Definition of the Domain for Summative Evaluation

MTH-4105-1

Mathematics
Exponents and Radicals

Québec

Definition of the Domain for Summative Evaluation

MTH-4105-1

Mathematics
Exponents and Radicals

Formation professionnelle et technique
et formation continue

Direction de la formation générale
des adultes

ISBN 2-550-43540-0

Legal deposit — Bibliothèque nationale du Québec, 2004

1. INTRODUCTION

This Definition of the Domain for Summative Evaluation describes and classifies the essential and representative elements of the secondary-level adult education Mathematics program and, more specifically, of the course entitled Exponents and Radicals. As such, it gives an overview of the program, but should by no means replace the program itself. The purpose of defining the domain is to ensure that all summative evaluation instruments are consistent with the overall program.

The Definition of the Domain for Summative Evaluation for each course in this program is organized in a similar manner; however, the content of this definition of the domain is specific to the course entitled Exponents and Radicals.

The goal of the Definition of the Domain for Summative Evaluation is to prepare examinations that are valid from one version to another or from one school board to another, taking into account the responsibilities shared by the Ministère de l'Éducation and the school boards.

2. PROGRAM ORIENTATIONS AND CONSEQUENCES FOR SUMMATIVE EVALUATION

ORIENTATIONS	CONSEQUENCES
The main objective of the secondary-level adult education Mathematics program is to help students fully understand mathematical concepts.	Evaluation should involve verifying whether the student has fully understood the different concepts.
The program also aims to improve the students' ability to clearly relate information using mathematical language.	Evaluation items should involve performing tasks that require the use of mathematical language. The appropriateness and clarity of the language used should be taken into account in the marking process.
The program is intended to help students develop a systematic work method.	Evaluation items should require the students to present their work in a clear and structured manner. This should be taken into account in the marking process.
The program will help students master the use of technological tools.	The use of a scientific calculator is permitted for the examinations related to this course.

3. CONTENT OF THE PROGRAM FOR PURPOSES OF SUMMATIVE EVALUATION

Concepts

Exponents

- multiplying the product of two monomials of the form $\left(a^m b^n c^p\right)^r$
- dividing two monomials of the form $\left(a^m b^n c^p\right)^r$
- simplifying an algebraic expression containing a numerator and a denominator of the form $a^m b^n c^p$
- multiplying two algebraic or numerical expressions that can be written in the form $\left(\frac{a^m}{b^n}\right)^p$ or $\left(\frac{b^n}{a^m}\right)^p$
- dividing two algebraic or numerical expressions that can be written in the form $\left(\frac{a^m}{b^n}\right)^p$ or $\left(\frac{b^n}{a^m}\right)^p$
- multiplying two numerical expressions, one written in scientific notation and one written in decimal form
- dividing two numerical expressions, one written in scientific notation and one written in decimal form
- determining whether two numerical expressions written in exponential form are equivalent
- determining whether two algebraic expressions written in exponential form are equivalent
- determining the sign and the value of expressions of the form a^m

Radicals

- adding or subtracting two or three terms of the form $a\sqrt{b}$
- multiplying two binomials of the form $\left(a\sqrt{b}+c\right)$
- dividing two terms of the form $a\sqrt{b}$
- dividing a binomial of the form $\left(a\sqrt{b}+c\right)$ divided by a monomial of the form $a\sqrt{b}$
- dividing a monomial of the form $a\sqrt{b}$ divided by a binomial of the form $\left(a\sqrt{b}+c\right)$
- determining whether two numerical expressions containing terms of the form $a\sqrt{b}$ are equivalent

Exponents and radicals

– determining whether two algebraic expressions that can be written in the form $a^{\frac{m}{n}}\sqrt{a^{p}}$ are equivalent
– determining whether two numerical expressions that can be written in the form $a\sqrt{b}$ *or* $\left(ab^{n}\right)^{m}$ are equivalent

Skills

Each skill is defined within the context of a mathematics program.

Operating — Performing a given operation or transformation.

Possible actions: to calculate, construct, break down, perform, estimate, evaluate, isolate, measure, reconstruct, solve, draw, transform, verify, and so on.

Analyzing — Demonstrating, in an organized fashion, the complex connections between concepts or definitions and their related actions and illustrations.

Possible actions: to conclude, correct, deduce, derive, demonstrate, explain, extrapolate, infer, justify, and so on.

4. TABLE OF DIMENSIONS

<table>
<tr><th>CONCEPTS

SKILLS</th><th>EXPONENTS

55%</th><th>RADICALS

45%</th></tr>
<tr><td rowspan="5">OPERATING
45%</td><td>Multiply two monomials of the form $\left(a^m b^n c^p\right)$.

1 5%</td><td></td></tr>
<tr><td>Divide two monomials of the form $\left(a^m b^n c^p\right)$.

2 5%</td><td>Add or subtract two or three terms of the form $a\sqrt{b}$.

11 5%</td></tr>
<tr><td>Simplify an algebraic expression containing a numerator and a denominator of the form $a^m b^n c^p$.

3 5%</td><td>Multiply two binomials of the form $\left(a\sqrt{b}+c\right)$.

12 5%</td></tr>
<tr><td>Multiply or divide two algebraic or numerical expressions that can be written in the form $\left(\frac{a^m}{b^n}\right)^p$ or $\left(\frac{b^n}{a^m}\right)^p$.

4 5%</td><td>Divide two terms of the form $a\sqrt{b}$

or

Divide a binomial of the form $\left(a\sqrt{b}+c\right)$ by a monomial of the form $a\sqrt{b}$.

13 5%</td></tr>
<tr><td>Multiply or divide, in scientific notation, two numerical expressions, one written in scientific notation and one written in decimal form.

5 5%</td><td>Divide a monomial of the form $a\sqrt{b}$ by a binomial of the form $\left(a\sqrt{b}+c\right)$.

14 5%</td></tr>
<tr><td rowspan="5">ANALYZING
55%</td><td>Determine whether two numerical expressions written in exponential form are equivalent.

6 10%</td><td></td></tr>
<tr><td>Given algebraic expressions that can be written in the form $a^m b^n$, determine which are equivalent.

7 10%</td><td></td></tr>
<tr><td>Determine whether statements related to the sign or the value of expressions of the form a^m, in which $0 < a < 1$ are true or false.

8 10%</td><td>Determine whether two numerical expressions containing terms of the form $a\sqrt{b}$ are equivalent.

15 10%</td></tr>
<tr><td colspan="2">Determine whether two algebraic expressions that can be written in the form $a^{\frac{m}{n}\sqrt{a^p}}$ are equivalent.

9 5%</td></tr>
<tr><td colspan="2">Given numerical expressions that can be written in the form $a\sqrt{b}$ or $\left(ab^n\right)^m$, determine which are equivalent.

10 10%</td></tr>
</table>

5. OBSERVABLE BEHAVIOURS

Examination items should be formulated on the basis of the observable behaviours listed below. The requirements and restrictions specified in the dimensions and the objectives of the program must be observed.

Dimension 1

Multiply two monomials of the form $\left(a^m b^n c^p\right)^r$, where a is an integer or a variable, b and c are variables and exponents m, n, p and r are rational numbers. If a is a number, it must have the same base in each monomial. The simplification process involves the application of no more than four laws of exponents. The exponents in the answer must be positive.
(operating) /5

Dimension 2

Divide two monomials of the form $\left(a^m b^n c^p\right)^r$, where a is an integer or a variable, b and c are variables and exponents m, n, p and r are rational numbers. If a is a number, it must have the same base in each monomial. The simplification process involves the application of no more than four laws of exponents. The exponents in the answer must be positive.
(operating) /5

Dimension 3

Simplify a rational algebraic expression containing a numerator and a denominator of the form $a^m b^n c^p$, where a is an integer or a variable, b and c are variables and exponents m, n and p are rational numbers. If a is a number, it must have the same base in each monomial. The expression will be assigned a rational exponent. The simplification process involves the application of no more than four laws of exponents. The exponents in the answer must be positive.
(operating) /5

Dimension 4

Multiply or divide two algebraic or numerical expressions. The expressions can be written in the form $\left(\frac{a^m}{b^n}\right)^p$ or $\left(\frac{b^n}{a^m}\right)^p$. Bases a and b are integers or variables. Exponents m, n and p are rational numbers. The operation involves the application of no more than four laws of exponents. The exponents in the answer must be positive.
(operating) /5

Dimension 5

Multiply or divide two numerical expressions by applying the laws of exponents. One of the expressions is written in scientific notation and the other, in decimal form. The operation must be performed in scientific notation. The operation involves the application of no more than four laws of exponents. The answer must be written in scientific notation. The students must clearly show all their work.
(operating) /5

Dimension 6

Determine whether two numerical expressions written in exponential form are equivalent by simplifying them using the laws of exponents. Each expression contains three factors that are of the form a^m or that can be written in this form; a is a rational number and m is an integer. The simplification process involves the application of no more than four laws of exponents. The students must clearly show all their work.
(analyzing) /10

Dimension 7

Given algebraic expressions that can be written in the form $a^m b^n$, determine which are equivalent. Base a is an integer less than or equal to 10 or is a variable; base b is a variable and exponents m and n are integers. The transformation of the expression must involve only one law of exponents. The students must clearly show all their work.
(analyzing) /10

Dimension 8

Determine whether statements related to the sign or the value of expressions of the form a^m, in which $0 < a < 1$ are true or false. The students must justify their answers with a numerical example.
(analyzing) /10

Dimension 9

Determine whether two algebraic expressions that can be written in the form $a^{\frac{m}{n}\sqrt{a^p}}$ are equivalent by converting each into exponential form. Together, the expressions have four factors. Index n is a natural number, exponent p is an integer and exponent m is a rational number. The students must clearly show all their work.
(analyzing) /5

Dimension 10

Given numerical expressions that can be written in the form $a\sqrt{b}$ or $\left(ab^n\right)^m$, where a is a rational number, b is a natural number and exponents m and n are rational numbers, determine which are equivalent. The transformation of the expressions should involve one or two steps. The students must clearly show all their work.
(analyzing) /10

Dimension 11

Add or subtract two or three terms of the form $a\sqrt{b}$, where a is a rational number and b is a natural number. Simplify the answer. The students must clearly show all their work.
(operating) /5

Dimension 12

Multiply two binomials of the form $\left(a\sqrt{b}+c\right)$, where a and c are integers and b is a natural number. Simplify the answer. The students must clearly show all their work.
(operating) /5

Dimension 13

Divide two terms of the form $a\sqrt{b}$, where a is an integer and b is a natural number. Simplify the answer and rationalize the denominator, if necessary. The students must clearly show all their work.

or

Divide a binomial of the form $\left(a\sqrt{b}+c\right)$ by a monomial of the form $a\sqrt{b}$, where a and c are integers and b is a natural number. Simplify the answer and rationalize the denominator. The students must clearly show all their work.
(operating) /5

Dimension 14

Divide a monomial of the form $a\sqrt{b}$ by a binomial of the form $\left(a\sqrt{b}+c\right)$, where a and c are integers and b is a prime number less than 10. Simplify the answer and rationalize the denominator. The students must clearly show all their work.
(operating) /5

Dimension 15

Determine whether two numerical expressions are equivalent by simplifying them. Each numerical expression contains no more than two binomials of the form $\left(a\sqrt{b}+c\right)$ or no more than four terms of the form $a\sqrt{b}$, where a and c are integers and b is a natural number. The operations performed must respect the specifications of dimensions 11 to 14. The students must clearly show all their work.
(analyzing) /10

6. JUSTIFICATION OF CHOICES

In the examination, 45% of the items test the students' **OPERATING** skills by verifying whether they have mastered certain operations or transformations:

– applying the laws of exponents
– multiplying or dividing two numerical expressions, one written in scientific notation and one written in decimal form
– simplifying expressions containing radicals

In the examination, 55% of the items test the students' skill in **ANALYZING** information; they involve verifying whether the students have the ability to make connections:

– by studying the sign or the value of exponential expressions
– by verifying the equivalence of algebraic or numerical expressions written in exponential form
– by verifying the equivalence of expressions containing radicals

7. DESCRIPTION OF THE EXAMINATION

A. TYPE OF EXAMINATION

The summative examination will be a written examination consisting of short- or extended-response items.

The items should take into account the restrictions and the requirements specified in the dimensions and the objectives of the program. The weighting of marks should be consistent with the percentages set out in the table of dimensions.

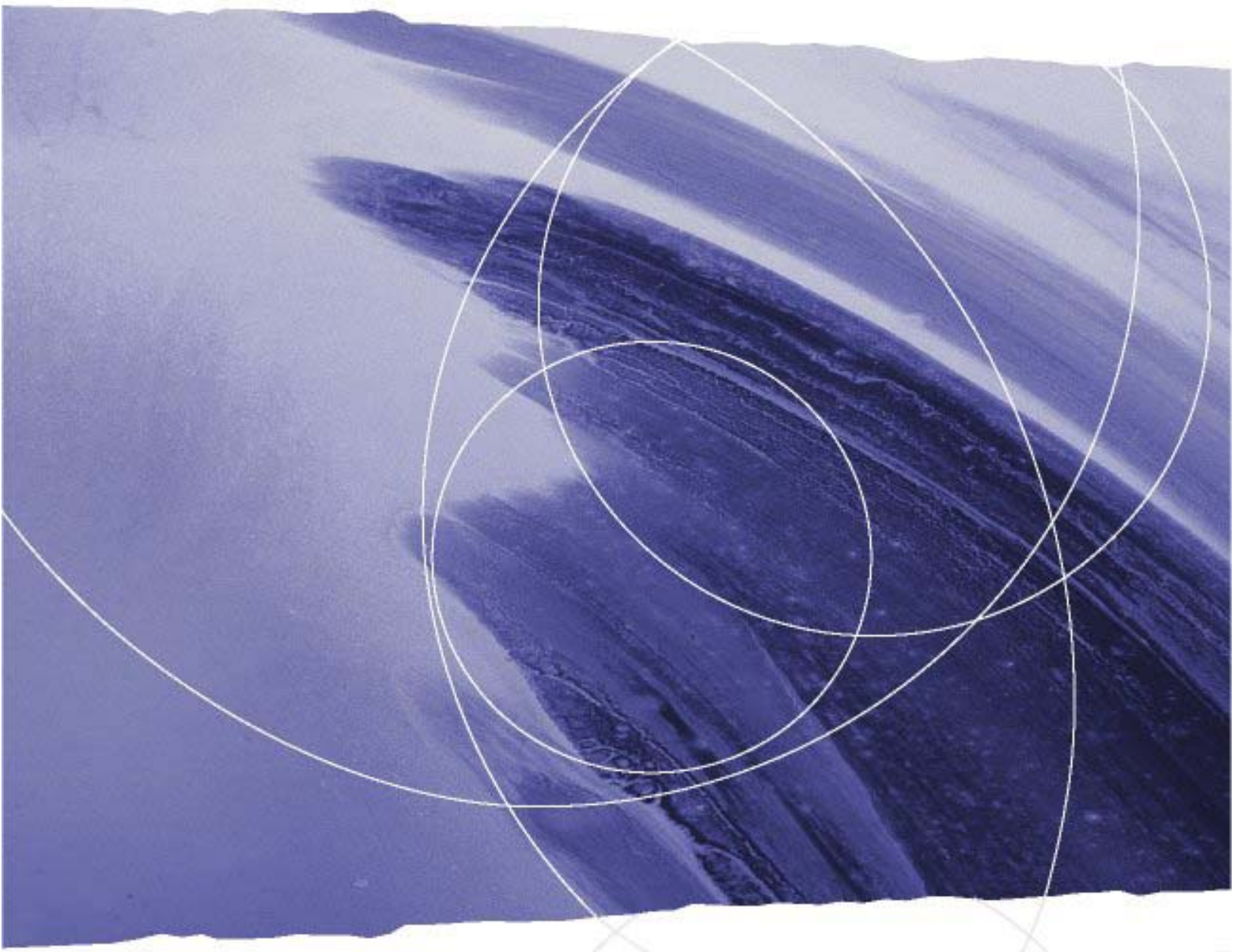
B. CHARACTERISTICS OF THE EXAMINATION

The examination will be administered in a single session lasting no more than two and a half hours.

Students are permitted to use a scientific calculator; however, they are not permitted to use a graphing calculator.

C. PASS MARK

The pass mark is set at 60 out of 100.



Éducation Québec

learning throughout life

41-3506-A