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Endogenous Changes in Tastes

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Endogenous Changes in Tastes

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Résumé/abstract

Les parents se soucient de leurs enfants, dépensent des ressources pour les éduquer, ils leur lèguent un capital physique et/ou humain. Ici, nous montrons que les actions des parents ont une influence, non seulement sur les possibilités de production ouvertes aux enfants (par le biais du capital dont ils héritent), mais aussi sur leurs goûts et leurs attitudes. Nous étudions la dynamique résultant de cette idée. Plusieurs influences sont identifiées mais un résultat surprenant est que, si l'héritage a une grande influence sur l'attitude des gens envers le travail, l'évolution du stock de capital et du travail devient totalement imprévisible après quelques générations.

Mots clés : évolution de préférences, legs, chaos.

Parents care about their children, spend resources on educating them and bequeath them some physical and/or human capital. Here we argue that the actions of the parents have an influence, not only on the productive possibilities open to the children (through the capital they inherit) but also on their tastes and attitudes. We investigate the resulting dynamics of this idea. Several influences are identified but one startling result is that, if inheritance has a large influence on people's attitude to work, the pattern of behavior becomes completely unpredictable after a few generations.

Key words: Endogenous preferences, bequests, chaos.

Codes JEL : D11

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1 Introduction

Parents care about their children, spend resources on educating them and bequeath them some physical and/or human capital. Here we argue that the actions of the parents have an influence, not only on the productive possibilities open to the children (through the capital they inherit) but also on their tastes and attitudes. This has been an accepted notion throughout history, although it was sometimes referred to as "good family background" or the reverse. We formalise this idea here and we investigate the resulting dynamics.¹

2 The Model

For simplicity we treat the parents as a single unit or individual, and the children also. In period t an individual has inherited capital b_t . They choose their consumption c_t , their labor l_t , and the bequests to their children b_{t+1} so as to maximize their own utility which takes the form of a modified Stone-Geary utility function where, for clarity, all parameters are positive

$$V_t = \ln(c_t - x - \delta b_t) - \gamma (b_t)^\epsilon l_t + \beta \ln(b_{t+1} - \theta b_t) \quad (1)$$

We restrict θ to be less than 1. If θ were greater than or equal to 1, the system would collapse unless the path of bequeathed wealth were forever increasing, precluding the possibility of fluctuations. The restriction $\theta < 1$ does not preclude the monotonicity of the time path of wealth, nor does it preclude fluctuations. The dynamic behavior of wealth depends on the relationship between θ and other parameters, as we shall see.

¹The notion that parents can intentionally or unintentionally influence the preferences of their children has received some attention in the economic literature; see e.g. Becker (1993), Bisin and Verdier (2001), Bala and Long (2005).

We model the effect of inherited wealth on tastes by assuming that what individuals consider the minimum acceptable level of consumption consists of a fixed consumption level x and a proportion δ of their own inherited wealth. This captures the idea that people born to riches are more demanding than people born poor. Wealth also affects their attitude to work: the wealthier they are the less they are inclined to work, or rather, they find it more painful or difficult. The last term in the utility function simply reflects the fact that they care for their children's well-being in the next period and wish to endow them; in so doing, they are also influenced by the inheritance they received from their parents. They are motivated by trying to improve their offsprings' situation relative to their own. Therefore the feelings of parents towards their children's wellbeing is a double-edged sword. Their caring entices them to endow their children well, but at the same time nourishes high expectations that may work to their detriment. There is, however, a very positive aspect to inherited high levels of capital. The children can earn more.

Income is earned from working and their capacity to earn is determined by their inherited wealth, b_t . However the new generation earns its living, it is undeniable that they obtain it from either skills or productive investments, usually a combination of both. Therefore the following formulation is appropriate

$$y_t = A(b_t)^\alpha l_t \quad (2)$$

where $0 < \alpha \leq 1$. One could argue as did Rebello (1990) to introduce his *AK* model, that b_t is a combination of human and physical capital and that α is equal to 1 under constant returns to scale.

Their budget constraint is

$$y_t = c_t + b_{t+1} \quad (3)$$

The first order conditions are

$$\frac{1}{c_t - x - \delta b_t} = \lambda_t \quad (4)$$

$$\frac{\beta}{b_{t+1} - \theta b_t} = \lambda_t \quad (5)$$

$$\gamma(b_t)^\epsilon = \lambda_t A(b_t)^\alpha \text{ or } \lambda_t = \frac{\gamma}{A}(b_t)^{\epsilon-\alpha} \quad (6)$$

From (5) and (6)

$$c_t - x - \delta b_t = A(b_t)^\alpha / \gamma(b_t)^\epsilon \quad (7)$$

$$b_{t+1} = \theta b_t + \frac{\beta A}{\gamma}(b_t)^{\alpha-\epsilon} \quad (8)$$

Note that it is necessary that $b_t - \theta b_{t-1} > 0$, for all t , for the utility function to be defined. This is always true in our formulation.

From (8) and (7) we have

$$c_t = x + (\delta - \theta/\beta)b_t + b_{t+1}/\beta = x + \delta b_t + \frac{A}{\gamma}(b_t)^{\alpha-\epsilon} \quad (9)$$

and with (3) we have

$$y_t = x + (\delta + \theta)b_t + \left(\frac{1+\beta}{\beta}\right)(b_t)^{\alpha-\epsilon} \quad (10)$$

and

$$l_t = \frac{y_t}{A(b_t)^\alpha} \quad (11)$$

$$l_t = \frac{x(b_t)^{-\alpha}}{A} + \frac{(\delta + \theta)(b_t)^{1-\alpha}}{A} + \left(\frac{1+\beta}{A\gamma}\right)(b_t)^{-\epsilon}$$

REMARKS: The dynamics of the model are given by the non-linear first-order difference equation (8). For parameter values such that $0 < \epsilon < \alpha \leq 1$, the path of b_t is monotone and the model quickly converges to a steady

state. It is not our purpose here to investigate cases where children are not much influenced by their education or inheritance, quite the opposite. For parameter values such that $\epsilon > \alpha$, the path of b_t may not be monotone. The paths of the other variables follow according to equations (9)-(11). Neither consumption nor income depend monotonically on inheritance (as long as $\epsilon > \alpha$) and labour does not always decrease with inheritance, unless $\alpha = 1$. However, for most parameter values, consumption and income are very strongly positively correlated with inheritance while work is very strongly negatively correlated.

3 Dynamics

The main relation is (8) and the one-period steady state is²

$$\bar{b} = \left(\frac{\beta A}{\gamma(1-\theta)} \right)^{\frac{1}{1-\alpha+\epsilon}} \quad \text{or} \quad (\bar{b})^{1-\alpha+\epsilon} = \frac{\beta A}{\gamma(1-\theta)} \quad (12)$$

The behaviour of the state variable around the steady state can be precisely analyzed by taking the derivative of (8), and evaluate it at the steady state.³ We obtain

$$\frac{db_{t+1}}{db_t} = \theta + \frac{\beta A}{\gamma}(\alpha - \epsilon)(b_t)^{\alpha-\epsilon-1}$$

When the slope equals -1 , the system becomes unstable. Using (12), we have the value for ϵ that does this

$$\begin{aligned} -1 &= \theta + \frac{\beta A}{\gamma}(\alpha - \epsilon) \frac{\gamma(1-\theta)}{\beta A} = \theta + (\alpha - \epsilon)(1-\theta), \\ 1 + \theta &= (\epsilon - \alpha)(1-\theta) \\ \epsilon &= \alpha + \frac{1+\theta}{1-\theta} \end{aligned} \quad (13)$$

²There is another steady state at $b = 0$, which we disregard.

³Chang et al. (1983) analysed the stability property of a simple overlapping generations model. They did not consider chaotic behavior.

The system is asymptotically stable when $\epsilon \in (\alpha, \alpha + \frac{1+\theta}{1-\theta})$. The bifurcation occurs at the upper bound. Note that the value of ϵ at which the bifurcation occurs is an increasing function of θ . Therefore the more parents care about their children's wealth (high θ), the more unlikely it is that chaotic behaviour would occur, because the range of value of ϵ that is consistent with stability is larger.

The two-period steady state $\bar{B}$ is described by

$$1 - \theta^2 = z [\theta + (\theta + z)^{\alpha-\epsilon}], \text{ where } z = \frac{\beta A}{\gamma} (\bar{B})^{\alpha-\epsilon-1} \quad (14)$$

This equation has no analytical solution for arbitrary parameter values and we need to resort to simulations to analyse the model.⁴

We choose $A = 10, \alpha = 1, \beta = \gamma = \theta = 0.5, \delta = 0.1$ and let ϵ take a range of values. Equation (14) has a single solution for small ϵ values and the paths of all variables are monotone after a few periods. Things change when ϵ reaches the bifurcation value, 4 in this example. There are now three steady states in (14). The solutions to that equation are illustrated in Figures 1-3 for ϵ values of 2, 4 and 6. The most interesting case is when $\epsilon = 6$ (or larger). Figures 4-7 show b_{t+1} through to b_{t+4} , respectively, against b_t . There clearly is period-doubling. In Figures 8 and 9, we depict b_{t+10} and b_{t+20} . The b -values are bounded and fill the space. These figures have been drawn using 1000 periods. This clearly points to chaotic behaviour of the system. Furthermore, these simulations have been done using the initial condition $b_0 = 1.01\bar{b}$, therefore very close to the one-period steady state. When altering the initial condition very slightly to $b_0 = 1.011\bar{b}$, there is a definite change of pattern after a few periods. This is illustrated in Figure 10, which is to be compared with Figure 8. Numerically, this slight change in the initial condition makes b_{t+10} change from 2.39 to 1.797 and b_{t+20} change

⁴For analysis of chaotic behavior in non-linear dynamical systems in discrete time, see Devalney (1983) and Whitley (1983).

from 1.5165 to 3.2956, in the opposite direction. This sensitivity to initial conditions is another indication of chaotic behaviour.⁵

We have experimented with other parameter values. Surprisingly the value of α (from 0.05 to 1) makes little difference to the results. The value of θ does in the sense that higher values of θ require higher values of ϵ for the chaotic behaviour to develop, as we indicated before the simulation; the pattern remains the same, though. Figure 11 shows the evolution of l_t against b_t . Larger bequests induce children to work less hard but this effect bottoms out after a while and the "mega wealthy" work about the same as the "very wealthy". Poorly endowed children work much harder. Consumption and inheritance move together as shown in Figure 12. In Figure 13 we have depicted the same pattern, with the slight change in initial condition indicated earlier. It confirms that the behaviour of the system is impossible to predict in practice.

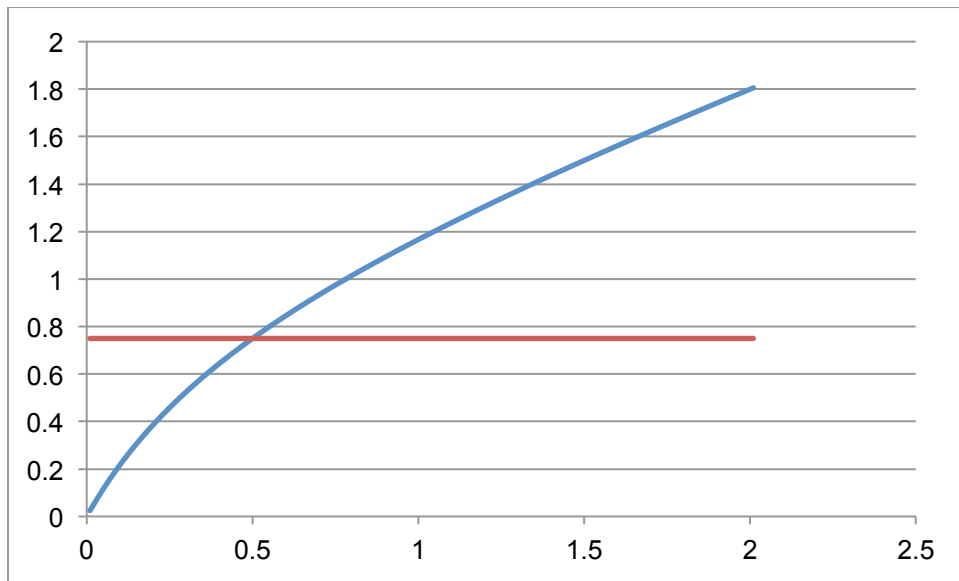
4 Conclusion

In conclusion, we have demonstrated that the changes in tastes due to the size of the inheritance profoundly affect the behaviour of future generations. The more poorly endowed work harder but the pattern soon becomes very irregular if the effect of the size of the inheritance on the attitude to work is large. To paraphrase a well known saying, "Even if we understand history, it is not condemned to repeat itself."

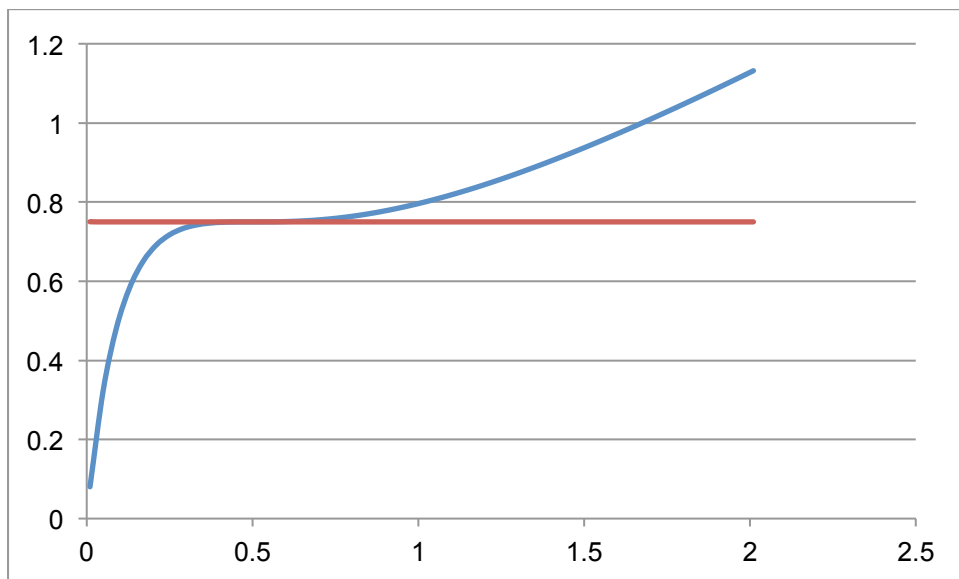
⁵Indeed, when doing simulations on Excel, which has substantial, but finite capacities of calculation, if we start at $\bar{b}$, the system does not stay there and goes into chaotic mode after 40 periods or so. This is because $\bar{b} = (20)^{\frac{1}{6}}$, which Excel cannot calculate with perfect accuracy.

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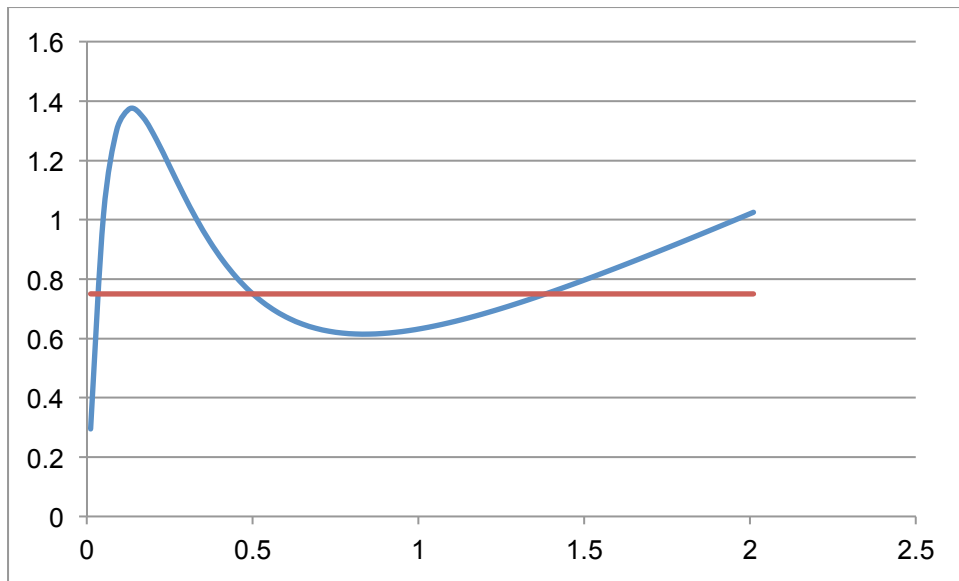
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SS2 with epsilon = 2
Figure1



SS2 with epsilon = 4
Figure2



SS2 with epsilon = 6
Figure3

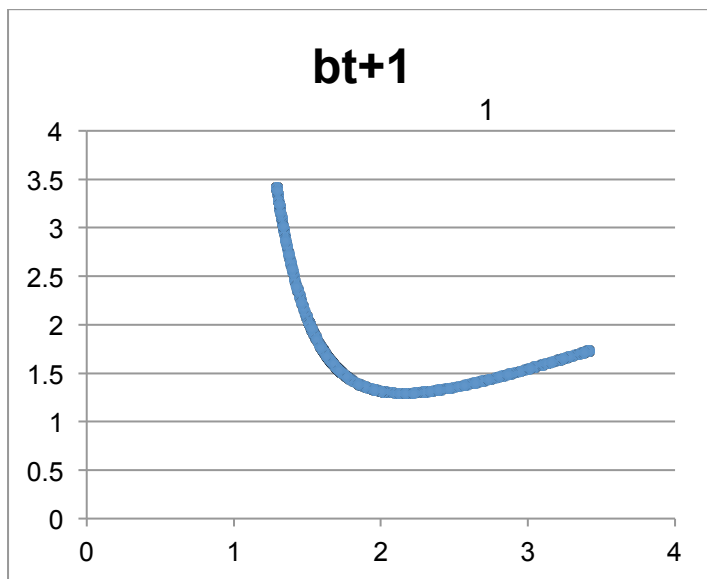


Figure 4

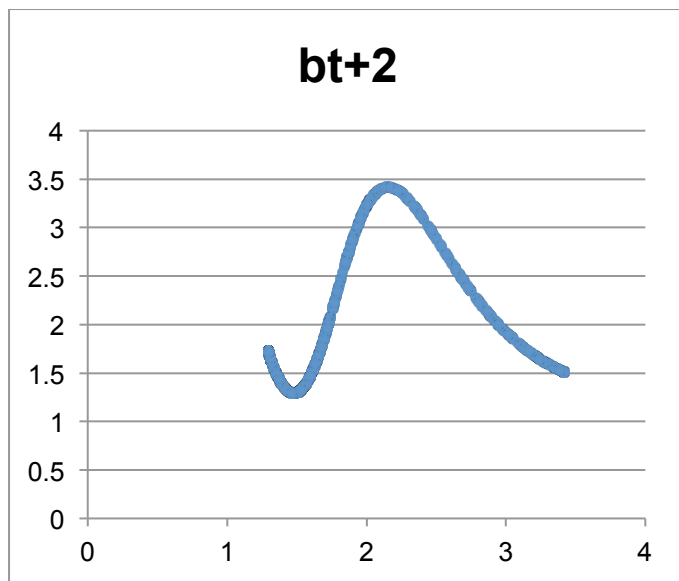


Figure 5

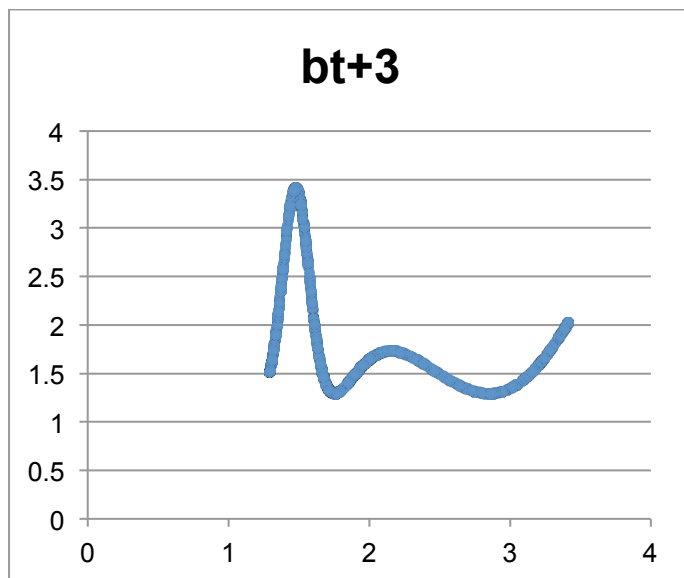


Figure 6

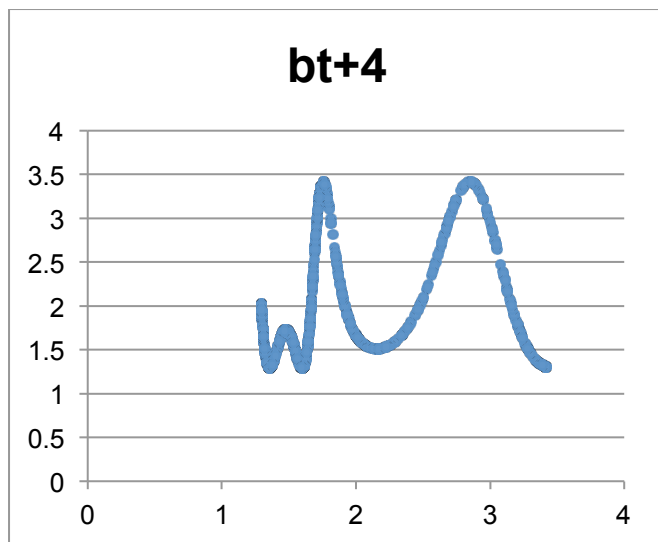


Figure 7

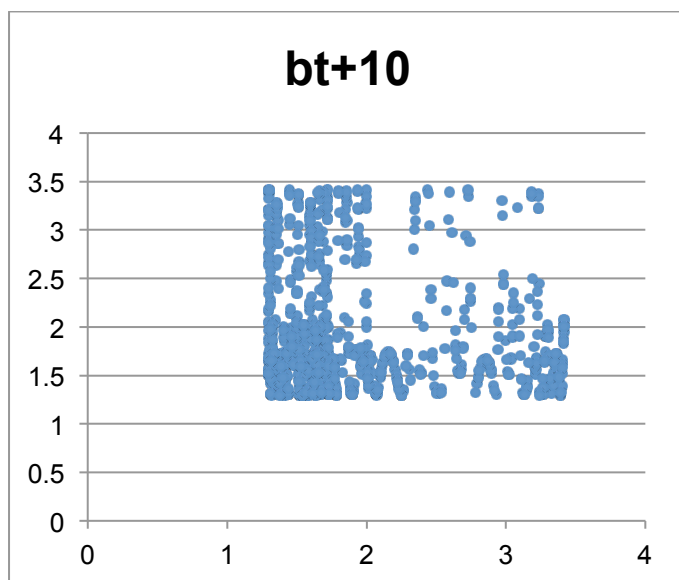


Figure 8

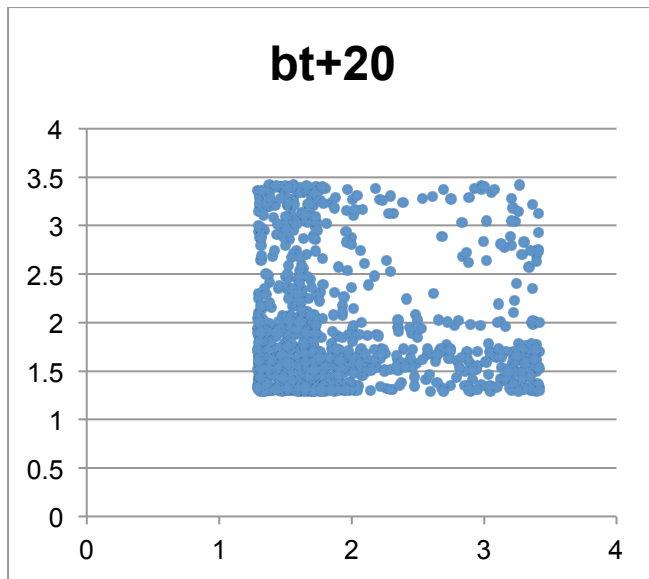
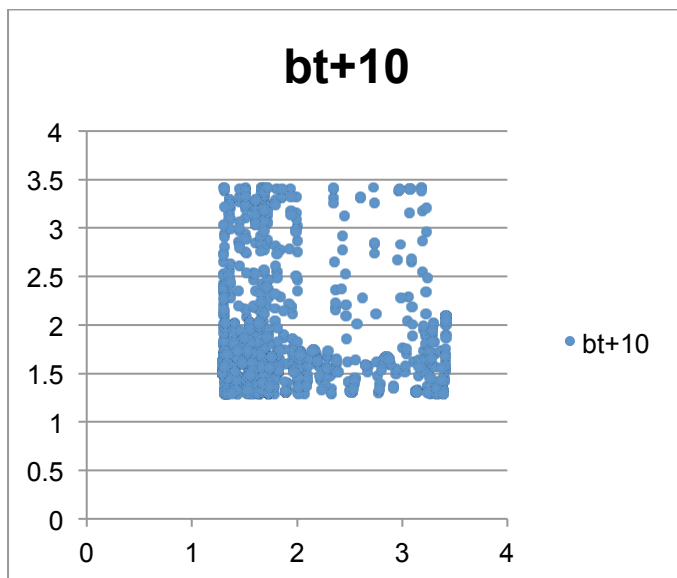


Figure 9



Slight change in initial condition

Figure 10

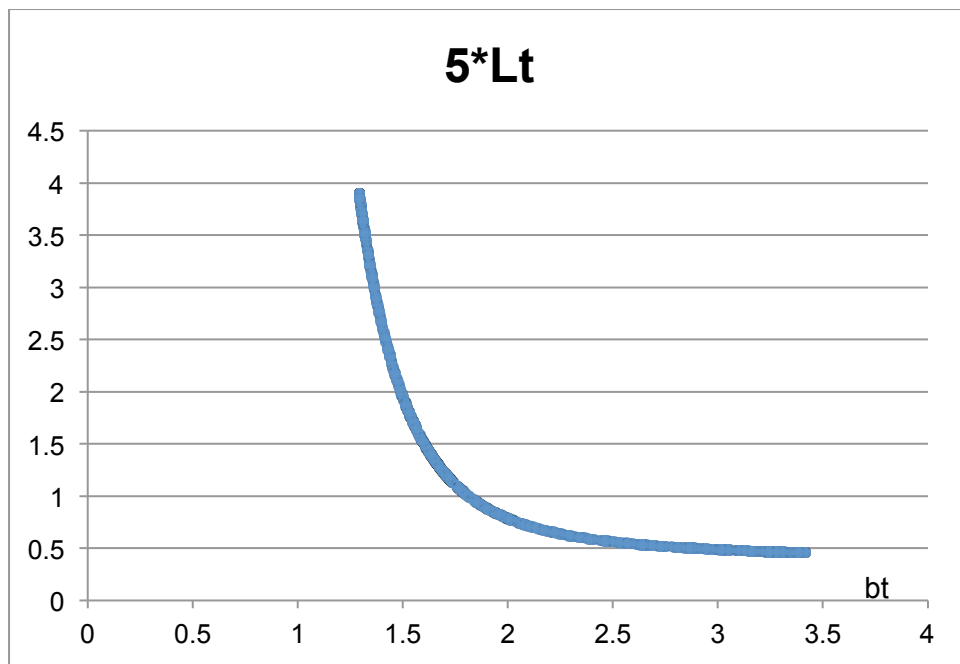


Figure 11

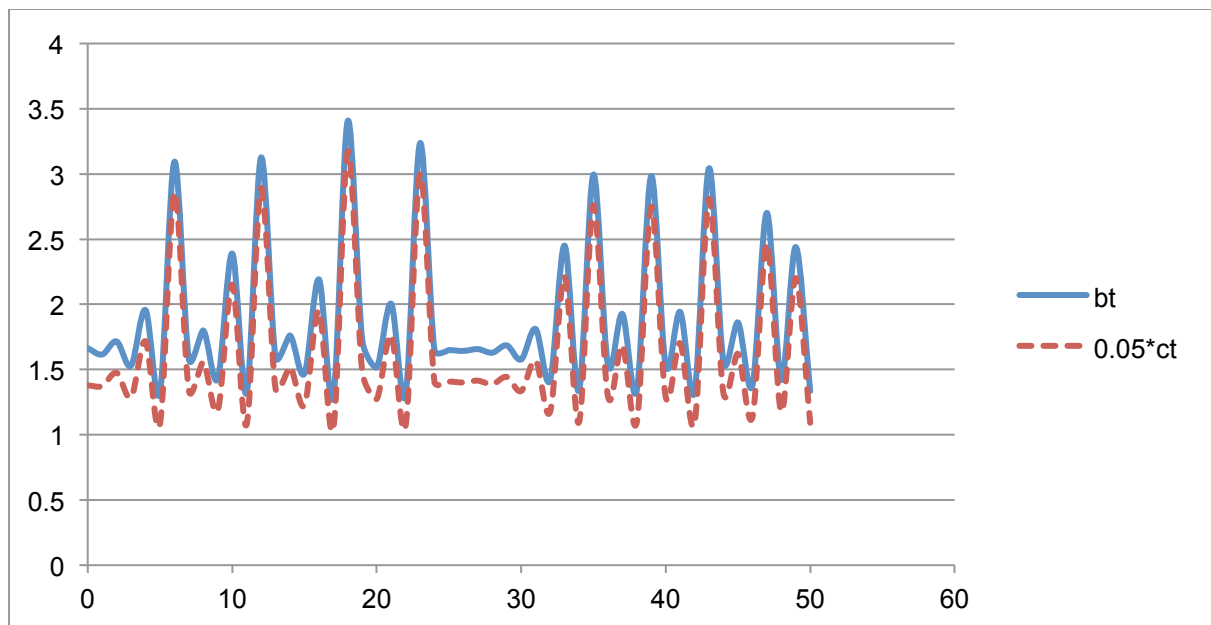
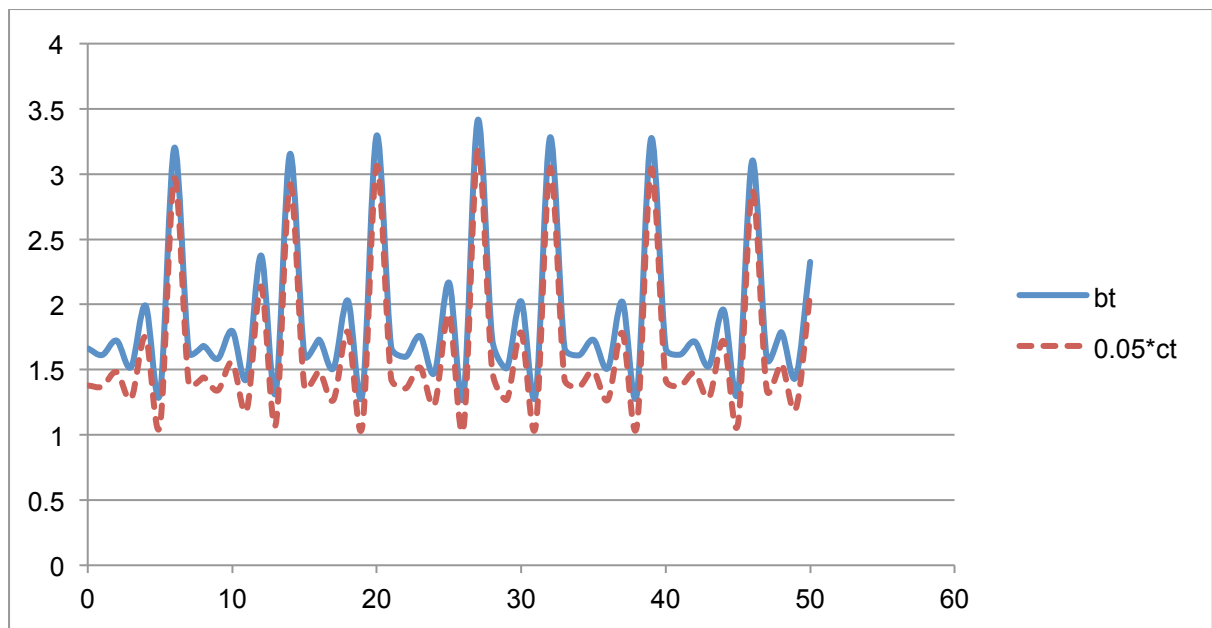


Figure 12



Slight change in initial condition

Figure 13