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Abstract

This paper studies the pricing strategies of firms belonging to a vertical channel structure where optional contingent products are sold. Optional contingent products are characterized by unilateral demand interdependencies. That is, the base product can be used independently of a contingent product. On the other hand, the contingent product's purchase is conditional on the possession of the base product.

We find that the retailer decreases the price of the base product to stimulate demand on the contingent-product market. Even a loss-leader strategy could be optimal, which happens when this contingent product is sufficiently profitable. The price reduction of the base product could also mitigate the double-marginalization problem, which is well known in a supply-chain setting with one manufacturer and one retailer. Competition on the contingent-product market reduces profits there. We find that this also hurts the manufacturer of the base product.

Key Words: Pricing, Complementary Products; Optional Contingent Products; Marketing Channels; Double Marginalization; Game Theory.

Résumé

Dans cet article, nous étudions les stratégies de tarification des entreprises appartenant à un circuit de distribution où des produits du type “optionnels contingents” sont vendus. Ces produits sont caractérisés par des relations d'interdépendance unilatérale dans les demandes. En d'autres termes, l'un des produits (le produit de base) peut être acheté et utilisé sans le produit contingent, alors que ce dernier ne sera acheté que par les consommateurs qui possèdent le produit de base.

Nos résultats indiquent que le détaillant baisse le prix de détail du produit de base afin de stimuler les ventes sur le marché du produit contingent. Une stratégie du type “Loss-leader”, où le détaillant vend le produit de base à perte pourrait être optimale quand le produit contingent est profitable. La réduction du prix du produit de base pourrait même atténuer les inefficacités liées à la double prise de marge dans les monopoles bilatéraux décentralisés. Finalement, nos résultats indiquent que la présence d'un concurrent sur le marché du produit contingent réduit les profits sur ce marché ainsi que les profits du manufacturier du produit de base.

Mots clés : Tarification, produits complémentaires, produits optionnels contingents, circuits de distribution, double prise de marge, théorie des jeux.

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1 Introduction

This paper investigates the problem of pricing optional contingent products in marketing channels. We are interested in the context where: (i) the sales of a *base (or primary) product* are unaffected by the sales of a *contingent product*, while the latter is useless to consumers who do not own the base product; and (ii) the two related products are controlled by different members of a marketing channel. An illustrative case of this setting is the digital-book industry. This industry is formed of manufacturers (e.g., Macmillan, Hachette or Penguin) that produce the base product (e-books) and retailers (e.g., Amazon, Barnes & Noble) that sell this base product and an optional contingent product (e-readers) via their e-libraries. An e-reader is a reading device intended exclusively to download and display e-books content for reading (e.g., Kindle or Nook). A 2012 report from eMarketer put the number of US e-reader users at 45.6 million in 2012 (about 19% of US consumers), and forecasted that the market will reach 56.9 million users in 2015.[1]

It is important to mention from the outset that our pricing context intrinsically differs from the highly studied cases of (i) pricing in an oligopoly (where the focus is on horizontal strategic interactions between a few firms competing with different brands); (ii) pricing of substitute products in marketing channels (where the focus is on vertical strategic interactions between manufacturers and retailers, with possible horizontal interactions at the manufacturing and/or retailing layer(s)); and (iii) pricing of complementary products marketed by the *same* firm. In this last example, the manager may find it optimal to practice a loss-leader pricing strategy, i.e., to use one of the products, often the durable product or the product with the lowest margin, as a springboard to increase sales of the second, which is often the frequently purchased product or the product with the higher unit margin. In our setting, where the base product is sold by the manufacturer through his retailer, which also offers the contingent product, there is no valid reason for the manufacturer to heavily discount the price of the base product, but there is a reason for the retailer to do so, namely, to boost sales of the high-margin contingent product. To be able to implement such a pricing policy, the retailer needs the manufacturer to provide the product at a sufficiently low wholesale price. This may be too much to ask the manufacturer, which means that all the ingredients for a conflict in the channel are in place.

Our study of this supply-chain configuration was motivated by the dispute between Macmillan and Amazon in January 2010. As these two companies could not agree on a pricing strategy, Amazon removed the buy button for all Macmillan books from its Kindle store. The conflict began with Macmillan accusing Amazon of practicing a loss-leader strategy by selling the e-books at very low prices in order to accelerate the diffusion of its e-reader. According to Macmillan, the low price set by Amazon "created the perception that e-books were a low-value item,"[2] and led to cannibalizing of the paper editions of their books. Amazon responded by accusing Macmillan of hurting consumer surplus by setting high margins for a digital product that has a very low production cost.

At the launch of the Kindle, Amazon had been the sole distributor of Macmillan e-books (80% of the US e-books market[3]) and almost the only distributor of reading devices.[4] At that time, Amazon and Macmillan had agreed on a wholesale-pricing model, under which the editor set the wholesale price of e-books and Amazon, the retail price. In practice, the wholesale price was set at 50% of hardcover prices, and the retail price was fixed at an average level of $9.99 (even for new bestsellers).[5] The dispute started when Macmillan changed this business model to an agency-type model, in which the editor sets the e-books retail price, and the retailer earns a fixed share of revenues.[6] Note that this business model is frequently used in platform selling,

[1]Source: Paul Verna, "Book ePublishing: The Price Disruption," eMarketer report 2012.

[2]Source: Paul Verna, "Book ePublishing: The Price Disruption," eMarketer report 2012.

[3]Source: *Wall Street Journal* (Online) April 1, 2010.

[4]Amazon was the dominant player in the reading-device market before 2010. Kindle eReader sales reached 8.2 million units in 2010 . Compared to a total market of 12.7 million e-reader users, Kindle represented almost 65% of this market. Source: eMarketer Report. September 11, 2012.

[5]Published February 2, 2010. MacMillan, D. "Amazon's E-Book Price Reversal: A Mixed Blessing." *Business Week* website (last visited September 30, 2012).

[6]More precisely, Macmillan decided to raise the eBook retail prices to between $12.99 and $14.99, and to fix Amazon's share of revenues to 30%. This business model is the one Apple proposed to book editors upon its entry to this market. Source: "Amazon's E-Book Price Reversal: A Mixed Blessing." *Business Week* website (last visited September 30, 2012). Published February 2, 2010. MacMillan, D.

where e-tailers give their customers access and control over retail prices to manufacturers in exchange for a fee. Actually, Macmillan's decision coincided with the launch of other reading devices by major competitors: the *i*Pad from Apple and the Nook from Barnes and Noble. In a sense, Macmillan wanted to take advantage of the arrival of new competitors to push Amazon to increase the e-books retail price. Clearly, the situation had become more complex for Amazon, which was losing some of its negotiation power with its suppliers.[7] A major consequence is that Amazon had to adjust its pricing strategy not only for e-books but also for e-readers in order to adapt to this new competitive situation.

Although our motivating example is taken from the digital-book industry, the problem we are investigating can easily be generalized to all marketing channels where the products sold are characterized by a unilateral dependency between demands. The digital-music and the entertainment industries are examples of industries facing similar pricing issues. Indeed, Apple and its music providers (Universal Music Group and Warner Music Group) experienced a similar disagreement upon the introduction of iTunes (2007), when the retailer decided to charge a flat 99 cents for all songs bought on iTunes.[8] In another dispute between Apple and one of its suppliers (NBC Universal), the latter announced clearly that Apple's practice of loss-leader pricing was hurting the other channel members' interests. The executive vice president of communications for NBC Universal, made the following statement: "It is clear that Apple's retail pricing strategy for its iTunes service is designed to drive sales of Apple devices at the expense of those who create the content that make these devices worth buying."[9]

These conflicts received considerable attention in the media, but have been overlooked by the literature despite the fact that they raise many interesting research questions that could contribute to our understanding of marketing channels and product-mix pricing.

In this paper, we wish to provide answers to some of these questions, namely:

1. What are the equilibrium wholesale and retail prices of the base and contingent products?
2. How does the competitive retail structure influence these prices and the players' profits?
3. Under which conditions is it optimal to use a loss-leader pricing strategy?
4. Can double-marginalization lead, as would be desired by the manufacturer, to higher base-product retail price?

To answer these questions, we develop a general model where a manufacturer sells its product to a retailer that handles both the base and the contingent product, and where competition exists in the contingent-product market. To assess the impact of competition in the contingent product on the price of the base product, we also study the case where there is no competition in the contingent-product market. For both scenarios, we consider that the retailer controls the retail price of the base product while the manufacturer controls its transfer price. The retailer also sets the retail price of the contingent product.

Our results confirm that the retailer has an incentive to discount the base product, and to practice a loss-leader strategy when the contingent product is highly profitable. We also obtain that competition in the contingent-product market hurts not only the retailer's profits but also those of the base-product manufacturer. Finally, our results indicate that the presence of a contingent product could mitigate the double marginalization-problem for the base product, provided that the base product market potential, relative to the contingent product, is sufficiently large.

The rest of the paper is organized as follows: In Section 2, we briefly review the related literature. In Section 3, we set the model, and in Section 4 we provide and discuss the results. Section 5 concludes.

[7] Kindle sales moved from 11.6 to 12.5 million units between 2011 and 2012: an 8% increase. If we compare this to the 325% increase observed between 2009 and 2010, we can clearly see the negative impact of competition on Amazon's dominant position in the e-reader market. Source: eMarketer Report, September 11, 2012.

[8] Ref: "Universal in Dispute With Apple Over iTunes." Published July 2, 2007. Leeds, Jeff. *The New York Times* website: www.nytimes.com (last visited Feb. 24, 2012).

[9] Statement by Cory Shields. Ref.: "Apple to stop selling NBC shows through iTunes: Pricing issues behind decision to drop all the network's shows in December." August 31, 2007, Rex Crum & David B. Wilkerson, MarketWatch. website: www.marketwatch.com (last visited Feb. 24, 2012).

2 Literature Review

Our work is related to two literature streams, namely, product-mix pricing and marketing channels. We briefly review these two streams, and point out our contribution.

2.1 Product-Mix Pricing

As the name suggests, product-mix pricing refers to strategies and tactics used to exploit mutual dependencies between brands, products or items (Tellis, 1986). Price bundling, customer-value pricing (premium pricing) and complementary-product pricing (including captive pricing, two-part tariff in services and loss leadership) are examples of strategies used for pricing consumer products. Peterson and Mahajan (1978) propose a classification of products based on the interdependency of their demands. Two products are:

(*i*) Independent: when the sales of one product are not affected, in a transparent way, by the sales of the other;

(*ii*) Substitutes: when an increase in the sales of one product lead to a decrease in the sales of the other. Competing brands in any product category are typically substitutes;

(*iii*) Complements: when the sales of one product positively affect the sales of the other because there is an associative "natural, functional relationship" between them and this relationship is symmetric (or bilateral). A hamburger and french fries is a typical example;

(*iv*) Contingent: when the sales of one product are conditional on the sales of another product. Mahajan and Muller (1991) distinguish between *captive contingent products* and *optional contingent products*. In the former case, we have bilateral or symmetric contingency, that is, neither product can be used without the other (e.g., a CD player and a CD). In the case of optional contingent products, the contingency is unilateral or asymmetric (e.g., a cell phone and its accessories). Obviously, a consumer would only consider buying the contingent product (accessories) if he buys (or has bought) the base product (cell phone).[10]

In most popular examples of optional contingent products, e.g., printer/ink cartridges and razor/blades, the base product is the durable one, and the optional product is the repeat-purchase product. In our example of the digital-book industry, it is (now) the other way around. Indeed, no consumer would purchase an e-reader if e-books did not exist. However, e-books can also be read by other devices, e.g., iPads and computers. Interestingly, the dependency between Kindle and Amazon's e-books was initially symmetric because these e-books were formatted to be read only by Kindle. It is competition in the e-reader market that led to a change in Amazon's strategy, that is, in developing an application that allows customers to read the Kindle e-books edition on competitors' devices.[11]

Noble and Gruca (1999) surveyed 270 pricing managers in order to identify the determinants of pricing strategies. They found that complementary product pricing is mainly used when the profitability of the contingent product is high. In such a context, the base product (printer, razor) is sold at cost or even given away for free, and money is made from the captive contingent product (ink cartridges, blades). Simon et al. (2005, p.231) recall that John Rockfeller, a pioneer in using complementary-product pricing, once said that oil lamps should be given away for free because, in any case, consumers have to buy the oil. This strategy, which is referred to as loss-leader pricing or the razor-and-blades business model,[12] is now practiced in many industries,[13] but typically for products manufactured or sold by the same firm (Kotler, 1988). Mahajan and Muller (1991) obtain that the prices of the primary and contingent product are lower when the two products

[10] Bayus et al. (2000) expanded the classification of demand interdependencies by developing a conceptual framework that captures all kinds of multi-product interactions. This model takes the form of a matrix where the rows and the columns represent the different levels of impact of one product on the other: negative, zero or strictly positive. This leads to 9 types of multi-product interaction categories. We do not consider this classification in our study because the subcategories have different labels, and many of them are particular cases related to the literature on the diffusion of successive product generations.

[11] Gütler (2009) examined the conditions under which a firm selling a base and a contingent product should consider protecting their complementary-product market when facing competition in this market.

[12] In reference to Gillette, a company that has often used this price strategy.

[13] The videogame industry is a heavy user of such a pricing strategy. The consoles are sold at a loss while profits are generated from software and accessories sold by the same company.

are managed by the same company, relative to the case where they are controlled by two separate firms. Mahajan and Muller (1991) is the closest study to what we are doing here, however with a huge difference, that is, in our case we also have vertical interactions between two independent players.

2.2 Pricing in Marketing Channels

The analytical marketing channel literature was initiated thirty years ago by two seminal papers, namely, Jeuland and Shugan (1983) and McGuire and Staelin (1983). In a nutshell, this literature is interested in coordination mechanisms that could achieve, in a decentralized way, the optimal-payoff result reminiscent of the vertically integrated channel. Examples of such mechanisms are quantity discounts, price leadership, cooperative advertising, two-part tariffs, revenue sharing, etc. Another important issue in this literature is the structure of the marketing channel itself, that is, when vertical integration is better/worse than decentralization from the point of view of different stakeholders. Since the publication of these two papers, the literature has expanded considerably in many directions, e.g., analyzing different marketing-channel structures, assessing the implementability of the above mechanisms in different channels, introducing dynamics to account for repeated interactions among players, etc. It is not our intention here to review this vast literature, but simply to point out the paper's contribution to it. (For an overview of the main results in static marketing channels, the interested reader may refer to the book by Ingene and Parry (2004), and to the book by Jørgensen and Zaccour (2004) for dynamic marketing channels. The recent survey by Ingene et al. (2012) fully reviews models and results for non-competitive marketing channels.)

In a bilateral monopoly (a dyad), considered to be the most parsimonious marketing-channel configuration that still allows a study of vertical interactions, the focus is typically on the impact of a lack of coordination between the manufacturer and retailer on wholesale and retail prices, and on individual and collective performance. Jeuland and Shugan (1983) suggest the study of a case where the retailer can handle many products, and where individual prices influence the sales of the other products. It would then be interesting to assess the impact of having multiple products on retailer profit.

The extension from the bilateral monopoly channel structure to multiple retailers and/or multiple manufacturers is mainly done in the spirit of assessing the generalizability of the findings in a dyadic channel to a competitive one. In any event, retailers are assumed to sell substitutable products offered by competing manufacturers. To the best of our knowledge, only a few papers study pricing issues in marketing channels where products are complementary, namely, Coughlan (1987), Moorthy (1988), Wang (2006), Dong et al. (2009) and Cai et al. (2012). However, their models and focuses are very different from ours. Indeed, Coughlan (1987) examines different channel arrangements to distribute complementary products, and assesses how coordination at different levels could increase profitability. Moorthy (1988) highlights the importance of considering the impact of strategic interactions between channel members on the efficiency of a channel involving two dyads. The latter can be vertically integrated or decentralized. In a situation with a high degree of demand complementarity between the manufacturers' products, the author proves that, given that one manufacturer decentralizes retailing activities, the other manufacturer is better off if he also decentralizes these activities. Nevertheless, having both dyads vertically integrated is a more efficient equilibrium compared to the case where both manufacturers decentralize. Wang (2006) considers a setting where n manufacturers produce as many perfectly complementary products, that is, products that must be purchased and consumed jointly. Although one retailer may be involved in the distribution system, the main issue is the determination by each manufacturer of the output and selling price for its product before the start of the selling season and in presence of demand uncertainty. Dong et al. (2009) consider a bilateral monopoly where the manufacturer sells its line of products through an exclusive retailer. The game is played in two stages: in the first stage, the two players decide to implement product-line pricing or not, and in the second stage, depending on the relative balance of power in the channel, the firms engage in either a Nash or a Stackelberg pricing game. Finally, Cai et al. (2012) study the impact of exclusive channels and revenue sharing in a competitive channel where complementary goods (a product and a related service) are present. The manufacturer of each product chooses between having an exclusive service with one retailer or selling through both retailers who compete for the service. In all the studies mentioned above, the products considered are complements. We differ from them mainly by analyzing a marketing channel with contingent products.

Our paper contributes to the literature in several ways. First, it extends the literature on the pricing of complementary products by looking at the case where these products are controlled by two different entities. Second, it offers some insight into the problem of double marginalization in marketing channels where members control two contingent products, a case which has not been considered before. Third, our study extends the current modeling literature in marketing channels to consider a real-world setting, which has not been done in most of the existing studies. We believe our model will shed a light on equilibrium pricing in digital industries, which will be of increasing importance in the economy, and that our results give guidelines to each stakeholder in the supply chain when it comes to elaborating its pricing strategy and assessing its bargaining power.

3 Model

We consider a marketing channel formed of one manufacturer (player M) and two retailers (players A and S). The manufacturer sells a frequently purchased product, the base product e, at the wholesale price w_e through retailer A. For product e to be used, the consumer can employ a dedicated-for-this-use complementary durable product (the contingent product), which is provided by retailer A under the brand name k, and by retailer S under the brand name r. The consumer can also use a more general item (think of, e.g., a personal computer, a tablet, or a smartphone) to enjoy product e. Retailer A sells product e at retail price P_e, and product k at price P_k. On the market under consideration, retailer S only offers the contingent product r at price P_r.[14] To remember the notation, the reader may think of M as Macmillan selling e (for eBook), of A as Amazon selling the e-reader k (for Kindle) and the product e to consumers, and of S as Sony selling a competing e-reader r (for Reader).

The demand for product $j \in \{e, k, r\}$ is denoted Q_j. The demand system is given by

$$\begin{aligned} Q_e &= f(P_e), \\ Q_k &= g(P_k, P_r)\, Q_e, \\ Q_r &= h(P_k, P_r)\, Q_e, \end{aligned}$$

with

$$\begin{aligned} \frac{\partial f}{\partial P_e} &< 0;\ \frac{\partial g}{\partial P_k} < 0,\ \frac{\partial h}{\partial P_r} < 0, \\ \frac{\partial g}{\partial P_r} &> 0,\ \frac{\partial h}{\partial P_k} > 0, \end{aligned}$$

that is, each demand is decreasing in its own price, and increasing in the competing product price. Sales of the contingent products k and r are equal to zero if the sales of the base product e are equal to zero. This captures the fact that the optional contingent product is useless without the base product. Note that the interdependence between the base-product and contingent-product demands is unidirectional. In particular, it holds that sales of products k and r depend on sales of product e, but not the other way around. This is because product e can also be used with other devices, e.g., a personal computer or a smartphone, so that it is not necessary to employ products k and r. The latter are considered optional contingent products since the consumer may or may not buy them in order to use the base product.

To keep the model tractable while still retaining all important interactions, we adopt a linear form for functions f, g and h, that is,

[14] Notice that this setting is still compatible with our example of the digital-book industry. Indeed, companies that sell the e-readers can either have their own eLibrary where they also sell the e-books, or use independent brick-and-mortar and online channels (e.g., eBay) to sell only their eReader. The first case may be illustrated by Amazon, which sells its e-reader (Kindle) and also offers e-books through its eLibrary. The second case corresponds to Sony, which used to compete with Amazon only on the reading-device market. Even though Sony launched a Reader store in 2006 to also sell e-books, this digital library was available only to the US and Canadian markets. In the European markets, Sony only sells the Reader.

$$\begin{aligned} Q_e(P_e) &= \alpha_e - P_e, \\ Q_k(P_k, P_r, Q_e) &= \left(\frac{\lambda_k}{1+\eta} - \frac{1}{1-\eta^2}P_k + \frac{\eta}{1-\eta^2}P_r\right)Q_e, \\ Q_r(P_k, P_r, Q_e) &= \left(\frac{\lambda_r}{1+\eta} - \frac{1}{1-\eta^2}P_r + \frac{\eta}{1-\eta^2}P_k\right)Q_e, \end{aligned}$$

where α_e denotes the market potential of the base product e. The parameter $\eta \in [0,1)$ captures the degree of horizontal differentiation between products k and r (see also the demand system in Kort and Zaccour (2011), p.881). If $\eta = 0$, this means that the two products are independent, and that the higher the value of η, the higher is the degree of competition between them. Note that, consistent with our context, we have $Q_k(P_k, P_r, 0) = Q_r(P_k, P_r, 0) = 0$.

The parameters λ_k and λ_r positively relate to the market potential of the two products. The difference between λ_k and λ_r is a measure for the vertical differentiation between products k and r. Observe that these specifications exhibit, as they should, a negative cross-price elasticity for the contingent-product demand with respect to the price of the base product. Indeed, for $Q_e > 0$, this cross-price elasticity ϵ is given by

$$\epsilon = \frac{\partial Q_k}{\partial P_e} \cdot \frac{P_e}{Q_k} = \frac{\partial Q_r}{\partial P_e} \cdot \frac{P_e}{Q_r} = -\frac{P_e}{Q_e} < 0.$$

It follows that retailer A can stimulate demand for product k by reducing the price of product e. In other words, the effect of such a price reduction is that more consumers will buy product e, which increases the number of potential users of product k. We come back to this issue in Section 4.3, where we investigate the circumstances under which a loss-leader strategy, i.e., retailer A selling product e at a loss to boost demand of product k, will be applied.

To account for the fact that only a fraction of product e's buyers will purchase products k or r, we need the following constraints to hold:

$$\begin{aligned} 0 &\leq \frac{\lambda_k}{1+\eta} - \frac{1}{1-\eta^2}P_k + \frac{\eta}{1-\eta^2}P_r < 1, \\ 0 &\leq \frac{\lambda_r}{1+\eta} - \frac{1}{1-\eta^2}P_r + \frac{\eta}{1-\eta^2}P_k < 1. \end{aligned}$$

We will not formally impose these restrictions in the optimization, but check ex-post for their satisfaction.

For tractability reasons, we do not incorporate costs. Admittedly, a unit cost is an important input for setting a price, but its impact is most likely to be simply an upward shift in price, which does not affect the results qualitatively. Finally, assuming profit-maximization behavior, the optimization problems of players M, A and S are as follows:

$$\begin{aligned} \pi_M &= \max_{w_e}(w_e Q_e(P_e)), \\ \pi_A &= \max_{P_e, P_k}((P_e - w_e)Q_e(P_e) + P_k \cdot Q_k(P_k, P_r, Q_e)), \\ \pi_S &= \max_{P_r}(P_r \cdot Q_r(P_k, P_r, Q_e)). \end{aligned}$$

Following a long tradition in the marketing-channel and supply-chain literatures (see, e.g., Ingene and Parry, 2004 and Jørgensen and Zaccour, 2004), we suppose that the non-cooperative game is played à la Stackelberg, with the manufacturer acting as leader and the retailers as followers.

4 Results

Section 4.1 characterizes the equilibria for the cases where there is competition in the retailer market and where there is only one retailer. The resulting pricing policies are analyzed in Section 4.2. Section 4.3 investigates when player A would apply a loss-leader pricing strategy, and Section 4.4 studies the effect of the presence of an optional contingent product on the double-marginalization problem, which is a well-known phenomenon occurring in supply chains under decentralized decision making.

4.1 Characterization of Equilibria

The following proposition characterizes the equilibrium where both retailers offer a contingent product.

Proposition 1 *In the scenario where there is competition in the contingent-product market, the unique Stackelberg equilibrium is given by*

$$\begin{aligned}
w_e^C &= \frac{1}{2}\left[\alpha_e + \frac{(1-\eta)}{(1+\eta)}\frac{(2\lambda_k+\eta\lambda_r)^2}{(4-\eta^2)^2}\right],\\
P_e^C &= \frac{1}{4}\left[3\alpha_e - \frac{(1-\eta)(2\lambda_k+\eta\lambda_r)^2}{(1+\eta)(4-\eta^2)^2}\right],\\
P_k^C &= \frac{1-\eta}{4-\eta^2}(2\lambda_k+\eta\lambda_r),\\
P_r^C &= \frac{1-\eta}{4-\eta^2}(2\lambda_r+\eta\lambda_k).
\end{aligned}$$

Proof. See Appendix. □

If η goes up, there is more competition on the contingent-product market, which makes it that, in most cases, contingent product prices P_k^C and P_r^C decrease with η.[15] If λ_k and λ_r are large, a large share of consumers will employ products k and r to consume product e. Therefore, the retailers will increase the price of their contingent products when λ_k and λ_r are larger. The increased profitability of the contingent-product market in this case makes it that the retailers want to enlarge their potential market size. Retailer A can do so by increasing sales of product e, which explains why P_e^C is decreasing in λ_k and λ_r. Because of this increase in sales, the manufacturer of the base product e also benefits from a profitable contingent-product market. This explains why w_e is increasing in both λ_k and λ_r. As expected, the wholesale price and the market price for product e are increasing in its market potential α_e.

Comparing the prices of the two competing contingent products, we get

$$P_k^C - P_r^C = \frac{1-\eta}{2+\eta}(\lambda_k - \lambda_r).$$

Clearly, the brand enjoying the higher market potential is priced, higher. In the absence of vertical differentiation, i.e., $\lambda_k = \lambda_r = \lambda$, we have that $P_k^C = P_r^C$. Note that the difference between the two prices is decreasing in the degree of horizontal differentiation η. Interestingly, this result is independent of the market potential of the e product, and therefore is independent of whether a retailer is selling both the base and the contingent products or only the contingent product. According to Mahajan and Muller (1991), we expect retailer A to set a lower price (P_k) for its e-reader (compared to P_r) since he controls the retail prices for both the base and the contingent products, while retailer S should set a higher price because he is in control of only one of the products.[16]

[15] It can be shown that the opposite result holds, i.e., $\frac{\partial P_k}{\partial \eta} > 0$ ($\frac{\partial P_r}{\partial \eta} > 0$), for $\eta \in [0, \bar{\eta})$,where $\bar{\eta}$ is the unique solution of

$$\bar{\eta}^4 - \bar{\eta}^3 + 3\bar{\eta}^2 + 4\bar{\eta} - 4 = 0,$$

and λ_r (λ_k) is large enough. The reason is that a large λ_r implies a large P_r, which in turn, implies a large P_k, where the positive impact of P_r on P_k is bigger when η is larger.

[16] Our result is probably a by-product of the multiplicatively separable functional forms of demands.

These prices lead to the following sales:[17]

$$\begin{aligned}
Q_e^C &= \frac{1}{4}\left[\alpha_e + \frac{(1-\eta)(2\lambda_k+\eta\lambda_r)^2}{(1+\eta)(4-\eta^2)^2}\right],\\
Q_k^C &= \left(\frac{2\lambda_k+\eta\lambda_r}{(1+\eta)(4-\eta^2)}\right)Q_e^C,\\
Q_r^C &= \left(\frac{2\lambda_r+\eta\lambda_k}{(1+\eta)(4-\eta^2)}\right)Q_e^C.
\end{aligned}$$

The quantity of product e forms the market potential of the contingent products. Therefore, sales of the contingent products go up with Q_e^C. Comparing the quantities of the contingent products, we get

$$Q_k^C - Q_r^C = \frac{(\lambda_k-\lambda_r)}{(1+\eta)(2+\eta)}Q_e^C.$$

The sign of this expression depends on the difference between both contingent products' market potentials. Interestingly, although it is sold at the highest price, the brand having the higher market potential enjoys the higher sales. Again, in the absence of vertical differentiation, we have that $Q_k^C = Q_r^C$.

Next, we analyze the case where there is no competition at the retailer level.

Proposition 2 *When only firm A supplies the contingent product, the bilateral monopoly equilibrium is given by*

$$w_e^B = \frac{1}{2}\left[\alpha_e + \frac{\lambda_k^2}{4}\right], \quad P_e^B = \frac{1}{4}\left(3\alpha_e - \frac{\lambda_k^2}{4}\right), \quad P_k^B = \frac{\lambda_k}{2}.$$

Proof. It suffices to set $\eta = 0$ in w_e^C, P_e^C and P_k^C to get the result. □

The equilibrium demands for products e and k are given by[18]

$$Q_e^B = \frac{1}{4}\left(\alpha_e + \frac{\lambda_k^2}{4}\right), \qquad Q_k^B = \frac{\lambda_k}{2}Q_e^B.$$

Here the same mechanisms play a role as when there is competition on the retailer market. Therefore, we can refrain from giving any additional explanations.

4.2 Comparing Equilibria

The following proposition shows what competition in the contingent-product market does to prices.

Proposition 3 *Consider the scenario where the two contingent products are not vertically differentiated, that is, $\lambda_k = \lambda_r = \lambda$. The wholesale and retail prices in the cases of 'competition' and 'no competition' on the contingent-product market compare as follows:*

$$\begin{aligned}
w_e^B &> w_e^C,\\
P_e^B &< P_e^C,\\
P_k^B &> P_k^C.
\end{aligned}$$

[17] The conditions for the coefficients of Q_e^C in Q_k^C and Q_r^C to be less than one can be summarized by

$$\max(2\lambda_k+\eta\lambda_r, 2\lambda_r+\eta\lambda_k) < (1+\eta)\left(4-\eta^2\right). \tag{1}$$

[18] A necessary condition for the coefficient of Q_e^B in Q_k^B to be less than one is $\lambda_k < 2$. We shall assume in the sequel that this condition is fulfilled.

Proof. Straightforward computations lead to

$$
\begin{aligned}
w_e^B - w_e^C &= \frac{\lambda^2 \eta \left[16 + (1+\eta)\,\eta^3 + 4\eta\,(1-\eta)\right]}{8\,(1+\eta)\,(4-\eta^2)^2} > 0, \\
P_e^B - P_e^C &= \frac{\eta\lambda^2 \left[-4 - \eta^2 + 3\eta\right]}{16\,(1+\eta)\,(2-\eta)^2} < 0, \qquad P_k^B - P_k^C = \frac{\lambda\eta}{2\,(2-\eta)} > 0.
\end{aligned}
$$

□

Results in Proposition 3 indicate that competition on the contingent-product market encourages retailer A to reduce the price of its contingent product k.

For the example of e-readers, we see in Table 1 the clear impact of the arrival of new competitors (Apple and Barnes & Noble) on the retail price of e-readers. Although part of this price decrease could be attributed to the introduction of a new product generation, this is not in fact the case. Indeed, even at their launch, the new products (e.g., Kindle Fire) were sold at lower than cost. This shows that retailers practiced a loss-leader pricing strategy on these contingent products.[19]

Table 1: Reader Cost and Retail Price

	Bilateral monopoly (Years 2007 to 2009)	**Competition at contingent-product level (Since year 2010)**
eReader (Kindle) cost	Not available \$185.49 (2$^{\text{nd}}$ generation)	\$84.00 (1$^{\text{st}}$ generation) \$209.63 (Fire, 4$^{\text{th}}$ generation)
Kindle retail price	\$399.00 (1$^{\text{st}}$ generation, 2007) \$359.00 (2$^{\text{nd}}$ generation, 2009) Not applicable Not applicable	\$79.00 or \$109.00 (1$^{\text{st}}$ generation) \$259.00 (2$^{\text{nd}}$ generation) \$189.00 (3$^{\text{rd}}$ generation) \$199.00 (Fire, 4$^{\text{th}}$ generation)
Nook retail price	At launch in 2009: \$259.00	\$130.00 in 2010

Since competition leads to a lower P_k, retailer A has less incentive to stimulate demand of product k by increasing sales of product e. This explains that a lower price P_k is accompanied by an increase in the price of the base product e.

So, in the competition case, the retailer orders less of product e from the manufacturer. The resulting reduced demand for product e explains why the manufacturer charges a lower wholesale price w_e compared to the non-competition situation.

Proposition 4 *Entry of firm S into the contingent-product market leads to lower profits for both players M and A.*

Proof. For player M, we have

$$
\begin{aligned}
Q_e^C - Q_e^B &= \frac{\eta\lambda^2}{16}\left[\frac{-4 - \eta\,(\eta-3)}{(1+\eta)\,(2-\eta)^2}\right] < 0 \quad \text{and } w_e^C - w_e^B < 0 \\
&\Rightarrow \quad \pi_M^C = Q_e^C w_e^C < \pi_M^B = Q_e^B w_e^B.
\end{aligned}
$$

For firm A, the profits in the two scenarios are given by

$$
\pi_A^C = \frac{1}{16}\left[\alpha_e + \frac{\lambda^2\,(1-\eta)}{(1+\eta)\,(2-\eta)^2}\right]^2, \qquad \pi_A^B = \frac{1}{16}\left(\alpha_e + \frac{\lambda^2}{4}\right)^2.
$$

[19] "Loss Leader Alert: \$79 Amazon Kindle Costs \$84 to Build" by S. Ludwing. November 10, 2011. Business Week Venture Beat website. www.http://venturebeat.com (last visited October 1, 2012).

The difference is given by

$$\pi_A^C - \pi_A^B = \frac{\lambda^2}{16}\left[\lambda^2 \frac{4(1-\eta)^2 - (1+\eta)^2(2-\eta)^4}{(1+\eta)^2(2-\eta)^4} - 2\alpha_e\eta\left(\frac{4+\eta(\eta-3)}{(1+\eta)(2-\eta)^2}\right)\right].$$

Clearly, $2\alpha_e\eta\left(\frac{4+\eta(\eta-3)}{(1+\eta)(2-\eta)^2}\right)$ is positive. To prove that $\pi_A^C - \pi_A^B < 0$, it remains to show that

$$4(1-\eta)^2 - (1+\eta)^2(2-\eta)^4 < 0.$$

Expanding yields

$$\begin{aligned} & 4(1-\eta)^2 - (1+\eta)^2(2-\eta)^4 \\ = \quad & -2(1-\eta)\left[6(1+\eta) + 4\eta(1-\eta) + 3\eta^4\right] - 3\eta^4 - \eta^6 < 0. \end{aligned}$$

□

Since the entry of player S implies that player A has to deal with competition, it makes sense that player A's profits become lower. The resulting price reduction of the contingent product k leads to, as explained above, a price increase for product e, which in turn, reduces sales of product e and thus, the profits of player M. This implies that competition on the contingent-product market has negative implications for the manufacturer.

To summarize, we showed that the arrival of a competitor in the contingent-product market leads to:

1. A higher wholesale price, and lower retail price for the base product;
2. A lower retail price for the contingent product;
3. Lower profits to all incumbent channel members.

The main takeaway is that competition in the contingent-product market not only affects prices and profits in that market, but also prices and payoffs in the base-product market. This completes our response to the second research question.

4.3 Loss-Leader Pricing

This section characterizes the conditions under which player A may pursue a loss-leader pricing strategy with product e. A loss-leader pricing strategy occurs when

$$w_e^C - P_e^C > 0,$$

i.e., the retailer pays a higher price to acquire product e from the manufacturer than the sales price it charges the consumer. Hence, the retailer loses on product e but it does so to stimulate demand of the contingent product k.

From Proposition 1 we note that the prices of the two products marketed by player A are related as follows:

$$\begin{aligned} P_e^C &= \frac{1}{4}\left[3\alpha_e - \frac{(1-\eta)(2\lambda_k + \eta\lambda_r)^2}{(1+\eta)(4-\eta^2)^2}\right] = \frac{3}{4}\alpha_e - \frac{\left(P_k^C\right)^2}{4(1-\eta^2)}, \\ P_k^C &= \frac{1-\eta}{4-\eta^2}(2\lambda_k + \eta\lambda_r). \end{aligned}$$

Clearly, retailer A can earn a lot of money from product k if its potential market size λ_k is large, because this raises the price P_k. This creates an incentive to enlarge the market for product k by increasing the sales Q_e of product e through a decrease in its price P_e. As prices P_k and P_r are strategic complements, a high value of P_r, accomplished by a high value of λ_r, raises P_k and thus lowers P_e. It is straightforwardly seen that the market potential of product e, that is α_e, has a positive effect on P_e.

Less straightforward is the effect of competition in the contingent-product market, measured by η, on the price P_e. To analyze this, we compute the derivative of P_e with respect to η to get

$$\frac{\partial P_e}{\partial \eta} = -\frac{P_k}{2\left(1-\eta^2\right)^2}\left(\left(1-\eta^2\right)\frac{\partial P_k}{\partial \eta} + \eta P_k\right).$$

To simplify, we consider the case of no vertical differentiation, i.e., $\lambda_k = \lambda_i = \lambda$, and keep in mind that the obtained signs of the derivatives will also hold if the difference between λ_k and λ_i is not too large. In this case, we get

$$\frac{\partial P_e^C}{\partial \eta} = \frac{\lambda P_k}{2\left(1-\eta^2\right)^2}\left(\frac{1-\eta}{\left(\eta^2-4\right)^2}\left(\eta^4+3\eta^3+\eta^2+4\right)\right) > 0.$$

The lower the competitive pressure in the contingent-product market (i.e., a lower value of η), the lower the price of the base product e. This is done to stimulate sales of the profitable contingent-product market.

Player M's wholesale price is given by

$$w_e^C = \frac{1}{2}\left[\alpha_e + \frac{(1-\eta)}{(1+\eta)}\frac{\left(2\lambda_k+\eta\lambda_r\right)^2}{\left(4-\eta^2\right)^2}\right].$$

The manufacturer especially charges a high wholesale price when the retailer purchases a large quantity from the manufacturer, and thus, when the retailer sets a low consumer price for product e. The retailer does this to expand sales of product e so as to increase the market potential of the contingent-product market. This is an attractive strategy when the profitability of the contingent-product market is high, which happens in case of a large λ_k , λ_r and a small η. On the other hand, the retailer chooses to make profits on market e when the market potential, α_e, is large. Then the wholesale price is also large but notice here that the dependence of P_e^C on α_e is larger than the dependence of w_e^C on α_e.

From the above, we get that whenever a loss-leader strategy occurs, it will do so in a scenario where λ_k and λ_r are large, while η and α_e are small. This characterizes the conditions where player A will practice a low price strategy for product e (relative to the cost of product e), and by the same token, explains Amazon's pricing strategy for Macmillan's e-books. In fact, Amazon practiced this strategy before Apple's entry onto the market, by offering product e at a price that is below the wholesale price. To illustrate, we report in Table 2 the different prices practiced in this industry under competitive and monopolistic market conditions. It clearly shows that Amazon used to lose about \$5 for each e-books sold in its e-library.[20]

Table 2: eBook Cost, Retail Price and Margin

	Bilateral monopoly (Years 2007 to 2009)	**Competition at contingent product level (Since year 2010)**
Cost of eBooks[7]	\$14.98	\$14.98
eBook retail price	\$9.99	\$15.99
Relailer's unit margin	\$(5.03)	30% of \$15.99 = \$4.80

Notice that since the arrival of Apple as a competitor and the implementation of the agency model, retailers no longer have control over the retail price of e-books. The latter are fixed at a higher level by the manufacturer. Our results state that even without following an agency model, and under the hypothesis that retailers keep control over the e-books retail price, retailers should increase these prices when facing competition.

The next proposition establishes the precise condition under which such an extreme loss-leader pricing strategy would be implemented by player A, and by the same token, answers our third research question.

[20]This example illustrates the case where the price of the hardcover version of a book is equal to \$29.99. Amazon negociated with Macmillan to buy the digital version of the book at a 50% discount on this price.

Proposition 5 *Consider the scenario where there is competition in the base-product market and where P_e^C is positive, i.e.,*

$$\frac{1}{3}\frac{(1-\eta)(2\lambda_k+\eta\lambda_r)^2}{(1+\eta)(4-\eta^2)^2}<\alpha_e.$$

If it holds that

$$\alpha_e<3\frac{(1-\eta)(2\lambda_k+\eta\lambda_r)^2}{(1+\eta)(4-\eta^2)^2},$$

then it is in the best interest of player A to sell product e at a positive price P_e^C that is below the wholesale price w_e^C.

Proof. Straightforward calculations give

$$\begin{aligned}
w_e^C-P_e^C &= \frac{1}{4}\left[-\alpha_e+\frac{3(1-\eta)}{(1+\eta)}\frac{(2\lambda_k+\eta\lambda_r)^2}{(4-\eta^2)^2}\right]>0,\\
&\Leftrightarrow \alpha_e<\frac{3(1-\eta)}{(1+\eta)}\frac{(2\lambda_k+\eta\lambda_r)^2}{(4-\eta^2)^2},\\
P_e^C &> 0\Leftrightarrow\frac{(1-\eta)(2\lambda_k+\eta\lambda_r)^2}{3(1+\eta)(4-\eta^2)^2}<\alpha_e.
\end{aligned}$$

□

In the absence of competition ($\eta=0$), the conditions for positive price and loss-leader price in the above proposition simplify to

$$\frac{\lambda_k^2}{12}<\alpha_e<\frac{3\lambda_k^2}{4}.$$

4.4 Effect of Demand Interdependencies on Double Marginalization

It is well known (see the literature review in Ingene et al., 2012), that in a supply chain formed of one manufacturer and one retailer, lack of price coordination leads to inefficiency, which is referred to as the double-marginalization problem. The inefficiency can be measured by the difference between the total channel profit under vertical integration (i.e., coordination) and decentralized decision making (i.e., non coordination).

The aim of this section is to check if the presence of a contingent product reduces this inefficiency. To be sure that the result is attributed to, and only to, the interdependencies between product demands, we shall assume away competition in this market, which implies that we are studying scenario B here.

To start with, consider the e product market in isolation and ignore the fact that the retailer also profits from the sales of the contingent product. It is easy to verify that if players M and A act as if they were vertically integrated (VI), i.e., maximize the sum of their profits

$$\begin{aligned}
\pi_M &= \max_{w_e} w_e(\alpha_e-P_e),\\
\pi_A &= \max_{P_e}(P_e-w_e)(\alpha_e-P_e),
\end{aligned}$$

then the retail price and the total channel profit would be given by

$$p_e^{VI}=\frac{1}{2}\alpha_e,\qquad \pi^{VI}=\frac{1}{4}\alpha_e^2.$$

In the decentralized vertical channel, the Stackelberg equilibrium prices and total channel profit would be given by

$$w_e^{NC}=\frac{1}{2}\alpha_e,\qquad p_e^{NC}=\frac{3}{4}\alpha_e,\qquad \pi^{NC}=\frac{3}{16}\alpha_e^2,$$

where the superscript NC stands for non coordination. Clearly, $\pi^{NC} < \pi^{VI}$, and the inefficiency due to double marginalization corresponds to

$$\pi^{VI} - \pi^{NC} = \frac{1}{16}\alpha_e^2.$$

Comparing the retail prices in the two scenarios, we see that p_e^{NC} is too high.

To obtain the e product channel profit in the presence of the contingent product, we insert the equilibrium strategies found in Proposition 2 in the individual profits of players M and A pertaining to only the e product, that is, in

$$\begin{aligned} \pi_M^B &= w_e^B\left(\alpha_e - P_e^B\right), \\ \pi_A^B &= \left(P_e^B - w_e^B\right)\left(\alpha_e - P_e^B\right). \end{aligned}$$

Then we get

$$\pi^B = \pi_M^B + \pi_A^B = \frac{1}{16}\left(3\alpha_e - \frac{\lambda_k^2}{4}\right)\left(\alpha_e + \frac{\lambda_k^2}{4}\right).$$

The inefficiency in the presence of the contingent-product market is equal to

$$\begin{aligned} \pi^{VI} - \pi^B &= \frac{1}{4}\alpha_e^2 - \frac{1}{16}\left(3\alpha_e - \frac{\lambda_k^2}{4}\right)\left(\alpha_e + \frac{\lambda_k^2}{4}\right), \\ &= \frac{1}{256}\left(4\alpha_e - \lambda_k^2\right)^2. \end{aligned}$$

This leads to the following proposition.

Proposition 6 *The presence of the optional-contingent-product market mitigates the double-marginalization problem if*

$$\alpha_e > \frac{\lambda_k^2}{8}. \tag{2}$$

Proof. Compute

$$\left(\pi^{VI} - \pi^{NC}\right) - \left(\pi^{VI} - \pi^B\right) = \frac{1}{256}\lambda_k^2\left(8\alpha_e - \lambda_k^2\right),$$

which is positive for $\alpha_e > \frac{\lambda_k^2}{8}$. □

In other words, the double-marginalization problem is mitigated if the potential market of product e, α_e, is large relative to the market potential of the contingent-product market, λ_k. Observe that the retail price of product e in the presence of a contingent product is lower than its level in the absence of such a product. Indeed, we have

$$P_e^B - P_e^{NC} = \frac{1}{4}\left(3\alpha_e - \frac{\lambda_k^2}{4}\right) - \frac{3}{4}\alpha_e = -\frac{\lambda_k^2}{16} < 0.$$

This means that the presence of an optional-contingent-product market also leads to a higher consumer surplus. Interestingly, player A may decrease the price of product e in scenario B even below its level in the fully integrated channel when there is no contingent product. This happens if $\alpha_e < \frac{\lambda_k^2}{4}$, that is, when the market potential of product e is low enough with respect to the market potential of product k. Indeed,

$$P_e^B - P_e^{VI} = \frac{1}{4}\left(\alpha_e - \frac{\lambda_k^2}{4}\right) \leq 0 \Leftrightarrow \alpha_e \leq \frac{\lambda_k^2}{4}.$$

We conclude that, whereas in the "classical" problem of double marginalization, the retail price is too high, here, the e product channel profits could be low because the retail price is too low. The reason for this low retail price is that, in this way, the firm stimulates demand for the contingent product. Note that the presence of the contingent product completely eliminates the double-marginalization problem if $\alpha_e = \frac{\lambda_k^2}{4}$, because exactly then it holds that $P_e^B = P_e^{VI}$. On the other hand, if $\alpha_e < \frac{\lambda_k^2}{8}$, the price for product e is so low that the presence of a contingent product leads to an even larger inefficiency compared to the non-coordination solution NC. This completes our answer to the last research question of the paper.

5 Concluding Remarks

This paper studies a channel framework with one manufacturer and two retailers. The manufacturer supplies a base product that is distributed to consumers by one of the retailers. In addition, both retailers offer an optional contingent product that consumers may employ to consume the base product. An obvious example is the market for e-books (base product) with the e-readers as optional contingent products.

We find that the retailer selling the base product has an incentive to reduce its price to stimulate demand on the contingent-product market. This incentive is larger the more profitable is this contingent-product market. Whenever this incentive is large, sales of the base product are large. Then the manufacturer benefits and can charge a large wholesale price. It follows that a loss-leader strategy, i.e., where the retailer offers the base product and sells it at a loss, occurs especially when the contingent-product market is profitable. The fact that the manufacturer benefits if the contingent-product market is profitable implies that manufacturer profits are higher if there is less competition on this market.

Finally, we study the implications of the presence of contingent products on the problem of double marginalization. This is the problem arising when both the manufacturer and the retailer want to obtain a positive margin on a certain product, and consequently, the price of this product becomes too large. However, in the case of contingent products being present, there is an incentive to reduce the price of the base product in order to stimulate demand on the contingent-product market. If this price reduction is not too large, the double-marginalization problem is mitigated on such a market.

Future research could analyze the problem of pricing optimal contingent products in the distribution channel while introducing competition at the base-product level too. Going back to our example with the digital-book industry, the problem of book editors (Macmillan) was the cannibalization effect of e-books on traditional printed books, due to the fact that the retailer (Amazon) offered e-books at a too-low price in order to stimulate demand for its e-reader (Kindle). Kannan et al. (2009) examined the problem of optimally pricing the different forms of products in the publishing market in a situation where these products could be substitutes or complements, but their study did not consider this issue in a vertical marketing structure. Our current model should then be extended by letting the manufacturer offer a second product that is a substitute to the current base product. This second product would then need to be directly sold to the consumers and/or supplied via different retailers.

6 Appendix

6.1 Proof of Proposition 1

To obtain a subgame-perfect equilibrium, we first determine the retailers' reactions functions. The two first-order conditions of retailer A are

$$\begin{aligned}
\pi_M &= \max_{w_e} \left(w_e\left(\alpha_e - P_e\right)\right), \\
\pi_A &= \max_{P_e, P_k} \left(\left(P_e - w_e\right)\left(\alpha_e - P_e\right) + P_k \left(\frac{\lambda_k}{1+\eta} - \frac{P_k}{1-\eta^2} + \frac{\eta P_r}{1-\eta^2} \right) \left(\alpha_e - P_e\right) \right), \\
\pi_S &= \max_{P_r} \left(P_r \left(\frac{\lambda_r}{1+\eta} - \frac{P_r}{1-\eta^2} + \frac{\eta P_k}{1-\eta^2} \right) \left(\alpha_e - P_e\right) \right).
\end{aligned}$$

$$-2P_e + w_e + \alpha_e - P_k \left(\frac{\lambda_k}{1+\eta} - \frac{P_k}{1-\eta^2} + \frac{\eta P_r}{1-\eta^2} \right) = 0, \tag{3}$$

$$\left(\frac{\lambda_k}{1+\eta} - \frac{2P_k}{1-\eta^2} + \frac{\eta P_r}{1-\eta^2} \right) \left(\alpha_e - P_e\right) = 0,$$

from which it follows that (note that $\alpha_e - P_e > 0$, otherwise all demands are zero)

$$\frac{\lambda_k}{1+\eta} - \frac{2P_k}{1-\eta^2} + \frac{\eta P_r}{1-\eta^2} = 0. \tag{4}$$

Retailer S's first-order condition is

$$\left(\frac{\lambda_r}{1+\eta}-\frac{2P_r}{1-\eta^2}+\frac{\eta P_k}{1-\eta^2}\right)(\alpha_e-P_e)=0,$$

i.e.,

$$\frac{\lambda_r}{1+\eta}-\frac{2P_r}{1-\eta^2}+\frac{\eta P_k}{1-\eta^2}=0. \tag{5}$$

From (5) it follows that

$$P_r=\frac{(1-\eta)\,\lambda_r}{2}+\frac{\eta}{2}P_k.$$

Inserting this into (4) results in

$$P_k=\frac{1-\eta}{4-\eta^2}\left(2\lambda_k+\eta\lambda_r\right), \tag{6}$$

so that

$$P_r=\frac{1-\eta}{4-\eta^2}\left(2\lambda_r+\eta\lambda_k\right). \tag{7}$$

We obtain P_e by substitution of (6) and (7) into (3):

$$\begin{aligned}
& -2P_e+w_e+\alpha_e \\
& -\left(\frac{1-\eta}{4-\eta^2}\left(2\lambda_k+\eta\lambda_r\right)\right)\left(\frac{\lambda_k}{1+\eta}-\frac{1}{1-\eta^2}\left(\frac{1-\eta}{4-\eta^2}\left(2\lambda_k+\eta\lambda_r\right)\right)\right) \\
& -\left(\frac{1-\eta}{4-\eta^2}\left(2\lambda_k+\eta\lambda_r\right)\right)\left(\frac{\eta}{1-\eta^2}\left(\frac{1-\eta}{4-\eta^2}\left(2\lambda_r+\eta\lambda_k\right)\right)\right) \\
= \;& 0,
\end{aligned}$$

which, since

$$\begin{aligned}
& \frac{\lambda_k}{1+\eta}-\frac{1}{1-\eta^2}\left(\frac{1-\eta}{4-\eta^2}\left(2\lambda_k+\eta\lambda_r\right)\right)+\frac{\eta}{1-\eta^2}\left(\frac{1-\eta}{4-\eta^2}\left(2\lambda_r+\eta\lambda_k\right)\right) \\
= \;& \frac{1}{1-\eta^2}\left(\frac{1-\eta}{4-\eta^2}\left(2\lambda_k+\eta\lambda_r\right)\right)=\frac{1}{1-\eta^2}P_k,
\end{aligned}$$

is equivalent to

$$-2P_e+w_e+\alpha_e-\frac{1}{1-\eta^2}P_k^2=0.$$

This condition has a nice interpretation. Increasing the e-books price by one unit raises its profits on the e-books market by $-2P_e+w_e+\alpha_e$. On the other hand, this price increase by one unit reduces the e-books quantity by one unit, and this in turn, reduces the revenue on the e-reader market by $\frac{1}{1-\eta^2}P_k^2$.

This results in

$$P_e\left(w_e\right)=\frac{1}{2}w_e+\frac{1}{2}\alpha_e-\frac{1}{2\left(1-\eta^2\right)}P_k^2.$$

Next, we go to the problem of the manufacturer:

$$\pi_e=\max_{w_e}\left(w_e\left(\alpha_e-P_e\left(w_e\right)\right)\right).$$

The first-order condition leads to

$$\begin{aligned}
w_e &= \frac{1}{2}\alpha_e+\frac{(1-\eta)}{2\,(1+\eta)}\left(\frac{\left(2\lambda_k+\eta\lambda_r\right)}{4-\eta^2}\right)^2. \\
P_k &= \frac{1-\eta}{4-\eta^2}\left(2\lambda_k+\eta\lambda_r\right)
\end{aligned}$$

By substitutions, we obtain the equilibrium values in the Proposition.

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