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Limit game models for climate change negotiation

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Abstract: This paper deals with a family of dynamic game models that represent schematically the interaction between groups of countries in achieving the necessary limitation of carbon atmospheric emissions in order to control climate change. We start from a situation where m coalitions of countries exist and behave as m players in a game of sharing a global emission budget through the establishment of an international emissions trading system. We characterize the Nash equilibrium solutions for this game in a deterministic context. Through a simple replication schemes, we increase the number of players, each one becoming infinitesimal and we characterize the limit games thus obtained. A stochastic version is proposed for this class of models and the limit games are characterized, using the recently defined concept of Mean Field Games.

1 Introduction

The Paris agreement, negotiated at COP-21 and signed by a majority of nations, is dedicated to limiting to less than 2 °C the surface average temperature (SAT) rise in the 21$^{\text{st}}$ century. To achieve this goal the participating countries must reduce their emissions of greenhouse gases (GHG). These emissions are mostly related to the use of fossil energy as an economic production factor. Recent research on mitigation policies have shown that a possible set of backstop technologies could be carbon dioxide removal (CDR), which refers to technologies that reduce the levels of carbon dioxide in the atmosphere. Among such technologies one finds, in particular, bio-energy with carbon capture and storage (BECCS), direct air capture, ocean fertilization, etc. [9, 16, 17, 19]. On the other hand, recent research on climate modeling tends to show that to limit to 2 °C the SAT rise with sufficiently high probability one should define a global limiting carbon budget of 1 trillion tons, over the whole period starting from the industrial revolution to the end of the 21$^{\text{st}}$ century [1, 14]. Up to now the world has emitted over 580 billion tons of carbon (GtC). So there is a remaining budget of a little more than 400 GtC to be shared among all the countries participating in the Paris agreement. Now the fundamental question which arises is 'how to share this remaining emissions budget among the more than 100 countries that signed the Paris agreement?'. This will certainly be a key element of the forthcoming negotiations on climate change. These countries are more or less formally regrouped in coalitions or groups that share similar economic conditions although not necessarily the same size nor the same economic power. Finally, in order to foster economic efficiency, an international emissions trading scheme with full banking and borrowing should be established. When countries are given a share of the remaining emission budget, they will tend to use these emission rights in a strategic way, in order to extract the most welfare benefits, taking into consideration the actions of the other countries that intervene on the international market (see [12] for a discussion of the strategic use of allowances). This implies that one should explore the equilibrium solutions of the dynamic game underlying these forthcoming burden sharing negotiations to decide what should be a fair allocation of this budget among the different coalitions or groups of countries. This question has been addressed in a stream of papers where a meta-game of burden sharing was built, using statistical emulation of a worldwide, multi-country, general equilibrium model, [3], [10], or an open-loop differential game model based on multi-coalition economic growth models with GHG emissions [4], [5]. In the present paper, we revisit this game theoretic approach with three important modifications introduced to take into account: (i) the fact that each coalition is composed of several countries that will also play non-cooperatively in the GHG emission games; (ii) the need to extend the theory to a fully stochastic environment; (iii) the important role of CDR in achieving the long term goal of climate sustainability.

We will work with a simple dynamic game paradigm that takes into consideration the fundamental aspects of the climate negotiations environment discussed above. We first introduce a deterministic open-loop differential game model and, in a second part, we extend the modeling to a stochastic diffusion game framework. In both cases we begin with a formulation of a Nash equilibrium played by a small number of coalitions and we look at the limit game obtained when each coalition is composed of an ensemble of independent small players who share a similar economy, defined by its productivity and rates of substitution factors. Our approach is strongly influenced by the recent developments of the theory of Mean Field Games (MFG) [7], [6], [13].

Caveat: *As our aim is to show how the concepts of limit or mean-field games can bring useful paradigms for the design of fair climate agreements, we will assume that all conditions for existence, convexity, convergence, etc, are satisfied for the deterministic and stochastic control systems that we introduce.*

2 An m-coalition deterministic model

In this section we propose a simple economic growth model with carbon emissions due to the use of fossil energy and possibility to invest in a CDR technology. We assume that the cumulative emission budget is shared among a small number m of coalitions of countries, like e.g. EU-28+Switzerland, USA-Canada-Australia-New Zealand, BRICs, Rest of the World. To realize efficiency an international emissions trading scheme is supposed to be installed with full banking and borrowing. Each coalition, considered as a 'big' player j, may then use the supply of emission permits on the market as a strategic variable in order to maximize returns from the emission budget share they control. The other strategic variable will be the investment in CDR technologies in order to reduce the amount of emissions to offset. The model will then be used to assess the welfare loss (WL) for each coalition j with reference to a Business as Usual (BAU) situation where there is no climate policy. We then compare the WLs associated with different possible allocation schemes of the cumulative carbon budget.

2.1 Model Formulation

2.1.1 Economy

Economic output (Y) occurs in the economies according to an extended Cobb-Douglas production function in three inputs, capital (K), labor (L) and fossil energy (the use of which is measured through emission level E):

$$Y(j,t;\cdot) = A(j,t)\,K(j,t)^{\alpha(j)}\,(\phi(j,t)\,E(j,t))^{\theta(j,t)}\,L(j,t)^{1-\alpha(j)-\theta(j,t)}. \tag{1}$$

Here $A(j,t)$ is the total factor productivity in the economy j at time t, $\alpha(j)$ is the elasticity of output with respect to a maleable stock of capital K, $\phi(j,t)$ is the energy conversion factor for emissions E in the economy$j$ at time t, and $\theta(j,t)$ is the elasticity of output with respect to emissions E. This production function is homogenous of degree 1.

Another stock of capital R is used to remove carbon. In this simple model we assume that the quantity $\varpi(j,t)$ of carbon removed at time t by coalition j is defined by a function

$$\varpi(j,t) = \Psi(j,t,R(j,t),L(j,t)), \tag{2}$$

which is assumed to be homogenous of degree 1. Economic output is used for consumption (C), investment (I) and the payment of energy costs:

$$Y(j,t) = C(j,t) + I_K(j,t) + I_R(j,t) + \pi(j,t)\,\phi(j,t)\,E(j,t), \tag{3}$$

where $\pi(j,t)$ is the energy price at period t in the economy of type j.

2.1.2 Emissions trading

One assumes that an international market for emissions trading exists. At each instant of time, given the quantities $\omega(j,t)$ put on the market by each player $j = 1,\dots,m$, the emission abatement decisions taken by the players are determined by the solution of a local optimization problem, where the maximized profit is given by:

$$Y(j,t) - \pi(j,t)\,\phi(j,t)\,\tilde{E}(j,t) + p(t,\Omega(t))\left[\omega(j,t) - \tilde{E}(j,t,p(t,\Omega(t)))\right],$$

where $p(t,\Omega(t))$ is the market carbon price, which depends on the the total permit supply $\Omega(t) = \sum_{j=1}^{m}\omega(j,t)$. $\tilde{E}(j,t;p(t,\Omega(t)))$ is the emission response in coalition j to carbon price $p(t,\Omega(t))$. The price $p(t,\Omega(t))$ is clearing the market, i.e., the following conditions hold:

$$\Omega(t) \geq \sum_{j=1}^{m}\tilde{E}(j,t,p(t,\Omega(t))), \tag{4}$$

$$0 = p(t,\Omega(t))\left[\Omega(t) - \sum_{j=1}^{m}\tilde{E}(j,t,p(t,\Omega(t)))\right]. \tag{5}$$

At time t the permit market price and emission response of each player j are thus defined by the equations:

$$0 = \frac{\partial Y(j,t;\cdot)}{\partial\tilde{E}(j,t,p(t,\Omega(t)))} - \pi(j,t)\phi(j,t) - p(t,\Omega(t)) \tag{6}$$

$$0 = \sum_{j=1}^{m}\tilde{E}(j,t,p(t,\Omega(t))) - \Omega(t). \tag{7}$$

Following the same developments as in [12], we can express the marginal influence of the supply of permits on the emission levels and market price. Taking derivatives of Equation (6) and Equation (7) w.r.t. w.r.t. $\Omega(t)$, denoted ', we obtain:

$$\begin{aligned} 0 &= \tilde{E}(j,t,p(t,\Omega(t)))' - \frac{p(t,\Omega(t))'}{\frac{\partial^2 Y(j,t;\cdot)}{\partial \tilde{E}(j,t,p(\Omega(t)))^2}} && (8) \\ 0 &= \sum_{j=1}^{m} \tilde{E}(j,t,p(\Omega(t)))' - 1. && (9) \end{aligned}$$

Therefore, the derivatives w.r.t. Ω of price and emission levels are given by:

$$\begin{aligned} p'(t,\cdot) \equiv \frac{d}{d\Omega} p(t,\cdot) &= \frac{1}{\sum_{j=1}^{m} \frac{1}{\frac{\partial^2 Y(j,t)}{\partial E(j,t)^2}}} && (10) \\ \tilde{E}(j,\cdot) \equiv \frac{d}{d\Omega} \tilde{E}(j,t,\cdot) &= \tilde{E}(j,t,p(t,\Omega))' \\ &= \frac{1}{\sum_{k=1}^{m} \frac{\frac{\partial^2 Y(j,t)}{\partial \tilde{E}(j,t)^2}}{\frac{\partial^2 Y(k,t)}{\partial \tilde{E}(k,t)^2}}} && (11) \end{aligned}$$

Since $\Omega(t) = \sum_{j=1}^{m} \omega(j,t)$ the partial derivatives w.r.t. $\omega(j,t)$ are the same as the derivatives w.r.t. $\Omega(t)$.

2.1.3 Capital and emission budaget dynamics

The dynamics of capital accumulation and GHG budget evolution is defined for $j = 1, \ldots, m$, by the following state equations:

$$\begin{aligned} \dot{K}(j,t) &= I_K(j,t) - \delta_K K(j,t) && (12) \\ \dot{R}(j,t) &= I_R(j,t) - \delta_R R(j,t) && (13) \\ \dot{B}(j,t) &= \Psi(j,t,R(j,t),L(j,t)) - \omega(j,t) && (14) \end{aligned}$$

where δ_K and δ_R are depreciation rates and initial conditions

$$\begin{aligned} K(j,0) &= K_j^o && (15) \\ R(j,0) &= R_j^o && (16) \\ B(j,0) &= B_j^o && (17) \end{aligned}$$

are given.

B_j^o is the share of the remaining cumulative emission budget, given to coalition j in the climate negotiations that take place at time $t = 0$. $\Psi(j,t,\cdot,\cdot)$ is a production function, assumed to be concave in both arguments, which determines how much dioxide carbon is removed in region j at time t, given the stock of CDR capital accumulated and the population in that region. In Equation (14) we have assumed that the CDR activity increases the budget. All state and control variables have to remain non negative, $K(j,t) \geq 0$, $R(j,t) \geq 0$, $B(j,t) \geq 0$.

2.1.4 Payoffs

The payoff for player j is the discounted sum of welfare over the planning horizon $[0,T]$, which can be written as follows:

$$\begin{aligned}\mathrm{WRG(j)} = \int_0^{\mathrm{T}} \mathrm{e}^{-\rho \mathrm{t}} \mathrm{L(j,t)} \log \Bigg[\frac{1}{\mathrm{L(j,t)}} \Big(& \mathrm{Y(j,t)} - \pi\mathrm{(j,t)}\phi\mathrm{(j,t)}\tilde{\mathrm{E}}\mathrm{(j,t,}\Omega\mathrm{(t))} \\ & - I_K(j,t) - I_R(j,t) + p(t,\Omega(t))\big(\omega(j,t) - \tilde{E}(j,t,\Omega(t))\big)\Big)\Bigg] \, dt \\ & + e^{-\rho T} \mathcal{V}(j,T,\mathbf{K}(T),\mathbf{R}(T),\mathbf{B}(T)) \end{aligned} \tag{18}$$

where $\mathcal{V}(j,T,\mathbf{K}(T),\mathbf{R}(T),\mathbf{B}(T))$ is the value at terminal time t for player j associated with the terminal state $(\mathbf{K}(T),\mathbf{R}(T),\mathbf{B}(T))$ of the m players. Here $\tilde{E}(j,t,\Omega(t))$ denotes the emission response of coalition j at time t, given the total supply $\Omega(t)$ of permits on the carbon market.

2.2 Open-loop Nash equilibrium conditions

For the ease of notation we introduce the utility function:

$$\begin{aligned} U_j(\cdot) &= L(j,t) \log \Bigg[\frac{1}{L(j,t)} \Big(Y(j,t) - \pi(j,t)\phi(j,t)\tilde{E}(j,t,\Omega(t)) - I_K(j,t) \\ &\quad - I_R(j,t) + p(t,\Omega(t))\big(\omega(j,t) - \tilde{E}(j,t,\Omega(t))\big)\Big)\Bigg] \end{aligned} \tag{19}$$

and the new consumption function, taking into consideration the payment for traded permits:

$$\begin{aligned} C(j,t) &= Y(j,t) - \pi(j,t)\phi(j,t)\tilde{E}(j,t,\Omega(t)) - I_K(j,t) \\ &\quad - I_R(j,t) + p(t,\Omega(t))\big(\omega(j,t) - \tilde{E}(j,t,\Omega(t))\big). \end{aligned} \tag{20}$$

For each player j define the Hamiltonian:

$$H_j(\cdot) = U_j(\cdot) + q_K(j,t)\dot{K}(j,t) + q_R(j,t)\dot{R}(j,t) + q_B(j,t)\dot{B}(j,t) - \vartheta(j,t)B(j,t), \tag{21}$$

where q_K, q_R and q_B are current valued costate (or adjoint) variables and $\vartheta(j,t) \geq 0$ is the multiplier associated with the constraint $B(j,t) \geq 0$. At each time t the Equations (6) and (7) define $\tilde{E}(j,t,\Omega(t))$ and $p(t,\Omega(t))$, and the following stationarity conditions must hold for a Nash equilibrium:

$$\begin{aligned} \omega(j,t) &\geq 0 \\ \omega(j,t)\frac{\partial}{\partial \omega(j,t)} H_j(\cdot) &= 0 \\ \frac{\partial}{\partial \omega(j,t)} H_j(\cdot) &= \frac{\partial}{\partial C(j,t)} U_j(\cdot) \Bigg[\frac{\partial Y(j,\cdot)}{\partial \tilde{E}(j,t,\Omega(t))} \tilde{E}(j,t,\Omega(t))' \\ &\quad - (\pi(j,t)\phi(j,t) + p(t,\Omega(t)))\tilde{E}(j,t,\Omega(t))' + p(t,\Omega(t)) \\ &\quad + p(t,\Omega(t))'\big(\omega(j,t) - \tilde{E}(j,t,\Omega(t))\big)\Bigg] - q_B(j,t) \leq 0 \end{aligned}$$

Using the market equilibrium condition (6), this last equation simplifies as:

$$\begin{aligned} \omega(j,t) &\geq 0 \\ \omega(j,t)\frac{\partial}{\partial \omega(j,t)} H_j(\cdot) &= 0 \\ \frac{\partial}{\partial \omega(j,t)} H_j(\cdot) &= \frac{\partial}{\partial C(j,t)} U_j(\cdot) \Bigg[p(t,\Omega(t)) \\ &\quad + p(t,\Omega(t))'\big(\omega(j,t) - E(j,t,\Omega(t))\big)\Bigg] - q_B(j,t) \leq 0 \end{aligned} \tag{22}$$

The other equilibrium conditions are:

$$
\begin{aligned}
\frac{\partial}{\partial I_K(j,t)} H_j(\cdot) &= -\frac{\partial}{\partial C(j,t)} U_j(\cdot) + q_K(j,t) = 0 && (23)\\
\frac{\partial}{\partial I_R(j,t)} H_j(\cdot) &= -\frac{\partial}{\partial C(j,t)} U_j(\cdot) + q_R(j,t) = 0 && (24)\\
\dot{q}_K(j,t) &= -\frac{\partial}{\partial K(j,t)} H_j(\cdot) + \rho q_K(j,t) - \frac{\partial}{\partial K(j,t)} Y(j,t;\cdot) + \rho q_K(j,t) && (25)\\
\dot{q}_R(j,t) &= -\frac{\partial}{\partial R(j,t)} H_j(\cdot) + \rho q_R(j,t) - q_B(j,t)\frac{\partial}{\partial R(j,t)} \Psi(j,t,\cdot) + \rho q_R(j,t) && (26)\\
\dot{q}_B(j,t) &= -\frac{\partial}{\partial B(j,t)} H_j(\cdot) + \rho q_B(j,t) \\
&= \vartheta(j,t) + \rho q_B(j,t) && (27)\\
\vartheta(j,t) B(j,t) &= 0 && (28)\\
B(j,t) &\geq 0 && (29)\\
\vartheta(j,t) &\geq 0. && (30)
\end{aligned}
$$

with terminal (transversality) conditions:

$$
\begin{aligned}
q_K(j,T) &= \frac{\partial}{\partial K(j,T)} \mathcal{V}(j,T;\cdot) && (31)\\
q_R(j,T) &= \frac{\partial}{\partial R(j,T)} \mathcal{V}(j,T;\cdot) && (32)\\
q_B(j,T) &= \frac{\partial}{\partial B(j,T)} \mathcal{V}(j,T;\cdot). && (33)
\end{aligned}
$$

Remark 1 *In this open-loop Nash equilibrium, each coalition has to take into account the marginal effect of its supply of emission permits on the carbon market, at each time t, as indicated by the derivatives $p(t,\Omega(t))'$ of the carbon price in Equation (22). This effect should become smaller if the number of players increases, each player becoming smaller.*

2.3 A limit game with many participants in each coalition

We consider the limit game that is obtained when one considers that each coalition is composed of many smaller countries that participate in the equilibrium solution. We propose the simple player replication scheme that has been used in [11] to show that a Wardrop equilibrium can be considered as the limit of a Nash equilibrium when the users of a congested network become infinitesimal. In this player replication scheme, each coalition 'size' is divided by n and these small agents are replicated n times.

The state and control variables for a small player of coalition j are denoted $l = \frac{1}{n}L$, $k = \frac{1}{n}K$, $b = \frac{1}{n}B$, $r = \frac{1}{n}R$, $i_k = \frac{1}{n}I_K$, $i_r = \frac{1}{n}I_R$, $e = \frac{1}{n}E$, $c = \frac{1}{n}C$, $y = \frac{1}{n}Y$ and $\omega_s = \frac{1}{n}\omega$, where:

$$
c(j,t) = y(j,t) - \pi(j,t)\phi(j,t)e(j,t) - i_k(j,t) - i_r(j,t) + p(t)\big(\omega_s(j,t) - e(j,t)\big). \tag{34}
$$

The hamiltonian for a small player is then:

$$
h_j(\cdot) = l(j,t)U_j(c(j,t)/l(j,t)) + \tilde{q}_k(j,t)\dot{k}(j,t) + \tilde{q}_r(j,t)\dot{r}(j,t) + \tilde{q}_b(j,t)\dot{b}(j,t) - \tilde{\vartheta}(j,t)b(j,t), \tag{35}
$$

where $U_j(\cdot)$ is still the utility of consumption for one individual, and $\tilde{q}_k = \frac{1}{n}q_K$, $\tilde{q}_r = \frac{1}{n}q_R$, $\tilde{q}_b = \frac{1}{n}q_B$ are the costate variables for a small player, while $\tilde{\vartheta}$ is the multiplier for the non-negativity constraint $b \geq 0$. The necessary conditions for an open-loop Nash-equilibrium can be rewritten for a small player in coalition j as follows:

$$
\begin{aligned}
\omega_s(j,t)\frac{\partial}{\partial \omega_s(j,t)} h_j(\cdot) &= 0 \\
\frac{\partial}{\partial \omega_s(j,t)} h_j(\cdot) &= -\frac{\partial}{\partial c(j,t)} U_j(\cdot)\Big[p(t,\Omega(t)) + p(t,\Omega(t))'\big(\omega_s(j,t) - e(j,t,\Omega(t))\big)\Big] \\
&\quad -\tilde{q}_b(j,t) \leq 0 && (36)\\
\frac{\partial}{\partial i_k(j,t)} h_j(\cdot) &= -\frac{\partial}{\partial c(j,t)} U_j(\cdot) + \tilde{q}_k(j,t) = 0 && (37)\\
\frac{\partial}{\partial i_r(j,t)} h_j(\cdot) &= -\frac{\partial}{\partial c(j,t)} U_j(\cdot) + \tilde{q}_r(j,t) = 0 && (38)\\
\dot{\tilde{q}}_k(j,t) &= -\frac{\partial}{\partial k(j,t)} h_j(\cdot) + \rho \tilde{q}_k(j,t) && (39)\\
\dot{\tilde{q}}_r(j,t) &= -\frac{\partial}{\partial r(j,t)} h_j(\cdot) + \rho \tilde{q}_r(j,t) && (40)\\
\dot{\tilde{q}}_b(j,t) &= -\frac{\partial}{\partial b(j,t)} h_j(\cdot) + \rho \tilde{q}_b(j,t) \\
&= \tilde{\vartheta}(j,t) + \rho \tilde{q}_b(j,t) && (41)\\
\tilde{\vartheta}(j,t) b(j,t) &= 0 && (42)\\
b(j,t) &\geq 0 && (43)\\
\tilde{\vartheta}(j,t) &\geq 0. && (44)
\end{aligned}
$$

Using 'coalition size' variables in Equation (45) one has:

$$
\frac{1}{n}\frac{\partial}{\partial \omega(j,t)} H_j(\cdot) = \frac{1}{n}\frac{\partial}{\partial C(j,t)} U_j(\cdot)\Big[p(t,\Omega(t)) + p(t,\Omega(t))'\big(\frac{1}{n}\omega(j,t) - \frac{1}{n}E(j,t,\Omega(t))\big)\Big] - \frac{1}{n} q_B(j,t) \tag{45}
$$

At the limit when $n \to \infty$ the terms $\frac{1}{n}E(j,t,\Omega(t))$ and $\frac{1}{n}\omega(j,t)$ tend to 0. Therefore Equation (45) becomes:

$$
0 = \omega(j,t)\left[\frac{\partial}{\partial C(j,t)} U_j(\cdot) p(t,\Omega(t)) - q_B(j,t)\right], \tag{46}
$$

or equivalently when $\omega(j,t) > 0$:

$$
\frac{L(j,t)}{C(j,t)} p(t,\Omega(t)) = q_B(j,t). \tag{47}
$$

The other equilibrium conditions (6) and (37)–(40) become:

$$
\begin{aligned}
\frac{\partial Y(j,t;\cdot)}{\partial E(j,t)} &= \pi(j,t)\phi(j,t) + p(t,\Omega(t)) && (48)\\
\frac{\partial}{\partial I_K(j,t)} H_j(\cdot) &= -\frac{\partial}{\partial C(j,t)} U_j(\cdot) + q_K(j,t) = 0 && (49)\\
\frac{\partial}{\partial I_R(j,t)} H_j(\cdot) &= -\frac{\partial}{\partial C(j,t)} U_j(\cdot) + q_R(j,t) = 0 && (50)\\
\dot{q}_K(j,t) &= -\frac{\partial}{\partial K(j,t)} H_j(\cdot) + \rho q_K(j,t) && (51)\\
\dot{q}_R(j,t) &= -\frac{\partial}{\partial R(j,t)} H_j(\cdot) + \rho q_R(j,t) && (52)\\
\dot{q}_B(j,t) &= -\frac{\partial}{\partial B(j,t)} H_j(\cdot) + \rho q_B(j,t) \\
&= \vartheta(j,t) + \rho q_B(j,t) && (53)
\end{aligned}
$$

$$\begin{aligned}\vartheta(j,t)B(j,t) &= 0 &(54)\\ B(j,t) &\geq 0 &(55)\\ \vartheta(j,t) &\geq 0. &(56)\end{aligned}$$

Remark 2 *In this limit-game Nash equilibrium, each coalition does not take into account the marginal effect of its supply of emission permits on the carbon market, at each time t, as indicated by the absence of the terms containing the derivatives $p(t,\Omega(t))'$ of the carbon price in Equation (46). This limit equilibrium is similar to the Wardrop equilibrium in a congested network [11].*

2.4 The use of the limit game to assess fair climate agreement

For each coalition j we may compute a BAU payoff, using m decoupled optimal control problems, without restrictions on emissions. Call $\Phi(j,0)$ the discounted sum of welfare over the planning horizon. Now allocate a share $\bar{B}^0_j$ of the safe cumulative emissions budget to coalition j, $j=1,\dots,m$.

Solving the limit game, as defined above, one obtains an expected welfare $W(j,0)\leq\Phi(j,0)$ and an expected price schedule $p(t,\Omega(t))$, expected emission schedule $E(j,t)$ and expected investment schedules in both types of capital and in supply of emission permits on the carbon market. We compute for each coalition the relative welfare loss (W_L):

$$W_L(j,0;\bar{B}(\cdot))=\frac{\Phi(j,0)-W(j,0)}{\Phi(j,0)}. \tag{57}$$

Now, changing the initial allocation $\bar{B}^0_j$ one solves the Rawlsian [18] distributive justice 'meta-game':

$$\min_{\bar{B}(\cdot)}\max_{j} W_L(j,0;\bar{B}(\cdot)). \tag{58}$$

Remark 3 *Through this approach we will define a budget allocation for the different coalitions that will have a similar implication in terms of relative welfare loss, when all the players in all coalitions use their allocations in a non-cooperative way, as price takers on the international carbon market.*

3 A stochastic Nash equilibrium model with m coalitions of players

As usual, when considering a dynamic game, one has to look at the possibility to define a subgame perfect equilibrium solution, for example by introducing feedback or closed-loop strategies for the players. The use of such strategies is mandatory if the dynamic system is stochastic. In this section we reformulate the model in a full probabilistic framework and we characterize a limit game, using a concept introduced in [13] and further developed under the name of Mean Field Games [7, 15].

The dynamics of capital accumulation and GHG budget evolution is defined for $j=1,\dots,m$, as controlled stochastic diffusions:

$$\begin{aligned}d\,K(j,t) &= \big(I_K(j,t)-\delta_K K(j,t)\big)dt+\sigma_K d\varepsilon_K(t), &(59)\\ d\,R(j,t) &= \big(I_R(j,t)-\delta_R R(j,t)\big)dt+\sigma_R d\varepsilon_R(t), &(60)\\ d\,B(j,t) &= \big(\Psi(j,t,R(j,t),L(j,t))-\omega(j,t)\big)dt+\sigma d\varepsilon_B(t), &(61)\end{aligned}$$

where $\varepsilon_K(t)$, $\varepsilon_R(t)$ and $\varepsilon_B(t)$ are independent Wiener processes (white noise).

The control variables must remain non negative, $I_K(j,t)\geq 0$, $I_R(j,t)\geq 0$, $\omega(j,t)\geq 0$. For the capital stocks $K(j,t),R(j,t)$ we may assume a reflecting boundary at 0. The budget $B(j,t)$ is not constrained to remain positive. A negative value indicates an over emission that will be penalized in the terminal value function.

The payoff for player j can thus be written as the expected value:

$$\begin{aligned}\mathrm{WRG(j)} = \mathrm{E}_\gamma \Bigg[\int_0^{\mathrm{T}} \mathrm{e}^{-\rho \mathrm{t}} \mathrm{L(j,t)} \log \Bigg[\frac{1}{\mathrm{L(j,t)}} \Big\{ &\mathrm{Y(j,t)} - \pi(\mathrm{j,t})\phi(\mathrm{j,t})\tilde{\mathrm{E}}(\mathrm{j,t},\Omega(\mathrm{t})) - \mathrm{I_K(j,t)} - \mathrm{I_R(j,t)} \\ &+ p(t,\Omega(t))\big(\omega(j,t) - \tilde{E}(j,t,\Omega(t))\big)\Big\}\Bigg] \, dt \\ &+ e^{-\rho T}\mathcal{W}(j,T;\mathbf{K}(T),\mathbf{R}(T),\mathbf{B}(T))\Bigg] \end{aligned} \qquad (62)$$

where $\mathcal{W}(j,T;\mathbf{K}(T),\mathbf{R}(T),\mathbf{B}(T))$ is the value at terminal time t for player j associated with the terminal state $(\mathbf{K}(T),\mathbf{R}(T),\mathbf{B}(T))$ of the m players. Here the expected value is taken with respect to the probability measure induced by the strategies γ chosen by all players. We shall assume that these strategies are state feedbacks, that is the investment decisions and the supply of permits of a coalition are functions of the state variables of all coalitions and of time. Indeed the players are interdependent through the emissions trading scheme, since their emissions $E(j,t,\Omega(t))$ and the carbon price $p(t,\Omega(t))$ are both depending on the total supply of permits:

$$\Omega(t) = \sum_{j=1}^{m} \omega(j,t). \qquad (63)$$

This implies that the value function for each player will be a function of the state variables of all players.

3.1 Nash equilibrium for the m coalitions

3.1.1 HJB equations

A Nash equilibrium for this stochastic game will be characterized by a set of m coupled HJB equations that we summarize below. For all $j = 1,\ldots,m$ the current valued payoff function $V(j,t;\mathbf{K},\mathbf{R},\mathbf{B})$ from time t, given state $(\mathbf{K},\mathbf{R},\mathbf{B})$ for all coalitions, satisfy the functional equations:

$$\begin{aligned}\rho V(j,t;\cdot) - \frac{\partial V(j,t;\cdot)}{\partial t} = \max_{\{I_K(j,t),I_R(j,t),\omega(j,t)\}} \Bigg\{ &L(j,t)\log\Bigg[\frac{1}{L(j,t)}\Big\{Y(j,t) \\ &- \pi(j,t)\phi(j,t)\tilde{E}(j,t,\Omega(t)) - I_K(j,t) - I_R(j,t) \\ &+ p(t,\Omega(t))\big(\omega(j,t) - \tilde{E}(j,t,\Omega(t))\big)\Big\}\Bigg] \\ &+ \sum_{\ell=1}^{m}\Bigg(\frac{\partial V(j,t;\cdot)}{\partial B(\ell,t)}\big(\Psi(\ell,t,R(\ell,t),L(j,t)) - \omega(\ell,t)\big) \\ &+ \frac{\partial V(j,t;\cdot)}{\partial K(\ell,t)}\big(I_K(\ell,t) - \delta_i K_i(\ell,t)\big) \\ &+ \frac{\partial V(j,t;\cdot)}{\partial R(\ell,t)}\big(I_R(\ell,t) - \delta_R R(\ell,t)\big) \\ &+ \frac{\sigma_K^2}{2}\frac{\partial^2 V(j,t;\cdot)}{\partial K(\ell,t)^2} + \frac{\sigma_R^2}{2}\frac{\partial^2 V(j,t;\cdot)}{\partial R(j,t)^2} + \frac{\sigma_{B_\ell}^2}{2}\frac{\partial^2 V(j,t;\cdot)}{\partial B(\ell,t)^2}\Bigg)\Bigg\} \end{aligned} \qquad (64)$$

with initial conditions:

$$\begin{aligned} K(j,0) &= \bar{K}_j^0 \\ B(j,0) &= \bar{B}_j^0, \\ j &= 1,\ldots,m, \\ \sum_{j=1}^{m}\bar{B}_j^0 &= \bar{B}; \end{aligned}$$

and terminal condition:

$$V(j,T;\mathbf{K}(T),\mathbf{R}(T),\mathbf{B}(T)) = \mathcal{W}(j,T;\mathbf{K}(T),\mathbf{R}(T),\mathbf{B}(T)), \quad j=1,\ldots,m. \tag{65}$$

3.1.2 Equilibrium strategies

The equilibrium feedback strategy is determined by the conditions of maximization of the RHS of the HJB equations. The optimality conditions are

Deriving w.r.t. $I_K(j,t)$**:**

$$0 \;=\; -\frac{L(j,t)}{C(j,t)} + \frac{\partial V(j,t;\cdot)}{\partial K(j,t)} \tag{66}$$

Deriving w.r.t. $I_R(j,t)$**:**

$$0 \;=\; -\frac{L(j,t)}{C(j,t)} + \frac{\partial V(j,t;\cdot)}{\partial R(j,t)} \tag{67}$$

Deriving w.r.t. $\omega(j,t)$**:**

$$0 = \frac{L(j,t)}{C(j,t)}\Big[p(t,\Omega(t)) + p(t,\Omega(t))'\big(\omega(j,t) - E(j,t,\Omega(t))\big)\Big] - \frac{\partial V(j,t;\cdot)}{\partial B(j,t)}. \tag{68}$$

Notice that the players are interdependent through the determination of the price of carbon and the emission levels as shown by the derivatives (10) and (11), which enter into Equation (68) determining the permit supply. They are also interdependent through the bequest function, which depends on all state variables for the m players.

Remark 4 *In this stochastic Nash equilibrium, the equilibrium strategies are feedback rules over the whole set of state variables and time, because the value functions are themselves dependent on all state variables and time.*

3.2 MFG limit

The HJB equations for a Nash equilibrium will be extremely difficult to solve, due to the curse of dimensionality in dynamic programming. As the number of players grows the difficulty increases also. However, if the players become “small”, or infinitesimal, the Nash equilibrium can be approximated by an equilibrium in an MFG as shown in this section.

- In each coalition j, one considers a measure space of countries. Let

  $$\Theta_j(L_j,K_j,R_j,B_j,t)$$

  be the pdf of the joint distribution of (L_j,K_j,R_j,B_j), at time t.
- The initial distribution $\Theta_j(L_j,K_j,R_j,B_j,0)$ is given. This defines in particular a distribution of the initial endowment $\bar{B}_j$ of emission permits for each category in coalition j.
- In each coalition a large number of infinitesimal players are competing. We consider that they become price takers on the carbon market.
- Each player will base its decision at time t on the observation of its own state $K(j,t), R(j,t), B(j,t)$ and the information provided by the m pdfs $\Theta_j(K_j,R_j,B_j,t)$.
- Hence, for each country (player) of coalition j the value function will satisfy a ‘local’ HJB equation:

$$
\begin{aligned}
\rho V(j,t;\cdot) - \frac{\partial V(j,t;\cdot)}{\partial t} = \max_{\{I_K(j,t),I_R(j,t),\omega(j,t)\}} \Bigg\{ & L(j,t)\log\Bigg[\frac{1}{L(j,t)}\Big\{Y(j,t) \\
& - \pi(j,t)\phi(j,t)\tilde{E}(j,t,\Omega(t)) - I_K(j,t) - I_R(j,t) \\
& + p(t,\Omega(t))\big(\omega(j,t) - \tilde{E}(j,t,\Omega(t))\big)\Big\}\Bigg] \\
& + \frac{\partial V(j,t;\cdot)}{\partial B(j,t)}\big(\Psi(j,t,R(j,t),L(j,t)) - \omega(j,t)\big) \\
& + \frac{\partial V(j,t;\cdot)}{\partial K(j,t)}\big(I_K(\ell,t) - \delta_i K_i(\ell,t)\big) \\
& + \frac{\partial V(j,t;\cdot)}{\partial R(j,t)}\big(I_R(j,t) - \delta_R R(j,t)\big) \\
& + \frac{\sigma_K^2}{2}\frac{\partial^2 V(j,t;\cdot)}{\partial K(j,t)^2} + \frac{\sigma_R^2}{2}\frac{\partial^2 V(j,t;\cdot)}{\partial R(j,t)^2} + \frac{\sigma_{B_j}^2}{2}\frac{\partial^2 V(j,t;\cdot)}{\partial B(j,t)^2}\Bigg\}. \qquad (69)
\end{aligned}
$$

- At time t, given a global supply of permits $\Omega(t)$, the carbon market price and emission response $\mathcal{E}(j,t,,\Omega(t);K_j,R_j,B_j)$ of the players of coalition j are defined by the equations:

$$
\begin{aligned}
0 &= \frac{\partial Y(j,t;\cdot)}{\partial \tilde{E}(j,t,\cdot)} - \pi(j,t)\phi(j,t) + p(t,\Omega(t)) \qquad (70) \\
\Omega(t) &= \sum_{j=1}^{m} \int \Theta_j(K_j,R_j,B_j,t)\tilde{E}(j,t,\Omega(t);K_j,R_j,B_j)\,d\Theta. \qquad (71)
\end{aligned}
$$

  Therefore, both the carbon price and the emission level of the players are now functionals of the pdfs $\Theta_j(K_j,R_j,B_j,t)$ for $j = 1,\dots,m$. Now, a given player, being infinitesimal, has no influence on the carbon market equilibrium by changing his level of supply $\omega(j,t)$.

- Hence, the equilibrium strategy defining the permit supply of a player of type j in Equations (68) for a Nash equilibrium is now defined by the simpler condition:

$$
\frac{L(j,t)}{C(j,t)}p(t,\Omega(t)) = \frac{\partial V(j,t;\cdot)}{\partial B(j,t)}, \qquad (72)
$$

  which corresponds to Equation (47) in the deterministic case.

- As $p(t,\Omega(t))$ and $E(j,t;\Omega(t))$ are defined by the market clearing conditions (70)–(71), we can summarize the optimal strategies as functions:

$$
\begin{aligned}
&\varphi_{I_K}(K(j,t),R(j,t),B(j,t);\Theta(\cdot,t)) \\
&\varphi_{I_R}(K(j,t),R(j,t),B(j,t);\Theta(\cdot,t))
\end{aligned}
$$

  and

$$
\psi_{\omega_j}(K(j,t),R(j,t),B(j,t));\Theta(\cdot,t)).
$$

- Finally, for each coalition j, the distribution $\Theta_j(x,t)$ evolves according to a Kolmogorov F-P (KFP) equation, where x stands for the vector of all state variables (K,R,B) and Q denote the covariance matrix:

$$
\begin{aligned}
\frac{\sigma^2}{2}\partial^2_{xx}\left[\|x\|^2_Q\Theta_j(x,t)\right] = \partial_t\Theta(x,t) \\
& + \partial_{K_i(j,t)}\Big[\Theta_j(x,t)\big(\varphi_{I_K}(K(j,t),R(j,t),B(j,t);\Theta(\cdot,t)) - \delta_i K_i(j,t)\big)\Big] \\
& + \partial_{R(j,t)}\Big[\Theta_j(x,t)\big(\varphi_{I_R}(K(j,t),R(j,t),B(j,t);\Theta(\cdot,t)) - \delta_i K_i(j,t)\big)\Big] \\
& - \partial_{B(j,t)}\Big[\Theta_j(x,t)\psi_{\omega_j}(K(j,t),R(j,t),B(j,t));\Theta(\cdot,t))\Big]. \qquad (73)
\end{aligned}
$$

- The terminal conditions must be reformulated as follows:

$$V(j,T;K(j,T),R(j,T),B(j,T);\Theta(\cdot,T)) = \mathcal{W}(j,T;K(j,T),R(j,T),B(j,T);\Theta(\cdot,T)). \tag{74}$$

  This means that the bequest function depends, for each coalition on its own state variables at time T and the distribution of all players at final time T.

In summary, this MFG equilibrium is defined by a fixed-point condition involving the pdfs $\Theta_j(x,t)$. Given a nominal set of m pdfs $\Theta_j(x,t)$, for each t, one can characterize, through the HJB Equations (69) and the carbon market conditions (8)–(9), a set of equilibrium strategies, which, injected in the KFP equations define a new set of pdfs $\bar{\Theta}_j(x,t)$. The equilibrium is reached when $\Theta_j(x,t) \equiv \bar{\Theta}_j(x,t)$ for $j = 1,\dots,m$.

Remark 5 *In this Mean-Field-Game limit for the stochastic Nash equilibrium, we have used the concept of a measure space of agents in each coalition j. This is a useful abstraction introduced initially by Robert Auman [2]. The challenge for the modeler will be to determine, from the economic data describing the coalitions, the pdf's at the initial time, which are part of the initial conditions for this system of dynamic equations. As the HJB equations are decoupled, the complexity for computing the equilibrium is much reduced. Once the value fuctions are obtained, the meta-game for a fair allocation of the emission budget to each coalition can be formulated as in Section 2.4.*

4 Conclusion

In this paper we have proposed a dynamic game paradigm to assess the fairness of the allocation of the safe cumulative emissions budget to the different coalitions of countries that are engaged in climate negotiations. We have treated both the deterministic and fully stochastic cases. By relying on the limit game formulations obtained using the MFG paradigms we propose an approach which circumvents the obstacle of dimensionality for computing equilibria.

Without the possibility to use CDR, the exploitation of the share of the emissions budget by a country or a group of countries would be very similar to the exploitation of a non-renewable resource, like oil for example. We would therefore rediscover a form of the Hoteling rule in the deterministic limit games discussed in the paper. In a stochastic context the developments would be very close to those of Giraud et al. [8] dealing with oil field exploitation.

The numerical solution for the stochastic case remains a challenge although one that can be managed, unlike the computation of many player stochastic Nash equilibria, which are notoriously still out of reach of existing numerical methods. The numerics will be the object of a further development of this research on the game theoretic dimension of climate change negotiations. The developments in this paper have shown that the MFG concepts could have interesting applications in the assessment of negotiations for the design of fair climate change policies.

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