathbb{1} p &= 1 \\ Ap &= \pi \\ p &\geq 0 \end{cases}$$

where  $\mathbb{1}$  denotes a unit row  $k$ -vector. Observe that a given instance  $(P)$  of the probabilistic logic problem is completely determined by the pair  $(S, \pi)$ .

Let us assume that the PSAT problem defined by  $(P)$  is consistent. Let  $S_{m+1}$  denote an additional logical sentence, with an unknown probability  $\pi_{m+1}$ . The Probabilistic Entailment problem [1] (also known as optimization version of PSAT), is to determine the range  $[\underline{\pi}_{m+1}, \bar{\pi}_{m+1}]$  of values of the probability  $\pi_{m+1}$  such that  $(S \cup \{S_{m+1}\}, (\pi, \pi_{m+1}))$  is consistent.

Consider the objective function  $A_{m+1}p$  (with  $A_{m+1} = (a_{m+1,j})$ , where  $a_{m+1,j}$  is equal to 1 if  $S_{m+1}$  is true for the possible world  $w^j$  and equal to 0 otherwise). The Probabilistic Entailment problem corresponds to the solution of the following linear programs

$$\begin{aligned} \text{and} \quad (P_{\min}) \quad & \underline{\pi}_{m+1} = \min\{A_{m+1}p : \text{constraints of } (P)\} \\ (P_{\max}) \quad & \overline{\pi}_{m+1} = \max\{A_{m+1}p : \text{constraints of } (P)\}. \end{aligned}$$

Numerical solution of both the probabilistic logic and entailment problems has been well studied [9; 11; 12; 13; 14; 15]. Using the powerful column generation technique of linear programming, instances with up to 1200 logical sentences  $S_i$ , 800 boolean variables  $x_k$  and billions of billions of variables  $p_j$  (i.e., up to  $2^{800}$ ), most of which are kept implicit, can now be solved exactly [15].

An analytical expression of the lower and upper bounds of the probability of an event to occur can be computed using the coordinates of the extreme points (or vertices) of the dual polyhedron of the linear program  $(P)$  which maximize or minimize the objective functions of the duals of  $(P_{\min})$  and  $(P_{\max})$ :

$$\begin{aligned} \text{and} \quad (D_{\max}) \quad & \max\{u_0 + \pi^t \cdot u : \mathbb{1} u_0 + A^t u \leq A_{m+1}^t\} \\ (D_{\min}) \quad & \min\{u_0 + \pi^t \cdot u : \mathbb{1} u_0 + A^t u \geq A_{m+1}^t\}. \end{aligned}$$

These expressions can be easily obtained using any enumeration algorithm for extreme points (e.g., Fukuda and Prodon [16]) as the constraints of the dual polyhedron of  $(P)$  are independent of  $\pi$ . The value of each vertex is associated with a linear expression in the probabilities  $\pi_i$  of the events to occur. Hailperin [3] showed that the best lower (resp. upper) bound on the probability of an additional sentence can be expressed as the largest (resp. smallest) value for all such linear expressions. This result was later completed by Hansen *et al.* [10] with the generation of the consistency conditions, which requires the enumeration of the extreme rays of the dual polyhedron  $(D)$  of  $(P)$ . The consistency conditions correspond to affine functions on the  $\pi_i (i = 1, \dots, m)$  such that the probabilistic logic problem is consistent if they are non-positive. These results are summarized, as follows:

**Theorem 1** (*Probabilistic Entailment, Hailperin [3]*).

The best lower (upper) bound for  $\underline{\pi}_{m+1}$  ( $\overline{\pi}_{m+1}$ ) is given by the following convex (concave) piecewise linear function of the probability assignment:

$$\underline{\pi}_{m+1}(\overline{\pi}_{m+1}) = \max_{j=1,2,\dots,k_{\max}} (1, \pi)^t \cdot u_{\max}^j \left( = \min_{j=1,2,\dots,k_{\min}} (1, \pi)^t \cdot u_{\min}^j \right)$$

where  $u_{\max}^j (u_{\min}^j)$ , for all  $j$ , represents the  $k_{\max} (k_{\min})$  extreme points of  $(D_{\max}) ((D_{\min}))$ .

**Theorem 2** (*Consistency Conditions, Hansen, Jaumard and Poggi de Aragão [10]*).

The Probabilistic Logic problem ( $P$ ) is consistent if and only if the inequality  $(1, \pi)^t \cdot r \leq 0$  holds for all extreme rays  $r$  of the dual ( $D$ ) of ( $P$ ).

Both the probabilistic logic and the probabilistic entailment problems can be easily extended to the case where probability intervals  $[\underline{\pi}, \bar{\pi}]$  are assigned to the logical sentences instead of the point values  $\pi_i$  for  $i = 1, 2, \dots, m$ , as shown by Hailperin [3]. The model so obtained is often more realistic in applications such as, e.g., medical or failure diagnosis, as the experts usually provide subjective probabilities for the  $\pi$  values and find it easier to specify intervals containing the  $\pi_i$  than exact values for these  $\pi_i$ . The width of the intervals then reflect the confidence of the experts in their probability estimates. Equations for PSAT are then modified as follows:

$$(PI) \quad \begin{cases} \mathbb{1} p = 1 \\ \underline{\pi} \leq Ap \leq \bar{\pi} \\ p \geq 0, \end{cases}$$

with an instance defined by a triplet  $(S, \underline{\pi}, \bar{\pi})$ , and by linear programs  $(PI_{\min})$  and  $(PI_{\max})$  for the corresponding probabilistic entailment problem, with objective as in  $(P_{\min})$  and  $(P_{\max})$ .

An even more significant extension is to introduce conditional probabilities in probabilistic logic and entailment. Indeed, it is often the case that probabilities of events are hard to estimate and uninformative unless some condition holds (e.g., the probability of having an illness given some symptom is of interest for diagnosis, while the probability of having that illness in general gives little information). This extension was already discussed by Boole [17] for particular examples, and much later by Hailperin [5], Chesnokov [18], Jaumard, Hansen and Poggi de Aragão [12], Frisch and Haddawy [19], Lukasiewicz [20], and others.

The conditional probability of the event  $E_i$ , given the event  $E_j$ , will be denoted  $\pi(E_i|E_j)$  and defined by the following equation, where  $\pi(E_j)$  and  $\pi(E_i \wedge E_j)$  are given:

$$\pi(E_i|E_j)\pi(E_j) = \pi(E_i \wedge E_j).$$

Observe that this definition coincides with the usual one, after dividing both sides by  $\pi(E_j)$ , if  $\pi(E_j) > 0$  and does not force  $\pi(E_i|E_j)$  to take any particular value if  $\pi(E_j) = \pi(E_i \wedge E_j) = 0$ .

Consider again a set  $S = \{S_1, S_2, \dots, S_m\}$  of  $m$  logical sentences. Let  $M = \{1, 2, \dots, m\}$  and  $Q = \{1, 2, \dots, q\}$  be the index set of the sentences with unconditional probabilities, where  $q < m$ . Let  $C \subseteq M \times M$  be the index set of the pair of sentences  $(S_i, S_j)$  involved in the definition of a conditional probability  $\pi_{i|j}$ . It can be written as follows, assuming

that each sentence  $S_i$  is associated with an event  $E_i$ :

$$\pi_{i|j}\pi_j = \pi_{i|j}A_i p = \pi(S_i|S_j)\pi(S_i) = \pi(S_i \wedge S_j) = (A_i \wedge A_j)p = A_{i \wedge j}p.$$

Considering furthermore that we want to compute best possible bounds on a conditional probability  $P(S_s|S_\ell)$  ( $s, \ell \in C$ ), in the objective function, leads to the most general case:

$$(PC) \quad \left\{ \begin{array}{ll} \min / \max & \frac{A_{s \wedge \ell} p}{A_\ell p} \\ \text{subject to:} & \\ & \mathbb{1} p = 1 \\ \underline{\pi}_i \leq & A_i p \leq \bar{\pi}_i \quad i \in Q \\ 0 \leq (A_{i \wedge j} - \underline{\pi}_{i|j} A_j) p & (i, j) \in C \\ (A_{i \wedge j} - \bar{\pi}_{i|j} A_j) p \leq 0 & (i, j) \in C \\ & A_\ell p > 0 \\ & p \geq 0. \end{array} \right.$$

Point probabilities and conditional probabilities correspond to the cases  $\pi_i = \underline{\pi}_i = \bar{\pi}_i$  and  $\pi_{i|j} = \underline{\pi}_{i|j} = \bar{\pi}_{i|j}$ . In the remainder of this note we examine how to solve  $(PC)$  analytically.

## 2 Reformulation

A first difficulty is the fractional objective function. However, a transformation due to Charnes and Cooper [21] yields a program  $(PC')$  with one more variable  $t$  and a linear objective:

$$(PC') \quad \left\{ \begin{array}{ll} \min / \max & z = A_{s \wedge \ell} p' \\ \text{subject to:} & \\ & A_\ell p' = 1 \\ & \mathbb{1} p' = t \\ \underline{\pi}_i t \leq & A_i p' \leq \bar{\pi}_i t \quad i \in Q \\ 0 \leq (A_{i \wedge j} - \underline{\pi}_{i|j} A_j) p' & (i, j) \in C \\ (A_{i \wedge j} - \bar{\pi}_{i|j} A_j) p' \leq 0 & (i, j) \in C \\ & p', t \geq 0, \end{array} \right.$$

where  $p' = p \cdot t$ , and with the same optimal value; the corresponding solution is obtained by dividing the optimal solution  $p'^*$  of  $(PC')$  by  $t^*$ . This transformation consist in multiplying all variables by a value  $t \geq 1$  chosen in such a way that the denominator of the objective is equal to 1, and hence disappears.

A second difficulty is that the constraints bounding conditional probabilities have parameter  $\underline{\pi}_{i|j}$  or  $\bar{\pi}_{i|j}$  in their left-hand sides (i.e., multiplying variables). This time, by

introducing new parameters and elementary transformations all parameters can be transferred to the right-hand sides:

$$(PC'') \quad \left\{ \begin{array}{l} \min / \max z = A_{s \wedge \ell} p' \\ \text{subject to:} \\ A_{\ell} p' = 1 \\ \mathbb{1} p' = t \\ \underline{\pi}_i t \leq A_i p' \leq \bar{\pi}_i t \quad i \in Q \\ \underline{\pi}_{i|j} \pi'_j \leq A_{i \wedge j} p' \leq \bar{\pi}_{i|j} \pi'_j \quad (i, j) \in C \\ A_j p' = \pi'_j \quad j : (i, j) \in C \\ p', t \geq 0. \end{array} \right.$$

While  $(PC'')$  has products of parameters in the right-hand sides, instead of single parameters as in the unconditional case, Theorems 1 and 2 still apply. This is the basis of the algorithm described in the next section.

Note that the order in which these two transformation are done can be reversed. This gives the following alternate form:

$$(PC''') \quad \left\{ \begin{array}{l} \min / \max z = A_{s \wedge \ell} p' \\ \text{subject to:} \\ A_{\ell} p' = 1 \\ \mathbb{1} p' = t \\ \underline{\pi}_i t \leq A_i p' \leq \bar{\pi}_i t \quad i \in Q \\ \underline{\pi}_{i|j} \pi'_j t \leq A_{i \wedge j} p' \leq \bar{\pi}_{i|j} \pi'_j t \quad (i, j) \in C \\ A_j p' = \pi'_j t \quad j : (i, j) \in C \\ p', t \geq 0. \end{array} \right.$$

### 3 Algorithm

Problem  $(PC)$  can be solved analytically by the following algorithm, called VRE (vertex and ray enumeration):

- (a) Introducing new parameters  $\pi'_j$  and  $t$ , express  $(PC)$  in the form  $(PC'')$ .
- (b) Write the dual  $(DC'')$  of  $(PC'')$ .
- (c) Enumerate all extreme points and extreme rays of the polyhedron defined by  $(DC'')$ .
- (d) Introducing values of these extreme points and extreme rays in the objective function of  $(DC'')$ , obtain best possible parametric expressions of bounds on the objective function value and consistency conditions.
- (e) Eliminate parameters  $\pi'_j$  and  $t$  from these bounds and conditions.

A variant of this algorithm consists in using  $(PC''')$  and its dual  $(DC''')$  instead of  $(PC'')$  and  $(DC'')$ . Correctness of algorithm VRE follows from the reformulations of the previous section and the duality theorem of linear programming as expressed in Theorems 1 and 2.

Several tools are available for implementing VRE, e.g., the vertex and extreme rays enumeration program of Fukuda and Prodon [16], already mentioned, for step (c) and the algebraic manipulation system REDUCE [22] for parameter elimination in step (e).

**Example 1** The following problem corresponds to rule (v) in Frisch and Haddawy [19]:

$$\pi(x_2) \in [\underline{\pi}_1, \bar{\pi}_2]; \quad \pi(x_1 \wedge x_2) \in [\underline{\pi}_2, \bar{\pi}_2]; \quad \pi(x_1|x_2) = ?$$

Setting  $p_1 = \pi(x_1x_2)$ ,  $p_2 = \pi(x_1\bar{x}_2)$ ,  $p_3 = (\bar{x}_1x_2)$  and  $p_4 = (\bar{x}_1\bar{x}_2)$ , problem  $(PC''')$ , in step (a) is:

$$\begin{aligned} & \min / \max \quad z = p'_1 \\ & \text{subject to:} \\ & \quad p'_1 \quad + \quad p'_3 \quad = 1 \\ & \quad p'_1 + p'_2 + \quad p'_3 \quad + p'_4 = t \\ & \quad p'_1 \quad + \quad p'_3 \quad \geq \underline{\pi}_1 t \\ & -p'_1 \quad - \quad p'_3 \quad \geq -\bar{\pi}_1 t \\ & \quad p'_1 \quad \geq \underline{\pi}_2 t \\ & -p'_1 \quad \geq -\bar{\pi}_2 t \\ & \quad p'_i, t \geq 0 \quad i = 1, 2, 3, 4. \end{aligned}$$

The dual  $(DC''')$  of this problem (step (b)) is:

$$\begin{aligned} & \max / \min \quad u_1 + tu_2 + \underline{\pi}_1 tu_3 - \bar{\pi}_1 tu_4 + \underline{\pi}_2 tu_5 - \bar{\pi}_2 tu_6 \\ & \text{subject to:} \\ & \quad u_1 + u_2 + \quad u_3 \quad - \quad u_4 \quad + \quad u_5 \quad - \quad u_6 \quad \leq / \geq 1 \\ & \quad \quad \quad u_2 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \leq / \geq 0 \\ & \quad u_1 + u_2 + \quad u_3 \quad - \quad u_4 \quad + \quad \quad \quad \quad \quad \quad \quad \quad \leq / \geq 1 \\ & \quad \quad \quad u_2 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \leq / \geq 0 \\ & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad u_3, u_4, u_5, u_6 \geq / \leq 0 \\ & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad u_1, u_2 \text{ unrestricted,} \end{aligned}$$

where the fourth constraint, which is equivalent to the second one, may be omitted.

For maximization the extreme points of the polyhedron defined by  $(DC''')$  are (step (c)):  $(0, 0, 0, 0, 1, 0)$  which corresponds to  $z \geq \underline{\pi}_2 t$  and  $(0, 0, 0, 0, 0, 0)$  which corresponds to  $z \geq 0$ , and for minimization:  $(0, 0, 0, 0, 0, -1)$  which corresponds to  $z \leq \bar{\pi}_2 t$ ,  $(1, 0, 0, 0, 0, 0)$  which corresponds to  $z \leq 1$ .

The extreme rays are (step (d)), for maximization:  $(0, 0, 0, 0, 0, 1)$  which corresponds to  $-\bar{\pi}_2 t \leq 0$ ,  $(-1, 0, 0, 0, 1, 0)$  which corresponds to  $-1 + \underline{\pi}_2 t \leq 0$ ,  $(0, 0, 0, 0, 1, 1)$  which

corresponds to  $\underline{\pi}_2 t - \bar{\pi}_2 t \leq 0$ ,  $(1, -1, 0, 0, 0, 0)$  which corresponds to  $1 - t \leq 0$ ,  $(-1, 0, 1, 0, 0, 0)$  which corresponds to  $-1 + \underline{\pi}_1 t \leq 0$ ,  $(1, 0, 0, 1, 0, 0)$  which corresponds to  $1 - \bar{\pi}_1 t \leq 0$  and for minimization:  $(0, 0, 0, 0, 0, -1)$  which corresponds to  $-\bar{\pi}_2 t \geq 0$ ,  $(1, 0, 0, 0, -1, 0)$  which corresponds to  $-1 + \underline{\pi}_2 t \geq 0$ ,  $(0, 0, 0, 0, -1, -1)$  which corresponds to  $-\underline{\pi}_2 t + \bar{\pi}_2 t \geq 0$ ,  $(-1, 1, 0, 0, 0, 0)$  which corresponds to  $-1 + t \geq 0$ ,  $(1, 0, 1, 0, 0, 0)$  which corresponds to  $1 - \underline{\pi}_1 t \geq 0$ , and  $(-1, 0, 0, -1, 0, 0)$  which corresponds to  $-1 + \bar{\pi}_1 t \geq 0$ .

After elimination of the parameter  $t$ , they give (step (e)) the lower bound on  $\pi(x_1|x_2)$ :  $L = \frac{\underline{\pi}_2}{\bar{\pi}_1}$  the upper bound  $U = \min \left\{ 1, \frac{\bar{\pi}_2}{\underline{\pi}_1} \right\}$  and the consistency conditions:  $\underline{\pi}_i \leq \bar{\pi}_i$ ,  $\underline{\pi}_i \leq 1$ ,  $\bar{\pi}_i \geq 0$ , (for  $i = 1, 2$ ), and  $\underline{\pi}_2 \leq \bar{\pi}_1$ .

Other solution methods than VRE could also be applied. One alternative is to use REDUCE directly on  $(PC')$ , eliminating all variables of  $p'$  and  $t$ , as well as  $z$ , which is needed to have all consistency conditions. This may be time-consuming as the number of possible worlds may be much larger than the number of constraints.

Note that Fourier-Motzkin elimination could be used here. This is what Boole [2] did, in a systematic way for the unconditional case of PSAT and somewhat less systematically for particular conditional PSAT problems. Instead of Fourier-Motzkin elimination, the REDLOG package of REDUCE uses the real quantifier method (see e.g., Dolzmann and Sturm [23], Dolzmann, Sturm and Weispfenning [24], Hearn [22]) which is more efficient. In particular the number of expressions grows during the solution process at most as an exponential in problem size versus a double exponential for Fourier-Motzkin's method.

Hailperin [5] proposes to find bounds on the objective function by enumeration of bases of the dual  $(DC')$  of  $(PC')$  where parameter  $t$  is viewed as a variable. Slack variables are added and all square submatrices considered in turn. For each of them it is checked analytically (as the constraints of  $(DC')$  contains parameters) whether the corresponding solution is feasible or not. Again, the approach may be time-consuming as there are many more bases of  $(DC')$  than extreme points of the corresponding polyhedron. Also, this method does not provide consistency conditions. It could, at least in principle, be extended to do so by enumerating analytically all edges from the extreme points found and checking whether they are extreme rays or not. This could be done by determining the intersection points of these edges with the constraints not active at the corresponding extreme points and verifying that they are all on the infeasible side.

We compare these three approaches on the next example.

**Example 2** The following problem a variants of one of Boole (1854, p.321) and was analyzed by Hailperin [5] (Theorem 5.52 p. 244):  $\pi(x_1) = \pi_1$ ;  $\pi(x_2) = \pi_2$ ;  $\pi(x_3|x_1) = \pi_{3|1}$ ;  $\pi(x_3|x_2) = \pi_{3|2}$ ;  $\pi(x_3|x_1 \vee x_2) = ?$

The eight truth assignments over  $X$ , i.e.,  $(1, 1, 1)$ ,  $(1, 1, 0)$ ,  $(1, 0, 1)$ ,  $(1, 0, 0)$ ,  $(0, 1, 1)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$ ,  $(0, 0, 0)$  lead to eight possible words over  $S = \{x_1, x_2, x_3|x_1, x_3|x_2, x_3|(x_1 \vee x_2)\}$  to which are associated the probabilities  $p_1, p_2, \dots, p_8$ . Problem ( $PC'''$ ) is as follows:

$$\begin{aligned} & \min / \max \quad z' = p'_1 + p'_3 + p'_5 \\ \text{subject to:} & \\ & p'_1 + p'_2 + p'_3 + p'_4 + p'_5 + p'_6 = 1 \\ & p'_1 + p'_2 + p'_3 + p'_4 + p'_5 + p'_6 + p'_7 + p'_8 = t \\ & p'_1 + p'_2 + p'_3 + p'_4 = \pi_1 t \\ & p'_1 + p'_2 + p'_5 + p'_6 = \pi_2 t \\ & p'_1 + p'_3 = \pi_1 \pi_{3|1} t \\ & p'_1 + p'_5 = \pi_2 \pi_{3|2} t \\ & t, p'_j \geq 0 \quad j = 1, 2, \dots, 8, \end{aligned}$$

where  $p' = p \cdot t$ .

Its dual ( $DC'''$ ), there are 3 extreme points and 10 extreme rays for maximization, 3 extreme points and 10 extreme rays for minimization. After eliminations of  $t$  we obtain

$$L = \max \left\{ \frac{\pi_2 \pi_{3|2}}{\pi_2 + \pi_1(1 - \pi_{3|1})}, \frac{\pi_1 \pi_{3|1}}{\pi_1 + \pi_2(1 - \pi_{3|2})}, \pi_2 \pi_{3|2}, \pi_1 \pi_{3|1}, \frac{\pi_1 \pi_{3|1}}{\pi_1 + \pi_2}, \frac{\pi_2 \pi_{3|2}}{\pi_1 + \pi_2} \right\}$$

which, keeping only non redundant expressions, is equivalent to:

$$L = \max \left\{ \frac{\pi_2 \pi_{3|2}}{\pi_2 + \pi_1(1 - \pi_{3|1})}, \frac{\pi_1 \pi_{3|1}}{\pi_1 + \pi_2(1 - \pi_{3|2})}, \pi_2 \pi_{3|2}, \pi_1 \pi_{3|1} \right\}$$

and

$$U = \min \left\{ \frac{\pi_1 \pi_{3|1} + \pi_2 \pi_{3|2}}{\pi_1 \pi_{3|1} + \pi_2}, \frac{\pi_1 \pi_{3|1} + \pi_2 \pi_{3|2}}{\pi_2 \pi_{3|2} + \pi_1}, (1 - \pi_2) + \pi_2 \pi_{3|2}, \right. \\ \left. (1 - \pi_1) + \pi_1 \pi_{3|1}, \frac{\pi_1 \pi_{3|1} + \pi_2}{\pi_1 + \pi_2}, \frac{\pi_1 + \pi_2 \pi_{3|2}}{\pi_1 + \pi_2} \right\}$$

which, again keeping only non redundant expressions, is equivalent to

$U = \min \left\{ \frac{\pi_1 \pi_{3|1} + \pi_2 \pi_{3|2}}{\pi_1 \pi_{3|1} + \pi_2}, \frac{\pi_1 \pi_{3|1} + \pi_2 \pi_{3|2}}{\pi_2 \pi_{3|2} + \pi_1}, (1 - \pi_2) + \pi_2 \pi_{3|2}, (1 - \pi_1) + \pi_1 \pi_{3|1} \right\}$ . The consistency conditions are:  $0 < \pi_1, \pi_2, \pi_1 \pi_{3|1}, \pi_2 \pi_{3|2} < 1$  and  $0 \leq L \leq U \leq 1$ .

This solution coincides with that obtained by Hailperin [5]. The time required for extreme points and rays enumeration was for both maximization and minimization of 0.01 second only on a Dell PC computer (233MHz). Applying REDLOG [23] immediately to ( $PC'''$ ) took 23 seconds, to obtain the same result. In order to apply Hailperin's approach

$t$  is viewed as a variable in the  $(PC')$  problem above. The dual  $(DC')$  of this problem is

$$\begin{array}{r}
 \max / \min u_1 \\
 \text{subject to:} \\
 u_1 + u_2 + u_3 + u_4 + u_5 + u_6 \leq / \geq 1 \\
 u_1 + u_2 + u_3 + u_4 \leq / \geq 0 \\
 u_1 + u_2 + u_3 + u_5 \leq / \geq 1 \\
 u_1 + u_2 + u_3 \leq / \geq 0 \\
 u_1 + u_2 + u_4 + u_6 \leq / \geq 1 \\
 u_1 + u_2 + u_4 \leq / \geq 0 \\
 u_2 \leq / \geq 0 \\
 u_2 \leq / \geq 0 \\
 -u_2 - \pi_1 u_3 |_1 - \pi_2 \pi_3 |_2 u_4 - \pi_1 \pi_3 |_1 u_5 - \pi_2 \pi_3 |_2 u_6 \leq / \geq 0,
 \end{array}$$

where the variables  $u_i$  are unrestricted in sign. As the right-hand side of the primal is  $(1, 0, 0, 0, 0, 0)$ , duality theorem allows reduction of the number of bases to consider. There are 14 variables (i.e., 6 dual variables plus 8 slack variables), 8 constraints, and 3003 bases. For maximization, we can limit the search to bases which contain  $u_1$ ; there are 1716 such bases. For minimization, we can limit the search to bases which do not contain  $u_1$ , this leads to 1287 bases.

Checking the feasibility of a given basis corresponds to checking the feasibility of a linear system. For both the maximization and the minimization cases, only 6 out of the respectively 1716 and 1287 bases are feasible. Using REDUCE [22] to find them required 182 seconds. Again, the same result as above was obtained, but without the consistency conditions.

## 4 Verifying and Improving Inference Rules

Frisch and Haddawy [19] have gathered a series of inference rules for conditional PSAT, to be used in their anytime deduction algorithm. Each of these rules had been proved by hand. We consider here those conditional rules where not all  $S_i$  are subject to the same condition. All results of this type were deduced again, in an automated way, with algorithm VRE. A few of them are presented in Table 1. In one case, i.e., rule (ix), the bounds obtained differ from those from the literature (Amarger, Dubois and Prade [25]), which are

$$\underline{\pi}(x_3|x_1) = \underline{\pi}_2 \cdot \max \left\{ 0, 1 - \frac{1 - \underline{\pi}_3}{\underline{\pi}_1} \right\}$$

and

$$\overline{\pi}(x_3|x_1) = \min \left\{ 1, 1 - \underline{\pi}_2 + \frac{\underline{\pi}_2 \overline{\pi}_3}{\underline{\pi}_1}, \frac{\overline{\pi}_2 \overline{\pi}_3}{\underline{\pi}_1 \underline{\pi}_4}, \frac{\overline{\pi}_2 \overline{\pi}_3}{\underline{\pi}_1 \underline{\pi}_4} (1 - \underline{\pi}_4) + \overline{\pi}_2 \right\}$$

Table 1: Derived inference rules with conditioning sentences.

| Inference Rules | Probability Assigned | Consistency Conditions                  | $\underline{\pi}_?$<br>Maximum of:   | $\overline{\pi}_?$<br>Minimum of:  |
|-----------------|----------------------|---|--|--|
|                 | $x_2$                | $[\underline{\pi}_1, \overline{\pi}_1]$ | $\underline{\pi}_i \leq 1 \quad i = 1, 2$  | 1  |
| (v)             | $x_1 \wedge x_2$     | $[\underline{\pi}_2, \overline{\pi}_2]$ | $\underline{\pi}_i, \overline{\pi}_i > 0 \quad i = 1, 2$   | $\frac{\underline{\pi}_2}{\overline{\pi}_1}$   |
|                 | $x_1 x_2$            | $\pi_?$                                 | $\underline{\pi}_i \leq \overline{\pi}_i \quad i = 1, 2$   | $\frac{\overline{\pi}_2}{\underline{\pi}_1}$   |
|                 |                      |   | $\underline{\pi}_2 \leq \overline{\pi}_1$  |  |
|                 | $x_2 x_1$            | $[\underline{\pi}_1, \overline{\pi}_1]$ | $\underline{\pi}_1 \leq 1 \quad \overline{\pi}_1 \geq 0$   | 1  |
| (vi)            | $x_1 \wedge x_2$     | $[\underline{\pi}_2, \overline{\pi}_2]$ | $\underline{\pi}_1 > 0 \quad \overline{\pi}_2 > 0$<br>$\underline{\pi}_i \leq \overline{\pi}_i \quad i = 1, 2$ | $\frac{\underline{\pi}_2}{\overline{\pi}_1}$   |
|                 | $x_1$                | $\pi_?$                                 | $\underline{\pi}_2 \leq \overline{\pi}_1$  | $\underline{\pi}_1$  |
|                 | $x_2$                | $[\underline{\pi}_1, \overline{\pi}_1]$ | $\underline{\pi}_i \leq 1 \quad i = 1, 2$  |  |
| (vii)           | $x_1 x_2$            | $[\underline{\pi}_2, \overline{\pi}_2]$ | $\overline{\pi}_i \geq 0 \quad i = 1, 2$<br>$\underline{\pi}_i \leq \overline{\pi}_i \quad i = 1, 2$           | $\overline{\pi}_1 \cdot \overline{\pi}_2$  |
|                 | $x_1 \wedge x_2$     | $\pi_?$                                 |  |  |
|                 | $x_1 x_2$            | $[\underline{\pi}_1, \overline{\pi}_1]$ | $\underline{\pi}_i < 1 \quad i = 1, \dots, 4$  | 0  |
| (ix)            | $x_2 x_1$            | $[\underline{\pi}_2, \overline{\pi}_2]$ | $\overline{\pi}_i < 1 \quad i = 1, 3, 4$   | $\frac{\underline{\pi}_1 \cdot (1 - \underline{\pi}_2) + \underline{\pi}_2 \overline{\pi}_3}{\underline{\pi}_1}$   |
|                 |                      |   | $\underline{\pi}_2 \cdot (\underline{\pi}_1 + \underline{\pi}_3 - 1)$  |  |
|                 | $x_3 x_2$            | $[\underline{\pi}_3, \overline{\pi}_3]$ | $\overline{\pi}_2 \leq 1$  | $\frac{\overline{\pi}_2 \cdot (\underline{\pi}_1 \underline{\pi}_4 - \underline{\pi}_4 \overline{\pi}_3 + \overline{\pi}_3)}{\underline{\pi}_1 \underline{\pi}_4}$ |
|                 | $x_2 x_3$            | $[\underline{\pi}_4, \overline{\pi}_4]$ | $\underline{\pi}_i, \overline{\pi}_i \geq 0 \quad i = 1, \dots, 4$   | $\frac{\overline{\pi}_2 \overline{\pi}_3}{\underline{\pi}_1 \underline{\pi}_4}$  |
|                 | $x_3 x_1$            | $\pi_?$                                 | $\underline{\pi}_i \leq \overline{\pi}_i \quad i = 1, \dots, 4$  | $\frac{\overline{\pi}_2 \cdot (\underline{\pi}_1 \underline{\pi}_4 + \underline{\pi}_4 + 1)}{\underline{\pi}_1 \underline{\pi}_4}$                                 |
|                 |                      |   |  | $\frac{\overline{\pi}_3}{\underline{\pi}_1 \underline{\pi}_4 + \overline{\pi}_3 \cdot (1 - \underline{\pi}_4)}$  |

and the upper bound found by VRE is tighter. Indeed, consider the following assignments:  $\underline{\pi}_1 = 0.20, \overline{\pi}_1 = 0.20, \underline{\pi}_2 = 0.15, \overline{\pi}_2 = 0.20, \underline{\pi}_3 = 0.11, \overline{\pi}_3 = 0.11, \underline{\pi}_4 = 0.10$  and  $\overline{\pi}_4 = 0.10$ , we find with our upper bound:  $\overline{\pi}(x_3|x_1) = 0.924$  and with the previous one  $\overline{\pi}(x_3|x_1) = 0.932$ . The lower bounds are the same.

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## Annexe

### Solution of the Inference Rule (ix)

$-\pi_1 z + \pi_{3|1} z \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \pi_{3|2} z \geq 0$  and  $\pi_2 < 1$  and  $-\pi_{3|1} z + \bar{\pi}_1 z \geq 0$  and  $-\pi_{3|1} z + \bar{\pi}_{3|1} z \geq 0$  and  $-\pi_{3|1} z + z \geq 0$  and  $\pi_{3|1} < 1$  and  $-\pi_{3|2} z + \bar{\pi}_{3|2} z \geq 0$  and  $-\pi_{3|2} z + z \geq 0$  and  $-\pi_{3|2} z + \bar{\pi}_2 \geq 0$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$  and  $z > 0$

or

$-\pi_1 \pi_{3|2}^2 \bar{\pi}_2 + \pi_1 \pi_{3|2}^2 z + \pi_1 \pi_{3|2} \bar{\pi}_2 - \pi_1 \pi_{3|2} z + \pi_{3|1} \pi_{3|2}^2 \bar{\pi}_2 - 2\pi_{3|1} \pi_{3|2} \bar{\pi}_2 + \pi_{3|1} \bar{\pi}_2 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $-\pi_{3|1} \pi_{3|2}^2 \bar{\pi}_2 + 2\pi_{3|1} \pi_{3|2} \bar{\pi}_2 - \pi_{3|1} \bar{\pi}_2 + \pi_{3|2}^2 \bar{\pi}_1 \bar{\pi}_2 - \pi_{3|2}^2 \bar{\pi}_1 z - \pi_{3|2} \bar{\pi}_1 \bar{\pi}_2 + \pi_{3|2} \bar{\pi}_1 z \geq 0$  and  $-\pi_{3|1} \pi_{3|2} \bar{\pi}_2 + \pi_{3|1} \pi_{3|2} z + \pi_{3|1} \bar{\pi}_2 - \pi_{3|1} z + \pi_{3|2} \bar{\pi}_2 \bar{\pi}_{3|1} - \pi_{3|2} \bar{\pi}_{3|1} z - \bar{\pi}_2 \bar{\pi}_{3|1} + \bar{\pi}_{3|1} z \geq 0$  and  $\pi_{3|2} \bar{\pi}_2 - \pi_{3|2} z - \bar{\pi}_2 + z > 0$  and  $-\pi_{3|1} \pi_{3|2} \bar{\pi}_2 + \pi_{3|1} \pi_{3|2} z + \pi_{3|1} \bar{\pi}_2 - \pi_{3|1} z + \pi_{3|2} \bar{\pi}_2 - \pi_{3|2} z - \bar{\pi}_2 + z \geq 0$  and  $\pi_{3|1} < 1$  and  $-\pi_{3|2}^2 \bar{\pi}_2 + \pi_{3|2}^2 z + \pi_{3|2} \bar{\pi}_2 \bar{\pi}_{3|2} + \pi_{3|2} \bar{\pi}_2 - \pi_{3|2} \bar{\pi}_{3|2} z - \pi_{3|2} z - \bar{\pi}_2 \bar{\pi}_{3|2} + \bar{\pi}_{3|2} z \geq 0$  and  $\pi_{3|2} \bar{\pi}_2 - \pi_{3|2} z - \bar{\pi}_2 + z > 0$  and  $\pi_{3|2}^2 z - \pi_{3|2} \bar{\pi}_2 - \pi_{3|2} z + \bar{\pi}_2 \leq 0$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $-\bar{\pi}_2 + z \geq 0$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$

or

$-\pi_1 \pi_{3|2} z + \pi_{3|1} \bar{\pi}_2 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $-\pi_{3|1} \bar{\pi}_2 + \pi_{3|2} \bar{\pi}_1 z \geq 0$  and  $-\pi_{3|1} \bar{\pi}_2 + \pi_{3|2} z \geq 0$  and  $-\pi_{3|1} z + \bar{\pi}_{3|1} z \geq 0$  and  $\pi_{3|1} < 1$  and  $-\pi_{3|2} z + \bar{\pi}_2 + z - 1 \leq 0$  and  $-\pi_{3|2} z + \bar{\pi}_2 \geq 0$  and  $-\pi_{3|2} z + \bar{\pi}_{3|2} z \geq 0$  and  $-\pi_{3|2} z + z \geq 0$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z > 0$

or

$-\pi_1 \pi_2 \pi_{3|2}^2 + \pi_1 \pi_2 \pi_{3|2} + \pi_1 \pi_{3|2}^2 z - \pi_1 \pi_{3|2} z + \pi_2 \pi_{3|1} \pi_{3|2}^2 - 2\pi_2 \pi_{3|1} \pi_{3|2} + \pi_2 \pi_{3|1} \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2 \pi_{3|1} \pi_{3|2} + \pi_2 \pi_{3|1} + \pi_2 \pi_{3|2} \bar{\pi}_{3|1} - \pi_2 \bar{\pi}_{3|1} + \pi_{3|1} \pi_{3|2} z - \pi_{3|1} z - \pi_{3|2} \bar{\pi}_{3|1} z + \bar{\pi}_{3|1} z \geq 0$  and  $-\pi_2 \pi_{3|1} \pi_{3|2}^2 + 2\pi_2 \pi_{3|1} \pi_{3|2} - \pi_2 \pi_{3|1} + \pi_2 \pi_{3|2}^2 \bar{\pi}_1 - \pi_2 \pi_{3|2} \bar{\pi}_1 - \pi_{3|2}^2 \bar{\pi}_1 z + \pi_{3|2} \bar{\pi}_1 z \geq 0$  and  $\pi_2 \pi_{3|2} - \pi_2 - \pi_{3|2} z + z > 0$  and  $-\pi_2 \pi_{3|1} \pi_{3|2} + \pi_2 \pi_{3|1} + \pi_2 \pi_{3|2} -$

$\pi_2 + \pi_{3|1}\pi_{3|2}z - \pi_{3|1}z - \pi_{3|2}z + z \geq 0$  and  $-\pi_2\pi_{3|2}^2 + \pi_2\pi_{3|2}\bar{\pi}_{3|2} + \pi_2\pi_{3|2} - \pi_2\bar{\pi}_{3|2} + \pi_{3|2}^2z - \pi_{3|2}\bar{\pi}_{3|2}z - \pi_{3|2}z + \bar{\pi}_{3|2}z \geq 0$  and  $\pi_2\pi_{3|2} - \pi_2 - \pi_{3|2}z + z > 0$  and  $\pi_2\pi_{3|2} - \pi_2 - \pi_{3|2}^2z + \pi_{3|2}z \geq 0$  and  $-\pi_2 + z \geq 0$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$

**or**

$-\pi_1\pi_{3|2}z + \pi_2\pi_{3|1} \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2\pi_{3|1} + \pi_{3|2}\bar{\pi}_1z \geq 0$  and  $-\pi_2\pi_{3|1} + \pi_{3|2}z \geq 0$  and  $\pi_2 - \pi_{3|2}z + z - 1 \leq 0$  and  $-\pi_2 + \pi_{3|2}z \leq 0$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $-\pi_{3|1}z + \bar{\pi}_{3|1}z \geq 0$  and  $\pi_{3|1} < 1$  and  $-\pi_{3|2}z + \bar{\pi}_{3|2}z \geq 0$  and  $-\pi_{3|2}z + z \geq 0$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z > 0$

**or**

$-\pi_1\pi_{3|1}z + \pi_1\pi_{3|1} + \pi_1z - \pi_1 + \pi_{3|1}^2 - 2\pi_{3|1} + 1 \geq 0$  and  $\pi_1 < 1$  and  $\pi_2 < 1$  and  $\pi_{3|1}^2 - \pi_{3|1}\bar{\pi}_1z + \pi_{3|1}\bar{\pi}_1 - 2\pi_{3|1} + \bar{\pi}_1z - \bar{\pi}_1 + 1 \leq 0$  and  $\pi_{3|1}^2z - \pi_{3|1}^2 - \pi_{3|1}\bar{\pi}_{3|1}z + \pi_{3|1}\bar{\pi}_{3|1} - \pi_{3|1}z + \pi_{3|1} + \bar{\pi}_{3|1}z - \bar{\pi}_{3|1} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1}^2 - \pi_{3|1}z - \pi_{3|1} + z \leq 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}\bar{\pi}_{3|2}z + \pi_{3|1}\bar{\pi}_{3|2} - \pi_{3|2}z + \pi_{3|2} + \bar{\pi}_{3|2}z - \bar{\pi}_{3|2} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}z + \pi_{3|1} - \pi_{3|2}z + \pi_{3|2} + z - 1 \leq 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $-\bar{\pi}_2 + 1 \leq 0$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$  and  $z \geq 0$

**or**

$-\pi_1\pi_{3|1}z + \pi_1\pi_{3|1} + \pi_1z - \pi_1 + \pi_{3|1}^2\bar{\pi}_2 - 2\pi_{3|1}\bar{\pi}_2 + \bar{\pi}_2 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1}^2\bar{\pi}_2 - \pi_{3|1}\bar{\pi}_1z + \pi_{3|1}\bar{\pi}_1 - 2\pi_{3|1}\bar{\pi}_2 + \bar{\pi}_1z - \bar{\pi}_1 + \bar{\pi}_2 \leq 0$  and  $\pi_{3|1}^2\bar{\pi}_2 - 2\pi_{3|1}\bar{\pi}_2 - \pi_{3|1}z + \pi_{3|1} + \bar{\pi}_2 + z - 1 \leq 0$  and  $-\pi_{3|1}^2\bar{\pi}_2 + \pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 + 2\pi_{3|1}\pi_{3|2}\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} + \pi_{3|1}z - \pi_{3|1} - \pi_{3|2}\bar{\pi}_2 + \pi_{3|2} \leq 0$  and  $\pi_{3|1}^2z - \pi_{3|1}^2 - \pi_{3|1}\bar{\pi}_{3|1}z + \pi_{3|1}\bar{\pi}_{3|1} - \pi_{3|1}z + \pi_{3|1} + \bar{\pi}_{3|1}z - \bar{\pi}_{3|1} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}\bar{\pi}_{3|2}z + \pi_{3|1}\bar{\pi}_{3|2} - \pi_{3|2}z + \pi_{3|2} + \bar{\pi}_{3|2}z - \bar{\pi}_{3|2} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}z + \pi_{3|1} - \pi_{3|2}z + \pi_{3|2} + z - 1 \leq 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_2 + z - 1 \geq 0$  and  $\bar{\pi}_2 \leq 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$

**or**

$-\pi_1\pi_{3|1}z + \pi_1\pi_{3|1} + \pi_1z - \pi_1 + \pi_2\pi_{3|1}^2 - 2\pi_2\pi_{3|1} + \pi_2 \geq 0$  and  $\pi_1 < 1$  and  $\pi_2\pi_{3|1}^2 - 2\pi_2\pi_{3|1} + \pi_2 - \pi_{3|1}\bar{\pi}_1z + \pi_{3|1}\bar{\pi}_1 + \bar{\pi}_1z - \bar{\pi}_1 \leq 0$  and  $\pi_2\pi_{3|1}^2 - 2\pi_2\pi_{3|1} + \pi_2 - \pi_{3|1}z + \pi_{3|1} + z - 1 \leq 0$  and  $-\pi_2\pi_{3|1}^2\pi_{3|2} + 2\pi_2\pi_{3|1}\pi_{3|2} - \pi_2\pi_{3|2} + \pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 + 2\pi_{3|1}\pi_{3|2}\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} + \pi_{3|1}z - \pi_{3|1} - \pi_{3|2}\bar{\pi}_2 + \pi_{3|2} \leq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1}^2z - \pi_{3|1}^2 - \pi_{3|1}\bar{\pi}_{3|1}z + \pi_{3|1}\bar{\pi}_{3|1} - \pi_{3|1}z + \pi_{3|1} + \bar{\pi}_{3|1}z - \bar{\pi}_{3|1} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}\bar{\pi}_{3|2}z + \pi_{3|1}\bar{\pi}_{3|2} - \pi_{3|2}z + \pi_{3|2} + \bar{\pi}_{3|2}z - \bar{\pi}_{3|2} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}z + \pi_{3|1} - \pi_{3|2}z + \pi_{3|2} + z - 1 \leq 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_2 + z - 1 \geq 0$  and  $\bar{\pi}_2 \leq 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$

**or**

$\pi_1\pi_{3|1}\bar{\pi}_2 - \pi_1\pi_{3|1}z - \pi_1\bar{\pi}_2 + \pi_1z + \pi_{3|1}^2\bar{\pi}_2 - 2\pi_{3|1}\bar{\pi}_2 + \bar{\pi}_2 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1}^2\bar{\pi}_2 + \pi_{3|1}\bar{\pi}_1\bar{\pi}_2 - \pi_{3|1}\bar{\pi}_1z - 2\pi_{3|1}\bar{\pi}_2 - \bar{\pi}_1\bar{\pi}_2 + \bar{\pi}_1z + \bar{\pi}_2 \leq 0$  and  $-\pi_{3|1}^2\bar{\pi}_2 + \pi_{3|1}^2z + \pi_{3|1}\bar{\pi}_2\bar{\pi}_{3|1} + \pi_{3|1}\bar{\pi}_2 - \pi_{3|1}\bar{\pi}_{3|1}z - \pi_{3|1}z - \bar{\pi}_2\bar{\pi}_{3|1} + \bar{\pi}_{3|1}z \leq 0$  and  $\pi_{3|1}\bar{\pi}_2 - \pi_{3|1}z - \bar{\pi}_2 + z < 0$  and  $\pi_{3|1}^2\bar{\pi}_2 - \pi_{3|1}\bar{\pi}_2 - \pi_{3|1}z + z \leq 0$  and  $-\pi_{3|1}\pi_{3|2}\bar{\pi}_2 + \pi_{3|1}\pi_{3|2}z + \pi_{3|1}\bar{\pi}_2\bar{\pi}_{3|2} - \pi_{3|1}\bar{\pi}_2z - \pi_2\bar{\pi}_{3|2} + \bar{\pi}_{3|2}z \leq 0$  and  $\pi_{3|1}\bar{\pi}_2 - \pi_{3|1}z - \bar{\pi}_2 + z < 0$  and  $-\pi_{3|1}\pi_{3|2}\bar{\pi}_2 + \pi_{3|1}\pi_{3|2}z + \pi_{3|1}\bar{\pi}_2 - \pi_{3|1}z + \pi_{3|2}\bar{\pi}_2 - \pi_{3|2}z - \bar{\pi}_2 + z \leq 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $-\bar{\pi}_2 + z \leq 0$  and  $\bar{\pi}_2 \leq 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \geq 0$

**or**

$-\pi_1 + \bar{\pi}_1 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $-\pi_{3|1}\bar{\pi}_2 + \bar{\pi}_1z - \bar{\pi}_1 + \bar{\pi}_2 \leq 0$  and  $-\pi_{3|1}\bar{\pi}_2 - \pi_{3|2}\bar{\pi}_1\bar{\pi}_2 + \pi_{3|2}\bar{\pi}_1z + \pi_{3|2}\bar{\pi}_2 \leq 0$  and  $-\pi_{3|1}\bar{\pi}_2 - \bar{\pi}_1\bar{\pi}_2 + \bar{\pi}_1z + \bar{\pi}_2 \geq 0$  and  $-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $-\pi_{3|1} - \bar{\pi}_1 + 1 \leq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$-\pi_1z + z \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2\pi_{3|1} + z \geq 0$  and  $\pi_2 < 1$  and  $-\bar{\pi}_1z + z \leq 0$  and  $-\pi_{3|1}\bar{\pi}_2 + z \leq 0$  and  $-\pi_{3|1}z + \bar{\pi}_{3|1}z \geq 0$  and  $-\pi_{3|1} + z \leq 0$  and  $-\pi_{3|1}z + z \geq 0$  and  $\pi_{3|1} < 1$  and  $-\pi_{3|2}z + \bar{\pi}_{3|2}z \geq 0$  and  $z > 0$  and  $-\pi_{3|2}z + z \geq 0$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \geq 0$

**or**

$\pi_1\pi_{3|1}z - \pi_1\pi_{3|1} - \pi_1z + \pi_1 - \pi_{3|1}z + \pi_{3|1} + z - 1 \leq 0$  and  $\pi_1 < 1$  and  $\pi_2\pi_{3|1}^2 - 2\pi_2\pi_{3|1} + \pi_2 - \pi_{3|1}z + \pi_{3|1} + z - 1 \leq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1}\bar{\pi}_1z - \pi_{3|1}\bar{\pi}_1 - \pi_{3|1}z + \pi_{3|1} - \bar{\pi}_1z + \bar{\pi}_1 + z - 1 \geq 0$  and  $\pi_{3|1}^2\bar{\pi}_2 - 2\pi_{3|1}\bar{\pi}_2 - \pi_{3|1}z + \pi_{3|1} + \bar{\pi}_2 + z - 1 \geq 0$  and  $\pi_{3|1}^2z - \pi_{3|1}^2 - \pi_{3|1}\bar{\pi}_{3|1}z + \pi_{3|1}\bar{\pi}_{3|1} - \pi_{3|1}z + \pi_{3|1} + \bar{\pi}_{3|1}z - \bar{\pi}_{3|1} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$

$z - 1 < 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 - 2\pi_{3|1}\pi_{3|2}z + \pi_{3|1}z - \pi_{3|1} + \pi_{3|2}z \leq 0$  and  $\pi_{3|1}^2 - \pi_{3|1}z - \pi_{3|1} + z \geq 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}\pi_{3|2}z + \pi_{3|1}\pi_{3|2} - \pi_{3|2}z + \pi_{3|2} + \pi_{3|2}z - \pi_{3|2} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 \leq 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$

or

$\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $-\pi_{3|1}\bar{\pi}_2 + \bar{\pi}_2 + z - 1 \leq 0$  and  $-\pi_{3|1}\bar{\pi}_2 + \pi_{3|2}z \leq 0$  and  $-\pi_{3|1}\bar{\pi}_2 + z \geq 0$  and  $-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $-\bar{\pi}_1 + 1 \leq 0$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

or

$-\pi_1z + z \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2\pi_{3|1} + \pi_{3|2}z \geq 0$  and  $\pi_2 < 1$  and  $-\bar{\pi}_1z + z \leq 0$  and  $-\pi_{3|1}\bar{\pi}_2 + \pi_{3|2}z \leq 0$  and  $-\pi_{3|1}\pi_{3|2}z + \pi_{3|1}z - \pi_{3|1} + \pi_{3|2}z \leq 0$  and  $-\pi_{3|1}z + \bar{\pi}_{3|1}z \geq 0$  and  $-\pi_{3|1}z + z \geq 0$  and  $\pi_{3|1} < 1$  and  $-\pi_{3|2}z + \bar{\pi}_{3|2}z \geq 0$  and  $-\pi_{3|2}z + z \geq 0$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z > 0$

or

$-\pi_1\pi_{3|1}\pi_{3|2}z + \pi_1\pi_{3|1}\pi_{3|2} + \pi_1\pi_{3|2}z - \pi_1\pi_{3|2} + \pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} + \pi_{3|1}z - \pi_{3|1} + \pi_{3|2} \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2\pi_{3|1}\pi_{3|2} + 2\pi_2\pi_{3|1}\pi_{3|2} - \pi_2\pi_{3|2} + \pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} + \pi_{3|1}z - \pi_{3|1} + \pi_{3|2} \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 - \pi_{3|1}\pi_{3|2}\bar{\pi}_1z + \pi_{3|1}\pi_{3|2}\bar{\pi}_1 - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} + \pi_{3|1}z - \pi_{3|1} + \pi_{3|2}\bar{\pi}_1z - \pi_{3|2}\bar{\pi}_1 + \pi_{3|2} \leq 0$  and  $-\pi_{3|1}^2\pi_{3|2}\bar{\pi}_2 + \pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 + 2\pi_{3|1}\pi_{3|2}\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} + \pi_{3|1}z - \pi_{3|1} - \pi_{3|2}\bar{\pi}_2 + \pi_{3|2} \leq 0$  and  $\pi_{3|1}^2z - \pi_{3|1}^2 - \pi_{3|1}\bar{\pi}_{3|1}z + \pi_{3|1}\bar{\pi}_{3|1} - \pi_{3|1}z + \pi_{3|1} + \bar{\pi}_{3|1}z - \bar{\pi}_{3|1} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 - 2\pi_{3|1}\pi_{3|2}z + \pi_{3|1}z - \pi_{3|1} + \pi_{3|2}z \leq 0$  and  $2\pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2\pi_{3|2} - \pi_{3|1}^2z + \pi_{3|1}^2 - 3\pi_{3|1}\pi_{3|2}z + \pi_{3|1}\pi_{3|2} + \pi_{3|1}z - \pi_{3|1} + \pi_{3|2}z \geq 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}\bar{\pi}_{3|2}z + \pi_{3|1}\bar{\pi}_{3|2} - \pi_{3|2}z + \pi_{3|2} + \bar{\pi}_{3|2}z - \bar{\pi}_{3|2} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}z + \pi_{3|1} - \pi_{3|2}z + \pi_{3|2} + z - 1 \leq 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$

or

$\pi_1\pi_{3|1}\pi_{3|2}\bar{\pi}_2 - \pi_1\pi_{3|1}\pi_{3|2}z - \pi_1\pi_{3|2}\bar{\pi}_2 + \pi_1\pi_{3|2}z + \pi_{3|1}^2\bar{\pi}_2 - 2\pi_{3|1}\pi_{3|2}\bar{\pi}_2 + \pi_{3|2}^2\bar{\pi}_2 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1}^2\bar{\pi}_2 + \pi_{3|1}\pi_{3|2}\bar{\pi}_1\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}\bar{\pi}_1z - 2\pi_{3|1}\pi_{3|2}\bar{\pi}_2 - \pi_{3|2}^2\bar{\pi}_1\bar{\pi}_2 + \pi_{3|2}^2\bar{\pi}_1z + \pi_{3|2}^2\bar{\pi}_2 \leq 0$  and  $-\pi_{3|1}^2\bar{\pi}_2 + \pi_{3|1}^2z + \pi_{3|1}\pi_{3|2}\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}z + \pi_{3|1}\bar{\pi}_2\bar{\pi}_{3|1} - \pi_{3|1}\bar{\pi}_{3|1}z - \pi_{3|2}\bar{\pi}_2\bar{\pi}_{3|1} + \pi_{3|2}\bar{\pi}_{3|1}z \leq 0$  and  $\pi_{3|1}\bar{\pi}_2 - \pi_{3|1}z - \pi_{3|2}\bar{\pi}_2 + \pi_{3|2}z < 0$  and  $-\pi_{3|1}^2\bar{\pi}_2 + \pi_{3|1}^2z + \pi_{3|1}\pi_{3|2}\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}z + \pi_{3|1}\bar{\pi}_2 - \pi_{3|1}z - \pi_{3|2}\bar{\pi}_2 + \pi_{3|2}z \leq 0$  and  $\pi_{3|1}^2\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}z + \pi_{3|2}^2z \leq 0$  and  $-\pi_{3|1}^2\pi_{3|2}\bar{\pi}_2 + \pi_{3|1}^2\pi_{3|2}z + \pi_{3|1}^2\bar{\pi}_2 + \pi_{3|1}\pi_{3|2}^2\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}^2z - \pi_{3|1}\pi_{3|2}\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}z + \pi_{3|2}^2z \geq 0$  and  $-\pi_{3|1}\pi_{3|2}\bar{\pi}_2 + \pi_{3|1}\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 + \pi_{3|1}\pi_{3|2}^2\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}^2z + \pi_{3|1}\pi_{3|2}\bar{\pi}_2 + \pi_{3|1}\pi_{3|2}z - 2\pi_{3|1}\pi_{3|2} - \pi_{3|2}^2\bar{\pi}_2 + \pi_{3|2}^2 \geq 0$  and  $-\pi_{3|1}\pi_{3|2}\bar{\pi}_2 + \pi_{3|1}\pi_{3|2}z + \pi_{3|1}\bar{\pi}_2\bar{\pi}_{3|2} - \pi_{3|1}\bar{\pi}_{3|2}z + \pi_{3|2}^2\bar{\pi}_2 - \pi_{3|2}^2z - \pi_{3|2}\bar{\pi}_2\bar{\pi}_{3|2} + \pi_{3|2}\bar{\pi}_{3|2}z \leq 0$  and  $\pi_{3|1}\bar{\pi}_2 - \pi_{3|1}z - \pi_{3|2}\bar{\pi}_2 + \pi_{3|2}z < 0$  and  $-\pi_{3|1}\pi_{3|2}\bar{\pi}_2 + \pi_{3|1}\pi_{3|2}z + \pi_{3|1}\bar{\pi}_2 - \pi_{3|1}z + \pi_{3|2}^2\bar{\pi}_2 - \pi_{3|2}^2z - \pi_{3|2}\bar{\pi}_2 + \pi_{3|2}z \leq 0$  and  $-\pi_{3|1} + \pi_{3|2} < 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

or

$\pi_1\pi_2\pi_{3|1} - \pi_1\pi_2 - \pi_1\pi_{3|1}z + \pi_1z + \pi_2\pi_{3|1}^2 - 2\pi_2\pi_{3|1} + \pi_2 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2\pi_{3|1}^2 + \pi_2\pi_{3|1}\bar{\pi}_{3|1} + \pi_2\pi_{3|1} - \pi_2\bar{\pi}_{3|1} + \pi_{3|1}^2z - \pi_{3|1}\bar{\pi}_{3|1}z - \pi_{3|1}z + \bar{\pi}_{3|1}z \leq 0$  and  $\pi_2\pi_{3|1}^2 + \pi_2\pi_{3|1}\bar{\pi}_1 - 2\pi_2\pi_{3|1} - \pi_2\bar{\pi}_1 + \pi_2 - \pi_{3|1}\bar{\pi}_1z + \bar{\pi}_1z \leq 0$  and  $\pi_2\pi_{3|1} - \pi_2 - \pi_{3|1}z + z < 0$  and  $\pi_2\pi_{3|1}^2 - \pi_2\pi_{3|1} - \pi_{3|1}z + z \leq 0$  and  $-\pi_2\pi_{3|1}\pi_{3|2} + \pi_2\pi_{3|1}\bar{\pi}_{3|2} + \pi_2\pi_{3|2} - \pi_2\bar{\pi}_{3|2} + \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\bar{\pi}_{3|2}z - \pi_{3|2}z + \bar{\pi}_{3|2}z \leq 0$  and  $\pi_2\pi_{3|1} - \pi_2 - \pi_{3|1}z + z < 0$  and  $-\pi_2\pi_{3|1}\pi_{3|2} + \pi_2\pi_{3|1} + \pi_2\pi_{3|2} - \pi_2 + \pi_{3|1}\pi_{3|2}z - \pi_{3|1}z - \pi_{3|2}z + z \leq 0$  and  $-\pi_2 + z \leq 0$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \geq 0$

or

$\pi_1 < 1$  and  $-\pi_2\pi_{3|1} + \pi_2 + z - 1 \leq 0$  and  $-\pi_2\pi_{3|1} + \pi_{3|2}z \leq 0$  and  $-\pi_2\pi_{3|1} + z \geq 0$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $-\bar{\pi}_1 + 1 \leq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

or

$-\pi_1 + \bar{\pi}_1 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2\pi_{3|1} + \pi_2 + \bar{\pi}_1z - \bar{\pi}_1 \leq 0$  and  $-\pi_2\pi_{3|1} - \pi_2\pi_{3|2}\bar{\pi}_1 + \pi_2\pi_{3|2} + \pi_{3|2}\bar{\pi}_1z \leq 0$  and  $-\pi_2\pi_{3|1} - \pi_2\bar{\pi}_1 + \pi_2 + \bar{\pi}_1z \geq 0$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $-\pi_{3|1} - \bar{\pi}_1 + 1 \leq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

or

$$\begin{aligned} & \pi_1 \pi_2 \pi_3 |1 \pi_3 |2 - \pi_1 \pi_2 \pi_3 |2^2 - \pi_1 \pi_3 |1 \pi_3 |2 z + \pi_1 \pi_3 |2^2 z + \pi_2 \pi_3 |1^2 - 2\pi_2 \pi_3 |1 \pi_3 |2 + \pi_2 \pi_3 |2^2 \geq 0 \text{ and } \pi_1 < 1 \text{ and } -\pi_2 \pi_3 |1 + \\ & \pi_2 \pi_3 |1 \pi_3 |2 + \pi_2 \pi_3 |1 \bar{\pi}_3 |1 - \pi_2 \pi_3 |2 \bar{\pi}_3 |1 + \pi_3 |1^2 z - \pi_3 |1 \pi_3 |2 z - \pi_3 |1 \bar{\pi}_3 |1 z + \pi_3 |2 \bar{\pi}_3 |1 z \leq 0 \text{ and } \pi_2 \pi_3 |1 + \pi_2 \pi_3 |1 \pi_3 |2 \bar{\pi}_1 - \\ & 2\pi_2 \pi_3 |1 \pi_3 |2 - \pi_2 \pi_3 |2 \bar{\pi}_1 + \pi_2 \pi_3 |2^2 - \pi_3 |1 \pi_3 |2 \bar{\pi}_1 z + \pi_3 |2 \bar{\pi}_1 z \leq 0 \text{ and } \pi_2 \pi_3 |1 - \pi_2 \pi_3 |2 - \pi_3 |1 z + \pi_3 |2 z < 0 \text{ and } \pi_2 \pi_3 |1 - \\ & \pi_2 \pi_3 |1 \pi_3 |2 - \pi_3 |1 \pi_3 |2 z + \pi_3 |2^2 z \leq 0 \text{ and } -\pi_2 \pi_3 |1 + \pi_2 \pi_3 |1 \pi_3 |2 + \pi_2 \pi_3 |1 - \pi_2 \pi_3 |2 + \pi_3 |1^2 z - \pi_3 |1 \pi_3 |2 z - \pi_3 |1 z + \pi_3 |2 z \leq \\ & 0 \text{ and } -\pi_2 \pi_3 |1 \pi_3 |2 + \pi_2 \pi_3 |1 + \pi_2 \pi_3 |1 \pi_3 |2 - \pi_2 \pi_3 |1 \pi_3 |2 + \pi_3 |1 \pi_3 |2 z - \pi_3 |1 \pi_3 |2 z - \pi_3 |1 \pi_3 |2 z + \pi_3 |2 z \geq 0 \text{ and } - \\ & \pi_2 \pi_3 |1 \pi_3 |2 + \pi_2 \pi_3 |1 \pi_3 |2 + \pi_2 \pi_3 |1 \pi_3 |2 - \pi_2 \pi_3 |2 + \pi_3 |1 \pi_3 |2 z - \pi_3 |1 z + \pi_3 |1 - \pi_3 |1 \pi_3 |2 z + \pi_3 |1 \pi_3 |2 z - 2\pi_3 |1 \pi_3 |2 + \pi_3 |2^2 \geq \\ & 0 \text{ and } -\pi_2 \pi_3 |1 \pi_3 |2 + \pi_2 \pi_3 |1 \bar{\pi}_3 |2 + \pi_2 \pi_3 |2^2 - \pi_2 \pi_3 |2 \bar{\pi}_3 |2 + \pi_3 |1 \pi_3 |2 z - \pi_3 |1 \bar{\pi}_3 |2 z - \pi_3 |2 z + \pi_3 |2 \bar{\pi}_3 |2 z \leq 0 \text{ and } \pi_2 \pi_3 |1 - \\ & \pi_2 \pi_3 |2 - \pi_3 |1 z + \pi_3 |2 z < 0 \text{ and } -\pi_2 \pi_3 |1 \pi_3 |2 + \pi_2 \pi_3 |1 + \pi_2 \pi_3 |2^2 - \pi_2 \pi_3 |2 + \pi_3 |1 \pi_3 |2 z - \pi_3 |1 z - \pi_3 |2 z + \pi_3 |2 z \leq 0 \text{ and } - \\ & \pi_2 + \bar{\pi}_2 \geq 0 \text{ and } \pi_2 < 1 \text{ and } -\pi_3 |1 + \pi_3 |2 < 0 \text{ and } \pi_3 |1 < 1 \text{ and } \pi_3 |2 < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_3 |1 < 1 \text{ and } \bar{\pi}_3 |2 < 1 \end{aligned}$$

or

$$-\pi_1 + \bar{\pi}_1 \geq 0 \text{ and } \pi_1 < 1 \text{ and } \pi_2 < 1 \text{ and } -\pi_3 |1 + \bar{\pi}_1 z - \pi_3 |1 + \bar{\pi}_3 |1 \geq 0 \text{ and } -\pi_3 |1 + \bar{\pi}_1 z \leq 0 \text{ and } -\pi_3 |2 + \bar{\pi}_3 |2 \geq 0 \text{ and } \pi_3 |1 < 1 \text{ and } \pi_3 |2 < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } -\bar{\pi}_2 + 1 \leq 0 \text{ and } \bar{\pi}_3 |1 < 1 \text{ and } \bar{\pi}_3 |2 < 1 \text{ and } z \leq 1 \text{ and } z \geq 0$$

or

$$-\pi_1 + \bar{\pi}_1 \geq 0 \text{ and } \pi_1 < 1 \text{ and } -\pi_2 + \bar{\pi}_2 \geq 0 \text{ and } \pi_2 < 1 \text{ and } -\pi_3 |1 \bar{\pi}_2 + \bar{\pi}_1 z - \bar{\pi}_1 + \bar{\pi}_2 \geq 0 \text{ and } -\pi_3 |1 + \bar{\pi}_3 |1 \geq 0 \text{ and } -\pi_3 |1 \bar{\pi}_2 + \bar{\pi}_1 \bar{\pi}_2 + \bar{\pi}_1 z - \bar{\pi}_1 \leq 0 \text{ and } \pi_3 |1 \pi_3 |2 \bar{\pi}_2 - \pi_3 |1 \bar{\pi}_2 - \pi_3 |2 \bar{\pi}_1 \bar{\pi}_2 + \pi_3 |2 \bar{\pi}_1 \leq 0 \text{ and } -\pi_3 |2 + \bar{\pi}_3 |2 \geq 0 \text{ and } \pi_3 |1 < 1 \text{ and } \pi_3 |2 < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_2 + z - 1 \geq 0 \text{ and } \bar{\pi}_2 \leq 1 \text{ and } \bar{\pi}_3 |1 < 1 \text{ and } \bar{\pi}_3 |2 < 1 \text{ and } z \leq 1$$

or

$$\begin{aligned} & \pi_1 \pi_3 |1 z - \pi_1 \pi_3 |1 - \pi_1 z + \pi_1 - \pi_3 |1 \bar{\pi}_1 z + \pi_3 |1 \bar{\pi}_1 + \bar{\pi}_1 z - \bar{\pi}_1 \leq 0 \text{ and } \pi_1 < 1 \text{ and } \pi_2 \pi_3 |1^2 - 2\pi_2 \pi_3 |1 + \pi_2 - \pi_3 |1 \bar{\pi}_1 z + \\ & \pi_3 |1 \bar{\pi}_1 + \bar{\pi}_1 z - \bar{\pi}_1 \leq 0 \text{ and } \pi_2 < 1 \text{ and } \pi_3 |1 \bar{\pi}_1 z - \pi_3 |1 \bar{\pi}_1 - \pi_3 |1 z + \pi_3 |1 - \bar{\pi}_1 z + \bar{\pi}_1 + z - 1 \leq 0 \text{ and } \pi_3 |1^2 z - \pi_3 |1^2 + \\ & \pi_3 |1 \bar{\pi}_1 z - \pi_3 |1 \bar{\pi}_1 - 2\pi_3 |1 z + 2\pi_3 |1 - \bar{\pi}_1 z + \bar{\pi}_1 + z - 1 \geq 0 \text{ and } \pi_3 |1^2 \bar{\pi}_2 - \pi_3 |1 \bar{\pi}_1 z + \pi_3 |1 \bar{\pi}_1 - 2\pi_3 |1 \bar{\pi}_2 + \bar{\pi}_1 z - \bar{\pi}_1 + \bar{\pi}_2 \geq \\ & 0 \text{ and } \pi_3 |1 \pi_3 |2 z - \pi_3 |1^2 z + \pi_3 |1^2 - \pi_3 |1 \pi_3 |2 \bar{\pi}_1 z + \pi_3 |1 \pi_3 |2 \bar{\pi}_1 - \pi_3 |1 \pi_3 |2 z - \pi_3 |1 \pi_3 |2 + \pi_3 |1 z - \pi_3 |1 + \pi_3 |2 \bar{\pi}_1 z - \pi_3 |2 \bar{\pi}_1 + \pi_3 |2 \leq \\ & 0 \text{ and } \pi_3 |1 - \pi_3 |1 \bar{\pi}_1 z + \pi_3 |1 \bar{\pi}_1 - 2\pi_3 |1 + \bar{\pi}_1 z \pi_3 |1^2 z - \pi_3 |1^2 - \pi_3 |1 \bar{\pi}_3 |1 z + \pi_3 |1 \bar{\pi}_3 |1 - \pi_3 |1 z + \pi_3 |1 + \bar{\pi}_3 |1 z - \bar{\pi}_3 |1 \leq \\ & 0 \text{ and } -\pi_3 |1 z + \pi_3 |1 + z - 1 < 0 \text{ and } \pi_3 |1 \pi_3 |2 z - \pi_3 |1 \pi_3 |2 - \pi_3 |1 \bar{\pi}_3 |2 z + \pi_3 |1 \bar{\pi}_3 |2 - \pi_3 |2 z + \pi_3 |2 + \bar{\pi}_3 |2 z - \bar{\pi}_3 |2 \leq \\ & 0 \text{ and } -\pi_3 |1 z + \pi_3 |1 + z - 1 < 0 \text{ and } \pi_3 |1 < 1 \text{ and } \pi_3 |2 < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_3 |1 < 1 \text{ and } \bar{\pi}_3 |2 < 1 \text{ and } z \leq 1 \end{aligned}$$

or

$$-\pi_1 + \bar{\pi}_1 \geq 0 \text{ and } \pi_1 < 1 \text{ and } -\pi_2 \pi_3 |1 + \pi_2 + \bar{\pi}_1 z - \bar{\pi}_1 \geq 0 \text{ and } -\pi_2 \pi_3 |1 + \pi_2 \bar{\pi}_1 + \bar{\pi}_1 z - \bar{\pi}_1 \leq 0 \text{ and } \pi_2 \pi_3 |1 \pi_3 |2 - \pi_2 \pi_3 |1 - \pi_2 \pi_3 |2 \bar{\pi}_1 + \pi_3 |2 \bar{\pi}_1 \leq 0 \text{ and } \pi_2 + z - 1 \geq 0 \text{ and } -\pi_2 + \bar{\pi}_2 \geq 0 \text{ and } -\pi_3 |1 + \bar{\pi}_3 |1 \geq 0 \text{ and } -\pi_3 |2 + \bar{\pi}_3 |2 \geq 0 \text{ and } \pi_2 < 1 \text{ and } \pi_3 |1 < 1 \text{ and } \pi_3 |2 < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_3 |1 < 1 \text{ and } \bar{\pi}_3 |2 < 1 \text{ and } z \leq 1$$

or

$$-\pi_1 + \bar{\pi}_1 \geq 0 \text{ and } \pi_1 < 1 \text{ and } -\pi_2 + \bar{\pi}_2 \geq 0 \text{ and } \pi_2 < 1 \text{ and } -\pi_3 |1 \bar{\pi}_2 - \bar{\pi}_1 \bar{\pi}_2 + \bar{\pi}_1 z + \bar{\pi}_2 \geq 0 \text{ and } -\pi_3 |1 + \bar{\pi}_3 |1 \geq 0 \text{ and } -\pi_3 |1 \bar{\pi}_2 + \bar{\pi}_1 z \leq 0 \text{ and } -\pi_3 |2 + \bar{\pi}_3 |2 \geq 0 \text{ and } \pi_3 |1 < 1 \text{ and } \pi_3 |2 < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } -\bar{\pi}_2 + z \leq 0 \text{ and } -\bar{\pi}_2 + 1 \geq 0 \text{ and } \bar{\pi}_3 |1 < 1 \text{ and } \bar{\pi}_3 |2 < 1 \text{ and } z \geq 0$$

or

$$-\pi_1 + \bar{\pi}_1 \geq 0 \text{ and } \pi_1 < 1 \text{ and } -\pi_2 + \bar{\pi}_2 \geq 0 \text{ and } \pi_2 < 1 \text{ and } -\pi_3 |1 + \bar{\pi}_3 |1 \geq 0 \text{ and } \pi_3 |1 \pi_3 |2 \bar{\pi}_2 - \pi_3 |1 \bar{\pi}_2 - \pi_3 |2 \bar{\pi}_1 \bar{\pi}_2 + \pi_3 |2 \bar{\pi}_1 z \leq 0 \text{ and } -\pi_3 |1 + \bar{\pi}_1 \leq 0 \text{ and } -\pi_3 |2 + \bar{\pi}_3 |2 \geq 0 \text{ and } \pi_3 |1 < 1 \text{ and } \pi_3 |2 < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } -\bar{\pi}_2 + z \geq 0 \text{ and } \bar{\pi}_3 |1 < 1 \text{ and } \bar{\pi}_3 |2 < 1 \text{ and } z \leq 1$$

or

$$-\pi_1 z + \bar{\pi}_1 z \geq 0 \text{ and } \pi_1 < 1 \text{ and } -\pi_2 \pi_3 |1 + \bar{\pi}_1 z \geq 0 \text{ and } \pi_2 < 1 \text{ and } -\bar{\pi}_1 z + z \geq 0 \text{ and } -\pi_3 |1 \bar{\pi}_2 + \bar{\pi}_1 z \leq 0 \text{ and } -\pi_3 |1 z + \bar{\pi}_1 z \geq 0 \text{ and } -\pi_3 |1 + \bar{\pi}_1 z \leq 0 \text{ and } -\pi_3 |1 z + \bar{\pi}_3 |1 z \geq 0 \text{ and } \pi_3 |1 < 1 \text{ and } -\pi_3 |2 z + \bar{\pi}_3 |2 z \geq 0 \text{ and } z > 0 \text{ and } -\pi_3 |2 z + z \geq 0 \text{ and } \pi_3 |2 < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_3 |1 < 1 \text{ and } \bar{\pi}_3 |2 < 1$$

or

$$\begin{aligned} & \pi_1 \pi_3 |1 z - \pi_1 \pi_3 |1 - \pi_1 \bar{\pi}_1 z + \pi_1 \bar{\pi}_1 - \pi_3 |1 \bar{\pi}_1 z + \pi_3 |1 \bar{\pi}_1 + \bar{\pi}_1^2 z - \bar{\pi}_1^2 \leq 0 \text{ and } \pi_1 < 1 \text{ and } \pi_2 \pi_3 |1^2 - 2\pi_2 \pi_3 |1 \bar{\pi}_1 + \pi_2 \bar{\pi}_1^2 - \\ & \pi_3 |1 \bar{\pi}_1 z + \pi_3 |1 \bar{\pi}_1 + \bar{\pi}_1^2 z - \bar{\pi}_1^2 \leq 0 \text{ and } \pi_2 < 1 \text{ and } \pi_3 |1^2 z - \pi_3 |1^2 - \pi_3 |1 \bar{\pi}_1 z + \pi_3 |1 \bar{\pi}_1 - \pi_3 |1 \bar{\pi}_3 |1 z + \pi_3 |1 \bar{\pi}_3 |1 + \bar{\pi}_1 \bar{\pi}_3 |1 z - \\ & \bar{\pi}_1 \bar{\pi}_3 |1 \leq 0 \text{ and } -\pi_3 |1 z + \pi_3 |1 + \bar{\pi}_1 z - \bar{\pi}_1 < 0 \text{ and } \pi_3 |1 \bar{\pi}_1 z - \pi_3 |1 \bar{\pi}_1 - \pi_3 |1 z + \pi_3 |1 - \bar{\pi}_1^2 z + \bar{\pi}_1^2 + \bar{\pi}_1 z - \bar{\pi}_1 \leq \end{aligned}$$



0 and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\bar{\pi}_{3|2}z + \pi_{3|2}^2\bar{\pi}_1z - \pi_{3|2}^2z - \pi_{3|2}\bar{\pi}_1\bar{\pi}_{3|2}z + \pi_{3|2}\bar{\pi}_{3|2}z \leq 0$  and  $-\pi_{3|1}z - \pi_{3|2}\bar{\pi}_1z + \pi_{3|2}z <$   
 0 and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}z + \pi_{3|2}^2\bar{\pi}_1z - \pi_{3|2}^2z - \pi_{3|2}\bar{\pi}_1z + \pi_{3|2}z \leq 0$  and  $-\pi_{3|1} - \pi_{3|2}\bar{\pi}_1 + \pi_{3|2} <> 0$  and  $\pi_{3|1} <$   
 1 and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$-\pi_1 + \bar{\pi}_1 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2\pi_{3|1} - \pi_2\pi_{3|2}\bar{\pi}_1 + \pi_2\pi_{3|2} + \pi_{3|2}\bar{\pi}_1z \geq 0$  and  $\pi_2\pi_{3|1}\pi_{3|2} - \pi_2\pi_{3|1} - \pi_2\pi_{3|2}\bar{\pi}_1 +$   
 $\pi_{3|2}\bar{\pi}_1z \leq 0$  and  $-\pi_2\pi_{3|1} + \pi_{3|2}\bar{\pi}_1z \leq 0$  and  $\pi_2\pi_{3|1}\pi_{3|2} - \pi_2\pi_{3|1} + \pi_{3|2}\bar{\pi}_1z \geq 0$  and  $\pi_2\pi_{3|1}\pi_{3|2} - \pi_2\pi_{3|1} - \pi_2\pi_{3|2}\bar{\pi}_1 +$   
 $\pi_{3|2}\bar{\pi}_1z \geq 0$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} <$   
 1 and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$\pi_1\pi_{3|1}z + \pi_1\bar{\pi}_1z - \pi_1z - \pi_{3|1}\bar{\pi}_1z - \bar{\pi}_1^2z + \bar{\pi}_1z \leq 0$  and  $\pi_1 < 1$  and  $\pi_2\pi_{3|1}^2 + 2\pi_2\pi_{3|1}\bar{\pi}_1 - 2\pi_2\pi_{3|1} + \pi_2\bar{\pi}_1^2 - 2\pi_2\bar{\pi}_1 +$   
 $\pi_2 - \pi_{3|1}\bar{\pi}_1z - \bar{\pi}_1^2z + \bar{\pi}_1z \leq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1}^2z + \pi_{3|1}\bar{\pi}_1z - \pi_{3|1}\bar{\pi}_{3|1}z - \pi_{3|1}z - \bar{\pi}_1\bar{\pi}_{3|1}z + \bar{\pi}_{3|1}z \leq 0$  and  $-\pi_{3|1}z - \bar{\pi}_1z + z <$   
 $0$  and  $\pi_{3|1}^2z + \pi_{3|1}\bar{\pi}_1z - 2\pi_{3|1}z - \bar{\pi}_1z + z \leq 0$  and  $\pi_{3|1}\bar{\pi}_1z - \pi_{3|1}z + \bar{\pi}_1^2z - 2\bar{\pi}_1z + z \leq 0$  and  $\pi_{3|1}^2\bar{\pi}_2 +$   
 $2\pi_{3|1}\bar{\pi}_1\bar{\pi}_2 - \pi_{3|1}\bar{\pi}_1z - 2\pi_{3|1}\bar{\pi}_2 + \bar{\pi}_1^2\bar{\pi}_2 - \bar{\pi}_1^2z - 2\bar{\pi}_1\bar{\pi}_2 + \bar{\pi}_1z + \bar{\pi}_2 \geq 0$  and  $\pi_{3|1}^2 - \pi_{3|1}\bar{\pi}_1z + 2\pi_{3|1}\bar{\pi}_1 - 2\pi_{3|1} - \bar{\pi}_1^2z +$   
 $\bar{\pi}_1^2 + \bar{\pi}_1z - 2\bar{\pi}_1 + 1 \geq 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\bar{\pi}_{3|2}z + \pi_{3|2}\bar{\pi}_1z - \pi_{3|2}z - \bar{\pi}_1\bar{\pi}_{3|2}z + \bar{\pi}_{3|2}z \leq 0$  and  $-\pi_{3|1}z - \bar{\pi}_1z + z <$   
 $0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}z + \pi_{3|2}\bar{\pi}_1z - \pi_{3|2}z - \bar{\pi}_1z + z \leq 0$  and  $-\pi_{3|1} - \bar{\pi}_1 + 1 <> 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} <$   
 1 and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \geq 0$

**or**

$-\pi_1 + \bar{\pi}_1 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $-\pi_{3|1}\bar{\pi}_2 + \bar{\pi}_1z - \bar{\pi}_1 + \bar{\pi}_2 \leq 0$  and  $-\pi_{3|1}\bar{\pi}_2 -$   
 $\pi_{3|2}\bar{\pi}_1\bar{\pi}_2 + \pi_{3|2}\bar{\pi}_1z + \pi_{3|2}\bar{\pi}_2 \leq 0$  and  $-\pi_{3|1}\bar{\pi}_2 - \bar{\pi}_1\bar{\pi}_2 + \bar{\pi}_1z + \bar{\pi}_2 \geq 0$  and  $-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $-\pi_{3|1} - \bar{\pi}_1 + 1 \leq$   
 $0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $-\pi_{3|1} - \bar{\pi}_1 + 1 <> 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$\pi_1\pi_{3|1}z + \pi_1\pi_{3|2}\bar{\pi}_1z - \pi_1\pi_{3|2}z - \pi_{3|1}\bar{\pi}_1z - \pi_{3|2}\bar{\pi}_1^2z + \pi_{3|2}\bar{\pi}_1z \leq 0$  and  $\pi_1 < 1$  and  $\pi_2\pi_{3|1}^2 + 2\pi_2\pi_{3|1}\pi_{3|2}\bar{\pi}_1 -$   
 $2\pi_2\pi_{3|1}\pi_{3|2} + \pi_2\pi_{3|2}^2\bar{\pi}_1^2 - 2\pi_2\pi_{3|2}^2\bar{\pi}_1 + \pi_2\pi_{3|2}^2 - \pi_{3|1}\pi_{3|2}\bar{\pi}_1z - \pi_{3|2}^2\bar{\pi}_1^2z + \pi_{3|2}^2\bar{\pi}_1z \leq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1}^2z +$   
 $\pi_{3|1}\pi_{3|2}\bar{\pi}_1z - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\bar{\pi}_{3|1}z - \pi_{3|2}\bar{\pi}_1\bar{\pi}_{3|1}z + \pi_{3|2}\bar{\pi}_{3|1}z \leq 0$  and  $-\pi_{3|1}z - \pi_{3|2}\bar{\pi}_1z + \pi_{3|2}z < 0$  and  $\pi_{3|1}^2z +$   
 $\pi_{3|1}\pi_{3|2}\bar{\pi}_1z - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}z - \pi_{3|2}\bar{\pi}_1z + \pi_{3|2}z \leq 0$  and  $\pi_{3|1}\bar{\pi}_1z - \pi_{3|1}z + \pi_{3|2}\bar{\pi}_1^2z - 2\pi_{3|2}\bar{\pi}_1z + \pi_{3|2}z \leq 0$  and  $\pi_{3|1}^2z +$   
 $\pi_{3|1}\pi_{3|2}\bar{\pi}_1z - \pi_{3|1}\pi_{3|2}z + \pi_{3|1}\bar{\pi}_1z - \pi_{3|1}z + \pi_{3|2}\bar{\pi}_1^2z - 2\pi_{3|2}\bar{\pi}_1z + \pi_{3|2}z \geq 0$  and  $\pi_{3|1}^2\bar{\pi}_2 + 2\pi_{3|1}\pi_{3|2}\bar{\pi}_1\bar{\pi}_2 - \pi_{3|1}\pi_{3|2}\bar{\pi}_1z -$   
 $2\pi_{3|1}\pi_{3|2}\bar{\pi}_2 + \pi_{3|2}^2\bar{\pi}_2 - \pi_{3|2}^2\bar{\pi}_1z - 2\pi_{3|2}^2\bar{\pi}_1\bar{\pi}_2 + \pi_{3|2}^2\bar{\pi}_1z + \pi_{3|2}^2\bar{\pi}_2 \geq 0$  and  $\pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 + \pi_{3|1}\pi_{3|2}^2\bar{\pi}_1z -$   
 $\pi_{3|1}\pi_{3|2}^2z - 2\pi_{3|1}\pi_{3|2}\bar{\pi}_1z + 2\pi_{3|1}\pi_{3|2}\bar{\pi}_1 + \pi_{3|1}\pi_{3|2}z - 2\pi_{3|1}\pi_{3|2} - \pi_{3|2}^2\bar{\pi}_1^2z + \pi_{3|2}^2\bar{\pi}_1^2 + \pi_{3|2}^2\bar{\pi}_1z - 2\pi_{3|2}^2\bar{\pi}_1 + \pi_{3|2}^2 \geq$   
 $0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\bar{\pi}_{3|2}z + \pi_{3|2}^2\bar{\pi}_1z - \pi_{3|2}^2z - \pi_{3|2}\bar{\pi}_1\bar{\pi}_{3|2}z + \pi_{3|2}\bar{\pi}_{3|2}z \leq 0$  and  $-\pi_{3|1}z - \pi_{3|2}\bar{\pi}_1z + \pi_{3|2}z <$   
 $0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}z + \pi_{3|2}^2\bar{\pi}_1z - \pi_{3|2}^2z - \pi_{3|2}\bar{\pi}_1z + \pi_{3|2}z \leq 0$  and  $-\pi_{3|1} - \pi_{3|2}\bar{\pi}_1 + \pi_{3|2} <> 0$  and  $-\pi_{3|1} - \bar{\pi}_1 + 1 <>$   
 $0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$\pi_1\pi_{3|1}z - \pi_1\pi_{3|1} - \pi_1z + \pi_1 - \pi_{3|1}\bar{\pi}_1z + \pi_{3|1}\bar{\pi}_1 + \bar{\pi}_1z - \bar{\pi}_1 \leq 0$  and  $\pi_1 < 1$  and  $\pi_2\pi_{3|1}^2 - 2\pi_2\pi_{3|1} + \pi_2 - \pi_{3|1}\bar{\pi}_1z +$   
 $\pi_{3|1}\bar{\pi}_1 + \bar{\pi}_1z - \bar{\pi}_1 \leq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1}\bar{\pi}_1z - \pi_{3|1}\bar{\pi}_1 - \pi_{3|1}z + \pi_{3|1} - \bar{\pi}_1z + \bar{\pi}_1 + z - 1 \leq 0$  and  $\pi_{3|1}^2z - \pi_{3|1}^2 +$   
 $\pi_{3|1}\bar{\pi}_1z - \pi_{3|1}\bar{\pi}_1 - 2\pi_{3|1}z + 2\pi_{3|1} - \bar{\pi}_1z + \bar{\pi}_1 + z - 1 \geq 0$  and  $\pi_{3|1}^2\bar{\pi}_2 - \pi_{3|1}\bar{\pi}_1z + \pi_{3|1}\bar{\pi}_1 - 2\pi_{3|1}\bar{\pi}_2 + \bar{\pi}_1z - \bar{\pi}_1 + \bar{\pi}_2 \geq$   
 $0$  and  $\pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 - \pi_{3|1}\pi_{3|2}\bar{\pi}_1z + \pi_{3|1}\pi_{3|2}\bar{\pi}_1 - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} + \pi_{3|1}z - \pi_{3|1} + \pi_{3|2}\bar{\pi}_1z - \pi_{3|2}\bar{\pi}_1 + \pi_{3|2} \leq$   
 $0$  and  $\pi_{3|1}^2 - \pi_{3|1}\bar{\pi}_1z + \pi_{3|1}\bar{\pi}_1 - 2\pi_{3|1} + \bar{\pi}_1z + \pi_{3|1}^2z - \pi_{3|1}^2 - \pi_{3|1}\bar{\pi}_{3|1}z + \pi_{3|1}\bar{\pi}_{3|1} - \pi_{3|1}z + \pi_{3|1} + \bar{\pi}_{3|1}z - \bar{\pi}_{3|1} \leq$   
 $0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}\bar{\pi}_{3|2}z + \pi_{3|1}\bar{\pi}_{3|2} - \pi_{3|2}z + \pi_{3|2} + \bar{\pi}_{3|2}z - \bar{\pi}_{3|2} \leq$   
 $0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $-\pi_{3|1} - \bar{\pi}_1 + 1 <> 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} <$   
 1 and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$

**or**

$-\pi_1 + \bar{\pi}_1 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2\pi_{3|1} + \pi_2 + \bar{\pi}_1z - \bar{\pi}_1 \leq 0$  and  $-\pi_2\pi_{3|1} - \pi_2\pi_{3|2}\bar{\pi}_1 + \pi_2\pi_{3|2} + \pi_{3|2}\bar{\pi}_1z \leq 0$  and  $-\pi_2\pi_{3|1} - \pi_2\bar{\pi}_1 + \pi_2 + \bar{\pi}_1z \geq 0$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $-\pi_{3|1} - \bar{\pi}_1 + 1 \leq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq$   
 $0$  and  $\pi_2 < 1$  and  $-\pi_{3|1} - \bar{\pi}_1 + 1 <> 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**





or

$$-\pi_1\pi_{3|2}z + \pi_2\pi_{3|1} \leq 0 \text{ and } -\pi_1\pi_{3|2}z + \pi_2\pi_{3|1} \geq 0 \text{ and } -\pi_1 + \bar{\pi}_1 \geq 0 \text{ and } \pi_1 < 1 \text{ and } \pi_2 - \pi_{3|2}z + z - 1 \leq 0 \text{ and } -\pi_2 + \pi_{3|2}z \leq 0 \text{ and } -\pi_2 + \bar{\pi}_2 \geq 0 \text{ and } \pi_2 < 1 \text{ and } \pi_{3|1} < 1 \text{ and } -\pi_{3|2}z + \bar{\pi}_{3|2}z \geq 0 \text{ and } -\pi_{3|2}z + z \geq 0 \text{ and } \pi_{3|2} < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_{3|1} < 1 \text{ and } \bar{\pi}_{3|2} < 1 \text{ and } z > 0$$

or

$$-\pi_1\pi_{3|2}\bar{\pi}_2 + \pi_1\pi_{3|2}z - \pi_{3|1}\bar{\pi}_2 + \pi_{3|2}\bar{\pi}_2 \geq 0 \text{ and } \pi_1\pi_{3|2}\bar{\pi}_2 - \pi_1\pi_{3|2}z - \pi_{3|2}\bar{\pi}_2 + \bar{\pi}_2\bar{\pi}_{3|1} \geq 0 \text{ and } \pi_1\pi_{3|2}\bar{\pi}_2 - \pi_1\pi_{3|2}z - \pi_1\bar{\pi}_2\bar{\pi}_{3|2} + \pi_1\bar{\pi}_{3|2}z - \pi_{3|2}\bar{\pi}_2 + \bar{\pi}_2\bar{\pi}_{3|2} \geq 0 \text{ and } \pi_1\pi_{3|2}\bar{\pi}_2 - \pi_1\pi_{3|2}z + \pi_1z - \pi_1 - \pi_{3|2}\bar{\pi}_2 + \bar{\pi}_2 \leq 0 \text{ and } \pi_1\pi_{3|2}\bar{\pi}_2 - \pi_1\pi_{3|2}z - \pi_1\bar{\pi}_2 + \pi_1z - \pi_{3|2}\bar{\pi}_2 + \bar{\pi}_2 \geq 0 \text{ and } \pi_1\pi_{3|2}\bar{\pi}_2 - \pi_1\pi_{3|2}z - \pi_{3|2}\bar{\pi}_2 + \bar{\pi}_2 \geq 0 \text{ and } \pi_1\pi_{3|2}\bar{\pi}_2 - \pi_1\pi_{3|2}z - \pi_1\bar{\pi}_2 - \pi_{3|2}\bar{\pi}_2 + \bar{\pi}_2 \leq 0 \text{ and } -\pi_1 + \bar{\pi}_1 \geq 0 \text{ and } -\pi_1\bar{\pi}_2 + \pi_1z + \bar{\pi}_2 > 0 \text{ and } \pi_1 < 1 \text{ and } -\pi_2 + \bar{\pi}_2 \geq 0 \text{ and } \pi_2 < 1 \text{ and } \pi_{3|1} < 1 \text{ and } \pi_{3|2} < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_{3|1} < 1 \text{ and } \bar{\pi}_{3|2} < 1$$

or

$$-\pi_1\pi_2\pi_{3|2} + \pi_1\pi_{3|2}z - \pi_2\pi_{3|1} + \pi_2\pi_{3|2} \geq 0 \text{ and } \pi_1\pi_2\pi_{3|2} - \pi_1\pi_{3|2}z - \pi_2\pi_{3|2} + \pi_2\bar{\pi}_{3|1} \geq 0 \text{ and } \pi_1\pi_2\pi_{3|2} - \pi_1\pi_2\bar{\pi}_{3|2} - \pi_1\pi_{3|2}z + \pi_1\bar{\pi}_{3|2}z - \pi_2\pi_{3|2} + \pi_2\bar{\pi}_{3|2} \geq 0 \text{ and } \pi_1\pi_2\pi_{3|2} - \pi_1\pi_{3|2}z + \pi_1z - \pi_1 - \pi_2\pi_{3|2} + \pi_2 \leq 0 \text{ and } \pi_1\pi_2\pi_{3|2} - \pi_1\pi_2z - \pi_1\pi_{3|2}z + \pi_1z - \pi_2\pi_{3|2} + \pi_2 \geq 0 \text{ and } \pi_1\pi_2\pi_{3|2} - \pi_1\pi_{3|2}z - \pi_2\pi_{3|2} + \pi_2 \geq 0 \text{ and } \pi_1\pi_2\pi_{3|2} - \pi_1\pi_2z - \pi_2\pi_{3|2} + \pi_2 \leq 0 \text{ and } -\pi_1\pi_2 + \pi_1z + \pi_2 > 0 \text{ and } -\pi_1 + \bar{\pi}_1 \geq 0 \text{ and } \pi_1 < 1 \text{ and } -\pi_2 + \bar{\pi}_2 \geq 0 \text{ and } \pi_2 < 1 \text{ and } \pi_{3|1} < 1 \text{ and } \pi_{3|2} < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_{3|1} < 1 \text{ and } \bar{\pi}_{3|2} < 1$$

or

$$-\pi_1\pi_{3|1}\pi_{3|2}z + \pi_1\pi_{3|2}z + \pi_{3|1}\pi_{3|2}z - \pi_{3|1}z - \pi_{3|2}z + z \geq 0 \text{ and } -\pi_1\pi_{3|2}^2z + \pi_1\pi_{3|2}\bar{\pi}_{3|2}z + \pi_{3|2}^2z - \pi_{3|2}\bar{\pi}_{3|2}z - \pi_{3|2}z + \bar{\pi}_{3|2}z \geq 0 \text{ and } \pi_1\pi_{3|2}z - \pi_{3|2}z + z > 0 \text{ and } -\pi_1\pi_{3|2}^2z + \pi_1\pi_{3|2}z + \pi_{3|2}^2z - 2\pi_{3|2}z + z \geq 0 \text{ and } \pi_1^2\pi_{3|2}^2z - \pi_1^2\pi_{3|2}z - 2\pi_1\pi_2\pi_{3|2}^2 + 2\pi_1\pi_2\pi_{3|2} + \pi_1\pi_{3|2}^2z - \pi_1\pi_{3|2}z + \pi_2\pi_{3|2}^2 - 2\pi_2\pi_{3|2} + \pi_2 \leq 0 \text{ and } \pi_1^2\pi_{3|2}^2\bar{\pi}_2 - \pi_1^2\pi_{3|2}^2z - 2\pi_1\pi_{3|2}^2\bar{\pi}_2 + \pi_1\pi_{3|2}^2z + 2\pi_1\pi_{3|2}\bar{\pi}_2 - \pi_1\pi_{3|2}z + \pi_{3|2}^2\bar{\pi}_2 - 2\pi_{3|2}\bar{\pi}_2 + \bar{\pi}_2 \geq 0 \text{ and } -\pi_1\pi_{3|2}\bar{\pi}_{3|1}z + \pi_1\pi_{3|2}z + \pi_{3|2}\bar{\pi}_{3|1}z - \pi_{3|2}z - \bar{\pi}_{3|1}z + z \leq 0 \text{ and } -\pi_1^2\pi_{3|2}z + \pi_1\pi_{3|2}\bar{\pi}_1z + \pi_1\pi_{3|2}z - \pi_1z - \pi_{3|2}\bar{\pi}_1z + \bar{\pi}_1z \geq 0 \text{ and } \pi_1\pi_{3|2}z - \pi_{3|2}z + z > 0 \text{ and } -\pi_1^2\pi_{3|2}z + 2\pi_1\pi_{3|2}z - \pi_1z - \pi_{3|2}z + z \geq 0 \text{ and } \pi_1\pi_{3|2}z - \pi_{3|2}z + z \geq 0 \text{ and } \pi_1\pi_{3|2} - \pi_{3|2} + 1 <> 0 \text{ and } \pi_1 < 1 \text{ and } \pi_2 < 1 \text{ and } \pi_{3|1} < 1 \text{ and } \pi_{3|2} < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_{3|1} < 1 \text{ and } \bar{\pi}_{3|2} < 1 \text{ and } z \leq 1$$

or

$$\pi_1^2\pi_{3|2}z - \pi_1^2\pi_{3|2}^2 - \pi_1\pi_{3|1}\pi_{3|2}^2z + \pi_1\pi_{3|1}\pi_{3|2}z - \pi_1\pi_{3|1}\pi_{3|2} - \pi_1\pi_{3|2}^2z + 2\pi_1\pi_{3|2} + \pi_1\pi_{3|2}z - \pi_1\pi_{3|2} + \pi_{3|1}\pi_{3|2}^2z - 2\pi_{3|1}\pi_{3|2}z + \pi_{3|1}\pi_{3|2} + \pi_{3|1}z - \pi_{3|1} - \pi_{3|2}^2 + \pi_{3|2} \geq 0 \text{ and } \pi_1^2\pi_{3|2}^2z - \pi_1^2\pi_{3|2}^2 - \pi_1\pi_{3|2}^2\bar{\pi}_{3|1}z - \pi_1\pi_{3|2}^2z + 2\pi_1\pi_{3|2}^2 + \pi_1\pi_{3|2}\bar{\pi}_{3|1}z - \pi_1\pi_{3|2}\bar{\pi}_{3|1} + \pi_1\pi_{3|2}z - \pi_1\pi_{3|2} + \pi_{3|2}^2\bar{\pi}_{3|1}z - \pi_{3|2}^2 - 2\pi_{3|2}\bar{\pi}_{3|1}z + \pi_{3|2}\bar{\pi}_{3|1} + \pi_{3|2} + \bar{\pi}_{3|1}z - \bar{\pi}_{3|1} \leq 0 \text{ and } \pi_1^2\pi_{3|2}^2z - \pi_1^2\pi_{3|2}z + \pi_1^2\pi_{3|2} - \pi_1\pi_{3|2}^2\bar{\pi}_1z - \pi_1\pi_{3|2}^2z + \pi_1\pi_{3|2}\bar{\pi}_1z - \pi_1\pi_{3|2}\bar{\pi}_1 + 2\pi_1\pi_{3|2}z - \pi_1\pi_{3|2} - \pi_1z + \pi_1 + \pi_{3|2}^2\bar{\pi}_1z - 2\pi_{3|2}\bar{\pi}_1z + \pi_{3|2}\bar{\pi}_1 + \bar{\pi}_1z - \bar{\pi}_1 \leq 0 \text{ and } -\pi_1\pi_{3|2}^2z + \pi_1\pi_{3|2}z - \pi_1\pi_{3|2} + \pi_{3|2}^2z - 2\pi_{3|2}z + \pi_{3|2} + z - 1 < 0 \text{ and } \pi_1^2\pi_{3|2}^2z - \pi_1^2\pi_{3|2}z + \pi_1^2\pi_{3|2} - 2\pi_1\pi_{3|2}^2z + 3\pi_1\pi_{3|2}z - 2\pi_1\pi_{3|2} - \pi_1z + \pi_1 + \pi_{3|2}^2z - 2\pi_{3|2}z + \pi_{3|2} + z - 1 \leq 0 \text{ and } 2\pi_1^2\pi_{3|2}^2z - \pi_1^2\pi_{3|2}^2 - \pi_1^2\pi_{3|2}z + \pi_1^2\pi_{3|2} - 3\pi_1\pi_{3|2}^2z + 2\pi_1\pi_{3|2}^2 + 4\pi_1\pi_{3|2}z - 3\pi_1\pi_{3|2} - \pi_1z + \pi_1 + \pi_{3|2}^2z - \pi_{3|2}^2 - 2\pi_{3|2}z + 2\pi_{3|2} + z - 1 \geq 0 \text{ and } -\pi_1^2\pi_{3|2}^2z + \pi_1^2\pi_{3|2}^2 + \pi_1^2\pi_{3|2}\bar{\pi}_{3|2}z - \pi_1^2\pi_{3|2}\bar{\pi}_{3|2} + \pi_1\pi_{3|2}^2z - 2\pi_1\pi_{3|2}^2 - \pi_1\pi_{3|2}\bar{\pi}_{3|2}z + 2\pi_1\pi_{3|2}\bar{\pi}_{3|2} - \pi_1\pi_{3|2}z + \pi_1\pi_{3|2} + \pi_1\bar{\pi}_{3|2}z - \pi_1\bar{\pi}_{3|2} + \pi_{3|2}^2 - \pi_{3|2}\bar{\pi}_{3|2} - \pi_{3|2} + \bar{\pi}_{3|2} \geq 0 \text{ and } \pi_1^2\pi_{3|2}z - \pi_1^2\pi_{3|2} - \pi_1\pi_{3|2}z + 2\pi_1\pi_{3|2} + \pi_1z - \pi_1 + \pi_2\pi_{3|2}^2 - 2\pi_2\pi_{3|2} + \pi_2 \leq 0 \text{ and } \pi_1^2\pi_{3|2}^2\bar{\pi}_2 - \pi_1^2\pi_{3|2}^2z + \pi_1^2\pi_{3|2}z - \pi_1^2\pi_{3|2} - 2\pi_1\pi_2\pi_{3|2}^2 + 2\pi_1\pi_2\pi_{3|2} + \pi_1\pi_{3|2}^2z - 2\pi_1\pi_{3|2}z + \pi_1\pi_{3|2} + \pi_1z - \pi_1 + \pi_2\pi_{3|2}^2 - 2\pi_2\pi_{3|2} + \pi_2 \leq 0 \text{ and } \pi_1^2\pi_{3|2}^2\bar{\pi}_2 - \pi_1^2\pi_{3|2}^2z + \pi_1^2\pi_{3|2}z - \pi_1^2\pi_{3|2} - 2\pi_1\pi_2\pi_{3|2}^2 + 2\pi_1\pi_2\pi_{3|2} + \pi_1\pi_{3|2}^2z - \pi_1\pi_{3|2}^2z + 2\pi_1\pi_{3|2}\bar{\pi}_2 - 2\pi_1\pi_{3|2}z + \pi_1\pi_{3|2} + \pi_1z - \pi_1 + \pi_{3|2}^2\bar{\pi}_2 - 2\pi_{3|2}\bar{\pi}_2 + \bar{\pi}_2 \geq 0 \text{ and } -\pi_1^2\pi_{3|2}^2z + \pi_1^2\pi_{3|2}^2 + \pi_1^2\pi_{3|2}z - \pi_1^2\pi_{3|2} + \pi_1\pi_{3|2}^2z - 2\pi_1\pi_{3|2}z + 3\pi_1\pi_{3|2} + \pi_1z - \pi_1 + \pi_{3|2}^2 - 2\pi_{3|2} + 1 \geq 0 \text{ and } \pi_1\pi_{3|2} - \pi_{3|2} + 1 <> 0 \text{ and } \pi_1 < 1 \text{ and } \pi_2 < 1 \text{ and } \pi_{3|1} < 1 \text{ and } \pi_{3|2} < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_{3|1} < 1 \text{ and } \bar{\pi}_{3|2} < 1 \text{ and } z \leq 1$$

or

$$-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0 \text{ and } -\pi_{3|2} + \bar{\pi}_{3|2} \geq 0 \text{ and } \pi_1z - \pi_1 - \pi_1z + \bar{\pi}_{3|1} \geq 0 \text{ and } -\pi_1 + \bar{\pi}_1 \geq 0 \text{ and } \pi_1 < 1 \text{ and } \pi_2 < 1 \text{ and } \pi_{3|1} < 1 \text{ and } \pi_{3|2} < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } -\bar{\pi}_2 + 1 \leq 0 \text{ and } \bar{\pi}_{3|1} < 1 \text{ and } \bar{\pi}_{3|2} < 1 \text{ and } z \leq 1 \text{ and } z \geq 0$$

or

$$-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0 \text{ and } -\pi_{3|2} + \bar{\pi}_{3|2} \geq 0 \text{ and } \pi_1\pi_{3|2}\bar{\pi}_2 - \pi_1\pi_{3|2} - \pi_{3|2}\bar{\pi}_2\bar{\pi}_{3|1} + \bar{\pi}_2\bar{\pi}_{3|1} \geq 0 \text{ and } \pi_1z - \pi_1 - \bar{\pi}_2\bar{\pi}_{3|1} + \bar{\pi}_2 \geq 0 \text{ and } -\pi_1\bar{\pi}_2 - \pi_1z + \pi_1 + \bar{\pi}_2\bar{\pi}_{3|1} \geq 0 \text{ and } -\pi_1 + \bar{\pi}_1 \geq 0 \text{ and } \pi_1 < 1 \text{ and } -\pi_2 + \bar{\pi}_2 \geq 0 \text{ and } \pi_2 < 1 \text{ and } \pi_{3|1} < 1 \text{ and } \pi_{3|2} < 1 \text{ and } \bar{\pi}_1 < 1 \text{ and } \bar{\pi}_2 + z - 1 \geq 0 \text{ and } -\bar{\pi}_2 + 1 \geq 0 \text{ and } \bar{\pi}_{3|1} < 1 \text{ and } \bar{\pi}_{3|2} < 1 \text{ and } z \leq 1$$



$\pi_1^2 z - \pi_1^2 - 2\pi_1 \bar{\pi}_2 \bar{\pi}_{3|1} - \pi_1 \bar{\pi}_{3|1} z + \pi_1 \bar{\pi}_{3|1} + \bar{\pi}_2 \bar{\pi}_{3|1}^2 \geq 0$  and  $\pi_1^2 z - \pi_1 \bar{\pi}_{3|1} z - \pi_1 \bar{\pi}_{3|1} + \bar{\pi}_{3|1}^2 \geq 0$  and  $-\pi_1 + \bar{\pi}_{3|1} <> 0$  and  $\pi_1 < 1$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$

**or**

$\pi_1 \pi_{3|1} z - \pi_1 \bar{\pi}_{3|1} z + \pi_{3|1} \bar{\pi}_{3|1} z - \pi_{3|1} z - \bar{\pi}_{3|1}^2 z + \bar{\pi}_{3|1} z \leq 0$  and  $\pi_1 \pi_{3|2} z - \pi_1 \bar{\pi}_{3|2} z + \pi_{3|2} \bar{\pi}_{3|1} z - \pi_{3|2} z - \bar{\pi}_{3|1} \bar{\pi}_{3|2} z + \bar{\pi}_{3|2} z \leq 0$  and  $-\pi_1 z - \bar{\pi}_{3|1} z + z < 0$  and  $\pi_1 \pi_{3|2} z - \pi_1 z + \pi_{3|2} \bar{\pi}_{3|1} z - \pi_{3|2} z - \bar{\pi}_{3|1} z + z \leq 0$  and  $\pi_1^2 z - \pi_1 \bar{\pi}_{3|1} z + \pi_1 \bar{\pi}_{3|1} z - \pi_1 z - \bar{\pi}_1 \bar{\pi}_{3|1} z + \bar{\pi}_1 z \leq 0$  and  $-\pi_1 z - \bar{\pi}_{3|1} z + z < 0$  and  $\pi_1^2 z + \pi_1 \bar{\pi}_{3|1} z - 2\pi_1 z - \bar{\pi}_{3|1} z + z \leq 0$  and  $\pi_1 \bar{\pi}_{3|1} z - \pi_1 z + \bar{\pi}_{3|1}^2 z - 2\bar{\pi}_{3|1} z + z \leq 0$  and  $\pi_1^2 \pi_2 - \pi_1^2 z + 2\pi_1 \pi_2 \bar{\pi}_{3|1} - 2\pi_1 \pi_2 - \pi_1 \bar{\pi}_{3|1} z + \pi_1 z + \pi_2 \bar{\pi}_{3|1}^2 - 2\pi_2 \bar{\pi}_{3|1} + \pi_2 \leq 0$  and  $\pi_1^2 \bar{\pi}_2 - \pi_1^2 z + 2\pi_1 \bar{\pi}_2 \bar{\pi}_{3|1} - 2\pi_1 \bar{\pi}_2 - \pi_1 \bar{\pi}_{3|1} z + \pi_1 z + \bar{\pi}_2 \bar{\pi}_{3|1}^2 - 2\bar{\pi}_2 \bar{\pi}_{3|1} + \bar{\pi}_2 \geq 0$  and  $-\pi_1^2 z + \pi_1^2 - \pi_1 \bar{\pi}_{3|1} z + 2\pi_1 \bar{\pi}_{3|1} + \pi_1 z - 2\pi_1 + \bar{\pi}_{3|1}^2 - 2\bar{\pi}_{3|1} + 1 \geq 0$  and  $-\pi_1 - \bar{\pi}_{3|1} + 1 <> 0$  and  $\pi_1 < 1$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \geq 0$

**or**

$-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $\pi_1 \pi_{3|2} \bar{\pi}_2 - \pi_1 \pi_{3|2} z - \pi_{3|2} \bar{\pi}_2 + \bar{\pi}_2 \bar{\pi}_{3|1} \geq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $\pi_1 z - \pi_1 - \bar{\pi}_2 \bar{\pi}_{3|1} + \bar{\pi}_2 \leq 0$  and  $-\pi_1 \bar{\pi}_2 + \pi_1 z - \bar{\pi}_2 \bar{\pi}_{3|1} + \bar{\pi}_2 \geq 0$  and  $-\pi_1 - \bar{\pi}_{3|1} + 1 \leq 0$  and  $-\pi_1 + \bar{\pi}_1 \geq 0$  and  $-\pi_1 - \bar{\pi}_{3|1} + 1 <> 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $\pi_1 \pi_2 \pi_{3|2} - \pi_1 \pi_{3|2} z - \pi_2 \pi_{3|2} + \pi_2 \bar{\pi}_{3|1} \geq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $\pi_1 z - \pi_1 - \pi_2 \bar{\pi}_{3|1} + \pi_2 \leq 0$  and  $-\pi_1 \pi_2 + \pi_1 z - \pi_2 \bar{\pi}_{3|1} + \pi_2 \geq 0$  and  $-\pi_1 - \bar{\pi}_{3|1} + 1 \leq 0$  and  $-\pi_1 + \bar{\pi}_1 \geq 0$  and  $-\pi_1 - \bar{\pi}_{3|1} + 1 <> 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$-\pi_1 \pi_{3|2} \bar{\pi}_{3|1} z + \pi_1 \pi_{3|2} \bar{\pi}_{3|1} + \pi_1 \pi_{3|2} z - \pi_1 \pi_{3|2} + \pi_{3|2} \bar{\pi}_{3|1}^2 z - \pi_{3|2} \bar{\pi}_{3|1} z - \pi_{3|2} \bar{\pi}_{3|1} + \pi_{3|2} - \bar{\pi}_{3|1}^2 z + \bar{\pi}_{3|1}^2 + \bar{\pi}_{3|1} z - \bar{\pi}_{3|1} \leq 0$  and  $\pi_1 \bar{\pi}_{3|1} z - \pi_1 \bar{\pi}_{3|1} - \pi_1 z + \pi_1 - \bar{\pi}_1 \bar{\pi}_{3|1} z + \bar{\pi}_1 \bar{\pi}_{3|1} + \bar{\pi}_1 z - \bar{\pi}_1 \leq 0$  and  $\pi_1 \bar{\pi}_{3|1} z - \pi_1 \bar{\pi}_{3|1} - \pi_1 z + \pi_1 - \bar{\pi}_{3|1} z + \bar{\pi}_{3|1} + z - 1 \leq 0$  and  $\pi_1 \bar{\pi}_{3|1} z - \pi_1 \bar{\pi}_{3|1} - \pi_1 z + \pi_1 + \bar{\pi}_{3|1}^2 z - \bar{\pi}_{3|1}^2 - 2\bar{\pi}_{3|1} z + 2\bar{\pi}_{3|1} + z - 1 \geq 0$  and  $-\pi_1 \bar{\pi}_{3|1} z + \pi_1 \bar{\pi}_{3|1} + \pi_1 z - \pi_1 + \pi_2 \bar{\pi}_{3|1}^2 - 2\bar{\pi}_2 \bar{\pi}_{3|1} + \bar{\pi}_2 \geq 0$  and  $-\pi_1 \bar{\pi}_{3|1} z + \pi_1 \bar{\pi}_{3|1} + \pi_1 z - \pi_1 + \bar{\pi}_{3|1}^2 - 2\bar{\pi}_{3|1} + 1 \geq 0$  and  $-\pi_1 - \bar{\pi}_{3|1} + 1 <> 0$  and  $\pi_1 < 1$  and  $\pi_2 < 1$  and  $\pi_{3|1} \bar{\pi}_{3|1} z - \pi_{3|1} \bar{\pi}_{3|1} - \pi_{3|1} z + \pi_{3|1} - \bar{\pi}_{3|1}^2 z + \bar{\pi}_{3|1}^2 + \bar{\pi}_{3|1} z - \bar{\pi}_{3|1} \leq 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} \bar{\pi}_{3|1} z - \pi_{3|2} \bar{\pi}_{3|1} - \pi_{3|2} z + \pi_{3|2} - \bar{\pi}_{3|1} \bar{\pi}_{3|2} z + \bar{\pi}_{3|1} \bar{\pi}_{3|2} + \bar{\pi}_{3|2} z - \bar{\pi}_{3|2} \leq 0$  and  $-\pi_{3|1} z + \bar{\pi}_{3|1} + z - 1 < 0$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $-\pi_{3|1} z + \bar{\pi}_{3|1} + z - 1 < 0$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$

**or**

$-\pi_1 \pi_{3|1} \pi_{3|2} z + \pi_1 \pi_{3|2} \bar{\pi}_{3|1} z + \pi_{3|1} \pi_{3|2} z - \pi_{3|1} \bar{\pi}_{3|1} z - \pi_{3|2} \bar{\pi}_{3|1} z + \bar{\pi}_{3|1}^2 z \geq 0$  and  $\pi_1^2 \pi_{3|2}^2 z - \pi_1^2 \pi_{3|2}^2 - \pi_1 \pi_{3|2}^2 \bar{\pi}_{3|1} z - \pi_1 \pi_{3|2}^2 z + 2\pi_1 \pi_{3|2}^2 + 2\pi_1 \pi_{3|2} \bar{\pi}_{3|1} z - 2\pi_1 \pi_{3|2} \bar{\pi}_{3|1} + \pi_{3|2}^2 \bar{\pi}_{3|1} z - \pi_{3|2}^2 - \pi_{3|2} \bar{\pi}_{3|1}^2 z - \pi_{3|2} \bar{\pi}_{3|1} z + 2\pi_{3|2} \bar{\pi}_{3|1} + \bar{\pi}_{3|1}^2 z - \bar{\pi}_{3|1}^2 \leq 0$  and  $-\pi_1 \pi_{3|2}^2 z + \pi_1 \pi_{3|2} \bar{\pi}_{3|1} z + \pi_{3|2}^2 z - \pi_{3|2} \bar{\pi}_{3|1} z - \pi_{3|2} \bar{\pi}_{3|1} z + \bar{\pi}_{3|1} \bar{\pi}_{3|2} z \geq 0$  and  $\pi_1 \pi_{3|2} z - \pi_{3|2} z + \bar{\pi}_{3|1} z > 0$  and  $-\pi_1 \pi_{3|2}^2 z + \pi_1 \pi_{3|2} z + \pi_{3|2}^2 z - \pi_{3|2} \bar{\pi}_{3|1} z - \pi_{3|2} z + \bar{\pi}_{3|1} z \geq 0$  and  $\pi_1^2 \pi_2 \pi_{3|2}^2 - \pi_1^2 \pi_{3|2}^2 z - 2\pi_1 \pi_2 \pi_{3|2}^2 + 2\pi_1 \pi_2 \pi_{3|2} \bar{\pi}_{3|1} + \pi_1 \pi_{3|2}^2 z - \pi_1 \pi_{3|2} \bar{\pi}_{3|1} z + \pi_2 \pi_{3|2}^2 - 2\pi_2 \pi_{3|2} \bar{\pi}_{3|1} + \pi_2 \bar{\pi}_{3|1}^2 \leq 0$  and  $\pi_1^2 \pi_{3|2}^2 \bar{\pi}_2 - \pi_1^2 \pi_{3|2}^2 z - 2\pi_1 \pi_{3|2}^2 \bar{\pi}_2 + \pi_1 \pi_{3|2}^2 z + 2\pi_1 \pi_{3|2} \bar{\pi}_2 \bar{\pi}_{3|1} - \pi_1 \pi_{3|2} \bar{\pi}_{3|1} z + \pi_{3|2}^2 \bar{\pi}_2 - 2\pi_{3|2} \bar{\pi}_2 \bar{\pi}_{3|1} + \bar{\pi}_2 \bar{\pi}_{3|1}^2 \geq 0$  and  $-\pi_1^2 \pi_{3|2} z + \pi_1 \pi_{3|2} \bar{\pi}_1 z + \pi_1 \pi_{3|2} z - \pi_1 \bar{\pi}_{3|1} z - \pi_{3|2} \bar{\pi}_1 z + \bar{\pi}_1 \bar{\pi}_{3|1} z \geq 0$  and  $\pi_1 \pi_{3|2} z - \pi_{3|2} z + \bar{\pi}_{3|1} z > 0$  and  $-\pi_1^2 \pi_{3|2} z + 2\pi_1 \pi_{3|2} z - \pi_1 \bar{\pi}_{3|1} z - \pi_{3|2} z + \bar{\pi}_{3|1} z \geq 0$  and  $-\pi_1 \pi_{3|2} \bar{\pi}_{3|1} z + \pi_1 \pi_{3|2} z + \pi_{3|2} \bar{\pi}_{3|1} z - \pi_{3|2} z - \bar{\pi}_{3|1}^2 z + \bar{\pi}_{3|1} z \geq 0$  and  $-\pi_1^2 \pi_{3|2} z - \pi_1 \pi_{3|2} \bar{\pi}_{3|1} z + 2\pi_1 \pi_{3|2} z - \pi_1 \bar{\pi}_{3|1} z + \pi_{3|2} \bar{\pi}_{3|1} z - \pi_{3|2} z - \bar{\pi}_{3|1}^2 z + \bar{\pi}_{3|1} z \leq 0$  and  $\pi_1 \pi_{3|2} - \pi_{3|2} + \bar{\pi}_{3|1} <> 0$  and  $-\pi_1 - \bar{\pi}_{3|1} + 1 <> 0$  and  $\pi_1 < 1$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$\pi_1 \pi_{3|2} z - \pi_{3|2} \bar{\pi}_{3|1} z + \bar{\pi}_{3|1} z - \bar{\pi}_{3|1} \leq 0$  and  $-\pi_1 \pi_{3|2} z + \pi_2 \bar{\pi}_{3|1} \leq 0$  and  $-\pi_1 \pi_{3|2} z + \bar{\pi}_2 \bar{\pi}_{3|1} \geq 0$  and  $-\pi_1 z + \bar{\pi}_1 z \geq 0$  and  $-\pi_1 z + z \geq 0$  and  $-\pi_1 z + \bar{\pi}_{3|1} z \leq 0$  and  $\pi_1 < 1$  and  $\pi_2 < 1$  and  $-\pi_{3|1} z + \bar{\pi}_{3|1} z \geq 0$  and  $\pi_{3|1} < 1$  and  $-\pi_{3|2} z + \bar{\pi}_{3|2} z \geq 0$  and  $-\pi_{3|2} z + z \geq 0$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z > 0$

**or**

$-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $\pi_1 \pi_{3|2} \bar{\pi}_2 - \pi_1 \pi_{3|2} z - \pi_{3|2} \bar{\pi}_2 + \bar{\pi}_2 \bar{\pi}_{3|1} \leq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $-\pi_1 \pi_{3|2} z + \bar{\pi}_2 \bar{\pi}_{3|1} \geq 0$  and  $\pi_1 \pi_{3|2} \bar{\pi}_2 - \pi_1 \pi_{3|2} z - \pi_{3|2} \bar{\pi}_2 \bar{\pi}_{3|1} + \bar{\pi}_2 \bar{\pi}_{3|1} \geq 0$  and  $-\pi_1 \pi_{3|2} z - \pi_{3|2} \bar{\pi}_2 \bar{\pi}_{3|1} + \bar{\pi}_2 \bar{\pi}_{3|1} \leq 0$  and  $\pi_1 \pi_{3|2} \bar{\pi}_2 -$







$\pi_{3|1}z - \pi_{3|2}^2z + \pi_{3|2}z \leq 0$  and  $-\pi_1\pi_{3|2} - \pi_{3|1} + \pi_{3|2} \langle \rangle 0$  and  $\pi_1 < 1$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$\pi_1\pi_{3|1}z - \pi_1\bar{\pi}_{3|1}z + \pi_{3|1}^2z - \pi_{3|1}\bar{\pi}_{3|1}z - \pi_{3|1}z + \bar{\pi}_{3|1}z \leq 0$  and  $\pi_1^2z + \pi_1\pi_{3|1}z - \pi_1\bar{\pi}_1z - \pi_1z - \pi_{3|1}\bar{\pi}_1z + \bar{\pi}_1z \leq 0$  and  $-\pi_1z - \pi_{3|1}z + z < 0$  and  $\pi_1^2z + \pi_1\pi_{3|1}z - 2\pi_1z - \pi_{3|1}z + z \leq 0$  and  $\pi_1\pi_{3|1}z - \pi_1z + \pi_{3|1}^2z - 2\pi_{3|1}z + z \leq 0$  and  $\pi_1^2\pi_2 - \pi_1^2z + 2\pi_1\pi_2\pi_{3|1} - 2\pi_1\pi_2 - \pi_1\pi_{3|1}z + \pi_1z + \pi_2\pi_{3|1}^2 - 2\pi_2\pi_{3|1} + \pi_2 \leq 0$  and  $\pi_1^2\bar{\pi}_2 - \pi_1^2z + 2\pi_1\pi_{3|1}\bar{\pi}_2 - \pi_1\pi_{3|1}z - 2\pi_1\bar{\pi}_2 + \pi_1z + \pi_{3|1}^2\bar{\pi}_2 - 2\pi_{3|1}\bar{\pi}_2 + \bar{\pi}_2 \geq 0$  and  $-\pi_1^2z + \pi_1^2 - \pi_1\pi_{3|1}z + 2\pi_1\pi_{3|1} + \pi_1z - 2\pi_1 + \pi_{3|1}^2 - 2\pi_{3|1} + 1 \geq 0$  and  $\pi_1\pi_{3|2}z - \pi_1\bar{\pi}_{3|2}z + \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\bar{\pi}_{3|2}z - \pi_{3|2}z + \bar{\pi}_{3|2}z \leq 0$  and  $-\pi_1z - \pi_{3|1}z + z < 0$  and  $\pi_1\pi_{3|2}z - \pi_1z + \pi_{3|1}\pi_{3|2}z - \pi_{3|1}z - \pi_{3|2}z + z \leq 0$  and  $-\pi_1 - \pi_{3|1} + 1 \langle \rangle 0$  and  $\pi_1 < 1$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \geq 0$

**or**

$-\pi_1\pi_{3|2}\bar{\pi}_2 + \pi_1\pi_{3|2}z - \pi_{3|1}\bar{\pi}_2 + \pi_{3|2}\bar{\pi}_2 \leq 0$  and  $-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $\pi_1z - \pi_1 - \pi_{3|1}\bar{\pi}_2 + \bar{\pi}_2 \leq 0$  and  $-\pi_1\bar{\pi}_2 + \pi_1z - \pi_{3|1}\bar{\pi}_2 + \bar{\pi}_2 \geq 0$  and  $-\pi_1 - \pi_{3|1} + 1 \leq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $-\pi_1 - \pi_{3|1} + 1 \langle \rangle 0$  and  $-\pi_1 + \bar{\pi}_1 \geq 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$-\pi_1\pi_2\pi_{3|2} + \pi_1\pi_{3|2}z - \pi_2\pi_{3|1} + \pi_2\pi_{3|2} \leq 0$  and  $-\pi_{3|1} + \bar{\pi}_{3|1} \geq 0$  and  $\pi_1z - \pi_1 - \pi_2\pi_{3|1} + \pi_2 \leq 0$  and  $-\pi_1\pi_2 + \pi_1z - \pi_2\pi_{3|1} + \pi_2 \geq 0$  and  $-\pi_1 - \pi_{3|1} + 1 \leq 0$  and  $-\pi_{3|2} + \bar{\pi}_{3|2} \geq 0$  and  $-\pi_1 + \bar{\pi}_1 \geq 0$  and  $-\pi_1 - \pi_{3|1} + 1 \langle \rangle 0$  and  $\pi_1 < 1$  and  $-\pi_2 + \bar{\pi}_2 \geq 0$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$\pi_1\pi_{3|1}\pi_{3|2}z - \pi_1\pi_{3|2}\bar{\pi}_{3|1}z + \pi_{3|1}^2z - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\bar{\pi}_{3|1}z + \pi_{3|2}\bar{\pi}_{3|1}z \leq 0$  and  $\pi_1^2\pi_{3|2}z + \pi_1\pi_{3|1}z - \pi_1\pi_{3|2}\bar{\pi}_1z - \pi_1\pi_{3|2}z - \pi_{3|1}\bar{\pi}_1z + \pi_{3|2}\bar{\pi}_1z \leq 0$  and  $-\pi_1\pi_{3|2}z - \pi_{3|1}z + \pi_{3|2}z < 0$  and  $\pi_1^2\pi_{3|2}z + \pi_1\pi_{3|1}z - 2\pi_1\pi_{3|2}z - \pi_{3|1}z + \pi_{3|2}z \leq 0$  and  $\pi_1\pi_{3|1}\pi_{3|2}z - \pi_1\pi_{3|2}z + \pi_{3|1}^2z - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}z + \pi_{3|2}z \leq 0$  and  $-\pi_1^2\pi_{3|2}^2z + \pi_1^2\pi_{3|2}^2 + \pi_1\pi_{3|1}\pi_{3|2}z - 2\pi_1\pi_{3|1}\pi_{3|2}z + 2\pi_1\pi_{3|1}\pi_{3|2}z + \pi_1\pi_{3|2}^2z - 2\pi_1\pi_{3|2}^2z + \pi_{3|1}^2z + \pi_{3|2}^2z \geq 0$  and  $-\pi_1^2\pi_{3|2}^2z + \pi_1^2\pi_{3|2}^2 + \pi_1\pi_{3|1}\pi_{3|2}z - 2\pi_1\pi_{3|1}\pi_{3|2}z + 2\pi_1\pi_{3|1}\pi_{3|2}z + \pi_1\pi_{3|2}^2z - 2\pi_1\pi_{3|2}^2z + \pi_{3|1}^2z + \pi_{3|2}^2z \geq 0$  and  $\pi_1^2\pi_2\pi_{3|2}^2 - \pi_1^2\pi_{3|2}^2z + 2\pi_1\pi_2\pi_{3|1}\pi_{3|2} - 2\pi_1\pi_2\pi_{3|2}^2 - \pi_1\pi_{3|1}\pi_{3|2}z + \pi_1\pi_{3|2}^2z + \pi_2\pi_{3|1}^2 - 2\pi_2\pi_{3|1}\pi_{3|2} + \pi_2\pi_{3|2}^2 \leq 0$  and  $\pi_1^2\pi_{3|2}^2\bar{\pi}_2 - \pi_1^2\pi_{3|2}^2z + 2\pi_1\pi_{3|1}\pi_{3|2}\bar{\pi}_2 - \pi_1\pi_{3|1}\pi_{3|2}z - 2\pi_1\pi_{3|2}\bar{\pi}_2 + \pi_1\pi_{3|2}^2z + \pi_{3|1}^2\bar{\pi}_2 - 2\pi_{3|1}\pi_{3|2}\bar{\pi}_2 + \pi_{3|2}^2\bar{\pi}_2 \geq 0$  and  $\pi_1\pi_{3|2}^2z - \pi_1\pi_{3|2}\bar{\pi}_{3|2}z + \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\bar{\pi}_{3|2}z - \pi_{3|2}z + \pi_{3|2}\bar{\pi}_{3|2}z \leq 0$  and  $-\pi_1\pi_{3|2}z - \pi_{3|1}z + \pi_{3|2}z < 0$  and  $\pi_1\pi_{3|2}^2z - \pi_1\pi_{3|2}z + \pi_{3|1}\pi_{3|2}z - \pi_{3|1}z - \pi_{3|2}^2z + \pi_{3|2}z \leq 0$  and  $-\pi_1\pi_{3|2} - \pi_{3|1} + \pi_{3|2} \langle \rangle 0$  and  $-\pi_1 - \pi_{3|1} + 1 \langle \rangle 0$  and  $\pi_1 < 1$  and  $\pi_2 < 1$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$

**or**

$\pi_1\pi_{3|1}z - \pi_1\pi_{3|1} - \pi_1z + \pi_1 - \pi_{3|1}\bar{\pi}_1z + \pi_{3|1}\bar{\pi}_1 + \bar{\pi}_1z - \bar{\pi}_1 \leq 0$  and  $\pi_1\pi_{3|1}z - \pi_1\pi_{3|1} - \pi_1z + \pi_1 - \pi_{3|1}z + \pi_{3|1} + z - 1 \leq 0$  and  $\pi_1\pi_{3|1}z - \pi_1\pi_{3|1} - \pi_1z + \pi_1 + \pi_{3|1}^2z - \pi_{3|1}^2 - 2\pi_{3|1}z + 2\pi_{3|1} + z - 1 \geq 0$  and  $-\pi_1\pi_{3|1}\pi_{3|2}z + \pi_1\pi_{3|1}\pi_{3|2} + \pi_1\pi_{3|2}z - \pi_1\pi_{3|2} + \pi_{3|1}^2\pi_{3|2}z - \pi_{3|1}^2z + \pi_{3|1}^2 - \pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} + \pi_{3|1}z - \pi_{3|1} + \pi_{3|2} \leq 0$  and  $-\pi_1\pi_{3|1}z + \pi_1\pi_{3|1} + \pi_1z - \pi_1 + \pi_2\pi_{3|1}^2 - 2\pi_2\pi_{3|1} + \pi_2 \leq 0$  and  $-\pi_1\pi_{3|1}z + \pi_1\pi_{3|1} + \pi_1z - \pi_1 + \pi_{3|1}^2\bar{\pi}_2 - 2\pi_{3|1}\bar{\pi}_2 + \bar{\pi}_2 \geq 0$  and  $-\pi_1\pi_{3|1}z + \pi_1\pi_{3|1} + \pi_1z - \pi_1 + \pi_{3|1}^2 - 2\pi_{3|1} + 1 \geq 0$  and  $-\pi_1 - \pi_{3|1} + 1 \langle \rangle 0$  and  $\pi_1 < 1$  and  $\pi_2 < 1$  and  $\pi_{3|1}^2z - \pi_{3|1}^2 - \pi_{3|1}\bar{\pi}_{3|1}z + \pi_{3|1}\bar{\pi}_{3|1} - \pi_{3|1}z + \pi_{3|1} + \bar{\pi}_{3|1}z - \bar{\pi}_{3|1} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1}\pi_{3|2}z - \pi_{3|1}\pi_{3|2} - \pi_{3|1}\bar{\pi}_{3|2}z + \pi_{3|1}\bar{\pi}_{3|2} - \pi_{3|2}z + \pi_{3|2} + \bar{\pi}_{3|2}z - \bar{\pi}_{3|2} \leq 0$  and  $-\pi_{3|1}z + \pi_{3|1} + z - 1 < 0$  and  $\pi_{3|1} < 1$  and  $\pi_{3|2} < 1$  and  $\bar{\pi}_1 < 1$  and  $\bar{\pi}_{3|1} < 1$  and  $\bar{\pi}_{3|2} < 1$  and  $z \leq 1$