MATHEMATICS PROGRAM

SECONDARY LEVEL

ADULT EDUCATION

Preliminary Version
January 1993

Direction de la formation générale des adultes

ACKNOWLEDGMENTS

The Direction de la formation générale des adultes would like to thank all those who contributed to the development of this program and especially those who helped implement it.

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FOREWORD

This new program provides adult students in Québec with a new method of learning mathematics.

The work of the various commitees that have been set up by the Direction générale de l'Éducation des Adultes (DGEA) and the Direction de la formation générale des adultes (DFGA) since 1984, as well as the work carried out by working groups over the last three years have led to the development of a coherent program that meets the needs of adults.

This program must be dynamic and effective and more than just a list of objectives. Fully individualized instructional materials designed specifically for adults have been developed for a number of modules in this program.

To make it easier for students to transfer from one sector to another, the modules in this program have been developed according to the learning sequence outlined in the mathematics program for the youth sector.

The program and related materials presuppose that the students are the prime architects of their own education; however, this instructional package also makes it possible for the teacher and the student to work together effectively to achieve the objectives of the program.

TABLE OF CONTENTS

1. GUIDING PRINCIPLES

The adult education secondary-level *Mathematics* program is based on the three guiding principles given below. The aim of the program is to ensure that:

1- adults acquire the mathematical knowledge they will need for everyday activities, jobs or job-training, and admission to college-level programs.

2- adults have access to recognized, high-quality mathematical training.

3- adults are regarded as the prime architects of their own education by recognizing their past scholastic and experiential achievements and providing them with materials that enable them to learn at their own pace.

This program was structured in accordance with these principles and with the goals and general objectives described on the next page.

2. GOALS AND GENERAL OBJECTIVES

2.1 Goals

1. **TO ENCOURAGE** adults to develop positive attitudes towards mathematics and its applications.

2. **TO HELP** adults learn how to view mathematics as a practial tool that can be used in various aspects of their lives (e.g. occupational, social, educational, daily and family life).

3. **TO SHOW** adults how to view the world in terms of quantities and relations.

2.2 General Objectives

1. **TO MASTER** mathematical concepts essential to a better understanding of their environment.

2. **TO MASTER** the use of certain instruments developed in mathematics for applications in scientific and technical work, and in trades.

3. **TO PROCESS** information using mathematical models.

4. **TO DEVELOP** a systematic approach to problem solving.

5. **TO APPLY** their mathematical knowledge in practical situations.

3. INSTRUCTIONAL APPROACH

3.1 General Features

The adult education *Mathematics* program for adults takes a modular approach to learning and teaching. The learning/teaching process associated with this modular approach is systematic and standardized and can be easily understood by the teacher and student. This process is outlined below:

- evaluation of experiential learning and prerequisites
- organization of content in a logical and progressive order
- progressive learning based on the mastery of small segments of the learning content
- systematic formative evaluation
- remedial activities for students experiencing difficulties
- individualized instruction

These elements facilitate the learning process and are consistent with the mastery learning approach, which is designed to minimize the risk of failure. Above all, the program is pragmatic, since it focuses on phenomena and problems of interest to adults, and helps them learn how to mathematize a situation, solve the ensuing mathematical problem and interpret the results.

Thus, this pedagogical model is based on two fundamental concepts: the modular approach and the learning/teaching process.

3.2 The Modular Approach

A program developed according to a modular approach consists of a series of learning modules to be mastered successively in a continuous process. To deal with the material in any given module, the student must first have mastered the skills learned in preceding modules.

A module is a section of a program in the form of a self-contained package of learning activities and instructions designed to help adult students master the material on their own. The shorter the duration of each module, the easier it will be for teachers to provide an accurate assessment of the students' knowledge and skills. Thus, each module has been divided into small segments that take between 25 and 50 hours to complete.

The modules that make up the program are self-contained, yet interrelated. Because they are self-contained, modules or groups of modules can be incorporated into various learning contexts, as needed.

3.3 The Learning/Teaching Process

The learning/teaching process should reflect the individualized learning method used in the adult education sector. This process differs from that of self-instruction, since individualized learning implies that the teacher and the adult student are actively involved in a joint effort to achieve the program objectives.

The term *individualized learning* has further implications. It means that the teacher spends less time merely conveying knowledge (instructional materials supplying most of the necessary factual information), and focuses instead on helping adults make sense of the learning process. With the teacher's support, the students learn how to learn, organize their work and identify what contributes to or hinders their progress. This process is known as metalearning.

In short, adult students must not only master the subject matter, but also learn how to organize their work and assess their work methods critically.

4. LIST OF MODULES AND STRUCTURE OF THE PROGRAM

4.1 List of Modules

The adult education *Mathematics* program consists of modules that deal with all the mathematical concepts and skills normally covered in secondary school.

Code	Title	No. of Credits
GSM 211	The Four Operations on Integers	2
GSM 212	The Four Operations on Fractions	2
GSM 213	Decimals and Percent	2
GSM 221	Equations and Inequalities I	2
GSM 222	Geometry I	2
GSM 223	Statistics and Probability I	2
GSM 231	The Four Operations on Polynomials	2
GSM 232	Geometry II	2
GSM 241	Straight Lines I	2
GSM 242	Geometry III	1
GSM 243	Equations and Inequalities II	2
GSM 244	Trigonometry I	1
GMO 241	Factoring	1
GMO 242	The Four Operations on Algebraic Fractions	1
GMO 243	Straight Lines II	1
GMO 244	Logic, Set Theory and Relations	2
GSM 251	Optimization	1
GSM 252	Statistics and Probability II	2
GSM 253	Geometry IV	1
GMO 251	Conics I	1
GMO 252	Equations and Inequalities III	1
GMO 253	Functions	1
GMO 254	Conics II	1
GMO 255	Exponential and Logarithmic Functions	1
GMO 256	Trigonometry II	2
GMO 257	Geometry V	2

4.2 Structure of the Program

As shown in the diagram on the next page, the modules are divided into four paths that enable adult students to acquire the mathematical skills they will need for different career options.

The first path, consisting of modules GSM 211 to GSM 213, GSM 221 to GSM 223, as well as GSM 231 and GSM 232, provides adult students with the mathematical skills required to enter programs leading to a Secondary School Vocational Certificate (SSVC).

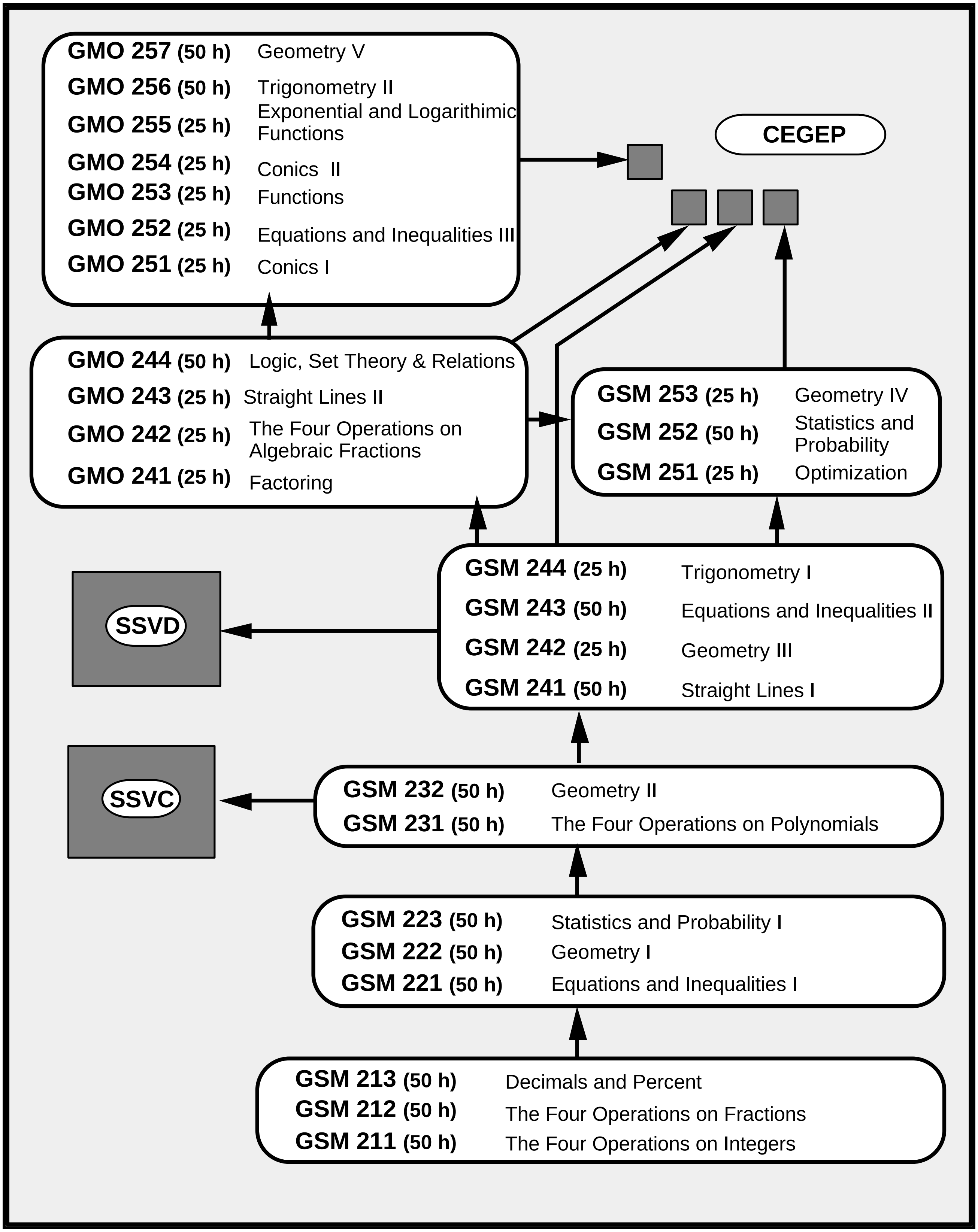
The second path, composed of modules GSM 241 to GSM 244, provides adult students with the mathematical skills required to obtain a Secondary School Vocational Diploma (SSVD).

The third path, consisting of either modules GSM 251 to GSM 253 or GMO 241 to GMO 244, provides adult students with the mathematical skills required to enter certain CEGEP programs.

The fourth path, which leads to a Secondary School Diploma (SSD), provides adult students with the mathematical training required to enter all CEGEP programs that call for an in-depth knowledge of mathematics. This path includes modules GMO 241 to GMO 244 as well as GMO 251 to GMO 257. Note that successful completion of modules GSM 251, GSM 252 and GSM 253 is not a prerequisite for admission into the fourth path.

The modules in the GSM series correspond to the compulsory courses for each level of secondary school and the modules in the GMO series correspond to the optional courses.

STRUCTURE OF THE PROGRAM



5. INSTRUCTIONAL MATERIALS

5.1 Individualized Learning Activity Packages

A number of the modules in this program include an individualized learning activity package characterized by seven principal features listed below:

Each package:

- is complete and covers all the subject matter related to that module;
- is composed of as many learning units as there are program objectives for that module; these units are to be mastered successively in a continuous process;
- is consistent with the three-step learning process described by Gagné[1]**:** motivation, acquisition and performance;
- is divided into three parts: an introduction to the module, a progressive series of learning units and a conclusion;
- is completed according to the following systematic learning procedure:
  - in the introduction to the module, the students do specific review activities to determine whether they have mastered the prerequisites, and are introduced to the material they will be studying;
  - students cover the subject matter of the module by doing a series of activities that are presented in a logical and progressive order;
  - at the conclusion of the module, formative evaluation is carried out on a criterion-referenced basis;
- is designed to be as self-contained as possible.

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1 R. M. Gagné and L.J. Briggs, *Principles of Instructional Design* (New York: Holt, Rinehart and Winston), 1974.

Note : For a more detailed description of a typical module, refer to the document entitled *Cadre de référence pour l'élaboration de matériel didactique en contexte d'enseignement-apprentissage*, DGEA, 1988.

5.2 Other Instructional Materials

A pocket calculator will prove to be essential for many learning activities. In fact, the program has been designed to encourage students to make maximum use of this instrument .

6. CERTIFICATION OF ACHIEVEMENT

Students are evaluated at the end of each module. Evaluation for purposes of certification is based on the results of ministry examinations. For guidelines regarding the certification of achievement, refer to the *Guide de la gestion de la sanction des acquis*, published annually by the Direction de la sanction des études (DSE).

7. OPERATIONAL OBJECTIVES FOR THE MODULES IN THE PROGRAM:

GSM 211 to GSM 213, GSM 221 to GSM 223, GSM 231 and GSM 232, GSM 241 to GSM 244, GMO 241 to GMO 244, GSM 251 to GSM 253, GMO 251 to GMO 257

Each objective in these 26 modules consists of a clear and precise description of the skills that the students must develop. The description of each objective is preceded by the title of the corresponding learning unit. Terminal objectives are printed in bold type to distinguish them from intermediate objectives.

The table preceding each list of operational objectives contains information regarding teaching time and the relative weight (%) assigned to each terminal objective for purposes of summative evaluation.

7.01 GSM 211 — The Four Operations on Integers

Module GSM 211 consists of seven objectives and requires 50 hours of study as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 211-01 to **GSM 211-06**	36	75 %
GSM 211-07	12	25 %

* Two hours are allotted for the final evaluation.

GSM 211-01 *Comparing Two Integers*

Compare two integers by locating them on the number line and by using the appropriate symbol: is greater than (>), is less than (<) or is equal to (=). The numbers to be compared are quantities used in everyday situations.

GSM 211-02 *Adding Two Integers*

Add two integers that are greater than -30 and less than +30. These integers represent quantities used in everyday situations.

GSM 211-03 *Subtracting Two Integers*

Subtract two integers that are greater than -30 and less than +30. These integers represent quantities used in everyday situations.

GSM 211-04 *Multiplying Two Integers*

Multiply two integers that are greater than -30 and less than +30. These integers represent quantities used in everyday situations.

GSM 211-05 *Dividing Two Integers*

Divide two integers that are greater than -30 and less than +30. These integers represent quantities used in everyday situations.

GSM 211-06 ***Order of Operations on Integers***

Evaluate an arithmetic expression containing integers by performing the appropriate operations and by following the order of operations. The arithmetic expression contains up to five sets of parentheses or brackets. The steps in the solution must be shown.

GSM 211-07 ***Problems Related to Everyday Situations***

Perform the operations required to solve problems related to everyday situations that can be represented by arithmetic expressions containing integers only. The steps in the solution must be shown.

7.02 GSM 212 **The Four Operations on Fractions**

Module GSM 212 consists of twelve objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 212-01 to **GSM 212-05**	12	15 %
GSM 212-06 to **GSM 212-11**	25	55 %
GSM 212-12	11	30 %

* Two hours are allotted for the final evaluation.

GSM 212-01 *Writing Ratios in Fractional Form*

Express the ratio of two quantities as a fraction, a mixed number or an improper fraction, as the case may be. These quantities are found in statements or diagrams relating to everyday situations.

GSM 212-02 *Finding Equivalent Representations for a Given Fraction*

Apply the method for converting a fraction to an equivalent fraction in order to solve ratio and proportion problems relating to everyday situations.

GSM 212-03 *Reducing Fractions to Their Lowest Terms*

Simplify fractions, reducing them to equivalent fractions in lowest terms. The numerator and denominator of each given fraction must be less than 225.

GSM 212-04 *Converting Mixed Numbers to Improper Fractions and Vice-versa*

Convert improper fractions to mixed numbers and mixed numbers to improper fractions. In each case, the result must be reduced to its lowest terms.

GSM 212-05 ***Locating Fractions, Improper Fractions and Mixed Numbers on a Number Line***

Locate a set of six positive or negative fractions, improper fractions or mixed numbers on a number line. The interval on the number line ranges from -2 to +2, the subdivisions are equal to the value of the common denominator and the denominators are less than or equal to 12.

GSM 212-06 *Comparing Two Fractions*

Compare two fractions, mixed numbers or improper fractions using the appropriate symbol (>, < or =).

GSM 212-07 *Multiplying Fractions*

Multiply three fractions, improper fractions or mixed numbers and reduce the product to its lowest terms. The problems are presented as statements or arithmetic expressions.

GSM 212-08 *Dividing Two Fractions*

Divide two fractions, improper fractions or mixed numbers and reduce the quotient to its lowest terms. The problems are presented as statements or arithmetic expressions.

GSM 212-09 *Adding Fractions*

Add three fractions, improper fractions or mixed numbers and reduce the sum to its lowest terms. The problems are presented as statements or arithmetic expressions.

GSM 212-10 *Subtracting Fractions*

Subtract different fractions, improper fractions or mixed numbers and reduce the difference to its lowest terms. The problems are presented as statements or arithmetic expressions containing two or three terms.

GSM 212-11 ***The Four Operations on Fractions***

Evaluate arithmetic expressions containing up to six positive or negative fractions, improper fractions or mixed numbers, by performing the appropriate operations, by following the order of operations and by applying the law of signs. The arithmetic expression should contain no more than three sets of parentheses or brackets, no more than three types of operations and denominators less than or equal to 12. The steps in the solution must be shown and the answer must be reduced to its lowest terms.

GSM 212-12 ***Problems Related to Everyday Situations***

Solve word problems that can be written as arithmetic expressions containing fractions, improper fractions or mixed numbers, by following the order of operations and by applying the law of signs. Evaluating the arithmetic expression involves performing up to five operations and no more than two types of operations (i.e. any two of the following: addition, subtraction, multiplication and division). The steps in the solution must be shown and the answer must be reduced to its lowest terms.

7.03 GSM 213 **Decimals and Percent**

Module GSM 213 consists of thirteen objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 213-01 to **GSM 213-08**	25	40 %
GSM 213-09	8	30 %
GSM 213-10 to **GSM 113-13**	15	30 %

* Two hours are allotted for the final evaluation.

GSM 213-01 *Finding Equivalences Between Fractions or Mixed Numbers and Decimals*

Convert a decimal (limited to thousandths) to a fraction or mixed number reduced to its lowest terms, and convert a fraction or mixed number to a decimal limited to thousandths. When the resulting decimal is a repeating or periodic decimal, standard notation is required. The numbers to be converted represent quantities used in everyday situations.

GSM 213-02 *Rounding Decimals*

Round a decimal limited to ten thousandths to the nearest tenth, hundredth or thousandth.

GSM 213-03 Multiplying or Dividing Decimals by 10, 100, 1 000 or 10 000

Multiply and divide a decimal (the decimal is limited to ten thousandths) by 10, by 100, by 1 000 and by 10 000 . The given decimals represent quantities used in everyday situation***.***

GSM 213-04 *Adding Two Decimals*

Add two positive or negative decimals limited to thousandths. The problems are presented as statements or arithmetic expressions.

GSM 213-05 *Subtracting Two Decimals*

Subtract two positive or negative decimals limited to thousandths. The problems are presented as statements or arithmetic expressions.

GSM 213-06 *Multiplying Two Decimals*

Multiply two positive or negative decimals limited to thousandths. The product must be rounded to the nearest thousandth if the decimal part contains more than three digits. These decimals represent quantities used in everyday situations.

GSM 213-07 *Dividing Two Decimals*

Divide two positive or negative decimals limited to thousandths. The quotient must be rounded to the nearest thousandth if the decimal part contains more than three digits. These decimals represent quantities used in everyday situations

GSM 213-08 ***Order of Operations on Decimals and Mixed Numbers***

Evaluate an arithmetic expression containing up to four decimals and two mixed numbers, whether positive or negative, by performing the appropriate operations, following the order of operations and applying the law of signs. The arithmetic expression should contain up to three sets of parentheses or brackets and no more than three types of operations (i.e. any three of the following: addition, subtraction, multiplication and division). The decimals are limited to thousandths and the mixed numbers have a denominator less than 13. The steps in the solution must be shown and the answer must be written as a decimal rounded to the nearest thousandth.

GSM 213-09 ***Solving Real-Life Problems by Means of Calculations Involving Decimals and Mixed Numbers***

Solve word problems that can be written as arithmetic expressions containing decimals and mixed numbers, by following the order of operations and applying the law of signs. The decimals are limited to thousandths and the mixed numbers have a denominator less than 13. Evaluating the arithmetic expression involves performing up to five operations and no more than three types of operations (i.e. any three of the following: addition, subtraction, multiplication and division). The steps in the solution must be shown and the answer must be written as a decimal rounded to the nearest thousandth.

GSM 213-10 *Converting Given Percentages to Ratios with a Denominator of 100*

Convert a given percentage to a ratio with a denominator of 100. The given percentage is limited to tenths of a percent. The problems deal with everyday situations.

GSM 213-11 *Changing Percentages to Decimals and Vice-Versa*

Convert a percentage to a decimal (the percentage is limited to tenths of a percent) and convert a decimal to a percentage (the decimal is limited to thousandths). The problems deal with everyday situations.

GSM 213-12 *Converting Percentages to Fractions and Vice-Versa*

Convert a percentage to a fraction reduced to its lowest terms and convert a fraction to a percentage. The percentage is usually limited to tenths of a percent. The problems deal with everyday situations.

GSM 213-13 ***Solving Real-Life Problems by Means of Calculations Involving Percentages***

Solve word problems that can be written as arithmetic expressions containing integers, decimals, mixed numbers and percentages. Solving the problem involves:

- **calculating part of the whole when the ratio is given in percentage form,**
- **and/or calculating the ratio of a part to the whole and expressing it in percentage form, given the value of the part and the whole.**

Solving the problem involves performing up to five operations and no more than three types of operations (i.e. any three of the following: addition, subtraction, multiplication and division). The decimals are limited to thousandths. The problems deal with everyday situations. The steps in the solution must be shown.

7.04 GSM 221 — Equations and Inequalities I

Module GSM 221 consists of seven objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 221-01 to **GSM 221-03**	10	20 %
GSM 221-04	10	20 %
GSM 221-05	9	20 %
GSM 221-06	9	20 %
GSM 221-07	10	20 %

* Two hours are allotted for the final evaluation.

GSM 221-01 *Adding and Subtracting Algebraic Expressions*

Simplify an algebraic expression containing similar terms and addition or subtraction operations. The initial algebraic expression consists of up to five terms containing no more than three first-degree variables. It contains no parentheses, brackets or braces.

GSM 221-02 *Distributive Property of Multiplication over Addition or Subtraction*

Apply the distributive property of multiplication over addition or over subtraction to an algebraic expression of the form $a(bx + cy)$, where a, b and c are rational numbers and where x and y represent variables. The initial algebraic expression contains up to four terms.

GSM 221-03 ***Solving First-Degree Equations in One Variable***

Solve a first-degree equation that can be expressed in the form $ax + b = 0$, where a and b are rational numbers and where x represents the variable. The initial equation contains up to six terms and is defined within a given universe ($\mathbb{N}$, $\mathbb{Z}$ or $\mathbb{Q}$). The solution set and the steps involved in solving the problem must be clearly indicated.

GSM 221-04 ***Solving First-Degree Inequalities in One Variable***

Solve a first-degree inequality that can be converted to one of the following forms:

- $ax + b \geq 0$
- $ax + b \leq 0$
- $ax + b > 0$
- $ax + b < 0$

The constants a and b are rational numbers and x represents the variable. The initial equation contains up to six terms and is defined within a given universe ($\mathbb{N}$, $\mathbb{Z}$ or $\mathbb{R}$). The solution set, its graphical representation on the number line and the steps involved in solving the inequality must be clearly indicated.

GSM 221-05 ***Ratios and Proportions***

Solve a first-degree equation in one variable by applying the fundamental property of proportions: *the product of the extremes is equal to the product of the means*. The given equation is stated as a proportion. The exercises consist of mathematical expressions containing up to six terms or word problems related to everyday situations. The solution set and the steps involved in solving the problem must be clearly indicated.

GSM 221-06 ***Formulas***

Solve an equation representing a given scientific formula which can be expressed as an equation containing an unknown variable. The formula and the value of the other variables are provided. The solution set and the steps involved in solving the problem must be clearly indicated.

GSM 221-07 ***Problems Involving First-Degree Equations in One Variable***

Solve word problems that can be written as first- degree equations in one variable. The problems can involve up to three unknowns. The solution set and the steps involved in solving the problem must be clearly indicated.

7.05 GSM 222 Geometry I

(Straight Lines, Angles, Pythagorean Theorem)

Module GSM 222 consists of eight objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 222-01 and **GSM 222-02**	10	20 %
GSM 222-03 and **GSM 222-04**	12	20 %
GSM 222-05 and **GSM 222-06**	13	30 %
GSM 222-07	8	20 %
GSM 222-08	5	10 %

* Two hours are allotted for the final evaluation.

GSM 222-01 *Basic Geometric Concepts*

(1) Distinguish between the following geometric figures:

- line
- ray
- line segment
- angle
- acute angle
- obtuse angle
- right angle
- straight angle

The problems involve distinguishing between figures that have already been drawn. (2) Using a protractor, measure a given angle between 0° and 180° to the nearest 2°.

GSM 222-02 ***Constructing an Angle***

Using a protractor, construct an angle of n degrees to the nearest 2°. The measure of the angle to be drawn is an integer between 0° and 180°.

GSM 222-03 *Types of Lines*

Distinguish between the following pairs of lines:

- parallel lines
- perpendicular intersecting lines
- non-perpendicular intersecting lines.

GSM 222-04 ***Categories of Angles***

Determine the measure of one or more angles in a geometric figure containing two parallel lines cut by a transversal, given the measure of one of the angles of that figure. The measure(s) will be determined by applying the properties of the following pairs of angles:

- **complementary angles**
- **supplementary angles**
- **adjacent angles**
- **vertically opposite angles**
- **alternate interior angles**
- **alternate exterior angles**
- **corresponding angles**

The answer must be explained.

GSM 222-05 *Polygons*

Given a set of geometric figures that are polygons, identify those that are:

- triangles
- equilateral triangles
- isosceles triangles
- right triangles
- isosceles right triangles
- scalene triangles
- quadrilaterals
- parallelograms
- rhombuses
- squares
- rectangles
- trapezoids

These figures will be identified by applying the properties of the angles, sides and diagonals of each of these polygons. The use of a protractor and a ruler is permitted.

GSM 222-06 ***Measuring Polygons***

Determine the measures of angles and sides in a geometric figure containing any of the following polygons: equilateral triangle, isosceles triangle, right triangle, isosceles right triangle, scalene triangle, parallelogram, rhombus, square, rectangle and trapezoid. The measures will be determined by applying the properties of the angles, sides and diagonals of these polygons. Any information needed to find the required measures is indicated on the figure. The students must justify each step in the solution.

GSM 222-07 ***The Pythagorean Theorem***

Given the measures of two sides of a right triangle, calculate the measure of the third side by applying the Pythagorean Theorem. These triangles illustrate everyday situations. The steps in the solution must be shown.

GSM 222-08 ***Special Right Triangles and The Pythagorean Theorem***

Given the measure of one side of a right triangle in which one of the angles measures 30° or 45°, calculate the measure of one of the other two sides by applying the Pythagorean Theorem. These triangles illustrate everyday situations. The steps in the solution must be shown.

7.06 GSM 223 — Statistics and Probability I

Module GSM 223 consists of six objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 223-01 and **GSM 223-02**	22	50 %
GSM 223-03	8	10 %
GSM 223-04	8	15 %
GSM 223-05 and **GSM 223-06**	10	25 %

* Two hours are allotted for the final evaluation.

GSM 223-01 *Data Tables and Frequency Distribution Tables*

(1) Find the following information in a data table: title, quantified objects, data. (2) Find the following information in a frequency distribution table: title, data, frequencies. If applicable, each numerical value must be stated in the unit of measure indicated in the table. (3) Calculate the range of a given distribution.

GSM 223-02 ***Broken-Line Graphs, Vertical and Horizontal Bar Graphs and Pictographs***

(1) Derive information from a broken-line graph, a vertical bar graph, a horizontal bar graph or a pictogram. (2) Given a data table or a frequency distribution table, construct a broken-line graph, a vertical bar graph, a horizontal bar graph or a pictograph according to instructions. The given table contains between three and eight statistics or frequencies. The instructions concern the length of the axes, the creation of a scale break, the width of the bars in a bar graph and the numerical value of each symbol in a pictograph. The graph must be given a title and each axis must be properly identified and graduated. The bars in the bar graph must be separated from one another, and the numerical value of each symbol must be indicated in the top right-hand corner of the pictograph.

GSM 223-03 ***Circle Graphs***

(1) Collect information from a circle graph. (2) Given a data table or a frequency distribution table, construct a circle graph by using a compass, a protractor and a ruler. The given table contains between three and eight statistics or frequencies. The graph must be given a title, and each sector must be given a subheading and assigned an appropriate percentage value. All the calculations involved in constructing the graph must be shown.

GSM 223-04 ***Histograms***

(1) Derive information from a histogram. (2) Using a frequency distribution table in which the data is grouped into classes, construct a histogram, following any instructions regarding the creation of a scale break. The given table contains between three and eight classes along with their respective frequencies. The histogram must be given a title and each axis must be properly identified and graduated.

GSM 223-05 *Tree Diagrams*

(1) Construct a tree diagram of the set of all possible outcomes, given the description of a series of two or three random experiments. Each experiment should have between two and six possible outcomes. (2) On the basis of that tree diagram, indicate the following information:

- a particular outcome to be written as an ordered pair or ordered triple;
- the universe of possible outcomes (usually denoted by U) to be written as a set of ordered pairs or ordered triples;
- an event (usually denoted by E) to be written as a set of ordered pairs or ordered triples.

Set theory notation is used. The problems deal with everyday situations and involve a series of two or three random experiments leading to no more than 36 possible outcomes.

GSM 223-06 ***Calculating Probabilities***

Calculate the probability of a particular event or outcome, given a description of a series of two or three random experiments. Probability can be expressed in one of two ways: as a decimal number between 0 and 1, rounded to the nearest thousandth, or as a percentage whose value ranges from 0% to 100%. The problems involve a series of random experiments leading to no more than 36 possible outcomes. The students must show the steps in the solution, including a tree diagram.

7.07 GSM 231 The Four Operations on Polynomials

Module GSM 231 consists of six objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 231-01 to **GSM 231-06**	48	100 %

* Two hours are allotted for the final evaluation.

GSM 231-01 *Value of the Numerical Coefficient and of the Exponent of a Given Base*

(1) Determine the value of the numerical coefficient of a monomial as well as the value of the exponent assigned to each of the variables (bases) in the monomial. This monomial contains up to three variables. (2) Identify similar monomials.

GSM 231-02 *Monomials, Binomials, Trinomials and Polynomials*

Select the monomials, binomials, trinomials and polynomials from a list containing up to ten algebraic expressions and order a polynomial according to the increasing or decreasing powers of a given variable.

GSM 231-03 *Adding and Subtracting Two Polynomials*

Add or subtract two polynomials, each containing up to four terms. The terms of these polynomials contain no more than three variables. The numerical coefficients are rational numbers and the exponents are natural numbers. The resulting polynomial must be in simplified form and its terms must be ordered.

GSM 231-04 *Multiplying Two Polynomials*

Multiply two algebraic expressions: either a monomial by a monomial, a monomial by a polynomial containing up to three terms or a binomial by a binomial. Each term has a maximum of three variables. The numerical coefficients are rational numbers and the exponents are natural numbers. If the resulting product is a polynomial, its terms must be ordered.

GSM 231-05 *Dividing Two Polynomials*

Divide two algebraic expressions: either a monomial by a monomial, a binomial by a monomial, a trinomial by a monomial or a trinomial by a binomial. Each term contains up to three variables. The numerical coefficients are rational numbers and the exponents (of the divisor, the dividend and the quotient) are natural numbers. If the resulting quotient is a polynomial, its terms must be ordered.

GSM 231-06 ***The Four Operations on Polynomials***

Evaluate an algebraic expression by performing the required operations (addition, subtraction, multiplication and division) and by following the order of operations. The algebraic expression contains no more than three sets of parentheses, one set of brackets and ten terms. Each term contains up to two variables. The numerical coefficients are rational numbers and the exponents are natural numbers. The exponents assigned to the variables in the simplified form (solution) are all positive. The resulting polynomial must be in simplified form and its terms must be ordered. The steps involved in evaluating the algebraic expression must be shown.

7.08 GSM 232 **Geometry II**

(Perimeter, Area and Volume)

Module GSM 232 consists of nine objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	**Number of Hours***	**% (Evaluation)**
GSM 232-01 to **GSM 232-04**	12	25 %
GSM 232-05 and **GSM 232-06**	11	20 %
GSM 232-07	5	15 %
GSM 232-08	10	20 %
GSM 232-09	10	20 %

* Two hours are allotted for the final evaluation.

GSM 232-01 *Constructing Quadrilaterals*

Using a ruler, a set-square and a protractor, construct the following quadrilaterals:

- a square, given the measure of one of its sides
- a rectangle, given its length and width
- a parallelogram, given the following measures:
  - the long side, the short side and the altitude
  - the long side, the short side and one of the angles

  or
  - the altitude and one of the angles.

These geometric figures illustrate everyday situations.

GSM 232-02 *Constructing Triangles*

Using a ruler, a set-square and a protractor, construct the following triangles:

- an equilateral triangle, given the measure of a side
- an isosceles triangle, given the measure of a side and the base or given the measure of the base and one of the angles
- a right triangle, given the measure of the base and altitude.

These geometric figures illustrate everyday situations.

GSM 232-03 *Altitude, Perpendicular Bisector, Median and Bisector*

Identify the altitude, perpendicular bisector, median and bisector in a triangle.

GSM 232-04 ***Constructing Trapezoids and Rhombuses***

Using a ruler, a set-square and a protractor, construct the following geometric figures:

- **a rhombus, given the lengths of the long and short diagonals**
- **an isosceles trapezoid, given the lengths of the long base, the short base and one of the non-parallel sides**
- **a right trapezoid, given the lengths of the long base, the short base and one of the non-parallel sides (the altitude).**

These geometric figures illustrate everyday situations.

GSM 232-05 *Perimeter and Area of Familiar Polygons*

Using an appropriate formula, solve word problems involving the calculation of the perimeter or area of the following polygons:

- squares
- rectangles
- parallelograms
- triangles
- rhombuses
- trapezoids

The measures required to calculate the perimeter or area are given or can be deduced. The problems deal with everyday situations. The steps in the solution must be shown.

GSM 232-06 ***Perimeter and Area of Any Polygon***

Solve problems that involve calculating the perimeter and area of any polygon by breaking it up into simpler figures and by applying the formulas for calculating the perimeter and area of the following figures: squares, rectangles, parallelograms, triangles, rhombuses and trapezoids. A ruler and set square are required. The students are assigned word problems related to everyday situations and accompanied by illustrations. The steps in the solution of the problem must be shown and the answer must be stated in the appropriate units of measure.

GSM 232-07 ***Circles, Circumference and Area***

(1) Given the radius, construct a circle using a ruler and compass. (2) Solve problems that involve calculating the circumference and area of a circle. The students are asked to solve word problems dealing with everyday situations. The steps involved in calculating the circumference or the area must be shown and the answer must be stated in the appropriate units of measure.

GSM 232-08 ***Lateral Area and Total Area of Solids***

Solve word problems that involve calculating the lateral and total area of the following solids: cubes, right prisms, cones and cylinders. The calculations must be done by applying the appropriate formulas. A ruler is required. The problems deal with everyday situations. The steps in the solution must be shown and the answer must be stated in the appropriate units of measure.

GSM 232-09 ***Volume of Solids***

Solve word problems that involve calculating the volume and capacity of the following solids: cubes, right prisms, cones and cylinders. The calculations must be done by applying the appropriate formulas. A ruler is required. The problems deal with everyday situations. A conversion table showing units of volume and corresponding units of capacity is provided. The steps in the solution must be shown and the answer must be stated in the appropriate units of measure.

7.09 GSM 241 — Straight Lines I (Equations, Graphs, and Slope)

Module GSM 241 consists of nine objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 241-01 to **GSM 241-03**	10	20 %
GSM 241-04	16	30 %
GSM 241-05 to **GSM 241-08**	10	20 %
GSM 241-09	12	30 %

* Two hours are allotted for the final evaluation.

GSM 241-01 *Representing Points in the Cartesian Plane*

(1) Given the coordinates of a point, plot it in a graduated Cartesian plane. (2) Identify the coordinates of a point located in a graduated Cartesian plane. The coordinates are rational numbers, but only the most common fractions and decimals are used.

GSM 241-02 *Graphing a First-Degree Equation in Two Variables*

Graph an equation of the form $y = mx + b$ after completing a table of values containing at least five pairs of coordinates. These equations represent everyday situations. The Cartesian plane used to graph the equation is already drawn; the axes are labelled and their gradations are shown.

GSM 241-03 ***Determining Points, Given the Graph of a First Degree Equation in Two Variables***

Solve word problems that involve drawing a graph in a Cartesian plane to determine one of the coordinates of the third point on a line, given the coordinates of two points on the line and one of the coordinates of the third point. These problems deal with everyday situations. The given numbers are rational numbers, but only the most common fractions and decimals are used.

GSM 241-04 ***Graphing an Equation of the Form Ax + By + C = 0***

Graph an equation of the form Ax + By + C = 0. Three points, including the x- and y-intercepts if any, must be plotted on the graph. The numbers used are rational numbers.

GSM 241-05 *Determining the Slope (Rate of Change) of a Line, Given Two Points on a Graph*

Calculate the slope of a line illustrating an everyday situation, given two points on that line. The slope must be stated in the appropriate unit of measure.

GSM 241-06 *Determining the Slope (Rate of Change) of a Line, GivenTwo Points*

(1) Calculate the slope of a line passing through points (x_1, y_1) and (x_2, y_2) by applying the formula: $m = \frac{y_2-y_1}{x_2-x_1}$. (2) Indicate whether this slope is positive, negative, equal to zero, or undefined.

GSM 241-07 *Graphing a Line, Given Its Slope and One of Its Points*

Graph a line in the Cartesian plane, given the slope of the line and one of its points.

GSM 241-08 ***Determining the Slope (Rate of Change) of a Line, Given Its Equation***

Given the equation of line, calculate its slope by converting the equation to the form $y = mx + b$.

GSM 241-09 ***Determining a First-Degree Equation in Two Variables***

Determine the equation of a line, given two of its points or its slope and one of its points. The resulting equation must be expressed in one of the following forms: $y = mx + b$ or $Ax + By + C = 0$. The steps in the solution must be shown and the graph must be drawn if it is not already provided.

7.10 GSM 242 — Geometry III (Isometry, Similarity Transformations)

Module GSM 242 consists of eight objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 242-01	4	10 %
GSM 242-02	4	10 %
GSM 242-03	5	20 %
GSM 242-04	4	25 %
GSM 242-05	2	5 %
GSM 242-06 to **GSM 242-08**	5	30 %

* One hour is allotted for the final evaluation.

GSM 242-01 ***Isometric Transformations: Translation, Rotation and Reflection***

(1) Given a set of diagrams representing isometric transformations of geometric figures, identify:

- **those illustrations that represent a translation**
- **those illustrations that represent a rotation**
- **those illustrations that represent a reflection**

(2) Using a ruler, a set-square, a compass or a protractor, draw the images of simple geometric figures under the following transformations:

- **a translation t, given the length and the direction of displacement;**
- **a rotation r, given the location of the centre of rotation and the measure of the angle of rotation;**
- **a reflection s, given the location of the line of reflection.**

GSM 242-02 ***Similarity Transformations (Size Transformations) and Similar Figures***

(1) Using a ruler and a set-square, draw the image of a geometric figure under a size transformation (h), given the location of the centre of similitude (o) and the ratio of similitude (k). The value of k can be either positive or negative. (2) Given diagrams of geometric transformations, indicate those that represent a size transformation (h).

GSM 242-03 ***Congruent Triangles and Similar Triangles***

(1) Using a ruler, a protractor and a compass, construct the triangle defined by one of the following groups of measures:

- **the measure of an angle and the lengths of the two sides that form this angle;**
- **the measures of two angles and the length of the side contained between these angles;**
- **the measures of the three sides.**

(2) Given the measures of some of the angles and some of the sides of two triangles, determine whether these triangles are congruent or similar by applying the properties of congruent and similar triangles. Reasons must be given for each conclusion.

GSM 242-04 ***Calculating the Lengths of Sides of Two Similar Triangles***

Calculate the length of one or more sides in one of two similar triangles, given:

- **the length of one or more corresponding sides of one of the triangles;**
- **the ratio of similitude (k) or the lengths required to calculate k.**

The steps in the solution must be shown.

GSM 242-05 ***Calculating the Lengths of Sides of Two Similar Polygons***

Calculate the length of one or more sides in one of two similar polygons, given:

- **the length of one or more corresponding sides of one of the polygons;**
- **the ratio of similitude (k) or the lengths required to calculate k.**

These polygons have a maximum of eight sides. The steps in the solution must be shown.

GSM 242-06 *Calculating Actual Dimensions, Given a Scale Diagram*

Given a scale diagram representing an everyday situation, apply the properties of similar figures to solve word problems that involve calculating actual dimensions.

GSM 242-07 *Drawing Scale Diagrams, Given Actual Dimensions*

Given a drawing that illustrates an everyday situation and using a ruler and a set-square, apply the properties of similar figures to solve problems that involve drawing a diagram to a given scale. Only 90° angles will be represented in the original drawing.

GSM 242-08 ***Solving Real-Life Problems Involving the Concepts of Similarity or Congruence in Geometric Figures***

(1) Given the measures on a plan, apply the properties of congruent and similar figures to solve problems that involve calculating actual dimensions and measures of angles. (2) Given actual dimensions, apply these same properties to solve problems that involve calculating measures to be used in a scale diagram. A ruler, a set-square and a protractor are required. Drawings illustrating various real-life situations will be provided. The steps in the solution must be justified on the basis of the above-mentioned properties.

7.11 GSM 243 — Equations and Inequalities II

Module GSM 243 consists of seven objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 243-01	6	10 %
GSM 243-02 to **GSM 243-05**	18	40 %
GSM 243-06	14	30 %
GSM 243-07	10	20 %

* Two hours are allotted for the final evaluation.

GSM 243-01 ***Graphing a System of Equations***

Solve a system of two first-degree equations in two variables by graphing that system. The equations are of the form $Ax + By + C = 0$, where the coefficients A, B and C are rational numbers. The steps in the solution must be shown.

GSM 243-02 *Solving a System of Equations by Comparison*

Solve a system of two first-degree equations in two variables by applying the comparison method. The equations are of the form $Ax + By + C = 0$, where the coefficients A, B and C are rational numbers. The steps in the solution must be shown.

GSM 243-03 *Solving a System of Equations by Substitution*

Solve a system of two first-degree equations in two variables by applying the substitution method. The equations are of the form $Ax + By + C = 0$, where the coefficients A, B and C are rational numbers. The steps in the solution must be shown.

GSM 243-04 *Solving a System of Equations by Elimination Through Addition*

Solve a system of two first-degree equations in two variables by applying the method of elimination through addition. The equations are of the form $Ax + By + C = 0$, where the coefficients A, B and C are rational numbers. The steps in the solution must be shown.

GSM 243-05 ***Solving a System of Equation: Four Possible Methods***

Solve a system of two first-degree equations in two variables by one of the following methods:

- **graphing the system**
- **solving by comparison**
- **solving by substitution**
- **solving by elimination through addition**

The equations are of the form $Ax + By + C = 0$, where the coefficients A, B and C are rational numbers. The steps in the solution must be shown.

GSM 243-06 ***Solving Everyday Problems***

Solve word problems that involve translating an everyday situation into a system of two first-degree equations in two variables and then solving that system of equations. The numbers used are rational numbers. The solution set and the steps involved in solving the problem must be clearly indicated.

GSM 243-07 ***Graphing a System of Inequalities***

Solve a system of two first-degree inequalities in two variables by graphing that system. These inequalities are of one of the following forms:

$$Ax + By + C < 0$$
$$Ax + By + C > 0$$
$$Ax + By + C \leq 0$$
$$Ax + By + C \geq 0.$$

The coefficients A, B and C are rational numbers.

7.12 GSM 244 — Trigonometry I (Trigonometric Ratios in Triangles)

Module GSM 244 consists of six objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 244-01 to **GSM 244-04**	7	40 %
GSM 244-05	9	44 %
GSM 244-06	8	16 %

* One hour is allotted for the final evaluation.

GSM 244-01 *Right Triangles*

Determine the measures of the angles and the sides of a right triangle, using the Pythagorean theorem and given the measures of some of its angles. There are two possible cases:

- a right triangle, given the measure of one acute angle and the lengths of two sides
- a right triangle, given an angle that measures either 30° or 45° and the length of one side.

The students are asked to solve word problems dealing with everyday situations. The steps in the solution must be shown.

GSM 244-02 *Trigonometry and Trigonometric Ratios*

Given the lengths of the three sides of right triangle ABC (angle C= 90°), evaluate any of the following trigonometric ratios of an angle A :

- sine A (sin A)
- cosine A (cos A)
- tangent A (tan A)

GSM 244-03 *Determining the Measure of an Angle, Given a Trigonometric Ratio*

Given the lengths of two of the sides of a right triangle, determine the measures of its acute angles by applying the definitions of trigonometric ratios and by using a trigonometric table or scientific calculator.

GSM 244-04 ***Solving Right Triangles***

Determine the unknown dimensions of a right triangle by using the definitions of trigonometric ratios, the Pythagorean theorem and the relationships between the angles in this triangle. There are two possible situations:

- **The lengths of two sides of the triangle are given.**
- **The length of one side and the measure of one acute angle are given.**

These problems can be solved with a trigonometric table or a scientific calculator. The steps in the solution must be shown.

GSM 244-05 ***Everyday Problems***

Solve word problems by applying the definitions of the sine, cosine, and tangent ratios in right triangles. The problems deal with everyday situations and involve determining the measures of angles, sides, or both angles and sides in a right triangle that is given or is to be drawn. A trigonometric table or a scientific calculator is required and the steps in the solution must be shown.

GSM 244-06 ***Solving Any Given Triangle***

Solve word problems by applying one of the two laws given below:

- **Law of Sines:** $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- **Law of Cosines:** $a^2 = b^2 + c^2 - 2\,bc \cos A.$

The problems deal with everyday situations and involve determining the measures of angles, sides, or both angles and sides in any type of triangle that is given or is to be drawn. A trigonometric table or a scientific calculator is required and the steps in the solution must be shown.

7.13 GMO 241 Factoring

Module GMO 241 consists of six objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GMO 241-01 to **GMO 241-06**	24	100 %

* One hour is allotted for the final evaluation.

GMO 241-01 *Factoring by Removing the Common Factor*

Find the common factor of all the terms of a polynomial containing up to six terms linked by + or - signs. The result must be expressed as the product of a monomial and a polynomial which is placed in parentheses. The numerical coefficients of the terms of the polynomial are rational numbers and the exponents of the variables are natural numbers.

GMO 241-02 *Factoring by Grouping*

To factor a polynomial of up to six terms linked by + or - signs by applying the method of grouping. The result must be expressed as the product of two binomials or as the product of a binomial and a trinomial. The terms of the polynomial may have to be rearranged before being grouped and factored. The numerical coefficients of the terms of the polynomial are rational numbers and the exponents of the variables are natural numbers. The steps in the solution must be shown.

GMO 241-03 *Factoring a Trinomial of the Form $x^2 + bx + c$ or $x^2 + bxy + cy^2$*

To factor a trinomial of the form $x^2 + bx + c$ or $x^2 + bxy + cy^2$, where b and c are integers. For trinomials of the form $x^2 + bx + c$, the result must be expressed as the product of two binomials of the form $(x + d)(x + e)$. For trinomials of the form $x^2 + bxy + cy^2$, the result must be expressed as the product of two binomials of the form $(x + dy)(x + ey)$. In both cases, d and e are integers. The steps in the solution must be shown.

GMO 241-04 *Factoring a Trinomial of the Form $ax^2 + bx + c$ or $ax^2 + bxy + cy^2$*

To factor a trinomial of the form $ax^2 + bx + c$ or $ax^2 + bxy + cy^2$, where a, b and c are integers. For trinomials of the form $ax^2 + bx + c$, the result must be expressed as the product of two binomials of the form $(kx + l)(mx + n)$. For trinomials of the form $ax^2 + bxy + cy^2$, the result must be expressed as the product of two binomials of the form $(kx + ly)(mx + ny)$. In both cases, k, l, m and n are integers. The steps in the solution must be shown.

GMO 241-05 *Factoring a Difference of Squares*

To factor the difference of two squares as the product of two binomials formed by the sum and by the difference of the square roots of each term of the initial algebraic expression. The difference of squares is of the form $(ax^{2n} - by^{2m})$, where a and b are squares of rational numbers, x and y are variables, and n and m are natural numbers less than or equal to 4.

GMO 241-06 ***Factoring a Polynomial Completely***

To factor a polynomial containing up to six terms as the product of at least three prime factors by applying the appropriate factoring methods selected from the list below:

- **factoring by removing the common factor**
- **factoring by grouping**
- **factoring trinomials of the form $x^2 + bx + c$ or $x^2 + bxy + cy^2$**
- **factoring trinomials of the form $ax^2 + bx + c$ or $ax^2 + bxy + cy^2$**
- **factoring a difference of squares.**

The steps in the solution must be shown.

7.14 GMO 242 — The Four Operations on Algebraic Fractions

Module GMO 242 consists of five objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	**Number of Hours***	**% (Evaluation)**
GMO 242-01 to **GMO 242-05**	24	100 %

* One hour is allotted for the final evaluation.

GMO 242-01 *Simplifiying Algebraic Fractions*

Reduce a rational algebraic fraction to its lowest terms. The numerator and the denominator are factorable polynomials that contain up to three terms each and each term contains no more than two variables. The steps involved in simplifying the fraction must be shown.

GMO 242-02 *Multiplying and Dividing Algebraic Fractions*

Multiply three rational algebraic fractions and divide two rational algebraic fractions. The polynomials in the numerators and denominators are factorable and contain up to three terms. Each term contains no more than two variables. The product and the quotient must be reduced to their lowest terms and the steps in the solution must be shown.

GMO 242-03 *Simplifying Algebraic Expressions Containing Algebraic Fractions That Are Multiplied and Divided*

Simplify an algebraic expression containing up to four rational algebraic fractions that are multiplied and divided. The numerators and denominators are factorable polynomials that contain up to three terms each and each term contains no more than two variables. The steps involved in simplifying the expression must be shown.

GMO 242-04 *Simplifying Algebraic Expressions Containing Algebraic Fractions That Are Added and Subtracted*

Simplify an algebraic expression containing up to three rational algebraic fractions that are added and subtracted. The numerators and denominators are factorable polynomials that contain up to three terms each and each term contains no more than two variables. The steps involved in simplifying the expression must be shown.

GMO 242-05 ***Series of Operations Involving Algebraic Fractions***

Simplify an algebraic expression containing up to three rational algebraic fractions by performing the appropriate operations and by following the order of operations. This algebraic expression contains no more than two sets of parentheses. The numerators and denominators are factorable polynomials that contain up to three terms each and each term contains no more than two variables. The steps involved in simplifying the expression must be shown.

7.15 GMO 243 — Straight Lines II (Parallel and Perpendicular Lines and Distance)

Module GMO 243 consists of four objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GMO 243-01 and **GMO 243-02**	10	35 %
GMO 243-03	7	35 %
GMO 243-04	7	30 %

* One hour is allotted for the final evaluation.

GMO 243-01 *Determining the Equation of a Line*

Determine the equation of a line, given any of the following:

- the slope and the y-intercept of the line
- the slope of the line and one of its points
- two points on the line

GMO 243-02 ***Perpendicular and Parallel Lines***

Determine the equation of a line, given any of the following:

- **one of its points and the equation of a line parallel to it**
- **one of its points and the equation of a line perpendicular to it.**

The coefficients of these linear equations and the coordinates of the points are rational numbers. The resulting equation must of the form $y = mx + b$ and the steps in the solution must be shown.

GMO 243-03 ***Distance Between Two Points***

Determine the distance between two given points in the Cartesian plane. The coordinates of these points and the distance are rational numbers and the problems deal with everyday situations. The steps in the solution must be shown and the distance must be stated in a unit of measure.

GMO 243-04 ***Coordinates of a Point That Divides a Line Segment in a Particular Ratio***

Determine the point that divides a line segment in a particular ratio, given the endpoints of the line segment. The coordinates of the endpoints and the ratio are rational numbers and the problems deal with everyday situations. The ratio in which the segment is divided must be derived from the information given in the problem. The steps in the solution must be shown.

7.16 GMO 244 — Logic, Set Theory and Relations

Module GMO 244 consists of 27 objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GMO 244-01 to **GMO 244-03**	3	5 %
GMO 244-04 to **GMO 244-06**	3	5 %
GMO 244-07	2	5 %
GMO 244-08 and **GMO 244-09**	4	10 %
GMO 244-10 to **GMO 244-12**	4	10 %
GMO 244-13 and **GMO 244-14**	4	10 %
GMO 244-15	3	5 %
GMO 244-16 and **GMO 244-17**	4	10 %
GMO 244-18 and **GMO 244-19**	4	10 %
GMO 244-20 and **GMO 244-21**	3	5 %
GMO 244-22 and **GMO 244-23**	3	5 %
GMO 244-24 and **GMO 244-25**	4	10 %
GMO 244-26	4	5 %
GMO 244-27	3	5 %

* Two hours are allotted for the final evaluation.

GMO 244-01 *Identifying Propositions*

Identify the propositions in a list of five to ten simple verbal and mathematical sentences.

GMO 244-02 *Different Types of Propositions and Connectives*

(1) Given a proposition expressed as a verbal or mathematical sentence, determine whether it is a negation, a conjunction, a disjunction (inclusive or exclusive), a conditional statement or a biconditional statement on the basis of its logical connective. (2) Transcribe this proposition, using one of the following symbols to represent the logical connective:

- $\neg$ for a negation (not)
- $\wedge$ for a conjunction (and)
- $\vee$ for a disjunction (or)
- $\rightarrow$ for a conditional statement (if...then)
- $\leftrightarrow$ for a biconditional statement (if and only if).

Simple statements must be selected.

GMO 244-03 ***Truth Value of a Compound Proposition***

Given the truth table for each type of proposition (negation, conjunction, disjunction, conditional statement and biconditional statement), determine the truth value of a proposition consisting of up to three simple propositions by following the order of logical operations. The truth value of each simple proposition is known. The given compound proposition must be written symbolically and should contain no more than three logical connectives. The steps in the solution must be shown.

GMO 244-04 *Tautologies and Contradictions*

Set up a truth table for a proposition consisting of up to three simple propositions and three logical connectives in order to determine if that proposition is a tautology or a contradiction. The proposition is a tautology if it is always true whatever the truth value of each of its simple propositions. The proposition is a contradiction if it is always false whatever the truth value of each of its simple propositions. These given compound propositions must be written symbolically and all possibilities must be included in the truth table. The steps in the solution must be shown.

GMO 244-05 *Logical Implication (=>)*

Given two compound propositions linked together to form a conditional statement, set up a truth table and determine if that conditional proposition is always true whatever the truth value of each of its simple propositions. If such is the case, connect the two compound propositions using the symbol for a logical implication (=>). These given compound propositions must be written symbolically and each should contain no more than three simple propositions and three logical connectives. All possibilities must be included in the truth table and the steps in the solution must be shown.

GMO 244-06 ***Logical Equivalence (<=>)***

Given two compound propositions linked together to form a biconditional statement, set up a truth table and to determine if that biconditional proposition is always true whatever the truth value of each of its simple statements. If such is the case, connect the two compound propositions using the symbol for a logical equivalence (<=>). These given compound propositions must be written symbolically and each should contain no more than three simple propositions and three logical connectives. All possibilities must be included in the truth table and the steps in the solution must be shown.

GMO 244-07 ***Negation of a Compound Proposition***

Determine the negation of a compound proposition written symbolically. The negation is determined by rewriting the given proposition so that only its simple propositions bear the negation symbol. Each compound proposition should contain no more than three simple propositions and five logical connectives. The steps in the solution must be shown.

GMO 244-08 *Propositions and Propositional Forms*

Identify the propositions and the propositional forms in a list of five to ten verbal and mathematical sentences, some of which contain variables.

GMO 244-09 ***Solution Sets of Propositional Forms***

Given a universe containing five to ten elements, list the elements of the solution set of a simple propositional form or the elements of the solution set of a propositional form containing two simple propositional forms linked together by a logical connective. In the latter case, the answer must also include the solution set of each simple propositional form. The propositional forms must be expressed mathematically.

GMO 244-10 *Existential and Universal Quantifiers*

(1) Given a list of five to ten quantified verbal sentences, indicate those that contain an existential quantifier and those that contain a universal quantifier. (2) Transcribe these statements, using one of the following symbols to represent the quantifier:

- $\exists$ for the existential quantifier (There is at least one...),
- $\exists$**!** for the unique existential quantifier (There is only one...),
- $\forall$ for the universal quantifier (For all...).

GMO 244-11 *Negation of a Compound Propositional Form Containing a Quantifier*

Determine the negation of a quantified compound propositional form expressed either verbally, mathematically or symbolically. The negation is determined by rewriting the given proposition so that only its simple statements bear the negation symbol. This compound propositional form should contain no more than three simple propositional forms and three logical connectives. The steps in the solution must be shown.

GMO 244-12 ***Truth Value of a Propositional Form Containing a Quantifier***

Given a universe containing five to ten elements, determine the truth value of a quantified compound propositional form by following the order of logical operations. This compound propositional form should contain no more than three simple propositional forms expressed mathematically and three logical connectives. The steps in the solution and the solution set of each propositional form must be indicated.

GMO 244-13 *Sets of Numbers and Membership in a Set*

(1) Given the definition of various sets of numbers:

- natural ($\mathbb{N}$)
- integers ($\mathbb{Z}$)
- rational ($\mathbb{Q}$)
- irrational ($\mathbb{Q}'$)
- real ($\mathbb{R}$)

indicate whether a given element belongs to a particular set. The problems can involve one of the sets listed below:

- one of the following sets of numbers: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{Q}'$ or $\mathbb{R}$
- a finite or infinite set of integers or natural, rational, irrational or real numbers; the content of this set is determined by an inequality of the form:

  $x < n$, $x > n$, $n_1 < x < n_2$

  $x \leq n$, $x \geq n$ or $n_1 \leq x \leq n_2$
- a finite set of natural numbers or integers that contains one of the following: prime numbers, even numbers, odd numbers, square numbers, cubed numbers, multiples of a given number or factors of a given number.

(2) Indicate whether or not the element belongs to a particular set by using the appropriate symbol (i.e. $x \in E$ if element x belongs to set E or $x \notin E$ if element x does not belong to set E).

GMO 244-14 ***Describing a Set by Listing Its Elements, Using Set-Builder Notation or Drawing a Venn Diagram***

(1) Describe a given finite or infinite set of natural, whole, rational, irrational, or real numbers by listing its elements, using set-builder notation or drawing a Venn Diagram. The content of this set should correspond to one of the following definitions:

- **the set is determined by an inequation of the form**

  $x < n,\quad x > n,\quad n_1 < x < n_2$

  $x \leq n,\quad x \geq n\quad \text{or}\quad n_1 \leq x \leq n_2$

- **the set contains one of the following types of numbers: prime numbers, even numbers, odd numbers, square numbers, cubed numbers, multiples of a given number, or factors of a given number.**

(2) Given a set whose elements are listed or are described by means of set-builder notation or a Venn Diagram, convert the given description to one of the other descriptive forms.

GMO 244-15 ***Set Relations: Inclusion and Equality***

(1) Determine whether two given sets are equal or whether one is included within the other. (2) Express the relationship between each pair of sets by means of the following symbols: $\subseteq$ if one set is included within the other; $\not\subseteq$ if a given set is not included within the other; = if the given sets are equal; ≠ if the given sets are not equal. (3) Given a list of sets, indicate those that are the subsets of a particular set. In most cases, the elements of these sets are listed.

GMO 244-16 *Set Operations: Union, Intersection, Difference and Complementation*

Given a universal set U, find the union, intersection or difference of two given sets or find the complement of a particular set by correctly applying the definitions listed below:

- $A \cup B = \{x \in U \mid (x \in A) \vee (x \in B)\}$
- $A \cap B = \{x \in U \mid (x \in A) \wedge (x \in B)\}$
- $A \setminus B = \{x \in U \mid (x \in A) \wedge (x \notin B)\}$
- $A' = \{x \in U \mid x \notin A\}$

Sets A and B are finite or infinite sets whose elements are listed, or are defined by means of set-builder notation or a Venn Diagram. The members of each finite set belong to a universe containing no more than twelve elements. The result of the operation must be indicated by listing its elements, using set-builder notation or drawing a Venn Diagram.

GMO 244-17 ***Series of Operations on Sets***

Given a universe U and two or more finite or infinite sets whose elements are listed or defined by means of set-builder notation or a Venn Diagram, perform a series of up to four set operations consisting of any of the following: $\cup$, $\cap$, \ , ′. Each series of operations should involve no more than three sets of parentheses. The content of each finite set is derived from a universe containing no more than twelve elements. The result of each series of operations must be indicated by listing its elements, using set-builder notation or drawing a Venn Diagram. The steps in the solution must be shown.

GMO 244-18 *Representing Intervals of Real Numbers*

(1) Given an interval of real numbers described by means of set-builder notation, graph it on the number line or indicate it using the appropriate symbolic notation to be selected from the list below:

[a,b], [a, b [,] a,b],] a, b [
[a, ∞,] a, ∞, - ∞, a] and - ∞, a [

In this case, a and b are the endpoints of the interval. (2) Given an interval written in brackets, graph it on a number line. (3) Write an interval in brackets, given its graph on the number line.

GMO 244-19 ***Set Operations Involving Intervals of Numbers***

Perform set operations ($\cup$, $\cap$, \ ,′) on intervals of real numbers that are written in brackets, graphed on a number line or described by means of set-builder notation. The problems may involve up to four operations and a maximum of three sets of parentheses. The result of the operation or series of operations must be written in brackets, graphed on a number line or described by means of set-builder notation. The steps in the solution must be shown.

GMO 244-20 *Definition of an Ordered Pair*

Given the definition of an ordered pair, mathematize simple relations representing everyday situations, using ordered pairs.

GMO 244-21 ***The Cartesian Product of Two Sets***

Find the Cartesian product of two sets whose elements are listed, or are defined by means of set-builder notation or a Venn Diagram. These two sets are finite subsets of $\mathbb{Z}$ and each has a cardinal number less than or equal to 4. The Cartesian product must be indicated by listing its elements, drawing a Cartesian graph or drawing an arrow diagram.

GMO 244-22 *Subset of a Cartesian Product*

Given the Cartesian product of two sets and a rule of correspondence that determines how the elements of these two sets are related, form a subset of this Cartesian product with all the elements for which that rule holds. The rule of correspondence must be related to some area of human activity or to a simple mathematical situation.

GMO 244-23 ***Defining a Relation, Its Domain and Its Range***

(1) Given a relation defined by means of set-builder notation, describe it by listing its elements. The source and target sets are finite subsets of $\mathbb{N}$ or $\mathbb{Z}$ and the rule of correspondence can be expressed as a linear or quadratic equation. (2) Find the domain and range of this relation.

GMO 244-24 *Graphic Representation of a Relation*

(1) Draw a Cartesian graph or an arrow diagram of a relation defined by means of set-builder notation in a finite subset of $\mathbb{Z} \times \mathbb{Z}$. The rule of correspondence must be expressed as a simple mathematical relation involving no more than twelve ordered pairs. (2) Draw a Cartesian graph of a relation defined by means of set-builder notation in $\mathbb{R} \times \mathbb{R}$. The rule of correspondence must be expressed as a first- or second- degree equation or inequality in one variable. (3) Find the domain and the range of this relation.

GMO 244-25 ***Defining a Relation by Using Set-Builder Notation***

Given a relation whose elements are listed or represented graphically, define it using set-builder notation. The rule of correspondence must be expressed as a first- or second-degree equation or inequality in one variable.

GMO 244-26 ***The Inverse of a Relation***

Find the inverse of a relation defined in one of the following ways:

- **Its elements are listed; there are a maximum of twelve ordered pairs.**
- **It is defined by means of set-builder notation. The rule of correspondence is expressed as a simple mathematical relation in $\mathbb{R} \times \mathbb{R}$, $\mathbb{Z} \times \mathbb{Z}$, or $\mathbb{N} \times \mathbb{N}$ or in a Cartesian product of two finite sets each with a cardinal number less than or equal to 4.**
- **It is represented by a Cartesian graph in $\mathbb{R} \times \mathbb{R}$, $\mathbb{Z} \times \mathbb{Z}$, $\mathbb{N} \times \mathbb{N}$ or in a finite subset of points.**
- **It is represented by an arrow diagram containing up to six arrows.**

The inverse must be expressed in the same form as the given relation.

GMO 244-27 ***The Composition of Relations***

Find the composition $r \circ s$ of two given relations r and s and express it in algebraic form. The two given relations are defined by means of set-builder notation and the rule of correspondence is expressed as a simple mathematical relation.

7.17 GSM 251 **Optimization**

Module GSM 251 contains five objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	**Number of Hours***	**% (Evaluation)**
GSM 251-01 and **GSM 251-02**	8	30 %
GSM 251-03	5	20 %
GSM 251-04 and **GSM 251-05**	11	50 %

* One hour is allotted for the final evaluation.

GSM 251-01 *Interpreting Different Parts of a Problem*

(1) For a given verbal optimization problem, indicate the elements that make it possible to define the function to be optimized, or objective function (i.e. the function that makes it possible to determine the required values). (2) Indicate the elements that make it possible to define the constraints imposed on the variables of the function to be optimized. Each problem should be no more than 150 words long.

GSM 251-02 ***Translating a Problem into Mathematical Language***

(1) Given the required elements of a verbal optimization problem, express the function to be optimized (i.e. the objective function) as an equation of the form $Ax + By + C = 0$, where A, B and C are integers. (2) Write the inequalities that correspond to the constraints imposed on the variables x and y of the function to be optimized. (3) Mathematize the constraints that affect each variable individually and the constraints that affect all the variables simultaneously. (4) Clearly indicate the variables. Each problem should be no more than 150 words long.

GSM 251-03 ***Polygon of Constraints***

Given a system of inequalities representing all the constraints imposed on the variables of a function to be optimized, graph all the given constraints in one Cartesian plane, draw the polygon of constraints defined by this system of inequations and find the coordinates of the vertices of that polygon.

GSM 251-04 *Verifying Whether a Point Belongs to the Polygonal Region of Constraints*

Given a system of inequalities representing all the constraints imposed on the variables of a function to be optimized, peform algebraic operations to verify whether a given point belongs to the polygonal region of constraints defined by that system of inequalities. The steps in the solution must be shown.

GSM 251-05 ***Solving an Optimization Problem***

For a given optimization problem, calculate the value of the objective function (function to be optimized) at each vertex of the polygon of constraints and determine the optimal solution. Each problem should be no more than 150 words long and the steps in the solution must be shown.

7.18 GSM 252 **Statistics and Probability II**

Module GSM 252 contains thirteen objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 252-01 to **GSM 252-05**	10	15 %
GSM 252-06 and **GSM 252-07**	8	15 %
GSM 252-08	5	10 %
GSM 252-09 and **GSM 252-10**	12	25 %
GSM 252-11 to **GSM 252-13**	13	35 %

*Two hours are allotted for the final evaluation.

GSM 252-01 *Nature of the Data in a Distribution*

Determine if the data in a distribution is **qualitative** or **quantitative**. If the data is quantitative, determine if the variable in the distribution is **discrete** or **continuous**. The given distribution must contain between ten and fifteen data items related to a concrete situation that has been clearly defined.

GSM 252-02 *Constructing a Data Table or a Distribution Table*

Construct a data table or a distribution table, using data from a survey or a study dealing with only one variable. The data table is constructed using ten to fifteen given data items, and the distribution table is constructed using forty to fifty data items and five to ten classes. The data must consist of integers related to a concrete situation that has been clearly defined.

GSM 252-03 *Arithmetic Mean of a Distribution*

Calculate the arithmetic mean of a given distribution containing ten to fifteen elements and, if necessary, round the result to the nearest tenth. The data must consist of rational numbers related to a concrete situation that has been clearly defined.

GSM 252-04 *Median and Mode of a Distribution*

Determine the median and the mode of a given distribution containing ten to fifteen elements. The data must consist of rational numbers related to a concrete situation that has been clearly defined.

GSM 252-05 ***Determining the Most Appropriate Measure of Central Tendency***

Determine the most appropriate measure of central tendency (arithmetic mean, median or mode) for a set of data from a survey or study dealing with only one variable. The given distribution must contain ten to fifteen rational numbers related to a concrete situation that has been clearly defined.

GSM 252-06 *Calculating a Summation* (Σ)

Given a one-variable mathematical expression containing the summation symbol (Σ), evaluate this expression for the values assigned to the variable. A maximum of fifteen successive numerical values must be substituted for that variable.

GSM 252-07 ***Calculating the Variance and Standard Deviation of a Distribution***

(1) Given data from a survey or a study dealing with only one variable, calculate the variance and standard deviation of the distribution by applying the appropriate formula and, if necessary, round these values to the nearest tenth. (2) Write a brief explanation, indicating the significance of these measures of dispersion in the situation under consideration. The given distribution must contain ten to fifteen rational numbers related to a concrete situation that has been clearly defined.

GSM 252-08 ***Quintile and Percentile Rank***

Assign a quintile or percentile rank to a specific data item in a given distribution. The distribution must contain between 20 and 120 elements when a quintile rank is required and at least 200 elements when a percentile rank is required. The data must consist of rational numbers related to a concrete situation that has been clearly defined.

GSM 252-09 *Determining the Best Graph for a Given Distribution*

Given four or five different graphs of a given distribution (broken-line graph, vertical or horizontal bar graphs, histogram, pictogram, circle graph), describe the advantages and disadvantages of each and select the one most appropriate for the situation under consideration. The distribution must contain between 5 and 50 rational numbers related to a concrete situation that has been clearly defined.

GSM 252-10 ***Comparing Two Distributions on the Basis of Their Respective Graphs***

On the basis of their respective graphs and in 25 to 75 words, compare two distributions obtained under the same circumstances and containing the same number of elements in order to delineate the variable under consideration. For the purposes of this exercise, calculate and use the given measures of dispersion and central tendency. The given distribution must contain between five and fifty rational numbers related to a concrete situation that has been clearly defined.

GSM 252-11 *Dependent and Independent Outcomes*

Given a description of two successive random experiments, determine whether the outcomes are dependent or independent.

GSM 252-12 *Complementary Events*

Determine if two given events are complementary, given the description of a series of two or three random experiments each of which can result in at least two and no more than six posssible outcomes. Successive experiments can have dependent or independent outcomes. The universe of possible outcomes should contain no more than 36 elements.

GSM 252-13 ***Calculating Probabilities***

Given a description of a series of random experiments, calculate the probability of a particular event or outcome by applying the fundamental theorem of counting to determine the number of favorable outcomes and the number of possible outcomes. Probability must be expressed as a decimal number between 0 and 1, rounded to the nearest thousandth, or as a percentage whose value ranges from 0% to 100%. Successive experiments can have dependent or independent outcomes.

7.19 GSM 253 Geometry IV

Module GSM 253 consists of six objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GSM 253-01 to **GSM 253-04**	12	50 %
GSM 253-05 and **GSM 253-06**	12	50 %

* One hour is allotted for the final evaluation.

GSM 253-01 *Identifying Various Elements in Circles*

On diagrams of circles in which several elements are represented and labelled with upper case letters, identify the following: a radius, a diameter, a chord, an arc, a secant, a tangent, a point of tangency, a central angle, an inscribed angle, an interior angle and an exterior angle.

GSM 253-02 *Relationships Governing the Measure of Length in a Circle*

Given a list of theorems and corollaries as well as a diagram of one or two circles with the measures required to solve a particular problem, determine the measure of a radius, a diameter, a circumference, an area, a chord, an arc or a segment tangent, indicating the theorems or corollaries used to support each step in the solution. The following is a list of the given theorems and corollaries:

Relationships Within a Circle

- Any perpendicular bisector of a chord is a diameter of a circle.
- The longest chord of a circle is a diameter.
- In a circle, any radius perpendicular to a chord divides that chord into two congruent segments.
- In a circle, any radius perpendicular to a chord divides the subtended arc into two congruent arcs.
- In a circle, arcs located between two parallel chords are congruent.
- Two chords are congruent if they are equidistant from the center of the circle.
- In a circle, congruent chords subtend congruent arcs and, conversely, congruent arcs are subtended by congruent chords.
- Any line tangent to a circle is perpendicular to the radius that shares the point of tangency.
- For any circle, two tangent segments originating from the same exterior point are congruent. (The segments are measured from that exterior point to their respective points of tangency).
- Two parallel lines, be they tangents or secants, intercept congruent arcs of a circle.

Relationships Involving Two Circles

- The circumferences of two circles have the same ratio as their radii.
- The areas of two circles have the same ratio as the squares of their radii.
- The measures of similar arcs of two circles have the same ratio as their radii.

Each solution should involve the application of no more than three theorems or corollaries.

GSM 253-03 *Relationships Governing Angular Measures in a Circle*

Given a list of theorems and corollaries as well as a diagram of a circle with the measures required to solve a particular problem, determine the measure of a central angle, an inscribed angle, an interior angle, an exterior angle or the measure of an arc in degrees, indicating the theorems or corollaries used to support each step in the solution. The following is a list of the given theorems and corollaries:

- In a circle, the measure of a central angle is equal to the measure of its intercepted arc.
- In a circle, the measure of an inscribed angle is one-half the measure of its intercepted arc.
- The measure of an angle formed by two chords intersecting in the interior of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
- The measure of an angle formed by two tangents, a tangent and a secant, or two secants is one-half the difference of the measures of the intercepted arcs.

Each solution should involve the application of no more than three theorems or corollaries.

GSM 253-04 ***Solving Real-Life Problems Involving the Relationships Governing Measures in a Circle***

Given diagrams of one or two circles with the dimensions and angular measures that make it possible to draw relevant conclusions as well as a list of theorems and corollaries pertaining to the relationships governing measures in a circle, solve problems related to a variety of human activities (e.g. carpentry, land surveying, architecture and technical drawing). The measure of the required angle, radius, diameter, segment, perimeter or area must be stated in the appropriate unit. The steps in the solution must be shown and the theorems and corollaries used to support the answer must be indicated.

GSM 253-05 *Relationships Governing Measures in a Right Triangle*

Given a list of theorems and corollaries as well as a diagram of a right triangle with the measures required to solve a particular problem, determine the measure of an angle, a side, a median, an altitude, the hypotenuse, the perimeter or the area of a right triangle, indicating the theorems or corollaries used to support each step in the solution. The following is a list of the given theorems and corollaries:

- The hypotenuse of a right triangle inscribed in a circle is always a diameter of that circle.
- In a right triangle, the length of the median to the hypotenuse is one-half the length of the hypotenuse.
- In a right triangle with a 30° angle, the length of the side opposite this angle is one-half the length of the hypotenuse.
- A right triangle and the altitude to its hypotenuse form two right triangles that are similar to the given triangle and to each other.
- The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse.
- The length of a leg of a right triangle is the geometric mean between the length of the hypotenuse and the length of the adjacent segment of the hypotenuse formed by the altitude to the hypotenuse.
- The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse.

Each solution should involve the application of no more than three theorems or corollaries.

GSM 253-06 ***Solving Real-Life Problems Involving the Relationships Governing Measures in a Right Triangle***

Given the diagram of a right triangle with the dimensions and angular measures that make it possible to draw relevant conclusions as well as a list of theorems and corollaries pertaining to the relationships governing measures in a right triangle, solve problems related to a variety of human activities (e.g. carpentry, land surveying, architecture and technical drawing). The measure of the required angle, side, perimeter or area must be stated in the appropriate unit. The steps in the solution must be shown and the theorems and corollaries used to support the answer must be indicated.

7.20 GMO 251 — Conics I

7.20 GMO 251 **Conics I**

Module GMO 251 consists of nine objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GMO 251-01 to **GMO 251-06**	4	20 %
GMO 251-07	10	40 %
GMO 251-08	5	20 %
GMO 251-09	5	20%

* One hour is allotted to the final evaluation.

GMO 251-01 *Determining the Maximum*

Given a word problem dealing with an everyday situation that can be expressed as an equation of the form $y = ax^2 + bx + c$, determine the value of the variables x and y that corresponds to a required maximum quantity (maximum output, maximum profit, maximum height) by using one of the following methods:

- filling in a partially completed table of values
- substituting different values for x in a second-degree equation. The equation and the values of x are given.

The values of a, b and c are rational numbers and $a \neq 0$. The given values of x are usually natural numbers. The answer must be written as an ordered pair (x, y) and the steps in the solution must be shown.

GMO 251-02 *Second-Degree Equations Representing an Everyday Situation*

Using a partially completed table of values, formulate a second-degree equation for a word problem dealing with an everyday situation. The equation must be of the form $y = ax^2 + bx + c$, where a, b and c are rational numbers and $a \neq 0$.

GMO 251-03 *Graphing an Equation of the Form* $y = ax^2$

Graph a second-degree equation of the form $y = ax^2$, where a is a rational number between -5 and 5 ($a \neq 0$). The result should be the graph of a parabola with the vertex, the axis of symmetry and the equation of the axis of symmetry clearly indicated. The scale for each of the two axes must also be indicated.

GMO 251-04 *Graphing an Equation of the Form* $y = ax^2 + c$

(1) Graph a second-degree equation of the form $y = ax^2 + c$, where a is a rational number between -5 and 5 ($a \neq 0$) and where c is a rational number. The result should be the graph of a parabola with the vertex, the axis of symmetry and the equation of the axis of symmetry clearly indicated. The scale for each of the two axes must also be indicated. (2) Indicate whether the vertex of the parabola is a maximum or a minimum.

GMO 251-05 *Solving a Second-Degree Equation by Factoring*

(1) Using the appropriate factoring method (removing the common factor, factoring by grouping, factoring a trinomial of the form $ax^2 + bx + c$, factoring a difference of squares) as well as the zero product property, solve a second-degree equation of the form $ax^2 + bx + c = 0$, where a, b and c are rational numbers and $a \neq 0$. The steps in the solution must be shown. (2) Given the roots of a quadratic equation, determine the points on the graph of the equation $ax^2 + bx + c = y$ that correspond to these given values.

GMO 251-06 Solving a Second Degree Equation by Means of the Quadratic Formula

(1) Find the value of the discriminant, Æ $= b^2 - 4ac$, to determine the number of roots (0, 1 or 2) of a second-degree equation of the form $ax^2 + bx + c = 0$, where a, b and c are rational numbers and $a \neq 0$. (2) If necessary, solve this equation using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

. The resulting values are real numbers and the steps in the solution must be shown. (3) Given the graph of a second-degree equation of the form $y = ax^2 + bx + c$, determine the number of zeros in this equation and indicate the points that correspond to these zeros.

GMO 251-07 ***Graphing a Second-Degree Equation***

Graph a second-degree equation of the form $y = ax^2 + bx + c$, where a, b and c are rational numbers and $a \neq 0$. The result should be the graph of a parabola with the following information clearly indicated: the vertex, the axis of symmetry and its equation, the y-intercept, the point symmetric with the y-intercept and, if necessary, the zeros of this equation. The scale for each axis and the calculations involved in finding each of these points must also be indicated.

GMO 251-08 ***Determining the Maximum and Minimum, Given a Second-Degree Equation***

Determine the abscissa and the ordinate of the maximum point or minimum point of a parabola, given a second-degree equation of the form $y = ax^2 + bx + c$ where a, b and c are rational numbers and $a \neq 0$. The problems focus on situations related to science or business. The steps in the solution must be shown.

GMO 251-09 ***Mathematizing and Solving a Problem That Can Be Written As a Second-Degree Equation***

Using the factoring method or the quadratic formula, solve a word problem that can be written as a second- degree equation of the form $ax^2 + bx + c = 0$, where a, b and c are rational numbers and $a \neq 0$. Solving the problem involves rejecting inappropriate values and finding a maximum of two values. The problems deal with computation, geometry or everyday situations and the steps in the solution must be shown.

7.21 GMO 252 — Equations and Inequalities III

Module GMO 252 consists of four objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	**Number of Hours***	**% (Evaluation)**
GMO 252-01 and **GMO 252-02**	10	40 %
GMO 252-03	7	30 %
GMO 252-04	7	30%

* One hour is allotted for the final evaluation.

GMO 252-01 *Solving a First-Degree Equation in One Variable*

Solve first-degree equations in one variable by converting them to the form ax + b = 0, where the constants a and b are rational numbers. These first-degree equations in one variable contain up to ten terms with or without denominators, and may also contain parentheses. The steps in the solution must be shown.

GMO 252-02 ***Solving First- or Second-Degree Inequalities***

Solve an inequality or a system of inequalities of the first or second degree in one variable. The first-degree inequalities can be converted to one of the following forms:

- $ax + b < 0$
- $ax + b > 0$
- $ax + b \leq 0$
- $ax + b \geq 0$

The second-degree inequalities are written as a product of first-degree factors in one variable. The different possibilities are listed below:

$(ax + b)(cx + d) < 0$ $(ax + b)(cx + d) > 0$
$(ax + b)(cx + d) \leq 0$ $(ax + b)(cx + d) \geq 0$

The constants a, b, c and d are rational numbers and $a \neq 0$. The solution set must be graphed or indicated using interval or set-builder notation. The steps in the solution must be shown.

GMO 252-03 ***Solving an Absolute Value Equation or Inequality of the First Degree in One Variable***

Solve an absolute value equation or inequality of the first-degree in one variable. The given equations and inequalities are written in one of the following forms:

- $k\,|ax + b| = c$
- $k\,|ax + b| > c$
- $k\,|ax + b| < c$
- $k\,|ax + b| \geq c$
- $k\,|ax + b| \leq c$

The constants k, a, b and c are rational numbers and $a \neq 0$. The solution set must be graphed or indicated using interval or set-builder notation. The steps in the solution must be shown.

GMO 252-04 ***Solving a Second-Degree Equation or Inequality in One Variable***

Solve a second-degree equation or inequality in one variable and graph the solution set or indicate it using interval or set-builder notation. The given equations or inequalities are written in one of the following forms:

- $ax^2 + bx + c = 0$
- $ax^2 + bx + c > 0$
- $ax^2 + bx + c < 0$
- $ax^2 + bx + c \geq 0$
- $ax^2 + bx + c \leq 0$

The constants a, b and c are real numbers and $a \neq 0$. The steps in the solution must be shown.

7.22 GMO 253 Functions

Module GMO 253 consists of seven objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	Number of Hours *	% (Evaluation)
GMO 253-01 and **GMO 253-02**	12	45 %
GMO 253-03 to **GMO 253-06**	10	45 %
GMO 253-07	2	10 %

* One hour is allotted for the final evaluation.

GMO 253-01 *The Image of an Element in a Functional Relation*

(1) Determine whether a given relation is a function. The elements of the relation are listed or are defined by means of set-builder notation, a Cartesian graph or an arrow diagram. (2) If the relation is a function, find the image of a specific numerical, verbal or symbolic element.

GMO 253-02 ***Graphing Functions***

Graph the following types of functions:

- **a linear function of the form $f(x) = mx + b$**
- **a quadratic function of the form $f(x) = ax^2 + bx + c$**
- **an absolute value function of the form $f(x) = k \mid ax + b \mid + c$**
- **a greatest integer function of the form $f(x) = k [ax + b] + c$**
- **an inverse variation function of the form $f(x) = \frac{b}{cx}$, where $c \neq 0$**
- **a square root function of the form $f(x) = \sqrt{ax + b}$ or $f(x) = -\sqrt{ax + b}$.**

The constants a, b, c, k and m are rational numbers and $a \neq 0$. The characteristics of each curve must be indicated on the graph (i.e. the vertex, zero(s), slope, y-intercept and asymptote(s), as the case may be).

GMO 253-03 *The Domain and Range of a Function*

Find the domain and range of a function defined in $\mathbb{R} \times \mathbb{R}$. The elements of the function are listed, or are defined by means of set-builder notation, a Cartesian graph or an arrow diagram. The problems deal with the functions that have already been studied (i.e. linear, quadratic, absolute value, greatest integer, inverse variation and square root functions). The domain and range must be subsets of $\mathbb{R}$ and must be indicated using interval or set-builder notation.

GMO 253-04 *The Maximum or Minimum of a Function*

Find the maximum or minimum of a function defined by means of set-builder notation. The problems deal with quadratic, absolute value, and square root functions.

GMO 253-05 *Increasing and Decreasing Functions in a Given Interval*

Determine if a function graphed in the Cartesian plane is increasing or decreasing in a given interval. The problems deal with the functions that have already been studied (i.e. linear, quadratic, absolute value, greatest integer, inverse variation and square root functions).

GMO 253-06 ***The Inverse of a Function***

Find the inverse of a function whose elements are listed, or are defined by means of set-builder notation, a Cartesian graph or an arrow diagram and determine if this inverse is a function. If the given function is defined by means of set-builder notation, the problem can involve only linear, quadratic, inverse variation or square root functions. The elements of the inverse must be listed, graphed or indicated by of means set-builder notation.

GMO 253-07 ***The Composition of Two Functions***

Find the composition of two functions f and g defined by means of set-builder notation and give the image of any element for the function fog or f(g(x)).

7.23 GMO 254 **Conics II**

Module GMO 254 consists of ten objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GMO 254-01	3	10 %
GMO 254-02 and **GMO 254-03**	3	15 %
GMO 254-04	1	5 %
GMO 254-05	3	10 %
GMO 254-06 and **GMO 254-07**	3	15 %
GMO 254-08	3	10 %
GMO 254-09	3	10%
GMO 254-10	5	25 %

* One hour is allotted for the final evaluation.

GMO 254-01 ***Graphing a Relation That Defines a Circle***

(1) Graph the region determined by a relation defining a circle. The given relation can be written in one of the following forms:

$$x^2 + y^2 + Dx + Ey + F = 0 \text{ or } (x - h)^2 + (y - k)^2 = r^2,$$
$$x^2 + y^2 + Dx + Ey + F < 0 \text{ or } (x - h)^2 + (y - k)^2 < r^2,$$
$$x^2 + y^2 + Dx + Ey + F \leq 0 \text{ or } (x - h)^2 + (y - k)^2 \leq r^2,$$
$$x^2 + y^2 + Dx + Ey + F > 0 \text{ or } (x - h)^2 + (y - k)^2 > r^2,$$
$$x^2 + y^2 + Dx + Ey + F \geq 0 \text{ or } (x - h)^2 + (y - k)^2 \geq r^2,$$

The parameters D, E, F, h, k and r are rational numbers. (2) Indicate the centre of the circle and its radius clearly on the graph. (3) Find the domain and range of this relation and indicate them using interval or set-builder notation.

GMO 254-02 *Translating a Circle Centred at the Origin*

Find and graph the equation of the circle obtained after a given circle centred at the origin (i.e. $x^2 + y^2 = r^2$) has been translated so that its centre is (h, k). The parameters h, k and r are integers.

GMO 254-03 ***General Form of the Equation of a Circle***

(1) Find the general form of the equation of a circle (i.e. $x^2 + y^2 + Dx + Ey + F = 0$), given its centre (h, k) and its radius r. The parameters D, E, F, h, k and r are rational numbers and are usually integers. (2) Find the centre (h, k) and the radius r of a circle, given its equation in general form.

GMO 254-04 ***Equation of a Line Tangent to a Circle***

Find the equation of a line tangent to a circle, given the point of tangency (x_1, y_1) and the equation of the circle in standard form [i.e. $(x - h)^2 + (y - k)^2 = r^2$] or in general form [i.e. $x^2 + y^2 + Dx + Ey + F = 0$]. The parameters x_1, y_1, h, k, r, D, E and F are integers.

GMO 254-05 ***Graphing a Relation Defining a Parabola***

(1) Graph a region determined by a relation defining a parabola. The relation can be written in one of the following forms:

$(y - k)^2 = 4a(x - h)$ or $(x - h)^2 = 4a(y - k)$,
$(y - k)^2 < 4a(x - h)$ or $(x - h)^2 < 4a(y - k)$,
$(y - k)^2 \leq 4a(x - h)$ or $(x - h)^2 \leq 4a(y - k)$,
$(y - k)^2 > 4a(x - h)$ or $(x - h)^2 > 4a(y - k)$,
$(y - k)^2 \geq 4a(x - h)$ or $(x - h)^2 \geq 4a(y - k)$,

The parameters a, h and k are rational numbers that are usually integers and $a \neq 0$. (2) Clearly indicate the following on the graph: the vertex, the focus, the axis of symmetry and the directrix of the parabola. (3) Find the domain and range of this relation and indicate them using interval or set-builder notation.

GMO 254-06 *Translating a Parabola with Vertex at the Origin*

Find and graph the equation of a parabola obtained after a given parabola with vertex at the origin (i.e. $y^2 = 4ax$ or $x^2 = 4ay$) has been translated so that its vertex is (h, k). The constants h and k are integers and a is a rational number other than 0.

GMO 254-07 ***Equation of a Parabola***

Find the standard form of the equation of a parabola [i.e. $(y - k)^2 = 4a(x - h)$ or $(x - h)^2 = 4a(y - k)$], given its vertex (h, k) and its focus (x_1, y_1). The parameters a, h, k, x_1 and y_1 are integers and $a \neq 0$.

GMO 254-08 ***Graphing an Ellipse Centred at the Origin***

(1) Graph a region determined by a relation defining an ellipse centred at the origin. This ellipse can be of one of the following forms:

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} > 1$
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} \geq 1$
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$

The parameters a and b are integers other than 0. (2) Indicate the axes and the two foci of the ellipse clearly on the graph. (3) Find the domain and the range of this relation and indicate them using interval or set-builder notation.

GMO 254-09 ***Graphing a Hyperbola Centred at the Origin***

(1) Graph a region determined by a relation defining a hyperbola centred at the origin. This hyperbola can be of one of the following forms:

- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
- $\frac{x^2}{a^2} - \frac{y^2}{b^2} < 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} < 1$
- $\frac{x^2}{a^2} - \frac{y^2}{b^2} \leq 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} \leq 1$
- $\frac{x^2}{a^2} - \frac{y^2}{b^2} > 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} > 1$
- $\frac{x^2}{a^2} - \frac{y^2}{b^2} \geq 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} \geq 1$

The parameters a and b are integers other than 0. (2) Indicate the asymptotes, vertices and foci of the hyperbola clearly on the graph. (3) Find the domain and range of this relation and indicate them using interval or set-builder notation.

GMO 254-10 ***Equation or Inequality Associated with the Graph of a Conic Section***

Find the equation or inequality associated with the graph of one of the following conic sections: a circle, a parabola, an ellipse centred at the origin or a hyperbola centred at the origin. The distinctive features of each curve are clearly indicated on the given graph (i.e. the radius, the centre, one or more vertices, one or more foci, one or more axes of symmetry, the directrix, the asymptotes, as the case may be). The equation or inequality must be written in standard form.

7.24 GMO 255 Exponential and Logarithmic Functions

Module GMO 255 consists of seven objectives and requires 25 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GMO 255-01 and **GMO 255-02**	6	25 %
GMO 255-03 and **GMO 255-04**	4	25 %
GMO 255-05	3	15 %
GMO 255-06	8	25 %
GMO 255-07	3	10%

* One hour is allotted for the final evaluation.

GMO 255-01 *Converting an Expression Containing a Radical to Exponential Form*

Convert an arithmetic expression containing a radical to an exponential expression with a simplified base and a positive exponent. The given arithmetic expression is of the form $a^m \sqrt[n]{b^k}$ where a and b are numbers that can be reduced to a lowest common base. The bases a and b are positive fractions or positive integers other than 1, the index n is a whole number other than 0 and 1 and the exponents m and k are either fractions or integers. Converting these expressions involves applying the following Laws of Exponents:

- $\sqrt[n]{a^m} = a^{m/n}$
- $(a^m)^n = a^{mn}$
- $(a\ b\ c)^m = a^m\ b^m\ c^m$
- $a^{-m} = \dfrac{1}{a^m}$
- $a^m \times a^n = a^{(m+n)}$
- $\dfrac{a^m}{a^n} = a^{(m-n)}$
- $a^0 = 1$
- $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

The steps involved in converting the expression must be shown.

GMO 255-02 ***Graphing an Exponential Function***

(1) Graph an exponential function of the form

$$\{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = b^{x+c} + d\}$$

where b is a positive fraction or a whole number other than 0 and 1 and where c and d are integers. (2) Indicate the asymptote clearly on the graph and determine if the curve is an increasing or decreasing function. (3) Find the domain and range of this function and indicate them using interval or set-builder notation.

GMO 255-03 *Converting an Exponential Expression to Logarithmic Form and Vice Versa*

Convert a given exponential expression to logarithmic form and convert a given logarithmic expression to exponential form. The exponential expressions are of the form $b^x = y$ and the logarithmic expressions are of the form $\log_b x = y$, where x and b are positive fractions or integers, $b \neq 0$ or 1 and y is a rational number.

GMO 255-04 ***Graphing a Logarithmic Function***

(1) Graph a logarithmic function of the form $\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = K \log_b(ax+c) + d\}$, where b is a positive fraction or a whole number other than 0 and 1 and a, c and d are integers. In addition, $a \neq 0$ and K is a rational number. (2) Indicate the asymptote clearly on the graph and determine if the curve is an increasing or decreasing function. (3) Find the domain and range of this function and indicate them using interval or set-builder notation.

GMO 255-05 ***Inverse of an Exponential Function and of a Logarithmic Function***

(1) Find and graph the inverse of a logarithmic function of the form $y = \log_b x$. (2) Find and graph the inverse of an exponential function of the form $y = b^x$. The base b is a positive fraction or a whole number other than 0 and 1.

GMO 255-06 ***Applying the Laws of Logarithmic Computation***

Simplify a given logarithmic expression by applying the laws of logarithmic computation listed below.

- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b b^n = n$
- $\log_b M^n = n \log_b M$
- $\log_{1/b} M = -\log_b M$
- $\log_b M = \dfrac{\log_a M}{\log_a b}$
- $\log_b(M \times N) = \log_b M + \log_b N$
- $\log_b\left(\dfrac{M}{N}\right) = \log_b M - \log_b N$

In these cases, a, b, b^n, $M \times N$, $\frac{M}{N}$ and M are positive real numbers with $a \neq 1$ and $b \neq 1$. The given expression should contain no more than three terms, each of which is written in logarithmic form. In addition, the expression to be simplified may contain numbers or variables. The result should be a simplified logarithmic expression or a numerical value. The steps in the solution must be shown.

GMO 255-07 ***Evaluating Logarithms to the Base 10***

Determine the logarithm to the base 10 of a given number, using a table of logarithms or a calculator. The given number is a positive real number other than 0. The result should be indicated by adding the characteristic and the mantissa.

7.25 GMO 256 — Trigonometry II (Trigonometric Functions and Identities)

Module GMO 256 consists of ten objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours*	% (Evaluation)
GMO 256-01 and **GMO 256-02**	5	5 %
GMO 256-03 to **GMO 256-05**	13	30 %
GMO 256-06	8	15 %
GMO 256-07 to **GMO 256-10**	22	50 %

* Two hours are allotted for the final evaluation.

GMO 256-01 *Converting from Degrees to Radians and from Radians to Degrees*

(1) Convert angular measures from degrees to radians and from radians to degrees, given central angles in a unit circle. The degree measures range 0° to 720° and are stated in sexagesimal form; the radian measures range from 0 to 4π (2) Define the unit of angular measure known as a **grad**.

GMO 256-02 ***Calculating the Measure of a Central Angle or the Measure of an Arc***

(1) Determine the degree or radian measure of a central angle, given the radius of a circle and the measure of the arc intercepted by that angle. (2) Determine the measure of an arc intercepted by a central angle, given the radius of a circle and the measure of that angle. The length of the radius must be a natural number and the measure of the arc must be a rational number rounded to the nearest hundredth, if necessary. The measure of the central angle must be an integer stated in degrees or a certain number of radians rounded to the nearest hundredth, if necessary.

GMO 256-03 *The Wrapping Function*

(1) Find the coordinates (x, y) on the unit circle that correspond to a real number under the wrapping function. (2) In a designated interval under the wrapping function, find the real number that corresponds to a given set of coordinates (x, y) on the unit circle. The trigonometric points are of the form $a\pi/b$, where $a \in \mathbb{Z}$ and $b \in \{1,2,3,4,6\}$. The intervals are of the form $[0,2\pi] + 2k\pi/n$, where $k \in \mathbb{Z}$ and $n \in \{1,2,3,4,6,8,12\}$.

GMO 256-04 *Evaluating a Trigonometric Function For a Number Expressed in Radians*

For a given trigonometric function, find the image of a real number expressed in radians and make sure the result bears the appropriate sign. The problems involve one of the following trigonometric functions: sine, cosine, tangent, cotangent, secant or cosecant. The real number is expressed in the form $a\pi/b$, where $a \in \mathbb{Z}$ and $b \in \{1,2,3,4,6\}$, or as a decimal number rounded to the nearest hundredth, if necessary. A scientific calculator or a trigonometric table must be used.

GMO 256-05 ***Graphing a Trigonometric Function***

(1) Graph a trigonometric function in a given interval and specify the various characteristics of the resulting curve (i.e. the zeros, the period, the maxima and minima if applicable, the asymptotes if applicable, and the intervals in which the function is increasing and decreasing). (2) Find the domain and range of the function and indicate them in interval notation. The problems involve graphing the following trigonometric functions:

- **y = sin x**
- **y = cos x**
- **y = tan x**
- **y = cotan x**
- **y = sec x**
- **y = cosec x**

The interval can be open, closed or half-open.

GMO 256-06 ***Graphing a Sine Function***

Graph a sinusoidal function in a given interval and specify the amplitude, period and phase shift of the resulting curve. The sinusoidal function is of the form $y = A \sin (Bx - h)$ or $y = A \cos (Bx - h)$, where A and B are integers other than 0 and h is a rational number of the form $a\pi/b$ with $b \neq 0$.

GMO 256-07 *Fundamental Trigonometric Identities*

Given the value of a trigonometric function at a point in a designated interval, find the value of another trigonometric function at that point, using one of the fundamental trigonometric identities listed below:

- $\sin^2 x + \cos^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $\text{cotan}^2 x + 1 = \text{cosec}^2 x$

The sign of the required function value must correspond to the sign required in the designated interval.

GMO 256-08 *Proving Simple Trigonometric Identities*

Prove a simple trigonometric identity by making algebraic transformations and using the definitions of trigonometric functions as well as the fundamental trigonometric identities. There should be no more than two terms on each side of the identity and each term should contain no more than two trigonometric functions. The definitions of the trigonometric functions and the fundamental trigonometric identities are not provided. The steps in the solution must be shown.

GMO 256-09 *Trigonometric Functions Involving the Sum or Difference of Two Real Numbers*

For a given trigonometric function, find the image of a real number that can be expressed as a sum or difference of two real numbers of the form $a\pi/b$, where $a \in \mathbb{Z}$ and $b \in \{1,2,3,4,6\}$. These real numbers can also be converted to their equivalent value in degrees. The problems involve the sine, cosine and tangent functions and must be solved without using a calculator or trigonometric tables. The following basic formulas required to evaluate trigonometric functions involving a sum or difference of two real numbers are provided:

- $\sin (A + B) = \sin A \cos B + \cos A \sin B$
- $\sin (A - B) = \sin A \cos B - \cos A \sin B$
- $\cos (A + B) = \cos A \cos B - \sin A \sin B$
- $\cos (A - B) = \cos A \cos B + \sin A \sin B$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad (1 - \tan A \tan B \neq 0)$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \qquad (1 + \tan A \tan B \neq 0)$$

- $\sin 2A = 2 \sin A \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
- $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A} \qquad (1 - \tan^2 A \neq 0)$

The following formulas are derived:

- $\cos (-A) = \cos A$
- $\sin (-A) = -\sin A$
- $\sin (\pi/2 - A) = \cos A$
- $\cos (\pi/2 - A) = \sin A$.

The steps in the solution must be shown.

GMO 256-10 ***Proving Complex Trigonometric Identities***

Prove a complex trigonometric identity by making algebraic transformations and using the definitions of trigonometric functions, the fundamental trigonometric identities and the basic formulas needed to transform trigonometric functions involving the sum or difference of two values. There should be no more than two terms on each side of the identity and each term should contain no more than three trigonometric functions. At least one of the terms must contain a trigonometric function involving the sum or difference of two values. The following list of basic formulas is provided:

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$
- $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ $(1 - \tan A \tan B \neq 0)$
- $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$ $(1 + \tan A \tan B \neq 0)$
- $\sin 2A = 2 \sin A \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
- $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$ $(1 - \tan^2 A \neq 0)$

The following formulas will be derived:

- **cos (-A) = cos A**
- **sin (-A) = -sin A**
- **sin (π/2 - A) = cos A**
- **cos (π/2 - A) = sin A.**

The definitions of trigonometric functions and the fundamental trigonometric identities are not provided. The steps in the solution must be shown.

7.26 GMO 257 **Geometry V**

Module GMO 257 consists of eleven objectives and requires 50 hours of study organized as indicated in the table below:

Objectives	Number of Hours *	% (Evaluation)
GMO 257-01 to **GMO 257-04**	12	25 %
GMO 257-05 and **GMO 257-06**	12	25 %
GMO 257-07 and **GMO 257-08**	11	20 %
GMO 257-09 to **GMO 257-11**	13	30 %

* Two hours are allotted for the final evaluation.

GMO 257-01 *Identifying Various Elements in Circles*

On diagrams of circles in which several elements are represented and labelled with upper case letters, identify the following: a radius, a diameter, a chord, an arc, a secant, a tangent, a point of tangency, a central angle, an inscribed angle, an interior angle and an exterior angle.

GMO 257-02 *Relationships Governing the Measure of Length in a Circle*

Given a list of theorems and corollaries as well as a diagram of one or two circles with the measures required to solve a particular problem, determine the measure of a radius, a diameter, a circumference, an area, a chord, an arc or a segment tangent, indicating the theorems or corollaries used to support each step in the solution. The following is a list of the given theorems and corollaries:

Relationships Within a Circle

- Any perpendicular bisector of a chord is a diameter of a circle.
- The longest chord of a circle is a diameter.
- In a circle, any radius perpendicular to a chord divides that chord into two congruent segments.
- In a circle, any radius perpendicular to a chord divides the subtended arc into two congruent arcs.
- In a circle, arcs located between two parallel chords are congruent.
- Two chords are congruent if they are equidistant from the center of the circle.
- In a circle, congruent chords subtend congruent arcs and, conversely, congruent arcs are subtended by congruent chords.
- Any line tangent to a circle is perpendicular to the radius that shares the point of tangency.
- For any circle, two tangent segments originating from the same exterior point are congruent. (The segments are measured from that exterior point to their respective points of tangency).
- Two parallel lines, be they tangents or secants, intercept congruent arcs of a circle.

Relationships Involving Two Circles

- The circumferences of two circles have the same ratio as their radii.
- The areas of two circles have the same ratio as the squares of their radii.
- The measures of similar arcs of two circles have the same ratio as their radii.

Each solution should involve the application of no more than three theorems or corollaries.

GMO 257-03 *Relationships Governing Angular Measures in a Circle*

Given a list of theorems and corollaries as well as a diagram of a circle with the measures required to solve a particular problem, determine the measure of a central angle, an inscribed angle, an interior angle, an exterior angle or the measure of an arc in degrees, indicating the theorems or corollaries used to support each step in the solution. The following is a list of the given theorems and corollaries:

- In a circle, the measure of a central angle is equal to the measure of its intercepted arc.
- In a circle, the measure of an inscribed angle is one-half the measure of its intercepted arc.
- The measure of an angle formed by two chords intersecting in the interior of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
- The measure of an angle formed by two tangents, a tangent and a secant, or two secants is one-half the difference of the measures of the intercepted arcs.

Each solution should involve the application of no more than three theorems or corollaries.

GMO 257-04 ***Solving Real-Life Problems Involving the Relationships Governing Measures in a Circle***

Given diagrams of one or two circles with the dimensions and angular measures that make it possible to draw relevant conclusions as well as a list of theorems and corollaries pertaining to the relationships governing measures in a circle, solve problems related to a variety of human activities (e.g. carpentry, land surveying, architecture and technical drawing). The measure of the required angle, radius, diameter, segment, perimeter or area must be stated in the appropriate unit. The steps in the solution must be shown and the theorems and corollaries used to support the answer must be indicated.

GMO 257-05 *Relationships Governing Measures in a Right Triangle*

Given a list of theorems and corollaries as well as a diagram of a right triangle with the measures required to solve a particular problem, determine the measure of an angle, a side, a median, an altitude, the hypotenuse, the perimeter or the area of a right triangle, indicating the theorems or corollaries used to support each step in the solution. The following is a list of the given theorems and corollaries:

- The hypotenuse of a right triangle inscribed in a circle is always a diameter of that circle.
- In a right triangle, the length of the median to the hypotenuse is one-half the length of the hypotenuse.
- In a right triangle with a 30° angle, the length of the side opposite this angle is one-half the length of the hypotenuse.
- A right triangle and the altitude to its hypotenuse form two right triangles that are similar to the given triangle and to each other.
- The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse.
- The length of a leg of a right triangle is the geometric mean between the length of the hypotenuse and the length of the adjacent segment of the hypotenuse formed by the altitude to the hypotenuse.
- The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse.

Each solution should involve the application of no more than three theorems or corollaries.

GMO 257-06 ***Solving Real-Life Problems Involving the Relationships Governing Measures in a Right Triangle***

Given the diagram of a right triangle with the dimensions and angular measures that make it possible to draw relevant conclusions as well as a list of theorems and corollaries pertaining to the relationships governing measures in a right triangle, solve problems related to a variety of human activities (e.g. carpentry, land surveying, architecture and technical drawing). The measure of the required angle, side, perimeter or area must be stated in the appropriate unit. The steps in the solution must be shown and the theorems and corollaries used to support the answer must be indicated.

GMO 257-07 *Congruent Polygons*

Given measures of angles and sides of two polygons, determine if these polygons are congruent on the basis of the following properties of congruent figures:

- corresponding angles are congruent
- corresponding sides are congruent
- the perimeters are equal
- the areas are equal

The problems deal with the following polygons: triangles, quadrilaterals, pentagons, hexagons or octogons.

GMO 257-08 ***Solving Real-Life Problems Involving the Properties of Congruent Polygons***

Given a list of theorems and corollaries as well as diagrams of polygons with the dimensions and angular measures that make it possible to draw relevant conclusions, solve congruence problems related to a variety of human activities (e.g. carpentry, land surveying, architecture and technical drawing). The measure of the required angle, side, diagonal, segment, perimeter or area must be stated in the appropriate unit. The steps in the solution must be shown and the theorems and corollaries used to support the answer must be indicated. The following is a list of the given theorems and corollaries:

- **If two lines are perpendicular to a third line, then they are parallel to each other.**
- **Any point on the perpendicular bisector of a segment is equidistant from the endpoints of that segment.**
- **Any point on the bisector of an angle is equidistant from the sides of that angle.**
- **The angles of a linear pair are supplementary.**
- **Vertical angles are congruent.**
- **If two parallel lines are intersected by a transversal:**
  - **a) alternate interior angles are congruent**
  - **b) alternate exterior angles are congruent**
  - **c) corresponding angles are congruent**
  - **d) interior angles on the same side of the transversal are supplementary**

e) exterior angles on the same side of the transversal are supplementary.

- **In any isosceles triangle, the angles opposite the congruent sides are congruent.**
- **In any isosceles triangle, the perpendicular bisector of the side adjacent to the congruent angles is also the bisector, median and altitude to that side.**
- **The line segment having the midpoints of two sides of a triangle as its endpoints is parallel to the third side and its length is one-half the length of the third side.**
- **The sum of the measures of the interior angles of a triangle is 180°.**
- **The sum of the measures of the interior angles of a polygon is equal to 180° times the number of sides minus two (i.e. 180° (n - 2), where n is the number of sides in the polygon).**
- **The sum of the measures of the exterior angles of a convex polygon is 360°.**
- **The opposite (or non-consecutive) angles of a parallelogram are congruent.**
- **The opposite (or non-consecutive) sides of a parallelogram are congruent.**
- **The diagonals of a parallelogram bisect each other.**
- **The diagonals of a rectangle are congruent.**
- **The diagonals of a rhombus are perpendicular to each other.**
- **The diagonals of a square are perpendicular bisectors of each other.**

GMO 257-09 *Similar Polygons*

Given measures of angles and sides of two polygons, determine if these polygons are similar on the basis of the following properties of congruent figures:

- corresponding angles are congruent
- the ratios of the corresponding sides are equal
- the ratio of the corresponding sides is equal to the ratio of the perimeters
- the ratio of the areas is equal to the square of the ratio of the corresponding sides

The problems deal with the following polygons: triangles, quadrilaterals, pentagons, hexagons or octogons.

GMO 257-10 *Regular Polygons*

Given sufficient data, construct a three-, four-, five-, six- or eight-sided regular polygon and indicate its apothem.

GMO 257-11 ***Solving Real-Life Problems Involving The Properties of Similar Polygons***

Given a list of theorems and corollaries as well as diagrams of two similar polygons with the dimensions and angular measures that make it possible to draw relevant conclusions, solve problems related to a variety of human activities (e.g. carpentry, land surveying, architecture and technical drawing). The measure of the required angle, side, diagonal, segment, perimeter or area must be stated in the appropriate unit. The steps in the solution must be shown and the theorems and corollaries used to support the answer must be indicated. The following is a list of the given theorems and corollaries:

- **Transversals intersected by parallel lines are divided into proportional segments.**
- **A line parallel to one side of a triangle and passing through an interior point of that triangle determines a second triangle similar to the first.**
- **The lengths of the corresponding segments of two similar triangles are proportional.**
- **Two polygons are similar if their corresponding angles are congruent and if the measures of their corresponding sides are proportional.**
- **The perimeters of similar polygons have the same ratio as the lengths of the corresponding diagonals or the lengths of the corresponding sides.**
- **The perimeters of regular similar polygons have the same ratio as their apothems or as the radii of their circumscribed circles.**
- **The areas of regular similar polygons have the same ratio as the the squares of their apothems or as squares of the radii of their circumscribed circles.**
- **The areas of similar polygons have the same ratio as the square of their ratio of similitude (i.e. the same ratio as the squares of the lengths of any two corresponding segments).**