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Viability of a multi-parcel agroecological system

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Abstract : To satisfy a rising demand, agricultural practices have shifted from the organic fertilization of soils to intensive and highly specialized farming that uses chemical fertilization. The resulting short-term increase in soil productivity has lead to serious ecological drawbacks over time, e.g., degradation of soil quality, pollution of water and air, and loss of biodiversity. Given this state of affairs, it is urgent to find alternative practices that preserve soil quality and, at the same time, ensure farmers receive acceptable revenues. In this work, we rely on viability theory to determine a set of policies that make it possible to reach this dual objective. The proposed multi-parcel land model is applied to data from the archipelago of Guadeloupe, located in the French West Indies.

Keywords: Viability theory, agriculture system, mixed-integer programming, multi-parcel farm, French West Indies

Résumé : Pour satisfaire une demande croissante, les pratiques agricoles sont passées de la fertilisation organique des sols à une agriculture intensive et hautement spécialisée et basée sur la fertilisation chimique. À court terme, cela a résulté en une augmentation de la productivité des sols mais elle a entraîné de graves inconvénients écologiques au fil du temps comme la dégradation de la qualité des sols, la pollution de l'eau et de l'air, et la perte de biodiversité. Compte tenu de cet situation, il est urgent de trouver des pratiques alternatives qui préservent la qualité des sols tout en garantissant aux agriculteurs des revenus acceptables. Dans ce travail, nous nous appuyons sur la théorie de la viabilité pour déterminer un ensemble de politiques permettant d'atteindre ce double objectif. Le modèle multi-parcelles proposé est appliqué aux données de l'archipel de Guadeloupe, situé aux Antilles françaises.

Mots clés : Théorie de la viabilité, système agricole, programmation en nombres entiers mixtes, ferme à parcelles multiples, Antilles françaises

1 Introduction

This paper deals with the long-term management of farms and their ability to restore and maintain the quality of their soil. The evolution over time of this quality depends on numerous factors, some being controllable, by the farmer, e.g., the choice of crops and farming method, and others not, such as climatic events. Our main research question can be framed as follows: given a farm of a certain size and a planning horizon, what choices should the farmer make to simultaneously achieve an ecological objective and an economic sustainability one. Roughly speaking, the ecological objective corresponds to soil quality while the economic one refers to revenues.

To answer our question, we rely on the mathematical theory of viability (Aubin [3]). In a nutshell, a viability problem involves a dynamical system whose evolution depends on state and control variables, and possibly on some random events. Given a set of constraints and an initial state of the system, one looks for viable solutions, that is, evolutions (or trajectories) of these variables that satisfy the constraints.

Viability theory (VT) has been successfully used to determine sustainable policies in the management of ecosystems and renewable resources, e.g., fisheries and forests; see Oubraham and Zaccour [11] for a literature review. In the specific context of farming and agroecological systems, most applications of VT have dealt with herd- and grassland-management problems (see, e.g., Tichit et al. [16], Sabatier et al. [13], Sabatier et al. [14], Mouysset et al. [10], Tichit et al. [17], Baumgärtner and Quaas [4], Martin et al. [9], Sabatier et al. [15], and Durand et al. [8]). Very few have addressed soil quality management problems. Durand et al. [8] proposed a deterministic model describing the evolution of a single land parcel and looked for agricultural strategies and crop planting sequences that restore soil quality to an acceptable level while preserving the farm's economic profitability. Oubraham et al. [12] extended the model to a stochastic environment to account for major climatic events that affect the evolution of the dynamical system describing the farm's state. In this paper, we extend the deterministic model in Durand et al. [8] to a multi-parcel case to assess the potential advantages of a multi-crop strategy. Our empirical study concerns the archipelago of Guadeloupe, located in the French West Indies.

It is well-established that soil has been poorly protected and overexploited for decades, which has resulted in its deterioration worldwide (see Figure 1). One of the most important human-induced factors in soil's deterioration is modern agriculture. Indeed, to meet the ever-growing demand, agronomic systems have had to drastically change during the last decades and have migrated to agricultural practices based on the chemical fertilization of soil and intensive and specialized farming practices. This has increased soil productivity in the short term, but in the long term has caused serious ecological drawbacks (degradation of soil quality, pollution of water and air, loss of biodiversity, erosion, etc.) and even reversed the trend of agricultural productivity. In contrast to traditional agricultural systems based on natural fertilization and diversified crop production, modern agriculture relies on intensive single-crop production that progressively degrades soil quality and reduces its productivity by changing its physical, chemical, and biological composition. Fertilization and irrigation with low-quality water unbalance the chemical composition of the soil; plowing, tillage, removal of vegetative cover, and overgrazing make it more vulnerable to wind and water erosion; intensive and specialized cultivation leaches minerals and water from it and damages its microfauna (Blanco and Lal [5]). In fact, modern agricultural systems have now fallen into a vicious cycle, where the use of chemical fertilizers is increased to compensate for the loss of productivity, causing more damage to the soil and more water and air pollution.

Given the state of affairs just described, it is urgent to take action to replace current agricultural practices with more ecoresponsible ones based on crop rotation and mixed crop-livestock associations, which are healthier for the soil. We need to establish an agroecological transition that would, in the medium term, allow a return to more environmentally friendly agricultural practices and restore soil quality to an acceptable level, while taking into account the socioeconomic aspects of that sector. This is particularly true for island regions, given the importance of agriculture in their economies (Angeon et al. [2]). A survey of farmers in the French West Indies revealed that the population is aware of the problem and that farmers are now placing soil quality at the center of their concerns and are willing to make efforts and even to sacrifice some of their financial benefits to restore the quality of their land (Angeon et al. [1]).

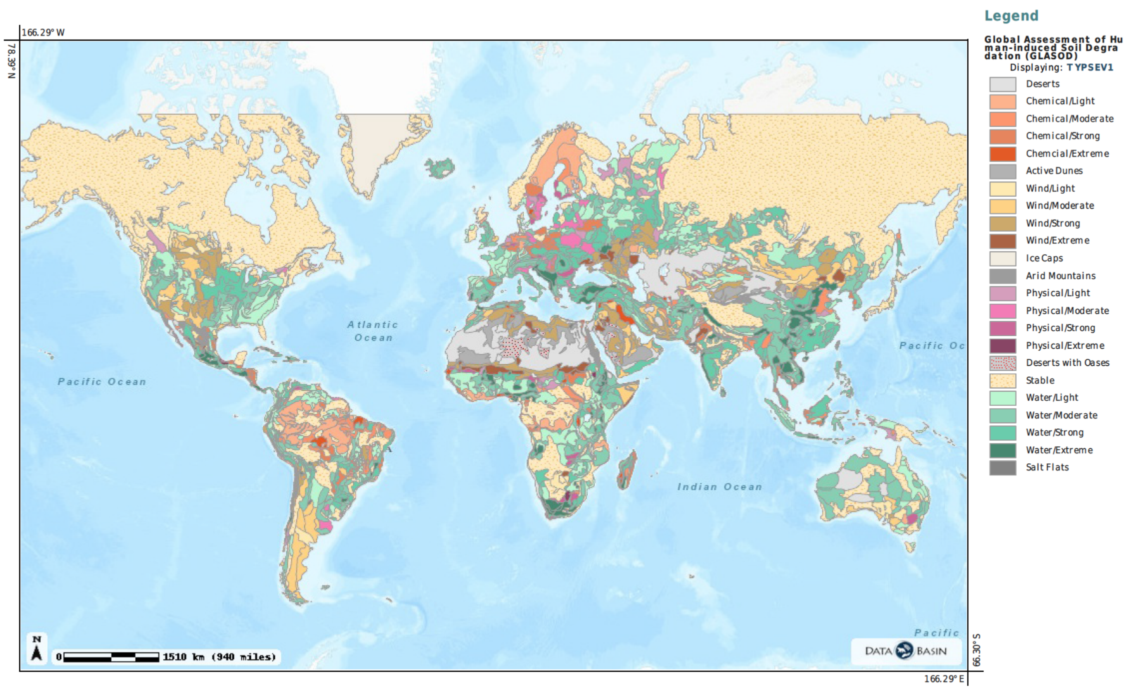

Figure 1: Global assessment of human-induced soil degradation. Source: https://databasin.org/datasets/7254137cabb042298cae0b769cba589f.

In our study, we consider a series of cases representing different farming systems practiced in the French West Indies, and analyze their impact on soil quality and the economic well-being of the farmer. Also, we conduct an extensive sensitivity analysis to assess the impact on the results of main parameters, namely, initial soil quality, planning horizon, and various costs.

Our contribution is threefold. First, to the best of our knowledge, our model is the first one to consider a multi-parcel land. Clearly, such a framework provides additional flexibility in the search for sustainable ecological and economical solutions. Second, extending the single-parcel model in Durand et al. [8] to multiple parcels, adds a huge complexity in determining viable solutions. In this paper, we design an algorithmic approach that is not only drastically novel with respect to Durand et al. [8], but also to the whole literature in viability theory. We believe that this approach could be helpful in other applications of viability theory. Finally, we contribute empirically by answering questions that are on the agenda of farmers and decision makers in the French West Indies.

The rest of the paper is organized as follows: In Section 2, we extend the single-parcel model in Durand et al. [8] to multiple parcels. In Section 3, we present the approach for obtaining viable solutions. In Section 4; we present our empirical results, and we briefly conclude in Section 5.

2 A multi-parcel bioeconomic model

In this section, we extend the model in Durand et al. [8] to a multi-parcel piece of land.

Consider a farm with N_P land parcels managed over time by a single agent. The planning horizon is T and the current time is denoted by $\tau \in \mathcal{T} = \{0, ..., T + T_\Delta\}$ where T_Δ is an extra time period by which the planning horizon can be extended to complete an ongoing agricultural cycle.

Each parcel $p \in \{1, ..., N_P\}$ is characterized by its area α_p (in ha) and its soil quality, which is an agronomic measure. Evaluating a soil's quality is a complex operating involving a series of physical, chemical, and biological characteristics. Here, we adopt the *General Indicator of Soil Quality* (GISQ), which has values in the interval $[0, 1]$ and is based on 54 variables measuring these characteristics (Velásquez et al. [18] and Camacho et al. [6]). Let $\mathcal{I}_p$ be the GISQ value of parcel p, with $\mathcal{I}_p \in \mathcal{I} = \{I_0, ..., I_{N_\mathcal{I}-1}\}$,[1] that is, $\mathcal{I}$ can take a finite number of values.

The main variables describing the farm's state are (i) the cash flow $\mathcal{W} \in \mathbb{R}$, a continuous variable characterizing the financial situation; and (ii) the soil quality of all its parcels $\mathcal{I} = (\mathcal{I}_1, ..., \mathcal{I}_{N_P})$. The evolution of the soil quality of parcel $p \in \{1, ..., N_P\}$ depends on its current quality $\mathcal{I}_p$ and the following two control variables: (i) the crop σ_p grown on the parcel, with $\sigma_p \in \Sigma_p$; and (ii) the agricultural practice $\pi_p \in \Pi_p$. In the rest of the paper, we consider the crop and practice choices to be the same for all parcels, that is, $\Sigma_p = \Sigma = \{\sigma_1, ..., \sigma_{N_\sigma}\}$ and $\Pi_p = \Pi = \{\pi_1, ..., \pi_{N_\pi}\}$ $\forall p \in \{1, ..., N_P\}$.[2] To clarify, in the case studies to follow, the set of crops includes either all or some of the following crop types: plantain, export banana, sugar cane, yam (yellow), yam (Grosse Caille), tomato, eggplant, lettuce, carrot, green bean, cabbage, cassava, melon, cucumber, and turban squash. The set of agricultural practices only includes two elements, i.e., conventional or agroecological practices.

Let the evolution of the soil quality be described by the following function:

$$\begin{array}{rcrcl} \phi & : & \mathscr{I} \times \Sigma \times \Pi & \rightarrow & \mathscr{I}, \\ & & (I, \sigma, \pi) & \mapsto & \phi(I, \sigma, \pi). \end{array}$$

Denote by $\delta(\sigma, \pi)$ the duration of the whole agricultural cycle of a crop σ using agricultural practice π. Then, to each pair (σ, π), we can associate production cycle ψ defined as follows:

$$\begin{array}{rcrcl} \psi & : & \Sigma \times \Pi & \rightarrow & [0, \delta(\sigma, \pi)], \\ & & (\sigma, \pi) & \mapsto & \psi(\sigma, \pi). \end{array}$$

Moreover, to each crop and practice (σ, π), we associate its first harvest time $f_c(\sigma, \pi)$, and the amount of time between two successive harvests $p_c(\sigma, \pi)$. The duration of the first production cycle is $f_c(\sigma, \pi)$ and the following cycles last $p_c(\sigma, \pi)$. If a crop only has a single harvest, then $f_c(\sigma, \pi) = \delta(\sigma, \pi)$ and $p_c(\sigma, \pi) = 0$. Also, we suppose that the produce is sold immediately after the harvest.

The revenue from each harvest depends on the crop and the agricultural practice, as well as on the soil quality at the beginning of the cycle. Formally, the revenue function ℓ is defined by

$$\begin{array}{rcrcl} \ell & : & \mathscr{I} \times \Sigma \times \Pi & \rightarrow & \mathbb{R}, \\ & & (I, \sigma, \pi) & \mapsto & \ell(I, \sigma, \pi). \end{array}$$

Denote by $s \in \mathcal{S} = \{1, ..., 12\}$ the current month. This discrete variable is needed to deal with the seasonality of crops that can be planted or sown only at specific times of the year. Let $\Sigma(s) \subset \Sigma$ be the set of all crops that can be planted or sown during season s.

In the single-parcel case studied in Durand et al. [8], the only event that must be accounted for over time is the beginning/end of a production cycle. If we refer by n to an event and by $\tau(n)$ to its timing, then the next event will happen at

$$\tau(n + 1) = \tau(n) + \psi(\sigma, \pi). \tag{1}$$

This formulation does not hold when we have multiple parcels, as each one has its own crop sequence and agricultural practice. This implies that, at any time, there are as many production cycles (independent, simultaneous, and potentially of different durations) as there are parcels in the farm, and the next evolution of the system should coincide with the nearest event on the parcels. Consequently, (1) needs to be modified to account for such events. To do so, we introduce the following new variables:

[1] $I_0 = 0$ and $I_{N_\mathcal{I}-1} = 1$

[2] Differences within the choices sets can be due to geographical or physical variations between parcels. For instance, slope or wind exposure can make it impossible to grow certain crops, and parcel accessibility can make it impossible to use some types of equipment.

- $e_p, p \in \{1, ..., N_P\}$: the remaining time before the next event on parcel $p \in \{1, ..., N_P\}$.
- $\epsilon = \min_{p=\overline{1,N_P}} e_p$: The remaining time before the next change in the system's state (the remaining time before the nearest event on the parcels).

Denote by e the vector of events $e = (e_1, ..., e_{N_p}, \epsilon)$.

Let $a_p(n)$ be the age of the crop on parcel p at step n, and $b_p(n)$ the state of the soil at the beginning of the agricultural cycle of $\sigma(n)$. The control variable on parcel p at that step is

$$v_p(n) = (\sigma_p(n), \pi_p(n), a_p(n), b_p(n)) \in \Sigma \times \Pi \times [0, \delta(\sigma_p, \pi_p)] \times \mathscr{I}.$$

The new parcel's GISQ value is given by

$$\mathcal{I}_p(n+1) = \zeta\left(\mathcal{I}_p(n), v_p(n), \epsilon(n)\right),$$

and the earnings generated by control $v(n)$ on a parcel during the time lapse $\epsilon(n)$ is defined by

$$\mathcal{L}(\mathcal{I}, v(n), \epsilon(n)) = g(\sigma(n), \pi(n), a(n), b(n), \epsilon(n)),$$

that is, the earnings depend on the type of crop $\sigma(n)$, its age $a\,(n)$, the state of the soil when it was planted $b(n)$, the agricultural practice $\pi(n)$, and the time elapsed since the last event $\epsilon(n)$.

2.1 Dynamical system

Denote by $v = (v_1, ..., v_{N_P}) \in V^{N_P}$ the control vector, with

$$v_p(n) = (u_p(n), a_p(n), b_p(n)), v_p(n), \quad \text{for } p \in \{1, ..., N_P\},$$

with $u_p = (\sigma_p, \pi_p) \in U = \Sigma \times \Pi$. To account for crop seasonality, we define by

$$U(s) \subset U, \quad U(s) = \Sigma(s) \times \Pi$$

the set of possible controls on parcels during a particular season s.

Denote by $x = (\mathcal{I}, \mathcal{W}, e, s, \tau)$ the state vector. The evolution of the farm is then governed by the following discrete-time dynamical system $\mathcal{F}$:

$$\mathcal{F}: \begin{cases} \tau(n+1) = \tau(n) + \epsilon(n) & \text{(2a)} \\ s(n+1) = (s(n) + \epsilon(n)) \; mod \; 12 & \text{(2b)} \\ \mathcal{I}_p(n+1) = \begin{cases} \zeta(\mathcal{I}_p(n), v_p(n), \epsilon(n)) & \text{If } \epsilon(n) = e_p(n) \\ \mathcal{I}_p(n) & \text{Otherwise} \end{cases} & \text{(2c)} \\ \mathcal{W}(n+1) = \mathcal{W}(n) + \displaystyle\sum_{p=1}^{N_P} \mathcal{L}(\mathcal{I}_p(n), v_p(n), \epsilon(n)) & \text{(2d)} \\ v_p(n+1) \in \begin{cases} \{(\sigma, \pi, 0, \mathcal{I}(n+1)) | (\sigma, \pi) \in U(s(n+1))\} & \text{if } \epsilon(n) = e_p(n) \text{ and} \\ & a_p(n) = \delta(\sigma_p(n), \pi_p(n)), \\ \{(\sigma_p(n), \pi_p(n), a_p(n) + \epsilon(n), b_p(n))\} & \text{otherwise,} \end{cases} & \text{(2e)} \\ e_p(n+1) = \begin{cases} p_c(\sigma_p(n), \pi_p(n)) & \text{If } \epsilon(n) = e_p(n) \text{ and } a_p(n) < \delta(\sigma_p(n), \pi_p(n)) \\ f_c(\sigma_p(n+1), \pi_p(n+1)) & \text{If } \epsilon(n) = e_p(n) \text{ and } a_p(n) = \delta(\sigma_p(n), \pi_p(n)) \\ e_p(n) - \epsilon(n) & \text{Otherwise} \end{cases} & \text{(2f)} \\ \epsilon(n+1) = \min_{p=\overline{1,N_P}} e_p(n+1) & \text{(2g)} \end{cases}$$

The initial state is given by $x(0) = (\mathcal{I}_1(0), ..., \mathcal{I}_{N_P}(0), \mathcal{W}(0), e(0), s(0), \tau(0))$,where $(\mathcal{I}_p(0))_{p=\overline{1,N_P}}$, $\mathcal{W}(0)$ and $s(0)$ are given parameters; $\epsilon(0) = \min_{p=\overline{1,N_P}} e_p(0)$, $\tau(0) = e_p(0) = 0$ and $b_p(0) = \mathcal{I}_p(0)$ for all $p \in \{1, ..., N_P\}$.

Equation (2a) sets the clock at each step $(n+1)$, advancing it by $\epsilon(n)$ (the time that elapsed between steps n and $n+1$). Equation (2b) does the same thing with the season while taking into account its cyclicality. Equation (2g) serves to update the event vector at each step.

The update of the other variables depends on where current events take place. At any step n, if the remaining time until the next event on a certain parcel p is equal to the remaining time until the next event on the whole system ($e_p(n) = \epsilon(n)$), then this means the event in question (at step $n+1$) will occur on that parcel, indicating the end of a production cycle on it and leading to the modification of the soil quality at that place (see (2c)). If the production cycle that just ended on the parcel is not the final one ($a_p(n) < \delta(\sigma_p(n), \pi_p(n))$), then the crop and practice applied on that parcel remain the same. We only update the age of the crop in the control variable (see (2e)) and a new production cycle starts right away, with the remaining time until the next event on that same parcel being $p_c(\sigma_p(n), \pi_p(n))$ (see (2f)). Otherwise, if the whole agricultural cycle ended ($a_p(n) = \delta(\sigma_p(n), \pi_p(n))$), then a new crop and agricultural practice $v(n+1)$ have to be chosen from the set $U(s(n+1))$ for this parcel, as expressed in (2e). The new time until the next event on that parcel is equal to the length of the first production cycle of the new crop, that is, $f_c(\sigma_p(n+1), \pi_p(n+1))$; (see (2f)).

Now, if the event does not occur on that parcel ($e_p(n) \neq \epsilon(n)$), then the cycle already in progress cannot be interrupted. Consequently, the control and the soil quality of this parcel remain the same, and only the age of the crop is updated (see (2c) and (2e)). The remaining time until the next event just has to be reduced by the time that elapsed between steps n and $n+1$ (see (2f)).

Finally, Equation (2d) updates the economic state of the farm by accumulating at each step the earnings generated by the parcels where a production cycle has ended.

2.2 A viability problem

When choosing crops and agricultural practices on the different parcels, the farmer seeks to achieve the following two objectives:

1. Ensuring an acceptable amount of income throughout the whole planning horizon.
2. Restoring the quality of the soil to a desired level by the terminal date of the planning horizon.

The first objective is economic, and can clearly take different forms depending on how one defines "acceptable" income. One practical option is to define a minimum threshold W_{min} below which cash flow should not fall at any step. Consequently, we have the following admissible set:

$$\mathcal{K} = \{(\mathcal{I}, \mathcal{W}, e, s, \tau) \| \tau \leq T, \mathcal{W} - W_{min} \geq 0\}. \tag{3}$$

In the numerical applications, we set $W_{min} = 0$ and then require a nonnegative treasury $\mathcal{W}$ over time.

The second objective is bioecological and embeds the long-term concern to maintain the resource (land) in an acceptable state. For instance, the objective may consist in restoring the quality of all farm parcels to a certain desired level $\mathcal{I}^*$ by the end of the time horizon. This can then be translated into a target to reach within a prescribed timeframe, i.e.,

$$\mathcal{C} = \{(\mathcal{I}_1, ..., \mathcal{I}_{N_P}, \mathcal{W}, e, s, \tau) \| \tau \geq T, \mathcal{I}_p \geq \mathcal{I}^* \ \forall p = \overline{1, N_P}\}. \tag{4}$$

Restoring the quality of all parcels to a desired level may be too ambitious in some situations, especially when the initial quality of each parcel varies greatly and when the time horizon is not long enough. In such cases, one can adopt a less demanding target to improve the quality of all parcels by a certain level (percentage) d. The target set is then defined as follows:

$$\mathcal{C}_d = \left\{(\mathcal{I}_1, ..., \mathcal{I}_{N_P}, \mathcal{W}, e, s, \tau) \| \tau \geq T, \mathcal{I}_p \geq \min\{\frac{d}{100}\mathcal{I}_p(0) \ ; \ 1\} \ \forall p = \overline{1, N_P}\right\}. \tag{5}$$

Finding the controls that satisfy some constraints, taking into account the evolution of the dynamical system, can be framed as a viability problem. More specifically, we aim to determine the initial states

$$X_0 = (\mathcal{I}_1(0), ..., \mathcal{I}_{N_P}(0), \mathcal{W}(0), e(0), s(0), \tau(0)),$$

of system $\mathcal{F}$, for which there exists at least one viable evolution, i.e., one that remains in $\mathcal{K}$ (3) during the entire planning horizon and reaches the target $\mathcal{C}$ (4) or $\mathcal{C}_d$ (5) at the end of that time horizon. This set is the *viability kernel*, and is formally defined as follows:

$$Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) = \{X_0 \in \mathcal{K} \| \exists x(.) \text{ s.t. } x(0) = X_0 \text{ \& } \exists t > 0 \text{ s.t. } x(t) \in \mathcal{C} \text{ \& } \forall n \in [0, t], x(n) \in \mathcal{K}\}. \quad (6)$$

3 Solution method

To compute the viability kernel defined in (6), we exploit the following property established in Proposition 1 and Corollary 1: if it is possible (not possible) to reach the target with a certain initial treasury, then it is possible (not possible) to reach it with a higher (lower) treasury.

As $\tau(0) = 0$, $s(0) = 1$, and $e(0) = 0$ are given data, we can characterize any initial state X_0 only by $\mathcal{I}(0) = (\mathcal{I}_1(0), ..., \mathcal{I}_{N_P}(0))$ and $\mathcal{W}(0)$ values, and write

$$X_0 = (\mathcal{I}(0), \mathcal{W}(0)).$$

Proposition 1 *Let $(\mathcal{I}, \mathcal{W})$ be an initial state from the viability kernel $Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C})$; then any initial state $(\mathcal{I}, \mathcal{W}')$ with $\mathcal{W}' \geq \mathcal{W}$ belongs to $Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C})$. Formally,*

$$(\mathcal{I}, \mathcal{W}) \in Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) \Rightarrow (\mathcal{I}, \mathcal{W}') \in Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) \quad \forall \;\; \mathcal{W}' \geq \mathcal{W}.$$

Proof. Let $B(x, I)$ be the minimum budget needed for the evolution $x(\cdot)$ to be viable, i.e.,

$$B(x, \mathcal{I}) = \min\{\mathcal{W} \| x(0) = (\mathcal{I}, \mathcal{W}) \text{ \& } \exists t > 0 \text{ s.t. } x(t) \in \mathcal{C} \text{ \& } \forall n \in [0, t], x(n) \in \mathcal{K}\}$$

We have

$$(\mathcal{I}, \mathcal{W}) \in Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) \quad \Leftrightarrow \quad \exists \; x(\cdot) \text{ s.t. } B(x, \mathcal{I}) \leq \mathcal{W}.$$

Therefore,

$$\forall \; \mathcal{W}' \geq \mathcal{W}, \quad B(x, \mathcal{I}) \leq \mathcal{W}',$$

and consequently

$$(\mathcal{I}, \mathcal{W}') \in Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}).$$

□

Corollary 1 *If it is not possible to reach the target from an initial state $(\mathcal{I}, \mathcal{W})$, then it is not possible to reach it with less initial treasury.*

$$(\mathcal{I}, \mathcal{W}) \notin Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) \Rightarrow (\mathcal{I}, \mathcal{W}') \notin Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) \quad \forall \;\; \mathcal{W}' \leq \mathcal{W}$$

The results in Proposition 1 and Corollary 1 imply that in order to completely characterize the viability kernel, it is sufficient to identify its border or, in other words, to find for each initial GISQ vector $\mathcal{I}$, the minimum initial treasury $\mathcal{W}_{inf}(\mathcal{I})$ needed for the restoration, i.e.,

$$\mathcal{W}_{inf}(\mathcal{I}) = \inf_{(\mathcal{I}, \mathcal{W}) \in Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C})} \mathcal{W} = \min_x B(x, \mathcal{I}).$$

Therefore, the problem amounts to finding a viable evolution of the system $x(\cdot)$ with the minimum possible restoration budget $B(x, \mathcal{I})$. This problem can be formulated as a shortest-path problem, where the objective is to minimize the restoration budget, and which can be solved by mixed-integer programming (MIP). Now, we introduce the network describing the system and the MIP modelling. We use Cplex software with a column generation approach to solve the MIP.

3.1 The network

The network representing the possible viable evolutions of a parcel $p \in \{1, ..., N_p\}$ is an oriented graph $G^p = (V^p, A^p)$ where:

V^p is the set of vertices. Each vertex $v^p = (t, i, u, e) \in V^p$ is identified by:

- its time : $T(v^p) = t$;
- its GISQ : $I(v^p) = i$;
- its control: $U(v^p) = u = (\sigma(v^p), \pi(v^p)) \in \Sigma \times \Pi$;
- its type: $E(v^p) = e \in \{\alpha, \beta, \gamma, SN, TN\}$, where
  - α : start of an agricultural cycle (start of the first production cycle);
  - β : intermediate harvest (start/end of an intermediate production cycle);
  - γ : final harvest (end of the agricultural cycle, i.e., last production cycle);
  - SN : source node;
  - TN : sink node.

Remark 1 *There is no need to identify the γ vertices by their control. They are common to all cycles ending at the same time, with the same GISQ. The source and sink nodes are particular vertices that do not have any of the characteristics of the other vertices.*

A^p is the set of arcs. Each arc $a^p_{i,j} \in A^p$ links the vertex $v^p_i \in V^p$ to $v^p_j \in V^p$ and has the following characteristics:

- $b^p_{i,j}(t_1, t_2)$: cost on arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$;
- $m^p_{i,j}(t_1, t_2)$: wealth generated by arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$;
- $d^p_{i,j}(t_1, t_2)$: change in the GISQ induced by arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$.

The network $G^p = (V^p, A^p)$ is constructed as follows:

Step 1. Initialize $V^p = \{v = (0, I_p(0), u, \alpha)$ such that $u = (\sigma, \pi) \in \Sigma(1) \times \Pi\}$.

Step 2. For each vertex $i \in V^p$, create its outgoing arcs $a_{i,j}$. An arc $a_{i,j}$ is created if it satisfies one of the following conditions:

C1. $i \in V^p$ with $E(i) = \alpha$ and $p_c(U(i)) = 0$, and $E(j) = \gamma$, $T(j) = T(i) + f_c(U(i))$, and $I(j) = \min\{\max\{I(i) + d_{i,j}(T(i), T(j))\ ;\ 0\}\ ;\ 1\}$.

C2. $i \in V^p$ with $E(i) = \alpha$ and $p_c(U(i)) \neq 0$, and $E(j) = \beta$, $T(j) = T(i) + f_c(U(i))$, $I(j) = \min\{\max\{I(i) + d_{i,j}(T(i), T(j))\ ;\ 0\}\ ;\ 1\}$, and $U(j) = U(i)$.

C3. $i \in V^p$ with $E(i) = \beta$ and $a(i) = \delta(U(i) - p_c(U(i)))$, and $E(j) = \gamma$, $T(j) = T(i) + p_c(U(i))$, and $I(j) = \min\{\max\{I(i) + d_{i,j}(T(i), T(j))\ ;\ 0\}\ ;\ 1\}$.

C4. $i \in V^p$ with $E(i) = \beta$ and $a(i) < \delta(U(i) - p_c(U(i)))$, and $E(j) = \beta$, $T(j) = T(i) + p_c(U(i))$, $I(j) = \min\{\max\{I(i) + d_{i,j}(T(i), T(j))\ ;\ 0\}\ ;\ 1\}$, and $U(j) = U(i)$.

C5. $i \in V^p$ with $E(i) = \gamma$ and $T(i) < T$, and $E(j) = \alpha$, $T(j) = T(i)$, $I(j) = I(i)$, $U(j) \in \Sigma(s(T(i))) \times \Pi$ and $T(i) + \delta(U(j)) \leq T + T_\Delta$.

Step 3. For each new arc $a_{i,j}$ added to A^p, if $j \notin V^p$, then add it to V^p and go to Step 2.

Step 4. The source and sink nodes are added to V^p. The source node is linked with arcs to all the initial vertices ($v \in V^p$ *s.t* $T(v) = 0, I(v) = I_p(0)$,and $E(v) = \alpha$). All terminal vertices reaching the target are linked with arcs to the sink node ($v \in V^p$ *s.t.* $T(v) \geq T, I(v) \geq I^*_p$,and $E(v) = \gamma$).

Step 5. Delete from the network all vertices from which there exists no path to the sink node and also their incoming and outgoing arcs.

Step 1 ensures that the evolution of a parcel starts at the beginning of an agricultural cycle at time $t = 0$ with initial GISQ $I_p(0)$. Steps 2 and 3 build the network forward to include all the possible evolutions of the parcel (viable and nonviable). Conditions (C1-C3) ensure the proper functioning of the process: A transition from an agricultural cycle-start vertex (of type α) goes to an intermediate harvest vertex (of type β) if the crop has multiple harvests or to a cycle-end vertex (of type γ) if the control has a single harvest (see conditions C1 and C2). If it is the final harvest, the transition from an intermediate harvest node goes to a cycle end; otherwise, the transition is to another intermediate harvest node (see conditions C3 and C4). Finally, at the end of a production cycle, if the time horizon is reached, then the process stops; otherwise, a transition to any cycle start is possible provided that the crop can be planted at that point, and that the remaining time (including the extension time T_Δ) is sufficient to complete the new production cycle.

To illustrate the network building process, we consider a simple example of a parcel with an initial GISQ of 0.5 and a time horizon of one year ($T = 12$ months) and $T_\Delta = 1$ month. Suppose we have the choice between two possible controls:

- u_1: can be planted at any time except $s = 7$ and $s = 8$, $\delta(u_1) = 3$, $f_c(u_1) = 2$, $p_c(u_1) = 1$, and it improves the GISQ by 0.2 (0.1 after the first harvest and 0.1 after the second one);
- u_2: can be planted at any time, $\delta(u_2) = 2$, $f_c(u_2) = 2$, $p_c(u_2) = 0$, and it deteriorates the GISQ by 0.1.

In the figures below, the graphs are represented on a grid where the columns represent time and the rows the GISQ. Thus, representing a vertex v at the intersection of column t and row i means that $I(v) = i$ and $T(v) = t$. Vertices of type α (start of an agricultural cycle) are represented by circles, those of type β (intermediate harvests) are represented by triangles, and finally those of type γ (end of an agricultural cycle) are represented by squares (orange if the time horizon is reached, and gray otherwise). We use the color red to represent the control u_1 and green to represent the control u_2.

Figure 2 displays the result of applying Steps 1 to 3 of the network building process. The first vertices (one for control u_1 and one for control u_2) are located at $t = 0$ and GISQ $I(0) = 0.5$, and they represent agricultural cycle starts (type α). The node of u_1 goes to an intermediate harvest node located two units of time later ($f_c = 2$) and one unit of GISQ up (u_1 improves the GISQ by 0.1 after the first harvest). Then, the intermediate harvest transits to a cycle end one unit of time later ($p_c = 1$) and improves the GISQ by 0.1. The same thing is repeated for all the agricultural cycle-start vertices of control u_1. Vertices of control u_2 transit directly to a cycle-end vertex two time units later and deteriorates the GISQ by 0.1. All the cycle-end vertices that did not reach the time horizon transit to cycle-start vertices of both controls u_1 and u_2 with no change in time or GISQ. The only exceptions are for times $t = 6$, $t = 7$, and $t = 11$, because u_1 cannot be planted in seasons $s(6) = 7$ or $s(7) = 8$; and the remaining time horizon at $t = 11$ is not sufficient to grow control u_1 ($t + \delta(u_1) = 11 + 3 = 14 > 13 = T + T_\Delta = 12 + 1$).

Figure 3 displays the result of applying Step 4. The source node is linked to the vertices of $t = 0$, and the end nodes that reached the target ($t \geq T$ and $GISQ \geq I^* = 0.6$) were linked to the sink node.

Finally, Figure 4 displays the actual network $G^p = (V^p, A^p)$ resulting from the application of Step 5, keeping only the viable paths.

3.2 The mixed-integer program

Let $G^p = (V^p, A^p)$ be the network representing parcel $p \in \{1, ..., N_P\}$. Let A be the set of all arcs, and V the set of all vertices, that is,

$$A = \bigcup_{p \in \{1,\dots,N_P\}} A^p \qquad \text{and} \qquad V = \bigcup_{p \in \{1,\dots,N_P\}} V^p$$

Let N be the set of steps and $\tau(n)$ the timing of step $n \in N$

$$\tau(0) = 0 \qquad \text{and} \qquad \tau(n+1) = \min_{v \in V \text{ s.t } T(v) > \tau(n)} T(v).$$

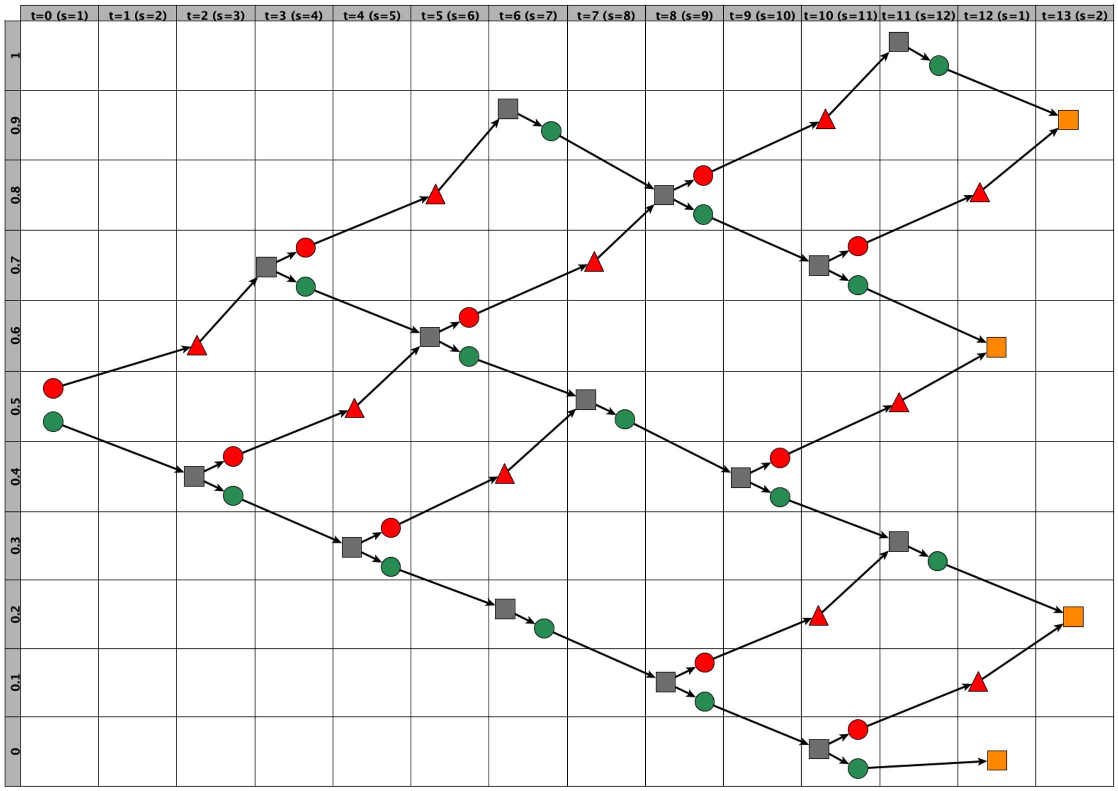

Figure 2: Steps 1 to 3.

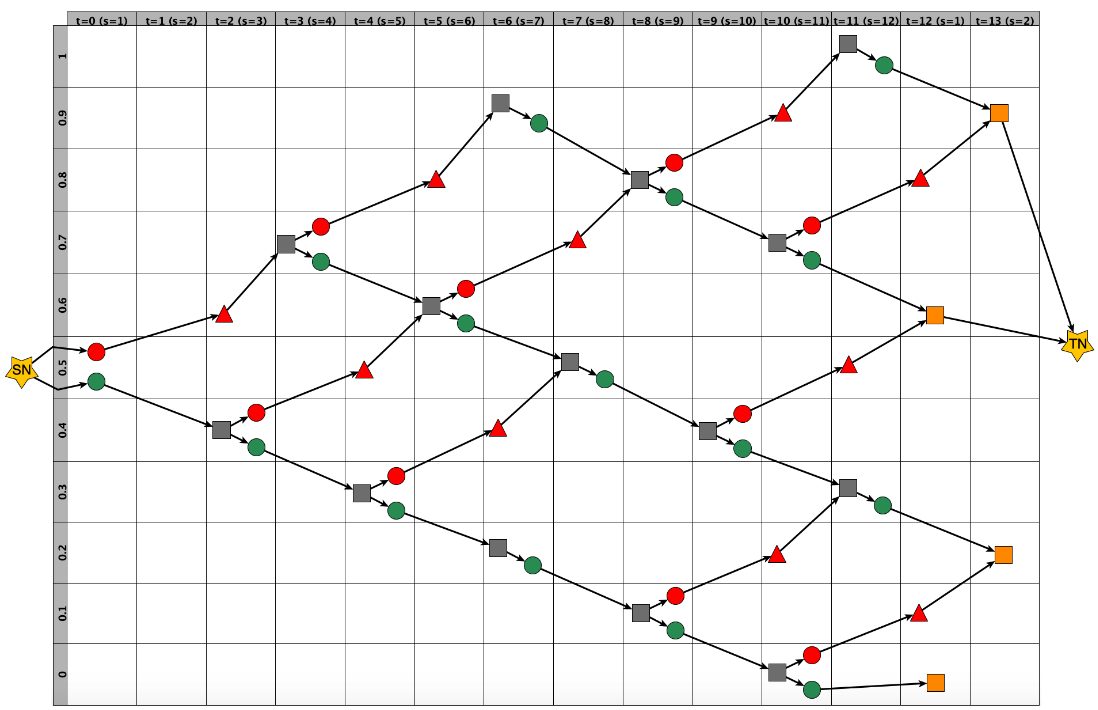

Figure 3: Step 4.

Recall that, in the dynamic system $\mathcal{F}$ ((2a)–(2g)), each step $n \in N$ coincides with the nearest event on the parcels.

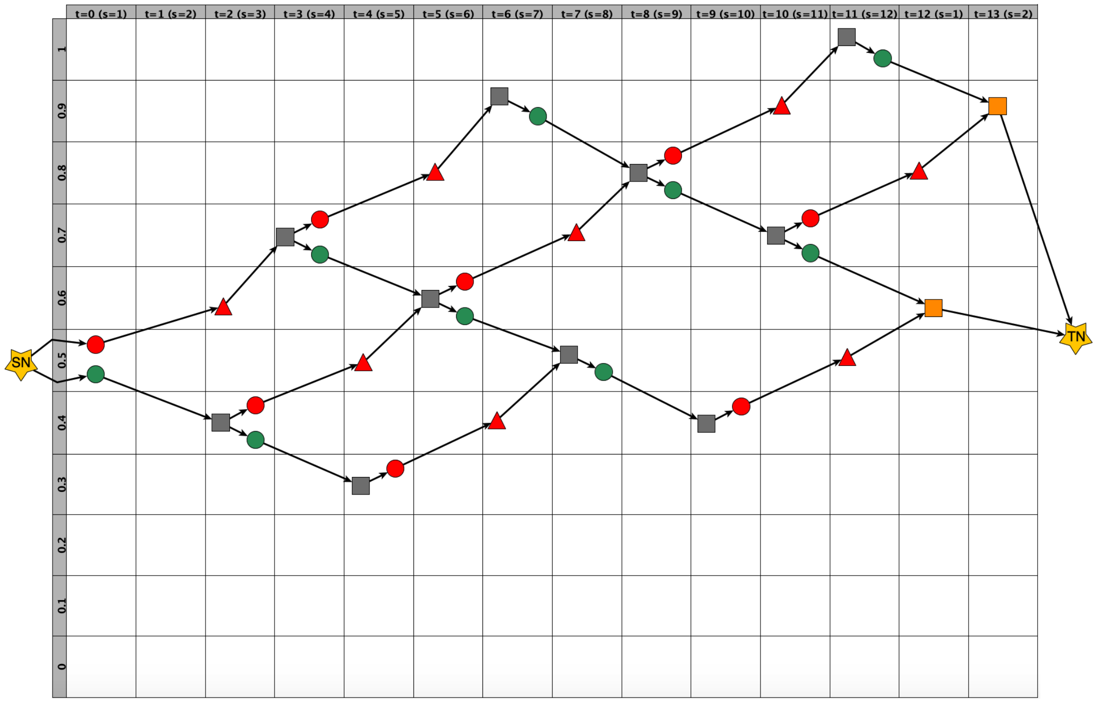

Figure 4: Step 5.

Introduce the following variables:

- $x^p_{i,j} = \begin{cases} 1, & \text{if arc } a^p_{i,j} \text{ is chosen,} \\ 0, & \text{otherwise.} \end{cases}$
- M_n: Treasury available between steps $n-1$ and n, and defined by

$$M_n = M_{n-1} - S_{n-1} + \sum_p \sum_{(i,j)\in A^p_n} x^p_{i,j} m^p_{i,j}(\tau(n-1), \tau(n)), \quad \forall n \in N \text{ and } M_0 = 0.$$

- S_n: Expenses during the period between steps $n-1$ and n, with the constraint

$$0 \leq S_n \leq M_n \quad \forall n \in N.$$

- C_n: Extra budget needed to cover expenses during the period between steps $n-1$ and n ($C_n \geq 0,\ \forall n \in N$), which is given by

$$C_n = \sum_p \sum_{(i,j)\in A^p_n} x^p_{i,j} b^p_{i,j}(\tau(n-1), \tau(n)) - S_n,\ \forall n \in N.$$

- $A^p(t_1, t_2)$: The set of arcs of G^p that are active between t_1 and t_2. Formally,

$$A^p(t_1, t_2) = \{a^p_{i,j} \in A^p | T(i) \leq t_1 \text{ and } T(j) \geq t_2\}.$$

- A^p_n: The set of arcs of G^p that are active between $\tau(n-1)$ and $\tau(n)$, i.e.,

$$A^p_n = A^p(\tau(n-1), \tau(n)).$$

The mixed-integer program consisting in minimizing the total restoration budget over the entire planning horizon is given by

$$
MIP \left\{
\begin{array}{ll}
\min & \sum_{n \in N} C_n \\
s.t. & \left|
\begin{array}{l}
\sum_{i|(i,j)\in A^p} x_{ij}^p - \sum_{k|(j,k)\in A^p} x_{jk}^p = \left\{ \begin{array}{rl} -1 & \text{if } j = \text{ SN}, \\ 1 & \text{if } j = \text{ TN}, \\ 0 & \text{otherwise}, \end{array} \right. \qquad \forall p \text{ and } \forall j \in V^p \\
C_n = \sum_p \sum_{(i,j)\in A_n^p} x_{i,j}^p b_{i,j}^p(\tau(n-1), \tau(n)) - S_n \qquad \forall n \in N \\
M_n = M_{n-1} - S_{n-1} + \sum_p \sum_{(i,j)\in A_n^p} x_{i,j}^p m_{i,j}^p(\tau(n-1), \tau(n)) \qquad \forall n \in N \\
S_n \leq M_n \qquad \forall n \in N \\
S_n \geq 0 \qquad \forall n \in N \\
C_n \geq 0 \qquad \forall n \in N \\
M_n \geq 0 \qquad \forall n \in N \\
x_{i,j}^p \in \{0,1\} \qquad \forall p \text{ and } \forall a_{i,j}^p \in A^p
\end{array}
\right.
\end{array}
\right. \tag{7}
$$

The first constraint represents the flow conservation over the network.

3.3 Column generation approach to solve the MIP

To solve the MIP defined in the previous subsection, we use Cplex with a column generation approach (Desaulniers et al. [7]). The column generation problem (CGP) is as follows:

$$
CGP \left\{
\begin{array}{ll}
\min & B_0 \\
s.t. & \left|
\begin{array}{l}
M_0 = B_0 - \sum_{i=0}^{N_P} p_j^i c_j^i(0,1) \\
M_t = M_{t-1} - \sum_{i=0}^{N_P} p_j^i c_j^i(t, t+1) + \sum_{i=0}^{N_P} p_j^i m_j^i(t-1, t) \qquad \forall t \in \{1, 2, \dots T\} \\
M_t \geq 0 \qquad \forall t \in \{0, 1, 2, \dots T\} \\
\sum_{j \in P(i)} p_j^i = 1 \qquad \forall i \in \{1, 2, \dots N_P\} \\
p_j^i \in \{0,1\} \qquad \forall i \in \{1, \dots, N_P\} \text{ and } \forall j \in P(i)
\end{array}
\right.
\end{array}
\right. \tag{8}
$$

Here, we generate paths $j \in P(i)$ that respect the flow conservation constraints from the source node to the sink node in each parcel i, and minimize the initial budget required to reach the sink on all parcels. Note that p_j^i is a binary variable, which is equal to 1 if the path j on parcel i is chosen, and 0 otherwise.

$M(t)$ represents the state of the treasury at time t. Its evolution depends on the cost and revenue contributions (c_j^i and m_j^i, respectively) on the active arcs at any time. We want this treasury to be nonnegative at any time t.

4 Numerical results and discussion

In this section, we illustrate our model and the solution approach with a series of farming systems, to which we will refer as cases. Table 1 lists the crops considered in these cases. Each crop can be used with either a conventional or an agroecological practice. Also, farmers can choose to leave their land fallow coupled with an agroecological practice (short fallow and free long fallow) or a conventional practice (improved short fallow and improved long fallow). Each of the listed cases represents a type of farming system practiced in the French West Indies: The export sector is specialized in banana and sugar cane (cases C1 and C2); the local-market-oriented sector is based on diversified vegetable farming (C3 and C4); and finally, C5 is a theoretical case that combines all the crops.

Table 1: Description of the different cases.

Case	Crops
Case 1 Banana	Plantain, Export banana
Case 2 Banana & sugar cane	Plantain, Export banana, Sugar cane
Case 3 Tomato & yam	Yam (Yellow), Yam (Grosse Caille), Tomato
Case 4 Multicrop	Yam (Yellow), Yam (Grosse Caille), Tomato, Eggplant, Lettuce, Carrot, Green bean, Cabbage, Cassava, Melon, Cucumber, Turban squash
Case 5 All crops	Plantain, Export banana, Sugar cane, Yam (Yellow), Yam (Grosse Caille), Tomato, Eggplant, Lettuce, Carrot, Green bean, Cabbage, Cassava, Melon, Cucumber, Turban squash

In order to analyze the effect of each parameter, the computations have been made for different parcels areas, initial GISQ levels, and time horizons. We always require that the farm remain self-sufficient, i.e., we set the economic constraint to 0 ($W_{min} = 0$) and we impose that the retained solution(s) allow for a positive treasury.

All the case studies concern the archipelago of Guadeloupe. For the common elements, we use the data and parameter values in Durand et al. [8] and Oubraham et al. [12]. The other required data and parameter values have been estimated and are displayed in Appendices B and C.

For the executions, we considered two cases regarding the number of parcels: the **single-parcel case**, where the whole land is considered one parcel over which the farmer should apply one control at a time, and the **multi-parcel case**, where the land is divided into two independent parcels, on which the farmer can apply different controls. To have comprehensive comparisons, we impose that the total area in the multi-parcel case equal the area in the single-parcel one. In the executions, we considered a 3-ha parcel in the single-parcel case, and two parcels of 1 ha and 2 ha, respectively, in the multi-parcel case.

The rest of the section is divided into two parts. In the first (Section 4.1), we focus on the viability kernel and the minimum restoration budgets. In particular, we analyze the impact of the time horizon (in Section 4.1.1), the impact of the initial GISQ (in Section 4.1.2), and the impact of crop diversification (in Section 4.1.3). In the second part (Section 4.2), we look at viable evolutions. More specifically, we discuss the main characteristics of such evolutions and how they are affected by such features as time horizon and GISQ improvement level.

4.1 The viability kernels

Denote by $Ker_{C_i,T}(I^*, N)$ the viability kernel of Case i for a time horizon T (in years), with I^* being the GISQ target and N the number of parcels. It represents the set of all pairs of initial GISQ I_0 and budget (initial treasury) that make it possible to reach the target while respecting the imposed constraints.

In Figure 5, each panel represents a superposition of viability kernels of the same problem case for different time horizons with one or two parcels. Each color represents a particular time horizon. The lightest and darkest shades of each color represent the single-parcel case and the two-parcel case, respectively.

In all cases, and for all time horizons, the viability kernel with a single parcel is always included in the two-parcel case, that is,

$$Ker_{C_i,T}(I^*, 1) \subseteq Ker_{C_i,T}(I^*, 2).$$

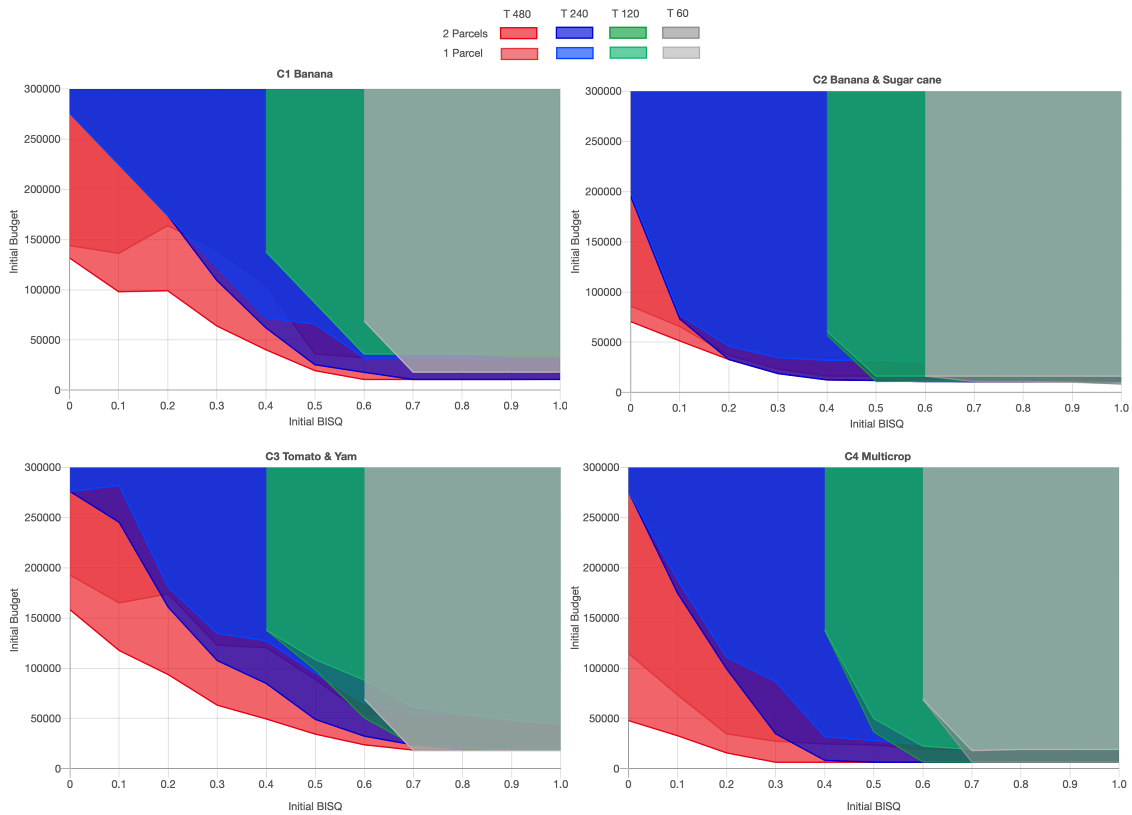

Figure 5: Viability kernels by scenario time horizons and number of parcels($I^* = 0.8$).

This result can be generalized to any number of parcels. Formally, we have

$$Ker_{C_i,T}(I^*, N_1) \subseteq Ker_{C_i,T}(I^*, N_2), \forall N_1 < N_2.$$

The reason for the above inclusion is as follows: splitting the land into multiple parcels does not involve any additional costs; further, it is always feasible to apply to these parcels the same controls implemented in the single-parcel case for the same cost. Therefore, the minimum restoration budget in the multi-parcel case will be at most equal to the one in the single-parcel case.

4.1.1 Impact of the time horizon

Figure 5 shows the viability kernels for different planning horizons (40, 20, 10, and 5 years). In all cases and for any number of parcels, the viability kernels shrink and slide to the right as the time horizon gets shorter. This indicates that the shorter time is, the more difficult and expensive it gets to restore soil quality. Further, restoring soil with a very low initial quality to a given desired level may be impossible to achieve.

Intuitively, one expects the cost of soil restoration to be higher when the horizon is shorter. This is observed in the results, but not when the initial GISQ is very close to the target (or is higher than the target). Indeed, in case C1, the kernel's border for $T = 5$ is lower than that for $T = 10$. Similarly, in cases C2 and C4, the borders of the kernels for $T = 5$ and $T = 10$ are below that at $T = 20$ with a single parcel. And, in case C3, the frontier of the kernel for $T = 5$ is lower than those for $T = 10$ and $T = 20$. One explanation is that when the difference in initial and target values of the GISQ is large, it requires a significant effort to improve the GISQ, and this is more difficult and pricy when the time horizon is short. However, when we start from a GISQ that is sufficiently close to or greater than the target, the effort is not to improve the GISQ but to maintain it over time. Maintaining the GISQ at a certain level over a short period of time is easier and cheaper than doing so over a long period.

Further, we notice that the gap between the viability kernel of the single-parcel case and the multi-parcel case gets bigger as the time horizon expands. To obtain some insight into this result, consider Figure 6, which represents the minimum budget needed for the restoration of soil with initial GISQ of 0.6 to a GISQ target of 0.8, for various planning horizons. The solid lines represent the budgets for the multi-parcel case, whereas the dotted lines show those for the single-parcel case.

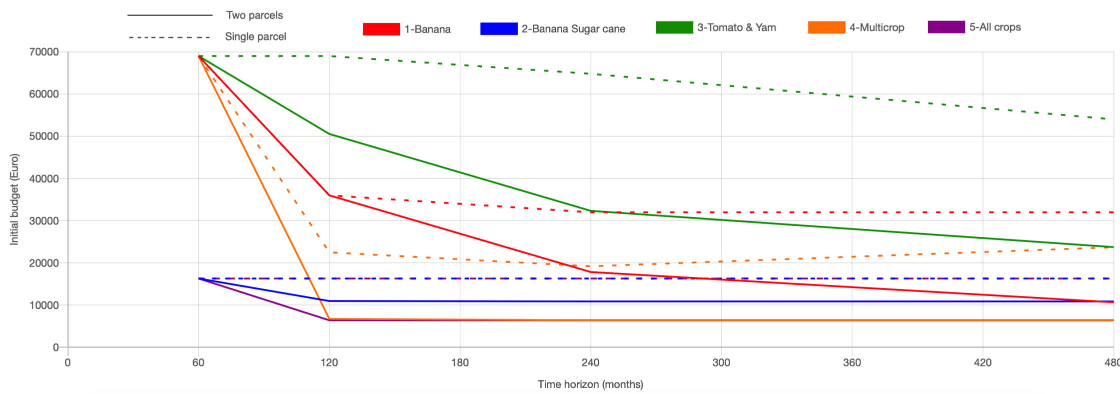

Figure 6: Total restoration cost for different time horizons.

Figure 6 shows that, in all cases and for any number of parcels, the restoration budget first decreases with T and next stabilizes when the time horizon reaches a certain value. Moreover, the restoration budget in the multi-parcel case is always less than or equal to the budget in the single-parcel case, confirming the result established previously, with the gap widening with the time horizon until it stabilizes after a certain value of time horizon. This result indicates that having multiple parcels never increases the restoration costs, with the benefits being larger when the time horizon is longer.

To wrap up, for each initial data set (problem case, and initial and target GISQ), we can determine a threshold terminal date after which the benefits of having multiple parcels are significant. Note that we assumed away any additional costs (e.g., management fees) induced by dividing a land into parcels. If such costs have to be paid, then the threshold will be farther away. In terms of computation effort, finding a solution in a single-parcel setting is much easier and faster than in the multi-parcel scenario.

4.1.2 Impact of the initial GISQ

Figure 7 displays the evolution of the total cost of a GISQ improvement by 0.2 for various initial GISQ values, for all cases in a single-parcel setting (dotted lines) and in a multi-parcel setting (solid lines), with $T = 40$ years.

The cost of improving the soil quality by given level depends on the initial GISQ value. Figure 7 shows that, in general, the cost is lower when the initial GISQ is higher. The reason is that a crop planted in higher-quality soil generates a higher income. Consequently, a lower minimum budget is needed, as the income covers a larger portion of the operating costs. However, we observe a different behavior for GISQ values close to 0 and 1. For instance, in cases C1 and C4, the restoration cost curve is increasing near initial the GISQ values of 0 and 1 for a single parcel. This result is even more visible in the representations of the viability kernels of cases C1 and C3 in Figure 5, where we observe that the costs increase a little for certain values of the initial GISQ close to 0 before decreasing. One explanation is that when the GISQ is too low, the degradation caused by a crop or a harmful agricultural practice cannot be too high. On the other hand, starting with a soil of medium quality leaves room for improvements that exceed the target, in anticipation of future damage caused by more profitable crops. However, if the initial GISQ is too high, then such an opportunity is not available, which increases the restoration cost in some cases.

To illustrate the above findings, we report in Figure 8 the evolution in GISQ induced by the same control sequence starting from different initial GISQ values (0.0 in orange, 0.3 in blue, 0.5 in gray and

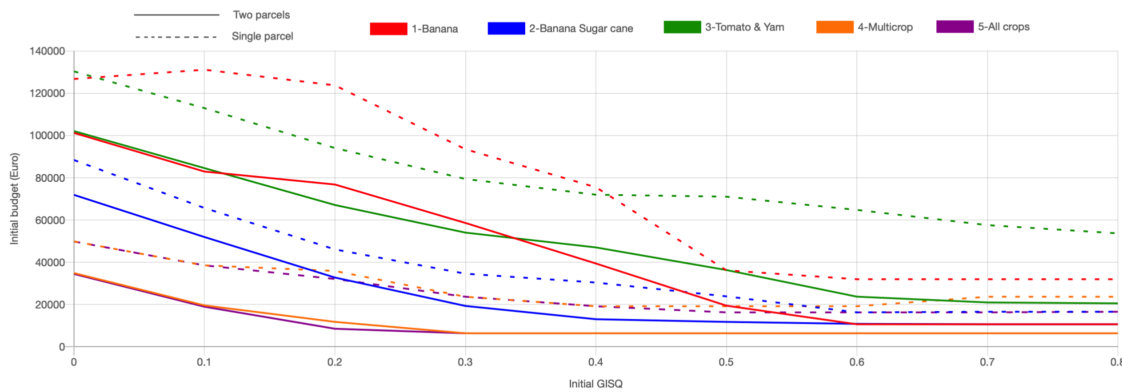

Figure 7: Total cost for a GISQ improvement of 0.2 for different initial GISQ values and $T = 40$.

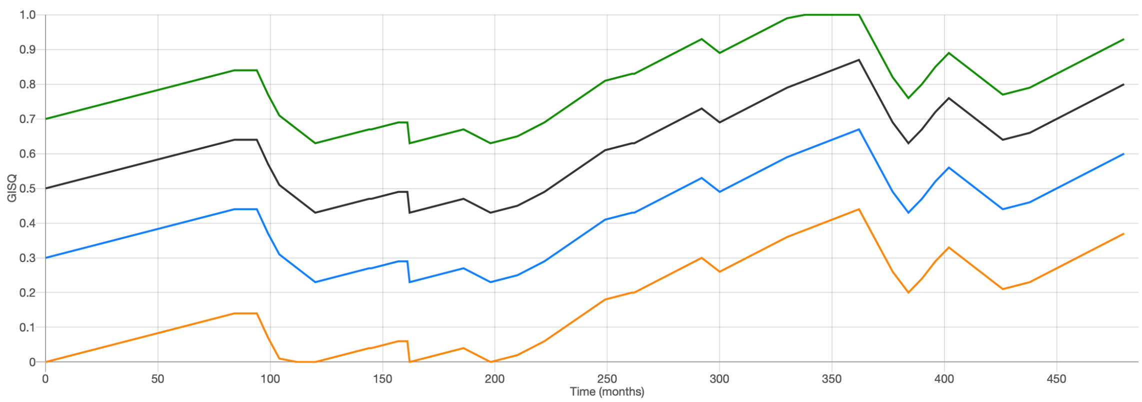

Figure 8: GISQ evolutions induced by the same control sequence for different initial GISQs.

0.7 in green) over 480 months. We can see that this strategy leads to a 0.3 improvement in the GISQ (from 0.3 to 0.6 and from 0.5 to 0.8) when starting from an average initial GISQ level (0.3 and 0.5). However, if we start from a higher initial GISQ level (0.7), the improvement is only 0.23 units (from 0.7 to 9.3). On the other hand, the same strategy is more efficient when the initial GISQ is lowest. Indeed, here, the improvement is equal to 0.37 units (from 0.0 to 0.37). Finally, we note that when the initial GISQ is low, the GISQ values over time can exceed the target, which provides a buffer against possible future degradation.

Finally, we note that the results are independent of the number of parcels, as no significant difference is observed between single and multiple parcels. Moreover, the cost gap between the single-parcel and multi-parcel settings is more or less the same regardless of the initial GISQ. Therefore, the benefits of having multiple parcels are invariable with respect to the initial GISQ level.

4.1.3 Impact of crop diversification

From Figures 5, 6, and 7 ,we observe that the viability kernel of a case with fewer crop options is always included in any viability kernel corresponding to a scenario with more crops. That is, if the set of possible controls U_i in case C_i is included in the set U_j in case C_j, then the viability kernel of C_i is contained in that of C_j for any planning horizon, i.e.,

$$U_i \subset U_j \Rightarrow Ker_{C_i,T}(I^*, N) \subseteq Ker_{C_j,T}(I^*, N), \quad \forall\, T \text{ and } N.$$

Furthermore, Figure 7 indicates that the restoration budget in a case with a larger number of crops is at most equal to the budget for a lower number of crops. Formally,

$$U_i \subset U_j \Rightarrow Budget_{(C_i,T)}(I^*, N) \geq Budget_{(C_j,T)}(I^*, N), \forall\, T \text{ and } N.$$

In fact, adding a control can only help in meeting the objective. However, adding new crops to the set of possible controls does not necessarily enlarge the viability kernel or decrease the restoration budget. Indeed, if the initial set already contains the right mix of crops, adding new crops will certainly not deteriorate the solution, but will not improve it either. For example, in almost all instances, the performance of C5 is identical to that of C4, even if it has more crop choices.

Finally, Figures 6 and 7 reveal that the lower is the number of crops, the more beneficial is the use of multiple parcels. Indeed, if we define by GAP_i the gap between the solid and dotted lines in case C_i , then we clearly have

$$U_i \subset U_j \Rightarrow GAP_i > GAP_j.$$

For instance, the gap between the red solid and dotted lines in these figures (representing case C1) is larger than the gap between the blue solid and dotted lines (representing problem case C2, which contains more crops).

4.2 Viable evolutions

In this section, we present the viable solutions with a minimum initial budget that lie on the boundary of the viability kernels. Figures 9, 10, 11, and 12 display the solutions for cases C1, C2, C3, and C4, respectively, for a single parcel and for multiple parcels. The time horizon is $T = 40$ (480 months), the initial GISQ is $I_0 = 0.6$, and the GISQ target is $I^* = 0.8$. The upper part of each figure shows the treasury and GISQ evolutions over time, and the lower part displays the crop recommendations.

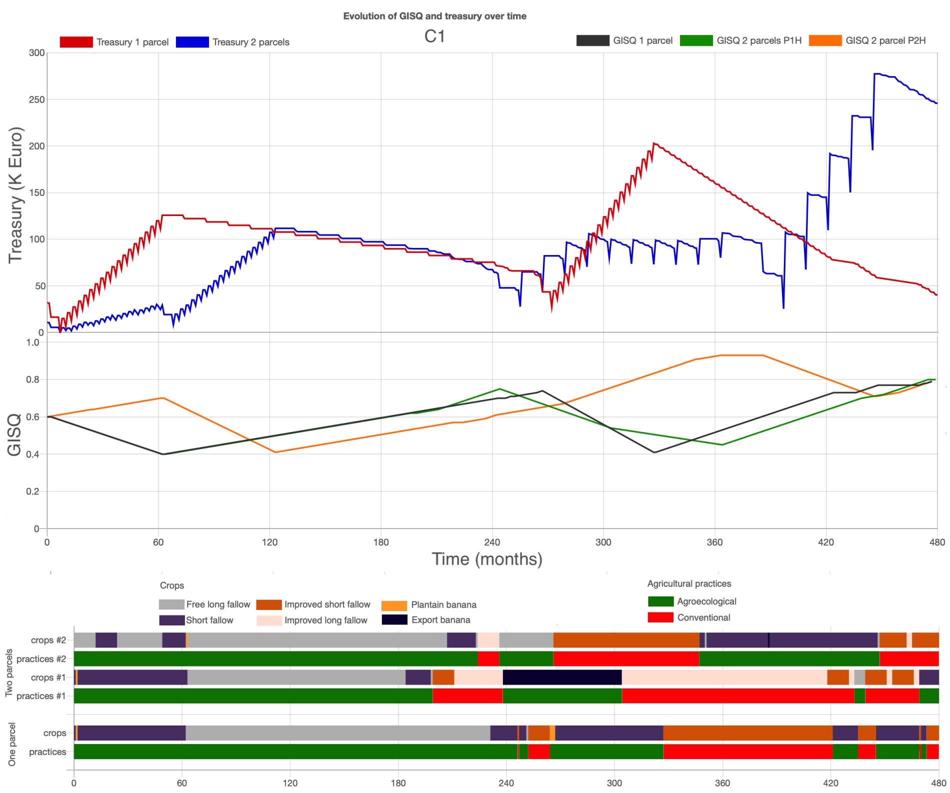

Figure 9: GISQ and treasury evolutions and crop recommandations for case C1.

A first result is that, in almost all cases, the initial treasury (i.e., the restoration budget) is lower, and the final treasury significantly higher, for multiple parcels then for the corresponding values for a single parcel. Also, in most cases, the GISQ evolution for one of the multiple parcels is quite similar to the one obtained for a single parcel. In terms of GISQ trajectories, we often observe that the GISQ of at least one of the parcels, in a multiple-parcel scenario, exceeds the target value at some intermediate

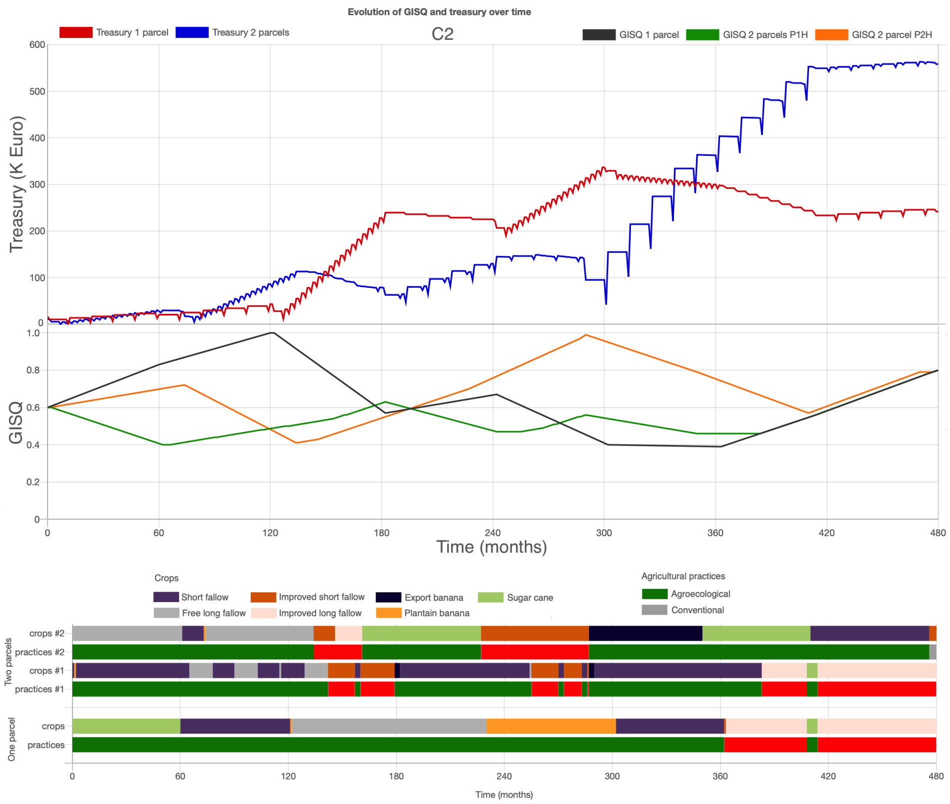

Figure 10: GISQ and treasury evolutions and crop recommandations for case C2.

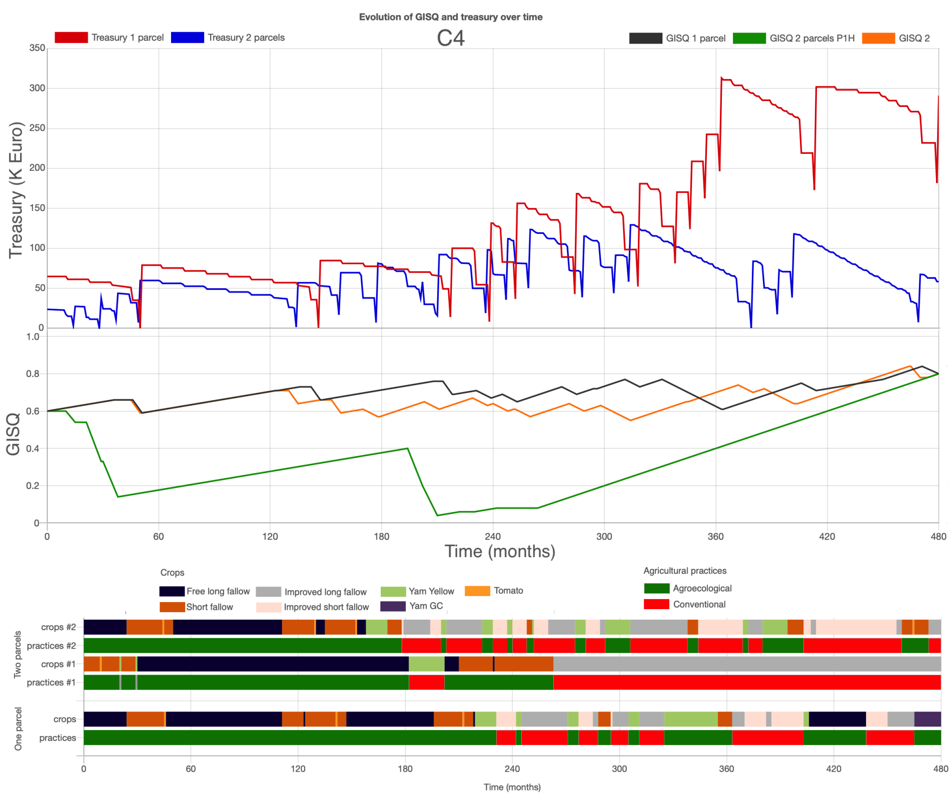

Figure 11: GISQ and treasury evolutions and crop recommandations for case C3.

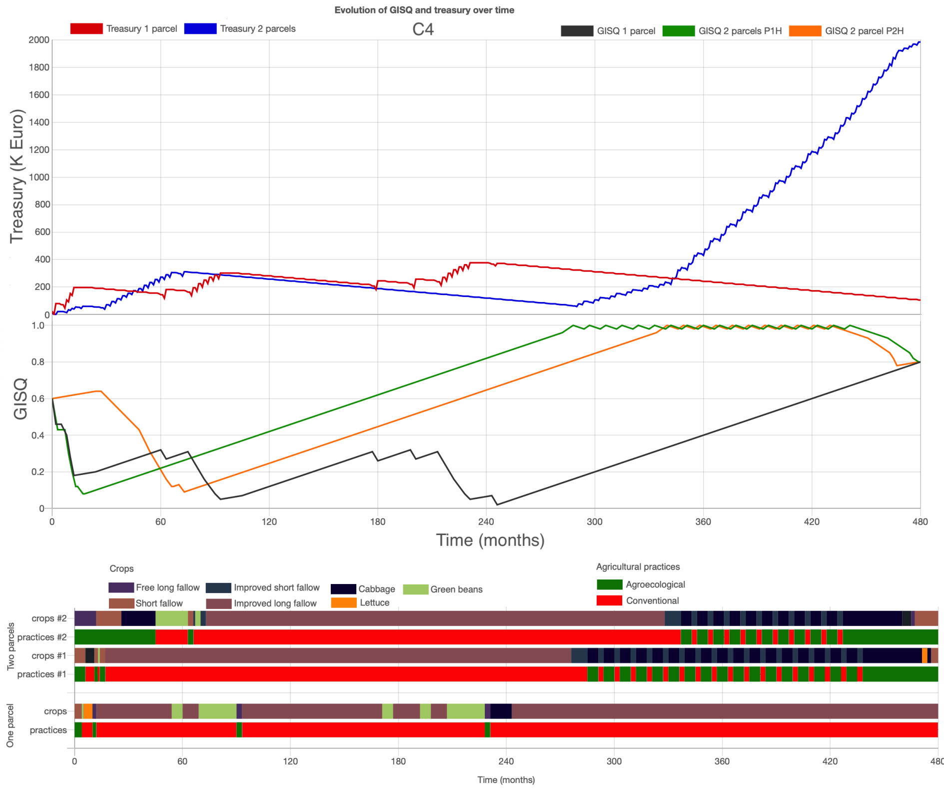

Figure 12: GISQ and treasury evolutions and crop recommandations for case C4.

date before decreasing towards the target value at terminal date T. In the single-parcel case, we see the opposite behavior, that is, in most cases, the GISQ level first drops and then increases towards the target value at T. When the farm is a single parcel, then the whole land is initially used to boost the treasury needed for the soil restoration, before investing the accumulated amount to reach the GISQ target. In the multi-parcel setting, the farmer can dedicate one parcel to generate revenues, while the other parcel is restored. A portfolio of assets (parcels) allows for more flexibility than a single asset.

Finally, we make the following remarks on the viable evolutions and the effect of some parameters:

Crop recommendations: In all viable evolutions, there is a prevalence of fallow as a recommended control. This is due to the fact that the solutions lie on the boundary of the viability kernel. Even if investing a bit more could make it possible to obtain a higher revenue, the returned solution remains the one with the minimum budget. As the cheapest option is to fallow, this is most likely to be chosen. However, we notice that in the multi-parcel setting, more controls other than fallow are recommended. This result is due to the above-mentioned flexibility of parcelization.

Time horizon: When the time horizon is short, fallow becomes the prevalent recommended control (except when the initial GISQ is very close to or higher than the target GISQ). Further, the recommended controls for both parcels in the multi-parcel setting are almost the same as the ones for a single parcel. As the time horizon gets longer, more non-fallow controls are recommended, and the differences between the two scenarios (multiple parcels and one parcel) become more pronounced.

Level of improvement: Increasing the level of improvement in the GISQ has two main effects. First, it calls for a more frequent implementation of fallow controls. Second, the differences between the two scenarios (one and multiple parcels) are less visible. The reason for this is that a small level of improvement in the GISQ requires less effort and time to achieve, leaving more possible paths open to achieve it and more time to improve the treasury. However, a higher level of improvement considerably reduces the number of possible paths to achieve that improvement (generally only paths involving many fallow periods are left), which makes the recommendations in both settings quite similar, the initial budget being high and the final treasury low.

5 Conclusion

The main qualitative results obtained in our study are as follows:

1. The export-oriented sector (cases C1 and C2) is globally more sensitive to the number of parcels. The local-market-oriented sector also benefits from parcelization, but the gap between the results for one and two parcels is larger in the export-oriented sector.
2. Having multiple parcels leads to better results than having one parcel, in all cases. However, the benefit is only noticeable when the time horizon is long enough.
3. Parcelization's benefits are higher when the number of crop choices is lower.
4. If we take into consideration the final treasury generated, the multi-parcel setting is globally much better than the single-parcel one. Although we are not optimizing a specific performance index in this study (and in viability theory in general), this result can be used as a guideline for making a choice.

Note that even if the results are based on a particular choice of crops, they confirm the intuition that alternating fallow between two plots can be beneficial and more profitable. Moreover, these calculations open up the possibility of using other, more judicious crop choices rather than fallow. Indeed, intuition may help predict that parcelization would be advantageous, but it does not allow for any conclusions regarding what crop sequence or alternation would be better.

Future developments include collecting additional data and refining the estimations, in particular the additional costs generated by parcelization (management fees for example) and other agricultural aspects like the effects of monoculture. A second extension would be to run executions on different cutouts of the land (varying the number of parcels and their areas). Finally, it would be interesting (and challenging) to take into consideration uncertainties related to market parameters, climatic events, crop diseases, etc.

A Lexicon

Viability kernel: The set of all initial states of the system, from which there exists at least one evolution that satisfies the viability constraints and reaches the target at the end of the time horizon.

Agricultural practice / type of agriculture: The type of practices adopted for agricultural activities. Here, we consider two types: **conventional** and **agroecological**.

Conventional practice: Agricultural practice based on modern practices/tools and on chemical fertilization.

Agroecological practice: Traditional practice based on organic fertilization of the soil or other practices for improved management of agroecosystems. Wezel et al. [19]

Fallow: Leaving land fallow consists in leaving it unsown for a certain period of time. We considered two types of fallow: one coupled with conventional practices (**conventional fallow**) and one coupled with agroecological practices (**agroecological fallow**).

Agroecological fallow (short fallow and free long fallow): the land is simply left fallow.

Conventional fallow (improved short fallow and improved long fallow): the fallow is improved with chemical additives and fertilizers.

B Assumptions

We made the following assumptions in our study:

A1: The sum of the parcels areas in the multi-parcel setting is equal to the area of the single parcel.

A2: Parcelization does not add any costs (management fees or others).

A3: The different parcels in the multi-parcel setting are completely independent. The only thing they have in common is the treasury, which concerns the whole farm.

C Parameter values and data

C.1 Agronomic data and parameters

For any control $(\sigma, \pi) \in \Sigma \times \Pi$, we have listed the agronomic parameters and transition functions in this section. The values of all the parameters used in this section are displayed in the tables of Section C.4

C.1.1 Crop yields

- $R_M = R_M(\sigma, \pi)$: Mean yield (tons/ha). Corresponds to the mean yield of the crop when the soil is of average quality (GISQ = 0.5).
- $r_i = r_i(\sigma, \pi)$: The control's sensitivity to the quality of the soil.
- $R(I, \sigma, \pi)$: The yield of the control.

If $r_i \neq 0.5$ the yield is given by

$$R(I,\sigma,\pi) = \begin{cases} 2R_M\left[\frac{I(1-2r_i)+r_i-\sqrt{(r_i)^2+2I(1-2r_i)}}{2r_i-1}\right], & \text{if } I \leq 0.5, \\ R_M\left[1+2r_i\frac{(1-2r_i)(I-0.5)-(1-r_i)+\sqrt{(1-r_i)^2-2(I-0.5)(1-2r_i)}}{2r_i-1}\right], & \text{otherwise.} \end{cases} \tag{9}$$

If $r_i = 0.5$ the yield is given by

$$R(I,\sigma,\pi) = \begin{cases} 2IR_M, & \text{if } I \leq 0.5, \\ R_M(I+0.5), & \text{otherwise.} \end{cases} \tag{10}$$

C.1.2 GISQ transition functions

- r_d: The soil's sensitivity to the crop loss of organic matter.
- r_p: The damage or improvement to the soil caused by the agricultural practice (effects that are dependent on the initial soil quality).
- r_{ap}: The damage or improvement to the soil caused by the agricultural practice (effects that are independent of the initial soil quality).
- $\Delta I_b(I, \sigma, \pi)$: The change in GISQ induced by the crop σ

$$\Delta I_b(I,\sigma,\pi) = \begin{cases} \frac{R(I,\sigma,\pi)}{2R_M(\sigma,\pi)} r_d(\sigma,\pi) & \text{If } R_M(\sigma,\pi) \neq 0, \\ 0 & \text{Otherwise} \end{cases} \tag{11}$$

- $\Delta I_p(I, \sigma, \pi)$: The change in GISQ induced by agricultural practice π

$$\Delta I_p(I,\sigma,\pi) = I.r_p(\sigma,\pi) + r_{ap}(\sigma,\pi). \tag{12}$$

- $\phi(I, \sigma, \pi)$: The transition function of the GISQ (for the whole agricultural cycle)

$$\phi(I,\sigma,\pi) = \min\{\max\{I - \Delta I_b(I,\sigma,\pi) + \Delta I_p(I,\sigma,\pi), 0\}, 1\}. \tag{13}$$

- $\delta(\sigma,\pi)$: The cycle length of control (σ,π) (for the whole agricultural cycle).
- $\zeta(I,v,\epsilon)$: The transition function of the GISQ at a step where the control $v=(\sigma,\pi,a,b)$ is applied

$$\zeta(I,v,\epsilon)=\min\{\max\{I+\kappa(v),0\},1\}, \tag{14}$$

where

$$\kappa(v)=\begin{cases}\frac{f_c(\sigma,\pi)}{\delta(\sigma,\pi)}\phi(b,\sigma,\pi), & \text{if } a+\epsilon=f_c,\\ \frac{p_c(\sigma,\pi)}{\delta(\sigma,\pi)}\phi(b,\sigma,\pi), & \text{if } a+\epsilon>f_c \text{ and } (a+\epsilon-f_c) \mod p_c=0,\\ 0, & \text{otherwise.}\end{cases}$$

C.2 Economic data and parameters

The values of all the parameters used in this section are displayed in the tables of Section C.4.

For any control $(\sigma,\pi)\in\Sigma\times\Pi$, we have the following economic parameters and transition functions:

- S_a: Subsidy for the crop (€/ha /month).
- S_p: Subsidy for the quantity produced (€/ton).
- C_t: Monthly labor cost (€/ha).
- C_i: Installation input cost (seeds, fertilizer, pesticides) (€/ha).
- C_e: Other expenses (€/ha).
- C_c: Harvesting cost (€/ha).
- C_m: Maintenance cost (€/ha/month).
- C_f: Fixed costs that are not related to a particular crop (€/ha/month).
- P: Selling price (€/ton).
- f_c: Date of the first sale.
- p_c: The time lapse between successive sales.
- ρ_0: The monthly recurring payoff, i.e.,

$$\rho_0=S_a-C_0, \tag{15}$$

where C_0 is the monthly recurring cost

$$C_0=C_f+C_t+C_m.$$

- $\ell(I,\sigma,\pi)$: The earnings generated by each harvest sale and given by

$$\ell(I,v)=R_h(I,u)-C_h(I,u), \tag{16}$$

where $R_h(I,u)$ is the revenue generated by the harvest and $C_h(I,u)$ is the harvesting cost, that is,

$$R_h(I,u)=R(I,\sigma,\pi)(P+S_p) \quad \text{and} \quad C_h(I,u)=C_c\frac{R(I,\sigma,\pi)}{R_M(\sigma,\pi)}.$$

- $\mathcal{L}(I,v,\epsilon)$ The earnings generated by v during the time lapse ϵ.

$$\mathcal{L}(I,v,\epsilon)=\begin{cases}\alpha_p(\epsilon\rho_0-C_i), & \text{if } a=0 \text{ and } a+\epsilon<f_c,\\ \alpha_p(\epsilon\rho_0+\ell(I,v)), & \text{if } a=f_c \text{ or } (a-f_c) \mod p_c=0,\\ \alpha_p\epsilon\rho_0, & \text{otherwise.}\end{cases} \tag{17}$$

C.3 MIP-related data

Let ρ^+ be the monthly recurring income and ρ^- the monthly recurring cost of a control $u = (\sigma, \pi)$ in a 1-ha parcel:

$$\rho^+(u) = \max\{0\ ;\ S_a(u) - C_0(u)\} \quad \text{and} \quad \rho^-(u) = \max\{0\ ;\ C_0(u) - S_a(u)\}$$

For any arc $a^p_{i,j} \in A^p$, we have the following quantities:

- $b^p_{i,j}(t_1, t_2)$: The cost of arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$, i.e.,

$$b^p_{i,j}(t_1,t_2) = \begin{cases} 0, & \text{if } E(i) \in \{\gamma, SN\} \\ \alpha_p\Big((t_2 - t_1)\rho^-(U(i)) + C_a\Big) & \text{otherwise,} \end{cases} \tag{18}$$

where $C_a(u)$ is the additional costs related to the following events:

If $E(i) = \alpha$ and $E(j) = \beta$:

$$C_a(u) = \begin{cases} C_i(U(i)) + C_h(I(i), U(i)), & \text{if } t_1 = T(i) \text{ and } t_2 = T(j), \\ C_i(U(i)), & \text{if } t_1 = T(i) \text{ and } t_2 < T(j), \\ C_h(I(i), U(i)), & \text{if } t_1 > T(i) \text{ and } t_2 = T(j), \\ 0, & \text{otherwise.} \end{cases}$$

If $E(i) = \alpha$ and $E(j) = \gamma$:

$$C_a(u) = \begin{cases} C_i(U(i)) + C_h(I(i), U(i)) + C_e(U(i)), & \text{if } t_1 = T(i) \text{ and } t_2 = T(j), \\ C_i(U(i)), & \text{if } t_1 = T(i) \text{ and } t_2 < T(j), \\ C_h(I(i), U(i)) + C_e(U(i)), & \text{if } t_1 > T(i) \text{ and } t_2 = T(j), \\ 0, & \text{otherwise.} \end{cases}$$

If $E(i) = \beta$:

$$C_a(u) = \begin{cases} C_h(I(i), U(i)), & \text{if } E(j) = \beta \text{ and } t_2 = T(j), \\ C_h(I(i), U(i)) + C_e(U(i)), & \text{if } E(j) = \gamma \text{ and } t_2 = T(j), \\ 0, & \text{otherwise.} \end{cases}$$

- $m^p_{i,j}(t_1, t_2)$: The wealth generated by arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$, i.e.,

$$m^p_{i,j}(t_1,t_2) = \begin{cases} 0, & \text{if } E(i) \in \{\gamma, SN\}, \\ \alpha_p\Big((t_2 - t_1)\rho^+(U(i)) + R_h(I(i), U(i))\Big), & \text{if } E(j) \in \{\beta, \gamma\} \text{ and } t_2 = T(j), \\ \alpha_p(t_2 - t_1)\rho^+(U(i)), & \text{otherwise.} \end{cases} \tag{19}$$

- $d^p_{i,j}(t_1, t_2)$: The change in the GISQ induced by arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$

$$d^p_{i,j}(t_1,t_2) = \begin{cases} \frac{f_c(U(i))}{\delta(U(i))}\phi(b, U(i)), & \text{if } E(i) = \alpha \text{ and } t_2 = T(j), \\ \frac{p_c(U(i))}{\delta(U(i))}\phi(b, U(i)), & \text{if } E(i) = \beta \text{ and } t_2 = T(j), \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

C.4 Crop data

All the parameters are normalized for one parcel of unit area (1 ha); production data are in tons per ha; economic data are provided by experts, in euros per ha or per ton produced (fixed and variable input costs, depreciations, subsidies, labor, sale prices) or per month (labor). Data are extracted from Durand et al. [8].

Table 2 displays the planting/sowing seasons of the seasonal crops.

Table 3 displays the values of the agronomic parameters, and Table 4 those of the economic parameters.

Table 2: Planting months of crops.

Crop	01	02	03	04	05	06	07	08	09	10	11	12
Plantain			x	x	x	x	x					
Export banana			x	x	x	x	x					
Eggplant										x	x	
Tomato										x	x	

Table 3: Agronomic parameter values.

Crop and practice	$\delta(\sigma, \pi)$	r_i	r_d	r_p	r_{ap}	f_c	p_c	R_M
Export banana Agro	60	0.75	0.2	0.12	0.02	12	12	24
Export banana Conv	60	0.3	0.5	-0.08	0	12	12	30.6
Plantain Agro	60	0.7	0.19	0.15	0	6	3	5
Plantain Conv	24	0.25	0.45	-0.02	-0.01	6	3	7.85
Cabbage Agro	3	0.5	0.21	0.15	0	3	0	13
Cabbage Conv	3	0.25	0.3	-0.02	0	3	0	18
Carrot Agro	4	0.65	0.14	0.07	0	4	0	11
Carrot Conv	4	0.05	0.15	-0.03	-0.005	4	0	14
Cassava Agro	12	0.5	0.15	0.07	0	12	0	22
Cassava Conv	12	0.1	0.2	-0.01	-0.01	12	0	25
Cucumber Agro	2	0.8	0.28	0.15	0	2	0	13
Cucumber Conv	2	0.05	0.25	-0.05	-0.025	2	0	18
Eggplant Agro	7	0.8	0.2	0.12	0	7	0	12
Eggplant Conv	7	0.15	0.2	-0.02	-0.075	7	0	20
Green beans Agro	4	0.6	0.09	0.01	0.002	4	0	10
Green beans Conv	3	0.05	0.1	-0.01	0	3	0	13
Lettuce Agro	2	0.6	0.18	0.12	0	2	0	7
Lettuce Conv	2	0.3	0.25	-0.01	0	2	0	13
Melon Agro	3	0.7	0.17	0.1	0	3	0	9
Melon Conv	2	0.2	0.35	-0.02	-0.015	2	0	16
Sugar cane Agro	60	0.3	0.08	0.08	0.035	12	12	60.5
Sugar cane Conv	60	0.1	0.05	-0.01	-0.01	12	12	63.75
Tomato Agro	5	0.8	0.2	0.13	0.002	5	0	11
Tomato Conv	4	0.1	0.4	-0.05	-0.06	4	0	15
Turban squash Agro	3	0.7	0.17	0.099	0	3	0	14
Turban squash Conv	3	0.1	0.15	-0.05	-0.02	3	0	22
Yam (yellow) Agro	8	0.6	0.18	0.11	0.002	8	0	11.7
Yam (yellow) Conv	8	0.15	0.4	-0.01	0	8	0	15
Yam (Grosse Caille) Agro	9	0.7	0.13	0.08	0.004	9	0	11
Yam (Grosse Caille) Conv	8	0.14	0.25	-0.02	-0.02	8	0	14
Short fallow	1	0	0	0	0	1	0	0
Free long fallow	12	0	0	0	0.025	12	0	0
Improved short fallow	3	0.1	0	0.01	0.01	3	0	0
Improved long fallow	6	0	0	0	0.025	6	0	0

References

[1] V Angeon, S Bates, E Chia, J.L Diman, A Fanchone, H Ozier-Lafontaine, and P Saint-Pierre. Détermination des contraintes de viabilité des exploitations agricoles : application aux antilles françaises. Communication presented during the *50e colloque de l'association de science regionale de langue française*, Mons, Belgium, 8–11 Jully 2013.

[2] V Angeon, S Bates, et al. L'agriculture, facteur de vulnérabilité des petites économies insulaires? Region et Developpement, 42:105–131, 2015.

[3] J-P Aubin. A survey of viability theory. SIAM Journal on Control and Optimization, 28(4):749–788, 1990.

[4] S Baumgärtner and M.F Quaas. Ecological-economic viability as a criterion of strong sustainability under uncertainty. Ecological Economics, 68(7):2008–2020, 2009.

[5] H Blanco and R Lal. Soil and water conservation. Principles of Soil Conservation and Management; Blanco, H., Lal, R., Eds, pages 1–19, 2010.

[6] Nuria Ruiz Camacho, Velasquez Elena, Anne Pando, Decaëns Thibaud, Dubs Florence, and Lavelle Patrick. Indicateurs synthétiques de la qualité du sol. Etude et gestion des sols, 16(3/4):323–338, 2009.

[7] G Desaulniers, J Desrosiers, and M M Solomon. Column generation, volume 5. Springer Science & Business Media, 2006.

Table 4: Economic parameter values.

Crop and practice	S_a	S_p	C_t	C_i	C_m	P	C_c	C_e
Export banana Agro	0	400	400	15143	250	600	8680.7	1840
Export banana Conv	0	400	392.9	17143	366.83	550	8680.7	1840
Plantain Agro	0	0	270	5000	465	800	452	120
Plantain Conv	0	0	260	5500	465	600	452	160
Cabbage Agro	0	0	1000	2500	0	1300	1200	500
Cabbage Conv	0	0	850	2200	50	1200	1200	500
Carrot Agro	0	0	550	1000	24	1300	2800	503
Carrot Conv	0	0	500	150	54	1200	28500	503
Cassava Agro	0	0	220	530	10	650	3390	0
Cassava Conv	0	0	221.5	528	14	610	3393	0
Cucumber Agro	0	0	1800	2800	200	850	1800	0
Cucumber Conv	0	0	1300	2500	650	790	1832	0
Eggplant Agro	0	0	600	3500	0	1300	1900	500
Eggplant Conv	0	0	300	3500	50	1100	1600	500
Green beans Agro	0	0	900	3000	0	2000	3000	500
Green beans Conv	0	0	700	2500	50	1800	6000	500
Lettuce Agro	0	0	1700	4000	0	2200	1000	500
Lettuce Conv	0	0	1700	3200	50	2000	1000	500
Melon Agro	0	0	600	5600	200	1200	387	0
Melon Conv	0	0	450	5600	750	1000	397	0
Sugar cane Agro	0	13.23	40	1928	30	60	1464	150
Sugar cane Conv	0	14.76	40	1872	50	55	1362.5	200
Tomato Agro	0	0	1800	5300	0	1600	2200	0
Tomato Conv	0	0	1100	5300	400	1300	2200	0
Turban squash Agro	0	0	850	6000	70	1000	876	0
Turban squash Conv	0	0	800	6500	130	900	876	0
Yam (yellow) Agro	0	0	1500	14000	10	2700	2500	0
Yam (yellow) Conv	0	0	900	13200	100	2500	2500	0
Yam (Grosse Caille) Agro	0	0	1406	11550	0	2100	3192	0
Yam (Grosse Caille) Conv	0	0	1000	7815	82	2000	2400	0
Short fallow	0	0	0	0	0	0	0	0
Free long fallow	0	0	0	0	0	0	0	0
Improved short fallow	0	0	150	400	50	0	0	0
Improved long fallow	0	0	100	800	50	0	0	0

[8] M-H Durand, A Desilles, P Saint-Pierre, V Angeon, and H Ozier-Lafontaine. Viability modeling of agro-ecological transition: enhancing the provision of soil ecosystem services to support farm sustainability in small island developing states. a, To apear.

[9] S Martin, G Deffuant, and J.M Calabrese. Defining resilience mathematically: from attractors to viability. In Viability and Resilience of Complex Systems, pages 15–36. Springer, 2011.

[10] L Mouysset, L Doyen, and F Jiguet. From population viability analysis to coviability of farmland biodiversity and agriculture. Conservation biology, 28(1):187–201, 2013.

[11] A Oubraham and G Zaccour. A survey of applications of viability theory to the sustainable exploitation of renewable resources. Ecological Economics, 145:346–367, 2018.

[12] A Oubraham, P Saint-Pierre, and G Zaccour. Viability of agroecological systems under climatic uncertainty. Sustainability, 12(15):5880, 2020.

[13] R Sabatier, L Doyen, and M Tichit. Modelling trade-offs between livestock grazing and wader conservation in a grassland agroecosystem. Ecological Modelling, 221(9):1292–1300, 2010.

[14] R Sabatier, L Doyen, and M Tichit. Action versus result-oriented schemes in a grassland agroecosystem: A dynamic modelling approach. PLoS ONE, 7(4):1–12, 04 2012. doi: 10.1371/journal.pone.0033257. URL http://dx.doi.org/10.1371%2Fjournal.pone.0033257.

[15] R Sabatier, L.G Oates, and R.D Jackson. Management flexibility of a grassland agroecosystem: A modeling approach based on viability theory. Agricultural Systems, 139:76–81, 2015.

[16] M Tichit, B Hubert, L Doyen, and D Genin. A viability model to assess the sustainability of mixed herds under climatic uncertainty. Animal Research, 53(5):405–417, 2004.

[17] M Tichit, L Doyen, J.Y Lemel, O Renault, and D Durant. A co-viability model of grazing and bird community management in farmland. Ecological Modelling, 206(3):277–293, 2007.

[18] Elena Velásquez, Patrick Lavelle, and Mercedes Andrade. Gisq, a multifunctional indicator of soil quality. Soil Biology and Biochemistry, 39(12):3066–3080, 2007.

[19] Alexander Wezel, Marion Casagrande, Florian Celette, Jean-François Vian, Aurélie Ferrer, and Joséphine Peigné. Agroecological practices for sustainable agriculture. a review. Agronomy for sustainable development, 34(1):1–20, 2014.