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How the Minimum Number of Periods Between Regeneration Harvests Induces Modeling Mistakes in the Well-Known Model II Forest Management[†]

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Abstract. The Model II is widely used in forest management models and different variants of it are implemented in various software. Overlooking the minimum number of periods between regeneration harvests can cause some modeling mistakes. We have observed two mistakes in the original formulation. The first is a mistake in the area constraints and the second in calculating one important parameter of the model representing discounted net revenue per hectare between periods. In this paper, we provide a slightly revised model together with comments on the computations of a parameter used in the model formulation. Then, in order to validate the problem identified, we solve the Model II with realistic data to address the modeling mistakes and explain how our revised formulation works with the same data. We also describe situations where the mistakes may have a larger impact and explain why they have not been identified earlier.

Keywords: Forest management planning, timber harvest, harvest scheduling, linear programming, Model II.

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1. Introduction

One of the most important models for forest management is the so-called Model II (Johnson and Scheurman 1977). Many variants of this model have been implemented in numerous planning systems within companies and government organizations. One of the main uses of the model is to evaluate the net present value (NPV) of a given forest under different silviculture treatment scenarios and discount rates over a set of time periods representing long-term planning. This model is straightforward and has been referenced in articles since its introduction without any revisions. It has also been implemented in various planning systems and it can be assumed that it is a standard tool to make evaluations of discounted forest values. However, a direct implementation of the original formulation may provide erroneous solutions under certain conditions on parameter values. The main reason is that a number of variables are created incorrectly when a minimum number of periods between regeneration harvests are used. Another mistake is in how the objective function coefficients are calculated. These mistakes may or may not have an impact depending on the silviculture options available and used. Also, as the model is often used for NPV values and not for operational planning, there is no actual need to analyze the actual harvest decision in detail. Hence, there has been little reason to verify or find that some variables have erroneous values. We describe the mistakes and propose a new formulation, which is tested on some illustrative examples.

Simple decision support systems cannot be created and applied universally because strategic planning of the forest value chain includes many different players in many different business contexts. Decision support for strategic planning helps decision makers assess the potential consequences of strategic business choices (Anthony 1965, Drucker 1995). For strategic planning, we should take decision makers' values, objectives, and their future business anticipations into account because these decisions will change the future by changing the flow of the resources and opportunities available to the company (Gunn 2005). At the strategic planning level, decisions, goals, and other constraints of decisions makers must be considered because they have a long-term impact on the company and its resources.

The goal of forest management strategy is to answer the following questions: what to supply, from where to supply to, to which market, and for what use to create value and jobs for local communities. It also has impacts on sustainability, carbon sequestration, wildlife habitat, and ecology, controlling invasive species,

and social values (like employment). Sustainable long-term supply is often depicted in terms of level flow or nondeclining yield. D'Amours et al. (2016) have stated that renewability is the key feature of the forest as a supplier of raw materials.

There are three specific parts in the forest management linear models: the process of forest growth and management, the sustainability of forest products, and the requirement to provide certain types of forest cover. Four distinctive modeling approaches are found for forest growth and management, including the well-known Model I and Model II (Johnson and Scheurman 1977) and Model III, which is less common (Garcia 1990); however, it forms the base of popular packages like FOLPI (Gunn 2007), and John and Tóth (2015) proposed a new model for spatial forest harvest scheduling called Model IV.

Woodstock software is capable of generating linear programming matrices by the use of a generalized Model II formulation which is markedly more powerful than other harvest scheduling models based on Model II, like MUSYC (Multiple-Use/Sustained Yield Calculation). FORPLAN (FOREst PLANning model) version 2 proposes the capabilities of the generalized Model II (Remsoft 1994).

A combination of Model I and Model II has been used as an optimization model to explore how different management regimes would affect the ability of forests to sequester carbon (Backéus et al. 2005). Martin et al. (2017) compared the efficiency of the spatial Model I and Model II and pointed out that Model I outperformed the Model II.

An optimization approach has been applied through a timber supply model which is an extension of the Model II formulation to estimate the cost of overlapping tenure constraint on forest management agreement areas in Northern Alberta (Nanang and Hauer 2006). A novel approach has been represented to simultaneously maximize carbon sequestration in both forest and wood products and abated emissions from product substitution using Remsoft Spatial Planning System (Hennigar et al. 2008). Note that Woodstock is on the basis of an optimized forest treatment scheduling using a model II LP formulation (Hennigar et al. 2008). Nanang and Hauer (2008) examined the long-term impacts of access road development, which is an important factor in determining harvesting and hunter preferences and non-timber benefits, and they used an extension of the Model II formulation. Model II was used for optimal harvest scheduling in a case study in Spain (Diaz-Balteiro et al. 2009). Model II has been utilized in the forestry portion of the FASOM-

GHG model which has been modified to simulate the effects of optional and mandatory participation in carbon offset sales programs (Latta et al. 2011). Model II has been applied in the forest sector model of a linked land-use and forest sector models which have been proposed to find how carbon offset sales can affect private forest owners' land-use and forest management decisions in Western Oregon (USA) (Latta et al. 2016). In order to analyze the impact of operational-level flexibility on long-term wood supply, a hierarchical planning, i.e. strategic, tactical, and operational, has been developed. The authors used a software called SilviLab to formulate the strategic-level model as a Model II linear program (Gautam et al. 2017). Model II has been used in a goal programming to analyze the long-term impact of policy and industry changes at the landscape level (Corrigan and Nieuwenhuis 2017).

The contribution of the paper is important as the Model II is used in many systems. It is difficult to know if any implementation has found and revised the modeling errors or not. However, we have not found any published article that addresses this and it is important for other researchers and users of the system to understand how they are impacted by the mistakes or how to identify if the implementation may provide erroneous results. This paper identifies and proposes a few modeling mistakes in the Model II formulation given in Johnson and Scheurman (1977). Model II is one of the most well-known forest management models, but the original formulation has two mistakes which may overestimate the objective function and mislead the forest manager or researchers over optimal harvest decisions in a specific context. The first mistake occurs in the first set of the area constraints, wherein some additional decision variables are created. These decision variables may take nonzero values and provide wrong information about the objective function and harvest decisions. The second mistake can be found in the way to calculate one of the key parameters of the model. This parameter will be explained in detail in the following sections.

The rest of this paper is organized as follows. Section 2 describes the forest management models, especially Model II in details with the mathematical formulation. In Section 3, we pose questions to Model II and propose a new mathematical formulation. In order to validate our new formulation, a problem would be represented with practical data and the results would be analyzed in Section 4. Finally, conclusions are drawn in Section 5.

2. Forest management models

In literature, four modeling approaches can be found for forest management planning, including the well-known Model I and Model II (Johnson and Scheurman 1977), Model III, which is less common (Garcia 1990), and John and Tóth (2015) proposed a new model for spatial forest harvest scheduling which is called Model IV. In Model I (Johnson and Scheurman 1977), the integrity of each age-class in the first period is kept throughout the planning horizon (see Model I in Figure 1). However, in Model II (Johnson and Scheurman 1977), the integrity of each age-class in the first period is kept until it is regeneration harvested and forms a new age-class until they are again regeneration harvested (see Model II in Figure 1). In Model III (Garcia 1990), in each period, the land in an age-class can be harvested or become one age-class older (see Model III in Figure 1). The aggregation of all stands in Model II is similar to the Model III; however, the network contains fewer nodes and arcs. Model I can be used to model either aggregated or individual stands (Gunn 2007). In the previous models, one decision variable is required for every applicable prescription for each forest management unit. The mentioned models are aspatial and also depend on static volume and revenue coefficients that must be calculated before starting optimization. Finally, John and Tóth (2015) introduced a new model which is called Model IV, using different equations and Boolean algebra for spatial forest harvest scheduling.

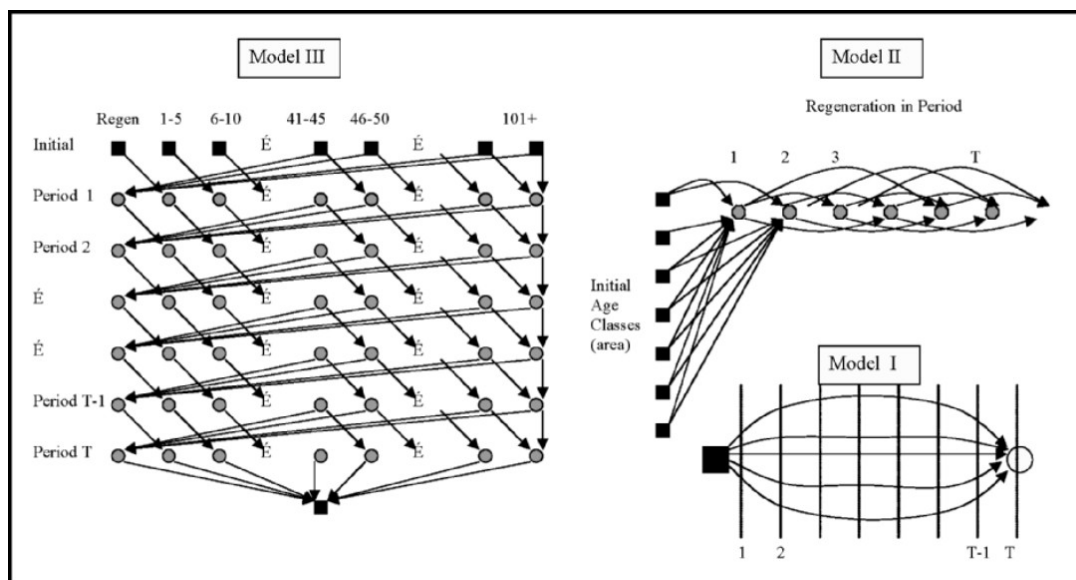


Figure 1: Models I, II, and III (Gunn 2007).

2.1. Model II

Forest management planning aims to schedule timber harvest and investment on an area of timberland under even-aged management. The goal is to maximize the volume or value produced from its timberland while encountering constant or decreasing prices in the volume of timber output (Johnson and Scheurman 1977). The manager may come across land availability limits for harvesting in each time period when the whole area is managed under one cultural treatment regime. A cultural treatment regime is any sequence of silvicultural practices such as planting, pre-commercial thinning, commercial thinning, and fertilization. In addition to area constraints, it may also consider flow constraints (harvest fluctuation and sustainability).

Seven simplifying assumptions have been stated as follows:

1. The forest has one type-site consisting of different age-classes.
2. The area of forestland is fixed during the planning horizon.
3. The number of years representing each time period in the planning horizon is consistent with the years of each age-class.
4. For regeneration harvest, we use clear-cutting.
5. Regeneration occurs in the same period as regeneration harvest.
6. Yield estimates take into account all uncertainties such as fire, insect, and diseases implicitly.
7. The only out-of-pocket costs that should be paid are the cultural treatment costs.

In Model II, each age-class forms a management unit that is harvested. Having regeneration harvested, new age-class is formed till they are again regeneration harvested. Each activity describes a possible management regime for a certain management unit from the time a unit is regenerated until it is regeneration harvested or left as ending inventory at the end of the planning horizon. A management regime includes two parts (Johnson and Scheurman 1977):

1. A regeneration harvest at some time during the planning horizon or an ending inventory at the end of the planning horizon.
2. An associated cultural treatment regime.

We require two sets of area constraints:

- One set on the areas that can be regeneration harvested from, or put aside as ending inventory in, each age-class that exists at the start of planning horizon (See Figure 2 and Constraint 2) (Johnson and Scheurman 1977). Figure 2 indicates that the areas cut from each age-class through different time periods plus the areas left as ending inventory from that age-class are equal to the total number of areas in that age-class at the beginning of planning horizon. For instance, Figure 2b indicates that the total area from age-class one (on the assumption that there is no minimum number of periods between regeneration harvests) at the beginning of the planning horizon can be harvested in different periods starting from one to N , and put aside as ending inventory.
- The second set is on the areas that can be regeneration harvested from, or put aside as ending inventory in, each age-class that is created throughout the planning horizon (See Figure 3 and Constraints 3) (Johnson and Scheurman 1977). Figure 3 illustrates that the areas cut from areas regenerated in period j plus the areas left as ending inventory from areas regenerated in period j are equal to the total number of areas regenerated in period j (j can vary between the first period and the end of the planning horizon). For example, in period j , different age-classes may be harvested, so these areas can be harvested in the following future periods and also put aside as ending inventory.

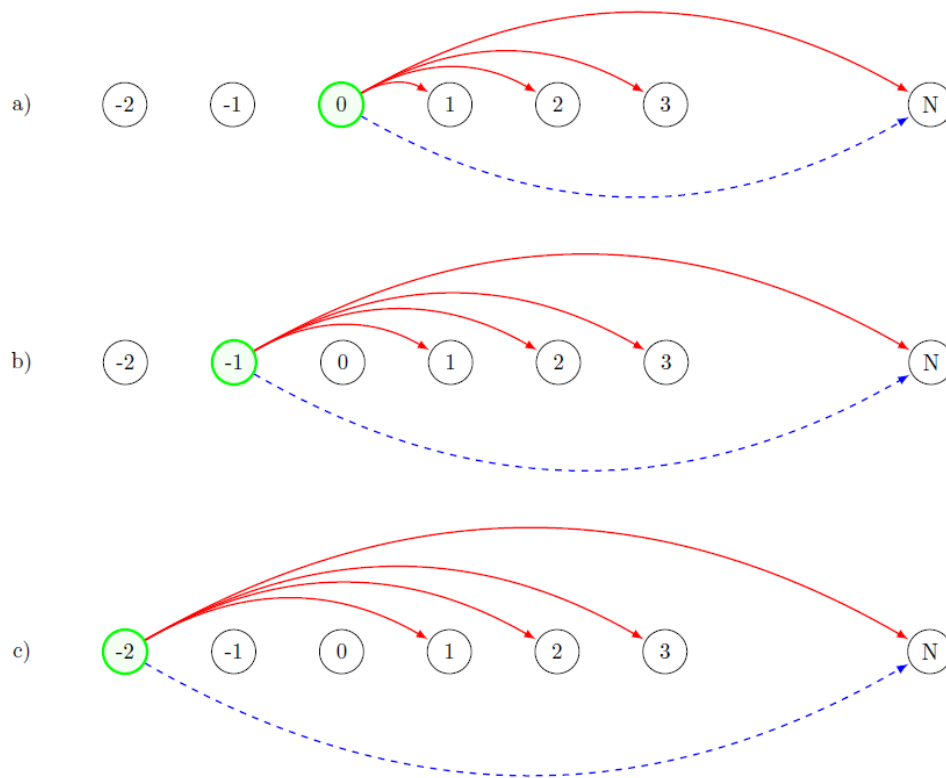


Figure 2: Balance constraint for areas regenerated or put aside as ending inventory at the start of the planning horizon for three different age-classes: 0, 1, and 2 can be seen in a, b, and c, respectively.

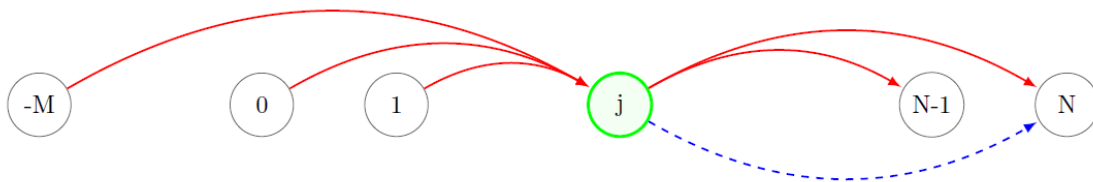


Figure 3: Balance constraint for areas regenerated or put aside as ending inventory throughout the planning horizon.

2.2. Mathematical formulation

The mathematical form of Model II is summarized as follows:

$$\max \sum_{j=1}^N \sum_{i=-M}^{j-z} D_{ij} x_{ij} + \sum_{i=-M}^N E_{iN} w_{iN} \quad (1)$$

Subject to

$$\sum_{j=1}^N x_{ij} + w_{iN} = A_i, \quad i = -M, \dots, 0 \quad (2)$$

$$\sum_{k=j+z}^N x_{jk} + w_{jN} = \sum_{i=-M}^{j-z} x_{ij}, \quad j = 1, 2, \dots, N \quad (3)$$

$$x_{ij} \geq 0, \quad i = -M, \dots, N, j = 1, 2, \dots, N \quad (4)$$

$$w_{iN} \geq 0, \quad i = -M, \dots, N \quad (5)$$

Constraint (2) expresses the land availability constraint for the beginning of the planning horizon (see Figure 2). The balance constraint for areas regenerated in period j can be found in Constraint (3) (see Figure 3). Constraint (4) and (5) show the non-negativity.

Authors defined the sets, data, and variables as follows, where:

N	Number of periods in the planning horizon
x_{ij}	Areas regenerated in period i and regeneration harvested again in period j
w_{iN}	Areas regenerated in period i and put aside as ending inventory in period N
A_i	Number of hectares present in period one that were regenerated in period i ($i = -M, \dots, 0$), with each A_i being a constant at the beginning of the planning horizon (period 1)
M	Number of periods before period zero in which the oldest age-class present in the period one was regenerated
z	Minimum number of periods between regeneration harvests (reasonably it is greater than one, i.e., $z \geq 1$)
D_{ij}	Discounted net revenue per hectare from areas regenerated in period i and regeneration harvested again in period j . It can be written as shown below:

$$D_{ij} = \sum_{k=\max(i,1)}^j \frac{P_{ikj}V_{ikj} - C_{ikj}}{\gamma^k}$$

Where

- P_{ikj} Unit price of the volume harvested in period k on areas regenerated in period i and regeneration harvested again in period j
- V_{ikj} Volume per hectare harvested in period k on areas regenerated in period i and regeneration harvested again in period j
- C_{ikj} Cultural treatment costs per hectare in period k on areas regenerated in period i and regeneration harvested again in period j
- γ^j Discount rate for period j
- E_{iN} Discounted net revenue per hectare during the planning horizon from areas regenerated in period i and put aside as ending inventory in period N plus discounted net value per hectare of leaving these areas as ending inventory. It can be written as shown below:

$$E_{iN} = \sum_{k=\max(i,1)}^N \frac{P_{ikN}V_{ikN} - C_{ikN}}{\gamma^k} + \frac{P'_{iN}}{\gamma^N}$$

Where

- P_{ikN} Unit price of the volume thinned in period k on areas regenerated in period i and put aside as ending inventory in period N
- V_{ikN} Volume per hectare thinned in period k on areas regenerated in period i and put aside as ending inventory in period N
- C_{ikN} Cultural treatment costs per hectare in period k on areas regenerated in period i and put aside as ending inventory in period N
- P'_{iN} Net value per hectare of leaving areas regenerated in period i as ending inventory in period N

3. Methods

In this section, we first represent the modeling mistakes in Model II and then propose our new model to overcome these mistakes.

3.1. Mistake in the first set of area constraints

In accordance with the definition of z , the minimum number of periods between regeneration harvests, it is not allowed to harvest an area unless at least z periods have been passed since the last regeneration harvest. Unfortunately, Constraint (2) has been mistakenly written in Johnson and Scheurman (1977). In the original formulation, there is a number of extra variables generated that should be forced to be 0 due to the requirement of periods between regeneration harvests. If coefficients of variables in the objective function are negative except for the additional decision variables, it is possible that those extra decision variables take values and if this is the case, we have an erroneous solution. There is a number of situations when this may happen, including partial cutting and thinning operations, where costs exceed revenue from sales.

In order to prove our claim and provide further clarification, consider the following example where we would like to schedule harvests for the next four time periods ($N = 4$) from a forest that now has three different age-classes aged 0, 1, and 2 (i.e. $A_0 = 300$, $A_{-1} = 200$, and $A_{-2} = 100$) ($M = 2$). There is a minimum of three time periods between regeneration harvests ($z = 3$).

Now we want to expand the objective function and first set of area constraints and take a closer look at them. As mentioned above, an area cannot be harvested unless a minimum of z periods have passed since the last regeneration. However, you can find variables in the constraints which are contrary to this law (x_{-11} , x_{01} , and x_{02}).

$$\begin{aligned} & \max \sum_{j=1}^4 \sum_{i=-2}^{j-3} D_{ij} x_{ij} + \sum_{i=-2}^4 E_{i4} w_{i4} \\ & = D_{-21} x_{-21} + \\ & \quad D_{-22} x_{-22} + D_{-12} x_{-12} + \end{aligned}$$

$$D_{-23}x_{-23} + D_{-13}x_{-13} + D_{03}x_{03} +$$

$$D_{-24}x_{-24} + D_{-14}x_{-14} + D_{04}x_{04} + D_{14}x_{14} +$$

$$E_{-24}w_{-24} + E_{-14}w_{-14} + E_{04}w_{04} + E_{14}w_{14} + E_{24}w_{24} + E_{34}w_{34} + E_{44}w_{44}$$

First area constraint

$$\sum_{j=1}^4 x_{ij} + w_{i4} = A_i, \quad i = -2, \dots, 0$$

$$x_{-21} + x_{-22} + x_{-23} + x_{-24} + w_{-24} = A_{-2}, \quad i = -2$$

$$x_{-11} + x_{-12} + x_{-13} + x_{-14} + w_{-14} = A_{-1}, \quad i = -1$$

$$x_{-01} + x_{-02} + x_{-03} + x_{-04} + w_{-04} = A_{-0}, \quad i = 0$$

As aforementioned, if the coefficients of variables in the objective function are negative except for the additional decision variables, it is possible that those extra decision variables take values. To clear it up, suppose the following values for parameters D_{ij} and E_{iN} in Tables 1 and 2 for the above-mentioned example.

Table 1: Values for parameters D_{ij}

		Next Harvesting Period (Period j)			
		1	2	3	4
Most Recent Harvesting Period (Period i)	-2	1	1	1	1
	-1	0	-1	-1	-1
	0	0	0	-1	-1
	1	0	0	0	1
	2	0	0	0	0
	3	0	0	0	0
	4	0	0	0	0

Table 2: Values for parameters E_{i4}

Period i	-2	-1	0	1	2	3	4
E_{i4}	1	-1	-1	1	1	1	1

We solved the model and the results can be found in Tables 3 and 4. Light gray cells indicate the forbidden periods for the harvesting of each age-class according to the definition of the z parameter. While the rule has been violated by x_{-11} and x_{01} , 200 and 300 are their values, respectively. The objective value is 300.

The repercussion will not be limited to this one. In addition to that, those values would be ignored for future harvest planning. For instance, when a management unit is regeneration harvested in period 1 ($x_{-11}=200$), it can be harvested in period 4 or taken into account as an ending inventory while it has been overlooked, likewise for the other unallowable variable ($x_{01}=300$).

Table 3: Outcomes for decision variables x_{ij} for original formulation

		Next Harvesting Period (Period j)			
		1	2	3	4
Most Recent Harvesting Period (Period i)	-2	100	0	0	0
	-1	200	0	0	0
	0	300	0	0	0
	1	0	0	0	100
	2	0	0	0	0
	3	0	0	0	0
	4	0	0	0	0

Table 4: Outcomes for decision variables w_{i4} for original formulation

Period i	-2	-1	0	1	2	3	4
w_{i4}	0	0	0	0	0	0	100

In order to correct the mistakes, a revised formulation for the first set of area constraints are:

$$\sum_{j=1}^N x_{ij} + w_{iN} = A_i, \quad i = -M, \dots, 1 - z$$

$$\sum_{j=z+i}^N x_{ij} + w_{iN} = A_i, \quad i = 2 - z, \dots, 0$$

We solved the example again by considering the new formulation. The outcomes can be found in Tables 5 and 6. As it can be seen, there is no breach of rule for harvesting. The objective value is -4200.

Table 5: Outcomes for decision variables x_{ij} for revised formulation

		Next Harvesting Period (Period j)			
		1	2	3	4
Most Recent Harvesting Period (Period i)	-2	100	0	0	0
	-1	0	0	0	200
	0	0	0	300	0
	1	0	0	0	100
	2	0	0	0	0
	3	0	0	0	0
	4	0	0	0	0

Table 6: Outcomes for decision variables w_{i4} for revised formulation

Period i	-2	-1	0	1	2	3	4
w_{i4}	0	0	0	0	0	300	300

3.2. Mistake in calculation of D_{ij}

The model uses the objective function coefficients D_{ij} and E_{iN} . To find the miscalculation of D_{ij} parameter tangibly, consider the following example with given parameters: $N = 7, M = 1, z = 3$.

The objective function is as below:

$$\begin{aligned}
 & \max \sum_{j=1}^7 \sum_{i=-1}^{j-3} D_{ij} x_{ij} + \sum_{i=-1}^7 E_{i7} w_{i7} \\
 & = D_{-12} x_{-12} + \\
 & \quad D_{-132} x_{-13} + D_{03} x_{03} + \\
 & \quad D_{-14} x_{-14} + D_{04} x_{04} + D_{14} x_{14} + \\
 & \quad D_{-15} x_{-15} + D_{05} x_{05} + D_{15} x_{15} + D_{25} x_{25} + \\
 & \quad D_{-16} x_{-16} + D_{06} x_{06} + D_{16} x_{16} + D_{26} x_{26} + D_{36} x_{36} + \\
 & \quad D_{-17} x_{-17} + D_{07} x_{07} + D_{17} x_{17} + D_{27} x_{27} + D_{37} x_{37} + D_{47} x_{47} + \\
 & \quad E_{-17} w_{-17} + E_{07} w_{07} + E_{17} w_{17} + E_{27} w_{27} + E_{37} w_{37} + E_{47} w_{47} + E_{57} w_{57} + E_{67} w_{67} + E_{77} w_{77}
 \end{aligned}$$

At each time period, two sets of timber flows are needed, including input (areas regenerated in previous time periods and going to be regeneration harvested again in this period) and output (areas may be regenerated in future or put aside as an ending inventory) flows. Figure 4 represents input and output flows in the aforementioned example ($N = 7, M = 1, z = 3$). For example, in Figure 4c, there are two timber inflows from areas regenerated harvested three and four periods ago (period -1 and 0, respectively) and three timber outflows, two of which will be regenerated again in periods 6 and 7, and the third outflow is related to areas left as ending inventory in period 7. Note that thinned volume obtained from stand thinnings of regeneration harvested areas are not shown in Figure 4.

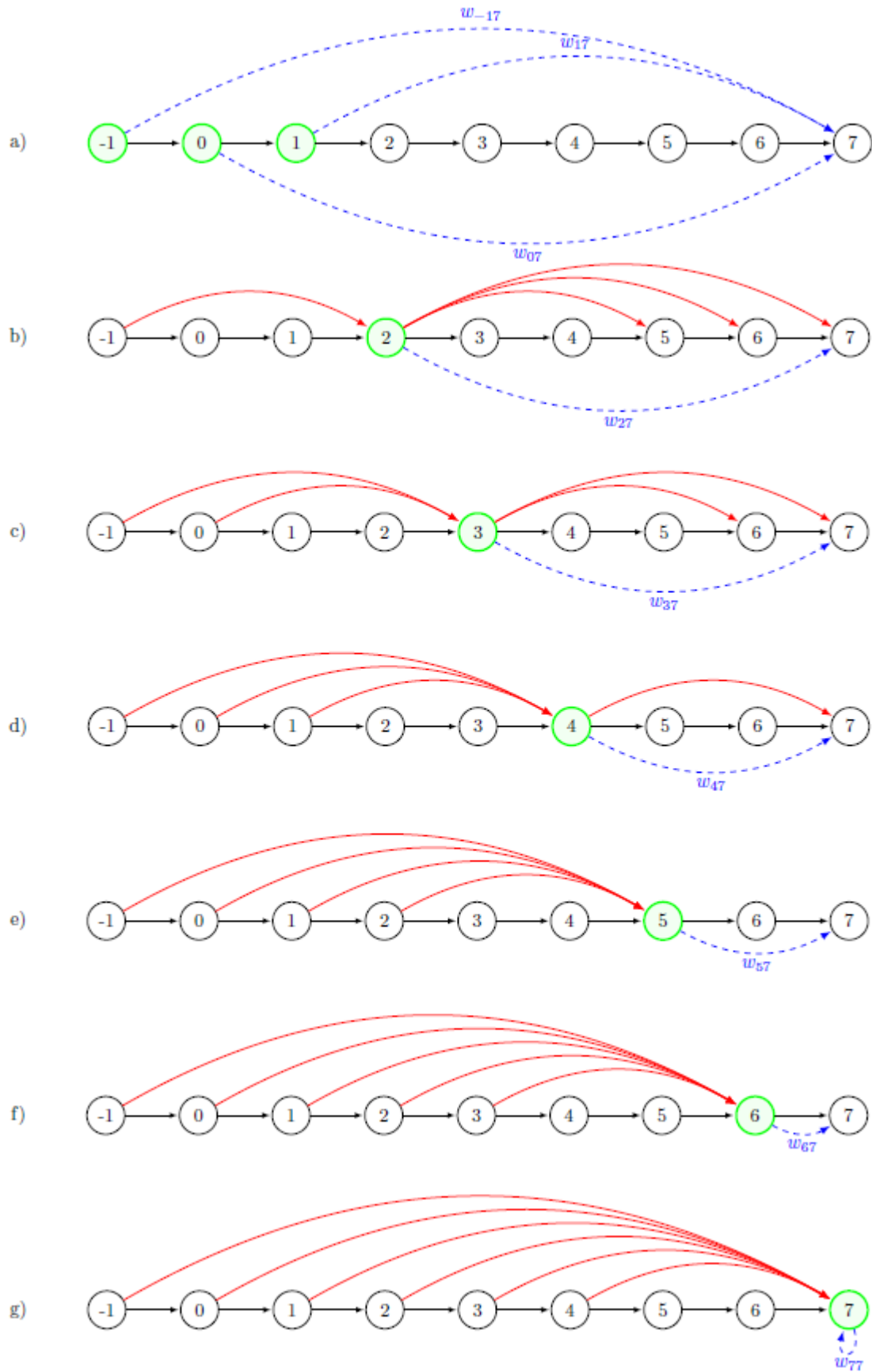


Figure 4: Timber flows for different time periods in an example with $N=7$, $M=1$, and $z=3$. Solid lines show the regenerated areas and dotted lines indicate the areas put aside as an ending inventory.

To represent the mistake which we will come across while we are calculating the coefficients with the use of original formulation, consider the following equations and figures:

$$\begin{aligned}
 D_{-12} &= \sum_{k=\max(-1,1)}^2 \frac{P_{-1k2}V_{-1k2} - C_{-1k2}}{\gamma^k} \\
 &= \frac{P_{-112}V_{-112} - C_{-112}}{\gamma} + \frac{P_{-122}V_{-122} - C_{-122}}{\gamma^2}
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 D_{-13} &= \sum_{k=\max(-1,1)}^3 \frac{P_{-1k3}V_{-1k3} - C_{-1k3}}{\gamma^k} \\
 &= \frac{P_{-113}V_{-113} - C_{-113}}{\gamma} + \frac{P_{-123}V_{-123} - C_{-123}}{\gamma^2} + \frac{P_{-133}V_{-133} - C_{-133}}{\gamma^3}
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 D_{-14} &= \sum_{k=\max(-1,1)}^4 \frac{P_{-1k4}V_{-1k4} - C_{-1k4}}{\gamma^k} \\
 &= \frac{P_{-114}V_{-114} - C_{-114}}{\gamma} + \frac{P_{-124}V_{-124} - C_{-124}}{\gamma^2} + \frac{P_{-134}V_{-134} - C_{-134}}{\gamma^3} \\
 &\quad + \frac{P_{-144}V_{-144} - C_{-144}}{\gamma^4}
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 D_{-15} &= \sum_{k=\max(-1,1)}^5 \frac{P_{-1k5}V_{-1k5} - C_{-1k5}}{\gamma^k} \\
 &= \frac{P_{-115}V_{-115} - C_{-115}}{\gamma} + \frac{P_{-125}V_{-125} - C_{-125}}{\gamma^2} + \frac{P_{-135}V_{-135} - C_{-135}}{\gamma^3} \\
 &\quad + \frac{P_{-145}V_{-145} - C_{-145}}{\gamma^4} + \frac{P_{-155}V_{-155} - C_{-155}}{\gamma^5}
 \end{aligned} \tag{9}$$

$$\begin{aligned}
 D_{-16} &= \sum_{k=\max(-1,1)}^6 \frac{P_{-1k6}V_{-1k6} - C_{-1k6}}{\gamma^k} \\
 &= \frac{P_{-116}V_{-116} - C_{-116}}{\gamma} + \frac{P_{-126}V_{-126} - C_{-126}}{\gamma^2} + \frac{P_{-136}V_{-136} - C_{-136}}{\gamma^3} \\
 &\quad + \frac{P_{-146}V_{-146} - C_{-146}}{\gamma^4} + \frac{P_{-156}V_{-156} - C_{-156}}{\gamma^5} + \frac{P_{-166}V_{-166} - C_{-166}}{\gamma^6}
 \end{aligned} \tag{10}$$

Note that the number of the first and last regeneration periods is constant; however, the middle harvested period (k) varies in the formulation. According to the definition of z , some timber flows (only the harvested volume and not the thinned volume of stand thinnings) are impossible; bold segments of the formulas refer to this point. Figure 5 illuminates the possible and impossible timber flows. For instance, as discovered in Figure 5d, D_{-15} is consisted of five timber flows such as $V_{-115}, V_{-125}, V_{-135}, V_{-145}$, and V_{-155} ; however, in accordance with the definition of the z parameters, some timber flows are impossible, like V_{-115}, V_{-135} , and V_{-145} . You should be aware that the mistake is not limited to impractical timber flows. In addition, there is an overlap between one fragment of the D_{-12} and D_{-15} . The fragments are as below:

$$\frac{P_{-122}V_{-122} - C_{-122}}{\gamma^2} \quad \& \quad \frac{P_{-125}V_{-125} - C_{-125}}{\gamma^2}$$

These two segments calculate the same timber flow and discount it for two periods. In other words, there is a timber flow in D_{-15} which has been computed in D_{-12} . Furthermore, two overlaps can be found between D_{-16}, D_{-12} , and D_{-13} .

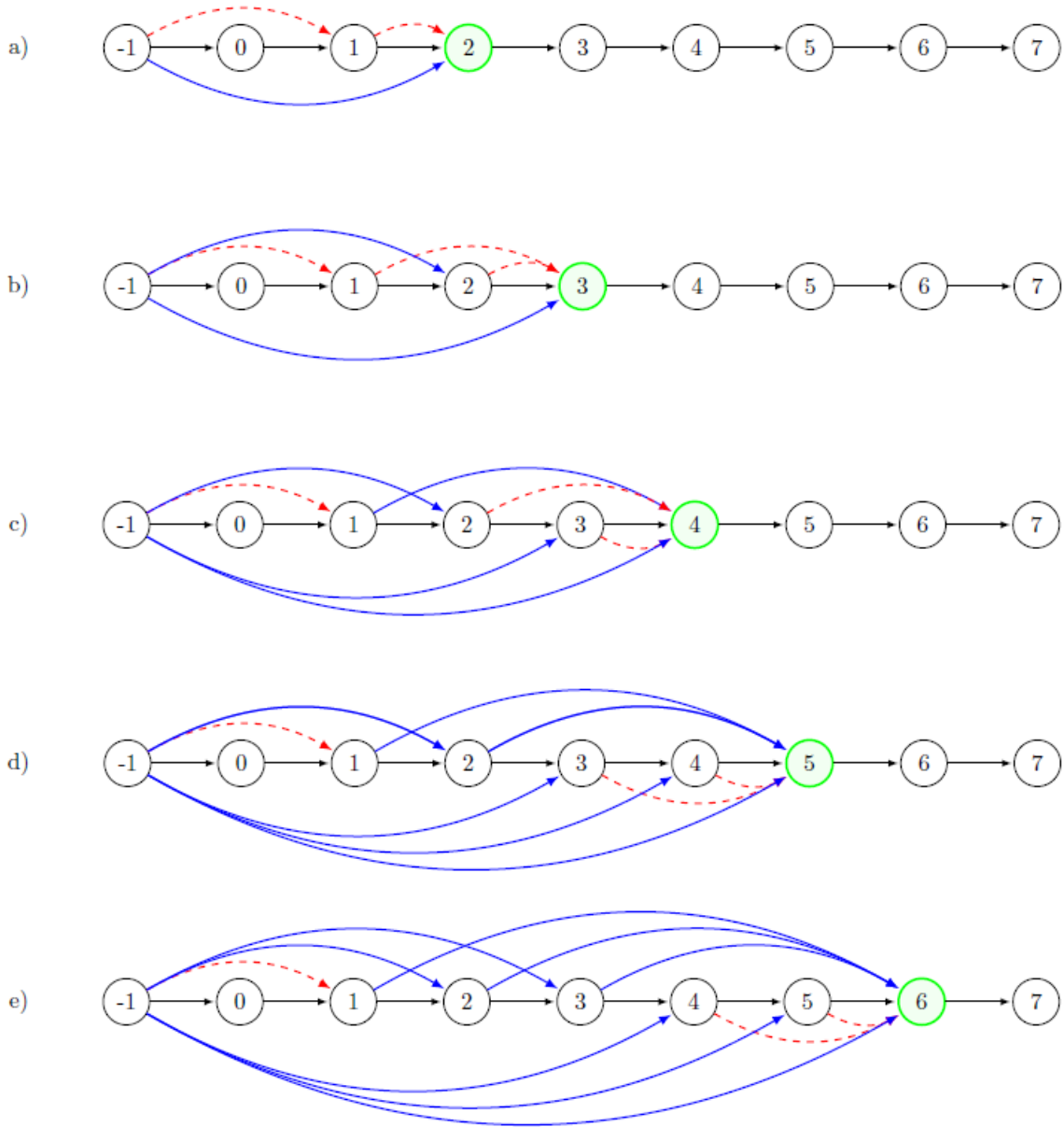


Figure 5: Timber flows used in calculation of discounted net revenue. Solid and dotted lines show the possible and impossible timber flows, respectively

To analyze the profitability of investment, NPV, which is the difference between the present value of cash inflows and outflows discounted by the discount rate, is used in capital budgeting. The coefficient D_{ij} is the discounted net revenue per hectare from areas regenerated in period i and regeneration harvested again in period j . However, in accordance with the definition of z , some timber flows (V_{ikj}) are impossible as was identified in the model constraint. Moreover, there is an overlap between different D_{ij} in the objective function, i.e., the V_{ikj} is counted multiple times in the D_{ij} which

results in an overestimation in the calculation of the net present value (NPV). Therefore, in order to correct the formula, D_{ij} should be broken into two segments to consider both revenues from harvested volume (D'_{ij}) and thinned volume (D''_{ij}) as below:

$$D'_{ij} = \frac{P'_{ij}V'_{ij} - C'_{ij}}{(1 + \gamma)^j} \quad j = 1, 2, \dots, N \text{ \& } i = -M, \dots, j - z \quad (11)$$

$$D''_{ij} = \sum_{k=\max(i,1)}^j \frac{P''_{ikj}V''_{ikj} - C''_{ikj}}{\gamma^k} \quad (12)$$

$$D_{ij} = D'_{ij} + D''_{ij} \quad (13)$$

Where

- P'_{ij} Unit price of the volume harvested in period i and regeneration harvested in period j
- V'_{ij} Volume per hectare harvested in period i and regeneration harvested in period j
- C'_{ij} Cultural treatment costs per hectare in period i and regeneration harvested in period j
- P''_{ikj} Unit price of the volume thinned in period k on areas regenerated in period i and regeneration harvested again in period j
- V''_{ikj} Volume per hectare thinned in period k on areas regenerated in period i and regeneration harvested again in period j
- C''_{ikj} Cultural treatment costs per hectare in period k on areas regenerated in period i and regeneration harvested again in period j

3.3. Full-revised Model II

Mathematically, the fully revised Model II would be as illustrated below:

$$\max \sum_{j=1}^N \sum_{i=-M}^{j-z} D_{ij} x_{ij} + \sum_{i=-M}^N E_{iN} w_{iN} \quad (14)$$

Subject to

$$\sum_{j=1}^N x_{ij} + w_{iN} = A_i, \quad i = -M, \dots, 1 - z \quad (15)$$

$$\sum_{j=z+i}^N x_{ij} + w_{iN} = A_i, \quad i = 2 - z, \dots, 0 \quad (16)$$

$$\sum_{k=j+z}^N x_{jk} + w_{jN} = \sum_{i=-M}^{j-z} x_{ij}, \quad j = 1, 2, \dots, N \quad (17)$$

$$x_{ij} \geq 0, \quad i = -M, \dots, N, j = 1, 2, \dots, N \quad (18)$$

$$w_{iN} \geq 0, \quad i = -M, \dots, N \quad (19)$$

$$D'_{ij} = \frac{P'_{ij} V'_{ij} - C'_{ij}}{(1 + \gamma)^j} \quad j = 1, 2, \dots, N \text{ \& } i = -M, \dots, j - z \quad (20)$$

$$D''_{ij} = \sum_{k=\max(i,1)}^j \frac{P''_{ikj} V''_{ikj} - C''_{ikj}}{\gamma^k} \quad (21)$$

$$D_{ij} = D'_{ij} + D''_{ij} \quad (22)$$

Where

P'_{ij}	Unit price of volume harvested in period i and regeneration harvested in period j
V'_{ij}	Volume per hectare harvested in period i and regeneration harvested in period j
C'_{ij}	Cultural treatment costs per hectare in period i and regeneration harvested in period j

- P''_{ikj} Unit price of volume thinned in period k on areas regenerated in period i and regeneration harvested again in period j
- V''_{ikj} Volume per hectare thinned in period k on areas regenerated in period i and regeneration harvested again in period j
- C''_{ikj} Cultural treatment costs per hectare in period k on areas regenerated in period i and regeneration harvested again in period j

4. Results

Having encountered problems and proposed a way to deal with them, we want to solve a real problem in order to validate our formulation. In order to solve the problem, first of all, we should compute the values of objective coefficients. Therefore, the price for timbers, volume harvested, and cost of cultural treatments will be discussed in the following paragraphs.

4.1. Case

The price of timber depends on different agents such as kind of tree, length, diameter, and quality. Quality is one of the chief agents of price change. Table 7 represents the stand volume before and after the commercial thinning, and the average unit price for Spruce lumbers based on age-classes (different diameter and length).

The growth rate is influenced by numerous variables, such as soil, local climate, light, fertility, and the care you provide. Each tree has its own growth rate curve concluding three phases. In the beginning, the tree is growing and the growth rate is increasing. Gradually, the growth rate decreases until the tree stops growing. Finally, the phase of decay starts and the growth rate reduces further to negative levels. Figure 6 shows the empirical yield tables for white spruce trees (Pothier and Savard 1998).

Table 7: Unit price of the wood based on the length and diameter of the wood

Age Class	Age	Height (m)	Diameter (cm)	Volume (m3/ha)	Volume after CT (m3/ha)	Price (\$/m3)
4	20	5.7	13.9	37	37	93
5	25	6.5	14.2	57	57	95
6	30	7.1	14.5	78	39	117
7	35	7.6	14.8	96	57	123
8	40	8.0	15.2	113	74	130
9	45	8.4	15.5	127	88	130
10	50	8.7	15.8	140	101	130
11	55	9.0	16.0	151	112	130
12	60	9.3	16.3	161	122	133
13	65	9.5	16.5	170	131	133
14	70	9.7	16.8	177	138	135
15	75	9.8	17.0	184	145	135
16	80	9.9	17.2	189	150	135
17	85	10.1	17.3	194	155	135
18	90	10.2	17.5	199	160	135
19	95	10.3	17.7	203	164	135
20	100	10.3	17.8	206	167	135
21	105	10.4	17.9	209	170	135
22	110	10.5	18.1	212	173	148
23	115	10.5	18.2	213	174	148
24	120	10.5	18.3	212	173	148
25	125	10.6	18.5	207	168	148
26	130	10.6	18.6	200	161	148
27	135	10.6	18.8	190	151	148
28	140	10.7	18.9	177	138	148
29	145	10.7	19.1	161	122	148
30	150	10.7	19.3	141	102	148

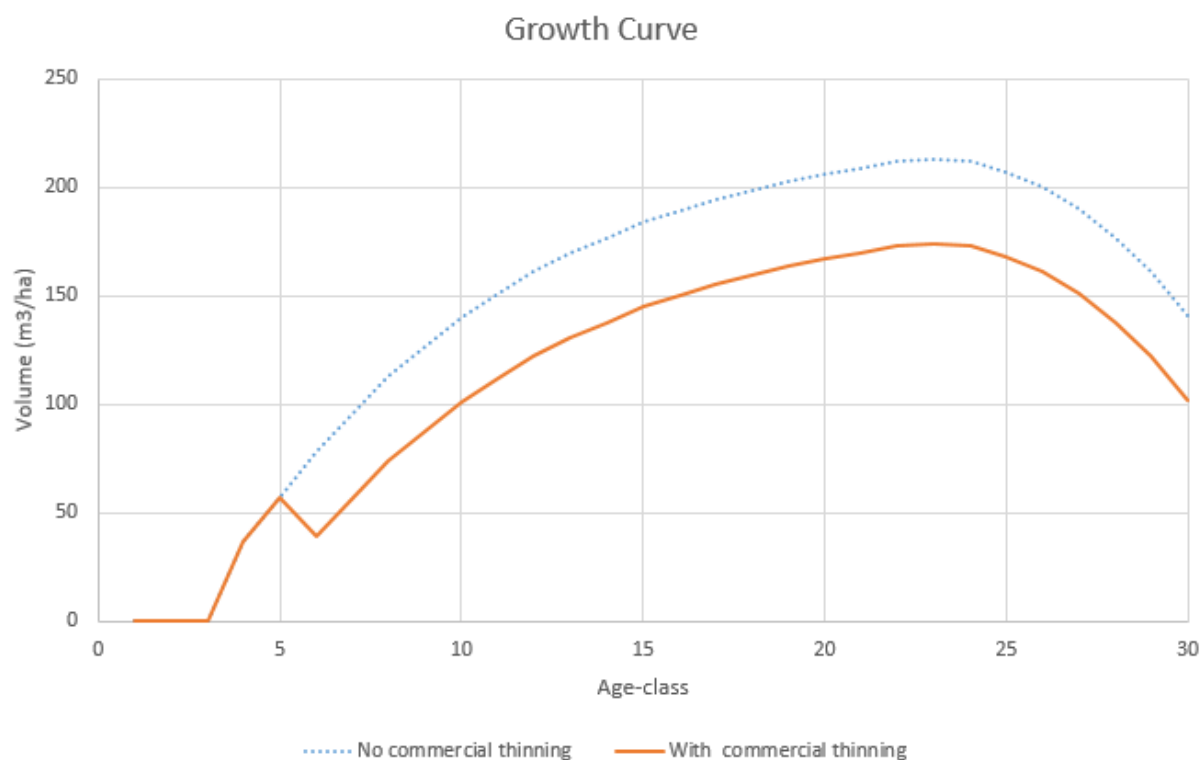


Figure 6: The growth curve for spruce stands used for calculation of discounted net revenue of timber flow.

Silvicultural treatment is an operational plan (a sequence of actions, including precommercial thinning (PCT), commercial thinning (CT), shelterwood, selection, buffer, clear-cut, and do nothing) which explains the forest management goals for an area.

In general, stands naturally regenerated are needed to be pre-commercially thinned. In Canada, there are no marketable wood materials during the pre-commercial thinning; it is a cost generator with no immediate income for the landowner. In order to minimize the cost, PCT should be performed within the first four years of the stand (Forest and Range 2004). Pre-commercial thinning is only conducted in even-aged forests around 15 years old. The trees are too small to be used in the mills and they are always left on site because their decomposition enriches the soil (Forêts, Faune et Parcs Gouvernement du Québec 2003). In this example, we assumed a PCT is performed in stands around 10 years old. Estimating the actual costs of pre-commercial thinning, labor, and equipment costs which vary depending on different issues should be known (De Franceschi and Boylen 1987). Hedin (1982) took into account the PCT costs of \$19.80 per hour based on brushsaw ownership, operating costs, and labor union wage. He also supposed that 15 hours should be spent to thin a hectare, i.e. \$297 per hectare. In this paper, we assumed $300 \frac{\$}{\text{ha}}$ for pre-commercial thinning.

The average cost of supply, with transportation to the state, in Quebec costs $65.24 \frac{\$}{\text{m}^3}$. Harvesting (34% of total cost), transportation (24.8%), and transfer to the state (20.2%) are the three most important expenditure items constituting 80% of the supply cost (Groupe DDM 2016). Therefore, harvesting cost is assumed $65.24 \frac{\$}{\text{m}^3}$.

Typically, in eastern Canada, when they do thinning they cut some trees to make more space for the rest of trees in the stand. It would lead to larger diameter for the rest of trees, in other words, the quality of the wood will increase. So, it is assumed that thinning doesn't change significantly the total volume in the forest stand, while it decreases the number of the trees and increases the wood in the forest stand. Figure 6 illustrates the effect of commercial thinning on the stand volume. Commercial thinning is an intermediate harvest process and can provide some intermediate return covering part or all of the cost of harvesting from salable wood. Commercial thinning is usually done in stands between 30 and 80 years old, with no regeneration objective (Forest Practices Branch, Ministry of Forests, British Columbia, Canada 1999). In this example, Commercial thinning is implemented at age 30. Usually the

commercial thinning cost is 10-15% higher than the harvesting cost which is assumed to be $75 \frac{\$}{m^3}$. Moreover, it is assumed that the thinned volume is 50% of the total volume of the stand.

To calculate the net value per hectare of leaving areas regenerated in period i as ending inventory in period N (P'_{iN}), we calculate the value of standing trees which is assumed to be equal to the potential income from wood volume available in the stand. We used the following formula to calculate P'_{iN} :

$$P'_{iN} = \frac{(P_{iN} - C_{iN})V_{iN}}{\gamma^N}$$

- P_{iN} Unit price of volume harvested in period i and regeneration harvested in period N (put aside as ending inventory)
- V_{iN} Volume per hectare harvested in period i and regeneration harvested in period N (put aside as ending inventory)
- C_{iN} Cultural treatment costs per hectare in period i and regeneration harvested in period N (put aside as ending inventory)

4.2. An illustrative example with realistic data

In this example, the length of each planning period is 5 years. Suppose we have a forest with six different age-classes of trees and given areas (See Table 8) and a minimum number of periods between regeneration harvests is 3 ($z = 3$, in other words, 15 years), as below:

Table 8: Values for parameters A_i

Period i	-6	-5	-4	-3	-2	-1	0
A_i	100	200	300	400	500	600	700

We assume that PCT is implemented at age 10, in other words, two planning periods after the regeneration harvest and CT is implemented at age 30 (6 periods after the regeneration harvest). In this example, annual interest rate is assumed to be 4% (Gouvernement du Québec Bureau de mise en marché des bois 2013) and constant throughout the planning horizon. According to the aforementioned cultural treatment costs, D_{ij} and E_{i6} values can be found in Table 9 and Table 10, respectively.

Table 9: Values for parameters D_{ij} obtained from revised formulation

		Next Harvesting Period (Period j)					
		1	2	3	4	5	6
Most Recent Harvesting Period (Period i)	-6	1,599	1,994	1,976	1,900	1,755	1,678
	-5	1,219	1,631	1,960	1,945	1,881	1,761
	-4	523	1,016	1,359	1,633	1,621	1,567
	-3	278	435	846	1,132	1,361	1,351
	-2	0	231	363	705	944	1,134
	-1	0	(667)	(474)	(364)	(79)	120
	0	0	0	(556)	(395)	(304)	(66)
	1	0	0	0	(463)	(329)	(253)
	2	0	0	0	0	(386)	(274)
	3	0	0	0	0	0	(322)
	4	0	0	0	0	0	0
	5	0	0	0	0	0	0
	6	0	0	0	0	0	0

Table 10: Values for parameters E_{i6}

Period i	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
E_{i6}	1678	1761	1567	1351	1134	120	(66)	(253)	(274)	(322)	(268)	0	0

If we solve the problem with two different formulations, the original one and our suggested formulation, there would be a gap between the objective values. The objective value for original and proposed formulations are 2,221,274.88 and 2,175,237.80 respectively. The new NPV is 2.12% less than the original formulation—the original formulation overestimate the objective function, that is:

$$\frac{(2,175,237.80 - 2,221,274.88)}{2,221,274.88} = -2.12\%$$

Note that it is hard to estimate the impact on general problems as the type of harvest operations included plays a role.

Tables 11 and 12 indicate the outcomes of decision variables if we solve the problem with original formulation. Tables 13 and 14 demonstrate the new values for the decision variables if we use the proposed formulation. According to the definition of the z parameter, some decision variables must not take a value (gray cells in tables 11 and 13); however, as it can be found in the Table 11, the value of x_{01} is equal to 700. This is the principal reason for the gap between two formulations. In the proposed formulation, the value of x_{01} is equal to 0 instead of 700 and it goes to E_{06} with coefficient of -267 in the objective function. Note that when discounted costs of cultural treatment are greater than the discounted income obtained from thinned volume plus discounted income from non-timber values, E_{iN} can have negative value.

Table 11: Original formulation - Outcomes for decision variables x_{ij}

		Next Harvesting Period (Period j)					
		1	2	3	4	5	6
Most Recent Harvesting Period (Period i)	-6	0	0	0	0	100	0
	-5	0	0	0	0	200	0
	-4	0	0	0	0	300	0
	-3	0	0	0	0	400	0
	-2	0	0	0	0	0	500
	-1	0	0	0	0	0	0
	0	700	0	0	0	0	0
	1	0	0	0	0	0	0
	2	0	0	0	0	0	0
	3	0	0	0	0	0	0
	4	0	0	0	0	0	0
	5	0	0	0	0	0	0
	6	0	0	0	0	0	0

Table 12: Original formulation - Outcomes for decision variables w_{i6}

Period i	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
w_{i6}	0	0	0	0	0	600	0	0	0	0	0	1000	500

Table 13: Outcomes for decision variables x_{ij} for revised formulation

		Next Harvesting Period (Period j)					
		1	2	3	4	5	6
Most Recent Harvesting Period (Period i)	-6	0	0	0	0	100	0
	-5	0	0	0	0	200	0
	-4	0	0	0	0	300	0
	-3	0	0	0	0	400	0
	-2	0	0	0	0	0	0
	-1	0	0	0	0	0	600
	0	0	0	0	0	0	700
	1	0	0	0	0	0	0
	2	0	0	0	0	0	0
	3	0	0	0	0	0	0
	4	0	0	0	0	0	0
	5	0	0	0	0	0	0
	6	0	0	0	0	0	0

Table 14: Outcomes for decision variables w_{i6} for revised formulation

Period i	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
w_{i6}	0	0	0	0	500	0	0	0	0	0	0	1000	1300

5. Concluding remarks

Models are considered as the basic tools of strategic forest planning by most foresters because they examine the long-term consequences of forest-management inputs (Gunn 2007). In this paper, we focused on Model II and how the minimum number of periods between regeneration harvests, i.e, z parameter, could lead to modeling mistakes. The first mistake came about in the first set of area constraints wherein some additional decision variables appeared. They had no coefficient in the objective function; however, in specific contexts, they could take nonzero values. The second one took place when we compute the D_{ij} parameter wherein overlaps and impossible timber flows could be found in the formulation.

As far as we know about the literature, these mistakes have not been identified by any researcher, since Model II was suggested by Johnson and Scheurman (1977). In addition, an example has been given with realistic parameters in order to authenticate our claim. It is well worth mentioning that mistakes had an impact on the real data. Some well-known software, such as Woodstock, FORPLAN, TigerMoth, and SilviLab are based on variants of Model II formulation. We have not verified that these applications use the formulation that was published in the original article by Johnson and Scheurman (1977). Furthermore, we have not verified that the models referenced to Johnson and Scheurman (1977) included the mistakes or that the mistakes were a publication error. It is, however, important to provide information to avoid that the mistakes are implemented in new models and can be corrected, if necessary, in old models. It can be very difficult to identify these errors in particular if the models are only used for computing forest values and it is not necessary to study the detailed harvest plan where the additional variables can be identified.

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