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strategic consumers and social contagion:
A mean-field game approach**

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New product diffusion in the presence of strategic consumers and social contagion: A mean-field game approach

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Abstract: In this paper, we introduce a framework for new product diffusion that integrates consumer heterogeneity and strategic interactions at individual level. Forward-looking consumers belong to two mutually exclusive segments: individualists, whose adoption decision is influenced by the price and reputation of the innovation, and conformists, whose adoption decision depends on social influences exerted by other consumers and on the price. We use a mean-field game approach to translate consumer strategic interactions into aggregate social influences that affect conformists' adoption decision. The game is played à la Stackelberg, with the firm acting as leader and consumers as followers. The firm determines its pricing and advertising strategies to maximize its profit over a finite planning horizon. We provide the conditions for existence and uniqueness of equilibrium and a numerical scheme to compute it. We conduct a series of numerical simulations to analyze firm's strategy and diffusion processes for different parameter constellations. Our results suggest that the firm adopts a penetration pricing strategy, even when the word-of-mouth effect is weak, in the presence of strategic consumers, whereas it implements a cyclic penetration-skimming policy in face of myopic consumers. Also, depending on the distribution of consumers, the diffusion curve can be S-shaped, concave or convex, which shows clearly that heterogeneity matters.

Keywords: New product diffusion, two-segment market, mean-field Stackelberg game, strategic consumers, social contagion

Résumé : Dans cet article, nous introduisons une nouvelle approche pour analyser le processus de diffusion de nouveaux produits qui tient compte de l'hétérogénéité des consommateurs et des interactions stratégiques au niveau individuel. Les consommateurs, supposés non myopes, appartiennent à deux segments mutuellement exclusifs : les individualistes, dont la décision d'adoption est influencée par le prix et la réputation de l'innovation, et les conformistes, dont la décision d'adoption dépend des influences sociales exercées par d'autres consommateurs, ainsi que du prix. Nous utilisons une approche de jeu à champ moyen pour traduire les interactions stratégiques des consommateurs en influences sociales agrégées qui affectent la décision d'adoption des conformistes. Le jeu est à la Stackelberg, avec la firme jouant le rôle de leader et les consommateurs, de followers. L'entreprise détermine ses stratégies de tarification et de publicité pour maximiser ses bénéfices sur un horizon de planification fini. Nous fournissons les conditions d'existence et d'unicité de l'équilibre et un schéma numérique pour le calculer. Nous effectuons une série de simulations numériques pour analyser la stratégie de l'entreprise et les processus de diffusion pour différentes constellations de paramètres. Nos résultats suggèrent que l'entreprise adopte une stratégie de prix de pénétration, même lorsque l'effet de bouche à oreille est faible, en présence de consommateurs stratégiques, alors qu'elle met en œuvre une politique cyclique de pénétration-écrémage face aux consommateurs myopes. De plus, selon la distribution des consommateurs, la courbe de diffusion peut être en forme de S, concave ou convexe, ce qui montre clairement que l'hypothèse d'hétérogénéité est importante.

Mots clés : Diffusion de nouveaux produits, deux segments de marché, jeu à champ moyen, consommateurs stratégiques, contagion sociale

1 Introduction

Since its inception more than half century ago by Rogers (1962) and Bass (1969), the theory of diffusion of innovations in a social system has remained constantly on the research agenda. Recent developments aimed at addressing some shortcomings of classical aggregate diffusion models by incorporating explicitly consumer behavior into the adoption process. Examining a new product adoption at individual level allows to connect a micro-founded consumer behavior to aggregate effects and firm decisions. Also, the growing impact of social networks in all areas of human activities has, at least implicitly, led to redefining the concept: “*Innovation diffusion is the process of the market penetration of new products and services, which is driven by social influences. Such influences include all of the interdependencies among consumers that affect various market players with or without their explicit knowledge*” (Peres et al., 2010). For instance, accounting for consumer heterogeneity and social interaction can explain why the diffusion rate does not necessarily follow a typical bell-shaped curve, as well as the presence of chasm or saddle phenomena in sales curve (Van den Bulte and Joshi, 2007; Goldenberg et al., 2002; Song and Chintagunta, 2003).

In this paper, we study how consumer heterogeneity and interaction at individual level frame market penetration of a new product and affect firm marketing strategies. More specifically, we consider a firm that launches a new product in a market composed of a large number of strategic (forward-looking) consumers divided into two mutually exclusive groups, namely individualists and conformists. The adoption decision of an individualist depends on the product price and reputation (goodwill), which is built through investment in advertising over time. Whereas an individualist choice to buy or not the product is independent of social pressure, a conformist’s decision is driven by other consumers’ behaviors, more precisely the percentage of adopters in the social system, and by the product price. In our framework, the firm’s paid-for advertising corresponds to information on the product emanating from outside the social system, while the fraction of adopters (in both consumers’ groups) captures the within social system communication.

When the number of consumers is large, it is intuitive to suppose that, while the aggregate behavior of the members of a social system has an important impact on any individual’s decision, each member has a negligible impact on the mass.¹ This assumption of weak interactions between individuals (players) is at the heart of *mean-field games* (MFGs) theory, which provides a natural methodological framework to study social influences in an adoption process. We assume that the firm, as a leader, plays a Stackelberg game with consumers, who act as followers, where the conformists play a mean-field game among each other in a Nash configuration.

Our objective is to address the following research questions:

1. How does a new product diffuse into a market composed of forward-looking consumers whose adoption behavior is framed by individual level dynamics and interactions in a two-segment structure?
2. How marketing strategies are influenced by market penetration of the product and consumers heterogeneity and interactions?
3. How does the consumer’s strategic or myopic behavior affect the equilibrium results?
4. What is the impact of the parameter values on the firm’s equilibrium strategy?

Our contributions are as follows. We propose a new game-theoretic model of innovation diffusion that involves a two-segment market of strategic consumers with heterogeneous adoption drivers and a strategic monopolist. We use the mean-field games theory to develop tractable solutions and predict the evolution of the consumers’ mass adoption behavior from the individual strategic reactions to the firm’s marketing strategies and Word-of-Mouth (WoM) communications. In particular, we develop a numerical scheme to compute the firm’s optimal pricing and advertising policies in face of a large

¹ To give an analogy, the driving time at a given hour between points A and B depends on the traffic density, which affects the decision of a driver to use or not this road. However, if an individual decides to use the road, her impact on the average driving time is clearly negligible.

population of heterogeneous consumers, and to predict the evolution of adoption rate. We generate new insights via numerical examples on how market penetration of new product is built from the strategic decisions of interacting small consumers and what are the marketing implications of such consumer heterogeneity and interactions. Our model is consistent with empirical findings in the literature and allows for different types of irregularities in diffusion curves. In particular, we find that the magnitude and rate of diffusion depend significantly on the consumer sensitivity towards mass media (external influences) and WoM (internal influences) as well as the two-segment structure of the market, and accordingly can exhibit different penetration curves. Moreover, our results generalize some findings on monopolist marketing strategies. For example, we obtain that, even when the WoM effect is weak, the firm implements a penetration pricing strategy to avoid that strategic consumers wait to adopt.

The rest of the paper is organized as follows: We discuss in Section 2 the relevant literature. The model is introduced in Section 3. In Section 4, we analyze the model and propose a numerical scheme to solve the game and compute the firm's optimal strategies. Section 5 provides a numerical example and sensitivity analysis. Finally, Section 6 concludes the paper and proposes some future directions.

2 Literature review

Our paper draws and contributes to two research streams in the marketing science literature on new product diffusion, namely, individual level diffusion models, and aggregate diffusion models taking either normative or two-segment structure approaches. We briefly review these two streams in this section.

Individual level diffusion modeling enables marketers to better set the marketing strategies by studying consumer behavior, various forms of consumer heterogeneity, and the relationship between micro level dynamics and aggregate behaviors. One common denominator in this literature stream is to let the consumer's utility be based on the new product performance under uncertainty (Chatterjee and Eliashberg, 1990; Roberts and Urban, 1988; Horsky, 1990; Lattin and Roberts, 1988; Oren and Schwartz, 1988). Following the reception over time of information from mass media or WoM communications, consumers update their uncertain perception of the innovation. A consumer adopts the product if her expected utility exceeds what she gets under non adoption. The penetration curve is next obtained by aggregating individual decisions. In a similar vein, Song and Chintagunta (2003) consider forward-looking consumers and account for consumer heterogeneity and price sensitivity to develop an individual diffusion-choice model. The authors neglect the role of WoM communications and assume that the price is exogenously given. Li (2020) deals with the pricing of new multi-product in which consumers are myopic and adopt in a two-stage process: First, affected by external and internal influences, consumers face some purchasing occasions, and next they select one innovation based on their individual price-dependent utility. This research finds different variants of pricing regimes for new multi-product diffusion depending on the significance of innovation versus imitation effects. We depart from this literature by considering forward-looking consumers in a two-segment structure where the conformists segment is affected by the adoption behavior of entire social system. Further, we examine how consumers heterogeneity and consumers sensitivity with respect to external and internal influences are tied to aggregate market and firm's marketing strategies and performance. The literature calls for transition from aggregate models to individual ones in order to better manage firms marketing activities in response to evolving individual level behaviors (Peres et al., 2010). Individual models allow to analyze the rich effects of social networks on innovation's market performance that have recently received a growing attention (see Muller and Peres (2019) for a review)

The second relevant stream of literature is concerned with aggregate diffusion models that either studies monopolist pricing and/or advertising policies given aggregate growth model or builds up new product diffusion through a two-segment market. The former line of this stream takes a normative approach to maximize the monopolist profit (see Nair (2019) for a recent review on new product pricing). It mainly considers myopic consumers and suggests skimming pricing strategy unless WoM communications are strong (Kalish, 1983, 1985; Dolan and Jeuland, 1981; Horsky, 1990). Kalish

(1983), for example, considers a general diffusion framework and finds that skimming strategy is optimal if WoM is not strong for two diffusion models: price-dependent multiplicative separable model and price-dependent market potential model. However, if market potential, planning horizon or other involved aspects lead to dominance of WoM effect compared to saturation one, then the penetration pricing strategy is the optimal choice for monopolist. Kalish (1985) finds a similar strategy for pricing and further suggests monotonically decreasing trend as optimal advertising strategy. Besanko and Winston (1990) also propose skimming pricing strategy for the monopolist who encounters forward-looking consumers in order to prevent them from waiting for more appealing deals in future periods, however, the paper did not account for the salient role of WoM in the diffusion process. Including a reference price in the model, Zhang and Chiang (2020) obtain that a myopic monopolist is better off implementing a skimming pricing strategy, whereas for a strategic seller, either penetration or skimming strategies could be optimal, depending on the potential market and reference effect. Additionally, some studies consider competition across firms in context of new product diffusion and examine marketing strategies along with other aspects such as cost learning or government subsidies for new technologies (see Jørgensen (2018) for a review).

In Rogers (1962), the distinction between the different groups (segments) of consumers is based on their timing of adoption, i.e., innovators, early adopters, early majority, late majority and laggards. In Bass (1969), and the literature that followed, the two groups of consumers (innovators and imitators) are differentiated in terms of what drive them to adopt. Whereas innovators are influenced by paid-for mass media communications emanating from outside the social system, imitators' adoption decision depends on social pressure, measured by WoM communications. The subsequent literature, however, argues that innovators are not necessarily the first adopters as defined by Rogers (1962); see, e.g., Mahajan et al. (1990). It also questions the common assumption that all consumers homogeneously have innovative and imitative tendency towards adoption regardless of their type and hence each potential adopter might be affected by both external and internal influences. This assumption is challenged by two-segment structure diffusion models that theoretically and empirically underpins the significance of such structure where each potential adopter is affected by corresponding adoption driver depending on her type. The notion of two-segment structure has been also adopted in other contexts such as established luxury products (e.g., Zhang et al. (2020)). Steffens and Murthy (1992) divide the population into innovators and imitators where the former segment is driven by external and internal influences and the latter by internal ones. Through an aggregate perspective, their model provides a better fit compared to Bass (1969) and further shows bimodal characteristics in the penetration curve. Van den Bulte and Joshi (2007) discuss five theories that articulate how potential adopters can be unbundled into a two-segment structure based on not only independent and imitative drivers but also the way the consumers imitate within or cross segments. They find that there is a chasm between early and later stages of penetration curve for a two-segment population that compete over the social status.

In our study, we name self-reliant consumers who make adoption decision independently from others and are affected by external influences as **individualists**, while those who tend to conform to social norms but also are sensitive to key external influences are termed as **conformists**. In contrast to the two-segment literature that uses aggregate approach, we introduce the individualist-conformist framework into the individual-based diffusion modeling in order to explore how heterogeneity of forward-looking consumers and their strategic interactions shape the adoption process. We use a mean-field games methodology to obtain the Nash equilibrium among conformists who collectively along with individualists act as followers in a Stackelberg game, where a profit-maximizing firm determines its pricing and advertising strategies.

3 Model

We consider a firm launching a new product in a market formed of N consumers, where N is a large number. Consumers are divided into two groups (or market segments): N^I individualists and N^i conformists, with $N = N^I + N^i$. Individualists are individuals whose adoption decision is based on the

product's characteristics, e.g., the price and brand reputation, whereas conformists' purchasing utility depends on the behavior of other consumers in addition to the prices.

At each instant of time $t \in \{0, \dots, T\}$, a consumer who has not yet adopted the product decides whether to purchase the product or not. The date T is interpreted as the end of the selling season, after which the product is not available. The state of individualist $j \in \{1, \dots, N^I\}$ at time t is $S_{jt} \in \{0, 1\}$, where $S_{jt} = 1$ means that individualist j has the product at time t , while $S_{jt} = 0$ refers to the opposite case. At time t , individualist j 's decision variable is $A_{jt} \in \{0, 1\}$, where $A_{jt} = 1$ and $A_{jt} = 0$ refer to adopting or not the product, respectively. We assume that each consumer buys the product at most once during the planning horizon, and consequently if a purchase is made at time t , then necessarily $A_{j\tau} = 0$ for all $\tau \neq t$. We define in a similar way the state $s_{kt} \in \{0, 1\}$ and decision variable $a_{kt} \in \{0, 1\}$ of conformist $k \in \{1, \dots, N^I\}$. Because the product is new, we suppose that $S_{j0} = s_{k0} = 0$.

At each time period t , the firm decides the price $p_t \in [0, M_p]$ of its product and its advertising (or marketing) investment $m_t \in [0, M_m]$, where M_p and M_m are positive scalars.² Advertising has a positive impact on the product's reputation or goodwill $G_t \in \mathbb{R}$. The goodwill dynamics are given by

$$G_{t+1} = h(t, G_t, m_t), \quad G_0 \text{ given}, \quad (1)$$

where G_0 is the initial goodwill's value, and h is a continuous function. A well-known instance of (1) is Nerlove and Arrow's model (Nerlove and Arrow, 1962), where $h(t, G_t, m_t) = m_t + (1 - \gamma)G_t$, and γ is consumer's forgetting rate. The advertising cost is given by the increasing non-negative function $c_m(m_t)$.

Consumers are described by random utility functions.³ That is, a consumer's utility for choosing an alternative is composed of two parts: a deterministic component encapsulating the observable consumer-alternative attributes that shape the consumer's choice; and a random component that depends on the idiosyncratic unobserved attributes representing the latent utility. In particular, at each time period t , a consumer who has not adopted the product faces a binary choice of buying it or waiting until the next purchasing occasion to make a decision. Without any loss of generality, we normalize the utility of not adopting to zero. This implies that a consumer will not adopt the product if she does not perceive a positive utility at least once during the planning horizon. Formally, at time t , an individualist j who does not yet possess the product ($S_{jt} = 0$) faces a binary choice, i.e., adopt or not. If she adopts the product ($A_{jt} = 1$), her future state $S_{j,t+1}$ switches to 1, and she enjoys the following utility:

$$U_{jt} = U^I(p_t, G_t) + \epsilon_{jt}, \quad (2)$$

where $U^I(p_t, G_t)$ is the deterministic component, and $\epsilon_{jt} \in [-M_\epsilon, M_\epsilon]$ is the random component. We assume that U^I is a continuous function increasing in the goodwill and decreasing in the price. If individualist j does not adopt at time t ($A_{jt} = 0$), her state at time $t + 1$ stays 0 and she gains a zero per-step utility. The random components ϵ_{jt} are assumed independent and identically distributed (i.i.d.). We summarize the individualist's dynamics and utility in Figure 1.

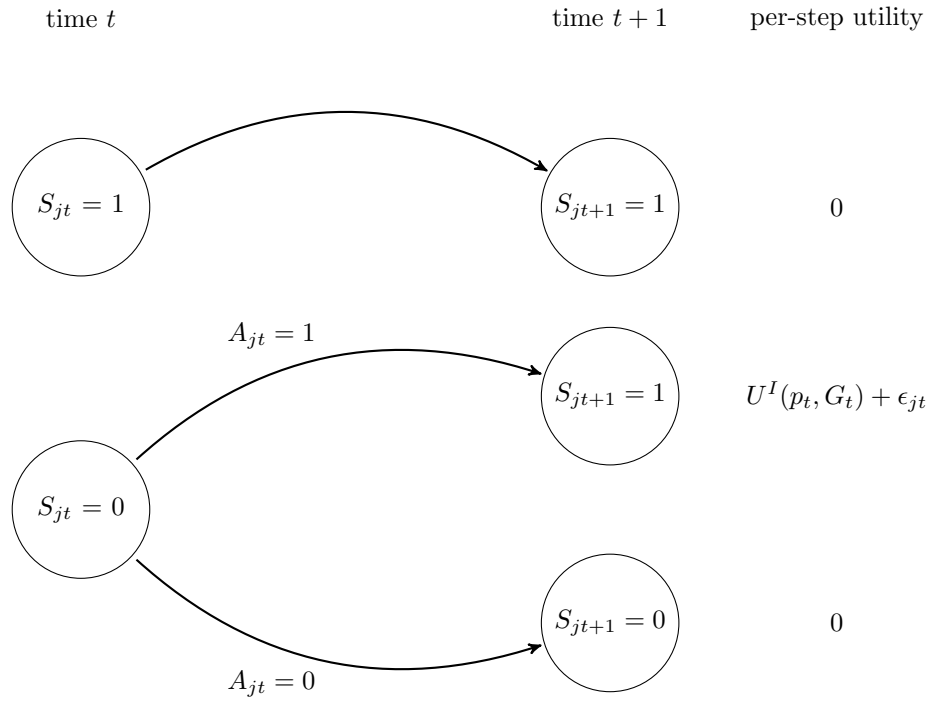
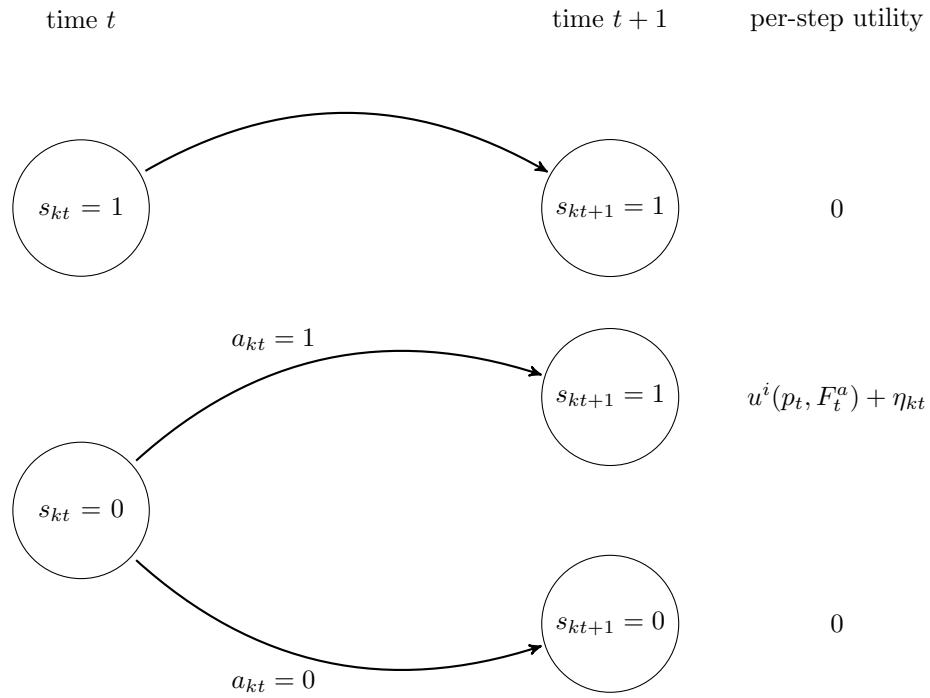
We assume, as for individualists, that conformists' utility function depends on the product's price. Differently from individualists, conformists' adoption decision is shaped by the degree of social acceptance of the product, which is measured by the percentage of consumers who have already acquired it, i.e., $F_t^a := \frac{1}{N} \left(\sum_{j=1}^{N^I} S_{jt} + \sum_{k=1}^{N^I} s_{kt} \right)$, for $t \in \{0, \dots, T\}$. Consequently, we have

$$u_{kt} = u^i(p_t, F_t^a) + \eta_{kt}, \quad (3)$$

where u^i is a continuous function, decreasing in p_t and increasing in F_t^a , and $\eta_{kt} \in [-M_\eta, M_\eta]$ are random utilities that are i.i.d. Note that we do not include the goodwill as an argument in the utility function, because G_t is already (indirectly) affecting u_{kt} through S_{jt} . We summarize the conformist's dynamics in Figure 2.

² Conceptually, we could only require to have $p_t \geq 0$ and $m_t > 0$. The upper bounds M_p and M_m , which can be arbitrary large, are needed for the existence and numerical computation of the equilibrium.

³ The theory of random utility was introduced by Thurstone (1927) and further developed by Lancaster (1966) and McFadden (1974). We refer the reader to Corstjens and Gautschi (1983) for an overview.

Figure 1: Individualist j dynamics and per-step utilityFigure 2: Conformist k dynamics and per-step utility

In determining the timing of adoption, consumers of both types act strategically, i.e., they are forward-looking. Whereas a myopic consumer purchases the product at the first date at which her utility is positive, a strategic consumer anticipates her future payoffs and adopts at the date that yields the highest positive utility. Technically speaking, each consumer solves an intertemporal optimization problem whose output is either no adoption throughout the planning horizon, or an adoption date. Strategic consumer behavior is well-documented in marketing (Su, 2007) and in operations management (see the survey in Wei and Zhang (2017)), and is significant in the context of introducing a new product (Besanko and Winston, 1990). One reason for postponing adoption can be the expectation of a drop in future prices, or to acquire more information about the product from the reviews posted by earlier adopters (Economist, 2009). These reviews are considered in the literature as a good proxy for WoM communications (Chevalier and Mayzlin, 2006).

Define by $f_t^a = \frac{1}{N} \left(\sum_{j=1}^{N^I} A_{jt} + \sum_{k=1}^{N^i} a_{kt} \right)$ the fraction of consumers who adopt the product at time t . The firm maximizes the following utility function:

$$U^f = \sum_{t=0}^{T-1} (p_t f_t^a - c_m(m_t)) + s(G(T)), \quad (4)$$

where $s(G(T))$ is an increasing salvage-value function.

Remark 1 It should be noted that $A_{jt} = 1$ (resp. $a_{kt} = 1$) if and only if $S_{jt+1} - S_{jt} = 1$ (resp. $s_{kt+1} - s_{kt} = 1$). Thus,

$$f_t^a = \frac{1}{N} \left(\sum_{j=1}^{N^I} S_{jt+1} + \sum_{k=1}^{N^i} s_{kt+1} \right) - \frac{1}{N} \left(\sum_{j=1}^{N^I} S_{jt} + \sum_{k=1}^{N^i} s_{kt} \right) = F_{t+1}^a - F_t^a.$$

We suppose that the firm pre-announces its price and advertising paths at the beginning of the planning horizon (Dasu and Tong, 2010). Aviv and Pazgal (2008) showed that in the presence of strategic consumers, pre-announcing prices is more profitable for the seller (up to 8.32% more profits) than is responsive pricing (see also Dasu and Tong (2010)). Examples of the implementation of pre-announced prices include Wanamaker's discount department store in Philadelphia, Pricetack.com, Tuesday Morning discount stores, Land's End Overstocks, Sam's club, Dress for Less, and TKTS ticket booths in London and New York City (Yin et al., 2009; Liu et al., 2019). The pre-announced advertising plan can be interpreted as the contract between the firm and a marketing agency defining the promotional activities to be implemented throughout the entire planning horizon.

We summarize the notation introduced above in Table 1.

Table 1: Notation

Consumers		Firm	
Notation	Description	Notation	Description
N^I	Number of individualists	p_t	Price decision variable
A_{jt}	Individualist adoption decision variable	m_t	Advertising decision variable
S_{jt}	Individualist adoption state variable	G_t	Goodwill state variable
U_{jt}, U^I and ϵ_{jt}	Individualist utilities	U_f	Profit
N^i	Number of conformists	c_m	Advertising cost
a_{kt}	Conformist adoption decision variable		
s_{kt}	Conformist adoption state variable		
u_{kt}, u^i and η_{kt}	Conformist utilities		
$N = N^I + N^i$	Number of consumers		
F_t^a	Fraction of adopters		
f_t^a	Instantaneous fraction of adopters		

3.1 A mean-field game

In making their adoption decision, consumers interact strategically with the firm and among themselves. We model this context as a game played à la Stackelberg with the firm acting as leader and consumers as followers.

Denote by $p_{0:T-1} = (p_0, \dots, p_{T-1})$ and $m_{0:T-1} = (m_0, \dots, m_{T-1})$ the firm's price and advertising strategies, respectively. We look for open-loop (global) Stackelberg solutions (Basar and Olsder, 1999), where the firm pre-announces its strategies $(p_{0:T-1}, m_{0:T-1})$ before the start of the game. The individualists react to these strategies and decide their optimal adoption time, and the conformists respond to the individualists' adoption rate while reacting to each others à la Nash. That is, the conformists play a non-cooperative game among each others, where a conformist's decision depends on the other conformists' decisions through the fraction of adopters. Therefore, the information states for the different players are as follows. Individualist j observes at time t her current state S_{jt} , the random utility ϵ_{jt} , the prices $p_{0:T-1}$ and goodwill profile $G_{0:T}$ and makes a decision. Hence, the information state of individualist j at time t is $\mathcal{I}_{jt}^I = \{S_{jt}, \epsilon_{jt}, p_{0:T-1}, G_{0:T}\}$. Conformist k observes her current state s_{kt} , the random utility η_{kt} , the prices $p_{0:T-1}$, the individualist fraction of adopters $\pi_{0:T}^{N^I} = \frac{1}{N^I} \sum_{j=1}^{N^I} S_{j0:T}$ and the current fraction of adopters F_t^a . Her information state is $\mathcal{I}_{kt}^I = \{s_{kt}, \eta_{kt}, p_{0:T-1}, \pi_{0:T}^{N^I}, F_t^a\}$. Since the firm pre-announces its strategies before the start of the game, its information state at time t is $\mathcal{I}_t^f = \{G_0\}$. As we shall see later, the MFG methodology allows the conformists to anticipate the fraction of adopters F_t^a , which will be dropped from their information state in the following sections.

To compute a Stackelberg equilibrium, one starts by determining the reaction functions of the followers to any announcement made by the leader. Typically in a hierarchical multistage game with many followers, one assumes that each follower's decision is affected by each of the other followers.⁴ In our case, a conformist does not need to know who precisely are the consumers who already adopted the product; she only needs to know the fraction of adopters in the social system. Similarly, the firm only needs to know the aggregate market reaction function to its policies and not the specific reaction of each of the N consumers, where (recall) N is a very large number. Consequently, such information structure points out towards the theory of mean-field games, which provides a powerful methodology to analyze dynamic games involving a large number of weakly couple players.

The basic ideas of a MFG approach are as follows: It starts by considering an infinite number of players and capitalizes on the weak coupling and law of large numbers to anticipate the coupling term (mean-field). Indeed, the mean-field under the players' Nash strategies satisfies two coupled backward-forward equations. The first, a backward dynamic programming equation, describes a generic player's best response to the mean-field. The second, a forward Chapman-Kolmogorov equation, propagates the mean-field under the players' best responses to it. The advantage of the infinite population case is that it reduces the game between an infinite number of players to that between a generic player and the mean-field. This is to be contrasted with the finite number of players case alluded to it above, where each individual is solely able to manipulate the game, and a Nash equilibrium is characterized by a number of equations proportional to that of the players. The infinite population assumption, however, makes the equilibrium less robust in face of unilateral deviant behaviors. In fact, when applied by a finite number of players, the mean-field strategies constitute an approximate Nash equilibrium. In particular, each player has a room to improve his profit by a unilateral deviation from the equilibrium. This improvement vanishes however as the number of players increases to infinite. The approximate Nash equilibrium is called ϵ -Nash (Huang et al., 2006, 2007) and is defined as follows:

Definition 1 Let S be a set. For all $N \in \mathbb{N}$, define on the set S^N the utility function of agent k , $1 \leq k \leq N$, $J_k^{(N)}(s_1, \dots, s_N)$. Let ϵ_N , $N \geq 0$, be a sequence of real numbers converging to 0 as $N \rightarrow \infty$. A strategy profile $\{s_i^*, i \in \mathbb{N}\}$ is called an ϵ_N -Nash equilibrium with respect to the utilities

⁴To illustrate, if the leader is a regulator and the followers are firms in an industry competing in prices, then each firm would take into account the price sets by each of the competitors.

$J_k^{(N)}$, if for all N , for any $1 \leq i \leq N$ and any $s_i \in S$, we have $J_i^{(N)}(s_i, s_{-i}^*) \leq J_i^{(N)}(s_i^*, s_{-i}^*) + \epsilon_N$, where $s_{-i}^* = (s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_N^*)$.

This ϵ_N -Nash equilibrium provides the reaction function to the leader, who then solves her optimization problem. Mean-field games theory was developed in a series of papers by Huang et al. (2003, 2007, 2006), and independently by Lasry and Lions (2006a,b, 2007). It has found applications ranging from transportation, energy systems and smart grids (Kizilkale et al., 2019) to finance. The first mean-field games marketing model was introduced by Salhab et al. (2018), whereby a producer makes advertising investments to sway consumer choices in favor of its product. Later, Salhab et al. (2019) propose a dynamic marketing and pricing MFG model involving a large number of online review sensitive consumers. For a comprehensive introduction to MFGs, see Caines et al. (2018).

3.2 Technical assumptions

In the following, we introduce our technical assumptions, which we assume to hold throughout the paper.

Assumption 1 *The random utilities $\{\epsilon_{jt}, \eta_{kt}, 1 \leq j \leq N^I, 1 \leq k \leq N^i, 0 \leq t \leq T\}$ are independent. The probability density functions (p.d.f) of ϵ_{jt} and η_{kt} are respectively f_ϵ and f_η . These p.d.f's are uniformly bounded by a positive scalar M_f .*

Assumption 1 is standard in the discrete-choice models literature (Rust, 1996, 1987; McFadden, 1974). An interesting class of probability distributions considered in this literature are the extreme value distributions.

Assumption 2 *The function u^i is Lipschitz continuous in F_t^a , with Lipschitz constant equal to L .*

Assumption 3 *As the size of population N increases to infinity, the individualists' and conformists' fractions N^I/N and N^i/N converge to $\Pi^I > 0$ and $\Pi^i = 1 - \Pi^I > 0$, respectively.*

Assumption 4 *The parameters f_ϵ , f_η , Π^I and Π^i are known by the firm and all the consumers.*

Assumptions 3, and 4 are standard in the MFGs literature.

4 Equilibrium results

We develop in this section a solution to our diffusion game using the MFGs methodology. The key feature of our model on which we rely is the form of interactions between the players. In particular, the consumers and firm interact only through the empirical distributions $(\pi_t^{N^I}, 1 - \pi_t^{N^I})$ and $(\pi_t^{N^i}, 1 - \pi_t^{N^i})$, where $\pi_t^{N^I} = \frac{1}{N^I} \sum_{j=1}^{N^I} S_{jt}$ and $\pi_t^{N^i} = \frac{1}{N^i} \sum_{k=1}^{N^i} s_{kt}$. Following the MFGs methodology, we assume throughout this section an infinite number of consumers. It is consistent to suppose that the couplings $\pi_t^I = \lim_{N \rightarrow \infty} \pi_t^{N^I}$ and $\pi_t^i = \lim_{N \rightarrow \infty} \pi_t^{N^i}$ in the infinite population are deterministic for the following reason. If all the consumers optimally respond to deterministic π_t^I and π_t^i , then their states at any time t are independent, and the Law of Large Numbers insures that $\pi_t^{N^I}$ and $\pi_t^{N^i}$ converge respectively to the deterministic probabilities $\pi_t^I = P(S_{jt} = 1)$ and $\pi_t^i = P(s_{kt} = 1)$. As a result, in the limiting problem, the conformists interact with each others through the cumulative fraction of adopters $F_t^a = \lim_{N \rightarrow \infty} \frac{1}{N} \left(\sum_{j=1}^{N^I} S_{jt} + \sum_{k=1}^{N^i} s_{kt} \right) = \Pi^I \pi_t^I + \Pi^i \pi_t^i$. The consumers, however, interact with the firm through the instantaneous fraction of adopters $f_t^a = F_{t+1}^a - F_t^a$.

4.1 The consumers' game

We suppose that the firm fixes and announces its strategies $(p_{0:T-1}, m_{0:T-1})$ before the start of the game. By the MFGs methodology, we assume that the consumers' flows of probabilities $\pi_{0:T}^I$ and

$\pi_{0:T}^i$ under Nash strategies are given for now. We show later that these flows always exist and can be computed by knowing f_ϵ , f_η , Π^I and Π^i . A generic individualist with information state $\mathcal{I}_t^I$ (we drop the index j to refer to a generic individualist in the infinite population) solves the following backward dynamic program (Bertsekas, 1995):

$$\begin{aligned} V_t^I(\mathcal{I}_t^I(1, \epsilon)) &= 0, \\ V_t^I(\mathcal{I}_t^I(0, \epsilon)) &= \max \left(u^I(p_t, G_t) + \epsilon, \int V_{t+1}^I(\mathcal{I}_{t+1}^I(0, \epsilon')) f_\epsilon(\epsilon') d\epsilon' \right). \end{aligned} \quad (5)$$

with $V_T^I = 0$. Here, $\mathcal{I}_t^I(S, \epsilon)$ refers to the realization of $\mathcal{I}_t^I$ when the vector (S_t, ϵ_t) takes the value (S, ϵ) , and V_t^I is the optimal utility-to-go at time t . The individualist's optimal choice at time t is then,

$$A_t^*(\mathcal{I}_t^I(0, \epsilon)) = \begin{cases} 0, & \text{if } \epsilon \leq \epsilon_t^*(\mathcal{I}_{1t}^I(0)), \\ 1, & \text{if } \epsilon > \epsilon_t^*(\mathcal{I}_{1t}^I(0)), \end{cases} \quad (6)$$

where $\mathcal{I}_{1t}^I = \mathcal{I}_t^I \setminus \{\epsilon_t\}$, $\mathcal{I}_{1t}^I(S)$ refers to the realization of $\mathcal{I}_{1t}^I$ when the vector S_t takes the value S , and

$$\epsilon_t^*(\mathcal{I}_{1t}^I(0)) = -u^I(p_t, G_t) + \int V_{t+1}^I(\mathcal{I}_{t+1}^I(0, \epsilon')) f_\epsilon(\epsilon') d\epsilon'. \quad (7)$$

It should be noted that once an individualist decides to adopt, she does not make any future decisions. For this reason, the optimal choice (6) is only defined on $\mathcal{I}_t^I(S, \epsilon) = \mathcal{I}_t^I(0, \epsilon)$. A generic conformist with information state $\mathcal{I}_t^i$ computes her best response to the cumulative fraction of adopters $F_{0:T}^a$ by solving the following backward dynamic program:

$$\begin{aligned} V_t^i(\mathcal{I}_t^i(1, \eta)) &= 0, \\ V_t^i(\mathcal{I}_t^i(0, \eta)) &= \max \left(u^i(p_t, F_t^a) + \eta, \int V_{t+1}^i(\mathcal{I}_{t+1}^i(0, \eta')) f_\eta(\eta') d\eta' \right), \end{aligned} \quad (8)$$

with $V_T^i = 0$. The conformist's best response is

$$a_t^*(\mathcal{I}_t^i(0, \eta)) = \begin{cases} 0, & \text{if } \eta \leq \eta_t^*(\mathcal{I}_{1t}^i(0)), \\ 1, & \text{if } \eta > \eta_t^*(\mathcal{I}_{1t}^i(0)), \end{cases} \quad (9)$$

where $\mathcal{I}_{1t}^i = \mathcal{I}_t^i \setminus \{\eta_t\}$, and

$$\eta_t^*(\mathcal{I}_{1t}^i(0)) = -u^i(p_t, F_t^a) + \int V_{t+1}^i(\mathcal{I}_{t+1}^i(0, \eta')) f_\eta(\eta') d\eta'. \quad (10)$$

We now turn to the problem of finding consistent flows of probabilities $\pi_{0:T}^I$ and $\pi_{0:T}^i$, i.e., equal to the generic individualist and conformist's distributions under their best responses (6) and (9) to these flows. Since an individualist's policy (9) does not depend on the flow of probabilities, then a consistent π^I is the unique solution of the following forward Chapman-Kolmogorov equation (Durrett, 2010):

$$\pi_{t+1}^I = \pi_t^I + (1 - \pi_t^I) \int_{\epsilon_t^*(\mathcal{I}_{1t}^I(0))}^{\infty} f_\epsilon(\epsilon') d\epsilon'^I, \quad \pi_0^I = 0. \quad (11)$$

This equation can be solved forward by knowing the firm's strategies. A consistent conformist's flow of probabilities $\pi_{0:T}^i$ must satisfy the following Chapman-Kolmogorov equation:

$$\pi_{t+1}^i = \pi_t^i + (1 - \pi_t^i) \int_{\eta_t^*(\mathcal{I}_{1t}^i(0))}^{\infty} f_\eta(\eta') d\eta'^i, \quad \pi_0^i = 0, \quad (12)$$

where $\mathcal{I}_{1t}^i(0)$ depends on $\pi_{0:T}^i$ itself. Thus, a consistent flow $\pi_{0:T}^i$ is a fixed point of the map $\pi_{0:T}^i \mapsto \mathcal{L}(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I)$, where $\mathcal{L} = \mathcal{L}_2 \circ \mathcal{L}_1$, $\mathcal{L}_1 : [0, 1]^{T+1} \times [0, M_p]^T \times [0, 1]^{T+1} \mapsto \mathbb{R}^T$ maps $(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I)$ to $(\eta_0^*(\mathcal{I}_{10}^i(0)), \dots, \eta_{T-1}^*(\mathcal{I}_{1T-1}^i(0)))$ defined by (10), and $\mathcal{L}_2 : \mathbb{R}^T \mapsto [0, 1]^{T+1}$ maps $(\eta_0, \dots, \eta_{T-1})$ to the probability flow $\pi_{0:T}^i$ generated by equation (12) with $\eta_t^*(\mathcal{I}_{1t}^i(0))$ replaced

by η_t , for $0 \leq t \leq T-1$. Unlike (11), equation (12) cannot be solved recursively since the threshold η_t^* depends on the entire trajectory $\pi_{0:T}^i$. Hence, the existence of solution is not trivial and is analyzed in Theorem 1 below.

Given the firm's strategy, the individualists react following a unique adoption rule given by (6). The fraction of adopters among the individualists is given then by the unique solution of (11). The conformists, however, may exhibit multiple adoption behaviors given by (9). Each behavior corresponds to a fraction of adopters among the conformists given by a fixed point of $\mathcal{L}$. As a result, the conformists' Nash equilibria are totally determined by the fixed points of $\mathcal{L}$.

In the following, we denote by $|x_s|_s = \max_s |x_s|$, where the maximum is taken over the domain of s . We now state the main result of this section which asserts that there always exists a Nash equilibrium for an infinite number of consumers.

Theorem 1 *The following statements hold:*

1. For all $\pi_{0:T}^i$ and $\bar{\pi}_{0:T}^i$ in $[0, 1]^{T+1}$, we have

$$|\mathcal{L}(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I)_t - \mathcal{L}(\bar{\pi}_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I)_t| \leq L_{\mathcal{L}} |\pi_t^i - \bar{\pi}_t^i|, \quad (13)$$

where $L_{\mathcal{L}} := M_f L \Pi^i \sum_{t=0}^{T-1} 2^{T-t-1} (T-t+1)$.

2. There exists at least one fixed point of $\mathcal{L}$. Equivalently, the limiting conformists' game has always a Nash equilibrium given by (9) for a fixed point $\pi_{0:T}^i$ of $\mathcal{L}$.
3. The mean field strategies (9), when applied by a finite number N of consumers, constitute an ϵ_N -Nash equilibrium (see Definition 1), where

$$|\epsilon_N| \leq L \left(\frac{1}{N} + \sqrt{\frac{4}{N} + \left(\frac{N^I}{N} - \Pi^I \right)^2 + \left(\frac{N^i}{N} - \Pi^i \right)^2} \right).$$

In the finite population, the Nash equilibrium is characterized by a set of $2N$ coupled equations. N backward dynamic program equations that describe the consumers' best responses, and N forward Chapman-Kolmogorov equations that propagate the consumer states' distributions under their best responses. When the number of consumers is large, solving for a Nash equilibrium becomes computationally intractable. The MFGs methodology leverages the game's symmetries to develop simple solutions described by only 4 equations (5), (8), (11), and (12). These solutions constitute a Nash equilibrium for the infinite population. When applied by a finite number of players, they induce a loss of performance in that the consumers can profit by unilateral deviation. According to the third point of Theorem 1, this loss of performance becomes negligible in large populations.

4.2 The firm's problem

As discussed in the previous section, the set of potential consumers' reactions to the firm's strategy is determined by the set of fixed points of $\mathcal{L}$. In order to design an optimal strategy, the firm needs to know how the consumers select a Nash equilibrium given her prices and advertising investments. To this end, we make the following assumption.

Assumption 5 *The conformists select a Nash equilibrium according to a predefined deterministic mechanism, which is given by a continuous "selection" function $S : [0, M_p]^T \times [0, 1]^{T+1} \mapsto [0, 1]^{T+1}$, that maps $(p_{0:T-1}, \pi_{0:T}^I)$ to a fixed point $\pi_{0:T}^i$ of $\mathcal{L}$.*

According to Assumption 5, the firm anticipates the consumers' reaction to its strategies. As a result, an optimal pricing and advertising strategy is a solution to the following optimization problem:

$$\begin{aligned}
 \max_{p_{0:T-1}, m_{0:T-1}} \quad & \sum_{t=0}^{T-1} (p_t f_t^a - c_m(m_t)), \\
 \text{s.t.} \quad & p_{0:T-1} \in [0, M_p]^T, \\
 & m_{0:T-1} \in [0, M_m]^T, \\
 & \pi_{0:T}^i = S(p_{0:T-1}, \pi_{0:T}^I), \\
 & f_t^a = \Pi^I(\pi_{t+1}^I - \pi_t^I) + \Pi^i(\pi_{t+1}^i - \pi_t^i),
 \end{aligned} \tag{14}$$

where $\pi_{0:T}^I$ satisfies (11). Under Assumption 5, the continuity of c_m , and the compactness of the firm's strategy set, problem (14) has at least one optimal solution.

4.3 Selection functions and numerical scheme

The existence of a firm's optimal policy requires the existence of a continuous selection function S . Recall that S maps $(p_{0:T-1}, \pi_{0:T}^I)$ to a root $\pi_{1:T}^i$ of the function

$$\Delta L(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I) := \pi_{0:T}^i - \mathcal{L}(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I).$$

We describe in the following a general procedure to construct an approximate continuous selection function. Let us assume that there exists an algorithm of the following form that converges to a root $\pi_{0:T}^i$,

$$(\pi_{0:T}^i)^{(n+1)} = (\pi_{0:T}^i)^{(n)} - \mathcal{F}\left((\pi_{0:T}^i)^{(n+1)}, p_{0:T-1}, \pi_{0:T}^I\right), \tag{15}$$

where $\mathcal{F}$ is a continuous function. Moreover, suppose that the algorithm converges uniformly, i.e., for every $\epsilon > 0$, there exists $n_\epsilon > 0$, such that for all $n > n_\epsilon$, $\sup_{p_{0:T-1}, \pi_{0:T}^I} |\Delta L((\pi_{0:T}^i)^{(n)}, p_{0:T-1}, \pi_{0:T}^I)| < \epsilon$.

Thus, for each $\epsilon > 0$, one can define an approximate selection function $S_\epsilon(p_{0:T-1}, \pi_{0:T}^I) = (\pi_{0:T}^i)^{(n_\epsilon)}$, which, following our assumptions, is a continuous function of $(p_{0:T-1}, \pi_{0:T}^I)$. S_ϵ is approximate in the sense that $\sup_{p_{0:T-1}, \pi_{0:T}^I} |\Delta L((S_\epsilon(p_{0:T-1}, \pi_{0:T}^I), p_{0:T-1}, \pi_{0:T}^I)| < \epsilon$, i.e. $S_\epsilon(p_{0:T-1}, \pi_{0:T}^I)$ is almost a fixed point. The family of algorithms in (15) includes a large number of members, such as Newton's and the fixed-point iterations methods (Ortega and Rheinboldt, 1970), which we discuss in details later.

In the following, we propose an algorithm to compute an optimal solution $(m_{0:T-1}, p_{0:T-1})$ of (14). The algorithm includes two nested loops. The external one is the projected gradient descent method (Bertsekas, 1999) that solves for a maximizer $(m_{0:T-1}^*, p_{0:T-1}^*)$ of the firm's utility, $U^f = \sum_{t=0}^{T-1} (p_t f_t^a - c_m(m_t))$, with $\pi_{0:T}^i = S_\epsilon(p_{0:T-1}, \pi_{0:T}^I)$, and $f_t^a = \Pi^I(\pi_{t+1}^I - \pi_t^I) + \Pi^i(\pi_{t+1}^i - \pi_t^i)$.

The iterations of the external loop are as follows:

$$\begin{aligned}
 m_t^{(n+1)} &= \pi_{[0, M_m]} \left(m_t^{(n)} + \xi \frac{\partial U^f}{\partial m_t} \left(m_t^{(n)}, p_t^{(n)} \right) \right), \\
 p_t^{(n+1)} &= \pi_{[0, M_p]} \left(p_t^{(n)} + \xi \frac{\partial U^f}{\partial p_t} \left(m_t^{(n)}, p_t^{(n)} \right) \right),
 \end{aligned} \tag{16}$$

for $1 \leq t \leq T-1$, where $\xi > 0$ and π_C is the Euclidean projection on the set C . Recall that the Euclidean projection of a point x on a cube $C = [a, b]^k$ is

$$\pi_C(x_1, \dots, x_k) = (\min(b, \max(a, x_1)), \dots, \min(b, \max(a, x_k))).$$

The partial derivatives are computed using the finite difference formulas. This involves computing the value of U^f at different $(m_{0:T-1}, p_{0:T-1})$, and more specifically, the fraction of adopters $f_{0:T-1}^a$, which is computed in an internal loop according to iterations (15). If the algorithm converges to

a global maximum $(m_{0:T-1}^*, p_{0:T-1}^*)$ of U^f , then $m_{0:T-1}^*$ and $p_{0:T-1}^*$ are the optimal marketing and pricing policies of the firm, respectively. In addition to the optimal strategies, one can anticipate the evolution of adoption or the cumulative fraction of adopters $F_{0:T}^a = \Pi^I \pi_{0:T}^I + \Pi^i \pi_{0:T}^i$, where $\pi_{0:T}^i = S_\epsilon(m_{0:T-1}^*, \pi_{0:T}^I)$, and $\pi_{0:T}^I$ is the solution of (11).

4.3.1 Newton's method

In Newton's method,

$$\mathcal{F}(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I) = (J(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I))^{-1} \Delta L(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I),$$

where J is the finite difference approximation of the derivative of ΔL with respect to $\pi_{0:T}^i$. The main challenge here is the non-smoothness of $\mathcal{F}$, which may result from the non-smoothness of the max operator in (5) and (8), and that of the functions u^I and u^i . A remedy would be to assume that u^I and u^i are smooth and to replace the function $\max(x, y) = \frac{1}{2}(x + y + |x - y|)$ by the smooth function $\frac{1}{2}(x + y + \sqrt{(x - y)^2 + \epsilon})$, where ϵ is a small positive number. The uniform convergence follows in this case from (i) the smoothness of $\mathcal{F}$, and (ii) the compactness of the firm's strategy set.

4.3.2 Fixed-point iterations method

In the fixed-point iterations method,

$$\mathcal{F}(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I) = \pi_{0:T}^i - \mathcal{L}(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I).$$

In this case, iterations (15) take the following form:

$$(\pi_{0:T}^i)^{(n+1)} = \mathcal{L}\left((\pi_{0:T}^i)^{(n)}, p_{0:T-1}, \pi_{0:T}^I\right). \quad (17)$$

The following assumption guarantees that $\mathcal{L}$ is a contraction in $\pi_{0:T}^i$, and by Banach fixed point Theorem (Berinde, 2007), the iterations (17) converge to the unique fixed point of $\mathcal{L}$.

Assumption 6 *We assume that $L_{\mathcal{L}} < 1$, where $L_{\mathcal{L}}$ is defined in Theorem 1.*

In this case, the selection map S sends $(p_{0:T-1}, \pi_{0:T}^I)$ to the unique fixed point of $\mathcal{L}$. The following Theorem shows that S is continuous.

Theorem 2 *The selection function S , defined as the unique fixed point of $\mathcal{L}$, is continuous.*

5 Numerical simulations

In this section, we illustrate our model with some numerical examples and provide insights on the interplay between the diffusion process, consumers behavior and firm's marketing strategies. In what follows, we specify the individual level consumer decision process as well as firms' decision problem, and start by determining the marketing strategies and penetration curves for each segment and aggregate market in a benchmark case. Next, we carry out sensitivity analysis with respect to the proportion of individualists to conformists, WoM and price parameters. Afterwards, we examine how the results differ in two cases. In the first case, we add a goodwill-dependent salvage value to the firm's optimization problem, and next suppose that consumers are myopic. The first case captures the impact of future benefits on the marketing strategies implemented during the current planning horizon. The second case allows us to see how the new product diffusion and marketing strategies vary with consumer's type (myopic or strategic).

5.1 Benchmark case

We consider a binary discrete choice model by which the consumers decide to whether buy the new product or not, and if yes, when to do so. As in Keeney and Raiffa (1993), we assume that the deterministic components in (2) and (3) of consumer utilities are additive, that is,

$$\begin{aligned} U^I(p_t, G_t) &= k_p^I U_p(p_t) + k_G U_G(G_t) \\ u^i(p_t, F_t^a) &= k_p^i u_p(p_t) + k_{F^a} u_{F^a}(F_t^a) \end{aligned} \quad (18)$$

where k_p^I, k_p^i, k_G and k_{F^a} are scaling coefficients that represent the significance of each uni-attribute utility function in the decision process. We consider linear price utilities, i.e., $U_p(p_t) = A^I - \beta^I p_t$ and $u_p(p_t) = A^i - \beta^i p_t$, and exponential forms utility with respect to goodwill and WoM, i.e., $U_G(G_t) = B^I - \alpha^I \exp(-r^I G_t)$ and $u_{F^a}(F_t^a) = B^i - \alpha^i \exp(-r^i F_t^a)$. Linear and exponential utilities are widely adopted in the literature (e.g., Chatterjee and Eliashberg (1990)) and capture constant risk-aversion, which was supported by empirical studies in marketing literature (e.g., Hauser and Urban (1977)). Without loss of generality, we assume that k_p^I, k_p^i, k_G and k_{F^a} are equal to 1, and A^I, A^i, B^I and B^i are equal to 0. We suppose that the random utilities ϵ_{it}^I and η_{it}^i are uniformly distributed on the unit interval $[0, 1]$.

The goodwill dynamics are given by a discrete time version of Nerlove-Arrow's model (Nerlove and Arrow, 1962), that is,

$$G_{t+1} = m_t + (1 - \gamma)G_t, \quad G(0) = G_0, \quad (19)$$

where $0 < \gamma < 1$ is the decay rate of goodwill stock, and G_0 the initial goodwill. The advertising cost is assumed quadratic and equal to $c_m(m_t) = \frac{1}{2}cm_t^2$, where $c > 0$. The values of the different parameters in the benchmark case are given in Table 2. According to the literature, the percentages of individualists Π^I and conformists Π^i depend on the type of product. Based on empirical studies for 11 durable products, Mahajan et al. (1990) adopt Bass framework and report that the relative impact of external influences to internal ones can range nearly from 1% to 20%. However, Van den Bulte and Joshi (2007) argue that the proportion of individualists to conformists can vary in large intervals from zero to 1 depending on the type of innovation. For instance, for a low-risk innovation, the size of conformists in a population can be relatively small. In the benchmark case, we consider the percentage of individualists to be 30% given the salient presence of imitative behavior in social system shown in the literature (e.g., Steffens and Murthy (1992) and Mahajan et al. (1990)).

Table 2: Benchmark

Consumer's parameters		Firm's parameters	
Parameter	Description	Parameter	Description
$\alpha^I = 0.9$	Scaling constant	$c = 0.01$	Advertising cost coefficient
$\alpha^i = 0.7$	Scaling constant	$\gamma = 0.95$	Decay rate
$\beta^I = 0.6$	Individualists' price sensitivity	$G_0 = 1$	Initial goodwill
$\beta^i = 0.6$	Conformists' price sensitivity		
$r^I = 0.1$	Goodwill sensitivity		
$r^i = 1$	WoM sensitivity		
$\Pi^I = 0.3$	Fraction of individualists		
$\Pi^i = 0.7$	Fraction of conformists		
$\pi_0^I = 0$	Initial individualist's flow probability		
$\pi_0^i = 0$	Initial conformist's flow probability		

Figures 3(a) and 3(b) report the pricing and advertising strategies in the benchmark case. Our results show that the monopolist adopts a penetration pricing strategy. This strategy is in response to consumers forward-looking behavior that can potentially cause adoption delay if future deals become more appealing. This is to be contrasted to the literature that widely finds skimming strategy for myopic consumers unless the WoM communications are strong whereas penetration or mixed strategies are recommended (e.g., Kalish (1983, 1985); Horsky (1990)).⁵ We find that the marketer maintains

⁵Note, however, that we do not account for cost learning in our model, which typically also leads to a decrease in price over time

this pricing strategy even when the WoM effect is weak in order to neutralize the effect of forward-looking behavior, which is shown later in Figure 9(a). From Figure 3(b), we see that advertising expenditures monotonically decrease over time, a result that has often been obtained in the literature (see the survey in Huang et al. (2012)). One conclusion here is that the firm behaves in the same way when we retain an individual-based diffusion model with strategic consumers. In fact, the rationale for advertising heavily at early stages remains the same, that is, to stimulate demand from individualists, which facilitates social contagion through WoM, and consequently incentivizes conformists to adopt the product.

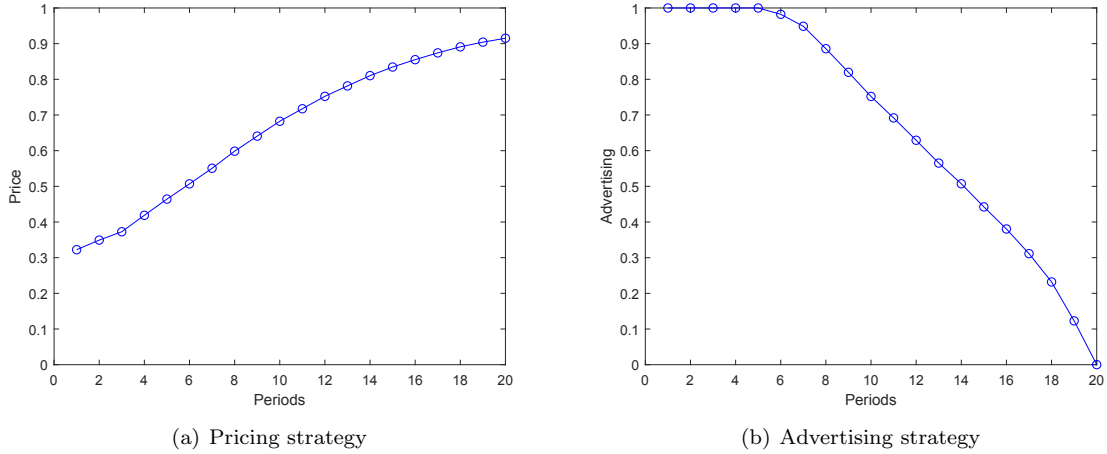


Figure 3: Marketing strategies in the benchmark case

Figures 4(a) and 4(b) exhibit the penetration curves in the individualists and conformists segment, respectively, and Figure 4(c) depicts the total penetration curve. While the general finding in the literature is an *S*-curve for penetration and a bell-shaped noncumulative distribution, these figures (and those to come later in the sensitivity analysis) show that different adoption curves can materialize, depending on consumer's response to external and internal influences as well as the composition of segments. In the benchmark case, we obtain a "flattened" *S*-curve and a slightly skewed bell-shaped noncumulative diffusion. The individualists' diffusion curve has an *S*-curve form due to the trade-off between goodwill effect and saturation effect over time, where the former has a function similar to WoM for the conformists (see Figure 4(a)). The early diffusion for this segment is slowly increasing given the low initial stock of goodwill till the point that aggressive advertising campaign markedly accelerates the individualists' diffusion. This trend continues until the saturation effect overcomes goodwill effect where the number of individualists not having adopted yet starts declining. Additionally, the individualists diffusion curve is consistent with Van den Bulte and Joshi (2007) findings, namely, that the magnitude of adoption does not decrease monotonically.

The trade-off between WoM and saturation effects act differently in our framework. Every forward-looking player in conformists segment anticipates the future outcomes and responds optimally to the mass adoption behavior of players. The individualists diffusion path is a priori in the conformist's intertemporal problem and hence she would focus on how to respond to mean-field equilibrium and firm's pricing strategy. When price and WoM have the same (with opposite sign) effect on conformists adoption decisions, then their total impact vanishes, and adoption is then solely driven by the random utility whose uniform distribution results in a linear cumulative diffusion curve as shown in Figure 4(b).

5.2 Sensitivity analysis

In this section, we run a series of sensitivity analysis to assess the impact of some parameter values on the results.

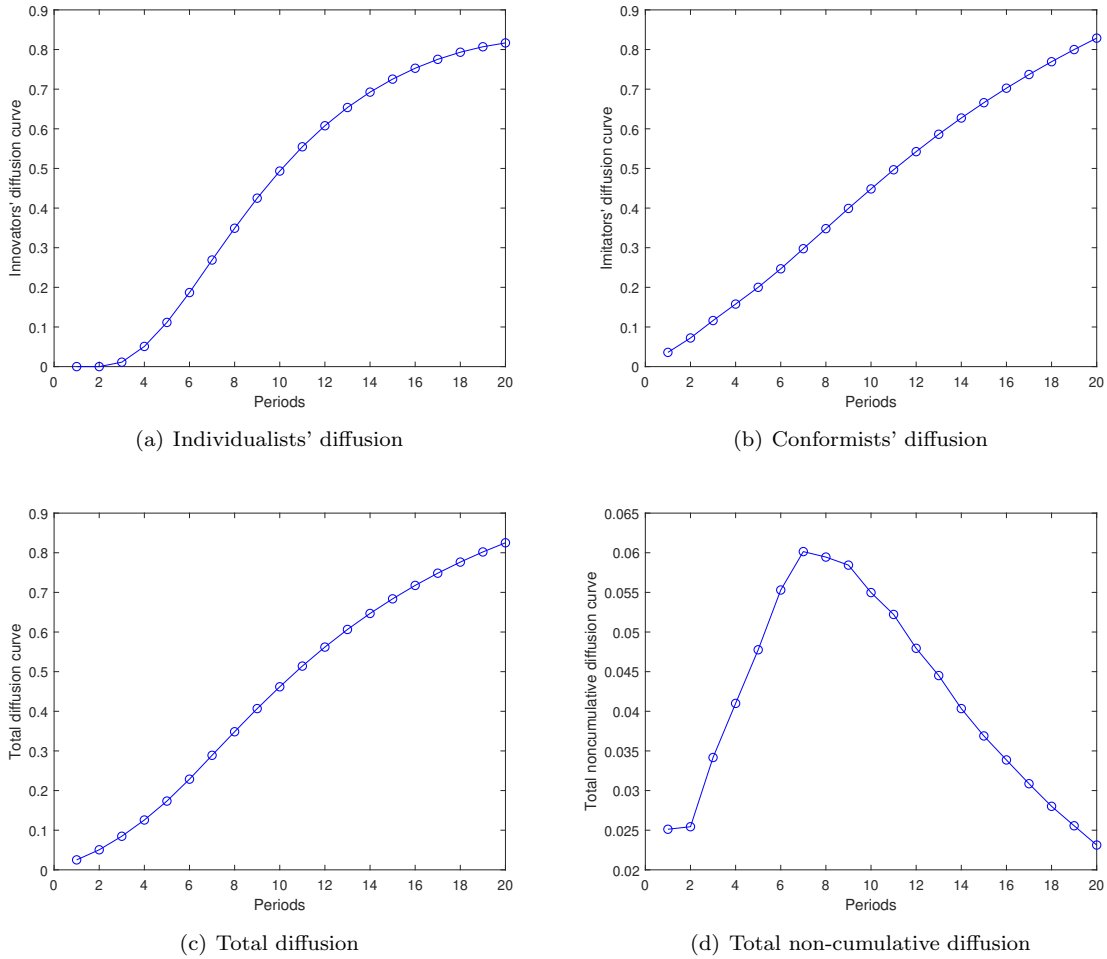


Figure 4: Penetration curves

5.2.1 Type of consumers

One important feature in our model is the distinction between conformists and individualists consumers. In the benchmark case, the fraction of individualists was set equal to 30%. In Figures 5(a) and 5(b) we provide the pricing and advertising trajectories, respectively, for the following fractions of individualists: 5%, 10%, 30%, 50% and 95%, that is, two below the benchmark value and two above it.

Figure 5(a) shows that a penetration pricing strategy remains the optimal choice for the firm regardless of the relative importance of the two segments. Note that the difference in price levels is very small. To illustrate, if we consider the price trajectories obtained with individualists accounting for 5% and 95%, respectively, that is, we multiply the former fraction by 19 to obtain the second one, we obtain a variation in price that is at most 25%. A main conclusion is that the price level is almost independent of consumers' types. It is as if the firm is counting on goodwill (respectively, WoM) to boost adoption when the fraction of individualists is high (respectively, low).

From Figure 5(b), we see that the shape of the advertising trajectory is the same for all considered configurations of the two segments, that is, the firm starts by advertising heavily in the early periods and decreases its expenditures over time. Interestingly, contrary to prices, the advertising levels vary significantly with the fraction of individualists, with the lower this fraction, the lower the advertising level. The reason is that advertising only targets individualists and the firm adapts its effort to the importance of their share in the society/market.

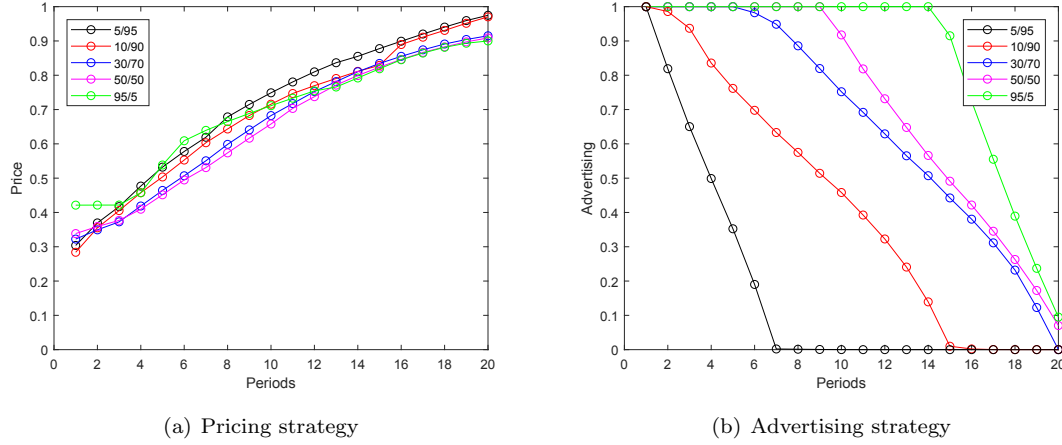
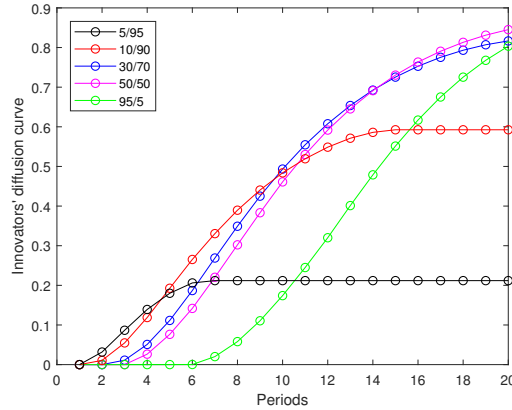


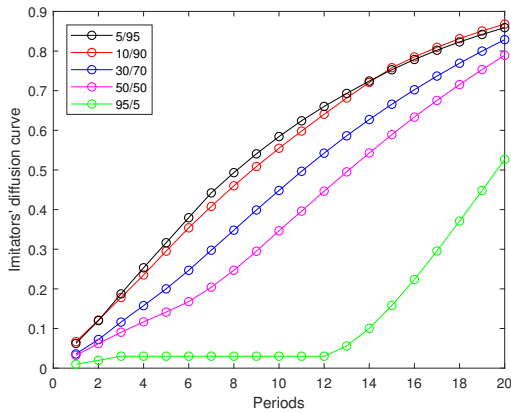
Figure 5: Marketing strategies for different fraction of individualists

The three panels in Figure 6(a) exhibit the adoption rate over time in the individualists and conformists segments and in the whole market, respectively. We can make the following two remarks: First, depending on the segments compositions, the adoption curves can be *S*-shaped, convex or concave, with the total penetration curve becoming more clearly *S*-shaped when the proportion of individualists to conformists is increased. These results highlight the importance of accounting explicitly for the two segments, and their different buying motives, in understanding the adoption dynamics. In particular, when individualists segment is small, their social contagion role, through WoM communication, is of lesser importance. In fact, here conformists themselves are the main contributor to this contagion. Second, if the planning horizon is long enough, as here with 20 periods, the final penetration percentage is almost the same for total diffusion across all configurations, however, it takes lower value in individualists and conformists diffusion curves for cases involving too low and too high fraction of individualists, respectively. When this fraction is below a certain threshold, the final penetration rate in individualists segment remains relatively low. Recall that in this case (of low fraction of individualists), the firm is investing little effort in advertising. Consequently, the goodwill, which is a main driver of individualists demand, is plateauing at a low level. For high fraction of individualists, the final penetration rate is lower in conformists' segment due to lower contribution of this group to WoM communications and to forward-looking effect. More precisely, most of diffusion trajectory in this case is predetermined in conformists intertemporal problem, which makes their early adoption less important for other consumers, and therefore they can enjoy from stronger WoM effect in later purchasing occasions.

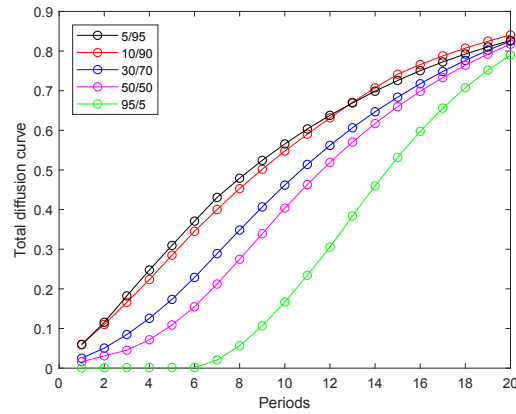
Finally, we look at the impact of segments composition on the firm's profit. Figure 7 shows that the total profit is a convex function in the fraction of individualists, with the minimum taking place at around one third. One conclusion is that if the firm has a say in the size of the two segments, then it would vote for a market populated as much as possible by individualists. Given our previous observation that prices vary little with the fraction of individualists, any explanation of this result should be related to the two forms of communication that influence adoption in the social system, i.e., paid-for advertising and WoM. Our results indicate that investing in goodwill pays off when the fraction of individualists exceeds a certain threshold, despite advertising being a costly activity. This can be explained by the fact that the firm deploys a more aggressive advertising campaign resulting in higher goodwill. The firm is offering, down the road, to the forward-looking consumers a better product (i.e., a product having a higher goodwill), but at a higher price, with the positive adoption effect of goodwill being larger (in absolute) value than the negative effect of a higher price (see, for example, 95/5 case in Figure 6(a)). On the other hand, when the fraction of individualists is sufficiently low, the firm needs to advertise less, and counts on WoM communications to stimulate the diffusion process (see, for example, the 5/95 case in Figure 6(b)).



(a) Individualists' diffusion



(b) Conformists' diffusion



(c) Total diffusion

Figure 6: Penetration curves for different proportions of individualists to conformists

5.2.2 Price, goodwill and WoM coefficients

Recall that the deterministic parts of the utility functions in the individualists and conformists segments are given, respectively, by

$$U^I(p_t, G_t) = -\left(\beta^I p_t + \alpha^I e^{-r^I G_t}\right),$$

$$u^i(p_t, F_t^a) = -\left(\beta^i p_t + \alpha^i e^{-r^i F_t^a}\right).$$

In what follows, we assess the impact of varying the price and WoM sensitivity parameters on the results.

As it is fully expected, increasing consumer's sensitivity to price, that is, having a higher β , leads to a lower price (see Figure 8(a)). Note that for all considered values of β , the pricing strategy remains of the penetration type, which is reminiscent to our assumption that consumers are strategic.

One important insight is that the firm uses a penetration pricing strategy even when WoM coefficient is low. As mentioned earlier, the firm uses this pricing strategy to avoid that forward-looking consumer waits to adopt, a result to be contrasted to the literature that widely finds skimming strategy, however, under the assumption that consumers are myopic. Furthermore, increasing r^i has two effects

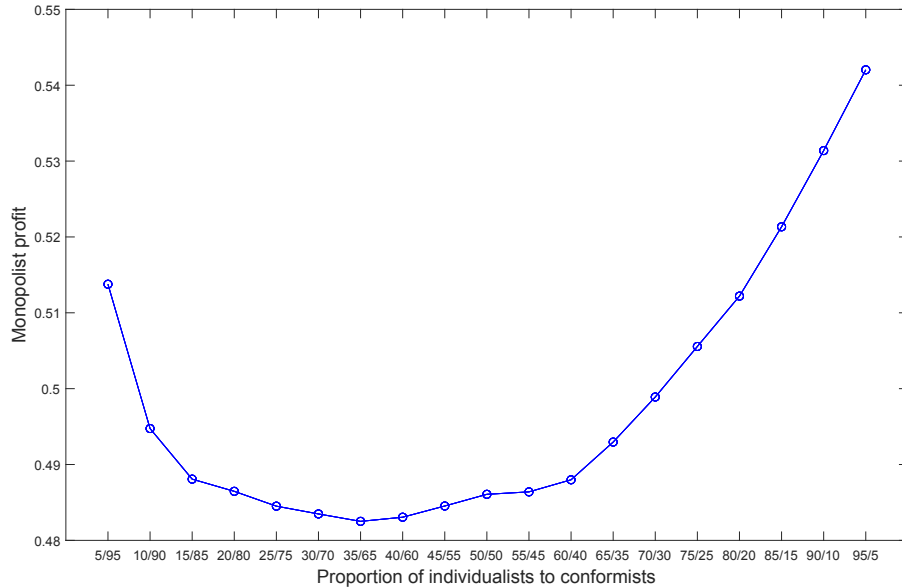


Figure 7: Monopolist profit for different compositions of market segments

on firm's strategy, namely, higher price and higher advertising spending (see Figure 9(b)). The intuition behind this result is straightforward. Indeed, the larger r^i , the larger the conformists' marginal utility of adoption, independently of the price level. This implies that the firm can afford to increase its price without much damage to demand. Further, to benefit from higher marginal contagion effect, the firm invests more in advertising (when r^i is higher) to increase buying by individualists who influence adoption by conformists. The impact of varying r^i on adoption over time is reported in Figures 10(a) and 10(b). As one could expect, the behavior of individualists is not much affected, whereas a higher r^i slows considerably adoption by conformists in early stages. The difference in adoption curves becomes smaller over time as the higher WoM effect compensates for the higher price.

Figures 11(a) and 11(b) show marketing strategies when individualists have different sensitivities towards goodwill effect. Like for high WoM coefficient, the firm can exploit the individualists' high marginal utility of adoption and charge higher price while maintaining the demand when individualists are more sensitive to goodwill. However, it can accelerate adoption rate with lesser advertising investments by relying on impact of high goodwill sensitivity that maintains the same final penetration rate.

5.3 Salvage value

All results so far have been obtained under the assumption that the firm does not account for any potential revenue after the end of the planning horizon. One reported impact of such assumption in the dynamic advertising models literature is the monotonic decline over time of advertising spending, reaching eventually zero at the last period (see the survey in Huang et al. (2012)). To verify this result in our context, we added a linear function of goodwill at terminal time to the firm's payoff function, given by $s(G(T)) = bG(T)$, where b is a positive parameter that is assumed to be equal to 20. As we can clearly see from Figure 13(a), having a positive salvage value does not have any significant impact on the pricing strategy. However, the advertising strategy is affected as the firm reverts towards the end of the planning horizon the decline in its advertising effort and increases expenditures sharply. In fact, the firm wants to raise its goodwill, i.e., invests in its future business. In short, the result is nothing but surprising.

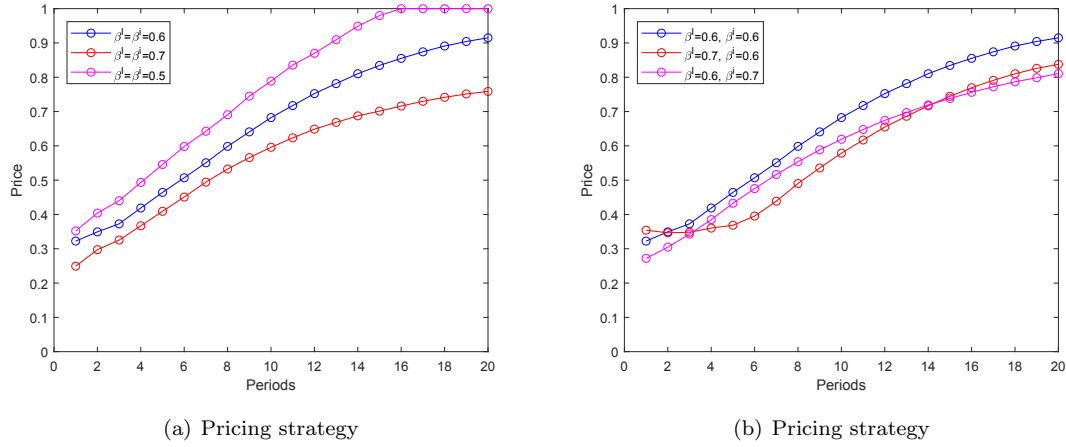


Figure 8: Pricing strategies under different price sensitivities

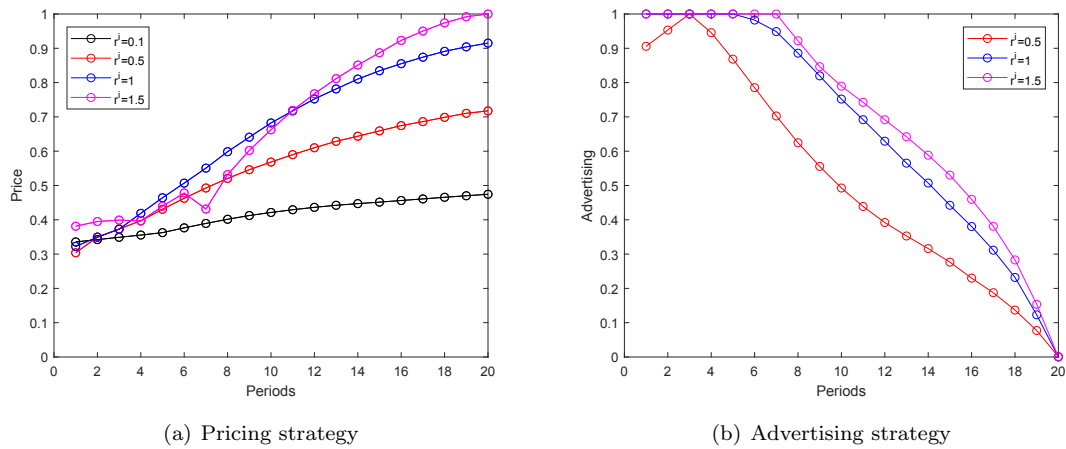


Figure 9: Marketing strategies under different WoM sensitivities

5.4 Myopic case

Up to now, we assumed that the consumers are strategic (or forward-looking), that is, they solve an inter-temporal optimization problem to decide whether and when to adopt the product. In this section, we consider the case where the consumers are myopic, which means that, at each period, they solve a static optimization problem, and adopt at the first occasion the expected utility turns out to be positive. By comparing the results to the benchmark case, we can shed a light on the impact of myopia on the firm's strategies and outcome and on the adoption curves in both market segments.

The pricing strategy is not monotone anymore. The firm starts by implementing a penetration strategy followed by a skimming one in a cyclic manner (see Figure 14(a)). The strong goodwill effect plays a similar role like WoM effect for the individualists and paves the way for marketer to implement penetration strategy to target individualist segment. It is followed by skimming one to gradually leaves the stage for WoM effect and favor emergence of second segment. The strong WoM effect then facilitates the penetration strategy till the point that combined WoM and goodwill effects dominate saturation one where the monopolist begins to cream skim the remaining untapped market. The cyclic penetration-skimming strategy is not optimal when consumers are forward-looking since they can delay their adoption for appealing future prices that can decelerate the magnitude and rate of diffusion at

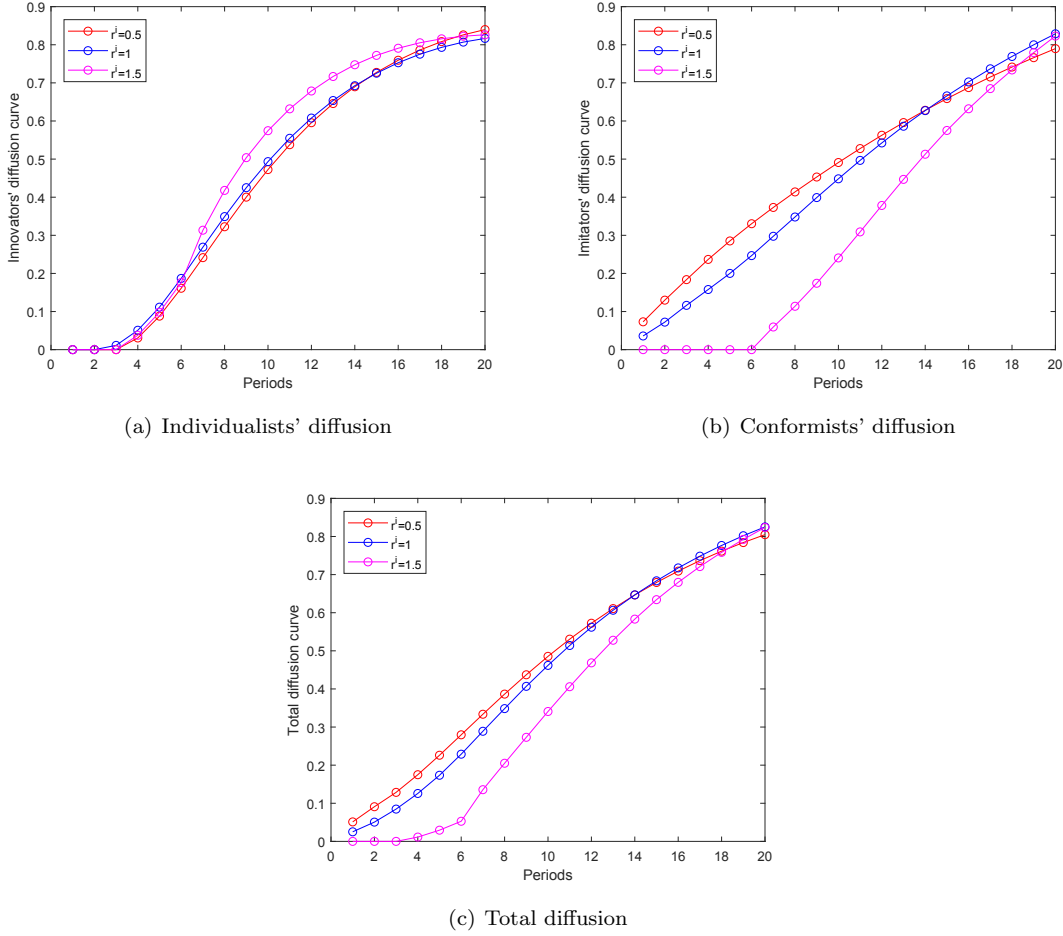


Figure 10: Penetration curves under different WoM sensitivities

both segments. During the first half of the game, the price in the myopic-consumer case is clearly higher than in the benchmark. These higher initial prices drive conformists demand to zero, and lead to a decrease in adoption by individualists with respect to the benchmark (see Figure 15(a)). Demand by individualists is positive because the negative effect of a high price is lessened by the product's goodwill, which is fed by the heavy advertising during these periods. Demand by conformists start picking up when the WoM effect starts compensating for the expensive price. In the early stages, the contagion effect is due to adoption by individualists.

A series of studies have reported that strategic consumers, who typically wait for bargains before buying, affect negatively a seller's profit. This is also the result we reach here. Whereas the firm makes a profit of 0.4835 when consumers are strategic, it realizes a profit of 0.5501 when consumers are myopic, that is an increase of 13.7%. The reason is clear from the discussion above, i.e., the firm exploits myopic consumers by raising its price.

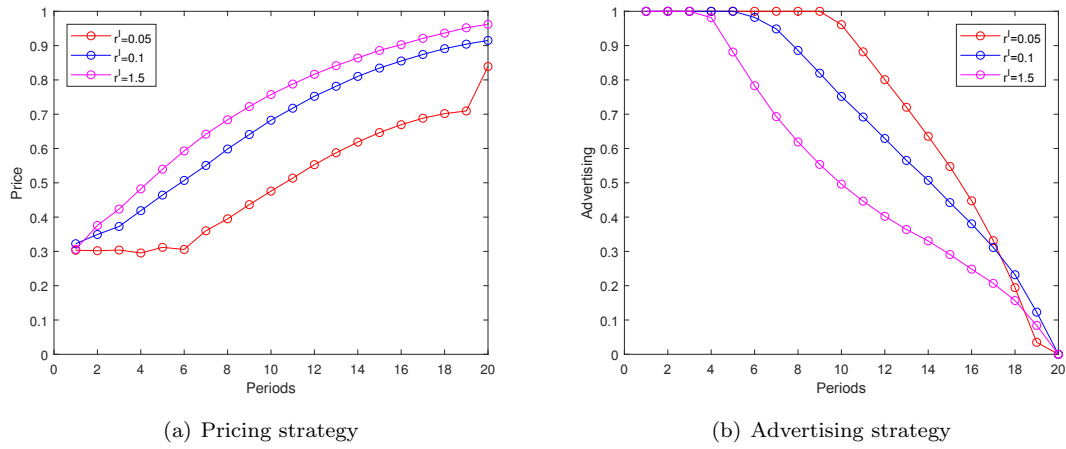


Figure 11: Marketing strategies under different goodwill sensitivities

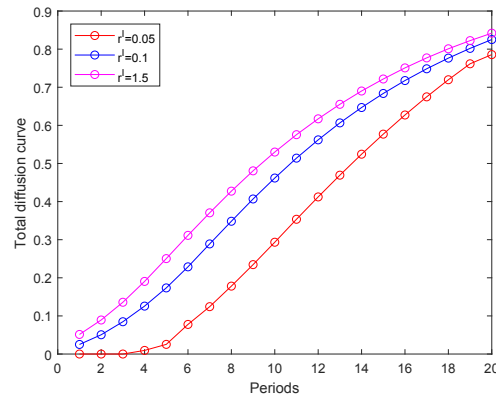


Figure 12: Total diffusion under different goodwill sensitivities

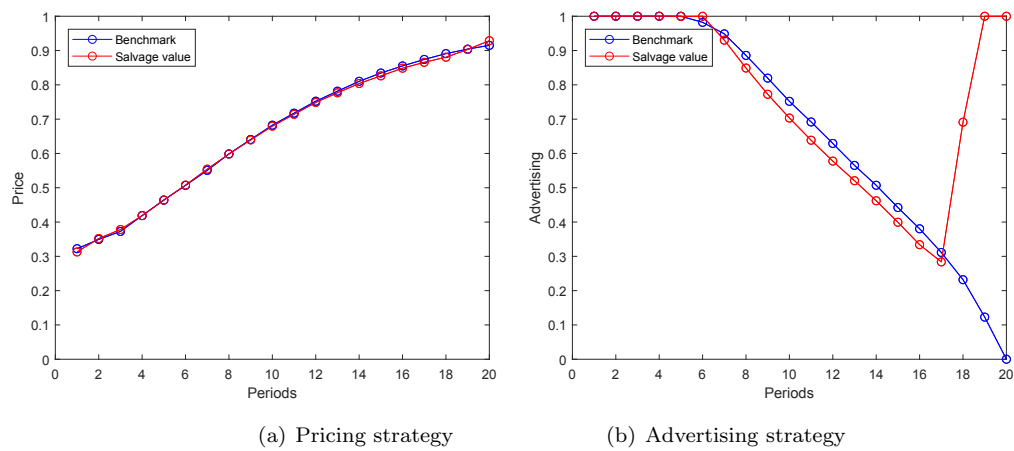


Figure 13: Marketing strategies

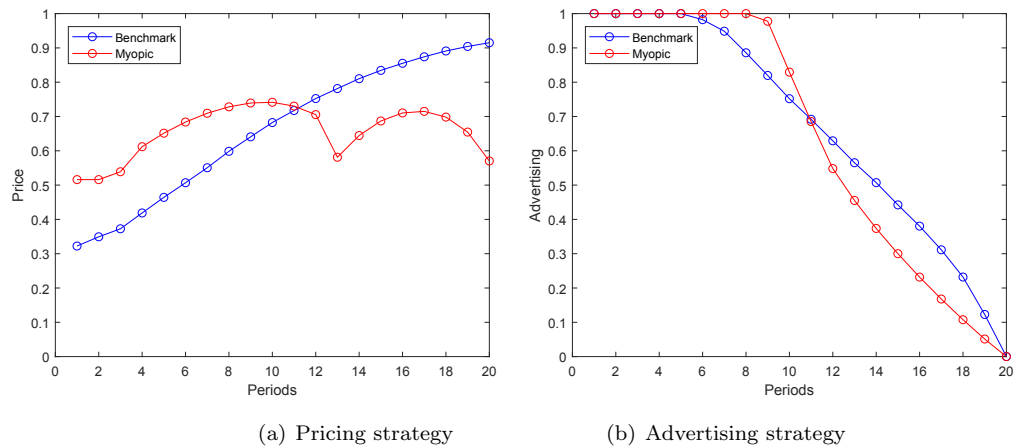


Figure 14: Marketing strategies in the myopic case

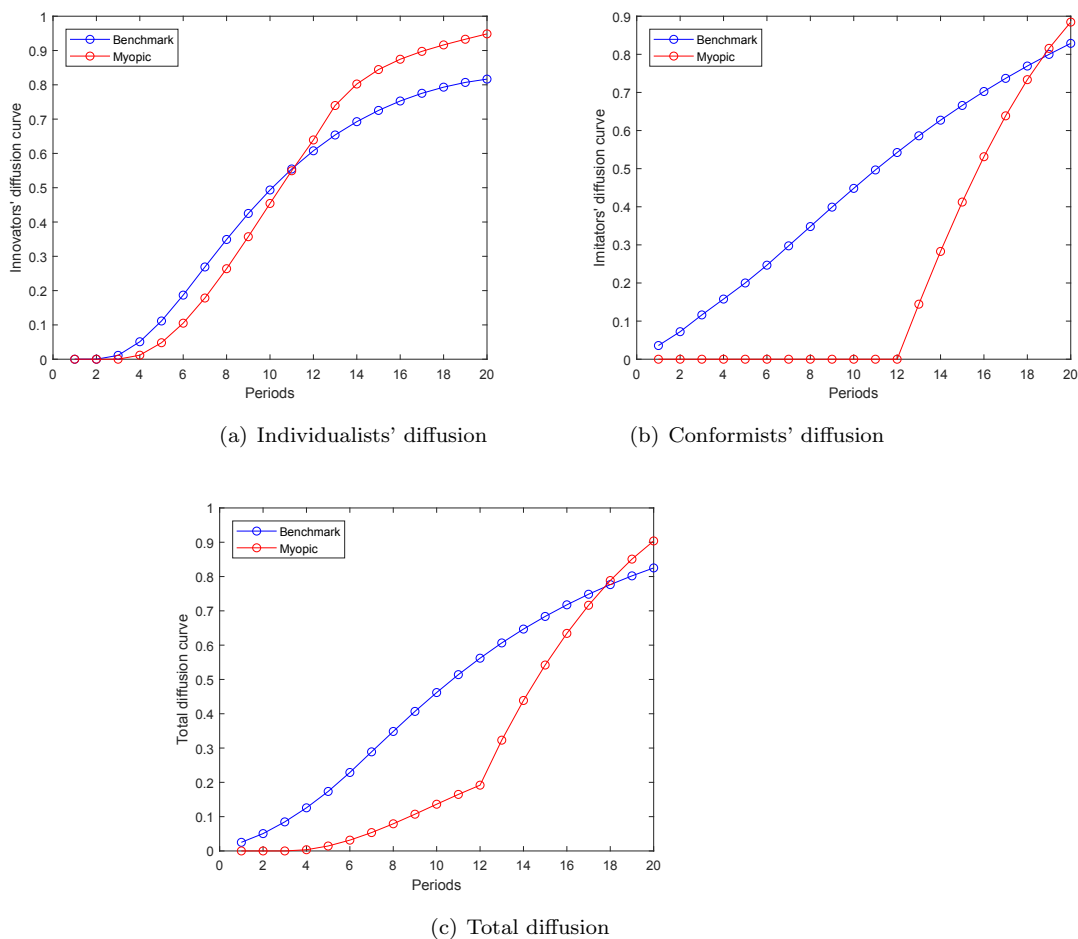


Figure 15: Penetration curves

6 Conclusion

This paper is an exploratory first step in understanding adoption dynamics in a context characterized by the presence of strategic consumers, who are either individualists and conformists, having different adoption drivers. The equilibrium among consumers is shaped by social contagion and the pricing and advertising of the firm. To the best of our knowledge, this is the first attempt to integrate in the same model forward-looking consumers, two-segment market structure, and pricing and advertising strategies in a fully dynamic framework. Also, it is the first application of mean-field games, an area witnessing an astonishing growth, to new product diffusion.

The main takeaways of this study are as follows:

1. The firm adopts a penetration pricing strategy when consumers are strategic. This result holds for all parameter constellations, even when WoM effect is low. Interestingly, a mix of penetration and skimming pricing strategies in a cyclic manner materialize when consumers are myopic.
2. Advertising strategy is highly intuitive: invest heavily in early stages to build the goodwill, incentivize individualists to adopt early, and trigger a social contagion effect.
3. The penetration curves can take different forms including S -curve, concave and convex depending on the mixture of individualists and conformists in the social system.
4. The firm earns higher profit when the market tends to be essentially populated by one type of consumers.
5. Individualists' and conformists' adoption processes are different, which fully justify our two-segment model.
6. The numerical results have been shown to be largely robust to variations in the parameter values.

As in any modelling work, we made some assumptions that should be relaxed in future work. Let us first consider two long-shot extensions. First, we assumed absence of competition. The new product diffusion literature is replete of monopoly models, which signals a methodological difficulty in introducing formally competition. This difficulty is huge in the context of MFGs, where a theory of multistage equilibria are yet to be developed. Second, we assumed that the firm preannounces its strategy from the outset. It is definitely a welcome move to attempt to consider feedback (state-dependent) strategies. Again, the characterization of feedback Stackelberg equilibria is still out of reach for the moment.

Modest but highly relevant extensions include (i) an attempt to have both strategic and myopic consumers in the market, and (ii) estimation of the parameter distributions (see Assumptions 4) and segment sizes from the game's output, for example, adoption rate data.

A Appendix: Proofs and preliminary results

Lemma 1 *The following statements hold:*

1. We have for all $\epsilon \in [0, M_\epsilon]$ and $\eta \in [0, M_\eta]$,

$$\begin{aligned} |V_t(\mathcal{I}_t^I(0, \epsilon))| &\leq |u^I(p, G)|_{p,G} + M_\epsilon, \\ |V_t(\mathcal{I}_t^i(0, \eta))| &\leq |u^i(p, F)|_{p,F} + M_\eta. \end{aligned} \quad (20)$$

2. For all $\pi_{0:T}^i$ and $\bar{\pi}_{0:T}^i$ in $[0, 1]^{T+1}$, we have,

$$|V_t^i(\mathcal{I}_t^i(0, \eta)) - V_t^i(\bar{\mathcal{I}}_t^i(0, \eta))|_\eta \leq L \sum_{\tau=t}^T |F_\tau^a - \bar{F}_\tau^a|, \quad (21)$$

$$|\eta_t^*(\mathcal{I}_{1t}^i(0)) - \eta_t^*(\bar{\mathcal{I}}_{1t}^i(0))| \leq L \sum_{\tau=t}^{T-1} |F_\tau^a - \bar{F}_\tau^a| \quad (22)$$

where F_t^a and $\mathcal{I}_t^i$ correspond to $\pi_{0:T}^i$, and $\bar{F}_t^a$ and $\bar{\mathcal{I}}_t^i$ correspond to $\bar{\pi}_{0:T}^i$.

Proof. The first point is a direct consequence of (5) and (8). We prove the second point using the equality $\max(a, b) = (a + b + |a - b|/2)$. We have

$$\begin{aligned} V_t^i(\mathcal{I}_t^i(0, \eta)) &= \frac{1}{2} \left(u^i(p_t, F_t^a) + \eta + \int V_{t+1}^i(\mathcal{I}_{t+1}^i(0, \eta')) f_\eta(\eta') d\eta' \right) \\ &\quad + \frac{1}{2} \left| u^i(p_t, F_t^a) + \eta - \int V_{t+1}^i(\mathcal{I}_{t+1}^i(0, \eta')) f_\eta(\eta') d\eta' \right|. \end{aligned} \quad (23)$$

Hence, for all $\pi_{0:T}^i$ and $\bar{\pi}_{0:T}^i$ in $[0, 1]^{T+1}$,

$$|V_t^i(\mathcal{I}_t^i(0, \eta)) - V_t^i(\bar{\mathcal{I}}_t^i(0, \eta))|_\eta \leq L|F_t^a - \bar{F}_t^a| + |V_{t+1}^i(\mathcal{I}_{t+1}^i(0, \eta)) - V_{t+1}^i(\bar{\mathcal{I}}_{t+1}^i(0, \eta))|_\eta. \quad (24)$$

(21) follows by induction, and (21) and (10) imply (22). $\square$

Proof of Theorem 1

1. Following (12) and (22), we get

$$\begin{aligned} &|\mathcal{L}(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I)_{t+1} - \mathcal{L}(\bar{\pi}_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I)_{t+1}| \leq \\ &2|\mathcal{L}(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I)_t - \mathcal{L}(\bar{\pi}_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I)_t| + M_f L \sum_{\tau=t}^T |F_\tau^a - \bar{F}_\tau^a|. \end{aligned} \quad (25)$$

This implies,

$$|\mathcal{L}(\pi_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I)_t - \mathcal{L}(\bar{\pi}_{0:T}^i, p_{0:T-1}, \pi_{0:T}^I)_t| \leq M_f L \sum_{t_2=0}^{t-1} \sum_{t_1=t_2}^T 2^{t-t_2-1} |F_{t_1}^a - \bar{F}_{t_1}^a|. \quad (26)$$

But, $F_{t_1}^a - \bar{F}_{t_1}^a = \Pi^i(\pi_{t_1}^i - \bar{\pi}_{t_1}^i)$. This implies the first point.

2. The second point follows from the continuity of $\mathcal{L}$ (first point of the Theorem) and Brouwer's fixed point theorem (Conway, 1985, Section V.9).
3. Fix $1 \leq k_0 \leq N^i$, and let $t_{k_0} \in \{0, \dots, T-1\}$ be the adoption time of conformist k_0 , and $a_{k_0 0:t_{k_0}}$ and $s_{k_0 0:T}$ the corresponding action and state, where $a_{k_0 0:t_{k_0}-1} = 0$, $a_{k_0 t_{k_0}} = 1$, $s_{k_0 t} = 0$ if $t \leq t_{k_0}$, and $s_{k_0 t} = 1$ if $t > t_{k_0}$. Denote by $A_{j 0:T-1}^*$ and $a_{k 0:T-1}^*$, $1 \leq j \leq N^I$, $1 \leq k \leq N^i$, the individualists' and conformists' mean field strategies (6) and (9) that correspond to the unique solution $\pi_{0:T}^I$ of (11) and a fixed point $\pi_{0:T}^i$ of $\mathcal{L}$. Denote by $S_{j 0:T}^*$ and $s_{k 0:T}^*$ the corresponding states. Define the fraction of adopters under the mean field strategies $F_t^{aN} = \frac{1}{N} \left(\sum_{j=1}^{N^I} S_{jt}^* + \sum_{k=1}^{N^i} s_{kt}^* \right)$, and the fraction of adopter when conformist k_0 deviates from the mean field strategies

$$F_{-k_0, t}^{aN} = \frac{1}{N} \left(\sum_{j=1}^{N^I} S_{jt}^* + \sum_{k=1, k \neq k_0}^{N^i} s_{kt}^* + s_{k_0 t} \right).$$

The utility of conformist k_0 when she deviates from the mean field strategies is

$$\begin{aligned} J_{k_0}^i(a_{k_0 0:t_{k_0}}, F_{-k_0, 0:T}^{aN}) &= \mathbb{E} \left(u^i(p_{t_{k_0}}, F_{-k_0, t_{k_0}}^{aN}) + \eta_{k_0 t_{k_0}} \right) \\ &= J_{k_0}^i(a_{k_0 0:t_{k_0}}, F_{-k_0, 0:T}^{aN}) - J_{k_0}^i(a_{k_0 0:t_{k_0}}, F_{0:T}^{aN}) &:= \xi_1 \\ &\quad + J_{k_0}^i(a_{k_0 0:t_{k_0}}, F_{0:T}^{aN}) - J_{k_0}^i(a_{k_0 0:t_{k_0}}, F_{0:T}^a) &:= \xi_2 \\ &\quad + J_{k_0}^i(a_{k_0 0:t_{k_0}}, F_{0:T}^a) - J_{k_0}^i(a_{k_0 0:T-1}^*, F_{0:T}^a) &:= \xi_3, \end{aligned}$$

where F_t^a is the mean field fraction of adopters, i.e $F_t^a = \Pi^i \pi_t^i + \Pi^I \pi_t^I$. By the definition of the mean field strategies, $\xi_3 \leq 0$. We have,

$$|\xi_1| = \left| \mathbb{E} \left(u^i(p_{t_{k_0}}, F_{-k_0, t_{k_0}}^{aN}) - u^i(p_{t_{k_0}}, F_{t_{k_0}}^{aN}) \right) \right| \leq L \mathbb{E} \left| F_{-k_0, t_{k_0}}^{aN} - F_{t_{k_0}}^{aN} \right| \leq \frac{L}{N}$$

The states S_{jt}^* , $1 \leq j \leq N^I$, are i.i.d., as well as the states s_{kt}^* , $1 \leq k \leq N^i$. Hence,

$$\begin{aligned} |\xi_2|^2 &= \left| \mathbb{E} \left(u^i(p_{t_{k_0}}, F_{t_{k_0}}^{aN}) - u^i(p_{t_{k_0}}, F_{t_{k_0}}^a) \right) \right|^2 \leq L^2 \mathbb{E} \left| F_{t_{k_0}}^{aN} - F_{t_{k_0}}^a \right|^2 \leq L \mathbb{E} \left| F_{t_{k_0}}^{aN} - F_{t_{k_0}}^a \right|^2 \\ &= L^2 \mathbb{E} \left(\frac{1}{N} \sum_{j=1}^{N^I} (S_{jt_{k_0}}^* - \mathbb{E} S_{jt_{k_0}}^*) \right)^2 + L^2 \mathbb{E} \left(\frac{1}{N} \sum_{k=1}^{N^i} (s_{kt_{k_0}}^* - \mathbb{E} s_{kt_{k_0}}^*) \right)^2 \\ &\quad + L^2 \left(\frac{N^I}{N} - \Pi^I \right)^2 \mathbb{E} (S_{jt_{k_0}}^*)^2 + L^2 \left(\frac{N^i}{N} - \Pi^i \right)^2 \mathbb{E} (s_{kt_{k_0}}^*)^2 \\ &\leq L^2 \left(\frac{4}{N} + \left(\frac{N^I}{N} - \Pi^I \right) + \left(\frac{N^i}{N} - \Pi^i \right)^2 \right). \end{aligned}$$

This shows the result.

Proof of Theorem 2

We have for all $(p_{0:T-1}, \pi_{0:T}^I)$ and $(p'_{0:T-1}, (\pi_{0:T}^I)')$,

$$\begin{aligned} &|S(p'_{0:T-1}, (\pi_{0:T}^I)')_t - S(p_{0:T-1}, \pi_{0:T}^I)_t|_t \\ &= |\mathcal{L}(S(p'_{0:T-1}, (\pi_{0:T}^I)'), p'_{0:T-1}, (\pi_{0:T}^I)')_t - \mathcal{L}(S(p_{0:T-1}, \pi_{0:T}^I), p_{0:T-1}, \pi_{0:T}^I)_t|_t \\ &\leq |\mathcal{L}(S(p'_{0:T-1}, (\pi_{0:T}^I)'), p'_{0:T-1}, (\pi_{0:T}^I)')_t - \mathcal{L}(S(p_{0:T-1}, \pi_{0:T}^I), p'_{0:T-1}, (\pi_{0:T}^I)')_t|_t \\ &\quad + |\mathcal{L}(S(p_{0:T-1}, \pi_{0:T}^I), p'_{0:T-1}, (\pi_{0:T}^I)')_t - \mathcal{L}(S(p_{0:T-1}, \pi_{0:T}^I), p_{0:T-1}, \pi_{0:T}^I)_t|_t. \end{aligned}$$

This implies that

$$\begin{aligned} &|S(p'_{0:T-1}, (\pi_{0:T}^I)')_t - S(p_{0:T-1}, \pi_{0:T}^I)_t|_t \leq \\ &\frac{1}{1 - L_{\mathcal{L}}} |\mathcal{L}(S(p_{0:T-1}, \pi_{0:T}^I), p'_{0:T-1}, (\pi_{0:T}^I)')_t - \mathcal{L}(S(p_{0:T-1}, \pi_{0:T}^I), p_{0:T-1}, \pi_{0:T}^I)_t|_t. \end{aligned}$$

The result follows from the continuity of $\mathcal{L}$, which can be shown using (5), (8), (11), and (12).

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