Definition of the Domain for Summative Evaluation

MTH-5107-2

Mathematics Exponential and Logarithmic Functions and Equations

Québec

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Formation professionnelle et technique
et formation continue

Direction de la formation générale
des adultes

ISBN 2-550-43699-7

Legal deposit — Bibliothèque nationale du Québec, 2004

1. INTRODUCTION

This Definition of the Domain for Summative Evaluation describes and classifies the essential and representative elements of the secondary-level adult education Mathematics program and, more specifically, of the course entitled Exponential and Logarithmic Functions and Equations. As such, it gives an overview of the program, but should by no means replace the program itself. The purpose of defining the domain is to ensure that all summative evaluation instruments are consistent with the overall program.

The Definition of the Domain for Summative Evaluation for each course in this program is organized in a similar manner; however, the content of this definition of the domain is specific to the course entitled Exponential and Logarithmic Functions and Equations.

The goal of the Definition of the Domain for Summative Evaluation is to prepare examinations that are valid from one version to another or from one school board to another, taking into account the responsibilities shared by the Ministère de l'Éducation and the school boards.

2. PROGRAM ORIENTATIONS AND CONSEQUENCES FOR SUMMATIVE EVALUATION

ORIENTATIONS	CONSEQUENCES
The main objective of the secondary-level adult education Mathematics program is to help students fully understand mathematical concepts.	Evaluation should involve verifying whether the student has fully understood the different concepts.
The program is designed to help students master the use of certain mathematical tools used in the field of science and technology or in different trades.	Evaluation items should pertain to situations in the field of science and technology or to situations related to trades.
The program aims to provide students with the skills they need to process information by applying mathematical models and appropriate strategies for solving problems.	Evaluation items should involve performing tasks that require the students to classify information, use mathematical models and solve problems.
The program also aims to improve the students' ability to clearly relate information using mathematical language.	Evaluation items should involve performing tasks that require the use of mathematical language. The appropriateness and clarity of the language used should be taken into account in the marking process.
The program is intended to help students develop a systematic work method.	Evaluation items should require the students to present their work in a clear and structured manner. This should be taken into account in the marking process.
The program will help students master the use of technological tools.	The use of a scientific calculator or graphing calculator is permitted for the examinations related to this course.

3. CONTENT OF THE PROGRAM FOR PURPOSES OF SUMMATIVE EVALUATION

Concepts

Exponential functions

– algebraically solving an exponential equation in which the two members of the equation can be expressed in the same base or in different bases
– inverse rule of an exponential function
– rule of an exponential function corresponding to a given context
– value and sign of the parameters of an exponential function, given the parametric equation and a graph
– connections between a change in two parameters of an exponential function and the transformation of a graph, given the parametric equation and two graphs
– examination of the characteristics of exponential functions
– rule of an exponential function of the form $f(x) = \pm c^x + k$, given the coordinates of a point and the equation of the asymptote
– problems involving exponential functions

Logarithmic functions

– properties of logarithms
– value of a logarithmic expression
– simplification of a logarithmic expression
– algebraically solving a logarithmic equation
– value and sign of parameters of a logarithmic function, given an equation and a graph
– connections between a change in two parameters of a logarithmic function and the transformation of a graph, given the parametric equation and two graphs
– examination of the characteristics of two logarithmic functions
– rule of a logarithmic function of the form $f(x) = log_c \pm (x - h)$, given the coordinates of a point and the equation of the asymptote
– problems involving logarithmic functions

Skills

Each skill is defined within the context of a mathematics program.

Mathematizing	Interpreting a given situation using a mathematical model (arithmetic, algebraic or graphical). Possible actions: to formalize, illustrate, represent, schematize, symbolize, translate, transpose, and so on.
Operating	Performing a given operation or transformation. Possible actions: to calculate, construct, break down, perform, estimate, evaluate, isolate, measure, reconstruct, solve, transform, verify, and so on.
Analyzing	Demonstrating, in an organized fashion, the complex connections between concepts or definitions and their related actions and illustrations. Possible actions: to conclude, correct, deduce, derive, demonstrate, explain, extrapolate, infer, justify, and so on.
Synthesizing	Effectively integrating a variety of concepts and skills to solve a problem. Possible actions: to solve a problem.

4. TABLE OF DIMENSIONS

<table>
<tr><th>CONCEPTS

SKILLS</th><th>EXPONENTIAL FUNCTIONS

15%</th><th>LOGARITHMIC FUNCTIONS

85%</th></tr>
<tr><td>MATHEMATIZING
5%</td><td>Determine the rule of an exponential function corresponding to a given context.

1 5%</td><td></td></tr>
<tr><td rowspan="4">OPERATING
40%</td><td>Algebraically solve an exponential equation where the two members of the equation are powers of the same base.

2 5%</td><td>Determine the value of a logarithmic expression.

11 5%</td></tr>
<tr><td>Algebraically solve an exponential equation where the two members of the equation are powers of different bases.

3 5%</td><td>Simplify a logarithmic expression.

12 10%</td></tr>
<tr><td></td><td>Algebraically solve a logarithmic equation where each member of the equation can be simplified so that the resulting expression contains only one logarithm.

13 10%</td></tr>
<tr><td colspan="2">Determine the inverse rule of an exponential function of the form $f(x) = \pm c^{x} + k$ or of a logarithmic function of the form $f(x) = log_{c} \pm (x - h)$.
4 5%</td></tr>
<tr><td rowspan="5">ANALYZING
35%</td><td></td><td>Given statements describing the properties of logarithms, determine which statements are false and correct those statements.
14 5%</td></tr>
<tr><td colspan="2">Determine the interval and the sign of certain parameters, given the parametric equation and a graph.
5 5%</td></tr>
<tr><td colspan="2">Examine the connections between a change in two parameters and the transformation of a graph, given the parametric equation and two graphs.
6 5%</td></tr>
<tr><td colspan="2">Compare the characteristics of two functions, given the rules of two exponential functions, two logarithmic functions or a function of each type.
7 10%</td></tr>
<tr><td colspan="2">Given statements describing certain characteristics of an exponential or logarithmic function, determine which statements are false and correct those statements.
8 5%</td></tr>
<tr><td colspan="2">Find the rule of an exponential function of the form $f(x) = \pm c^{x} + k$ or of a logarithmic function of the form $f(x) = log_{c} \pm (x - h)$, given the coordinates of a point and the equation of the asymptote.
9 5%</td></tr>
<tr><td>SYNTHESIZING
20%</td><td colspan="2">Solve two problems involving exponential or logarithmic functions.

10 20%</td></tr>
</table>

5. OBSERVABLE BEHAVIOURS

Examination items should be formulated on the basis of the observable behaviours listed below. The requirements and restrictions specified in the dimensions and the objectives of the program must be observed.

Dimension 1

Determine the rule of an exponential function corresponding to a given context.
(mathematizing) /5

Dimension 2

Algebraically solve an exponential equation where the two members of the equation are powers of the same base. The exponents are algebraic expressions of a degree less than 2.
(operating) /5

Dimension 3

Algebraically solve an exponential equation where the two members of the equation are powers of different bases. The exponents are algebraic expressions of a degree less than 2.
(operating) /5

Dimension 4

Determine the rule of the inverse of an exponential function of the form $f(x) = \pm c^x + k$ or of a logarithmic function of the form $f(x) = log_c \pm (x - h)$.
(operating) /5

Dimension 5

Given the parametric equation and the graph without any underlying grid of an exponential function or a logarithmic function, determine the interval containing parameter c, either $\in]0,1[$ or $\in]1,\infty$ and the sign of parameters a and k or b and h. The exponential function is of the form $f(x) = a \cdot c^x + k$ and the logarithmic function is of the form $f(x) = log_c(b(x-h))$. The questions are multiple choice.
(analyzing) /5

Dimension 6

Given the parametric equation and two graphs without any underlying grid of an exponential function or a logarithmic function, determine the change to two parameters of the equation that made it possible to transform the first graph to obtain the second. The changes may include a modification to the sign of parameters a or b or a significant change in parameters h or k. The exponential function is of the form $f(x) = a \bullet c^{bx} + k$ and the logarithmic function is of the form $f(x) = a \bullet log_c(b(x-h))$. The questions are multiple choice. The first graph corresponds to the base function: $f(x) = log_c x$ or $f(x) = c^x$.

or

Given the parametric equation and the graph without any underlying grid of an exponential or logarithmic function as well as the description of a change to two of its parameters, select the graph obtained following the change. The changes consist in an inversion of the value of c, a change to the sign of parameters a or b, or significant change in parameters h or k. The exponential function is of the form $f(x) = a \bullet c^{bx} + k$ and the logarithmic function is of the form $f(x) = a \bullet log_c(b(x-h))$.
(analyzing) /5

Note: Dimension 6 must involve a type of function other than the function used in Dimension 5.

Dimension 7

Given the rules and the graphs of two exponential functions, two logarithmic functions or a function of each type, compare the characteristics of these two functions. The exponential function is of the form $f(x) = a \bullet c^{b(x-h)} + k$ and the logarithmic function is of the form $f(x) = a \bullet log_c(b(x-h)) + k$.
(analyzing) /10

Dimension 8

Given a graph, the rule of an exponential or logarithmic function as well as statements describing certain characteristics of this function, determine which statements are false and correct them to make them true. The exponential function is of the form $f(x) = a \bullet c^{b(x-h)} + k$ or the logarithmic function is of the form $f(x) = a \bullet log_c(b(x-h)) + k$. No more than two of the statements are false.
(analyzing) /5

Dimension 9

Find the rule of an exponential function of the form $f(x) = \pm c^x + k$ or of a logarithmic function of the form $f(x) = log_c \pm (x - h)$. For the exponential function, the coordinates of a point whose *x*-coordinate is $\neq 0$ and the equation of the asymptote are given. For the logarithmic function, the coordinates of a point whose *y*-coordinate is $\neq 0$ and the equation of the asymptote are given. The information may be provided in the form of written statements or graphs. The students must clearly show all their work.
(analyzing) /5

Note: Dimension 9 must involve a type of function other than the function used in Dimension 8.

Dimension 10

Solve two problems involving exponential functions or logarithmic functions. For logarithmic functions, the rule is given. For exponential functions, solving the problems may require completing the rule by determining the value of a parameter. It may also require drawing the graph, determining certain characteristics of the function, identifying certain information based on the context or comparing certain characteristics of two functions. The students must clearly show all their work.
(synthesizing) /20

Dimension 11

Determine the value of a logarithmic expression. The expression should contain no more than three terms. Each term should be expressed in logarithmic form and can be evaluated separately. The terms to be evaluated may contain numbers or variables.
(operating) /5

Dimension 12

Simplify a logarithmic expression. The expression should contain no more than three terms. Each term must be expressed in logarithmic form. The expression to be simplified may contain numbers or variables. Simplifying the expression may involve a simple factorization of polynomials. The students must clearly show all their work.
(operating) /10

Dimension 13

Using the properties of logarithms, algebraically solve a logarithmic equation where each member can be simplified so that the resulting expression contains only one logarithm. The given equation should contain three terms and one of the terms may be an integer. One of the members of the equation may contain a second-degree expression. The students must clearly show all their work.
(operating) /10

Dimension 14

Given statements about equalities between two expressions illustrating the properties of logarithms, determine which statements are false and correct them to make them true. The bases are variables and each statement describes a single property. No more than two of the statements are false.
(analyzing) /5

Note: The examination should include at least one question on the base e.

6. JUSTIFICATION OF CHOICES

In the examination, 5% of the items test the students' **MATHEMATIZING** skills by verifying whether they are able to translate a given situation into a mathematical model:

– finding the rule of an exponential function that corresponds to a given context

In the examination, 40% of the items test the students' **OPERATING** skills by verifying whether they have mastered certain operations or transformations:

– algebraically solving an exponential equation
– finding the inverse rule of an exponential function or a logarithmic function
– determining the value of a logarithmic expression
– simplifying a logarithmic expression
– algebraically solving a logarithmic equation

In the examination, 35% of the items test the students' skill in **ANALYZING** information; they involve verifying whether the students have the ability to make connections:

– between a change in two parameters and the transformation of the graph, given the parametric equation and two graphs
– between the value and the sign of the parameters, the parametric equation and the graph
– between the characteristics of a function and its graph
– by comparing the characteristics of two functions
– by finding the rule of an exponential function or a logarithmic function
– by verifying statements describing the properties of logarithms

In the examination, 20% of the items test the students' **SYNTHESIZING** skills by verifying their ability to:

– solve problems
– use a rigorous work method
– communicate clearly using mathematical language

7. DESCRIPTION OF THE EXAMINATION

A. TYPE OF EXAMINATION

The summative examination will be a written examination consisting of multiple-choice, short-response or extended-response items.

The items should take into account the restrictions and the requirements specified in the dimensions and the objectives of the program. The weighting of marks should be consistent with the percentages set out in the table of dimensions.

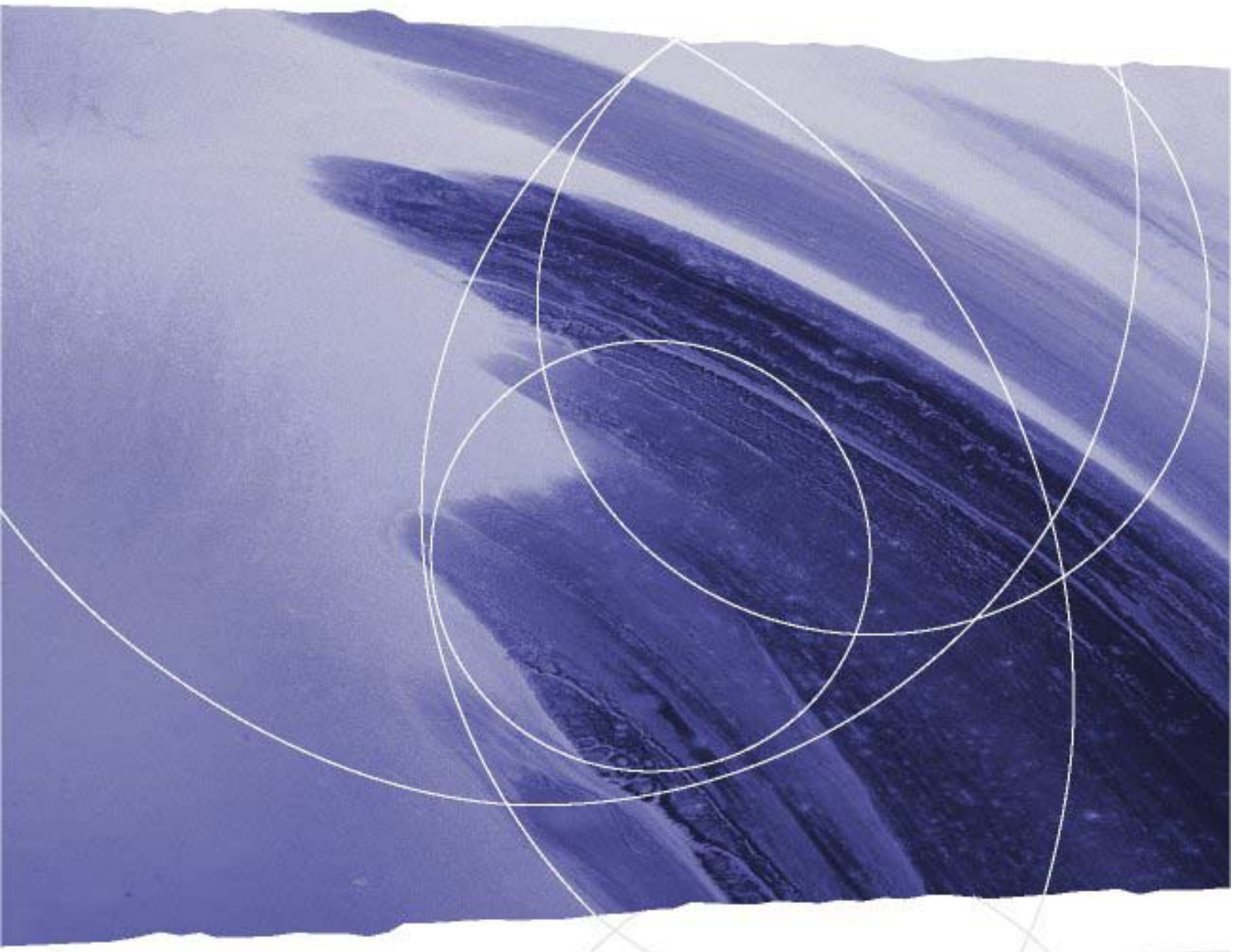
B. CHARACTERISTICS OF THE EXAMINATION

The examination will be administered in a single session lasting no more than two and a half hours.

Students are permitted to use a scientific calculator or a graphing calculator.

C. PASS MARK

The pass mark is set at 60 out of 100.



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41-3519-A