

MATHEMATICS

MTH-5081-2 — Trigonometry II

DEFINITION OF THE DOMAIN FOR
SUMMATIVE EVALUATION

SEPTEMBER 1997

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SUMMATIVE EVALUATION**

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Direction de la formation générale des adultes
Service de l'évaluation des acquis

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LES ÉVALUÉS...
MONTAGNE...

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1. Introduction

This definition of the domain for summative evaluation describes and classifies the essential and representative elements of the *Mathematics* program—specifically, for module *Trigonometry II*. It presents an overview of the program, but should by no means replace the program itself. The purpose of defining the domain is to ensure that all summative evaluation instruments are consistent with the overall program.

The organization of this definition of the domain is the same as that of those of other modules. The content of each section is, however, specific to this module.

The goal of the definition of the domain for summative evaluation is to permit the preparation of examinations that are valid from one version to another, from year to year and from one school board to another, taking into account the responsibilities shared by the ministère de l'Éducation and the school boards.

2. Program Orientations and Consequences for Summative Evaluation**Orientations**

The main purpose of the adult education secondary level *Mathematics* program is to help adults understand the mathematical concepts needed to solve problems related to everyday situations and expand their knowledge of mathematics. Consequently, in the program, mathematics is presented as a practical tool for solving common, real-life problems.

This module, however, is more concerned with the theoretical aspects of trigonometric functions and identities.

All the learning activities in the program emphasize the acquisition of a systematic work method.

The program places equal emphasis on mastering the use of a calculator.

Consequences

During evaluation, special attention should be paid to the analysis of a trigonometric function (e.g. solution set, increase, decrease) and to the appropriate application of trigonometric laws in the simplification and proof of simple or complex trigonometric identities.

Evaluation should measure the adult's ability to follow the steps involved in proving simple or complex trigonometric identities.

The use of a calculator is permitted.

3. Content of the Program for Purposes of Summative Evaluation**Concepts**

- **Wrapping function and measures of angles and arcs**
 - Conversion from degrees to radians and vice versa
 - Calculation of the measure of a central angle
 - Calculation of the measure of an arc intercepted by a central angle
 - Image of a trigonometric point

- **Trigonometric and sine functions**
 - Image of a real number expressed in radians
 - Graph of the two functions

- **Trigonometric identities**
 - Calculation of the value of a trigonometric function
 - Image of a real number that can be expressed as a sum or difference of two real numbers expressed in radians
 - Proof of simple trigonometric identities
 - Proof of complex trigonometric identities

Skills

Each skill is defined within the context of a mathematics program. Given that the adult education *Mathematics* program corresponds to the mathematics programs in the youth sector, the skills involved are the same for students in both sectors.

- **Operating:** Performing a given operation or transformation.
Possible actions: calculating, constructing, breaking down, performing, estimating, evaluating, isolating, measuring, reconstructing, solving, drawing, transforming, verifying, and so on

- **Analyzing or Synthesizing:** Establishing a link between a problem and a given solution or solving a given problem.
Possible actions: concluding, deducing, deriving, explaining, extrapolating, inferring, justifying, proving, solving, transferring, and so on

4. Table of Dimensions

SKILLS	CONCEPTS	WRAPPING FUNCTION AND MEASURE OF ANGLES AND ARCS 16%	TRIGONOMETRIC AND SINE FUNCTIONS 34%	TRIGONOMETRIC IDENTITIES 50%
OPERATING 64%	1 Conversion of angular measures from degrees to radians and vice versa 2%	5 Determination of the image of a real number expressed in radians for a trigonometric function 8%	8 Calculation of the value of a trigonometric function using fundamental trigonometric identities 7%	
	2 Calculation of the measure of a central angle given the radius of a circle and the measure of the arc intercepted by that angle 3%	6 Graph of a trigonometric function 12%	9 Proof of the image of a real number that can be expressed as a sum or difference of two real numbers 7%	
	3 Calculation of the measure of an arc intercepted by a central angle given the radius of a circle and the measure of that angle 3%	7 Graph of a sine function 14%		
	4 Proof of the image of a trigonometric point using the wrapping function 8%			
ANALYZING OR SYNTHESIZING 36%			10 Proof of a simple trigonometric identity 16 %	
			11 Proof of a complex trigonometric identity 20%	

Note: The numbers 1 to 11 identify the dimensions.

5. Observable Behaviours

Examination items should be formulated on the basis of the observable behaviours listed below. The requirements and restrictions specified in the objectives of the program must be observed.

Dimension 1

Converting angular measures from degrees to radians and from radians to degrees, given central angles in a unit circle. The degree measures range from 0° to 720° and the radian measures range from 0 to 4π .

Dimension 2

Determining the degree or radian measure of a central angle, given the radius of a circle and the measure of the arc intercepted by that angle.

Dimension 3

Determining the measure of an arc intercepted by a central angle, given the radius of a circle and the measure of that angle.

Dimension 4

Specifying, for an interval of the form $[0, 2\pi] + 2k\pi/n$, where $n \in \{1, 2, 3, 4, 6, 8, 12\}$ and $k \in \mathbb{Z}$, the image of a trigonometric point using the wrapping function. The trigonometric points are of the form $a\pi/b$, where $a \in \mathbb{Z}$ and $b \in \{1, 2, 3, 4, 6\}$.

Dimension 5

Finding the image of a real number expressed in radians for one of the following trigonometric functions: sine, cosine, tangent, cotangent, secant, cosecant. The real number is expressed in the form $a\pi/b$, where $a \in \mathbb{Z}$ and $b \in \{1, 2, 3, 4, 6\}$, or as a decimal number.

Dimension 6

Graphing one of the six trigonometric functions in a given interval and specifying the period, the maximum and minimum, the zeros, the intervals in which the function is increasing and decreasing, the asymptotes if applicable, the domain and the range.

Dimension 7

Graphing a sine function of the form $y = A \sin (Bx - h)$ or $y = A \cos (Bx - h)$ in a given interval, where A and $B \in \mathbb{Z}^*$ and h is a rational number of the form $a\pi/b$, where $b \neq 0$. The amplitude, period and phase shift of the function must be specified.

Dimension 8

Given the value of a trigonometric function at a point in a designated interval, finding the value of another trigonometric function at that point, using one of the following fundamental trigonometric identities: $\sin^2x + \cos^2x = 1$; $\tan^2x + 1 = \sec^2x$; $\cotan^2x + 1 = \operatorname{cosec}^2x$.

Dimension 9

For a given trigonometric function, finding the image of a real number that can be expressed as a sum or difference of two real numbers of the form $a\pi/b$, where $a \in \mathbb{Z}$ and $b \in \{1, 2, 3, 4, 6\}$. The problems involve the sine, cosine and tangent functions; the formulas required to evaluate trigonometric functions involving a sum or difference of two real numbers are provided.

Dimension 10

Proving a simple trigonometric identity by using the definitions of trigonometric functions. There should be no more than two terms on each side of the identity and each term should contain no more than two trigonometric functions.

Dimension 11

Proving a complex trigonometric identity by using the definitions of trigonometric functions. There should be no more than two terms on each side of the identity and each term should contain no more than two trigonometric functions. The basic formulas needed to transform trigonometric functions involving the sum or difference of sine, cosine and tangent functions are provided.

6. Explanation of Content and Weighting

Given the particular content of this module on the graphic representation of trigonometric functions and the application of the trigonometric laws to the wrapping function and the measure of angles and arcs, emphasis has been placed on the skill of operating.

However, in order to measure the students' understanding of certain concepts and laws related to trigonometric identities, the skills of analyzing and synthesizing have also been included.

Finally, in order to ensure that the students are able to follow a problem-solving method, emphasis has been placed on the process of proving simple and complex trigonometric identities.

The weighting of the skills listed below is based on the program itself and on the time normally required to master these skills.

OPERATING	64%
ANALYZING OR SYNTHESIZING	36%

In accordance with the program, the trigonometric identities have been weighted more heavily because of the importance given the skills of analyzing and synthesizing.

WRAPPING FUNCTION AND MEASURE OF ANGLES AND ARCS	16%
TRIGONOMETRIC AND SINE FUNCTIONS	34%
TRIGONOMETRIC IDENTITIES	50%

7. Description of the Examination**7.1 Type of Examination**

There will be a written examination consisting of items that will be scored subjectively (free-response or extended-response items). Some items may be scored objectively.

7.2 Characteristics of the Examination

- The examination must be taken in a single session lasting no more than two hours.
- The distribution of marks should be consistent with the percentages indicated in the table of dimensions.
- Students are permitted to use a scientific calculator, a trigonometric table and a "rader."
- The requirements and restrictions specified in the objectives of the program must be observed.
- A list of complex trigonometric identities will be provided (see appendix).

7.3 Pass Mark

To pass the module, students must obtain 60 out of 100 on the examination.

APPENDIX

TRIGONOMETRIC IDENTITIES INVOLVING THE SUM OR DIFFERENCE OF TWO REAL NUMBERS

$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad (\text{where } 1 - \tan A \tan B \neq 0)$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \quad (\text{where } 1 + \tan A \tan B \neq 0)$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \quad (\text{where } 1 - \tan^2 A \neq 0)$$