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Manufacturing Network Design with Reliable Promising Capabilities

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Abstract

This paper presents an optimization methodology for the design of manufacturing networks with reliable promising capabilities. The designed manufacturing network must have the capability to deliver products to customers within a promised combination of order-to-delivery time and service level. To cope with demand variability, the approach seeks an advantageous trade-off between service offered, standard parts inventory and manufacturing capacity deployment. Given the current state of the network, the facilities to open, close and reconfigure are optimally chosen from a set of potential facilities. Within each of these facilities, the selected production centers are configured so as to specify their assigned products, resources, capacity and lead time. The demand depends on the product-market offer selected. The model seeks to maximize profits related to the expected demand, revenues and costs of a selected set of product-market offers. A mixed integer programming model based on an engineering approach is presented. Results illustrating the practical use of the model as a tool for generating robust manufacturing networks are also presented.

Keywords

Manufacturing Network Design, Capacity Deployment, Order-To-Delivery Time, Service Level, Product-Market Offers, Mixed Integer Programming (MIP)

1. Problem context

This paper presents a methodology for the design of manufacturing networks with reliable promising capabilities. The enterprise makes order-to-delivery time and service level commitments for each of its product-market, while taking into account the stochastic nature of the demand. These commitments have an impact on the expected demand. When demand is stochastic and customers expect short lead times, a network capacitated for producing up to the average daily demand leads to many unsatisfied customers. On the other end, a network capacitated for producing up to the maximum forecasted daily demand leads to low resource utilization and high fixed costs. Therefore the enterprise needs to calibrate its network capacity in conjunction with its product-market offers development. For example, if the business aims to offer a 99%-reliability two-day delivery, it must capacitate its network so as to be able to fulfill this offer in face of stochastic daily demand. Different strategies may be used to achieve the capacity to cope with demand variability. Capacity can be calibrated through the amount of resources used, work profiles in terms of allowed shifts and overtime, standard parts inventory at decoupling points, and the externalization of some activities through subcontracting and outsourcing. Therefore, the enterprise needs to evaluate the required capacity and the deployment of this capacity in its network at the same time as it develops its product-market offers.

The supply chain network design problem integrates many decisions that have evolved over the last 30 years. Good starting points for the study of this problem are the reviews of Verter and Dincer (1992) and Geoffrion and Powers (1995). The decisions considered in this problem are related to facilities location, capacity acquisition and technology selection. Erengüç et al. (1999) discuss several of these decisions. Other relevant elements such as service levels and capacity planning related to the robustness of the supply chain are discussed by Martel (2005). However, the explicit consideration of many of these decisions in a single formulation leads to very complex mathematical models that are difficult to solve (Min and Zhou, 2002). The most comprehensive design models proposed in the literature include Cohen and Lee (1988), Cohen and Moon (1990), Cohen and Moon (1991) and Arntzen et al. (1995). These papers consider lead times, inventory decisions and economies of scale and scope in their proposed models. More recent papers integrate stochastic demand and lead time dependent demand with service levels (Rao et al., 2000;

Keskinocak et al., 2001; So and Zheng, 2003; Candas and Kutanoglu, 2004; Jeet and Kutanoglu, 2004; Slotnick and Sobel, 2005; Santoso et al., 2005). Escudero et al. (1999) and Vidal and Goetschalckx (2000) discuss different types of uncertainty in the supply chain. Strategic positioning of the inventory in the network is another important aspect to consider. Through a case study Billington et al. (2004) present a robust way to design supply-chain networks that determines the location of stock when considering service levels and demand uncertainty. Vidal and Goetschalckx (1997) present a review on several of the mathematical formulations available in the literature. A modeling framework integrating most of the decisions is presented in Martel (2005). To the best of our knowledge, all these papers consider the design problem in make-to-stock environment. The methodology proposed in this paper addresses the design of assemble-to-order and make-to-order supply chains, including the positioning of decoupling inventory in the network.

Our approach also covers some layout decisions for each facility. Benjaafar et al. (2002) present a review of recent progress on layout methodologies. Montreuil and Lefrançois (1996) present a framework for layout design based on responsibilities and Montreuil et al. (1998) propose a framework to design facilities using these types of organization. Key aspects for the design of layouts are presented by Montreuil (2000). The methodology also deals with capacity planning problems under demand uncertainty. Eppen et al. (1989), Karmarkar (1993), Verter and Dincer (1992), Verter and Dasci (2002), Benjaafar and Sheikhzadeh (2000), Orcun et al. (2003), Venkatadri et al. (2004), Zhang et al. (2004) and Cochran and Marquez Uribe (2005) propose different approaches and models for this problem.

The goal of our manufacturing network design methodology is to maximize the expected operating profits associated with the redeployment of an existing manufacturing network. The methodology proposed is appropriate for enterprises producing make-to-order and assemble-to-order products with high demand variability. Trade-offs between the expected demand and revenues stemming from the potential product-market offers on one side and the required network capacity and inventory on the other side are analyzed and the most profitable offer is identified for each product in each demand zone. The methodology is an engineering approach to design robust manufacturing networks that respect the offers made to demand zones with efficient resource utilization. It embeds an iterative solution method based on a mathematical programming model.

The paper is organized as follows. The next section describes the manufacturing network design methodology. The modeling framework and the proposed mathematical programming model are then presented, followed by a solution method and the experimental evaluation. The last section concludes the paper.

2. Methodology

The proposed design methodology is summarized in Figure 1. It starts in Step 1 with the definition of a set of potential locations specifying suppliers, plants, centers which could be implemented in plants, and demand zones for the supply chain network. Step 1 also specifies the bill-of-materials (raw, manufactured and finished product families) of the products sold. Step 2 elaborates a set of potential configurations for each center. A center is defined by a mission in terms of potential products to manufacture, and a center configuration is specified in terms of potential manufacturing resources (workers and processors) and service policy (lead time and service level). Many configurations can exist for a center, but only one configuration will eventually be implemented. Potential centers and their possible configurations must be defined prior to the network optimization phase. The resulting set of potential production center configurations must be rich enough to give flexibility to the design. These steps generate discrete sets of potential centers and configurations. Step 3 specifies the discrete set of potential product-market offers to consider. The order-to-delivery times, service levels and product selling prices offered in a demand zone have an impact on the demand in that zone. The result of the initial phase (Steps 1 to 3) is a potential supply and manufacturing network that must be optimized.

Capacity and inventory requirements depend on the allocation of the production to plants, centers and resources which are not known before the optimization. In order to optimize the network, some center configuration parameters must be estimated. These parameters, estimated in Step 4, are approximations of capacity and inventory requirements for a pre-selection of center configurations and resources in the potential network. These estimates are an input of the mixed integer programming model used in Step 5. This model is solved to determine the best network design using these estimates.

After the optimization, more precise capacity and inventory requirements can be computed with the current selection of center configurations, and the demand propagation in the network can be simulated to ensure that the promised service levels are respected. These computations give new values for some of the selected center configuration parameters and the network must be re-optimized. The iterative phase (Steps 4 and 5) stops when no changes occur in the selection of center configurations and in the values of their parameters. The methodology leads to a reliable manufacturing network which can be used as an advanced basis for further investigation before its final deployment.

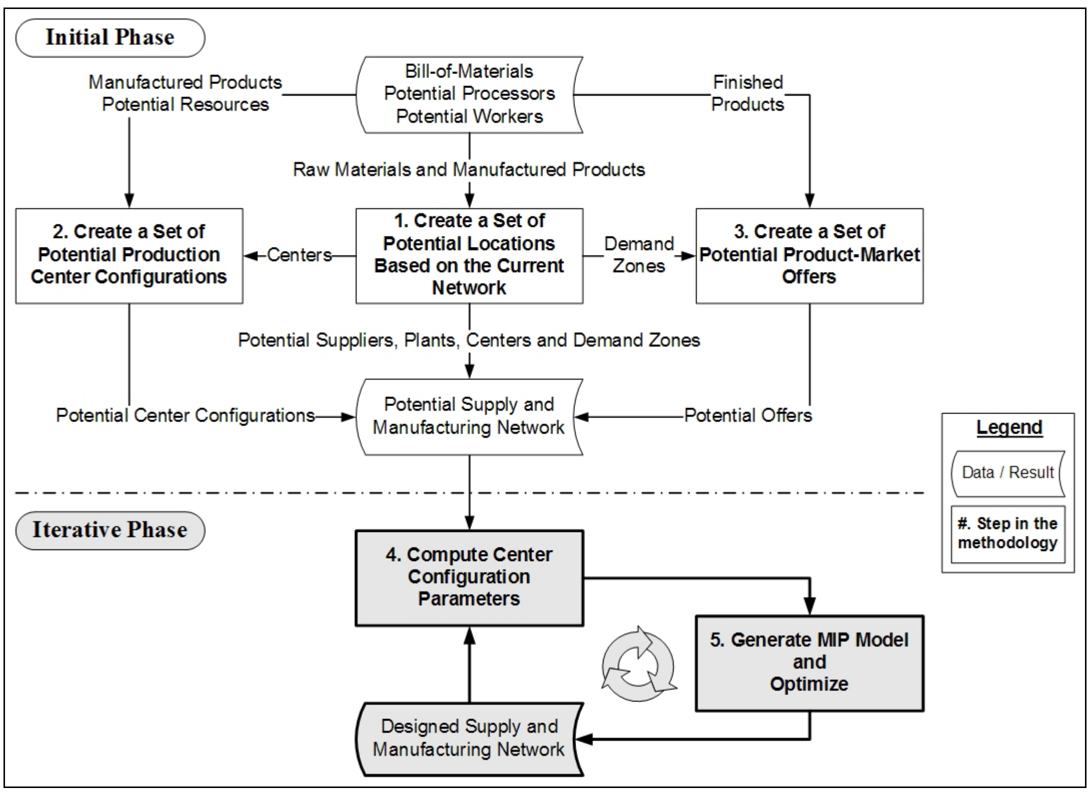

Figure 1: Manufacturing Network Design Methodology

3. Modeling Framework

The modeling framework elaborated in this section describes the components of the potential manufacturing network build in the initial phase of the methodology.

3.1. Products and Product-States

A product graph with the sequence of manufacturing operations for each product is illustrated for a simple case in Figure 2. Products 1, 2 and 3 are raw materials, products 4, 5, 6 and 7 are manufactured parts, and products 8 and 9 are finished products. A product-state graph is used to describe the production process of the product families. This graph is derived from the bill-of-materials of the products and their sequence of operations. A product-state $p \in P$ is defined as the state of a product when an operation is completed. Three types of product-states are distinguished:

P^R Raw material product-states ($P^R \subset P$).

P^M Manufactured product-states ($P^M \subset P$).

P^F Finished product-states ($P^F \subset P^M$).

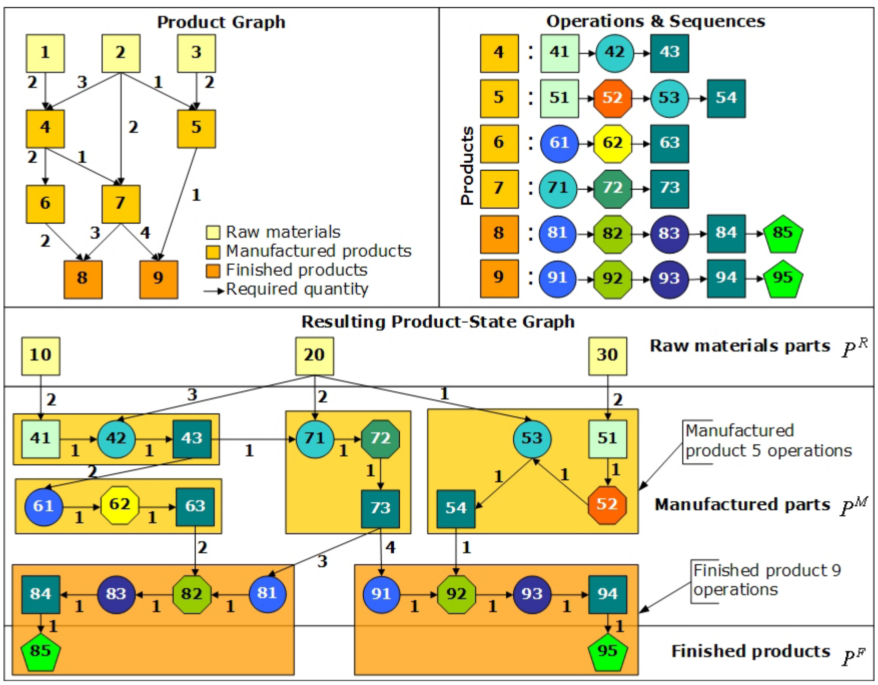

Figure 2: Product Graph, Operations & Sequences and Product-State Graph

All product-states in $P \setminus P^F$ are referred as parts. Parts 43, 54, 63 and 73 are the only manufactured product-states that can be stored and shipped between plants. The following notation is used to characterize elements of the product-state graph:

P_p Parts required in the manufacturing of product-state $p \in P^M$ ($P_p \subset P \setminus P^F$).

P_p^{-1} Manufactured product-states that require part $p \in P$ ($P_p^{-1} \subset P^M$).

$g_{pp'}$ Quantity of part $p \in P_{p'}$ required to make a unit of product-state $p' \in P^M$.

3.2. Potential Manufacturing Network

The manufacturing network is composed of existing and potential plants containing centers producing product-states. Some centers can stock raw materials and standard parts. The raw materials are supplied by a set of suppliers. Plants ship finished products directly to demand zones, which correspond to geographically aggregated customer locations. The various locations in the network are thus denoted as follows:

$l \in L$	Potential locations.
$v \in L^V$	Potential suppliers ($L^V \subset L$).
$u \in U$	Potential plants (plants are only containers for production centers).
$c \in L^C$	Potential centers ($L^C \subset L$). A center can be implemented in only one plant.
L_u^C	Set of centers which can be installed in plant $u \in U$ ($L_u^C \subset L^C$).
$d \in L^D$	Demand zones ($L^D \subset L$).

The following sets specify the capabilities or requirements of the locations:

L_p^V	Suppliers of raw material $p \in P^R$ ($L_p^V \subset L^V$).
L_p^C	Centers which can produce product-state $p \in P^M$ ($L_p^C \subset L^C$).
L_p^D	Demand zones requiring finished product $p \in P^F$ ($L_p^D \subset L^D$).

In order for a product to be allowed to visit a center more then one time, the set of nodes N correspond to location and product-state pairs (l, p) in the network. The set of arcs A corresponds to movements of parts between centers in a plant or to the transportation of product-states between locations. The set N includes the following types of nodes:

$(v, p) \in (L^V \times P^R)$	Raw material sourcing nodes ($(L^V \times P^R) \subset N$).
$(c, p) \in (L^C \times P^M)$	Production center nodes ($(L^C \times P^M) \subset N$).
$(d, p) \in (L^D \times P^F)$	Demand zone nodes ($(L^D \times P^F) \subset N$).

For a given design, a manufacturing network is represented by a directed graph (digraph) $E = (N, A)$, as illustrated in Figure 3.

The following additional sets are required to describe the digraph:

N_{lp}	Immediate predecessor nodes (l', p') of node $(l, p) \in N$, with $p' \in P_p$ ($N_{lp} \subset N$).
N_{lp}^{-1}	Immediate successor nodes (l', p') of node $(l, p) \in N$, with $p' \in P_p^{-1}$ ($N_{lp}^{-1} \subset N$).

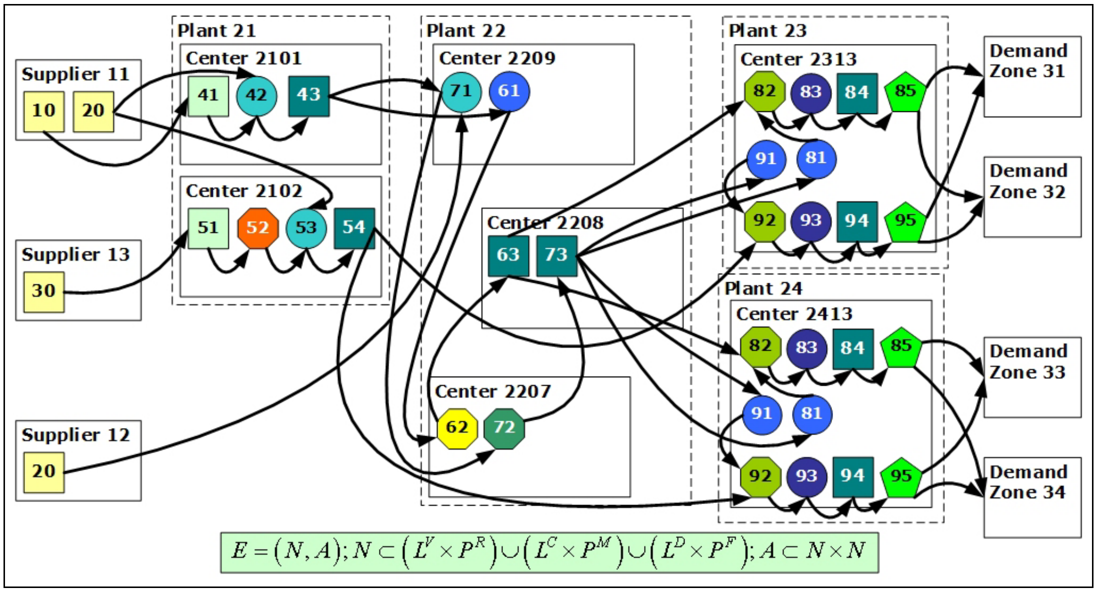

Figure 3: Digraph *E* of a Given Manufacturing Network

3.3. Potential Production Center Configurations

A potential production center is defined by a *mission* that specifies the product-states it is mandated to make, as shown in Figure 4. The mission of a center is defined by the following sets:

P_c^C Product-states manufactured by potential center $c \in L^C$ ($P_c^C \subset P^M$).

P_c^{CS} Product-states which can be shipped from potential center $c \in L^C$ ($P_c^{CS} \subset P_c^C$).

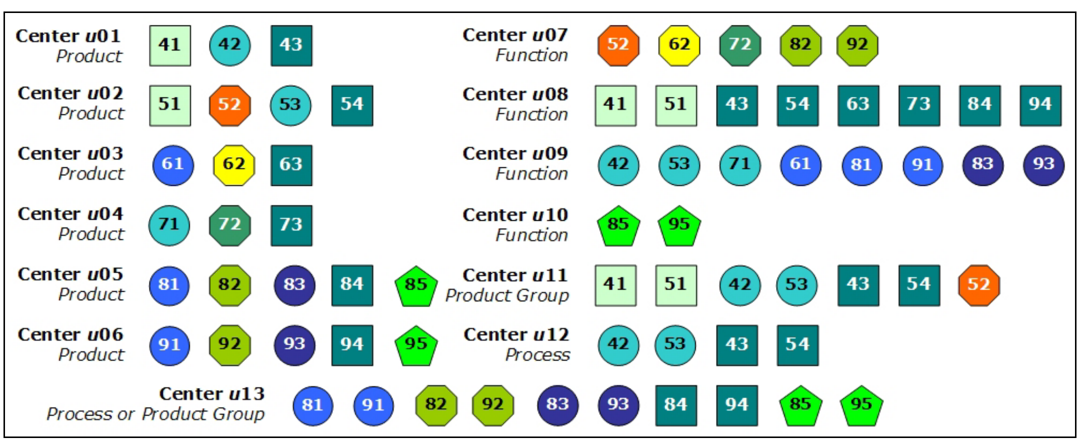

Figure 4: Example of Potential Production Center Missions

Centers can be classified as *Product Centers* (responsible to realize all operations on a product), *Function Centers* (responsible to realize a specific operation for all products), *Product Group Centers* (responsible to realize all operations on a set of products) or

*Process Center*s (responsible to realize a recurring sequence of operations on all products). A discussion of the various types of centers is available in Montreuil et al. (1998). The following sets are required to specify the focus of the center mission:

L^{CG} — Generic centers where different operations on one or more products are realized ($L^{CG} \subset L^{C}$). Typically for *Functions* centers.

L^{CS} — Specialized centers where sequences of operations on one or more products are realized ($L^{CS} \subset L^{C}$). Only the final product-states of the sequences can be ordered at the center. For *Product*, *Product Group* and *Process* centers.

A production center may have a set G_c of potential configurations *g*. However, only one configuration can be active for each selected center in a design. A configuration is first defined by a *resource configuration* that specifies the resource types, $r \in R$, which can be used to realize its mission. The following sets are required to characterize the types of resources used:

$m \in R^{M}$ — Processor types ($R^{M} \subset R$).

$w \in R^{W}$ — Worker types ($R^{W} \subset R$).

Different resources may be used to produce a given product-state, as shown in Figure 5.

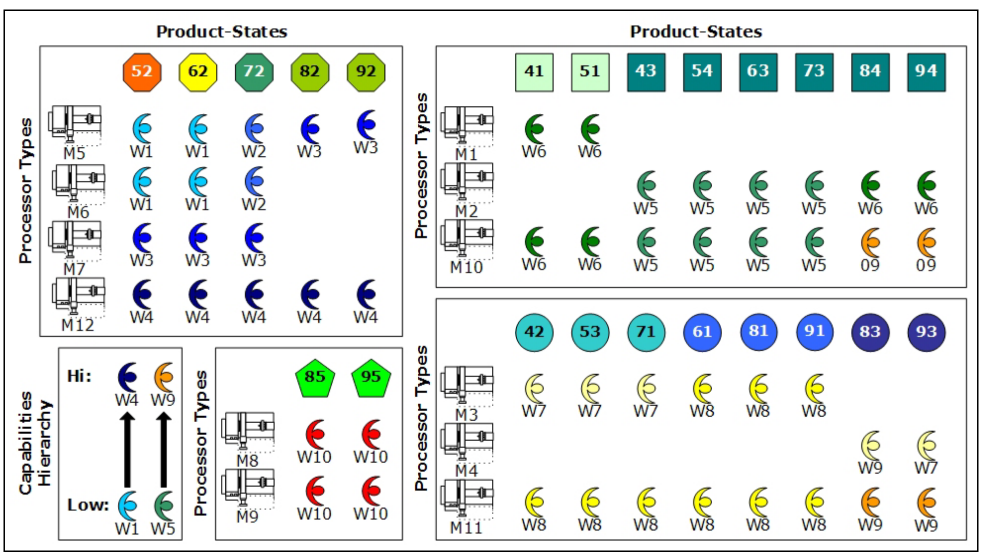

Figure 5: Potential Processor-Worker Combination Capabilities

A given resource can produce a limited set of product-states with distinct production times and costs. Highly qualified workers can perform the tasks of less qualified worker types (see Paquet et al. (2006)). For example, product-state 82 can be made by processor type M5 with worker types W3 or W4 and by processor type M12 with worker type W4. Workers can be laid-off, hired or transferred. The configuration specifies only the potential resource types that can be used in the center and not their quantity. The capacity consumption of the resources is assumed linear. A resource configuration is defined by the following sets:

R_{cg}^{MC} Processor types of center $c \in L^C$ for potential configuration $g \in G_c$ ($R_{cg}^{MC} \subset R^M$).

R_{cg}^{WC} Worker types of center $c \in L^C$ for potential configuration $g \in G_c$ ($R_{cg}^{WC} \subset R^W$).

The production times for the resource types are defined by these notations:

h_{wmp}^{M}, h_{wmp}^{W} Processor, and worker, production times for product-state $p \in P^M$ when processor type $m \in R^M$ is used by worker type $w \in R^W$.

A potential center configuration is characterized by a *service policy* defined by a production lead time target and a service level target applicable to all outgoing products treated by the center. These targets specify the desired center production lead time and its service level. The later is defined as the probability to produce the products within the lead time specified. For generic centers, the lead time is related to each single product-state produced in the center. For specialized centers, the lead time is related to the production of all product-states of a specific outgoing product. These targets are defined as follows:

τ_{cg}^{C} Production lead time target of center $c \in L^C$ under configuration $g \in G_c$.

α_{cg}^{C} Service level target (in %) of center $c \in L^C$ under configuration $g \in G_c$.

Many production centers can be developed with distinct mission and potential configurations. An example of a complete center configuration is shown in Figure 6.

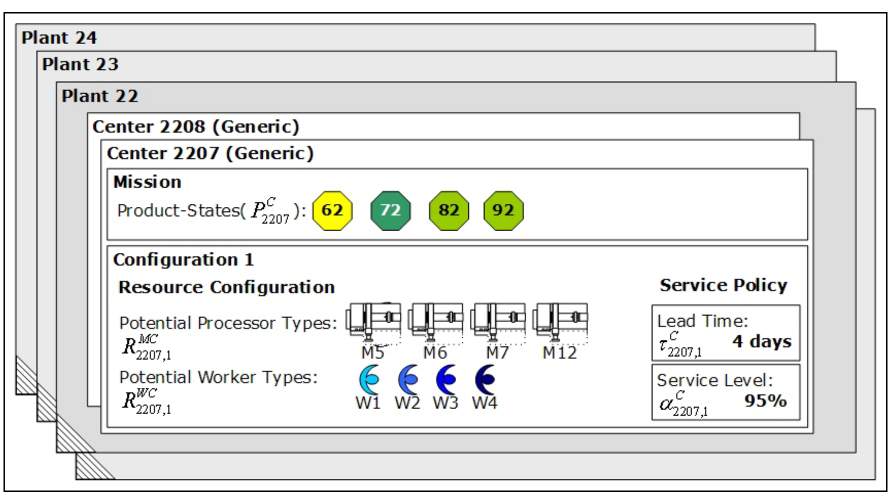

Figure 6: Example of a Potential Production Center Configuration

In the example of Figure 6, a generic center configuration is illustrated. Four types of processor and four types of worker are possible for the four product-states in the mission of the center. Since the center is generic, each product-state has a 4 days production lead time, with a 95% service level target, for this configuration.

3.4. Potential Product-Market Offers

A *product-market offer* corresponds to competitive characteristics that cause the customers to prefer the enterprise products to those of its competitors. The enterprise wants to determine which, among a set of potential offers, it should propose to its customers. For a product-market offer $o \in O_{dp}$ of finished product p for demand zone d, these competitive characteristics are the *order-to-delivery* (OTD) *time* for each demand zone, a *service level*, defined as the probability to respect the OTD time and the selling price of the finished product. Different OTD time and selling price may be offered to different demand zones but without loss of generality, we assume a unique service level for all demand zones. A product-market offer is described as follows:

τ_{dpo} Promised order-to-delivery time of offer $o \in O_{dp}$ for finished product $p \in P^F$ in demand zone $d \in L_p^D$.

α_{po} Promised service level of offer $o \in O_{dp}$ for finished product $p \in P^F$ for all demand zones.

s_{dpo} Selling price of product $p \in P^F$ at demand zone $d \in L_p^D$ under offer $o \in O_{dp}$.

The daily demand of finished product p for a demand zone d for a reference offer ($o = 0$) based on historical data, is assumed to be a stationary continuous random variable during a planning horizon of T days. The demand is also assumed independent and identically distributed for all demand zones. The demand processes are defined by these notations:

φ_{dpoi} Random demand of finished product $p \in P^F$ on day $i = 1, \ldots, T$ in demand zone $d \in L_p^D$, under offer $o \in O_{dp}$.

$f_{\varphi_{dpo}}(\cdot)$ Probability density function of φ_{dpoi}, $i = 1, \ldots, T$.

$F_{\varphi_{dpo}}(\cdot)$ Cumulative distribution function of φ_{dpoi}, with $F_{\varphi_{dpo}}(x) = P\left[\varphi_{dpo} \le x\right]$.

$\mu_{\varphi_{dpo}}$ Daily mean of the demand during the planning horizon of T days.

$\sigma^2_{\varphi_{dpo}}$ Daily variance of the demand during the planning horizon of T days.

The market response to an offer is characterized by a positive scale parameter (equal to 1 for the reference offer $o = 0$):

$\theta_{dpo} \geq 0$ Scale parameter reflecting the impact of offer $o \in O_{dp}$ on the demand of finished product $p \in P^F$ in demand zone $d \in L_p^D$ with respect to offer o = 0.

Any offer with better (worst) service than the reference offer has a scale parameter higher (lower) than 1. However, it is not realistic to try to evaluate a complete set of scale parameters. It is more suitable to develop offers for some probable scenarios. The use of a linear relationship between OTD time and demand is also reasonable (Yang and Geunes, 2004). The coefficient of variation $CV_{\varphi_{dpo}}$ of demand is kept constant for all offers for a given finished product and demand zone, as shown by Equation (1).

$$CV_{\varphi_{dpo}} = \frac{\sigma_{\varphi_{dpo}}}{\mu_{\varphi_{dpo}}} = \frac{\theta_{dpo}\sigma_{\varphi_{dp0}}}{\theta_{dpo}\mu_{\varphi_{dp0}}} = CV_{\varphi_{dp0}} = \text{constant} \tag{1}$$

The expected demands $\bar{x}_{dpo}$ of finished product $p \in P^F$ in demand zone $d \in L_p^D$ when $o \in O_{dp}$ is offered during the planning horizon are computed with Equation (2):

$$\bar{x}_{dpo} = T\theta_{dpo}\mu_{\varphi_{dp0}} = T\mu_{\varphi_{dpo}} \tag{2}$$

The order-to-delivery (OTD) time τ_{dpo} is composed of three different delay types:

τ_{lp}^O Order processing times (assumed deterministic) at supplier and center locations $l \in L_p^V \cup L_p^C$ for product-states $p \in P$.

$\tau_{ll'p}^T$ Transportation times (assumed deterministic) from supplier and center locations $l \in L_p^V \cup L_p^C$ to center and demand zone locations $l' \in L_p^C \cup L_p^D$ for products-states $p \in P$.

τ_{cg}^C Production lead times at center $c \in L^C$ with configuration $g \in G_c$.

Order processing times are assumed to be known for each product-state ordered from suppliers and by demand zones and centers. Shipping times for raw materials are known, i.e. it is assumed that suppliers ship products on time. Transportation times between plants and demand zones, and handling times between centers in a plant, are also assumed to be known. The only variable part is the lead time of production centers τ_{cg}^C. The capacity of the center must be sufficient to respect this lead time with a service level α_{cg}^C. When a product-market offer is selected, a combination of center configurations must be activated to make sure that the offer conditions are respected. A subdigraph E_{dp} of product $p \in P^F$ for demand zone $d \in L_p^D$, extracted from the designed network E, is used to implement an offer. Figure 7 presents the subdigraph to source and produce finished product 95 for demand zone 31. This subdigraph is extracted from the network of Figure 3. The demand is transmitted to the centers which produce the finished product. These centers order parts according to the product-state graph. In order to ensure that the promised OTD time is respected for the demand zone, the total lead time in the subdigraph is established. The computation of the total lead time starts from the demand zone node of the subdigraph and then moves recursively to all parent nodes until it finds sourcing nodes or part inventory at a decoupling point. The following notation describes the total lead time in the network:

T_{lp} Total lead time to obtain product-state $p \in P^M$ at location $l \in L_p^C \cup L_p^D$.

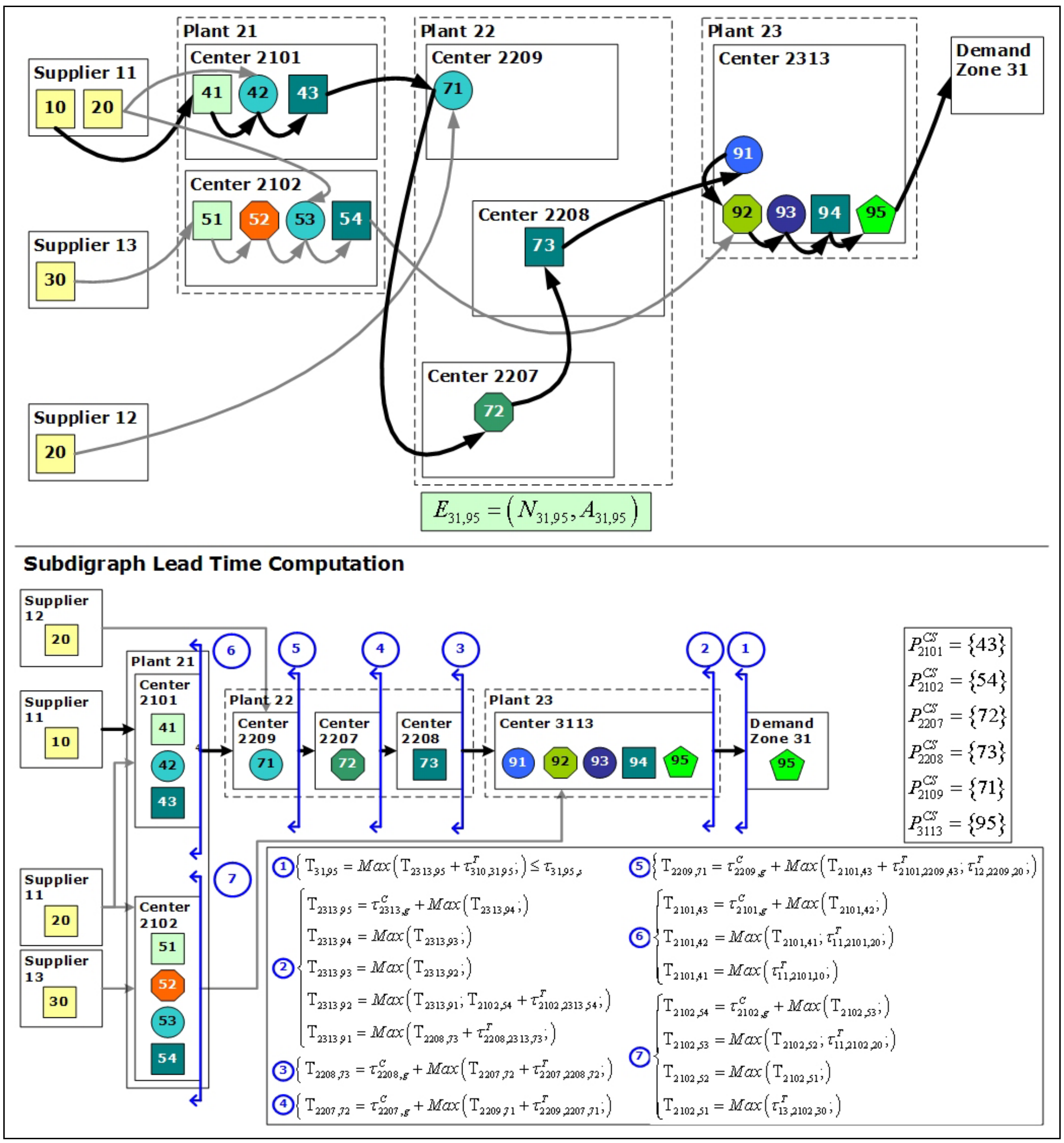

Figure 7: A Subdigraph in the Designed Manufacturing Network *E* and its Lead Time Computation

Equation (3) computes the maximum lead time for the shipping of finished product *p* to demand zone *d*. This lead time must be less than or equal to the promised OTD time. Equation (4) computes recursively the lead time for the production of product-state *p* at center *c*, including all shipping, production, transportation and handling of parts.

$$\mathrm{T}_{dp} = \underset{(l,p)\in N_{dp}}{Max}\left(\mathrm{T}_{lp} + \tau^{O}_{lp} + \tau^{T}_{ldp}\right) \le \tau_{dpo} \qquad \forall d \in L^{D}_{p}, p \in P^{F} \tag{3}$$

$$\mathrm{T}_{cp} = \begin{cases} \tau^{C}_{cg} + \underset{(l',c')\in N_{cp}}{Max}\left(\mathrm{T}_{l'p'} + \tau^{O}_{l'p'} + \tau^{T}_{l'cp'}\right) & \forall c \in L^{C}_{p}, p \in P^{CS}_{c} \\ \underset{(l',c')\in N_{cp}}{Max}\left(\mathrm{T}_{l'p'} + \tau^{O}_{l'p'} + \tau^{T}_{l'cp'}\right) & \forall c \in L^{CS}_{p}, p \in P^{C}_{c} \setminus P^{CS}_{c} \end{cases} \tag{4}$$

The longest path defined by Equations (3) and (4) corresponds to the *critical path* of the subdigraph. For a specific network, the maximum time available for production can be computed for all subdigraphs. This available production lead time must be distributed between the centers on the critical path of these subdigraphs. An assignment to the planned production lead time τ^{C}_{cg} at each center in the network must be realized simultaneously in all subdigraphs. For the example of Figure 7, the critical path of the subdigraph is represented by a set of bold arcs. This critical path starts from the demand zone and its associated lead time is represented by cut ①. The lead time is then recursively computed on the critical path from cut ② to cut ⑥. The number of possible lead time assignments at centers is a function of the available OTD time and the granularity of the centers lead time, usually a work shift or a day.

The lead time at a center allows a smoothing of the demand during this lead time. This smoothing decreases the variability of the demand at the center and permits to find an advantageous trade-off between capacity and lead time in the network (Ko et al., 2004). Equation (5) computes the resulting average daily demand during a lead time of *t* days.

$$\bar{\varphi}_{dpoi} = \frac{1}{t}\sum_{i=1}^{t}\varphi_{dpoi} \text{ with } \mu_{\bar{\varphi}_{dpo}} = \frac{1}{t}\sum_{i=1}^{t}\mu_{\varphi_{dpoi}} = \mu_{\varphi_{dpo}} \text{ and } \sigma^{2}_{\bar{\varphi}_{dpo}} = \frac{1}{t^{2}}\sum_{i=1}^{t}\sigma^{2}_{\varphi_{dpoi}} = \frac{1}{t}\sigma^{2}_{\varphi_{dpo}} \tag{5}$$

In order to give more time to centers which produce customized parts, it is possible to introduce an inventory of standard parts in some points of the network. These *decoupling points* (see Hadzilias (2005)) are defined as location (*c*, *p*) of center $c \in L^{C}_{p}$ stocking all parts $p' \in P_{p}$ required to make product-state *p*. A center can be a potential decoupling point for one or more of the product-states of its mission. Figure 8 illustrates a decoupling point defined by center 2209 and product-state 71 of the subdigraph presented at Figure 7. The decoupling point needs to keep all parts required to make product-state 71. Therefore, product-states 20 & 43 are kept in stock in order to remove the delays from supplier 12 and center 2101 (and upstream) from the total lead time computation. This changes the critical path of this subdigraph.

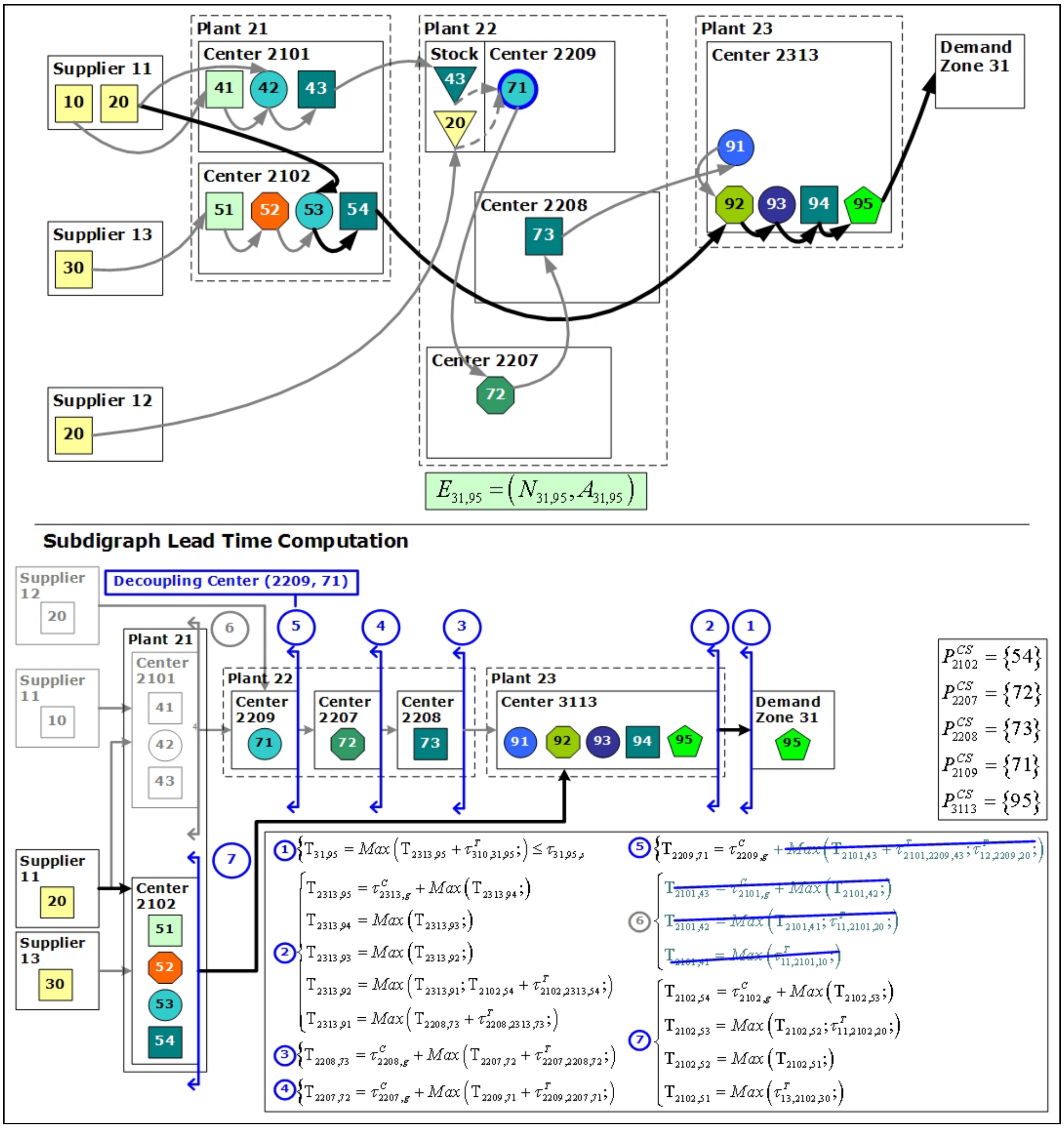

Figure 8: Example of a Decoupling Point and its Lead Time Computation

The methodology selects decoupling points to give more time to some centers in the digraph. This influences the capacity level at the centers and the stock level at the decoupling points. This helps reduce the lead time on the critical path and absorb the variability of the demand (Billington et al., 2004). This also eliminates the parts production and transportation delay from the computation of the finished product-state total lead time. A center is a decoupling point for a product-state in each subdigraph where it is present. The requirements of parts must be met directly from the on–hand inventory (Bashyam and Fu, 1998 and Bollapragada et al., 2004) and all required parts must be available in this stock (Benjaafar and ElHafsi, 2004). The selection of a decoupling point may modify the critical path in some subdigraph, as shown in Figure 8. For decoupling points, Equation (4) is replaced by Equation (6).

$$\mathrm{T}_{cp} = \tau_{cg}^{C} \quad \forall c \in L_p^C, p \in P_c^{CS} \tag{6}$$

We must then select a service level α_{cg}^{C} at each center in order to respect the global service level α_{po} promised to the demand zones. A higher service level will require a higher level of capacity. The promised service level is directly related to the capabilities of the centers along the critical path of each subdigraph to supply their successor nodes. At the end of a critical path, the resulting service level must be respected. The iterative phase of the methodology is used to ensure that the promised service level of each offer is respected.

4. Model Formulation

The optimization step of the iterative phase of the methodology is based on a mixed integer programming model used to obtain a robust initial manufacturing network. The first constraints of the model concern raw material procurement. The following decision variables and data are required to formulate these constraints:

X_{cwmp}^{P} Expected number of units of product-state $p \in P^M$ manufactured by center $c \in L_p^C$ with processor type $m \in R^M$ and worker type $w \in R^W$.

$X_{ll'p}^{T}$ Expected number of units of part or finished product $p \in P$ transported from location $l \in L_p^V \cup L_p^C$ to location $l' \in L^C \cup L_p^D$.

b_{vp}^{V} Upper bound on the amount of raw material $p \in P^R$ that can be provided by supplier $v \in L_p^V$.

The following constraints are required to ensure that the capacity of external suppliers is not exceeded and that the raw materials required are shipped to the centers.

$$\sum_{c\in L^C} X_{vcp}^T \le b_{vp}^V \quad \forall v\in L_p^V, p\in P^R \tag{7}$$

$$\sum_{w\in R^W}\sum_{m\in R^M}\sum_{p'\in P_p^{-1}} g_{pp'} X_{cwmp'}^P - \sum_{v\in L_p^V} X_{vcp}^T = 0 \qquad \forall c\in L_{p'}^C, p\in P^R \tag{8}$$

Constraints related to the manufacturing network configuration are also required. Let:

B_{cg}^G Binary variable equal to 1 if configuration $g \in G_c$ is used for center $c \in L^C$ and 0 otherwise.

B_{cp}^P Binary variable equal to 1 if product-state $p \in P^M$ is manufactured in center $c \in L_p^C$ and 0 otherwise.

B_u Binary variable equal to 1 if plant $u \in U$ is opened and 0 otherwise.

$B_u^{+/-}$ Binary variable equal to 1 if plant $u \in U$ is to be opened $^{(+)}$ or to be closed $^{(-)}$ and 0 otherwise.

M An arbitrary large number.

B_u^0 Parameter equal to 1 if plant $u \in U$ is initially opened and 0 otherwise.

Constraints (9) are related to the opening and closing of plants. Constraints (10) and (11) ensure that only one configuration is selected for an active center. Constraints (12) indicate if a product is manufactured at a center and constraints (13) ensure products flow equilibrium in plants.

$$B_u + B_u^- - B_u^+ = B_u^0 \qquad \forall u\in U \tag{9}$$

$$\sum_{g\in G_c} B_{cg}^G \le 1 \qquad \forall c\in L^C \tag{10}$$

$$\sum_{p\in P^M} B_{cp}^P - M\sum_{g\in G_c} B_{cg}^G \le 0 \qquad \forall c\in L^C \tag{11}$$

$$\sum_{w\in R^W}\sum_{m\in R^M} X_{cwmp}^P - M\left(B_{cp}^P\right) \le 0 \quad \forall c\in L_p^C, p\in P^M \tag{12}$$

$$\begin{aligned}\sum_{w\in R^W}\sum_{m\in R^M} X_{cwmp}^P + \sum_{c'\in L_{p'}^C} X_{c'cp}^T - \sum_{c'\in L_{p'}^C} X_{cc'p}^T - \sum_{d\in L_p^D} X_{cdp}^T \\ - \sum_{w\in R^W}\sum_{m\in R^M}\sum_{p'>p\in P^M} g_{pp'} X_{cwmp'}^P = 0 \quad \forall c\in L_p^C, p\in P^M\end{aligned} \tag{13}$$

The following constraints are related to the capacity provided by resources. Let:

Z_{cr} Integer number of resource of type $r \in R$ required in center $c \in L^C$.

Z_{crp} Time required from resources of type $r \in R$ for the manufacturing of product-state $p \in P^M$ in center $c \in L_p^C$.

Z_{cr}^O Overtime required in time units from resources of type $r \in R$ in center $c \in L^C$.

$Z_{uu'r}^T$ Number of resources of type $r \in R$ to transfer from plant $u \in U$ to plant $u' \in U$.

$Z_{ur}^{+/-}$ Number of resources of type $r \in R$ to add $^{(+)}$ or dispose $^{(-)}$ at plant $u \in U$.

Z_{cr}^0 Number of resources of type $r \in R$ initially available at center $c \in L^C$.

b_{cw}^O Maximum overtime that a worker of type $w \in R^W$ at center $c \in L^C$ can do during a period.

b_{cr}^R Capacity provided in time units by a resource of type $r \in R$ at center $c \in L^C$.

λ_{crg}^R Capacity inflation factor for resource type $r \in R$ at center $c \in L_p^C$ under configuration $g \in G_c$. This factor permits to adjust the capacity requirements related to the service policy of the center and the variability of the demand.

Constraints (14) and (15) compute resource requirements for each product-state. Constraints (16) establish a limit on the amount of overtime that can be done by a worker type. Constraints (17) ensure that the total time required does no exceed the capacity provided by each resource type. Parameter λ_{crg}^R is gradually adjusted during the iterative phase of the methodology. Constraints (18) ensure that resource movements, acquisitions and disposal are properly accounted.

$$\sum_{w \in R^W} h_{wmp}^M X_{cwmp}^P - Z_{cmp} = 0 \qquad \forall c \in L_p^C, m \in R_p^M, p \in P^M \tag{14}$$

$$\sum_{m \in R^M} h_{wmp}^W X_{cwmp}^P - Z_{cwp} = 0 \qquad \forall c \in L_p^C, w \in R^W, p \in P^M \tag{15}$$

$$Z_{cw}^O - b_{cw}^O Z_{cw} \le 0 \qquad \forall c \in L^C, w \in R^W \tag{16}$$

$$\lambda_{crg}^R \sum_{p \in P^M} Z_{crp} - b_{cr}^R Z_{cr} - Z_{cr}^O - M\left(1 - B_{cg}^G\right) \le 0 \qquad \forall c \in L^C, r \in R_c, g \in G_c \tag{17}$$

$$\sum_{c \in L_u^C} Z_{cr} + Z_{ur}^- - Z_{ur}^+ + \sum_{u' \in U} Z_{uu'r} - \sum_{u' \in U} Z_{u'ur} = \sum_{c \in L_u^C} Z_{cr}^0 \qquad \forall u \in U, r \in R \tag{18}$$

The next constraints are related to the demand. The following decision variable is required:

B_{dpo}^O Binary variable equal to 1 if product-market offer $o \in O_{dp}$ is active for finished product $p \in P^F$ and demand zone $d \in L_p^D$ and 0 otherwise.

Constraints (19) are expected demand constraints and constraints (20) ensure that one offer only is active for a finished product in a demand zone.

$$\sum_{c\in L_p^C} X_{cdp}^T - \sum_{o\in O_{dp}} \overline{x}_{dpo} B_{dpo}^O = 0 \qquad \forall p \in P^F, d \in L_p^D \tag{19}$$

$$\sum_{o\in O_{dp}} B_{dpo}^O = 1 \qquad \forall d \in L_p^D, p \in P^F \tag{20}$$

Constraints (21) to (26) are related to the computation of the lead times in the network and the OTD time of the offers. The following decision variables are required:

- B_{cp}^D Binary variable equal to 1 if center $c \in L_p^C$ is a decoupling point for product-state $p \in P^M$ and 0 otherwise.
- $B_{ll'p}^T$ Binary variable equal to 1 if product-state $p \in P$ is transported from location $l \in L_p^V \cup L_p^C$ to location $l' \in L^C \cup L_p^D$ and 0 otherwise.
- T_{lp} Maximum lead time to supply product-state $p \in P^M$ at location $l \in L^C \cup L_p^D$, including the production and shipping of its required parts.

Constraints (21) allow transportation on an arc only if this arc is selected. Constraints (22) to (25) compute the total lead time for product-states as described in section 3.4. Constraints (22) compute the lead time at centers when the parts required are shipped from suppliers while constraints (23) compute the lead time when the parts are shipped from other centers. Constraints (24) are for decoupling point locations. Constraints (25) compute the total lead time to demand zones and constraints (26) ensure that this total lead time respects the required OTD time of the selected offers.

$$X_{ll'p}^T - M\left(B_{ll'p}^T\right) \le 0 \qquad \forall l \in L_p^V \cup L_p^C, l' \in L^C \cup L_p^D, p \in P \tag{21}$$

$$\begin{aligned} &\mathrm{T}_{cp} - \tau_{cg}^C B_{cg}^G - \tau_{vcp'}^T B_{vcp'}^T + M\left(B_{cp}^D\right) + M\left(1 - B_{cp}^P\right) \ge \tau_{vp'}^O \\ &\qquad \forall c \in L_p^C, p \in P^M, g \in G_c, v \in L_{p'}^V, p' \in P^R \cap P_p^{-1} \end{aligned} \tag{22}$$

$$\begin{aligned} &\mathrm{T}_{cp} - \tau_{cg}^C B_{cg}^G - \mathrm{T}_{c'p'} - \tau_{c'cp'}^T B_{c'cp'}^T + M\left(B_{cp}^D\right) + M\left(1 - B_{cp}^P\right) \ge \tau_{c'p'}^O \\ &\qquad \forall c \in L_p^C, p \in P^M, g \in G_c, c' \in L_{p'}^C, p' \in P^M \cap P_p^{-1} \end{aligned} \tag{23}$$

$$\begin{aligned} &\mathrm{T}_{cp} - \tau_{cg}^C B_{cg}^G + M\left(1 - B_{cp}^D\right) + M\left(1 - B_{cp}^P\right) \ge \tau_{l'p'}^O \\ &\qquad \forall c \in L_p^C, p \in P^M, g \in G_c, c' \in L_{p'}^C \cap L_{p'}^V, p' \in P_p^{-1} \end{aligned} \tag{24}$$

$$\mathrm{T}_{dp} - \mathrm{T}_{cp} - \tau_{cdp}^T B_{cdp}^T \ge \tau_{cp}^O \qquad \forall c \in L_p^C, d \in L_p^D, p \in P^F \tag{25}$$

$$\mathrm{T}_{dp} - \sum_{o\in O_{dp}} \tau_{dpo} B_{dpo}^O \le 0 \quad \forall d \in L_p^D, p \in P^F \tag{26}$$

The last constraints are related to inventory and space requirements in the plants. The following decision variables and data are required:

$X^{I}_{p'cp}$ Expected inventory, including the safety stocks, in units of part $p \in P_{p'}$ stocked at center $c \in L^{C}_{p'}$ which are required to manufacture product-state $p' \in P^{M}$.

e_u Total space available for processors and stocks in plant $u \in U$.

e^{M}_{m} Working and buffer space required by a processor of type $m \in R^{M}$.

e^{P}_{p} Space required by one storage unit (a pallet for example) of part $p \in P$.

i_p Number of unit of product-state $p \in P$ to make one storage unit of $p \in P$.

$\lambda^{I}_{p'cpg}$ Inventory factor for part $p \in P_{p'}$ of product-state $p' \in P^{M}$ at center $c \in L^{C}_{p'}$ with configuration $g \in G_c$ used at decoupling point (c, p'). This factor permits to adjust the total inventory requirements related to the service policy of the center configuration and the variability of the demand.

Constraints (27) compute the required inventory at center c, including the safety stock, for the parts of a specific product p in order to eliminate the lead time upstream of the decoupling point (c, p). Parameter $\lambda^{I}_{p'cpg}$ is adjusted during the iterative phase of the methodology. Constraints (28) ensure that the space available in each plant is not exceeded.

$$X^{I}_{p'cp} - \lambda^{I}_{p'cpg} g_{pp'} \sum_{w \in R^{W}} \sum_{m \in R^{M}} X^{P}_{cwmp'} + M\left(1 - B^{D}_{cp'}\right) + M\left(1 - B^{G}_{cg}\right) \geq 0 \qquad \forall p \in P^{I}_{p'}, c \in L^{C}_{p'}, p' \in P^{M}, g \in G_c \tag{27}$$

$$\sum_{c \in L^{C}_{u}} \sum_{m \in R^{M}} e^{M}_{m} Z_{cm} + \sum_{p \in P^{I}_{p'}} \sum_{c \in L^{C}_{p'} \cap L^{C}_{u}} \sum_{p' \in P^{C}_{c}} \left(e^{P}_{p} / i^{P}_{p}\right) X^{I}_{pcp'} - e_u B_u \leq 0 \qquad \forall u \in U \tag{28}$$

The model's objective function computes the expected net revenues from the selected product-market offers and their corresponding demand, as well as the cost of the network design (utilization, opening and closing of plants; production, inventory and transportation of products; utilization, overtime, adding, disposing and transfer of resources). Without loss of generality, the transition costs and the variable operating costs are assumed to be paid during the planning period and the fixed costs are assumed to cover devaluation, opportunity costs and fixed operating costs. The following cost data are thus required to formulate the objective function:

a_u Fixed cost associated to the use of plant $u \in U$.

$a^{+/-}_{u}$ Fixed cost associated to the opening $^{(+)}$ or closing $^{(-)}$ of plant $u \in U$.

o_{cr}	Fixed cost associated to the use of a resource of type $r \in R$ at center $c \in L^C$.
$o_{ur}^{+/-}$	Unit cost to add $^{(+)}$ or remove $^{(-)}$ a resource of type $r \in R$ at plant $u \in U$.
o_{cr}^{O}	Variable cost associated to the use in time units of a resource of type $r \in R$ at center $c \in L^C$ in overtime.
$o_{uu'r}^{T}$	Unit cost to relocate a resource of type $r \in R$ from plant $u \in U$ to $u' \in U$.
c_{cp}^{I}	Unit inventory holding cost of a storage unit (a pallet for example) of part p at center c.
c_{cp}^{P}	Unit production cost of product-state $p \in P^M$ at center $c \in L_p^C$ (excluding resource costs).
$c_{ll'p}^{T}$	Unit transportation cost of part or finished product $p \in P$ from location $l \in L_p^V \cup L_p^C$ to location $l' \in L^C \cup L_p^D$.

Therefore, the problem can be formally represented by the following MIP model:

Maximize total expected operating profits:

$$\begin{aligned}
&\sum_{p\in P^F}\sum_{d\in L_p^D}\sum_{o\in O_{dp}} s_{dpo}\bar{x}_{dpo}B_{dpo}^{O} - \sum_{u\in U} a_u B_u - \sum_{u\in U} a_u^{+} B_u^{+} - \sum_{u\in U} a_u^{-} B_u^{-} \\
&-\sum_{c\in L_p^C}\sum_{w\in R^W}\sum_{m\in R^M}\sum_{p\in P^M} c_{cp}^{P} X_{cwmp}^{P} - \sum_{c\in L_{p'}^C}\sum_{p\in P_{p'}}\sum_{p'\in P^M} c_{cp}^{I} X_{p'cp}^{I} - \sum_{l\in L_p^V\cup L_p^C}\sum_{l'\in L^C\cup L_p^D}\sum_{p\in P} c_{ll'p}^{T} X_{ll'p}^{T} \\
&-\sum_{c\in L^C}\sum_{r\in R}\left(o_{cr}Z_{cr} + o_{cr}^{O}Z_{cr}^{O}\right) - \sum_{u\in U}\sum_{r\in R}\left(o_{ur}^{+}Z_{ur}^{+} + o_{ur}^{-}Z_{ur}^{-}\right) - \sum_{u\in U}\sum_{u'\in U}\sum_{r\in R} o_{uu'r}^{T} Z_{uu'r}^{T}
\end{aligned} \tag{29}$$

Subject to:

Constraints (7) to (28)

All variables ≥ 0, binary and integers as described in the notation

5. Solution Method

The solution method is related to the iterative phase of the methodology of Figure 1. This phase is used to compute the capacity inflation factor λ_{crg}^{R} of constraints (17) and the inventory factor $\lambda_{p'cpg}^{I}$ of constraints (27) of the model formulation. Figure 9 presents the detailed iterative phase algorithm. The first step initialize the values of these factors and sets the iteration count to $j = 0$. The MIP model (29) is generated and optimized with a state-of-the art solver. The resulting network E^0 is then analyzed. Product demands are simulated in the network to verify that the service levels associated to the selected offers are respected and, if necessary, new values of the factors are computed for the next iteration. This step is currently implemented in a spreadsheet and takes between 5 to 25 minutes to execute. These times can be reduced if implemented in a specialized environment. A new network E^j is then obtained by resolving the model. The iterative process stops when the resulting network E^j is stable (same plants, centers, decoupling points, resources, and capacity and inventory factors) between two iterations. When good starting values of the capacity and inventory factors can be computed, a small number of iterations is required.

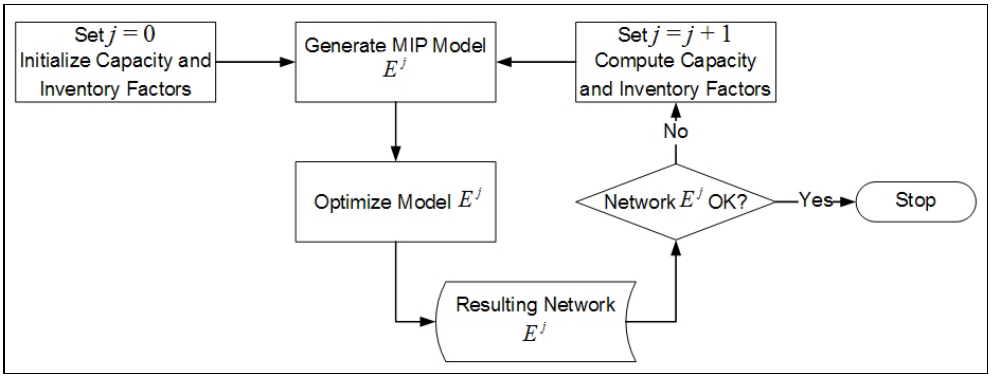

Figure 9: Iterative Phase Algorithm

In order to compute the resource capacity requirements for each center at iteration *j*, the effective demand at centers must be specified, starting from the downstream centers and recursively to all centers in the network. The daily random demand γ_{cpgi} at each downstream center is a linear combination of the random demand φ_{dpoi} for each demand zone *d* that the center *c* is responsible to supply in product-state *p* in the current network E^j. The following notation is required to characterize the demand at the centers:

γ_{cpgi}	Average random demand of product-state $p \in P^M$ for day *i* at center $c \in L_{up}^C$ with configuration $g \in G_c$, when a lead time of τ_{cg}^C days applies.
$\mu_{\gamma_{cpg}}$	Daily mean of the demand during the production lead time of τ_{cg}^C days. These means are a function of the variables X^P and X^T in the optimization model.
$\sigma_{\gamma_{cpg}}^2$	Daily variance of the demand during the lead time of τ_{cg}^C days.

Under a product-market offer *o*, the resulting demand and its mean and variance during the lead time are computed with Equations (30) and (31), first for downstream centers and then for all other centers following the product-state graph.

$$\gamma_{cpgi} = \begin{cases} \frac{1}{\tau_{cg}^C} \sum_{t=1}^{\tau_{cg}^C} \sum_{(d,p)\in N_{cp}^{-1}} \varphi_{dpot} & \forall p \in P^F \\ \frac{1}{\tau_{cg}^C} \sum_{t=1}^{\tau_{cg}^C} \sum_{(c',p')\in N_{cp}^{-1}} g_{pp'} \gamma_{c'p'g't} & \text{otherwise} \end{cases} \tag{30}$$

$$\mu_{\gamma_{cpg}} = \begin{cases} \sum_{(d,p)\in N_{cp}^{-1}} \sum_{o\in O_p} \mu_{\gamma_{dpo}} & \forall p \in P^F \\ \sum_{(c',p')\in N_{cp}^{-1}} g_{pp'} \mu_{\gamma_{c'p'g'}} & \text{otherwise} \end{cases} \qquad \sigma^2_{\gamma_{cpg}} = \begin{cases} \frac{1}{\tau_{cg}^C} \sum_{(d,p)\in N_{cp}^{-1}} \sum_{o\in O_p} \sigma^2_{\varphi_{dpo}} & \forall p \in P^F \\ \frac{1}{\tau_{cg}^C} \sum_{(c',p')\in N_{cp}^{-1}} g^2_{pp'} \sigma^2_{\gamma_{c'p'g'}} & \text{otherwise} \end{cases} \tag{31}$$

It is assumed that the centers can schedule their work so that the daily demand during a lead time is equal to the average daily demand. Therefore, the enterprise scheduling skills decrease the variability of the demand. This variability is also decreased by the smoothing during the lead time and by the pooling from many sources (Billington et al., 2004, Benjaafar et al., 2005).

For the current assignments of product-states to resource types in network E^j, the capacity, expressed in terms of time requirements, can be computed for each resource type at each active center. The following notations are used to describe these requirements:

γ^R_{crgi}	Random requirements in time units on resource type $r \in R$ ($m \in R^M$ or $w \in R^W$) for day i at center $c \in L^C_{up}$ with configuration $g \in G_c$.
$F^{-1}_{\gamma^R_{crg}}(\cdot)$	Inverse of the cumulative distribution function of γ^R_{crgi}.
$\mu_{\gamma^R_{crg}}$	Daily mean of the requirements.
$\sigma^2_{\gamma^R_{crg}}$	Daily variance of the requirements.

The requirements for the resource types (processors $m \in R^M$ and workers $w \in R^W$) are computed with Equation (32) and the mean and variance with Equation (33).

$$\gamma^R_{cmgi} = h^M_{wmp} \gamma_{cpgi}, \qquad \gamma^R_{cwgi} = h^W_{wmp} \gamma_{cpgi} \tag{32}$$

$$\begin{aligned} \mu_{\gamma^R_{cmg}} &= h^M_{wmp} \mu_{\gamma_{cpg}}, & \mu_{\gamma^R_{cwg}} &= h^W_{wmp} \mu_{\gamma_{cpg}} \\ \sigma^2_{\gamma^R_{cmg}} &= \left(h^M_{wmp}\right)^2 \sigma^2_{\gamma_{cpg}}, & \sigma^2_{\gamma^R_{cwg}} &= \left(h^W_{wmp}\right)^2 \sigma^2_{\gamma_{cpg}} \end{aligned} \tag{33}$$

Therefore, the required number Z_{cr} of resource type r at center c corresponds to the inverse of the cumulative distribution function of its requirements based on the service level of the active center configuration, divided by the time provided by a single resource of this type, as computed with Equation (34). This capacity can be expressed in terms of a *capacity inflation factor* and the mean of the distribution function of the requirements. If the actual mean of the distribution at a center is different than the mean used, the value of the factor will be recomputed for the next iteration and the iterative phase will continue until the difference between the two values is considered acceptable. The factor λ_{crg}^{R} can then be computed with Equation (35) and inserted in constraints (17) for the next iteration.

$$Z_{cr} = \frac{1}{b_{cr}^{R}} F_{\gamma_{crg}^{R}}^{-1}\left(\alpha_{cg}^{C}\right) = \frac{1}{b_{cr}^{R}} \lambda_{crg}^{R} \mu_{\gamma_{crg}^{R}} \tag{34}$$

$$\lambda_{crg}^{R} = F_{\gamma_{crg}^{R}}^{-1}\left(\alpha_{cg}^{C}\right) \Big/ \mu_{\gamma_{crg}^{R}} \quad \text{(default value set to 1.00)} \tag{35}$$

For the decoupling points, it is assumed that the enterprise keeps enough stock to be able to cover production during the parts replenishment delay (order processing time, production lead time and transportation time) for a specified service policy. It is also assumed that the enterprise has historical data on its inventory levels and demands from which it can derive empirical inventory turnover functions (see Ballou (2000)). Therefore, the *inventory factor* $\lambda_{p'cpg}^{I}$ of a decoupling point results from the inventory policy of the enterprise as reflected by the turnover ratio of each stocked part. The demand during the replenishment delay and the required stock are computed with Equation (36) and are defined by these notations:

$\gamma_{cp'glpi}^{I}$	Average random demand of product-state $p' \in P^{M}$ for day i at center $c \in L_{p'}^{C}$ with configuration $g \in G_{c}$ during the replenishment delay (order processing, production and transportation) of part $p \in P_{p'}$ from location $l \in L_{p}^{V} \cup L_{p}^{C}$.
$h(\mu,\tau,\alpha)$	Inventory turnover function based on the historical data of the enterprise. This function, developed by regression for specific replenishment delays τ and service levels α, return the average inventory required to support a given throughput μ (the demand at a center for a specific part). See Ballou (2000) for details and examples of inventory turnover functions.

$$\gamma_{cp'glpi}^{I} = g_{pp'} \sum_{t=1}^{\tau_{lp}} \gamma_{cpg't} \qquad \forall p \in P_{p'}, \text{ with } \tau_{lp} = \begin{cases} \tau_{lp}^{O} + \tau_{lg}^{C} + \tau_{lcp}^{T} & \forall l \in L_{p}^{C} \\ \tau_{lp}^{O} + \tau_{lcp}^{T} & \forall l \in L_{p}^{V} \end{cases} \tag{36}$$

The required inventory at the decoupling point is computed with Equation (37) and the inventory factor $\lambda^I_{p'cpg}$ to be used in constraints (27) is computed with Equation (38).

$$X^I_{p'cp} = \lambda^I_{p'cpg}\mu_{\gamma^I_{cp'glp}} = g_{pp'}\lambda^I_{p'cpg}\mu_{\gamma_{cp'g}} \text{ (according to the turnover curve)} \quad (37)$$

$$\lambda^I_{p'cpg} = \mu_{\gamma^I_{cp'glp}} \Big/ h(\mu_{\gamma^I_{cp'glp}}, \tau_{lp}, \alpha^C_{cg}) \text{ (default value set to 0.00)} \quad (38)$$

In order to respect the service level of the offer α_{po}, the service level α^C_{cg} of some center configurations can be modified during the simulation of the demand in the algorithm, leading to new values of the capacity inflation factor and the inventory factor.

6. Experimental Investigation

The objective of the experimental investigation is to demonstrate the feasibility of the approach and to develop managerial insights for typical uses of the model as a strategic decision making tool. The purpose of the methodology is to help enterprises in the redeployment of their existing manufacturing network. The methodology provides a robust feasible design which can be used as a good basis for further investigations. Therefore, the proposed methodology helps the designer in the testing of different supply chain strategies, under different scenarios, to generate efficient reliable networks in a what-if approach without necessarily redefining all aspects of the current network simultaneously. In our experiments, the models are solved with the CPLEX 10.0 solver (ILOG, 2006) on an AMD Athlon MP 2600+ processor. The simulator is built in VBA with Microsoft Excel 2003.

The characteristics of the initial manufacturing network used to validate and illustrate the design approach are given in Table 1.

Table 1: Starting Network Configuration

Initial Network Configuration	
Plants & Centers	Plant 21: Centers 2101, 2102 Plant 22: Centers 2207, 2208, 2209 Plant 23: Center 2313 Plant 24: Center 2413
Resources	Plant 21: 181 processors (4 types) & 181 workers (4 types) Plant 22: 132 processors (4 types) & 136 workers (6 types) Plant 23: 16 processors (5 types) & 15 workers (5 types) Plant 24: 36 processors (6 types) & 36 workers (5 types)

The initial manufacturing network is presented in Figure 3 and in Figure 10. The product-state graph for the case considered is presented in Figure 2 and the potential center missions and resources are those presented in Figure 4 and Figure 5. In this network, plant 21 is configured for the manufacturing of products 4 & 5 with *product centers* while plant 22 produce parts 6 & 7 with *function centers*. Plants 23 & 24 produce finished products 8 & 9, each with a single *product group center*.

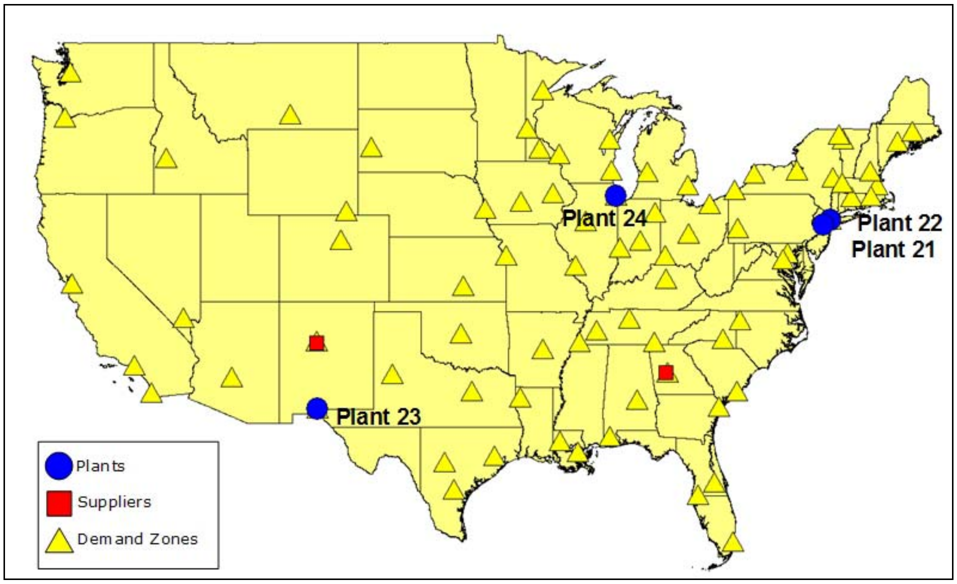

Figure 10: Initial Manufacturing Network

Using this network as a basis, different types of strategies are tested to illustrate the effect of lead times, service levels, and decoupling points on the required capacity deployment and profitability of the network. Source-to-order (STO) strategies without inventory, make-to-order (MTO) strategies using raw material inventories, and assemble-to-order (ATO) strategies with raw material and standard part inventories are tested. Strategies are also defined by potential product-market offers (selling prices, OTD times and service levels) and by the potential network used (plants, centers and center configurations with potential resources, lead times and service levels). For each offer, different sales response scenarios (values of the scale parameter θ_{dpo} for each demand zone) are tested. For example, in a scenario, a demand zone can be interested by low prices, but another by short OTD times and high service levels. The business scenario parameters considered are presented in Table 2. Not all combinations of these parameter values are tested; only those leading to meaningful what-if scenarios are considered. For the first sets of experiments, the potential modifications to the manufacturing network were limited. Consequently, for these scenarios, it was possible to obtain good initial parameter values in the first step of the method (capacity and inventory factors initialization) using factor values from the preceding experiments. With these good starting values, the method converges in only two iterations.

Table 2: Values considered for Business Scenario Parameters

Parameter	Parameter Values Considered
Production	Assemble-To-Order, Make-To-Order, Source-To-Order
OTD Time	14 days or less, 17 days or less, 20 days or less
Service Level	85%, 95%, 99.9%
Price	Conservative, Aggressive (15% discount)
Demand Level	Realistic, Pessimistic (15% decrease), Optimistic (15% increase)

The first experiments illustrate the effects of the parameters of Table 2 on the resulting operating profits. Figure 11 summarizes the results obtained, with offers involving 14, 17 and 20 days OTD time, aggressive pricing and a realistic demand level.

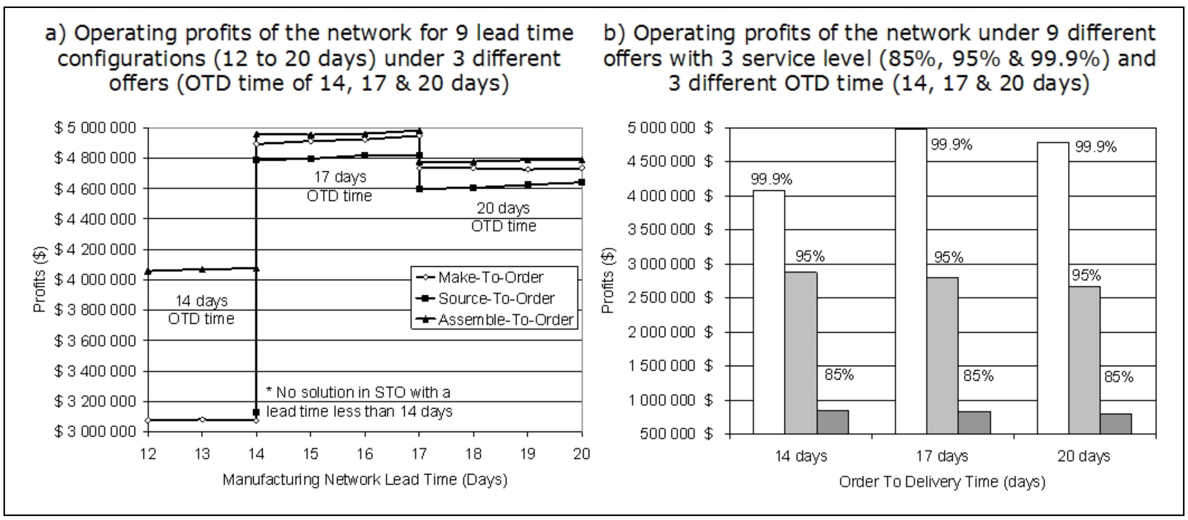

Figure 11: Effects of the Order-To-Delivery Time and Service Level on Operating Profits

In Figure 11a), the service level is set to 99.9% and the network lead time is set to nine different values, from 20 down to 12 days. It shows, as could be expected, that configuring the network lead times to values lower than the OTD time offered to customers is never optimal. When STO configurations only are used, the lead times at each center must be kept low to respect offers, which require more resources. Short OTD times (12 & 13 days) cannot be offered with this strategy. When MTO configurations are considered, a decoupling point (to stock raw materials 10 & 20) is introduced at center 2101 for product-states 41 and 42. These stocks are used to supply the center directly without waiting for suppliers. This gives more time to centers on the critical path, thus reducing their capacity requirements or allowing the network to respect shorter OTD times. Finally, when ATO configurations are considered, centers 2313 and 2413 become decoupling points for product-states 81 and 82 (to stock parts 63 & 73).

Figure 11b) presents the discounted operating profits for nine offers with three service levels and three OTD time for ATO configurations. When the service level is decreased from 99.9% to 85%, the demand and the resources required are reduced. Therefore, the total cost of the network is decreased but it yields lower profits. The profits are maximized ($ 4 982 514) for 17 days at 99.9% offers. Table 3 presents two networks related to Figure 11a).

Table 3: Sample Networks with Make-To-Order and Assemble-To-Order Configurations

Data	MTO – 14 days		ATO – 17 days	
Processors	Plant 21: 12 added Plant 22: 67 added Plant 24: 15 added	94 processors added	Plant 21: 27 added Plant 22: 18 added Plant 24: 12 added	57 processors added
Workers	Plant 21: 8 added Plant 22: 71 added Plant 23: 1 added Plant 24: 15 added	95 workers added	Plant 21: 26 added Plant 22: 20 added Plant 24: 10 added	56 workers added
Inventory	–		Part 63 (2 plants): 3542 units Part 73 (2 plants): 4528 units	
Revenues	$ 51 019 828		$ 49 736 075	
Profits	$ 3 073 955		$ 4 982 514	
Problem Size	2218 constraints 2285 variables (455 binaries)		2266 constraints 2295 variables (461 binaries)	
Simulation Time	7 min. per iteration		7 min. per iteration	
Solution Time	2 iterations: 191 + 159 = 350 s.		2 iterations: 178 + 132 = 310 s.	

The first network corresponds to make-to-order production with short lead times (14 days). The second network has medium lead time (17 days) and use part stocks. The first offer generates higher revenues, but the MTO system does not allow inventory and requires more capacity, especially with a short lead time. Therefore, this network has very low profits compared to the second network, which generates an additional $ 1 908 559 profit. Overall, offers at 20 days cannot capture enough market shares to be interesting, while offers at 14 days are too short for the manufacturing network to perform well. The best compromise is to make 17 day offers with an assemble-to-order strategy.

Figure 12 illustrates the effects of some of the scenarios defined in Table 2. These scenarios are for ATO configurations with a 99.9% service level. Figure 12a) shows the effect of the pricing on the network. The customers are more responsive to lower prices than shorter OTD times in these scenarios. It is apparent that a conservative pricing strategy in not advantageous in these cases, especially for long lead times.

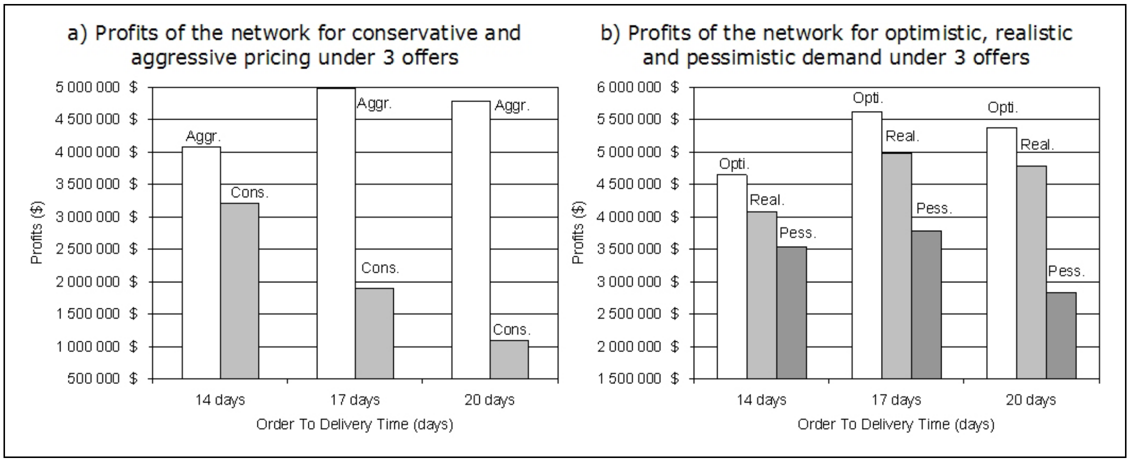

Figure 12: Effects of the Pricing and Demand Level on Operating Profits

Figure 12b) uses aggressive pricing and investigates different demand level scenarios. The realistic and optimistic demand levels correspond, respectively, to 5% and 21% increases over to the current demand. The pessimistic demand level corresponds to an 11% current demand decrease. The optimistic demand scenario generates more profits, but also requires more resources. Table 4 presents some aspects of two of the manufacturing networks of Figure 12b). Under the pessimistic scenario, only slight modifications to the initial manufacturing network are required but the profit drops ($ 3 779 307 vs. $ 4 982 514 for the realistic scenario). However, under the optimistic scenario, which adds 10% profits ($ 5 616 289), significant changes to the network resources and inventories are required. The three scenarios also lead to very different plant layouts. This indicates that the design is sensitive to the demand level and further analysis would be required to determine the best network to implement.

Table 4: Sample Pessimistic and Optimistic Networks with Assemble-To-Order Configurations

Data	ATO – 17 days – Pessimistic		ATO – 17 days – Optimistic	
Processors	Plant 21: 1 added Plant 23: 1 added Plant 24: 1 added	***3*** processors added	Plant 21: 58 added Plant 22: 43 added Plant 24: 19 added	***120*** processors added
Workers	Plant 21: 1 added Plant 23: 1 added Plant 24: 1 added	***3*** workers added	Plant 21: 54 added Plant 22: 46 added Plant 24: 19 added	***119*** workers added
Inventory	Part 63 (1 plant): 2888 units Part 73 (1 plant): 3713 units		Part 63 (2 plants): 3761 units Part 73 (2 plants): 4745 units	
Revenues	$ 42 277 957		$ 57 186 284	
Profits	$ 3 779 307		$ 5 616 289	
Problem Size	2266 constraints & 2295 variables (461 binaries)			
Simulation Time	7 min. per iteration		7 min. per iteration	
Solution Time	2 iterations: 16 + 39 = 55 s.		2 iterations: 68 + 94 = 162 s.	

The next experiments examine the effect of the demand variability on the resulting network. A higher value of the variance of the historical demand (the average coefficient of variation is 0.51 compared to 0.39) is used for these tests with the same offers and revenues. As could be expected, this greater variability requires more resources and thus decreases overall profits, as shown in Figure 13. This effect is slightly amplified for the STO strategy and the profit difference between high CV and medium CV is higher ($ 361 712 for ATO vs. $ 680 005 for STO for the 14 days offer scenario).

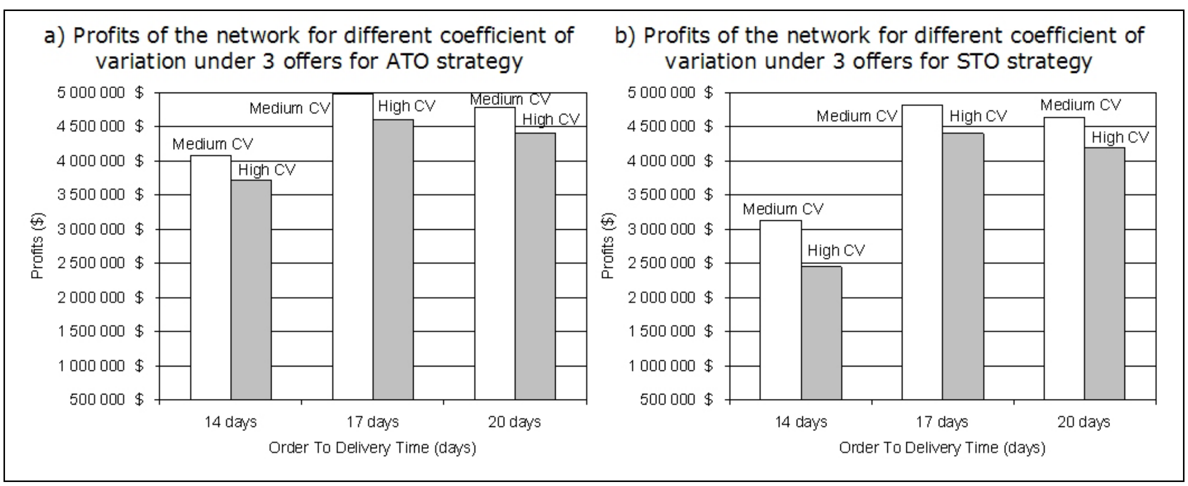

Figure 13: Effects of the Demand Variability on Profits

The last experiment corresponds to a what-if scenario that can be evaluated with the proposed methodology. The enterprise wants to test the effect of a high demand increase in its demand zones, especially where demand is currently low (90-95% increase when demand is low and 55-60% otherwise). This increase is motivated by the opportunity to sell more products in some demand zones if the OTD time is very short with high service level. Three new offers were developed for this case with new scale factors θ_{dpo} reflecting these new customer needs. These offers have 99.9% service level and: 1) conservative pricing for 14 days OTD time, 2) aggressive pricing for 17 days OTD time and, 3) conservative pricing for 17 days OTD time. Three new potential plants (plants 25, 26 & 27) are considered to assemble the finished products. Table 5 presents the optimal manufacturing network obtained and this network is illustrated in Figure 14.

Table 5: Optimal Manufacturing Network Design

Data	ATO – 14 days – 99.9% – Conservative Pricing
Plants & Centers	Plant 21: Centers 2101, 2102 Plant 22: Centers 2207, 2208, 2209 Plant 23: Center 2313 Plant 24: Center 2413 ***Plant 25***: Center 2513
Resources	Plant 21: 351 processors (4 types) & 348 workers (4 types) Plant 22: 344 processors (4 types) & 358 workers (6 types) Plant 23: 46 processors (5 types) & 47 workers (5 types) Plant 24: 36 processors (6 types) & 36 workers (5 types) ***Plant 25***: 31 processors (5 types) & 31 workers (5 types)
Inventory	Plant 24: 1380 units of part 63 & 3209 units of part 73 ***Plant 25***: 805 units of part 54, 1619 units of part 63 & 2054 units of part 73
Revenues	$ 96 333 954
Profits	$ 7 439 726
Problem Size	4359 constraints & 4717 variables (1097 binaries)
Simulation Time	25 min. per iteration
Solution Time	5 iterations: 4610 sec.

As in the initial network, parts are produced in plant 21 & 22 which keep their original mission, but with a high increase of parts manufactured and resources installed in centers. Plants 23, 24 and the new plant 25 produce finished products. Plants 24 and 25 have stock parts in order to remove the lead time of plants 21 & 22 from the computation of their OTD time to the demand zones. Offer 1) with a conservative pricing strategy, 14 days OTD time and 99.9% service level is the best option in this case with profits of $ 7 439 726.

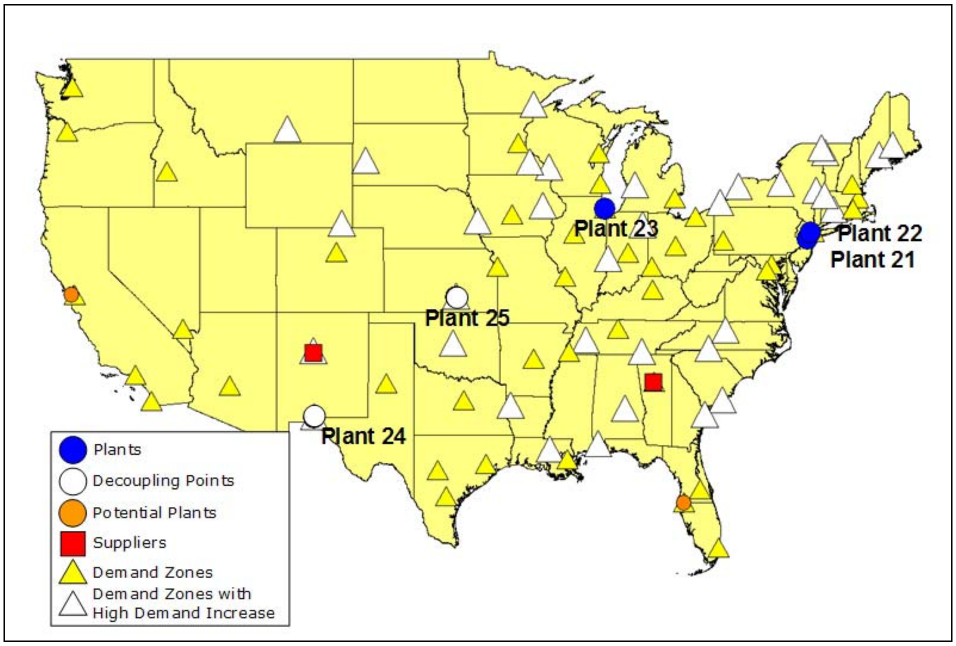

Figure 14: Resulting Manufacturing Network

Overall, on the computational side, the solution method for the 55 cases tested required 2 iterations when good capacity and inventory factor initialization was possible. This efficient initialization is possible when limited changes in the network are investigated. In this case, the average MIP resolution time is 257 seconds (between 10 and 1550 seconds), which is good for this type of models. For more complex cases, more iterations can be necessary (see Table 5). Each iteration also requires capacity and inventory factors computation, which takes between 5 and 25 minutes depending on the number of potential centers and configurations considered. The advances of commercially available solvers now permit the solution of large MIPs in a reasonable time. Therefore, the use of engineering approaches such as ours to solve manufacturing network design problems becomes feasible and attractive.

7. Conclusion

This paper presented an optimization methodology for the design of manufacturing networks with reliable promising capabilities. The designed network has the capability to deliver the products to the customers in a promised order-to-delivery time with a specific service level. Starting from the current state of the network, the facilities to open, close or reconfigure are chosen from a set of potential facilities. For each of these facilities, production centers are activated with a specific configuration specifying its mission in terms of products to produce, resources to use, production lead time and service level targets. A center can be selected as a decoupling point for some of its products which, for a given product-market offer, gives more time to some centers in the network. Taking into account the potential plants, centers and configurations, an offer for each finished product, yielding an expected demand for all demand zones, is selected to maximize the expected operating profits of the resulting network. The operations required for the manufacturing of each product and the capabilities of the human resources and processors used are taken into account in the process. A methodology is proposed to solve this complex problem in a heuristic way and to analyze what-if scenarios. When the tool developed to implement the approach is used with a rich set of potential manufacturing strategies and business scenarios, robust and efficient manufacturing network designs are obtained.

Research avenues following this work are related to the development of more powerful heuristic solution methods and simulation methods to support the methodology. Stochastic programming can also be investigated as a solution method. Modeling extensions may finally be necessary to cope with to the specific needs of different industrial contexts.

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