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Abstract. We introduce the location-or-routing problem (LoRP), which integrates the facility location and the vehicle routing problems by uncovering a new connection. In the LoRP, open facilities cover the customers in their neighborhood and the uncovered customers are transported to open facilities by capacitated vehicles. Each facility has a maximum coverage range and each vehicle route is constrained by a maximum length. In this setting, a customer can be covered either by 'location' or by 'routing', hence the name. We discuss several application areas of LoRP and present its relation to the location and routing problems. We develop a set covering model and a branch-and-price algorithm as an exact solution methodology. Our trade-off analyses on random graphs show that the total cost decreases almost linearly with increasing facility coverage range.

Keywords: Transportation, location, routing, location-routing, branch-and-price.

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1. Introduction

This paper proposes a generalized customer coverage model by integrating the location and vehicle routing problems. To this end, we introduce the location-or-routing problem (LoRP), in which the customers can be covered by facilities if they are located within the coverage range of any open facility or by a vehicle departing from an open facility subject to maximum route length and vehicle capacity constraints. The objective of LoRP is to minimize the total cost of opening facilities and vehicle routing.

Consider locating general service stores such as retailers, supermarkets or shopping malls in an urban area. In the covering location problems, a customer is assumed to be *covered* (or equivalently *served*) if the distance to the closest open facility is within a certain threshold. This threshold generally represents the tolerance of the customer to travel to the closest open facility. The customers that lie beyond the coverage range of any open facility are assumed to be uncovered in covering location problem. When customers cannot be served by open facilities, complimentary shuttle buses are offered in many large cities including Beijing (Kai-yan et al., 2013; Wang and Nie, 2020), Toronto (Tsawwassen Mills, 2020; Vaughan Mills, 2020; CrossIron Mills, 2020), Victoria (Victoria Transport Policy Institute, 2020) and Istanbul (Historia Shopping Mall, 2020; Starcity Outlet, 2020; Canpark Shopping Mall, 2020). Put differently, the open facilities cover a subset of customers and the uncovered customers are served by vehicles. The LoRP arises in a number of application contexts.

1. *Retail store, supermarket and shopping mall location,*

2. *School location and bus routing:* Generally, the school location (Antunes and Peeters, 2000) and school bus routing problems (Park and Kim, 2010) are solved hierarchically. When the two problems are considered simultaneously, the problem we deal with is LoRP, in which the students are assumed to be covered within a certain distance from the school and buses are used to transport students that lie beyond the coverage range,

3. *Urban delivery center location with drone operations:* When packages from urban delivery centers are transported by limited-range drones (Otto et al., 2018) and the uncovered customers are served by trucks located at these delivery centers, we encounter a LoRP application,

4. *Testing facility location in pandemics:* When there is a need for a large-scale testing due to a pandemic such as COVID-19, the LoRP can also be applied to the location optimization of testing centers in urban areas. The aim is to cover the population by providing them with an opportunity either to visit a nearby center or to get tested at their home by a mobile medical clinic vehicle.

In general, the LoRP is applicable when a facility provides public or private service to a neighborhood, and the customers beyond the facility coverage range receive service by vehicles either at their location or by being transported to the facility. Though each different application has its own peculiarities and side-constraints, the common understanding of service to a customer is similar: the location decisions induce the mode of service for different sets of customers. In LoRP, we capture this connection between the location and routing decisions.

Figure 1 demonstrates an example. There are a total of 30 customers. In Figure 1(a), when

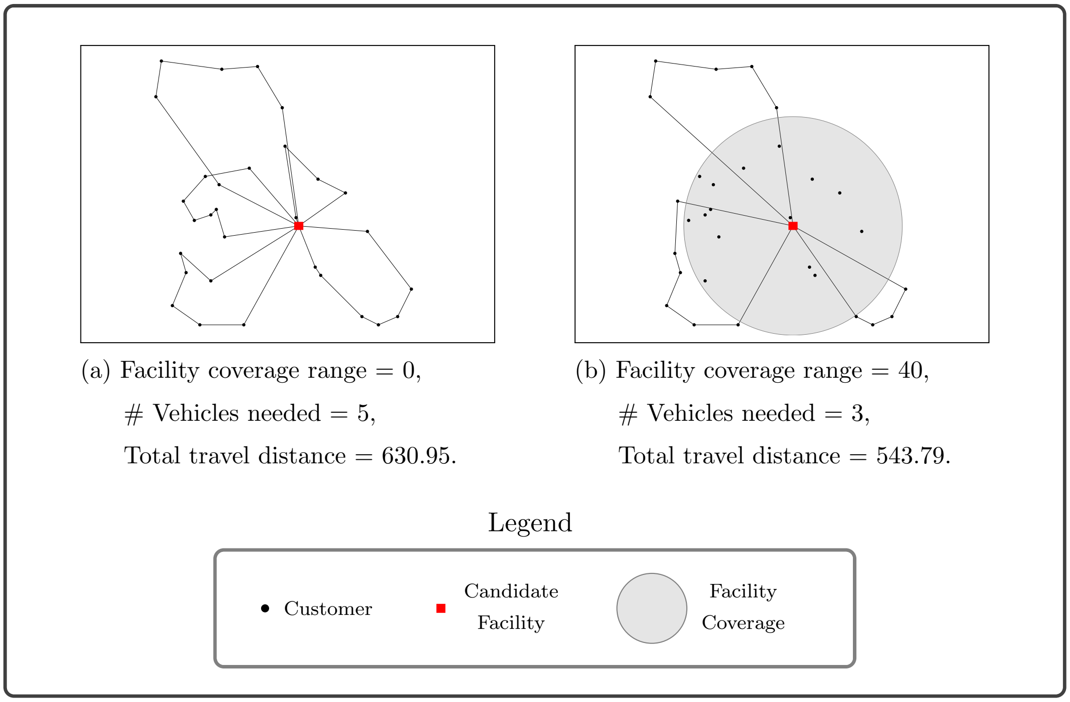

Figure 1: Vehicle routes and total distance change with and without facility coverage in parts (a) and (b), respectively.

facility coverage range is zero, five vehicles are needed and the total travel distance is 630.95. When we assume that the facility covers the customers in its neighborhood as in Figure 1(b), the required number of vehicles reduce to three and the total travel distance reduces to 543.79.

1.1. Literature review

The LoRP is closely related to the location-routing problem (LRP), in which the goal is opening a set of facilities and visiting every customer by a capacitated vehicle starting at one of the open facilities. Similar to LoRP, the objective of LRP is to minimize the total cost of opening facilities and vehicle routing. The facility prototype considered in LRP is a depot or a warehouse (Perl and Daskin, 1985). Opening a facility does not directly cover any customer.

The idea of combining the location and routing problems is rooted in the study by Von Boventer (1961) and evolved into the LRP over time (Laporte, 1988; Laporte et al., 1988; Min et al., 1998). Recent reviews on the LRP include Nagy and Salhi (2007) and Prodhon and Prins (2014). The survey by Drexl and Schneider (2015) focus on variants and extensions of LRP and the survey by Schneider and Drexl (2017) focus on the standard LRP.

Combining two NP hard problems, the LRP has intricate relationship between the location and routing decisions and often necessitates advanced solution techniques. To this end, Berger et al. (2007) and Akca et al. (2009) develop exact branch-and-price (B&P) algorithms for solving the LRP. Belenguer et al. (2011) add capacity constraints on depots and vehicles and develop an exact branch-and-cut algorithm. Baldacci et al. (2011) observe that the LRP can be decomposed into a limited set of multicapacitated depot vehicle-routing problems and build an algorithm by various bounding procedures. Contardo et al. (2014) combine these ideas and introduce several new valid inequalities to accelerate the solution process. Escobar et al. (2014) propose a novel granular tabu search within a variable neighborhood search. Ponboon et al. (2016) and Farham et al. (2018) also include time windows in routing.

Considering a heterogeneous fleet of vehicles is an important facet of the vehicle routing problem (Koç et al., 2016c). The mix LRP (mLRP) relaxes the homogeneity assumption of the vehicles and considers a heterogeneous fleet in location-routing problem (Koç et al., 2016a). This problem arises in the context of city logistics (Koç et al., 2016b). The mLRP is particularly important, because every vehicle type offers an alternative coverage option, similar to the location and routing decisions in LoRP. We further elaborate on this relationship in Section 2.

All of the studies described above consider the vehicle visits as the only way to serve a customer, and the location decisions are related to selection of the origin of the vehicle routes. Another closely related problem that does not have this assumption is the covering tour problem (CTP) (Gendreau et al., 1997). The CTP defines a coverage circle around a customer node and all customers in this circle are assumed to be covered when the node is visited. CTP then finds a minimum cost vehicle route such that all customers are within a certain radius from any node on the route. Unlike the vehicle routing problem, serving a customer is possible without actually visiting the customer by a vehicle. In CTP, the coverage circle is the same for all the nodes in the network. In LoRP, on the other hand, a customer does not have a coverage radius, but a candidate facility has. A customer can be covered by a facility only, but not by any other customer.

1.2. Scientific Contributions and Organization of the Paper

In our study, we take the location and routing decisions as alternative ways of serving the customers. As contributions of this study, we

- introduce the ‘location-or-routing problem’ and position it precisely in the vast body of vehicle routing and facility location literature,
- develop a B&P algorithm as an exact solution methodology,
- investigate the trade-off between the facility coverage range and the total cost and
- show computationally on random graphs that the total cost decreases almost linearly with increasing facility coverage range.

We present the formal problem definition and a set covering formulation in Section 2, develop an exact B&P algorithm in Section 3 and test its computational efficiency and discuss the results in Section 4. We conclude the study in Section 5.

2. Problem Definition and Formulation

In this section, we formally define the LoRP, investigate its relation with the LRP, develop a set covering formulation and valid inequalities.

2.1. Problem Definition

Let I be the set of candidate facilities, J be the set of customers and $G = (N, A)$ be a directed network where $N = I \cup J$ is the set of nodes and $A = \{(i, j) : i, j \in N, i \neq j\}$ is the set of arcs. The length of arc $(i, j) \in A$ is d_{ij}. Without loss of generality, we assume that the distance matrix satisfies the triangular inequality. Each candidate facility $i \in I$ has a setup cost f_i and a coverage range r_i. Each customer $j \in J$ has a demand q_j, which must be covered either by a facility or by a vehicle. A facility i covers a customer j if and only if $d_{ij} \leq r_i$. Let I_j be the set of facilities covering customer $j \in J$ and J_i be the set of customers covered by facility $i \in I$. There is a homogeneous fleet of vehicles with capacity Q and route lengths are constrained to be maximum T.

Definition 1. *Location-or-routing problem (LoRP) is defined as selecting a set of facilities to open and a set of vehicle routes that start at open facilities and respect the vehicle capacity and maximum length constraints such that every customer is covered either by a facility or by a vehicle route and the total cost of opening facilities and routing vehicles is minimized.*

Remark 1. *If the coverage range $r_i = 0$ for all $i \in I$, LoRP transforms into a LRP.*

Remark 2. *If the route length of vehicles $T = 0$, LoRP transforms into a set covering problem.*

Therefore, the problem introduced in this paper is a generalization of the LRP and the set covering problem.

Observe that covering customers by facilities can be represented by dummy vehicle routes. There are two ways to achieve this. First, the coverage of a customer j by a facility i can be represented by a dummy vehicle route $p_{ij} = (i, j, i)$ with $d_{p_{ij}} = 0$. Note that, this requires introducing a different vehicle type for every customer $j \in J$ and facility $i \in I_j$ with capacity q_j. Second, a dummy vehicle route p_i with $d_{p_i} = 0$ can be introduced for every facility $i \in I$ which visits all customers $j \in J_i$ in random order. Note that the such dummy vehicles must be uncapacitated. These two observations lead to the following result.

Remark 3. *LoRP is a special case of mLRP.*

Note that this observation enables us to use the formulations developed for mLRP for solving the LoRP. However, to the best of our knowledge, no exact algorithm has been developed yet for solving the mLRP.

2.2. Location-or-routing as a Problem Class

The problem we define here can also be referred to as the *set covering location-or-routing problem (SCLoRP)*, because it integrates the location-routing and the set covering problems. It is then natural to ask if other facility location problems can also be extended similar to set covering problem, and the answer is positive. A straightforward extension is to change the objective function as the maximization of the customer coverage and consider the cost as a constraint, as in maximum covering location problem. This would lead to the *maximum covering location-or-routing problem*

(MCLoRP). Another interesting extension is by considering the time that customers spend to reach to facilities. A customer can directly walk to the service facility or use a shuttle vehicle. When the objective is to minimize the total time that customers spend en route, we attain the *p-median location-or-routing problem (pMLoRP)*. If the objective is to minimize the maximum time a customer spends to reach to a facility, we then obtain the *p-center location-or-routing problem (pCLoRP)*. Therefore, the LoRP can be considered as a class of problems. In this paper, we restrict ourselves to the SCLoRP and refer to it as LoRP.

2.3. A Set Covering Formulation

We define a path p as an ordered set of directed arcs, starting and ending at the same candidate facility node $i \in I$. We use the terms route and path interchangeably. Set J_p represents the customers visited on path p, A_p represents the arcs in p and p_0 represents the starting node of p. We refer to a path as feasible if it respects the maximum route length and vehicle capacity constraints, that is, $\sum_{(i,j)\in A_p} d_{ij} \leq T$ and $\sum_{(i,j)\in A_p : j \neq p_0} q_j \leq Q$. Let $\widetilde{\mathcal{P}}_j$ be the set of paths visiting customer $j \in J$, $\overline{\mathcal{P}}_i$ be the set of paths starting from facility $i \in I$, $\widehat{\mathcal{P}}_{ij} = \widetilde{\mathcal{P}}_j \cap \overline{\mathcal{P}}_i$, and $\mathcal{P} = \bigcup_{i\in I} \overline{\mathcal{P}}_i$.

Proposition 1. *Without loss of generality, we set $\widehat{\mathcal{P}}_{ij} = \emptyset$ when $d_{ij} \leq r_i$, for all $i \in I$ and $j \in J$.*

Proof. Assume that $d_{i_0 j_0} \leq r_{i_0}$, for $i_0 \in I$ and $j_0 \in J$ and that $y_{p_0} = 1$ in the optimal solution for path $p_0 = (i_0, \ldots, i_1, j_0, i_2 \ldots, i_0)$ starting at i_0 and visiting customer j_0. Since customer j_0 can be covered at no cost by facility i_0, there exists a feasible path $p_1 = p_0 \setminus \{j_0\} = (i_0, \ldots, i_1, i_2 \ldots, i_0)$ with $d_{p_0} = d_{p_1}$ due to triangular inequality. Thus, an alternative optimal solution with $y_{p_0} = 0$ and $y_{p_1} = 1$ exists. Applying the same argument for every i and j pair with $d_{ij} \leq r_i$ gives the desired result. $\square$

Note that, due to Proposition 1, increasing coverage range implies smaller number of feasible routes. Let x_i equal 1 if and only if facility $i \in I$ is selected and y_p equal 1 if and only if path $p \in \mathcal{P}$ is selected. We formulate the LoRP as follows.

$$\text{minimize} \quad \sum_{i\in I} f_i x_i + \sum_{p\in\mathcal{P}} d_p y_p \tag{1}$$

$$\text{subject to} \quad \sum_{i\in I_j} x_i + \sum_{p\in\widetilde{\mathcal{P}}_j} y_p \geq 1 \qquad j \in J \tag{2}$$

$$|J|\, x_i - \sum_{p\in\overline{\mathcal{P}}_i} y_p \geq 0 \qquad i \in I \tag{3}$$

$$x_i, y_p \in \{0,1\} \qquad i \in I, p \in \mathcal{P} \tag{4}$$

The objective function minimizes the sum of facility opening and routing costs. Constraints (2) ensure that all customers are covered. Constraints (3) force a path to start from an open facility and constraints (4) are the domain restrictions.

2.4. Valid inequalities

The main purpose of constraints (3) is to force $x_i = 1$ when there exists a path in the optimal solution starting at facility i. Here $|J|$ is used as a big number, and such big-M type inequalities generally provide poor linear programming (LP) relaxations. Alternatively, constraints (3) can be replaced by

$$x_i - \sum_{p \in \widehat{\mathcal{P}}_{ij}} y_p \geq 0 \qquad i \in I \setminus I_j, j \in J, \tag{5}$$

which states that if a customer j is served by a vehicle route starting from a facility i, then facility i should be open. Constraints (5) are known to be stronger (Akca et al., 2009) than constraints (3).

Note that constraints (2) may have positive slack because a customer can potentially be covered by multiple facilities and the left-hand side can be greater than or equal to 2. Therefore, it is not necessarily satisfied at equality. We use the following inequality to put an upper bound on the summation of location and routing variables.

$$x_i + \sum_{p \in \widetilde{\mathcal{P}}_j} y_p \leq 1 \qquad i \in I_j, j \in J, \tag{6}$$

which implies that if a customer is covered by a facility, then all the variables corresponding to routes visiting customer j are set equal to zero.

Let v_i be an integer variable representing the number of vehicles starting their routes from facility $i \in I$. This variable type is not necessary for problem description, but it is mainly used to implement the branching rules in a B&P algorithm. The following inequalities are then valid.

$$\sum_{p \in \overline{\mathcal{P}}_i} y_p = v_i \qquad i \in I \tag{7}$$

$$\sum_{i \in I} v_i \geq \left\lceil \frac{\sum_{j \in J \setminus \cup_{i \in I} J_i} q_j}{Q} \right\rceil \tag{8}$$

Constraint (8) puts a lower bound on the number of vehicles to be used. This bound is generally weaker in LoRP than LRP because customers are not necessarily visited by vehicles and they may be covered by a facility.

Combining the initial model with these valid inequalities, we obtain the following formulation, which we refer to as the set covering (SC) model.

$$\begin{aligned} \text{(SC) minimize} \quad & \sum_{i \in I} f_i x_i + \sum_{p \in \mathcal{P}} d_p y_p \\ \text{subject to } & (2), (5) - (8) \\ & x_i, y_p, v_i \geq 0 \text{ and integer} \qquad i \in I, p \in \mathcal{P} \end{aligned}$$

Note that we relax binary variables x and y into integer variables without loss of generality due to the minimization type of the objective function. This eliminates the need to add an upper bound of 1 on the binary variables when solving the LP relaxation of the SC model.

3. Solution methodology

We now develop an exact B&P algorithm to solve the SC model. In this section, we present a column generation algorithm, the pricing problem, branching rules, and other implementation details including generation of initial set of columns, variable fixing and upper bound heuristics.

3.1. Column generation

Solving the LP relaxation of SC model, which we refer to as SC-R, is an integral part of the B&P algorithm. We start the column generation algorithm by solving the LP relaxation of a restricted SC model with an initial set of columns only. This provides us with the dual variables, which in turn allows us to obtain the reduced cost of all path variables in the SC model. We then add at least one variable with a negative reduced cost, if one exists, and resolve the LP relaxation. This iterative procedure is continued until no such variable with a negative reduced cost exists after solving the restricted SC model, which gives us a certificate that the solution obtained is optimal for the SC-R.

Let α_j, β_{ij} and δ_{ij} be the nonnegative dual variables associated with constraints (2), (5), (6), respectively, and γ_i be the dual variables unrestricted in sign, associated with constraints (7). The reduced cost of a path variable is then given in the following expression.

$$c_p = d_p - \sum_{j \in J_p} \alpha_j + \sum_{\substack{j \in J: \\ p_0 \notin I_j}} \beta_{p_0 j} + \sum_{j \in J} \sum_{i \in I_j} \delta_{ij} - \gamma_{p_0}. \tag{9}$$

Note that the reduced cost of a path variable contains information about the facilities that cover the customers on the path through the δ_{ij} variables.

Having $\min_{p \in \mathcal{P}}\{c_p\} \geq 0$ ensures that all path variables with nonnegative reduced costs are in the problem and the SC-R is solved optimally. If there exists a path p with $c_p < 0$, we add the corresponding variable y_p to the formulation. Therefore, the goal after solving the restricted SC-R is to identify a path with negative reduced cost. This problem is referred to as the pricing problem, which is the topic of the next section.

3.2. Pricing problem

For candidate facility i, consider graph $\hat{G}_i = (\hat{N}_i, \hat{A}_i)$, where $\hat{i}$ is a duplicate node of facility i, $\hat{N}_i = J \cup \{i, \hat{i}\}$ and $\hat{A}_i = \big\{(m,n) \in A : m,n \in J\big\} \bigcup \big\{(i,m) : m \in J\big\} \bigcup \big\{(m,\hat{i}) : m \in J\big\}$. The length of arc $(m,n) \in \hat{A}_i$ is

$$\hat{d}_{mn} = \begin{cases} d_{mi} - \gamma_i & \text{if } n = \hat{i} \\ d_{mn} - \alpha_n + \beta_{in} & \text{if } n \neq \hat{i} \text{ and } i \notin I_n \\ d_{mn} - \alpha_n + \sum_{k \in I_n} \delta_{kn} & \text{otherwise} \end{cases} \qquad (m,n) \in \hat{A}_i. \tag{10}$$

Traveling an arc $(m,n) \in \hat{A}_i$ consumes two types of resources, r^1_{mn} and r^2_{mn} from the vehicle capacity (Q) and the maximum route length (T), respectively.

$$r^1_{mn} = \begin{cases} q_n & \text{if } n \neq \hat{i} \\ 0 & \text{otherwise} \end{cases} \qquad (m,n) \in \hat{A}_i, \tag{11}$$

$$r^2_{mn} = \begin{cases} d_{mn} & \text{if } n \neq \hat{i} \\ d_{im} & \text{otherwise} \end{cases} \qquad (m,n) \in \hat{A}_i. \tag{12}$$

The pricing problem is then to find a shortest path in graph $\hat{G}_i$ for every candidate facility $i \in I$ subject to two side constraints associated with the vehicle capacity Q and the maximum route length T. Let z_{mn} equal 1 if and only if arc (m,n) is selected. The formulation presented below models the pricing problem for a given facility i in graph $\hat{G}_i$.

$$\text{minimize} \quad \sum_{(m,n) \in \hat{A}} \hat{d}_{mn} z_{mn} \tag{13}$$

$$\text{subject to} \quad \sum_{n:(m,n) \in \hat{A}} z_{mn} - \sum_{n:(n,m) \in \hat{A}} z_{nm} = \begin{cases} 1 & \text{if } m = i \\ -1 & \text{if } m = \hat{i} \\ 0 & \text{otherwise} \end{cases} \qquad m \in \hat{N}_i \tag{14}$$

$$\sum_{(m,n) \in \hat{A}} r^1_{mn} z_{mn} \leq Q \tag{15}$$

$$\sum_{(m,n) \in \hat{A}} r^2_{mn} z_{mn} \leq T \tag{16}$$

$$\sum_{(m,n) \in S} z_{mn} \leq |S| - 1 \qquad S \subset \hat{A}_i \tag{17}$$

$$z_{mn} \in \{0,1\} \qquad (m,n) \in \hat{A}_i. \tag{18}$$

The objective function minimizes the reduced cost. Constraints (14) are the node balance equations. Constraints (15) and (16) ensure that the selected path respects the resource constraints. Constraints (17) ensure that the selected path is elementary and constraints (18) are the domain restrictions.

The pricing problem itself is NP-hard, however several algorithms exist for solving a resource constrained shortest path problem (Pugliese and Guerriero, 2013). In this paper, we adopt a stateof-the-art algorithm developed by Lozano et al. (2016), referred to as the pulse algorithm. The algorithm solves an elementary resource constrained shortest path problem with time windows. In our implementation, we use r^2_{mn} as the time consumption on arc (m, n) and set the time window of a candidate facility as $[0, T]$. Resource r^1_{mn} counts towards the capacity Q. This ensures that the path given by the algorithm is elementary resource constrained shortest path respecting the vehicle capacity and the maximum route length constraints.

3.3. Determining Feasibility

Before starting the algorithm, we use the following observation to determine if an instance is feasible.

Proposition 2. *For $j \in J$, let ℓ_j be the distance to the closest candidate facility, that is, $\ell_j = \min_{i \in I} d_{ij}$. A given LoRP instance is feasible if and only if one of the following two conditions hold for every customer $j \in J$:*

- *there exists $i \in I$ such that $d_{ij} \leq r_i$,*
- *$q_j \leq Q$ and $\ell_j \leq T/2$.*

Proof. (Necessity) Assume that a given LoRP instance is feasible. Then, for each customer $j \in J$, there exists either a facility that covers j or a feasible path visiting j. The former provides the first condition above. Let p_j be the path visiting customer j, N_j be the set of nodes on path p_j and A_j be the set of arcs on path p_j. Due to feasibility, we have $\sum_{k \in N_j} q_k \leq Q$, which implies that $q_j \leq Q$. Similarly, we have $\sum_{(m,n) \in A_j} d_{mn} \leq T$ and due to triangular inequality, we have $2\ell_j \leq \sum_{(m,n) \in A_j} d_{mn} \leq T$, which provides the second condition above.

(Sufficiency) Assume that at least one of the two conditions hold for every $j \in J$. First, assume that the former condition holds. Then, there exists $i \in I$ such that $d_{ij} \leq r_i$ and customer j is covered. Now, assume that the second condition holds. Let i_j be the closest facility to customer $j \in J$. Since $q_j \leq Q$ and $\ell_j \leq T/2$, opening candidate facility i_j and selecting path (i_j, j, i_j) covers customer j. This solution with all customers covered either by an open facility or by a path is feasible for the given problem instance. □

3.4. Initial Set of Columns

The initial set of columns is needed to ensure that the first restricted problem is feasible and that the dual variables can be obtained. Therefore, there must exist a path variable in the formulation corresponding to each customer. Similarly, there must exist a path variable starting at every candidate facility. Furthermore, these paths must be feasible. We build a feasible solution as follows. Let i_j be the closest candidate facility to customer $j \in J$ and p_j be a path that visit customer j from i_j. That is, $p_j = (i_j, j, i_j)$ for every $j \in J$. We add a variable for path p_j for every customer $j \in J$, which ensures that the instance is feasible due to Proposition 2.

3.5. Branching Rules and Variable Fixing

If the optimal solution of SC-R is integer, it is also optimal for SC model. If it has fractional terms, we need to branch for an integer solution. We implement a three-stage hierarchical branching. In all levels, we select the most fractional variable to branch on. The first level branches on the facility variables x. At the master problem level, we only add a constraint to enforce the branching rule. We also do not need to solve the pricing problem associated with a facility i if $x_i = 0$. When all facility variables are binary, we branch on v variables. Similar to the facility location variables, we only add a single constraint to the master problem in order to enforce the branching rules. The pricing problem is not impacted by the branching rules on v variables. The third branching level is on flow variables. We branch on implicit arc variables which equals 1 if and only if an arc is used in the solution. To enforce an arc (i, j) to be used, we remove all the variables from the master problem corresponding to the paths that visit nodes i or j without using arc (i, j). In the pricing problem, we remove all arcs leaving node i and entering node j except for arc (i, j). To forbid using arc (i, j) is more straightforward, we remove all variables corresponding to paths using arc (i, j) from the master problem and forbid using arc (i, j) in the pricing problem.

We also implement variable fixing by reduced cost for facility location variables (Savelsbergh, 1994). Let ψ_i be the reduced cost of variable x_i for all $i \in I$ and $\overline{z}$ and $\underline{z}$ be the upper and lower bounds on the optimal objective function value, respectively, at any given node of the branch-and-bound (B&B) tree. The variable x_i is set equal to zero if $\psi_i > \overline{z} - \underline{z}$. The implementation is similar to first level branching rule.

3.6. Upper Bound Heuristic

At any node in the B&B tree, we can build a mixed integer linear program (MILP) from the SC-R model by imposing integrality constraints on the continuous variables in order to obtain a feasible solution for the SC model. If the solution generated improves the incumbent solution, we keep the new solution as the incumbent and continue exploring the B&B tree. We build the first MILP after solving the SC-R at the root node, which provides an upper bound when starting to explore the B&B tree. Note that any feasible solution of such a restricted MILP model is also feasible for the SC model. Therefore, we do not necessarily run the restricted MILP to optimality. In our implementation, we run the restricted model for at most 120 seconds. In our computational experiments, we observed that the MILP model generally runs much faster, in a few seconds. Similarly, we build the restricted MILP model every time 100 or more new path variables are generated. This allows us to obtain good upper bounds as we build the B&B tree.

4. Computational Study

We implemented our algorithms using Java, and all the experiments were conducted on the Cedar cluster of Compute Canada using single thread and 10GB of RAM under Linux environment. We used CPLEX 12.10.0 for solving linear programs. The time limit for all experiments is set to three hours.

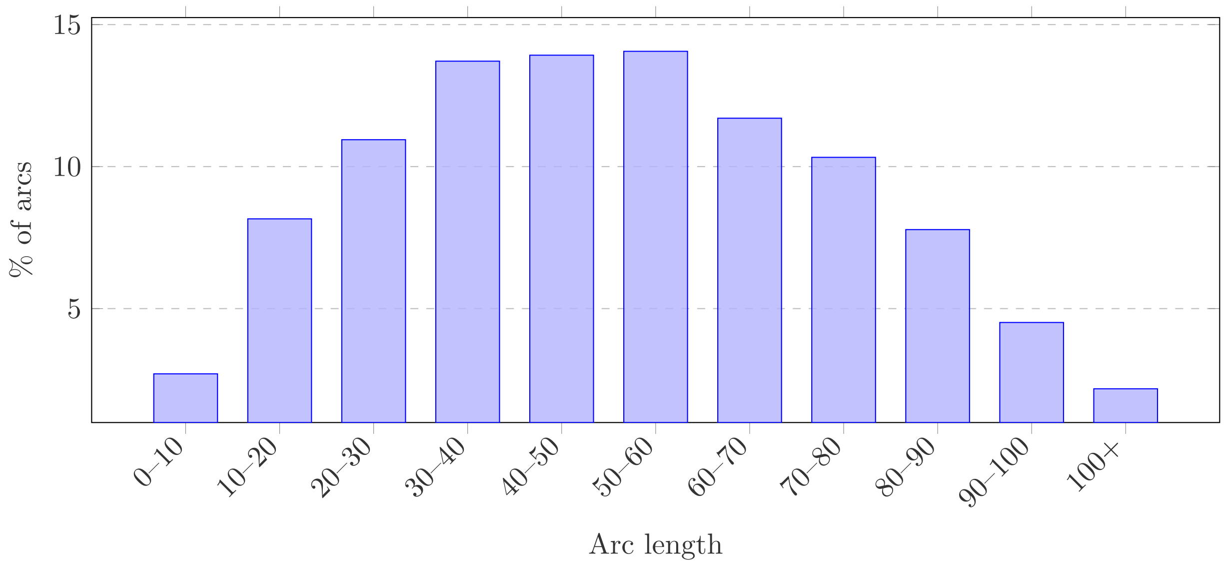

Figure 2: Arc length distribution in the 9 networks considered

4.1. Experimental Setting

We use the large instances of Akca et al. (2009), which contains 6 instances with 30 customers (namely, r30x5a-1, r30x5a-2, r30x5a-3, r30x5b-1, r30x5b-2, r30x5b-3) and 6 instances with 40 customers (namely, r40x5a-1, r40x5a-2, r40x5a-3, r40x5b-1, r40x5b-2, r40x5b-3), all of which have 5 candidate facilities. The fixed setup cost of facilities is 100 and the vehicle capacities vary between 275 and 390. Figure 2 plots the arc length distribution in all 12 networks considered. The horizontal axis is the arc length and the vertical axis is the percentage of the arcs in the graphs. The longest arc length is 130.1. In the design of experiments, we use a maximum route length (T) of 150, 160, 170, 180, 190 and 200. We use a fixed coverage range for every facility, which we refer to as R, and test for $R \in \{0, 10, 20, 30, 40, 50, 60\}$. This setting makes a total of 504 problem instances. In the following, we first present the computational results and then study the impacts of facility coverage range and maximum route length on the total costs.

4.2. Computational Results

Among the 504 instances, there are 7 infeasible instances corresponding to network ‘r30x5b-3’ with the maximum route length $T = 150$ and facility coverage range $R \in \{0, 10, 20, 30, 40, 50, 60\}$. Among the remaining 497 instances, 480 were solved to optimality within the time limit. The average optimality gap of the remaining 17 feasible instances is 0.77% and the gaps are between 0.10% and 2.04%. The average gap over all instances solved is 0.03%.

In the following, we refer to the optimality gap of the LP relaxation as ‘LP gap’. The LP gaps for varying T and R values are shown in Table 1. The average LP gaps are 1.60%, 1.57%, 1.28%, 1.92%, 1.15%, 0.06% and 0% when R equals 0, 10, 20, 30, 40, 50 and 60, respectively. When the facility range increases, the LoRP converges to set covering problem and the LP gaps are generally smaller. Generating *elementary* paths in the pricing problem is also another reason for such small LP gaps. The average LP gap over all instances is 1.08%.

Table 1: LP relaxation optimality gap (%)

Max. Route Length (T)	Facility Coverage Range (R) 0	10	20	30	40	50	60	Average
150	1.59	1.56	1.34	1.68	1.55	0.00	0.00	1.10
160	1.77	1.73	1.61	2.08	1.28	0.00	0.00	1.21
170	1.58	1.65	1.04	1.86	0.92	0.00	0.00	1.01
180	1.57	1.56	1.14	1.83	1.04	0.00	0.00	1.02
190	1.52	1.44	1.25	1.99	0.99	0.00	0.00	1.03
200	1.58	1.47	1.30	2.03	1.14	0.34	0.00	1.12
Average	1.60	1.57	1.28	1.92	1.15	0.06	0.00	1.08

The computational times are reported in Table 2. The solution times are 2848.2, 1433.0, 29.5, 15.3, 2.4, 0.8 and 0.5 seconds for facility coverage ranges of 0, 10, 20, 30, 40, 50 and 60, respectively. The average solution time is significantly affected by the facility coverage range. When the facility coverage range is long, several customers are automatically covered and this results in smaller number of feasible paths. Therefore the column generation does not spend much time for generating new paths and the solution times decrease. The maximum route length also has an impact on the solution times, but not as significant as the facility coverage range. The average solution time increases when the maximum route length increases, mainly because the pricing problem is less constrained, which leads to more time being spent for solving the pricing problem.

Table 2: Average computation time (s)

Max. Route Length (T)	Facility Coverage Range (R) 0	10	20	30	40	50	60	Average
150	2104.6	1078.5	23.2	5.6	2.1	0.5	0.4	459.3
160	2145.0	1123.1	32.8	8.1	1.8	0.7	0.4	473.1
170	2323.5	1297.9	19.4	12.9	2.0	0.7	0.5	522.4
180	3272.1	1363.6	27.4	15.1	2.5	0.7	0.5	668.9
190	3544.0	1344.2	34.3	20.2	2.7	0.9	0.5	706.7
200	3638.2	2361.0	39.3	28.9	3.2	1.1	0.5	867.5
Average	2848.2	1433.0	29.5	15.3	2.4	0.8	0.5	618.5

4.3. Discussion

We first demonstrate the effect of facility coverage range on the optimal solutions using the example in Figure 3. The network we consider is 'r30x5a-1' with a maximum route length $T = 200$. The optimal locations and vehicle routes for facility coverage range of 0, 20, 40, 60 are shown in Figures 3(a), (b), (c) and (d), respectively. When $R = 0$, the optimal cost is 730.95 and 5 vehicles are used (Figure 3(a)). When the range equals 20, the cost is not significantly affected, it only reduces 1.15% and 5 vehicles are used again (Figure 3(b)). The impact is more obvious when the

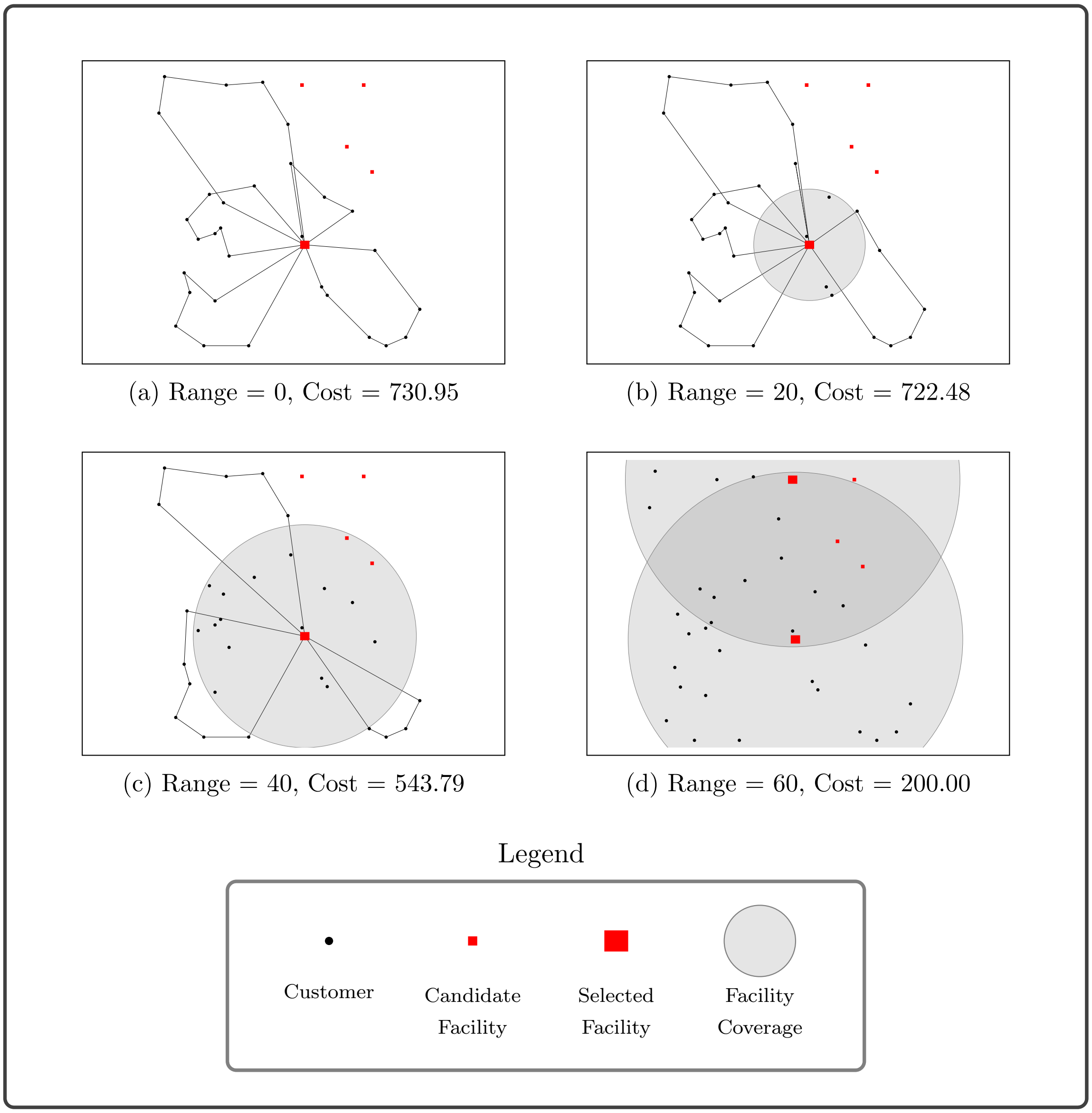

Figure 3: Selected facilities and vehicle routes in optimal solutions for network 'r30x5a-1', maximum route length $T = 200$ and facility coverage range $R \in \{0, 20, 40, 60\}$ in parts (a), (b), (c) and (d), respectively.

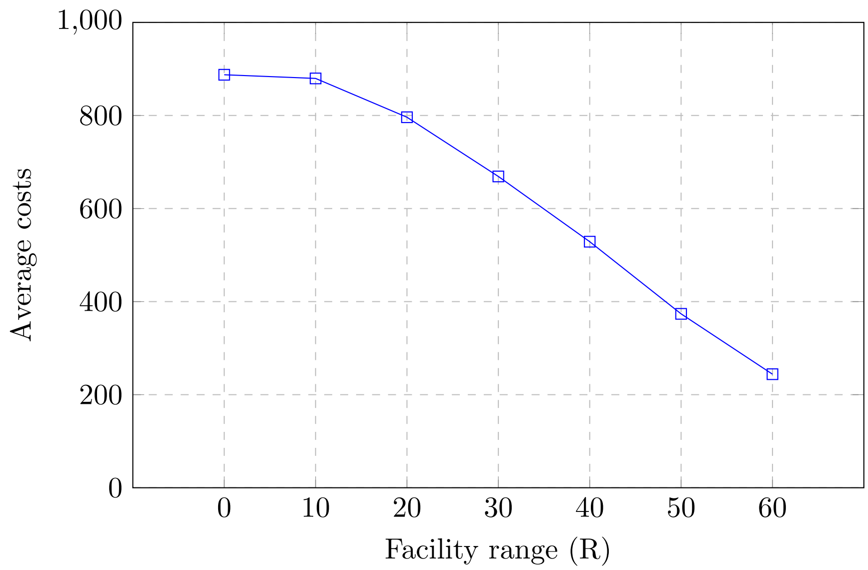

Figure 4: Average cost versus facility coverage range

facility coverage range increase to 40, in which case the cost reduction is more than 25% and 3 vehicles are used (Figure 3(c)). Finally, when the range increases to 60, two facilities are open and no vehicle is needed (Figure 3(d)). The problem effectively transforms into a set covering problem.

Table 3 reports the average costs for different facility coverage ranges and maximum route lengths. Note that we exclude the instances with $T = 150$, due to the infeasible instances. The impact of the facility coverage range on the total cost is stronger than the maximum route length. The average costs are 887.5, 879.6, 796.2, 669.0, 528.8, 373.7 and 244.0 for facility ranges of 0, 10, 20, 30, 40, 50 and 60, respectively. There is almost a linear relationship between these two factors, particularly when range is longer than 10 (Figure 4). The slope of the linear part of the curve is 12.7. That is, every unit increase in facility coverage range decreases the cost by 12.7. We suspect that the linear relationship between the cost and the range is rooted in the random distribution of customers in the plane.

Table 3: Average costs

Max. Route Length (T)	Facility Coverage Range (R)							Average
	0	10	20	30	40	50	60	
160	899.4	891.4	807.7	678.2	544.5	378.8	244.0	634.9
170	892.3	885.0	797.1	671.4	537.4	378.8	244.0	629.4
180	883.9	877.1	792.8	665.6	521.5	370.3	244.0	622.2
190	881.3	872.9	792.4	665.5	520.3	370.3	244.0	620.9
200	880.7	871.7	791.1	664.2	520.3	370.3	244.0	620.3
Average	887.5	879.6	796.2	669.0	528.8	373.7	244.0	625.5

The average number of open facilities are reported in Table 4. There is no clear relationship between the number of open facilities and the maximum route length or the facility coverage range. The number of open facilities varies between 1.67 and 2.83 and the overall average is 2.29.

Table 4: Average number of open facilities

Max. Route Length (T)	Facility Coverage Range (R)							Average
	0	10	20	30	40	50	60	
150	2.08	2.17	2.50	2.75	2.75	2.50	2.00	2.39
160	1.92	2.00	2.58	2.67	2.83	2.58	2.08	2.38
170	1.83	2.00	2.42	2.58	2.75	2.58	2.08	2.32
180	1.75	1.92	2.25	2.50	2.58	2.50	2.08	2.23
190	1.75	1.92	2.33	2.50	2.58	2.50	2.08	2.24
200	1.67	1.83	2.25	2.50	2.58	2.50	2.08	2.20
Average	1.83	1.97	2.39	2.58	2.68	2.53	2.07	2.29

Finally, Table 5 shows the number of vehicles used in the optimal solutions. The average number of vehicles used is 6.50, 6.26, 4.54, 3.14, 2.08, 0.93 and 0.24 when facility range is 0, 10, 20, 30, 40, 50 and 60, respectively. The clear relationship between these two factors is depicted in Figure 5. On average, the number of vehicles reduces by 1.2 for every 10 units of increase in the facility coverage range.

Table 5: Average number of vehicles

Max. Route Length (T)	Facility Coverage Range (R)							Average
	0	10	20	30	40	50	60	
150	6.17	5.83	4.33	3.00	2.08	0.83	0.17	3.20
160	6.67	6.33	4.75	3.33	2.25	1.00	0.25	3.51
170	6.58	6.42	4.58	3.25	2.08	1.00	0.25	3.45
180	6.58	6.42	4.58	3.17	2.08	0.92	0.25	3.43
190	6.50	6.33	4.50	3.00	2.00	0.92	0.25	3.36
200	6.50	6.25	4.50	3.08	2.00	0.92	0.25	3.36
Average	6.50	6.26	4.54	3.14	2.08	0.93	0.24	3.38

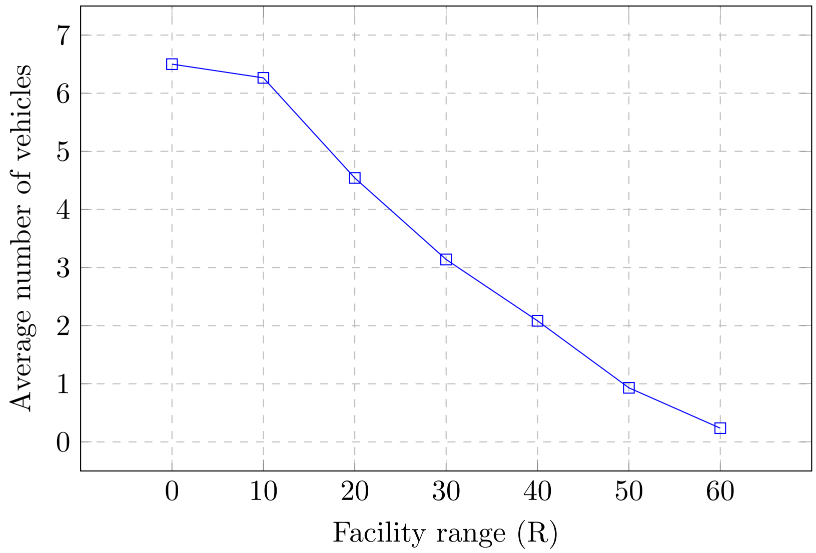

Figure 5: Average number of vehicles versus facility coverage range

5. Conclusion and Future Research

The facility location and vehicle routing are closely related problems in operations research. We have uncovered a new connection between the location and routing decisions by introducing the location-or-routing problem (LoRP), in which a customer can be covered either by a facility or by a vehicle visit. Each selected facility covers the customers in the neighborhood around itself defined by a coverage range, similar to the set covering problem. If a customer lies beyond the coverage range of any open facility, then a vehicle visit is required to cover the customer. This new problem has applications in location optimization of retail stores, supermarkets, shopping malls, schools, urban delivery centers and medical testing centers. We have presented a set covering model with an exponential number of variables for solving the LoRP. As a solution method, we have developed an exact branch-and-price algorithm and provided insights on the total costs. In particular, experiments on random graphs show that the total cost linearly decreases as the facility coverage range increases.

Several extensions of LoRP are possible. Basic extensions include considering capacitated facilities and stochastic nature of the demand. Additionally, the customer demand and the facility coverage range are assumed to be independent in this study. However, in retail store, supermarket or shopping mall location applications, the coverage of the demand may decay as the distance between a facility and a customer increases. Lastly, the LoRP offers a rich class of problems by integrating the location-routing problem with other facility location problems such as maximum covering, p-median and p-center problems. Therefore, the LoRP can be considered as a class of problems and the methodology presented in this paper can easily be adopted to model and solve such problems.

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