Curriculum

Mathematics 526
Transitional Version

Secondary School

Québec

CurriculuM

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Transitional Version

Secondary School

Direction de la formation générale des jeunes

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Mathematics 526:Transitional Version is a course designed for Secondary V students in accordance with the provisions of section 461 of the Education Act (R.S.Q.,c.I-13.3).This course will be implemented in all schools as of July 1,2000.

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Table of Contents

Introduction

Mathematics 526: Transitional Version is a course designed for Secondary V students who wish to pursue a social science, business or technical education at the postsecondary level and who have successfully completed *Mathematics 426: Transitional Version*.

To prepare young Quebecers for the demanding world of today and tomorrow, schools must focus on the students' cognitive growth and the development of basic skills (i.e. problem-solving and communication skills as well as the ability to work with technology).

Because of changes in society and developments in the field of mathematics education, it is important to stress the interconnection of knowledge, skills and attitudes when teaching this course.

Mathematics 526: Transitional Version is the second course in the intermediate sequence of mathematics courses, which falls between the basic sequence (i.e. Mathematics 416-514) and the advanced sequence (i.e. Mathematics 436-536) in terms of the amount of material covered, the detail involved and the complexity of the situations, problems and applications studied. In *Mathematics 526: Transitional Version*, students must use precise terminology and formal notation, always be rigorous and exact, and justify every step in their work.

In addition to ensuring that students are prepared to pursue a post-secondary education, mathematics education should provide fertile ground for the development of skills that will be useful to them in the future. As Resnick and Kloffer have noted, "Graduates must not only be literate; they must also be competent thinkers."[1]

1. L.B. Resnick and L.E. Kloffer, "Toward the Thinking Curriculum: An Overview," in *Toward the Thinking Curriculum: Current Cognitive Research, 1989 Yearbook of the Association for Supervision and Curriculum Development*, ed. Lauren B. Resnick and Leopold E. Kloffer (Alexandria, Va.: Association for Supervision and Curriculum Development, 1989), p. 1.

Three Major Guiding Principles

Current knowledge of the learning process and the focus of student learning have led to an emphasis on three principles intended to guide teachers in their work with students. These principles are as follows: to encourage the students to participate actively in the learning process, to encourage them to use a problem-solving approach at each stage of the learning process and to encourage them to use the appropriate technology for each task.

Encouraging Students to Take an Active Part in the Learning Process

A great deal of research has shown that students should play a central role in the learning process. In short, they should be ultimately responsible for their own education:

> The construction of a given concept is a complex process that depends first and foremost on the student. Concepts are not directly transmitted from a knowledgeable person to a student who supposedly knows nothing in a given field. Before they tackle new subject matter, students have already developed their own ideas, which are well organized, practical and sometimes fairly resistant to the changes targeted in a course of study.

> Thus, teaching involves creating situations in which students draw on their own knowledge. Teaching involves structuring the learning process around their strategies and thinking in order to try to get them to make progress in the construction of a given concept.[1]

To help students acquire the knowledge and skills targeted by this program, it is important to design learning situations that call upon their powers of observation and dexterity and that involve manipulations, exploration, construction and simulations. Through these activities, the students analyze hypotheses, actively look for solutions, discuss their approaches, analyze concepts or theories from their own point of view while taking into account other points of view, actively question the meaning and consequences of the procedures they use and relate the knowledge they have acquired to their own experience. These situations encourage the students to reflect, act, react and establish connections with what they have already learned.

Another way teachers can encourage students to participate in the learning process is by developing a suitable teaching approach. By asking students questions instead of giving them the answers, teachers will do more to help young people build their knowledge.

Any question that helps students get on the right track or answer their own questions encourages them to participate in their own learning.

1. Nadine Bednarz, "L'enseignement des mathématiques et le Québec de l'an 2000," in Richard Pallascio, ed., *Mathématiquement vôtre! Défis et perspectives pour l'enseignement des mathématiques* (Montréal: Les éditions Agence d'ARC inc., 1990), p. 69 (free translation).

Encouraging Students to Use a Problem-Solving Approach at Every Stage in the Learning Process

Problem solving is an essential teaching and learning tool in several general education programs (e.g. pure sciences, social studies) and is an integral part of any mathematical activity. Problem solving is not a separate theme, but rather a process that should be applied throughout the program and that provides a suitable context for learning concepts and acquiring skills.

> Problem solving is both a basic skill that students should develop and an effective teaching approach that promotes the development of mathematical knowledge, thinking skills, socio-affective attitudes and problem-solving strategies.[2]

Learning through problem solving calls for the active involvement of the students and the use of questions. The teacher should ask the students some questions, the students should ask the teacher some questions, the students should ask one another some questions and each student should ask himself or herself some questions.

2. Québec, Ministère de l'Éducation, *Mathematics Curriculum Guide, Elementary School, Booklet K, Problem Solving*, Code 16-2300-11A (Québec: Direction de la formation générale des jeunes, 1989), pp. 47-51.

Some problem-solving tasks may be more difficult than others and the problems themselves can be quite varied. For instance, the students may encounter the following:

> . . . problems with solutions requiring students to choose an appropriate combination of knowledge and skills from among several combinations seen in the past.[1]

They may even encounter the following:

> . . . problems requiring students to create a new combination of knowledge and skills, exercise a great deal of intellectual independence and use plausible reasoning in order to solve them.[2]

Problem solving is a very effective means of developing knowledge and skills. The quality of learning depends on the variety of problems assigned and on their level of difficulty. In a learning context, the students can even be presented with very challenging problems. By solving these problems, the students can discover such things as properties, relationships and strategies on their own. A wide variety of problems allows the students to conceptualize their knowledge and develop numerous problem-solving strategies. Problem solving is a way of learning and a way of teaching.

The problems can be related to the students' environment and used at various stages in the learning process. Problem solving can help students learn new concepts and develop skills or help them delve more deeply into what they already know and reinforce what they have learned.

Thus, problems provide an opportunity to:

- apply and integrate mathematical knowledge (e.g. concepts, properties, algorithms, techniques, procedures);
- develop intellectual skills (e.g. organizing, structuring, abstracting, analyzing, synthesizing, estimating, generalizing, deducing, justifying);
- develop positive attitudes (e.g. becoming aware of one's potential, respecting the opinions of others, and being imaginative and creative as well as rigorous and precise);
- use different problem-solving strategies (e.g. looking for patterns, representing a problem by means of a figure or a graph, constructing a table, referring to a known model, using a formula, formulating an equation, working backwards).

The emphasis on problem solving does not mean that exercises have no part in the teaching or in the learning of mathematics. Exercises play a different role, but one that is complementary to that of problem solving. For instance, exercises can help students consolidate skills and habits that they have already begun to develop. They can also give students the opportunity to apply definitions and properties that they have already learned in class. Exercises can neither replace nor be replaced by problems.

By using a problem-solving approach, the students become accustomed to referring to a known mathematical model and are thereby more likely to attain the terminal objectives. The teacher should also assist the students in using a procedure that will enable them to acquire more knowledge and generate other models. This will help the students attain the global objectives in accordance with the first guiding principle, namely, to encourage the students' active participation.

1. Québec, Ministère de l'Éducation, *Mathematics Curriculum Guide, Elementary School, Booklet K, Problem Solving*, Code 16-2300-11A (Québec: Direction de la formation générale des jeunes, 1989), p. 15.
2. Ibid., p. 15.

Students must have the opportunity to analyze their work methods and organize their thinking. In short, they must be able to learn how to learn.

Encouraging Students to Use the Appropriate Technology for Each Task

> All industrialized countries have experienced a shift from an industrial to an information society, a shift that has transformed both the aspects of mathematics that need to be transmitted to students and the concepts and procedures they must master if they are to be self-fulfilled, productive citizens in the next century.
>
> . . . This social and economic shift can be attributed, at least in part, to the availability of low-cost calculators, computers, and other technology. The use of this technology has dramatically changed the nature of the physical, life and social sciences; business; industry; and government. The relatively slow mechanical means of communication—the voice and the printed page—have been supplemented by electronic communication, enabling information to be shared almost instantly with persons—or machines—anywhere. . . . The impact of this technological shift is no longer an intellectual abstraction. It has become an economic reality. Today, the pace of economic change is being accelerated by continued innovation in communications and computer technology.[1]
>
> . . . changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. Davis and Hersh (1981) claim that we are now in a golden age of mathematical production, with more than half of all mathematics having been invented since World War II.[2]

Since technology has influenced mathematics and its applications, students must learn to use modern electronic tools like scientific calculators, graphing calculators, drawing software and utility software (e.g. spreadsheet programs, word processing systems and database managers).

Technology does not guarantee that students will do well in mathematics, since calculators and computers, like a word processor for a writer, are merely tools. Technology can, however, help students to understand and master new concepts more quickly.

Connection with Previous Courses

With continuity in learning, students can review topics they have already studied and further develop their conceptions and representations. This mathematics course enables students to build on the knowledge acquired in elementary school, Cycle One of secondary school and in Secondary IV.

This learning process will be dynamic if the learning activities allow the students to use their previously acquired knowledge and skills in new situations and help them become more proficient at applying what they have learned.

1. Thomas A. Romberg, ed. *Curriculum and Evaluation Standards for School Mathematics* (Reston, Va: National Council of Teachers of Mathematics, 1989), p. 3.

2. Ibid., pp. 7-8.

As they acquire new knowledge, the students will review the following skills and concepts acquired in previous courses:

- number, operation and spatial sense; proportionality; the concept of a variable;
- the habit of estimating;
- the type of dependence characterizing the relationship between the variables in a situation;
- translation from one mode of representation to another;
- the concept of function and its properties;
- the definitions, properties, theorems or corollaries related to different geometric concepts;
- the ability to organize and process statistical data;
- the simulation of random events and the concept of probability.

Evaluation of Learning

Orientations and Pratices Relating to the Evaluation of Learning

The evaluation of student learning has come in for a great deal of discussion in the Québec education system over the last decade and it is surely no exaggeration to say that this field has been and to some extent remains a subject of scrutiny. Teachers today are more knowledgeable about the evaluation of student learning than they were in the past. . . .[1]

It is important to draw on all the available expertise in evaluation and ensure that evaluation practices increasingly tie in with the essential learning pursued in the various courses. Thus, the aim should be to establish greater consistency between the spirit of these courses and evaluation practices.

Procedures for Evaluating Learning

When evaluating student learning, teachers should keep in mind the purpose of evaluation. Whether the goal is to give immediate educational feedback (formative evaluation) or to determine whether one or more terminal objectives have been attained (summative evaluation), evaluation provides individual students with useful information about their learning progress. It also helps teachers to assess the organization of course content and the effectiveness of teaching methods. Since the program is aimed at helping students acquire a solid basic education and the skills that will enable them to adapt to a constantly changing society,

> . . . the evaluation of learning should take into account the various components of human development and the complex nature of education, [and] be consistent with the learning activities carried out in the classroom.[2]

In this course, the students not only acquire knowledge, but also learn how to investigate, communicate, represent, reason and use a variety of approaches in order to solve problems. They also acquire other skills and attitudes.

Because we are evaluating the students' knowledge, skills and attitudes, all of which are evolving, it is necessary to create situations which will yield information that, after criterion- or norm-referenced interpretation, is likely to provide a reliable indication of each student's or group's knowledge.

1. Conseil supérieur de l'education, *Évaluer les apprentissages au primaire : un équilibre à trouver* (Québec: Direction des communications du CSE, 1992), p. 1 (free translation).

2. Ibid., p. 2 (free translation).

Since “paper-and-pencil” evaluation may not be appropriate to every aspect of this program, a certain amount of adaptation will be necessary. Depending on the specific goals, the following means of evaluation could be appropriate:

- Log
- Oral presentation of a solution or a mathematical topic
- Quiz
- Class discussion
- Group project
- Interview
- Comprehensive examination comprising a number of sections
- Evaluation during computer-assisted activities
- Observation checklist
- Self-evaluation

The different types of evaluation must also take into account the variety of learning activities:

- Manipulation activity
- Communication activity (oral or written, individual or group)
- Estimation activity
- Activity using a calculator
- Activity using a computer

When planning educational evaluation, it is important to vary the means of evaluation. However, this does not mean that evaluation should be carried out with only one purpose in mind (i.e. diagnostic, formative or summative). Choices must be made in this regard.

The evaluation of learning, be it formative or summative, is essentially aimed at improving both learning and teaching.

As Esther Paradis notes in *L'évaluation des apprentissages : valoriser sa mission pédagogique,* “Isn’t it essentially a matter of rediscovering the educational merit of evaluation?”[1]

Relative Importance of Each General Objective

The following table shows the relative importance of each general objective.

General Objective	%
1. To help the students develop their ability to use algebra.	68
2. To help the students develop their ability to analyze geometric situations.	20
3. To help the students develop their ability to analyze statistical data.	12

1. Esther Paradis, *L’évaluation des apprentissages : valoriser sa mission pédagogique* (Québec: Fédération des enseignantes et des enseignants de commissions scolaires, Centrale de l’enseignement du Québec, 1992), p. 26 (free translation).

Course Content

Structure of the Course

This course is made up of global, general, terminal and intermediate objectives. These objectives reflect the aims of mathematics education and the guiding principles mentioned previously.

Global Objectives

Objectives that summarize the role that mathematics plays in providing students with the basic education they need to integrate into our changing society. These global objectives remain the same throughout the five years of secondary school and form the nucleus around which the objectives for each level are structured.

General Objectives

Objectives that specify the context in which the global objectives will be pursued and that describe in general terms the expected educational outcomes associated with each course theme. General objectives can be broken down into a set of terminal objectives.

Terminal Objectives

Objectives that clarify the general objectives and describe the anticipated results. Each terminal objective is described in three paragraphs:

- The first paragraph indicates what the students have already learned.

- The second paragraph provides criteria for determining whether the students have attained the terminal objective.

- The third paragraph outlines activities relating to the terminal objective that are consistent with the general objective, the global objectives and the guiding principles. In this way, it reflects the spirit of the program.

The terminal objective is attained when the students are able to establish a link between a situation and acquired knowledge. This ability is directly related to attainment of the terminal objective and not to attainment of each of the underlying intermediate objectives, a complex object of knowledge being more than the sum of its parts. Hence, the primary goal is to have the students achieve the terminal objectives of the course. The degree to which the terminal objectives of the program are attained is directly related to the appropriateness of the measurement instruments, which must take into account the scope of the intermediate objectives and the context outlined by the general objective and the global objectives.

Intermediate Objectives

Objectives that specify the scope of a terminal objective, intermediate objectives might also be described as “reference objectives.” They are not intended as a series of steps to be completed one after the other. Such a process would give a very fragmented picture of teaching and learning. Rather, intermediate objectives are:

- aspects of a theme that have been chosen for the course;
- clarifications to ensure that the terminal objective is clearly understood;
- guidelines that indicate the connection between the terminal objective and student learning;
- prerequisites for attaining a terminal objective.

Course Objectives

Global Objectives

Establishing Connections

Increasing the students' ability to establish connections between the knowledge they are acquiring and the knowledge they already have of mathematics and other disciplines, and encouraging them to view their knowledge as a tool that can be useful to them in everyday life.

Problem Solving

Increasing the students' ability to analyze the data associated with a problem and to use appropriate strategies to arrive at a solution that they will be able to verify, interpret and generalize.

Communicating

Increasing the students' ability to grasp and transmit information and to express their thoughts clearly, using mathematical language.

Reasoning

Increasing the students' ability to formulate hypotheses and verify them using an inductive or a deductive method.

GENERAL OBJECTIVE 1

To help the students develop the ability to use algebra

We are living in the information age, marked by easy access to a large quantity and a great variety of information. Students should therefore be equipped to select, organize, understand, process and interpret this information using situation modelling.

In Secondary II, the students learned that algebra is a powerful and useful language or communication tool. They were introduced to different modes of representation (e.g. numerical expressions, images or drawings, tables of values, graphs or diagrams, algebraic expressions, equations, formulas) which highlighted certain aspects of problems they had to solve.

In Secondary III, the students used algebra to derive general rules from a number of specific situations. Conversely, they applied general rules to individual cases. The students discovered the type of dependence characterizing the relationship between certain variables, particularly the dependence represented by the graph of a straight line. They also continued learning about algebraic manipulations.

In *Mathematics 426: Transitional Version*, the students systematically broadened this knowledge. They analyzed different real functions and their properties (particularly polynomial functions of a degree less than three). They also acquired and developed the ability to perform operations, exploring those that involve the law of exponents, operations on algebraic expressions, factoring, and systems of first-degree equations in two variables. The students also used analytic geometry to study straight lines.

In *Mathematics 526: Transitional Version*, the students will build on this knowledge. They will begin working with inequalities and systems of inequalities involving real variables and then use them to solve optimization problems. They will then analyze functions involving one real variable (i.e. absolute value, square root, rational, exponential, logarithmic, sine, cosine, tangent and sinusoidal functions). The students will learn their principal properties, such as the existence of asymptotes, and will also study the inverse of a function. They will ultimately solve problems using these functions to devise models for different situations.

The use of computer programs or graphing calculators will save time during the exploration of these concepts and will make it easier for students to work toward the related objectives.

The students should learn and develop methods and procedures for solving different types of equations (involving absolute values, square roots, exponents, logarithms, or trigonometric expressions). They will use the properties of logarithms to transform logarithmic expressions. They will also prove trigonometric identities.

Finally, the students will study the geometric loci associated with first- and second-degree relations (especially conics).

The formal approach to mathematics introduced in *Mathematics 426: Transitional Version* will also be used in this course. As a result, teachers should use appropriate symbols, logical connectives and set notation as the need arises. The students should understand this language and be encouraged to use it whenever it clarifies the mathematical information they wish to present. With practice, the students will find it easier to understand mathematical symbolism.[1]

In addition, students in *Mathematics 526: Transitional Version* should always be required to justify the work needed to solve problems involving algebra and geometry. Since the students taking this course will probably enrol in post-secondary programs requiring a good knowledge of mathematics, it is important to prepare them for this by gradually introducing them to more sophisticated mathematical procedures.

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1. Québec, Ministère de l'Éducation, *Information Document, Graphs, Notation and Symbols Used in Secondary Mathematics,* (Québec: ministère de l'Éducation, 1997).

Terminal Objective 1.1

To solve problems using systems of inequalities

In Cycle One of secondary school, the students developed their understanding of the four operations involving rational numbers as well as their ability to perform these operations. Among other things, they used the symbols of inequality. They also learned to represent a situation by a first-degree equation. In *Mathematics 426: Transitional Version*, the students learned to represent a situation by a system of equations and then solve it (algebraically or graphically).

Students who have attained Terminal Objective 1.1 of this course will be able to represent a situation by a system of inequalities of the first degree in two variables, find the feasible set and use it to solve the problem. Since this is the first time the students will be working with inequalities, they will begin by studying procedures for solving first-degree inequalities in one real variable and then in two real variables. Only afterwards will they start to examine systems of these types of inequalities. They will learn how to solve simple optimization problems by determining the polygon of constraints, the vertices of the polygon, the *objective function* to be optimized, its value at each vertex and the optimum solution. The students should always be assigned problems involving mathematical, real, realistic and/or imaginary situations. Computers or graphing calculators may be used to help them find solutions.

Situations in which the students must analyze the relationships between the elements of a problem, formulate an inequality or a system of inequalities to solve the problem and then interpret the results are consistent with the global objectives, General Objective 1 and the guiding principles. A wide variety of examples should be used. The students should then grasp the connections, the similarities and the differences between the equations already studied and inequalities. It would be useful to employ different learning aids or methods (i.e. "paper-and-pencil" exercises, computers, graphing calculators).

1.1

Intermediate Objectives

- To represent a situation by an inequality in one or two real variables.
- To solve first-degree inequalities in one real variable.
- To graph first-degree inequalities in two real variables.
- To graph the feasible set of a system of first-degree inequalities in two real variables.
- To find the optimum solution in a given situation, taking the various constraints into account.

Terminal Objective 1.2

To solve problems by using functions involving one real variable to develop a model for a given situation

In Secondary II, the students had to use different modes of representation to describe and represent a situation. They learned to represent a situation by a first-degree equation. In studying ratios and proportions, they explored situations involving direct variation. In Secondary III, they studied situations in which the variables are directly or inversely proportional or in which one of the variables is proportional to the square of the other. In particular, they analyzed situations in which the relationship between the variables is linear. In *Mathematics 426: Transitional Version*, the students formally defined the concept of a function and observed the properties of functions. They examined several types, but concentrated on polynomial functions of degree 0, 1 and 2.

Students who have attained Terminal Objective 1.2 of this course will be able to analyze different types of functions involving one real variable (absolute value, square root, exponential, logarithmic, sine, cosine, tangent and sinusoidal functions) and to use these functions to develop models in order to solve problems. After examining these models, the students should then be able to characterize a function involving one real variable by referring to its rule of correspondence or its graph. They will analyze the role of parameters in the rule of correspondence of a function, their effects on a graph, and their connections with a given situation. The students will also explore the concept of an inverse function and analyze certain aspects of rational functions of the form $f(x) = \frac{ax+b}{cx+d}$. Use of computers or graphing calculators can help the students to attain this terminal objective.

Activities that bring about discussions and questions and in which students will develop their powers of observation and their ability to analyze and synthesize information are consistent with the global objectives, General Objective 1 and the guiding principles. A function is a fundamental concept in mathematics. Functions provide models that are used to study a wide variety of situations and give rise to numerous applications in computer science, technology and the sciences. The students will learn to classify a function according to its rule of correspondence or Cartesian graph and will come to understand the relationships between the two modes of representation. In addition, the students will be able to use their knowledge and skills to solve problems based on mathematical, real, realistic and/or imaginary situations. They will be able to explore simple situations that lead to the application of the concept of the composition of two functions. In this case, technology could be used to study other functions for the purpose of illustrating properties or giving clearer definitions of concepts such as continuity and periodicity.

1.2

Intermediate Objectives

The following objectives pertain to the absolute value, square root, exponential, logarithmic, sine, cosine, tangent and sinusoidal functions.

- To determine the relationships between the change in a parameter of the rule for a function and the change in the equivalent Cartesian graph (this objective does not cover rational functions).

- To draw the Cartesian graph of a function, given its rule.

- To describe the characteristics of the Cartesian graph of a function, given its rule.

- To determine, from its rule or its Cartesian graph, the following information about a function:

  - the domain and range, the element(s) of the domain associated with a given image;

  - if they exist, the extrema, the zeros, the *y*-intercept, the intervals within which the function is increasing or decreasing, the sign (graphically), the equations of its asymptotes;

  - the rule for and the graph of its inverse (this objective does not cover trigonometric or rational functions).

- To determine the rule for a function, given sufficient information (this objective does not cover rational functions).

- To determine the rule for the function that represents a situational problem to be solved, given the related model (this objective does not cover rational functions).

Terminal Objective 1.3

To transform mathematical expressions into equivalent expressions

In Secondary II, the students acquired certain skills that enabled them to represent a given situation by a first-degree equation and to then solve it. In Secondary III, they continued to learn how to perform operations on algebraic expressions. In *Mathematics 426: Transitional Version*, they developed additional algebraic skills (factoring and operations involving algebraic expressions containing integral exponents) and learned to solve second-degree equations.

Students who have attained Terminal Objective 1.3 of this course will be able to do the calculations involved in converting algebraic expressions to equivalent expressions. They should be able to solve simple equations involving absolute values, square roots, exponents, logarithms or trigonometric expressions. The students will be shown proofs of the properties of logarithms, with emphasis on the connection between exponents and logarithms. The students will apply these properties in solving simple exponential or logarithmic equations. They will then be introduced to reciprocal trigonometric ratios (secant, cosecant, cotangent) as well as the identities related to the Pythagorean theorem. These identities as well as the definitions of trigonometric ratios will be used to prove simple trigonometric identities and transform trigonometric equations to be solved. Teachers should note the close connection between this terminal objective and Terminal Objective 1.2; they could reinforce this connection by covering both parts of the course concurrently.

Activities in which the students gradually develop their knowledge of the structure of algebra, their understanding of the laws of algebra and their ability to apply its techniques are consistent with the global objectives, General Objective 1 and the guiding principles. While ensuring that the students understand this material, the teacher should also help them learn methods and procedures for manipulating algebraic expressions. The students will have many opportunities to use the skills they have already acquired or developed.

1.3

Intermediate Objectives

- To find the solution set of an equation in one real variable, which contains absolute values.
- To find the solution set of an equation in one real variable, which contains square roots.
- To find the solution set of an exponential or logarithmic equation by applying the properties of exponents or logarithms.
- To prove trigonometric identities.
- To find the solution set of a first- or second-degree trigonometric equation in one real variable, which can be transformed into an expression containing either a sine or a cosine.

Terminal Objective 1.4

To solve problems using geometric loci associated with first- and second-degree relations in a Cartesian plane

Since Secondary II, the students have acquired knowledge and skills in algebra (i.e. first-degree equations in one variable, operations on polynomials, the relationship between the variables of a given situation, functions, transformations of algebraic expressions, systems of equations and the use of analytic geometry to study straight lines). The students have also been developing their knowledge of geometry since Secondary I.[1]

Students who have attained Terminal Objective 1.4 of this course will be able to solve problems involving geometric loci in a Cartesian plane. First of all, they will be introduced to the concept of a geometric locus and of the equation associated with a given geometric locus. Next, through observation or exploration, they will try to identify the figure corresponding to a given locus by finding points that satisfy the definition of that locus. Then, they will try to find the equation of that locus using analytical methods. Conversely, given the equation of a locus, they will try to find the corresponding geometric figure. Computers or graphing calculators could be very useful in this case. The study of ellipses, parabolas and hyperbolas will be limited to those centred at the origin. The students will examine both circles at the origin and translated circles so that they can see that all these geometric loci can be translated.

Activities in which the students can solve problems involving geometric loci are consistent with the global objectives, General Objective 1 and the guiding principles. Students will be introduced to new mathematical models represented by second-degree equations. In exploring this new material, the students will be able to develop their powers of observation and their ability to analyze and synthesize information, while discovering the powerful methods that combine algebra and geometry. The use of technology could be very helpful in attaining this objective.

1. See Appendix 1, p. 32.

1.4

Intermediate Objectives

The following objectives pertain to parabolas, ellipses and hyperbolas centred at the origin, circles centred at the origin and translated circles.

- To determine the figure corresponding to the description of a given geometric locus.
- To determine the equation associated with the description of a given geometric locus.
- To determine the equation associated with a given conic section.
- To describe a conic section and its principle elements (i.e. centre, radius, directrix, vertices, foci, semi-axes or asymptotes), given its equation in standard form.

GENERAL OBJECTIVE 2

To help the students develop their ability to analyze geometric situations

One of the major reasons for teaching geometry "is to build the kind of strong geometric intuition that has been shown to be an important factor for success on the job and in college."[1]

The students gradually develop their geometric thinking skills: they first learn to recognize shapes and then analyze the different properties of these shapes before establishing relationships between the properties and making simple deductions. Through numerous activities involving exploration and observation, the students have created a system of relationships pertaining to triangles, quadrilaterals, circles, regular polygons, solids and trigonometric ratios and to the concepts of isometry and similarity.[2] The students have also learned to state propositions and justify assertions when solving problems. In this course, this system will be expanded to include the relationships governing the measurements in circles and right triangles.[3] Furthermore, the students will learn to use logical reasoning by proving theorems. "Students need to have many informal experiences that involve reasoning and arguing to support conjectures before they are likely to understand the need for, or the value of, a formal proof."[4] The students will then be given problems related to the ideas and concepts they have just studied, followed by problems that involve using their overall knowledge of geometry. At this point, the students will have to synthesize what they have learned in studying geometry.

Since the students taking this course will probably enrol in post-secondary level programs requiring a good knowledge of mathematics, it is important to prepare them for this by gradually introducing them to more sophisticated mathematical procedures. Moreover, the students will have to solve problems using the same systematic approach required to prove a theorem. Technology can be very useful for this topic. Indeed, some software can be used to explore geometry problems, thereby making it possible to formulate, discuss and test conjectures.

1. Arthur F. Coxford et al., "Geometry from Multiple Perspectives," in *Curriculum and Evaluation Standards for School Mathematics, Addenda Series Grades 9 - 12* (Reston, Va.: National Council of Teachers of Mathematics, 1991), p. v.
2. See Appendix 1, p. 32.
3. See Appendix 2, p. 36.
4. Arthur F. Coxford et al., p. 51.

Terminal Objective 2.1

To solve geometry problems

While studying geometry in the first cycle of secondary school, the students acquired and developed knowledge and skills related to plane figures, solids, isometries and similarity transformations.[1] In addition, they were always asked to explain their reasoning by justifying the steps in their solution to a problem. In *Mathematics 426: Transitional Version*, teachers worked towards the same objective by presenting students with new principles of geometry as well as some deductive reasoning in analytic geometry.

Students who have attained Terminal Objective 2.1 of this course will be able to prove propositions and use their knowledge and skills to solve geometry problems. They will be able to prove theorems related to circles and right triangles[2] and use this newly acquired knowledge to solve a variety of problems. These problems may be solved in different ways and may require the use of equations, functions, trigonometric ratios and other concepts. In every case, the students will solve problems by organizing their solutions and justifying the steps in their reasoning. The use of technology could be helpful in solving these problems. The students will have attained Terminal Objective 2.1 when they can use structured procedures involving sound, systematic argumentation in order to prove propositions or solve problems.

Activities in which students will have to identify properties or theorems and then prove them are consistent with the global objectives, General Objective 2 and the guiding principles. They will have to distinguish between a conjecture and a certainty and between a hypothesis and a conclusion. The students will have to determine the properties or theorems required to solve a problem and then organize and justify their solutions. Ultimately, the students should realize that all their knowledge (e.g. of algebra, geometry, statistics and the sciences) and all the means at their disposal (e.g. computers, calculators, instructional materials) can be used to solve problems. The students should understand the value of systematic reasoning in every branch of mathematics.

1. See Appendix 1, p. 32.
2. See Appendix 2, p. 36.

2.1

Intermediate Objectives

- To prove propositions related to circles and right triangles.[1]
- To determine measurements in circles and right triangles.
- To justify a statement used in solving a problem.[2]

1. See Appendix 2, p. 36.
2 .See Appendices 1 and 2, pp. 32, 36.

GENERAL OBJECTIVE 3

To help the students develop their ability to analyze statistical data

When dealing with all the statistical information now available, students must have a certain ability to interpret data and make intelligent decisions based on the quantitative and qualitative analysis of that data. When working with data that describes the characteristics of a population, one must go beyond the numbers involved in order to arrive at a more critical appraisal of the situation. To attain General Objective 3, the students will therefore learn how to ask pertinent questions and present the results of their analysis, while developing their critical sense.

In the first cycle of secondary school, the students organized and presented data in tables and graphs. They also saw that they could use certain descriptive measures (mean, median, mode, range) to synthesize data and thus provide information on various phenomena. In Secondary IV, the students learned about measures of position and began to study the concept of dispersion. In addition, the students were made aware of the problems involved in gathering data.

This course will briefly examine measures of dispersion (focusing on standard deviation) and then explore a new measure of position, namely the standard score (Z-score). The students will also study two-variable statistical distributions and, in the process, will learn to draw scatter plots, make tables of data, determine linear correlation coefficients and develop statistical models.

With this approach, the students will learn to use and interpret data rather than produce it. In addition, emphasis is placed on the analysis of situations rather than the calculation of parameters. Technology can be used for tedious tasks so that the students can concentrate on analysis and interpretation.

Terminal Objective 3.1

To solve problems related to situations involving a one- or two-variable statistical distribution

Since Secondary I, the students have learned how to organize data associated with situations involving a one-variable statistical distribution. They have also learned how to calculate the related parameters and to analyze and interpret data in terms of its qualitative and quantitative characteristics. In Secondary III, the students analyzed statistical data using measures of central tendency (mean, median, mode). In Secondary IV, they built upon this knowledge by examining measures of position. In addition, over these last two years, the students have become familiar with the dispersion of data by studying the concept of range as well as box-and-whisker plots.

Students who have attained Terminal Objective 3.1 of this course will be able to solve problems related to one- or two-variable statistical distributions, using the graphical and numerical tools they have to analyze information. The students will first compare the following measures of dispersion: the range (spread), the semi-interquartile range, the mean deviation and the standard deviation. They will learn about a new way of indicating the position of a data value in a distribution, namely the standard score (Z-score). When analyzing and interpreting the relationship between two variables in a statistical distribution, the students will learn to make contingency tables or draw scatter plots. By analyzing a scatter plot, they can obtain information about the correlation between the variables and also characterize that correlation [as positive, negative or null (zero), strong or weak]. The students should learn to estimate the correlation coefficient in different ways: they can estimate the dispersion of a scatter plot by comparing it with those that have already been studied, use the best methods of approximation (ellipse method or box plot) and then use technology. In studying the correlation between two variables, the main focus should be the analysis and interpretation of the situation rather than the numerical calculations. The students should learn to critically assess the methods for processing data as well as the conclusions drawn from it.

Activities in which the students analyze and interpret the relationship between two variables are consistent with the global objectives, General Objective 3 and the guiding principles. Instead of focusing on the application of formulas, the teacher should devote more time to ensuring that the students understand concepts such as standard deviation, standard score (Z-score), correlation coefficient and regression line. In this regard, the use of technology is recommended. The students will learn to develop suitable models for different situations and to assess a model critically. Here again, the focus should be on analyzing the situation and presenting the analysis.

3.1

Intermediate Objectives

- To compare different measures of dispersion in a given distribution.
- To determine the standard score (Z-score) for a data value in a distribution.
- To construct a two-variable distribution table (contingency table).
- To construct the scatter plot for a two-variable distribution.
- To describe, in their own words, the correlation between the two variables in a distribution.
- To estimate the (linear) correlation coefficient for the two variables in a distribution.
- To interpret the (linear) correlation coefficient for the two variables in a distribution.

Appendices

Appendix 1 Principles of Geometry Introduced in Earlier Courses

Between Secondary I and Secondary IV, the students learned about the properties of two- and three-dimensional figures as well as the properties of geometric transformations. These properties are summarized below and should be included with those to be introduced in *Mathematics 526: Transitional Version*. All these principles can be used to determine measurements in certain figures and to justify certain steps involved in solving problems.

I *PLANE FIGURES*

ANGLES

1. Adjacent angles whose external sides are in a straight line are supplementary.

2. Vertically opposite angles are isometric.

3. If a transversal intersects two parallel lines, then:
   _ the alternate interior angles are isometric;
   _ the alternate exterior angles are isometric;
   _ the corresponding angles are isometric.

4. If two corresponding angles (alternate interior or alternate exterior) are isometric, then they are formed by two parallel lines and a transversal.

TRIANGLES

5. The sum of the measures of the interior angles of a triangle is 180_.

6. In any triangle, the length of any side is less than the sum of the lengths of the other two sides.

7. In any triangle, the length of any side is greater than the difference of the lengths of the other two sides.

8. In any triangle, the longest side is opposite the largest angle.

9. In any isosceles triangle, the angles opposite the isometric sides are isometric.

10. In any equilateral triangle, each angle measures 60_.

11. In any right triangle, the acute angles are complementary.

12. In any isosceles right triangle, each acute angle measures 45_.

13. The axis of symmetry of an isosceles triangle contains a median, a perpendicular bisector, an angle bisector and an altitude of the triangle.

14. The axes of symmetry of an equilateral triangle contain the medians, perpendicular bisectors, angle bisectors and altitudes of the triangle.

15. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

Appendix 1 (cont'd)

16. A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other two sides.

17. In a right triangle, the length of the side opposite a 30_ angle is equal to half the length of the hypotenuse.

18. The Law of Sines:

    The sines of the angles of any triangle are proportional to the lengths of the sides opposite these angles:

    $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

19. The Law of Cosines:

    The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle:

    $$a^2 = b^2 + c^2 - 2bc\cos A$$

20. The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.

21. The three perpendicular bisectors of the sides of a triangle are concurrent in a point that is equidistant from the three vertices.

22. The three medians of a triangle are concurrent and trisect one another at the point of concurrency.

QUADRILATERALS

23. The opposite angles of a parallelogram are isometric.

24. The opposite sides of a parallelogram are isometric.

25. The diagonals of a parallelogram bisect each other.

26. The diagonals of a rectangle are isometric.

27. The diagonals of a rhombus are perpendicular to each other.

28. The segment joining the midpoints of the non-parallel sides of a trapezoid is parallel to the bases and its length is one-half the sum of the lengths of the bases.

29. The midpoints of the sides of any quadrilateral are the vertices of a parallelogram.

CIRCLES

30. Three non-collinear points determine one and only one circle.

31. All the perpendicular bisectors of the chords of a circle meet at the centre of the circle.

32. All the diameters of a circle are isometric.

33. In a circle, the measure of the radius is half the measure of the diameter.

34. The axes of symmetry of a circle contain its centre.

Appendix 1 (cont'd)

35. The ratio of the circumference of a circle to its diameter is a constant known as π.

36. In a circle, the measure of the central angle is equal to the degree measure of its intercepted arc.

37. In a circle, the ratio of the measures of two central angles is equal to the ratio of the measures of their intercepted arcs.

38. The area of a circle is equal to πr^2.

39. In a circle, the ratio of the areas of two sectors is equal to the ratio of the measures of their central angles.

POLYGONS AND REGULAR POLYGONS

40. The diagonals from one vertex of a convex polygon form n – 2 triangles, where *n* is the number of sides in that polygon.

41. In a convex polygon, the sum of the measures of the exterior angles, one at each vertex, is 360_.

42. The sum of the measures of the interior angles of a polygon is 180_ (n - 2), where *n* is the number of sides in the polygon.

II ***TRANSFORMATIONS OF PLANE FIGURES AND SOLIDS***

ISOMETRIES AND ISOMETRIC FIGURES

43. An isometry preserves collinearity, parallelism, the order of points, distances and the measures of angles. In addition, translations and rotations preserve the orientation of the plane.

44. Any translation will transform a straight line into another line parallel to it.

45. The corresponding parts of isometric plane figures or solids are equal in measure.

46. Plane figures are isometric if and only if there is an isometry that maps one figure onto the other.

47. If the corresponding sides of two triangles are isometric, then the triangles are isometric.

48. If two sides and the contained angle of one triangle and the two corresponding sides and contained angle of another triangle are isometric, then the triangles are isometric.

49. If two angles and the contained side of one triangle and the two corresponding angles and contained side of another triangle are isometric, then the triangles are isometric.

50. Plane figures with a scale factor of 1 are isometric.

SIMILIARITY TRANSFORMATIONS AND SIMILAR FIGURES

51. Any dilatation preserves collinearity, parallelism, the order of points, the orientation of the plane, the measures of angles and the ratio of the distances.

52. Any dilatation will transform a straight line into another line parallel to it.

53. Transversals intersected by parallel lines are divided into segments of proportional lengths.

54. The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.

55. Any straight line that intersects two sides of a triangle and is parallel to the third side forms a smaller triangle similar to the larger triangle.

56. Plane figures are similar if and only if there is a similarity transformation that maps one figure onto the other.

57. If two angles of one triangle are isometric to the two corresponding angles of another triangle, then the triangles are similar.

58. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles are similar.

59. If the lengths of two sides of one triangle are proportional to the lengths of two sides of another triangle and the contained angles are isometric, then the triangles are similar.

60. In similar plane figures:

    - the ratio of the measures of the corresponding angles is 1;
    - the ratio between the lengths of the corresponding segments is equal to the scale factor;
    - the ratio of the areas is equal to the square of the scale factor.

III *SOLIDS*

61. In any convex polyhedron, the sum of the number of vertices and the number of faces is equal to the number of edges plus two.

62. The lateral area of a right prism or of a right circular cylinder is equal to the product of the perimeter of its base and its height.

63. The lateral area of a regular pyramid or of a right circular cone is equal to one-half the product of the perimeter of its base and its slant height.

64. The area of a sphere is equal to the product of the circumference of its great circle and its diameter.

65. The volume of a prism or of a cylinder is equal to the product of the area of its base and its height.

66. The volume of a regular pyramid or of a cone is equal to one-third the product of the area of its base and its height.

67. The volume of a sphere is equal to two-thirds the product of the area of its great circle and its diameter.

IV *ANALYTIC GEOMETRY*

68. The distance between two points:
    Given $\mathrm{A}(x_1, y_1)$ and $\mathrm{B}(x_2, y_2)$,
    $\mathrm{d(A, B)} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

69. Two straight lines that are not parallel to the y-axis are parallel if and only if their slopes are equal.

70. Two straight lines that are not parallel to the y-axis are perpendicular if and only if their slopes are negative reciprocals.

Appendix 2 Deductive Reasoning Involving Circles and Right Triangles

Using their knowledge of geometry and the skills they have already acquired, the students will have to prove the following propositions. Students will not be required to prove propositions 1, 2, 3, 4 and 6 for purposes of summative evaluation.

1. The diameter is the longest chord of a circle.
2. Any diameter divides a circle into two isometric parts.
3. In a circle or in isometric circles, isometric arcs are subtended by isometric chords, and vice versa.
4. Any diameter perpendicular to a chord divides that chord and each of the arcs that it subtends into two isometric parts.
5. In a circle or in two isometric circles, two isometric chords are equidistant from the centre, and vice versa.
6. If a line is perpendicular to a radius of a circle at the endpoint of the radius in the circle, the line is tangent to the circle. The converse is also true.
7. Two parallel lines, be they secants or tangents, intercept isometric arcs of a circle.
8. If point P is located outside circle O, and if segments PA and PB are tangents to that circle at points A and B respectively, then OP bisects angle APB and $\overline{PA} \cong \overline{PB}$.
9. The measure of an inscribed angle is one-half the measure of its intercepted arc.
10. The measure of an angle formed by two chords intersecting in the interior of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
11. The measure of an angle whose vertex is located outside a circle is one-half the difference of the measures of the intercepted arcs.
12. In any triangle, the bisector of an angle divides the opposite side into two segments whose lengths are proportional to those of the adjacent sides.
13. If two chords intersect inside a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.
14. If secant PAB intersects a circle at points A and B and secant PCD intersects the circle at points C and D, then $m\,\overline{PA} \cdot m\,\overline{PB} = m\,\overline{PC} \cdot m\,\overline{PD}$.
15. If tangent PA intersects a circle at point A and secant PBC intersects the circle at points B and C, then $\left(m\,\overline{PA}\right)^2 = m\,\overline{PB} \cdot m\,\overline{PC}$.
16. The length of a leg of a right triangle is the geometric mean (mean proportional) between the length of the hypotenuse and the length of the adjacent segment of the hypotenuse formed by the altitude to the hypotenuse.
17. The length of the altitude to the hypotenuse of a right triangle is the geometric mean (mean proportional) between the lengths of the segments of the hypotenuse.

Appendix 2 (cont'd)

18. In a right triangle, the length of the hypotenuse multiplied by the length of the altitude to the hypotenuse is equal to the product of the lengths of the sides of the right angle.

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