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J.-F. Bégin, M. Boudreault,
G. Gauthier

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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Firm-specific credit risk modelling in the presence of statistical regimes and noisy prices

Jean-François Bégin [a]

Mathieu Boudreault [b]

Geneviève Gauthier [c]

[a] *Department of Decision Sciences, HEC Montréal, Montréal (Québec) Canada, H3T 2A7*

[b] *Department of Mathematics, UQAM, Montréal (Québec) Canada, H2X 3Y7*

[c] *GERAD & Department of Decision Sciences, HEC Montréal, Montréal (Québec) Canada, H3T 2A7*

jean-francois.begin@hec.ca
boudreault.mathieu@uqam.ca
genevieve.gauthier@hec.ca

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Abstract: Security prices are important inputs for estimating credit risk models. Yet, to obtain an accurate firm-specific credit risk assessment, one needs a reliable model and a methodology that filters all elements unrelated to the firm's fundamentals from observed market prices. Therefore, we first introduce a flexible hybrid credit risk model defined in a Markov-switching environment. It captures firm-specific changes in the leverage uncertainty during crises as well as the negative relationship between creditworthiness and recovery rates. Second, estimation is performed using maximum likelihood by accounting for latent regimes and unobserved noise included in security prices. Using CDS premiums for 225 firms of both CDX North American IG and HY indices, we perform two different empirical applications. The effects of stochastic recovery and the presence of regimes on theoretical credit spread curves are investigated. We also apply the model to corporate bond credit spreads to assess the importance of bond-specific liquidity in the bond-CDS basis.

Keywords: Risk analysis, credit risk, maximum likelihood estimation, regime-switching, filtering

1 Introduction and literature review

The Global Financial Crisis of 2008 is considered by various economists as the worst crisis since the Great Depression. Worldwide, one hundred and one corporate issuers rated by Moody's defaulted on a total of $281.2 billion of debt in 2008 (Moody's, 2009). For the sake of comparison, only 18 issuers defaulted in 2007 on a total of $6.7 billion of debt. Also, the year 2008 witnessed the largest defaulter in history: Lehman Brothers.

Indeed, defaults are rare events and the lack of direct observations brings an additional challenge to estimating credit risk. To overcome this challenge, a common practice is to use credit ratings as a proxy for firm-specific credit risk. However, in the aftermath of the crisis, credit ratings were criticized and many stakeholders started advocating for market-based valuation of credit risk.

Although market prices are forward-looking measures that are updated frequently by market participants, observed prices can diverge from their theoretical values as risks unrelated to their fundamentals could be priced (e.g. illiquidity, asymmetric information, price discreteness). These additional risks are hereafter referred to as market noise. Filtering techniques are powerful tools for extracting fundamental values from noisy security prices.

In the literature of credit risk, some recent estimation procedures allow for noisy observations. Indeed, Duan and Fulop (2009) propose an estimation technique for Merton's (1974) model that allows for trading noise in observed equity prices. The maximum likelihood estimation (MLE) is carried out by a particle filter. It is argued that ignoring noise could non-trivially inflate one's estimate of the asset volatility. Huang and Yu (2010) introduce an alternative estimation technique based on Markov-chain Monte Carlo (MCMC) methods for the same credit risk framework. According to the authors, noises are positively correlated with firms' values. Finally, in a hybrid credit risk framework, Boudreault et al. (2013) use the unscented Kalman filter (UKF) to extract the latent creditworthiness from credit default swap (CDS) premiums.[1]

Firm-specific estimation of credit risk in the previously cited papers is simplified because these frameworks are based on single-factor models. As the last financial crisis showed, credit spreads and other solvency indicators exhibit structural changes. Regime-switching dynamics are able to capture behaviour changes similar to those associated with a crisis by allowing for dynamic levels of uncertainty. Such regime-switching approaches are also very intuitive in the context of crises; in its most simplistic form, one could have a regime associated with 'good' times and another with 'bad' times. However, firm-specific MLE in the presence of regimes and noisy market prices is complicated because both regimes and noises are not directly observed.

This paper shall thus contribute to the current credit risk literature by proposing a Markov-switching model and a consistent estimation technique for the assessment of credit risk. In this spirit, we design a flexible model that is able to capture desired empirical facts. Our hybrid credit risk model includes a firm-specific statistical regime variable that accommodates for changes in the leverage uncertainty. The endogenous random recovery rates are negatively related to default probabilities as they both depend on the firm's financial health. Finally, the framework accounts for the presence of noise as market prices potentially include measurement errors.

Continuous-path structural models have failed to appropriately represent short-term credit spreads, mainly because the default is a predictable stopping time. To solve this issue, structural models with incomplete information were proposed; the presence of a surprise element that adds randomness to the default trigger is an important feature of these models. For instance, Duffie and Lando (2001) suppose that bond investors cannot observe the issuer's assets directly and receive periodic and imperfect accounting reports instead. Jarrow and Protter (2004) show that the incomplete knowledge of the firm's assets and liabilities leads to an inaccessible default time.

Other solutions combine ideas from both structural and reduced-form approaches to obtain hybrid models: firms' liabilities and assets are modelled as stochastic processes and the default time is given by the first jump of a Cox process for which the intensity depends on the firm's fundamentals. In this spirit,

[1] More recently, Kwon and Lee (2015) use MCMC to estimate the Black and Cox (1976) model, whereas Guarin et al. (2014) propose a filtering technique to extract the time-varying default risk from CDS premiums.

Madan and Unal (2000) use a two-factor hazard rate model which links the intensity to the value of the firm; this leads to a non-predictable default time. Bakshi et al. (2006a) use Duffie and Singleton's (1999) reduced-form framework with factors modelled as Vasicek (1977) processes; one of these processes is the firm's leverage. We use a similar approach: the default intensity of a firm is a convex function of its leverage.

As documented by various authors, CDS premium dynamics and credit spreads change during financial crises. It thus makes sense to include this risk in our framework, especially as our sample period includes the last crisis. The dynamics of CDS premiums are investigated by Huang and Hu (2012) during the 2008 crisis by applying a smooth-transition autoregressive model. Maalaoui Chun et al. (2014) study regimes by applying a regime-detection technique that distinguishes between level and volatility regimes in credit spreads and show that most breakpoints occur around economic downturns, thus linking the statistical regimes to financial crises. Furthermore, Alexander and Kaeck (2008) show that CDS premiums display pronounced regime-specific behaviour.

The negative correlation between the firm's leverage and the recovery rate also is of paramount importance: as the firm's financial health becomes precarious (i.e. the firm's leverage rises), its probability of default increases and the recovery rate decreases, respectively. According to Altman et al. (2005) and Acharya et al. (2007), a negative correlation between probabilities of default and recovery rates exists, and both variables seem to be driven by the same factor. Other contributions show the importance of the negative relationship between probabilities of default and recovery rates. Using BBB-rated corporate bonds, regression analyses and the information on all companies that have defaulted between 1981 and 1999, Bakshi et al. (2006b) find that, on average, a 4% increase in the risk-neutral hazard rate is associated with a 1% decline in risk-neutral recovery rates. Further, an econometric model developed by Bruche and González-Aguado (2010) tries to assess by how much one underestimates credit risk if the negative relationship between probabilities of default and recovery rates is ignored. Using a simple structural model, Bade et al. (2011) find that default and recovery are highly negatively correlated; the recovery is modelled as a stochastic quantity that depends on observable risk factors and a systematic random variable.

The contributions of this paper are manifold. First of all, as stated above, we propose a hybrid credit risk model that includes a firm-specific statistical regime, among other realistic features.

Second, the model is estimated by maximum likelihood using a filtering approach that we design for the issue at hand: two latent variables (i.e. leverage and regime) shall be filtered simultaneously in addition to the model's parameters, which need to be estimated for each firm. The proposed filter has numerous advantages. First, it is possible to use multiple data sources to help disentangle some effects that cannot be separated using a single product.[2] Second, using time series of derivatives captures the dynamics under both objective and risk-neutral probability measures simultaneously. Finally, its numerical implementation is fast and efficient because it is deterministic, as opposed to particle filters and MCMC approaches. Since the estimation is based on security prices, we also propose a numerical scheme based on trinomial lattices that allow an efficient pricing in the framework.

The model is then estimated with CDS premiums for each of the 225 firms of both CDX North American IG and HY indices. The estimated results are used in two empirical illustrations.

First, the effects of stochastic recovery and the presence of regimes on theoretical credit spread curves are investigated before, during and after the financial crisis. We find that recovery uncertainty and its negative relationship with default probability have a major impact on mid- and long-term credit spreads. In addition, the presence of regimes modifies the short-term shape of the average credit spread curves during the financial turmoil, especially for highly rated firms.

Second, hedging credit risk using CDSs is at the mercy of basis risk as different risk factors could impact bonds and CDS premiums. For instance, factors such as bond-specific liquidity could have an effect on bond spreads making the hedging strategy less effective. Therefore, using credit risk inferred from CDS premiums, we analyze the bond-CDS basis during the 2008 financial crisis. Based on regression tests and yield-to-

[2] For instance, liquidity issues in some tenors, recovery rate uncertainty, default risk, etc.

maturity spreads implied by our credit risk model, we find that bond-specific liquidity is an important driver of the bond-CDS basis and its contribution is even more significant during the past crisis.

The rest of this paper is organized as follows. The joint default and loss model is presented in Section 2. Section 3 explains the model estimation approach and the pricing methodology. The estimation results are discussed in Section 4. In Section 5, the impact on spreads of endogenous random recovery and regime-switching risks is analyzed. The bond-CDS basis is investigated during the last crisis in Section 6. Finally, Section 7 concludes.

2 Credit risk model

In this paper, the shell approach of Boudreault et al. (2013) is extended to capture the time-varying nature of the leverage volatility.[3] The shell is essentially an intensity process that depends on the firm's leverage, which is modelled by Markov-switching dynamics in this study. This flexible approach allows for an endogenous recovery rate that is both stochastic and negatively correlated with the firm's probability of default. Even though the extension looks straightforward, the estimation and CDS pricing procedures under the new Markov-switching generalization are cumbersome and require the implementation of sophisticated numerical methods.

Let L_t and A_t be the time t market value of the firm's liabilities and assets respectively. The leverage ratio is defined as the quotient of these two values.[4] Because the asset volatility is not constant in time, the model should be flexible enough to capture the changes in the state variable dynamics. Hence, the market value of the firm's log-leverage is characterized by the following regime-switching dynamics:

$$\log\left(\frac{L_t}{A_t}\right) \equiv x_t = x_{t-1} + \left(\mu^{\mathbb{P}}_{s_t} - \frac{1}{2}\sigma^2_{s_t}\right)\Delta + \sigma_{s_t}\sqrt{\Delta}\epsilon^{\mathbb{P}}_t, \tag{1}$$

where s_t is the regime prevailing during the time interval $((t-1)\Delta, t\Delta]$, Δ is the time step between two consecutive observations, $\{\epsilon^{\mathbb{P}}_t\}_{t=1}^{\infty}$ is a sequence of independent standardized Gaussian random variables under the statistical probability measure $\mathbb{P}$, and $\mu^{\mathbb{P}}_1$, ..., $\mu^{\mathbb{P}}_K$, σ_1, ..., σ_K are parameters that drive the leverage dynamics under the K possible regimes. The information structure is captured with the filtration $\{\mathcal{G}_t\}_{t=0}^{\infty}$ generated by the noise series $\{\epsilon^{\mathbb{P}}_t\}_{t=1}^{\infty}$ and the regimes $\{s_t\}_{t=0}^{\infty}$.

Because the leverage ratio is one of the potential drivers of default, it is incorporated in an intensity process $\{H_t\}_{t=0}^{\infty}$ that characterizes the potential default:

$$H_t = \beta + \left(\frac{L_t/A_t}{\theta}\right)^{\alpha} \tag{2}$$

where α, β and θ are positive constants. The default intensity increases with the leverage ratio, making the default more likely to happen. The intensity used in this study generalizes assumptions made by others. For instance, Bakshi et al. (2006a) use an intensity that depends linearly on the firm's leverage. In Doshi et al. (2013), the intensity is modelled as a quadratic function of the leverage.[5] As usual in intensity-based models, the default time arises as soon as the intensity accumulation reaches a random level determined by an exponentially distributed random variable E_1 of mean 1 independent of $\{\mathcal{G}_t\}_{t=0}^{\infty}$:

$$\tau = \inf\left\{t \in \{1, 2, ...\} : \sum_{u=0}^{t-1} H_u\Delta > E_1\right\}. \tag{3}$$

The recovery rate is determined by the firm's assets and liabilities at default. Following the intuition of structural models, it is characterized by the amount recovered by the creditors divided by the amount

[3] Volatility is important in credit risk models. For instance, Zhang et al. (2009) show that volatility risk predicts 48% of the variation in the CDS spreads using a Merton-type structural model and a calibration approach.

[4] The leverage ratio L_t/A_t is not constrained to lie within the unit interval since L_t is the liabilities value and not the debt value. The liabilities value L_t could thus be larger than A_t.

[5] The quadratic intensity case is a specific case of Equation (2) where $\alpha = 2$.

available upon default. At default, the creditors receive the amount the firm owe them (i.e. L_τ) or the asset value, minus legal and restructuration fees (i.e. $(1-\kappa)A_\tau$), depending on how much each quantity is worth. Thus, the recovery rate at default time is proxied by

$$R_\tau = \frac{\min\left((1-\kappa)A_\tau; L_\tau\right)}{L_\tau} = \min\left((1-\kappa)\frac{A_\tau}{L_\tau}; 1\right) = \min\left((1-\kappa)e^{-x_\tau}; 1\right), \tag{4}$$

where κ represents the legal and restructuration fees, expressed as a proportion of the asset value at default time. Consequently, an endogenous random recovery rate negatively correlated to default probabilities is implied by Equation (4).

The unobserved regime is modelled as a time-homogenous Markov chain with transition probabilities

$$p_{ij} = \mathbb{P}(s_t = j \,|\, s_{t-1} = i), \qquad i, j \in \{1, 2, ..., K\}. \tag{5}$$

The market model is incomplete, implying that there is an infinite number of pricing measures. Among these measures, we restrict the choices to those preserving the model structure:

$$x_t = x_{t-1} + \left(\mu^{\mathbb{Q}}_{s_t} - \frac{1}{2}\sigma^2_{s_t}\right)\Delta + \sigma_{s_t}\sqrt{\Delta}\epsilon^{\mathbb{Q}}_t \tag{6}$$

and

$$q_{ij} = \mathbb{Q}(s_t = j \,|\, s_{t-1} = i), \qquad i, j \in \{1, 2, ..., K\} \tag{7}$$

where $\{\epsilon^{\mathbb{Q}}_t\}_{t=1}^{\infty}$ is a sequence of independent standardized Gaussian random variables under $\mathbb{Q}$.

3 Estimation method and pricing technique

The model introduced in Section 2 is estimated on a firm-by-firm basis using MLE. However, its estimation is far from being straightforward since each firm's leverage ratios and statistical regimes are unobservable and should be inferred from observed quantities. Also, noisy observations lay additional stress upon the estimation procedure. To solve the problem at hand, a well-chosen combination of numerical methods and deterministic filters is employed.

First, a filtering method that simultaneously infers both latent quantities is needed. Theoretically, it would be possible to compute the filtered leverages given a regime path; however, this approach suffers from the curse of dimensionality. We extend Tugnait (1982) by allowing for non-linearities in the state-space representation, while considering only the most likely regime paths at each time step.

Second, the filter uses complex derivative prices (i.e. CDS) as measurements. Hence, a fast-pricing scheme must be implemented to ensure the feasibility of the estimation step.

3.1 Estimation technique

A state-space representation is commonly used to define a model for which the state is a Markov process and observed quantities (i.e. CDS premiums) are related to the state variable. The log-leverage dynamics is given by Equation (1):

$$x_t = x_{t-1} + \left(\mu^{\mathbb{P}}_{s_t} - \frac{1}{2}\sigma^2_{s_t}\right)\Delta + \sigma_{s_t}\sqrt{\Delta}\epsilon^{\mathbb{P}}_t.$$

The measurement equation becomes

$$y^{(i)}_t = \log\left(g^{(i)}\left(x_t, s_t\right)\right) + \nu^{(i)}_t, \tag{8}$$

with $y^{(i)}_t$ the observed i-year credit default swap log-premium, $\nu^{(i)}_t$ being a Gaussian random variable (having a standard deviation $\delta^{(i)}$) independent across maturities and $g^{(i)}$ the theoretical premium of an i-year credit default swap (in basis points). To reduce the number of parameters to be estimated, independence in the pricing error between each CDS has been assumed. Note that the CDS pricing formula (i.e. function $g^{(i)}$) is a nonlinear function of the log-leverage x_t. To maintain positive premiums, a logarithmic transformation is applied.

3.1.1 Unscented Kalman filter-based method

If leverage time series were observable, the regimes could be painlessly filtered (for a review of classic methods, see Elliott et al., 1995). However, it is not the case and filtering regimes based on a latent time series is not straightforward.

Tugnait (1982) considers the problem of state estimation and system structure detection for discrete stochastic systems with parameters that may change among a finite set of values. His detection-estimation algorithm (DEA) is designed to filter a state and a regime simultaneously in the context of linear state-space representations. In this paper, the DEA is generalized to account for nonlinearities by running the UKF of Julier and Uhlmann (1997) instead of the classic Kalman (1960) filter.[6] According to Christoffersen et al. (2014), the UKF may prove to be a good approach for a variety of problems in fixed income pricing. Moreover, for some applications in finance, it seems to significantly outperform the extended Kalman filter and behaves well when compared to the much more computationally intensive particle filter. To the best of our knowledge, this study is the first using the DEA coupled with the UKF.

The rationale behind the method is quite intuitive: if one could observe the regime path, the UKF could be applied directly. However, since it is not the case, the filtered value of the log-leverage would require the computation of a sum over each possible regime path. The number of terms in the summation would increase exponentially making its calculation futile. Since the regime process is Markovian and tends to forget its origin, a good approximation considers the most likely regime paths, which is Tugnait's (1982) idea. Precisely, we collapse from K^M to K^{M-1} regime paths by keeping the paths that yield the most likely sequences in terms of *a posteriori* probabilities. This would lead to minimum probability of error according to Van Trees (1968).

This method is similar to the one proposed by Kim (1994). The main difference between the DEA and Kim (1994) is the collapsing scheme. The latter proposes to take averages across the different regime paths to reduce the number of sequences instead of keeping the most likely ones. Another related method would be the one of Fearnhead and Clifford (2003). They propose an optimal procedure in which all paths with probability greater than some cutoff are kept and the others are resampled (as in a conventional particle filter). However, since Fearnhead and Clifford (2003) relies on particle filtering, the estimation using this stochastic filter would be more cumbersome.[7]

Suppose that at stage $t-1$, K^{M-1} regime paths are available. At stage t and given the K^{M-1} paths, all possible extensions of these regime sequences shall be considered. For instance, all the possible extensions $s^j_{0:t} \equiv \{s^j_0, s^j_1, ..., s^j_t\}$ of the a^{th} path $s^a_{0:t-1}$ are given by

$$s^j_{0:t} = \{s^a_{0:t-1}, s\}, \qquad s \in \{1, 2, ..., K\}.$$

Doing this for each of the K^{M-1} paths shall create K^M new regime paths. Then, using the UKF, it is possible to compute the filtered value of x_t based on the first t observations and the regime path $s^j_{0:t}$:

$$x^j_{t|t} \equiv \mathbb{E}^{\mathbb{P}}(x_t |\, \mathbf{y}_{1:t}, s^j_{0:t})$$

where $\mathbf{y}_{1:t} \equiv \{\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_t\}$ and $\mathbf{y}_t$ is a size n vector containing log-premiums $y^{(i)}_t$. Moreover, the posterior probability of observing this specific regime path based on the first t observations is

$$\mathbb{P}(s^j_{0:t} \,|\, \mathbf{y}_{1:t}) = \frac{f(\mathbf{y}_t | s^j_{0:t}, \mathbf{y}_{1:t-1})\mathbb{P}(s^j_t | s^j_{0:t-1}, \mathbf{y}_{1:t-1})\mathbb{P}(s^j_{0:t-1} | \mathbf{y}_{1:t-1})}{\sum_{k=1}^{K^M} f(\mathbf{y}_t | s^k_{0:t}, \mathbf{y}_{1:t-1})\mathbb{P}(s^k_t | s^k_{0:t-1}, \mathbf{y}_{1:t-1})\mathbb{P}(s^k_{0:t-1} | \mathbf{y}_{1:t-1})}$$

where $\mathbb{P}(s^j_t \,|\, s^j_{0:t-1}, \mathbf{y}_{1:t-1})$ can be computed using p_{ij} of Equation (5) and $s^k_{0:t}$ is the k^{th} extended path.

[6]The parameters of the UKF technique have been assumed to be $\kappa_{\text{UKF}} = 2$, $\alpha_{\text{UKF}} = 0.1$, and $\beta_{\text{UKF}} = 0$.

[7]Indeed, a change in the parameters can cause the filter to select a different set of particles inducing discontinuity in the likelihood function. The discontinuity is created from the resampling stage within the update state of the particle filter. For more details, see Creal (2012).

In addition,

$$f(\mathbf{y}_t \,|\, s^j_{0:t}, \mathbf{y}_{1:t-1}) = \exp\left(-\frac{1}{2}\log(\det \mathbf{V}^j_t) - \frac{1}{2}(\mathbf{e}^j_t)^\top(\mathbf{V}^j_t)^{-1}(\mathbf{e}^j_t) - \frac{n}{2}\log(2\pi)\right),$$

$\mathbf{e}^j_t = \mathbf{y}_t - \mathbf{y}^j_{t|t-1}$ is the difference between the observations and the forecasted value of the observations at stage t, and $\mathbf{V}^j_t = \mathbb{E}\left[(\mathbf{e}^j_t)(\mathbf{e}^j_t)^\top \,\middle|\, s^j_{0:t}, \mathbf{y}_{1:t-1}\right]$. This leads to the following state estimate:

$$x_{t|t} = \sum_{k=1}^{K^M} x^k_{t|t}\mathbb{P}(s^k_{0:t} \,|\, \mathbf{y}_{1:t}).$$

Finally, we collapse the K^M regime paths at time t to obtain K^{M-1} new regime paths according to Tugnait's (1982) idea. Simply put, we select the K^{M-1} regime paths yielding the highest posterior probabilities $\mathbb{P}(s^j_{0:t} \,|\, \mathbf{y}_{1:t})$ across the K^M remaining paths.

The quasi-likelihood function is given as a by-product of running the unscented Kalman filter:

$$\prod_{t=1}^{T}\left(\sum_{k=1}^{K^M} f(\mathbf{y}_t \,|\, s^k_{0:t}, \mathbf{y}_{1:t-1})\mathbb{P}(s^k_t \,|\, s^k_{0:t-1}, \mathbf{y}_{1:t-1})\mathbb{P}(s^k_{0:t-1} \,|\, \mathbf{y}_{1:t-1})\right).$$

In our application, quasi-likelihood means that the first two moments of the posterior distribution are matched and a posterior Gaussian distribution has been assumed.

A simulation study (see Appendix B) concludes that the DEA-UKF method yields virtually no differences between estimated and true parameter values on average. Moreover, the filtered values recovered by the filter are very close to their true values, with no differences between the two.

3.2 Credit-sensitive derivative pricing

The derivative securities priced within this model take into consideration default, recovery and regime risks. However, stochastic recovery rates and regime-switching dynamics prevent us from using closed-form solutions for CDS premiums and corporate coupon bond prices. Even though Monte Carlo methods could be applied, lattice-based methods are more suited for the problem at hand since these yield accurate prices in a timely fashion.

The use of lattice methods in Markov-switching environment dates back to Bollen (1998) who proposes a pentanomial lattice. However, more recently, Yuen and Yang (2010) use a different idea: they adopt identical trinomial lattices across every regime, but change the risk-neutral weights as the regime state shifts. This would allow the trinomial trees to recombine across the different regimes. It works for many kinds of options, but credit-sensitive derivatives could not be priced directly using this methodology. Yuen and Yang's (2010) method is thus extended to price credit-sensitive securities (i.e. bond and CDS). An additional branch is added at each node to account for the potential default of the firm. This idea is similar to the one proposed by Schönbucher (2002). Details about the pricing scheme are provided in Appendix A.

4 Estimation results

Even though the question of whether CDS premiums include liquidity effects or counterparty risk is controversial (see Tang and Yan, 2007; Bühler and Trapp, 2009b; Brigo et al., 2011; Arora et al., 2012; Qiu and Yu, 2012), a large number of authors still use them to capture credit risk. Longstaff et al. (2005) argue that CDSs are of a contractual nature that affords relative ease of transacting large notional amounts compared to the corporate bond market. Moreover, an investor can liquidate a position by entering into a new swap in the opposite direction instead of selling his current position. Therefore, liquidity is less relevant given the ability to replicate swap cash flows using another CDS. Moreover, Mahanti et al. (2008), Dionne and Maalaoui Chun (2013) and Guarin et al. (2014) use CDS premiums as pure measures of credit risk.

We follow this literature and use CDS premiums since they are good proxies for credit risk. Moreover, the filtering approach adopted in this paper allows for potential lack of liquidity in specific tenors to be absorbed by the noise terms. Thus, the selected methodology would help to reduce the impact of illiquidity in our dataset, if ever there is any. Also, to minimize the impact of such risk, the study also focuses on firms that are part of the widely followed CDX indices.

4.1 Data

The investigation is performed on 225 firms of the CDX North American Investment Grade (IG) and High Yield (HY) indices provided by the Markit Group on September 20, 2013.[8] The indices span multiple credit ratings and sectors. The weekly term structure of CDS premiums from January 5, 2005, to December 25, 2013, is also provided by Markit for a maximum of 469 weeks. CDS premiums up to the end of 2012 are used in the estimation; the last year of data (i.e. 2013) is kept for an out-of-sample analysis. We use Wednesday CDS premiums.[9] Prices for maturities of one, two, three, five, seven and ten years are available for most firms (125 IG and 100 HY).[10] This yields a grand total of 487,796 observations in our final sample of CDS premiums. The 'No Restructuring' clause is selected to capture only the credit risk of the firm.

To illustrate how CDS premiums move over time, the first panel of Figure 1 shows the evolution of the mean five-year CDS premium taken across firms for both IG and HY portfolios. Premiums were more or less stable during the pre-crisis era. They increased during the financial crisis: the mean five-year premiums jumped to a high of 321 basis points (bps) for IG firms and 1,422 bps for HY firms. In the post-crisis era, the premiums decreased, but did not reach their pre-crisis levels.

The second panel of Figure 1 shows the evolution of the average slope proxied by the difference between the average ten-year CDS premiums and the average one-year CDS premiums taken across firms for both IG and HY portfolios. The slope of the CDS premiums' term structure became negative during the crisis for IG and HY firms on average. According to Figure 1, both the level and the slope of the CDS premiums change during the financial crisis. The regime-switching component of the proposed framework is required to capture these important changes in behaviour.

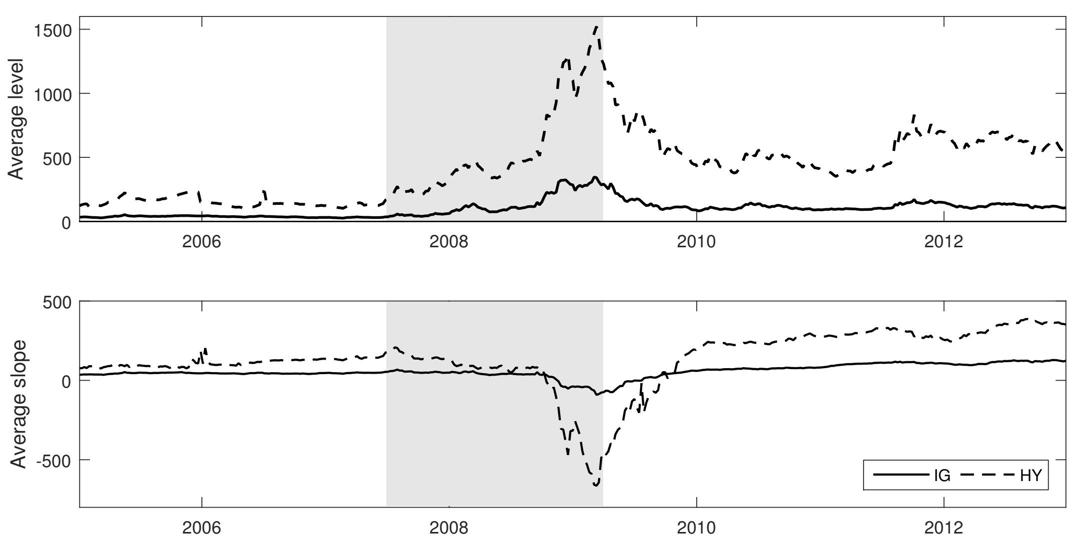

Figure 1: Evolution of the mean five-year CDS premiums (across firms) and of the difference between the average ten-year CDS premiums and the average one-year CDS premiums (across firms).
The CDS premiums were taken from both CDX.NA.IG.21.V1 and CDS.NA.HY.21.V1 portfolios, between January 2005 and December 2012. The grey surface corresponds to the financial crisis (July 2007 to March 2009).

[8] CDS.NA.IG.21.V1 and CDS.NA.HY.21.V1 respectively.

[9] We focus on Wednesday CDSs because it is the least likely day to be a holiday and it is least likely to be affected by weekend effects. For more details on the advantages of using Wednesday data, see Dumas et al. (1998).

[10] However, 15 firms (4 IG and 11 HY) were removed from the sample since there were not enough observations for the estimation procedure (i.e. less than 100 observations per maturity).

The initial state value x_0 has been set to the book value of the log-leverage using Compustat.[11] Also, to price CDS, we follow standard industry practice as well as Carr and Wu (2010) and we use the interest rate curve defined by the LIBOR and swap rates. One-, two-, three- and six-month LIBOR are selected as well as one-, two-, three-, four-, five-, seven- and ten-year swap rates.[12]

Saunders and Allen (2010) decompose the recent financial crisis in three periods. The first period corresponds to the credit crisis in the mortgage market (June 2006 to June 2007), the second one covers the period of the liquidity crisis (July 2007 to August 2008) and the third period covers the default crisis period (September 2008 to March 2009). This study focuses on the second and the third periods; thus, the financial crisis started in July 2007 and finished in March 2009 throughout this paper. Notice that these deterministic sample's subperiods are not related to the firm-specific regimes anyhow.

4.2 Estimated parameters

The leverage model used hereafter is a simplified version of Equation (1); we only consider two regimes. Considering more regimes is theoretically feasible; however, the number of parameters increase drastically and the numerical optimization of the firm-specific likelihood function becomes unmanageable. We also consider the same drift parameter across both regimes ($\mu^{\mathbb{P}} = \mu_1^{\mathbb{P}} = \mu_2^{\mathbb{P}}$ and $\mu^{\mathbb{Q}} = \mu_1^{\mathbb{Q}} = \mu_2^{\mathbb{Q}}$). Indeed, the drift parameter estimators of the latent variable is rather inaccurate and create numerical instability due to the short span of the time series used.[13] Besides this caveat, the regime-switching framework still captures the variation of volatility across the different regimes (i.e. uncertainty).

The parameters are estimated using a quasi-MLE on a firm-by-firm basis. Overall, the parameters to be estimated for each company are $\boldsymbol{\phi} = \{\mu^{\mathbb{P}}, \mu^{\mathbb{Q}}, \sigma_1, \sigma_2, p_{12}, q_{12}, p_{21}, q_{21}, \alpha, \beta, \theta, \kappa,\ \delta^{(1)}, \delta^{(2)}, \delta^{(3)}, \delta^{(5)},\ \delta^{(7)}, \delta^{(10)}\}$. Thus, a single set of parameters is used for each firm to explain the default risk and loss given default; this contrasts with calibration techniques where the credit default swap term structure is fitted at every available period.

Table 1 shows descriptive statistics on firm-specific estimated parameters. All the firms have a positive intensity process since β is always positive. The parameter α is always greater than 1.67, confirming the convex relationship between the intensity process and the leverage ratio. The volatility parameters σ_1 and σ_2 are very different one from another, on average. The low-volatility regimes normally correspond with the end of the pre-crisis era and the post-crisis period; its average value is around 11%. The dispersion across the different firm's σ_1 is rather small (i.e. 4%). For most of the firms, the high-volatility regimes correspond to the financial crisis. Its volatility is roughly 36% on average. The dispersion of the second regime volatility parameter is higher (i.e. 8%), which means that this parameter is quite dependent on the firm's condition during turmoil. The regimes are persistent since both $\mathbb{P}$- and $\mathbb{Q}$-transition probability matrices are concentrated over the main diagonal.

Noise term standard errors for mid- and long-term tenors are small on average: for instance, 13.2% for two-year maturity and 3.4% for five-year. It is the highest for the one-year CDS on average. Two reasons can explain this result: elements not necessarily related to the entity's true default and recovery risks, as well as fitting error due to model misspecification. Credit default swaps with a maturity of five years are the ones with the smallest noise standard errors on average. This makes sense empirically: five-year is known to be the most liquid tenor for CDS.

[11] Initial leverages are approximated using their book values. More precisely, total liabilities divided by total assets is taken. Both quantities are acquired by Compustat, which is available from Wharton Research Data Services (WRDS). The fourth quarter of 2004's accounting data is selected to compute the proxy since Q4 of 2004 predates January 2005 (the beginning of our sampling period). In the database, the total liabilities are identified by LTQ and the total assets by ATQ. In addition, the firm's ticker symbol is matched to that of Markit's CDS premiums through the reference entity's name to ensure that the right information for each firm is used. For one firm, no data is available; this firm is thus removed from the sample.

[12] These rates are provided by the Federal Reserve of St. Louis website via FRED (Federal Reserve Economic Data). The series IDs are USD1MTD156N, USD2MTD156N, USD3MTD156N, USD6MTD156N, DSWP1, DSWP2, DSWP3, DSWP4, DSWP5, DSWP7 and DSWP10.

[13] Even in a 'one-regime' framework where the log-leverage is assumed to be observed, the precision of the drift parameter estimate is proportional to the square root of the sampling period length. Hence, long-time series are required to pin down these parameters.

Table 1: Descriptive statistics on the distribution of firm-specific parameters and noise terms across the portfolio of firms of the CDX indices.

Panel A: Descriptive statistics on leverage dynamics parameters (under $\mathbb{P}$) and fees κ.							
	$\mu^{\mathbb{P}}$	p_{12}	p_{21}	σ_1	σ_2	κ	Obs.
Mean	0.0169	0.0128	0.0190	0.1147	0.3572	0.6099	210
SD	0.0485	0.0161	0.0223	0.0376	0.0806	0.1728	
10%	-0.0087	0.0028	0.0048	0.0738	0.2526	0.4159	
25%	-0.0012	0.0038	0.0075	0.0934	0.3253	0.5000	
50%	0.0000	0.0083	0.0110	0.1111	0.3541	0.5819	
75%	0.0144	0.0149	0.0203	0.1351	0.3773	0.7126	
90%	0.0709	0.0274	0.0459	0.1614	0.4656	0.8474	
IG	0.0084	0.0132	0.0194	0.1089	0.3738	0.5825	121
HY	0.0285	0.0123	0.0184	0.1226	0.3346	0.6471	89
Panel B: Descriptive statistics on leverage dynamics parameters (under $\mathbb{Q}$) and intensity.							
	$\mu^{\mathbb{Q}}$	q_{12}	q_{21}	α	θ	β	Obs.
Mean	0.0165	0.0061	0.0397	11.4807	1.4385	0.0088	210
SD	0.0302	0.0064	0.0287	7.9199	0.3090	0.0225	
10%	-0.0050	0.0016	0.0081	6.0294	1.0724	0.0002	
25%	-0.0002	0.0028	0.0136	7.5511	1.3113	0.0007	
50%	0.0104	0.0046	0.0382	9.9483	1.4823	0.0029	
75%	0.0258	0.0074	0.0579	12.2609	1.5215	0.0083	
90%	0.0447	0.0123	0.0760	17.1065	1.7277	0.0170	
IG	0.0034	0.0060	0.0547	12.3016	1.3643	0.0066	121
HY	0.0342	0.0062	0.0193	10.3646	1.5394	0.0117	89
Panel C: Descriptive statistics on error terms.							
	$\delta^{(1)}$	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(5)}$	$\delta^{(7)}$	$\delta^{(10)}$	Obs.
Mean	0.2503	0.1318	0.0810	0.0339	0.0470	0.0762	210
SD	0.0647	0.0362	0.0297	0.0263	0.0400	0.0460	
10%	0.1749	0.0904	0.0420	0.0002	0.0003	0.0282	
25%	0.2049	0.1063	0.0665	0.0138	0.0095	0.0366	
50%	0.2446	0.1300	0.0817	0.0353	0.0457	0.0690	
75%	0.2972	0.1559	0.1023	0.0452	0.0661	0.1001	
90%	0.3442	0.1763	0.1152	0.0706	0.1061	0.1425	
IG	0.2310	0.1253	0.0768	0.0335	0.0657	0.0964	121
HY	0.2765	0.1407	0.0867	0.0345	0.0217	0.0487	89

[1] For each of the 210 firms, the parameters of the model are estimated using weekly CDS premiums of maturities one, two, three, five, seven and ten years, using the DEA-UKF filtering technique. The mean, standard deviation (SD) and quantiles are computed across firms. The last two rows compute the mean across firms of CDX.NA.IG.21.V1 and CDX.NA.HY.21.V1 portfolios.
[2] The δs represent the standard deviation of the noise terms present in the observation equation of the filter.
[3] IG refers to Investment Grade and HY means High Yield.

4.3 In- and out-of-sample performances

The regime-switching hybrid default model (RS) is compared to three other models: the 'one-regime' (1R) equivalent of our model, a regime-switching structural version (SA) of the proposed framework, and a regime-switching reduced-form model (RFA).

The so-called 1R model is the one introduced in Boudreault et al. (2013); the main difference between this model and ours is that we use regime-switching dynamics to model the firm's leverage.

Letting $\beta = 0$ and $\alpha \to \infty$, the hybrid approach leads to a pure structural framework in which the log-leverage ratio is modelled by a regime-switching discretized version of an arithmetic Brownian motion. The structural model is nested in the hybrid framework; thus, its overall in-sample performance is expected to be weaker than the full model. However, it is possible that local performances dominate those of the full model.

The pure reduced-form version of the model (i.e. $\theta \to \infty$) is not the one used in this paper since this model would have constant intensity and this is obviously too restrictive. Instead, the log-intensity process is modelled by a regime-switching discretized version of an arithmetic Brownian motion under the $\mathbb{Q}$ measure:

$$\log(H_t) = \begin{cases} \log(H_{t-1}) + \left(\mu_H^{\mathbb{Q}} - \frac{1}{2}\sigma_{H,1}^2\right)\Delta + \sigma_{H,1}\sqrt{\Delta}\epsilon_t^{\mathbb{Q}} & \text{if } s_t = 1 \\ \log(H_{t-1}) + \left(\mu_H^{\mathbb{Q}} - \frac{1}{2}\sigma_{H,2}^2\right)\Delta + \sigma_{H,2}\sqrt{\Delta}\epsilon_t^{\mathbb{Q}} & \text{if } s_t = 2 \end{cases}$$

where $\mu_H^{\mathbb{Q}}$ is the drift parameter and $\sigma_{H,1}$ and $\sigma_{H,2}$ are volatility parameters for regimes 1 and 2, respectively. Moreover, $\{\epsilon_t^{\mathbb{Q}}\}_{t=1}^{\infty}$ is a sequence of independent standardized Gaussian random variables under $\mathbb{Q}$. Since endogenous recoveries no longer make sense in the context of this model, we opt for a constant exogenous recovery rate $R_t = R \in [0,1]$. This new parameter is estimated among all other parameters in the filtering procedure.

The models' performances are compared using the sum of squared errors:

$$\text{SSE} = \sum_{t=t_1}^{t_2}\sum_{i=1}^{N_t}(m_{t,i} - o_{t,i})^2 \tag{9}$$

where $m_{t,i}$ is the theoretical i-year CDS premium at time t, $o_{t,i}$ is the observed i-year CDS premium at time t, and N_t is the number of CDSs considered at time t. The parameter estimates are obtained using the entire sample and are kept fixed at any point in time. To standardize these fitting performances, the SSE is divided by the total variation of the observed CDS premiums:

$$\text{SST} = \sum_{t=t_1}^{t_2}\sum_{i=1}^{N_t}(o_{t,i} - \bar{o})^2 \text{ where } \bar{o} = \frac{1}{\sum_{t=t_1}^{t_2} N_t}\sum_{t=t_1}^{t_2}\sum_{i=1}^{N_t} o_{t,i}. \tag{10}$$

Consequently, the performance measure is the ratio SSE/SST. The closer it is to zero, the better the model is.[14]

4.3.1 In-sample performance

The in-sample study is performed over 417 weeks, starting in January 2005 and ending in December 2012.

Figure 2 shows the evolution of the SSE/SST ratio calculated by week. The measure used in these figures combines the six tenors. The in-sample performance of the full model is very good with an average SSE/SST of 3.33%. It is better when compared to the other models: indeed, the performance measure of the full model is 1.83 times lower than the one of the reduced-form approach, 2.05 than the structural model, and 3.45 than the 'one-regime' equivalent. During the financial crisis of 2008, the SSE/SST ratio spikes when we consider the three benchmark models. However, the full model does well during these turbulent times: there are no apparent spikes in SSE/SST for RS during the crisis era.

Panels A, B and C of Table 2 shows the ratio calculated by maturity and by period. The performance of the regime-switching hybrid model remains good, even during the crisis era. Systematically, the full model outperforms the other ones which means that each risk involved in the full model is important to capture credit risk. For IG firms, the performance is sound, even for one-year CDS premiums.

4.3.2 Out-of-sample: forecasting 2013

In our out-of-sample study, the parameters are estimated using CDS premiums from January 2005 to December 2012. Then, CDS premiums observed in 2013 are used to evaluate the out-of-sample fit of the model. The out-of-sample measures are calculated for every CDS premium observed in 2013.[15]

[14] We also apply a loose criterion to eliminate outliers: if the absolute difference between the model premium and the observed premium is more than 5 times the observed premium, it is considered an outlier: $\mid m_{t,i} - o_{t,i} \mid > 5o_{t,i}$. For the in-sample analysis, seven observations out of 489,796 observations are removed. Thus, we remove 0.0014% of our initial CDS sample.

[15] As before, observations satisfying the rejection criterion are removed from the analysis. Overall, 41 out of 67,373 observations are discarded since they are considered to be outliers by at least one of the four models.

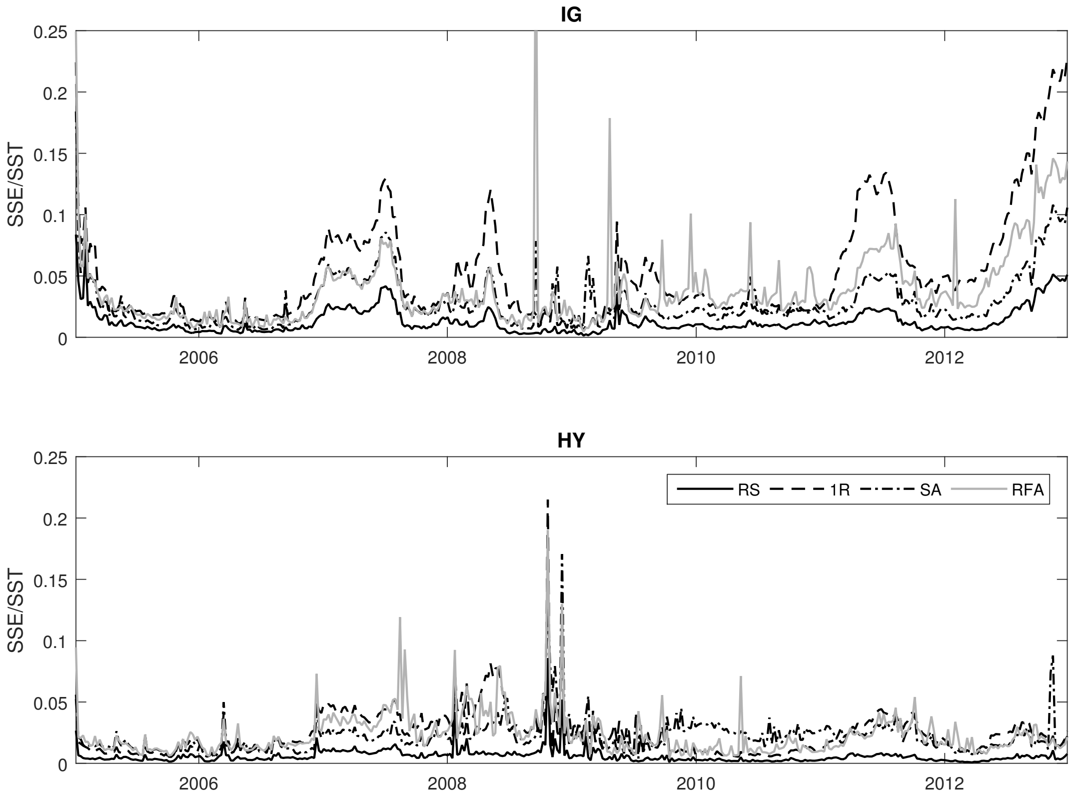

Figure 2: SSE/SST: evolution of the in-sample performance for IG and HY.
These figures show the evolution of the SSE/SST ratio calculated by week. For each firm, a single set of parameters is estimated using CDS premiums with maturities 1, 2, 3, 5, 7, and 10 years between January 2005 and December 2012. IG stands for Investment Grade and HY for High Yield. RS stands for regime-switching hybrid model, 1R means 'one-regime' equivalent of our model, SA is the regime-switching structural model and RFA refers to the regime-switching reduced-form model. Notice that on September 17, 2008, the three benchmark models seem to yield large pricing errors on IG firms. This is mainly caused by errors on AIG's credit default swap premiums (on September 15, 2008, AIG's credit rating was downgraded from AA- to A-; the downgrade had an important impact on CDS premiums).

Panel D of Table 2 shows that our new model produces adequate one-week-ahead forecasts. The full model's curves is lower than the other ones for most maturities and risk classes, which is good. For IG, SSE/SST ratios are higher for short maturities. The main reason for this behaviour is that 1-year SST is much smaller than the other SSTs, even though the 1-year SSE is somewhat smaller than those computed for other maturities. Therefore, it is natural to observe high SSE/SSTs for 1-year credit default swaps in 2013.

4.4 Most likely statistical regimes through time

The approach of Viterbi (1967) is adapted to the context of a hidden regime and a latent variable to extract the most probable regime path for each firm. The regime path that maximizes the likelihood function given the estimated parameters is constructed recursively. At this point, we find relevant to stress that the filtered statistical regimes depend on firm-specific information and shall not only be related to financial or economic cycles. Therefore, one should not misinterpret the notion of filtered regime: these are computed for each firm and are solely related to the firm's financial health. By taking the average proportion of firms in the high-volatility regime, we hope to find a systematic trend across the firms, although it is possible that some companies were only sparsely affected by the crisis or that some were in precarious positions during pre- and post-crisis eras.

Figure 3 shows the proportion of firms considered in the high-volatility regime for each week and for each risk class. In our context, being in the second (high-volatility) regime is synonymous with more uncertainty in the firm's leverage. The fast transition from the first (low-volatility) regime to the second (high-volatility) regime during 2007 and 2008 is obviously very natural due to the financial conditions at that time. At the

Table 2: SSE/SST: in- and out-of-sample performance for IG and HY.

Panel A: Pre-crisis (in-sample)								
	IG				**HY**			
	RS	1R	SA	RFA	RS	1R	SA	RFA
1	0.089	0.176	0.171	0.162	0.023	0.065	0.081	0.051
2	0.023	0.053	0.050	0.045	0.014	0.030	0.029	0.026
3	0.007	0.019	0.022	0.016	0.005	0.011	0.010	0.013
5	0.003	0.012	0.012	0.008	0.002	0.004	0.008	0.006
7	0.001	0.008	0.010	0.008	0.001	0.006	0.014	0.007
10	0.004	0.018	0.021	0.017	0.004	0.014	0.029	0.017
Panel B: Crisis (in-sample)								
	IG				**HY**			
	RS	1R	SA	RFA	RS	1R	SA	RFA
1	0.046	0.091	0.098	0.103	0.142	0.527	0.263	0.233
2	0.021	0.045	0.037	0.043	0.084	0.274	0.189	0.162
3	0.009	0.020	0.017	0.022	0.022	0.067	0.050	0.043
5	0.001	0.007	0.004	0.017	0.002	0.007	0.007	0.006
7	0.005	0.022	0.010	0.026	0.001	0.004	0.005	0.007
10	0.013	0.051	0.027	0.051	0.005	0.014	0.015	0.017
Panel C: Post-crisis (in-sample)								
	IG				**HY**			
	RS	1R	SA	RFA	RS	1R	SA	RFA
1	0.055	0.150	0.197	0.118	0.089	0.176	0.171	0.162
2	0.024	0.064	0.076	0.051	0.023	0.053	0.050	0.045
3	0.014	0.033	0.041	0.030	0.007	0.019	0.022	0.016
5	0.004	0.010	0.015	0.011	0.003	0.012	0.012	0.008
7	0.002	0.009	0.015	0.012	0.001	0.008	0.010	0.008
10	0.011	0.030	0.038	0.034	0.004	0.018	0.021	0.017
Panel D: 2013 (out-of-sample)								
	IG				**HY**			
	RS	1R	SA	RFA	RS	1R	SA	RFA
1	2.017	5.156	1.974	2.183	0.139	0.189	0.129	0.156
2	0.679	0.949	0.885	0.722	0.080	0.125	0.100	0.099
3	0.261	0.259	0.349	0.289	0.045	0.066	0.075	0.051
5	0.032	0.084	0.028	0.021	0.038	0.046	0.040	0.046
7	0.072	0.203	0.061	0.054	0.041	0.062	0.043	0.053
10	0.100	0.288	0.092	0.110	0.050	0.081	0.060	0.070

[1] This table shows the SSE/SST ratio calculated by maturity and period (i.e. pre-crisis, crisis, post-crisis, 2013). For each firm, a single set of parameters is estimated using CDS premiums with maturities of 1, 2, 3, 5, 7 and 10 years between January 2005 and December 2012.
[2] IG stands for Investment Grade and HY for High Yield.
[3] RS stands for regime-switching hybrid model, 1R means 'one-regime' equivalent of our model, SA is the regime-switching structural model and RFA refers to the regime-switching reduced-form model.

beginning of the sample, many HY firms are in the high-volatility regime, meaning that the filtered leverage is more uncertain for these firms. Note that uncertainty is typically related to the slope of the CDS term structure; for instance, in a study based on sovereign CDS, Augustin (2012) argues that slopes tend to be positive in good times and invert during economic distress. We compare the slopes of the term structure of CDS premiums (i.e. ten-year minus one-year CDS premiums) during the pre-crisis era and find that they tend to be smaller when firms are in the second regime.[16] Therefore, the filtered statistical regimes are consistent with this observation. Moreover, the beginning of our sample corresponds to a period of general widening in yield spreads in debt markets due to the downgrade of General Motors and Ford's ratings (Saldías, 2013).

For HY firms, the moment when the proportion of firms in the second regime increased corresponds to the beginning of the financial crisis (i.e. July 2007). During the first six months of the crisis, the proportion of firms in the high-volatility regime goes from 8% to almost 60%. For IG firms, the transition happens a little later and is consistent with the beginning of the NBER economic recession in the United States

[16] The interested reader may refer to the Internet Appendix of the paper.

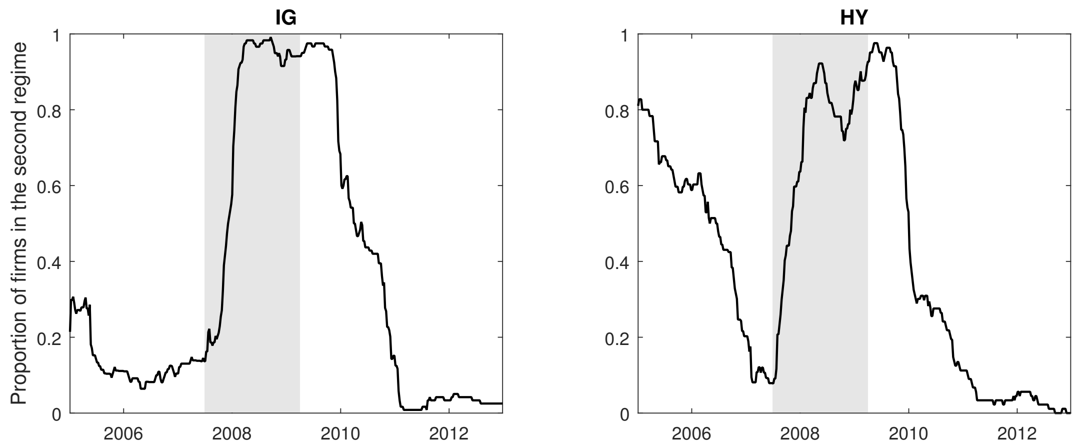

Figure 3: Proportion of firms in the high-volatility statistical regime.
Each week, the proportion of firms considered in the second regime is computed for the 121 IG and the 89 HY ones. The grey surface corresponds to the financial crisis (July 2007 to March 2009).

(i.e. December 2007). At the end of the financial crisis (or the recession), both IG and HY firms remain in the high-volatility regime for several weeks. These results are in line with the recent literature. Indeed, using a different approach, Maalaoui Chun et al. (2014) detect some persistence in the volatility regimes of credit spreads. They are also consistent with those of Garzarelli (2009) and Mueller (2008). These authors document that credit spread levels increase before the onset of the NBER recession and somehow persist until long after the recession is over.

5 Recovery rate versus regime-switching: Term structure comparison

This section assesses the relative importance of recovery rate and regime-switching risks in credit spread curves. Essentially, we compare the difference between risky and riskless zero-coupon yields for various model specifications of the model.

The time t value of risky zero-coupon bond (see Appendix A.2) is given by $V(t, T, \widehat{x}_t, \widehat{s}_t; R_\tau, 0)$ where $\widehat{x}_t$ is the time t filtered log-leverage and $\widehat{s}_t$ is the most probable regime at time t.[17]

To compare the relative importance of recovery-rate risk and regime-switching risk, four different formulations of the model are estimated. The first one (RS-ENDR) is described in Section 2. The second model (RS-EXOR) is a variation of the full model: instead of using endogenous recovery rates, a constant exogenous recovery rate $R_t = R \in [0,1]$ is incorporated into the set of parameters to be estimated. The third model is the 'one-regime' equivalent with endogenous recovery rates (1R-ENDR) presented in Boudreault et al. (2013). The last model (1R-EXOR) is a 'one-regime' equivalent of the full model with a constant exogenous recovery rate to be estimated.

In this section, the results are broken down by credit rating. These ratings are attributed by Standard and Poor's between January 31, 2005, and December 31, 2012, and are available from Compustat in WRDS. The rating used is identified as 'S&P Domestic Long Term Issuer Credit Rating' (SPLTICRM) in the database.[18]

Figure 4 shows average credit spreads across credit ratings and models.[19] There has been an important rise in average credit spreads during the financial turmoil that affects mainly the short and mid terms.

[17]Normally, we would need to take a weighted average of the different prices, given the regime at t; the weights correspond to $\mathbb{Q}(s_t = s \,|\, \mathbf{y}_{1:t})$, the risk-neutral probability of being in regime s at time t conditional on $\mathbf{y}_{1:t}$. Though, these probabilities cannot be computed readily, and this is why we use the most probable regime at time t as a proxy. We could also use the probability under the physical measure as a proxy; however, this shall lead to the same estimate since the probability of being in a regime is most of the time in the neighbourhood of 1 or 0.

[18]The firm's ticker symbol is matched to the data in Compustat. For two firms, no rating is available, leaving a sample of 208 firms whenever results are presented by credit rating.

[19]Curves obtained from AAA- and AA-rated firms are not shown in the figure since the number of firms with such rating is too small to be representative.

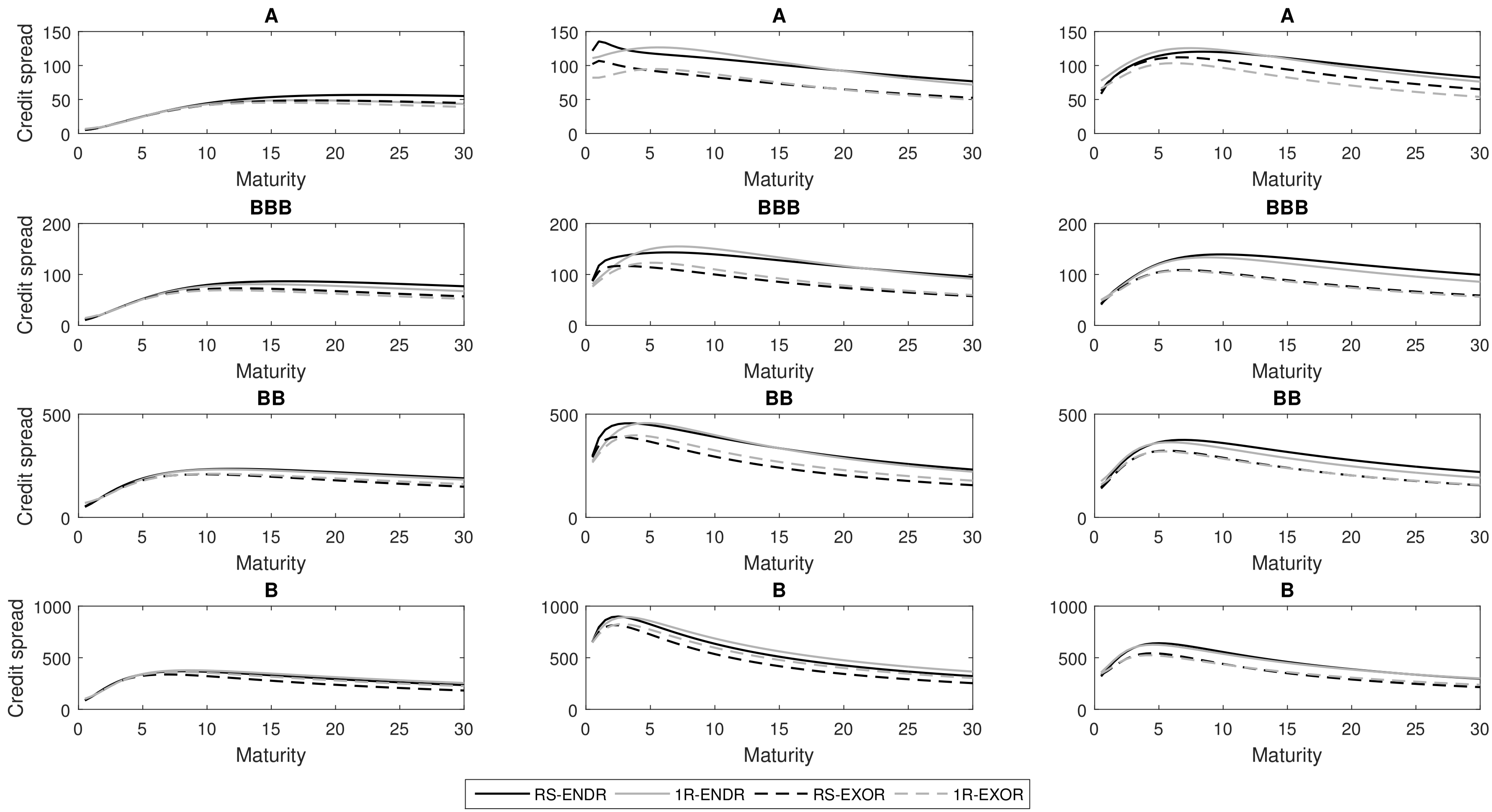

Figure 4: Credit spreads in basis points as a function of maturities, credit ratings and eras.
For each of the 208 firms, the parameters of the model have been estimated using weekly CDS and the DEA-UKF technique. The data is available for January 2005 to December 2012. The term structures are evaluated each week. Then, we average the spreads across the time periods (i.e. pre-crisis, crisis, post-crisis era). Each graph shows average term structures for four different models. Two versions of our regime-switching model are used: RS-ENDR and RS-EXOR. Moreover, two versions of the one-regime equivalent are used: 1R-ENDR and 1R-EXOR.

According to any of the tested models, the two-year average credit spread of A-rated firms is about 10 times larger during the crisis period than before. For riskier firms rated BBB, BB and B, the average credit spread is about 4 times larger. When considering the five-year credit spread, it is 4 times larger for the A-rated firms and about 2.5 times larger for the credit ratings BBB, BB and B. There is a reduction of the credit spread in the post-crisis period, yet it never reaches the pre-crisis levels.

Recovery uncertainty and its negative relationship with default probability have a major impact on mid- and long-term credit spreads. Indeed, in a regime-switching environment, the five-year credit spread increases by 11% (from 23 to 34%) with the endogenous recovery assumption when compared to an equivalent model with a constant recovery rate during the crisis. On a longer horizon, the effect is even more significant, ranging between 12% and 69% depending on the credit rating or the period considered. The dependence between default probabilities and recovery rates have been documented empirically (i.e. lower-rated firms have lower recovery rates), but most modelling approaches neglect the accrued risk associated with this negative relationship. Thus, assuming constant recovery rates seriously impacts credit spread curves, especially over the long run.

The presence of regimes has a second-order effect on average credit spread curves and is mainly attributed to how these averages are constructed. The parameters of the 'one-regime' model capture the behaviour during good and bad times. Unlike the previous approach, the two-regime model allows for a distinct set of parameters for each state. However, the curves presented in Figure 4 are constructed by averaging all weekly curves on a given period and correspond to a mix of various firms and regime weights. Even then, the presence of regimes affects the long-term credit spread of highly rated firms in the pre- and post-crisis periods. During the financial turmoil, the presence of regimes modifies the short-term shape of the average credit spread curves, especially for highly rated firms.

6 Bond-CDS basis

Many investors, such as insurers and pension funds, were considerably affected by the last crisis as their liabilities mainly comprise long-term commitments funded by long-term fixed income instruments like corporate bonds. According to a NAIC[20] special report on the insurance industry's use of derivatives, credit risk is hedged primarily with credit default swaps (NAIC, 2015). Generally, one would buy some single-name CDSs that hedge against default of a reference entity.[21] However, this scheme is subject to basis risk as distinct risk constituents could impact bond and CDS spreads.

The bond-CDS basis measures the extent to which these two spreads differ from one another. Broadly speaking, the basis is computed as the difference between measures of the CDS spread and the bond spread, both with the same maturity dates. Therefore, a deeper understanding of the bond-CDS basis starts with a better understanding of credit risk and how the latter is involved in the pricing of both bonds and CDSs. As noted in Section 4, the proposed model outperforms classic benchmarks and yields small errors on CDS premiums. In this section, we use the proposed model (along with the estimated parameters) to adequately capture CDS credit risk and compute accurate estimates of credit spreads.

In the literature, most researchers focused on market-wide and firm-specific factors. Indeed, Fontana (2010) finds that the basis is time-varying and negatively correlated with the LIBOR–OIS[22] spread and OIS–Treasury bill spread, which are proxies for the increased funding costs and flight-to-liquidity, respectively. Bühler and Trapp (2009a) explore the impact of bond-specific liquidity measures by using an indirect proxy. As a matter of fact, they use the bond yield volatility of a specific portfolio as a proxy for the portfolio's liquidity.

Yet, direct bond-specific drivers have not been investigated much. Bai and Collin-Dufresne (2011) use the bond yield bid-ask spread, bond liquidity beta and bond liquidity market beta as measures of the bond-specific liquidity. They find that the negative basis is mainly explained by bond trading liquidity risk during the crisis. In this spirit, we propose to evaluate the importance of bond-specific liquidity in the basis. However, opposed to Bai and Collin-Dufresne (2011), we use different bond-specific liquidity and bond-CDS basis measures.

[20] National Association of Insurance Commissioners.

[21] Fabozzi and Mann (2012) state that credit derivatives are mainly used to transfer and hedge credit risk.

[22] Overnight indexed swap.

6.1 Measuring the basis

As stated in Elizalde et al. (2009), the three most common bond spreads to compute the bond-CDS basis are the Z-spread, the par asset swap spread (ASW) and the par equivalent CDS spread (PECS). The first two have features that make them difficult to compare properly to CDS premiums. As a matter of fact, they do not take into account expected recovery rates and the term structure of the probabilities of default, while PECS does.

These three measures take the CDS premiums as-is and subtract a computed bond spread, given some assumptions. However, to benefit from our credit risk model, we need to compute the basis the other way around. Actually, using the model introduced above, bond-specific theoretical YTM spreads can readily be obtained.[23] Since the model is estimated exclusively on CDS premiums, this would give us a bond-equivalent of the risk embedded in the firm's CDSs, while using the exact bond maturity date. To obtain a measure of the bond-CDS basis, we subtract the bond observed YTM spread from the model's bond-equivalent YTM spread:

$$\text{Bond-CDS basis} = \underbrace{\text{Bond-equivalent YTM spread}}_{\text{From the model}} - \underbrace{\text{Bond YTM spread}}_{\text{From the market}} \tag{11}$$

This measure of the bond-CDS spread obviously accounts for recovery rates and the term structure of the default probabilities as does our credit risk model.

6.2 Data

We obtain bond information from Bloomberg and from Mergent Fixed Income Securities Database (FISD). The same 225 firms are considered. In order to find the right firms that match our CDS premiums, the tickers are manually matched. The selected bonds are senior, non-callable, non-putable bullet bonds with fixed coupon rates. Then, the trading data are acquired by the Trade Reporting And Compliance Engine (TRACE). We keep issues with at least 100 trades during the period considered (i.e. from 2005 to 2012).

Dick-Nielsen (2009)'s algorithm is used to filter out the errors in TRACE. Omitting this step might result in high liquidity biases: if TRACE data are not cleaned up before use, the number of transactions will be too high.[24] Our final bond subsample contains 1,046 issues from 97 firms (66 IG and 31 HY), for a total of 2,264,566 observations.

To obtain the spreads (in percent) from our bond sample, LIBOR and swap rates are used. We use one-, two-, three- and six-month LIBOR as well as one-, two-, three-, four-, five-, seven-, ten- and 30-year swap rates. We linearly interpolate the different rates to obtain the corresponding rate.[25] These rates are provided by the Federal Reserve of St. Louis website via FRED. Dick-Nielsen et al. (2012) also use swap rates to proxy risk-free rates.

Bond YTMs are computed on clean prices from which the corresponding maturity-matched risk-free rate is removed. Then, the daily spreads are averaged over each month. Note that we winsorize the 0.5% higher and lower values of observed YTM spreads.

Bond-equivalent YTM spreads are obtained using the regime-switching hybrid credit risk model. One spread is computed for every week (since the time step of our estimation method was $\Delta = 1/52$) and the monthly theoretical spread is the average of the weekly ones. Like CDS premiums, bond prices are numerically computed using the trinomial lattice (see Subsection 3.2).

The leftmost columns of Table 3 show the descriptive statistics for the bond-CDS basis as computed by Equation (11). The average basis is negative for both IG and HY bonds. The standard deviation is greater for HY bonds than for IG. Top panels of Figure 5 exhibits the average monthly bond-CDS basis for both IG and HY bonds. The basis is negative for IG firms throughout the sample; there is a decrease in the

[23] The difference between a yield-to-maturity of the corporate bond and the linearly interpolated maturity-matched risk-free rate calculated on the same day.

[24] The filter is divided into three steps. First, true duplicates are deleted (i.e. intra-day trades with the same unique message sequence number). Then, reversals (a trade cancellation for a trade report that was originally submitted to TRACE on a previous date) are also deleted. Finally, same-day corrections are deleted. These are identified using the report's trade status.

[25] The rate that matches the product's maturity.

basis during the 2005–2012 period, although this decline is somewhat small. For HY bonds, the basis is also negative for most months. There is a strong decrease in the basis during the crisis, on average. At the end of the crisis, the bond-CDS basis tends to increase to reach pre-crisis levels in 2011.

Table 3: Descriptive statistics on the bond-CDS basis and the bond-specific liquidity measure λ.

	Basis			$\boldsymbol{\lambda}$		
	All	IG	HY	All	IG	HY
Mean	-0.7483	-0.6689	-1.1839	-0.0178	-0.0162	-0.0261
SD	1.6004	1.2754	2.7325	3.0399	3.0138	3.1736
5%	-2.8854	-2.4519	-6.1766	-3.1579	-3.1709	-3.0787
10%	-1.9839	-1.8242	-3.6912	-2.7747	-2.7887	-2.7214
25%	-1.1719	-1.1283	-1.6904	-1.9154	-1.9080	-1.9506
50%	-0.6284	-0.6265	-0.6419	-0.7667	-0.7449	-0.8687
75%	-0.1686	-0.1807	-0.1103	0.9018	0.9198	0.7994
90%	0.4202	0.3993	0.5945	3.4023	3.4081	3.3875
95%	1.3282	1.2464	1.7583	5.7441	5.6507	6.1544

[1] This table shows statistics for the bond-CDS basis (in percent) and the bond-specific liquidity measure λ. The two quantities are calculated monthly for each bond from January 2005 to December 2012.
[2] The mean, standard deviation (SD) and quantiles are computed across firms.
[3] IG refers to Investment Grade and HY means High Yield.

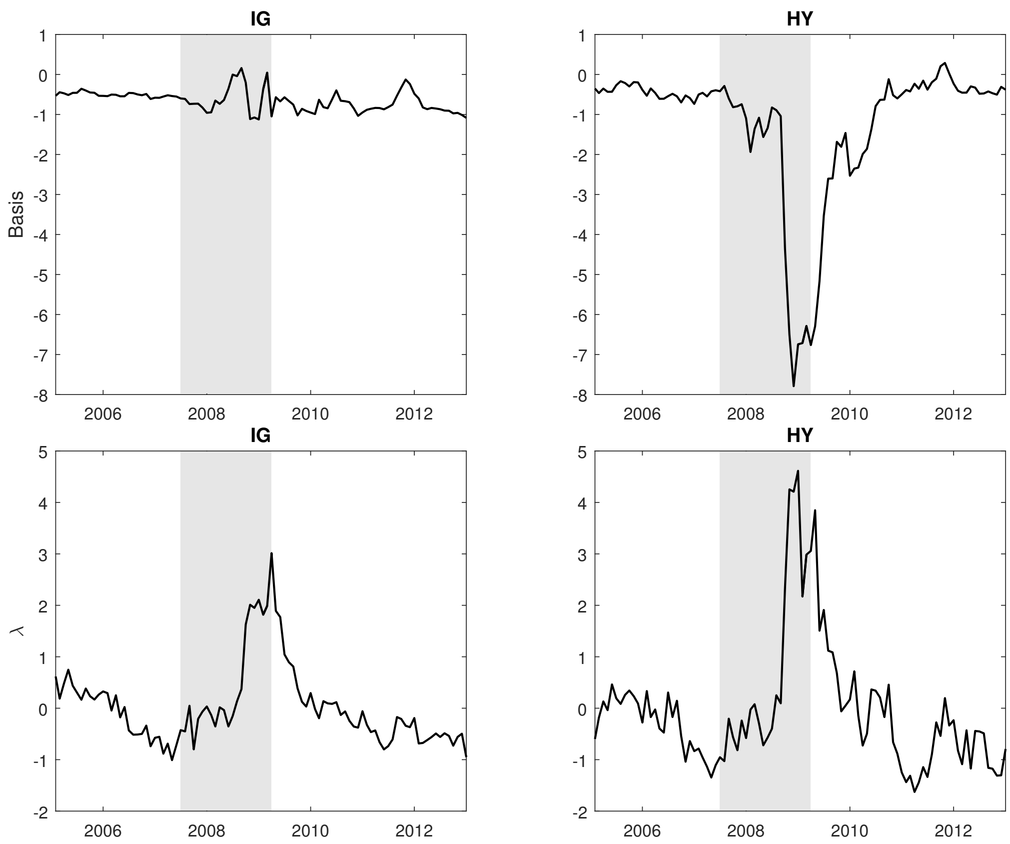

Figure 5: Time series of the average bond-CDS basis (in percent) and the average composite λ measure for IG and HY. For each month, the average over each variable is computed across our dataset of 1,046 bond issues (paying no attention to the maturity of the bonds). To proxy the bond-specific liquidity, we use Dick-Nielsen et al. (2012)'s λ measure in the study. The latter is a construction made of four different liquidity proxies: the Amihud (2002) measure, the Amihud risk, the imputed round-trip cost (IRC) and the IRC risk. The liquidity proxy is computed on a monthly basis from January 2005 to December 2012.

6.3 Bond-specific liquidity as driver of the basis

Liquidity is usually vaguely defined as the degree to which an asset or security can be bought or sold in the market quickly without affecting the asset's price (Tang and Yan, 2007). However, it has multiple facets and cannot be defined efficiently by a single statistic. There are three oft-cited dimensions of the liquidity risk. The first one is tightness: if the bid-ask spread is small, it is assumed that the market is liquid. The second dimension is depth: it is related to the amount of security that can be traded without affecting the price. Finally, the third dimension is called resiliency: a market is liquid in this specific dimension if price recovers quickly after a demand or supply shock.

Many recent papers focus on liquidity issues in the bond market. Using a reduced-form approach, Longstaff et al. (2005) use credit default swap premiums to measure the size of the default component in corporate spreads. The authors find that the majority of the spread is due to default risk and that the non-default component is time varying and strongly related to measures of bond-specific illiquidity. Dick-Nielsen et al. (2012) analyze corporate bond spreads during 2005–2009 using a new robust illiquidity measure based on four different proxies. They show that the spread contribution from illiquidity increases dramatically with the onset of the subprime crisis. This effect is slow and persistent for investment-grade bonds while it is stronger but shorter-lived for speculative-grade bonds.

To proxy the bond-specific liquidity, we use Dick-Nielsen et al. (2012)'s λ measure in the study. The latter is a construction made of four different liquidity proxies: the Amihud (2002) measure, the Amihud risk, the imputed round-trip cost (IRC) and the IRC risk. To be precise, each proxy is normalized and then summed to create the new λ measure. Note that the liquidity proxy is computed on a monthly basis.

To be certain that this measure is robust, seven different liquidity proxies used in Dick-Nielsen et al. (2012) are computed and a principal component analysis is applied on these variables.[26] The first component explains 77% of the variation and is positively correlated with Dick-Nielsen et al. (2012)'s λ measure (i.e. 55%). Therefore, it seems fair to use this variable as a liquidity proxy.

The rightmost columns of Table 3 show the descriptive statistics of the liquidity proxy λ. The composite λ measure has a zero mean by construction; however, its distribution seems highly asymmetrical. The 5th percentile is around -3.18 and the 95th percentile is at 5.74. Note that the same asymmetric behaviour is true for the four constituents of the λ measure. Bottom panels of Figure 5 shows the average value of the liquidity proxy through time for both IG and HY bonds. For both risk classes, the average λ increases during the crisis, although the increase for HY is much more severe. The increase for HY bonds is consistent with the decrease of the basis on average: as the average basis decreases, the bond-specific liquidity measure increases. For IG, the relationship is less clear when we aggregated issue-specific bases and proxies.

To assess the importance of bond-specific liquidity, a dummy variable regression using monthly observations is run. In the latter, we regress the liquidity proxy on the bond-CDS basis by issue:

$$\begin{aligned}(\text{Bond-CDS basis})_{it} = &\gamma_0^{(i)}\mathbb{I}_{\{\text{Pre-crisis}\}}(t) + \gamma_1^{(i)}\mathbb{I}_{\{\text{Pre-crisis}\}}(t)\lambda_{it} + \gamma_2^{(i)}\mathbb{I}_{\{\text{Crisis}\}}(t) + \gamma_3^{(i)}\mathbb{I}_{\{\text{Crisis}\}}(t)\lambda_{it} \\ &+ \gamma_4^{(i)}\mathbb{I}_{\{\text{Post-crisis}\}}(t) + \gamma_5^{(i)}\mathbb{I}_{\{\text{Post-crisis}\}}(t)\lambda_{it} + \epsilon_{it},\end{aligned} \tag{12}$$

where $\gamma_j^{(i)}$ are the issue-specific regressors. We only consider issues for which there are at least three data points in each era.

We report the average and the standard deviation of the regression estimates as well as the average R-squared in Table 4. For the three periods, the average coefficients are negative and statistically significant at a confidence level of 95%. The liquidity proxy average coefficient is –0.0080 in the pre-crisis era, when liquidity was less important in general. Then, in the crisis era, the average liquidity proxy coefficient jumped to –0.2369, which is 30 times higher than the pre-crisis average coefficient. In the post-crisis period, the coefficient increases, but does not reach pre-crisis levels meaning that liquidity is an important determinant of the bond-CDS spread. The R-squared values of these regressions are around 39% on average. Therefore, the unique liquidity factor accounts for a little less than half of the variations, on average.

[26] The Amihud (2002) measure, the Amihud risk, the imputed round-trip cost (IRC), the IRC risk, the Roll (1984) measure, the turnover rate of a bond and the proportion of zero trading days. For more details on these variables, see Dick-Nielsen et al. (2012).

Table 4: Bond-CDS basis regression results.

	Pre-crisis		Crisis		Post-crisis	
	Intercept	Liquidity	Intercept	Liquidity	Intercept	Liquidity
Average coefficient	**-0.6228**	**-0.0080**	**-0.9389**	**-0.2369**	**-1.3437**	**-0.0788**
SD	0.0281	0.0040	0.0972	0.1087	0.1621	0.0281
Average R^2	0.3942					
Number of regressions	268					

[1] Using our dataset of bond issues, the regressions

$$\begin{aligned}(\text{Bond-CDS basis})_{it} = {} & \gamma_0^{(i)}\mathbb{I}_{\{\text{Pre-crisis}\}}(t) + \gamma_1^{(i)}\mathbb{I}_{\{\text{Pre-crisis}\}}(t)\lambda_{it} + \gamma_2^{(i)}\mathbb{I}_{\{\text{Crisis}\}}(t) + \gamma_3^{(i)}\mathbb{I}_{\{\text{Crisis}\}}(t)\lambda_{it} \\ & + \gamma_4^{(i)}\mathbb{I}_{\{\text{Post-crisis}\}}(t) + \gamma_5^{(i)}\mathbb{I}_{\{\text{Post-crisis}\}}(t)\lambda_{it} + \epsilon_{it},\end{aligned}$$

are estimated for each bond issue. The conclusions of the statistical test $H_0 : \overline{\gamma}_j^{(i)} = 0$ against $H_1 : \overline{\gamma}_j^{(i)} \neq 0$, $j = 0, 1, ..., 5$, are reported. Estimates in bold are significant at a confidence level of 95%.
[2] On average, 60 data points are used in each regression (on a maximum of 84 months).
[3] The averages and SD are based on 268 regressions. These regressions needed at least 3 data points in each era to be considered.

A histogram of the regressions' R-squared values is given in Figure 6. About 60% of the regressions have an R-squared of more than 30% while using only the liquidity factor as a regressor (along with some dummies). Indeed, this is consistent with the idea that liquidity is a major determinant of the bond-CDS basis.

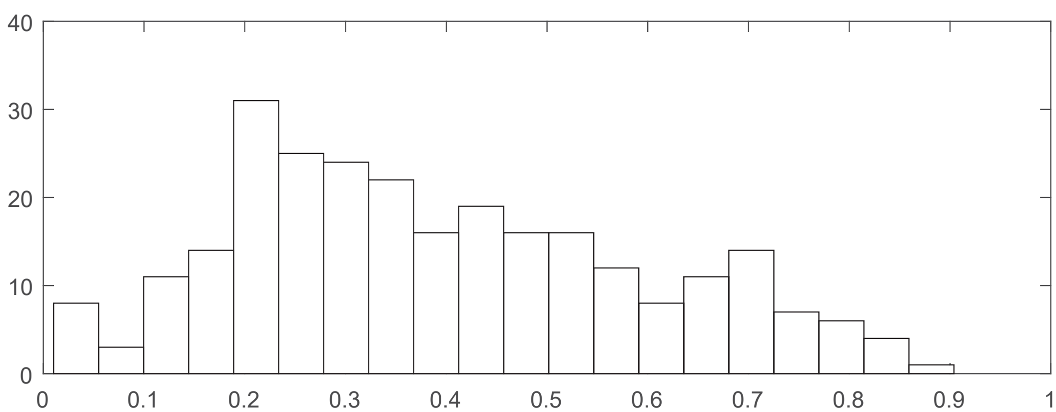

Figure 6: Histogram of the regressions' R-squared values.
For each issue, we run the regression of Equation (12). The R-squared are then computed from these regressions. The basis and liquidity proxy are computed on a monthly basis from January 2005 to December 2012.

To conclude this section, our regression-based tests are rather clear: bond-specific liquidity is an important driver of the bond-CDS basis and bond hedgers should account for this additional risk when hedging their positions. Every coefficient is statistically and economically significant, meaning that bond-specific liquidity was (and probably still is today) an important determinant of the basis. This result is consistent with Bai and Collin-Dufresne (2011), as their liquidity proxies are highly significant during the crisis.

7 Concluding remarks

Regime-switching dynamics are able to capture behaviour changes similar to those associated with a crisis. However, firm-specific MLE in the presence of regimes and noisy prices is problematic as both regimes and noises are not directly observed.

This paper contributes to the present credit risk literature by proposing a Markov-switching model and a filter-based estimation technique. A flexible credit risk model is designed to capture empirical evidences observed during the last decade. A regime-switching variable is included to accommodate behaviour changes during the financial crisis. A negative dependence between the endogenous recovery rates and the firm's default probability addresses Altman et al. (2005) empirical findings.

A firm-by-firm estimation procedure based on a filtering method and the principle of maximum likelihood deals with latent variables and noise. Tugnait (1982) is extended to allow for nonlinearities in the state-space representation and a fast lattice-based pricing scheme is implemented to ensure the feasibility of the estimation step.

The in-sample performance of the model reveals that it is flexible enough to adjust to various firms and financial cycles. An out-of-sample study concludes that the model is reliable and outperforms other benchmarks.

The negative dependence between the default probabilities and the recovery rates greatly impacts mid- and long-term credit spreads. Indeed, in a regime-switching environment, the five-year credit spread increases with the endogenous recovery assumption when compared to a model with a constant recovery rate. Also, during the financial turmoil, the short-term shape of the average credit spread curves is changed by the presence of regimes.

Based on model-implied yield-to-maturity spreads and regression tests, bond-specific liquidity appears to be a significant driver of the bond-CDS basis. For instance, during the crisis era, the average liquidity proxy coefficient reached –0.2369, which is 30 times higher than the pre-crisis average coefficient. This conclusion is consistent with our prior belief that bond hedgers must account for additional basis risk when hedging their positions in fixed income markets. The exact way bond-specific liquidity would change the hedging strategy is a question by itself and is left for future research.

Appendices

Appendix A Derivative pricing

A.1 Trinomial lattice approach

Yuen and Yang (2010) propose a trinomial lattice approach for Markov-switching dynamics. A 'up-across-down' branching structure is chosen with $x_u = xe^{\sigma\sqrt{\Delta}}$, $x_m = x$, and $x_d = xe^{-\sigma\sqrt{\Delta}}$ where x is the actual value of the log-leverage process at a typical node. Moreover, when the number of regimes is K, the value of σ is given by $\sigma = \max_{1\leq i\leq K}\sigma_i + (\sqrt{1.5}-1)\overline{\sigma}$ where $\overline{\sigma}$ is the arithmetic mean of σ_i. This suggestion is based on the values used in the binomial and trinomial trees in the literature.

The weights for the ith regime are

$$\pi_m^i = 1-\frac{1}{\lambda_i^2}, \quad \pi_u^i = \frac{e^{\mu_i^{\mathbb{Q}}\Delta} - e^{-\sigma\sqrt{\Delta}} - (1-1/\lambda_i^2)(1-e^{-\sigma\sqrt{\Delta}})}{e^{\sigma\sqrt{\Delta}} - e^{-\sigma\sqrt{\Delta}}}, \quad \pi_d^i = \frac{e^{\sigma\sqrt{\Delta}} - e^{\mu_i^{\mathbb{Q}}\Delta} - (1-1/\lambda_i^2)(e^{\sigma\sqrt{\Delta}}-1)}{e^{\sigma\sqrt{\Delta}} - e^{-\sigma\sqrt{\Delta}}},$$

where $\lambda_i = \frac{\sigma}{\sigma_i}$. These weights are different for each regime. The Schönbucher (2002) lattice that deals with credit-sensitive instruments is adapted in the trinomial lattice approach for Markov-switching dynamics by adding an additional branch at each node. According to Equations (2) and (3), the default probability is

$$p = 1 - \exp\left(-\left(\beta + \left(\frac{e^x}{\theta}\right)^{\alpha}\right)\Delta\right),$$

if the log-leverage value at this typical node is x.

A.2 Coupon bond prices

To price coupon bonds on a single firm, recovery specifications are crucial. Again, the endogenous recovery of Subsection 2 is utilized. Bond investors will recover a fraction R_τ of an equivalent Treasury bond at default time τ. Given a coupon rate of c, a maturity T, an initial log-leverage of x_t, an initial regime of s_t, and a face value of 1, the coupon bond price is given by

$$\begin{aligned}
V(t,T,x_t,s_t;R_\tau,c) &= \mathbb{E}^{\mathbb{Q}}\left[\frac{c}{2}\sum_{t_i^*} P(t,t_i^*)\mathbb{I}_{\{t\leq t_i^*<\tau\}} + P(t,T)\mathbb{I}_{\{\tau>T\}} + P(t,\tau)R_\tau P(\tau,T)\mathbb{I}_{\{t<\tau\leq T\}}\middle|\mathcal{F}_t\right]\\
&= \mathbb{E}^{\mathbb{Q}}\left[\frac{c}{2}\sum_{t_i^*} P(t,t_i^*)\exp\left(-\sum_{t\leq u<t_i^*} H_u\Delta\right)\middle|\mathcal{G}_t\right]\mathbb{I}_{\{\tau>t\}}\\
&+ \mathbb{E}^{\mathbb{Q}}\left[P(t,T)\exp\left(-\sum_{t\leq u<T} H_u\Delta\right)\middle|\mathcal{G}_t\right]\mathbb{I}_{\{\tau>t\}}\\
&+ \mathbb{E}^{\mathbb{Q}}\left[\sum_{t\leq k<T} R_k P(t,T)\exp\left(-\sum_{t\leq u<k} H_u\Delta\right)\ (1-\exp(-H_k\Delta))\middle|\mathcal{G}_t\right]\mathbb{I}_{\{\tau>t\}}
\end{aligned} \tag{A1}$$

where $P(t,T)$ is the time t value of a risk-free zero-coupon bond maturing at T and t_i^* are coupon payment dates. Note that a zero-coupon bond can be computed by replacing the coupon rate c by zero in Equation (A1).

A.3 Credit default swap premiums

A CDS is a credit derivative that compensates the buyer in the event of a default (or other credit events). In the most basic type of CDS, the protection seller provides a payment of par-minus-recovery (settled in cash) on default. This shall cover the loss incurred by a typical bondholder. In exchange, the protection buyer pays a periodic premium that ceases if a default occurs. Normally, these premiums are paid four times per year. The premium of such a contract is fixed by setting the expected present value of losses equals to the expected present value of the premiums.

The endogenous recovery rate defined in Section 2 introduces a negative relation between recovery and default risks. Given that the CDS matures at time T and that $P(t,T)$ is the time t value of a risk-free zero-coupon bond maturing at T, the protection leg (expected present value of the losses) is

$$\mathbb{E}^{\mathbb{Q}}\left[P(t,\tau)(1-R_\tau)\mathbb{I}_{\{t<\tau\leq T\}} \mid \mathcal{F}_t\right] = \mathbb{E}^{\mathbb{Q}}\left[\sum_{t\leq k<T}(1-R_k)P(t,k)\exp\left(-\sum_{t\leq u<k}H_u\Delta\right)(1-\exp(-H_k\Delta))\middle|\mathcal{G}_t\right]\mathbb{I}_{\{\tau>t\}}, \quad \text{(A2)}$$

where the filtration $\{\mathcal{F}_t\}_{t=1}^{\infty}$ is defined as $\mathcal{F}_t=\sigma(\mathcal{G}_t,\mathcal{H}_t)$ and $\mathcal{H}_t=\sigma(\mathbb{I}_{\{\tau\leq s\}}: s\leq t)$.

To simplify the presentation, assume that a premium of 1 is paid. In this case, the premium leg (expected present value of the premiums) is given by

$$\mathbb{E}^{\mathbb{Q}}\left[\sum_{t_i^*}P(t,t_i^*)\mathbb{I}_{\{t\leq t_i^*<\tau\}}\middle|\mathcal{F}_t\right] = \mathbb{E}^{\mathbb{Q}}\left[\sum_{t_i^*}\delta_i^* P(t,t_i^*)\exp\left(-\sum_{t\leq u<t_i^*}H_u\Delta\right)\middle|\mathcal{G}_t\right]\mathbb{I}_{\{\tau>t\}} \quad \text{(A3)}$$

where t_i^* are premium payment dates and $\delta_i^*=t_i^*-t_{i-1}^*$. The periodic premium is the ratio of (A2) over (A3).

The numerical scheme explained in Appendix A.1 shall come in handy to compute credit default swap premiums in the proposed framework. To price credit default swaps, a time step of 1 week (i.e. $\Delta=1/52$) is used in the tree.

Appendix B Simulation study for the DEA-UKF method

The efficiency of the method is assessed by two simulation studies.[27] The first Monte Carlo experiment compares the filtered leverages and regimes to their true values. To this end, 500 paths of 500 leverage observations are simulated. Then, we compute the logarithm of the CDS premiums for six tenors and add Gaussian random noises as stated in Equation (8).[28] To each path, we apply the DEA-UKF and compare the extracted filtered states to their true values, which are known because the data is simulated.

Panel A of Table B1 exhibits the summary statistics for the true and filtered states. Relative to the true states, the DEA-UKF filtered values of leverages are close to the true state values. Even for higher moments behaviour, the filtered states are virtually identical to the true ones. The root mean square error (RMSE) on filtered leverages (with respect to their true values) is 0.0034. With respect to the most probable regime, the DEA-UKF is capturing the right one 98.3% of the time.

In a second test, we investigate potential parameter estimation issues. The parameters are estimated with the DEA-UKF leading to a sample of 500 estimates.

Panel B of Table B1 reports true values, averages and standard deviations of parameter estimates for the DEA-UKF estimation method. The filter yields virtually no differences between estimated and true parameter values on average. The difference for each parameter is also inside the 99% margin of error constructed from the sample standard deviation for most parameters, except for $\mu^{\mathbb{Q}}$.

Table B1 also exhibits the coverage rate (CR) of the 95% confidence interval for each parameter and their t-statistics. The CR corresponds to the proportion of 95% confidence intervals containing the true parameter value. Confidence intervals for the DEA-UKF method are in line with theory (i.e. close to 95%). Also, most CR are not statistically different of 0.95 at a confidence level of 95%.

[27] In this paper, M is set to 5. This choice appears to be a good compromise between accuracy and efficiency. Moreover, the results seem to be robust to different choices of M greater than 4.

[28] As in our dataset, we use 1-, 2-, 3-, 5-, 7- and 10-year CDS premiums.

Finally, a multivariate confidence region is constructed by inverting the likelihood ratio statistic. Let $U(\mathbf{y}) = \{\hat{\boldsymbol{\phi}} : 2L(\hat{\boldsymbol{\phi}};\mathbf{y}) - 2L(\boldsymbol{\phi}_0;\mathbf{y}) < F_{\text{crit}}\}$, where L is the log-likelihood function, $\hat{\boldsymbol{\phi}}$ is an estimator, $\mathbf{y}$ denotes the data and F_{crit} is the critical value given by a chi-square distribution with 18 degrees of freedom (i.e. for a 95% confidence region, $F_{\text{crit}} = 28.86$). Notice that for the MLE, U should contain the true parameter vector 95% of the time asymptotically, under usual regularity conditions. According to our tests, the coverage rate is 94.4% for the filter. Therefore, the results obtained using the DEA-UKF are consistent with the theory.

Table B1: Simulation studies on the DEA-UKF methodology.

Panel A: Summary statistics for the true and filtered states.

	True	DEA-UKF
Filtered leverage		
Mean	0.6560	0.6561
SD	0.2092	0.2091
Skewness	0.5443	0.5448
Kurtosis	3.3402	3.3430

Panel B: Comparison of true and estimated parameters.

	True	Mean	SD	t-stat	CR	t-stat (CR)
$\mu^{\mathbb{P}}$	0.0100	0.0098	0.0025	-2.0340	0.9700	2.0520
$\mu^{\mathbb{Q}}$	0.0100	0.0101	0.0004	3.1893	0.9480	-0.2052
σ_1	0.0800	0.0802	0.0048	0.9666	0.9440	-0.6156
$p_{1,2}$	0.0500	0.0493	0.0131	-1.1558	0.9520	0.2052
$q_{1,2}$	0.0500	0.0499	0.0021	-1.1614	0.9340	-1.6416
σ_2	0.3000	0.3000	0.0033	-0.1028	0.9420	-0.8208
$p_{2,1}$	0.0500	0.0495	0.0140	-0.7612	0.9580	0.8208
$q_{1,2}$	0.0500	0.0497	0.0033	-2.1076	0.9160	-3.4883
α	10.0000	9.9955	0.3133	-0.3184	0.9520	0.2052
θ	1.2000	1.2010	0.0090	2.4801	0.9400	-1.0260
β	0.0100	0.0100	0.0010	-0.6704	0.9540	0.4104
κ	0.5000	0.5000	0.0002	0.9253	0.9660	1.6416
$\delta^{(1)}$	0.1500	0.1496	0.0052	-1.9237	0.9480	-0.2052
$\delta^{(2)}$	0.1000	0.0998	0.0035	-1.0133	0.9340	-1.6416
$\delta^{(3)}$	0.0500	0.0501	0.0022	0.6300	0.9440	-0.6156
$\delta^{(5)}$	0.0300	0.0299	0.0017	-1.7315	0.9500	0.0000
$\delta^{(7)}$	0.0500	0.0498	0.0018	-2.0463	0.9540	0.4104
$\delta^{(10)}$	0.1000	0.0999	0.0031	-0.3991	0.9480	-0.2052

[1] For each path, leverage paths were generated using Equation (1). The experiment consisted of 500 paths of 500 weekly observations.

[2] Mean corresponds to the mean estimated parameter. SD stands for standard deviation of the parameter sample. The column labeled t-stat represents the t-statistics of the following hypothesis testing: $H_0 : \overline{\phi} - \phi = 0$, $H_1 : \overline{\phi} - \phi \neq 0$. The columns labeled CR exhibit the coverage ratio of the 95% confidence interval of each individual parameters. The last column labeled t-stat (CR) shows the t-statistics of the following hypothesis testing: $H_0 : \text{CR} = 0.95$, $H_1 : \text{CR} \neq 0.95$.

Internet appendix

Appendix C Estimation (Additional results)

C.1 Results by ratings

Table C2 shows the average parameter values across credit ratings. In agreement with what has already been stated, both drift parameters seem to increase with riskiness. The same phenomenon is true for the parameter θ. The parameter β increases as the firms become riskier on average.

Table C2: Average parameter values and standard deviation of noise terms across credit ratings.

Panel A: Mean of leverage dynamics parameters (under $\mathbb{P}$) and recovery rate.							
	$\mu^{\mathbb{P}}$	p_{12}	p_{21}	σ_1	σ_2	κ	Obs.
AAA	-0.0047	0.0079	0.0187	0.1033	0.3707	0.6618	3
AA	-0.0002	0.0266	0.0397	0.0611	0.3317	0.6724	1
A	0.0155	0.0177	0.0252	0.1061	0.3653	0.5855	41
BBB	0.0084	0.0094	0.0170	0.1121	0.3811	0.5840	84
BB	0.0262	0.0115	0.0139	0.1256	0.3464	0.6745	52
B	0.0305	0.0187	0.0230	0.1187	0.2870	0.5848	27
Panel B: Mean of leverage dynamics parameters (under $\mathbb{Q}$) and intensity.							
	$\mu^{\mathbb{Q}}$	q_{12}	q_{21}	α	θ	β	Obs.
AAA	0.0078	0.0060	0.0435	8.6283	1.7265	0.0015	3
AA	-0.0099	0.0111	0.0987	13.5385	1.5033	0.0004	1
A	0.0003	0.0059	0.0553	11.7457	1.4193	0.0066	41
BBB	0.0067	0.0057	0.0513	11.9016	1.3503	0.0067	84
BB	0.0257	0.0053	0.0219	12.0393	1.5175	0.0080	52
B	0.0540	0.0085	0.0136	9.0010	1.5224	0.0203	27
Panel C: Mean of error terms.							
	$\delta^{(1)}$	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(5)}$	$\delta^{(7)}$	$\delta^{(10)}$	Obs.
AAA	0.2248	0.1304	0.0822	0.0438	0.0349	0.0606	3
AA	0.2451	0.1740	0.1273	0.0508	0.0476	0.0882	1
A	0.2349	0.1291	0.0771	0.0338	0.0654	0.0983	41
BBB	0.2346	0.1270	0.0788	0.0327	0.0647	0.0943	84
BB	0.2824	0.1411	0.0872	0.0351	0.0260	0.0554	52
B	0.2589	0.1276	0.0772	0.0346	0.0081	0.0311	27

[1] For each of the 208 firms, the parameters of the model are estimated using weekly CDS premiums of maturities 1-, 2-, 3-, 5-, 7-, and 10-year, using the DEA-UKF filtering technique. The mean is computed across firms for each credit rating.
[2] The δ's represent the standard deviation of the noise terms present in the observation equation of the filter.

C.2 Slopes and regimes

Figure C1 shows the CDS premiums term structure slopes during the pre-crisis period for HY firms. In general, slopes associated with regime 2 are lower than the ones of regime 1. When looking at slopes, blue dots are generally concentrated on the left side of each line, opposed to red dots which are more concentrated on the right side of the figure (always looking one line at a time).

Appendix D Recovery rate versus regime-switching: Term structure comparison (Additional material)

Table D3 shows average credit spreads across credit ratings and models.

Table D3: Average credit spreads (in basis points) across credit ratings and models.

		Pre-crisis					Crisis					Post-crisis				
		2-y	5-y	15-y	30-y	Obs.	2-y	5-y	15-y	30-y	Obs.	2-y	5-y	15-y	30-y	Obs.
AAA	RS-ENDR	5	12	29	33	3	137	139	110	78	3	-	-	-	-	0
	1R-ENDR	6	12	26	23		120	140	108	70		-	-	-	-	
	RS-EXOR	5	12	24	21		102	76	43	25		-	-	-	-	
	1R-EXOR	6	12	26	24		113	115	85	57		-	-	-	-	
AA	RS-ENDR	10	20	39	40	4	208	170	117	81	2	87	121	122	89	3
	1R-ENDR	11	20	34	28		168	177	128	80		86	122	107	71	
	RS-EXOR	10	20	39	40		174	135	95	71		75	86	49	27	
	1R-EXOR	10	20	31	25		139	115	74	46		85	110	90	59	
A	RS-ENDR	10	25	54	55	55	130	118	101	77	48	91	114	111	82	35
	1R-ENDR	10	25	49	43		118	126	105	72		97	121	110	76	
	RS-EXOR	10	25	48	45		103	93	73	52		90	110	94	65	
	1R-EXOR	11	25	45	39		86	95	75	50		83	102	82	54	
BBB	RS-ENDR	23	52	86	77	100	132	142	128	95	88	78	121	132	99	87
	1R-ENDR	23	51	81	67		114	150	133	92		75	120	122	86	
	RS-EXOR	24	51	72	57		116	114	85	57		73	105	89	59	
	1R-EXOR	24	50	67	52		104	123	92	60		70	104	87	56	
BB	RS-ENDR	106	189	232	189	40	443	447	335	232	47	256	364	317	220	48
	1R-ENDR	101	185	227	183		395	456	335	221		267	361	287	192	
	RS-EXOR	104	180	197	149		387	366	241	156		236	321	240	156	
	1R-EXOR	102	177	204	161		366	392	269	178		240	319	239	157	
B	RS-ENDR	201	341	330	235	16	893	823	510	322	29	516	641	459	296	36
	1R-ENDR	190	343	347	255		867	855	562	366		529	625	450	300	
	RS-EXOR	198	325	277	183		815	725	418	254		457	541	350	215	
	1R-EXOR	192	336	316	226		811	772	479	304		460	520	361	238	

[1] For each of the 208 firms, the parameters of the model are estimated using weekly CDS premiums of maturities one-, two-, three-, five-, seven- and ten-year, using the DEA-UKF filtering technique. The data is available for January 2005 to December 2012.
[2] The credit spreads (in basis points) are averaged across the models, the credit ratings and the periods.
[3] In the table, Obs. means the average number of observations (firms) per rating-period, 2-y stands for two-year, 5-y corresponds to five-year and so on.

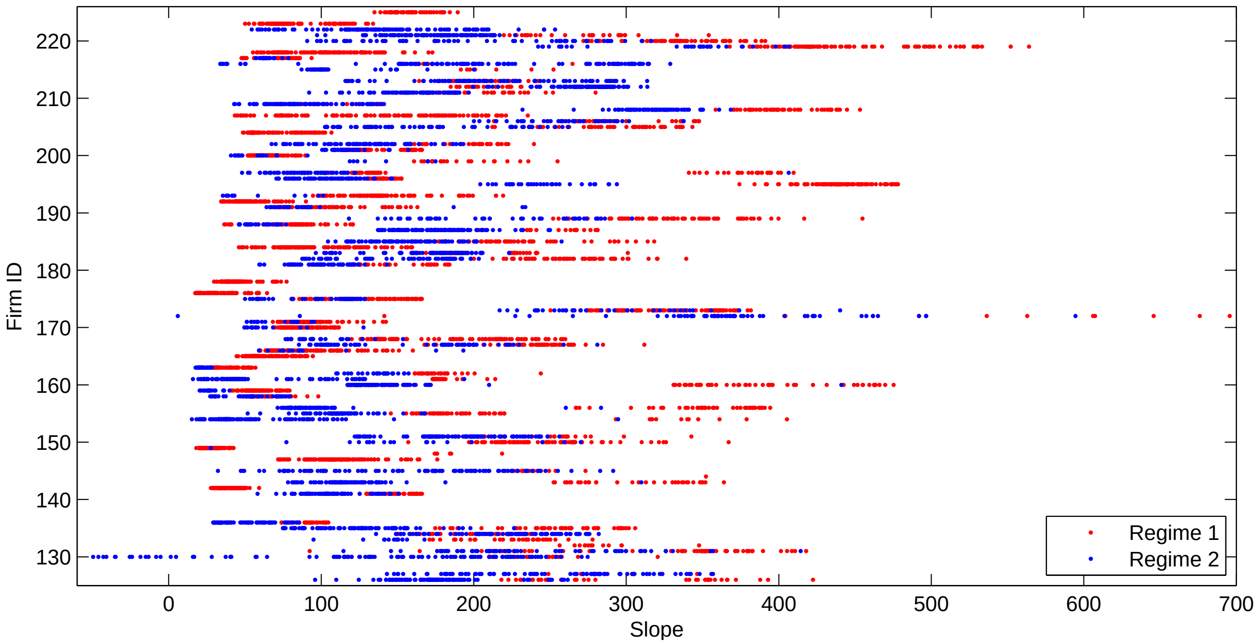

Figure C1: Slopes for HY firms during the pre-crisis period.
For the 100 HY firms during the pre-crisis period, CDS premiums slopes (proxied by the difference between 10-year and 1-year tenors) are plotted. Red dots correpond to slopes in regime 1 and blue in regime 2.

Appendix E Liquidity (Additional material)

In the spirit of Dick-Nielsen et al. (2012), eight different liquidity measures are presented: Amihud (2002) measure, Amihud risk, imputed roundtrip cost (IRC), IRC risk, Roll (1984)'s measure, turnover rate of a bond, proportion of zero trading days, and Dick-Nielsen et al. (2012)'s λ measure. Each measure is computed on a monthly basis; moreover, these measures are calculated for each bond considered in the sub-sample.

Amihud (2002) constructs an illiquidity measure based on a model proposed by Kyle (1985). The measure ascertains the price impact of a trade per unit traded. It is defined as the daily average of absolute returns r_j divided by the trade size Q_j (in million of dollars) of consecutive transactions:

$$\text{Amihud}_t = \frac{1}{N_t}\sum_{j=1}^{N_t}\frac{|r_j|}{Q_j} \tag{E4}$$

where N_t is the number of returns on day t, $r_j = (P_j - P_{j-1})/P_{j-1}$, and P_j is the price j-th trade of the day. The monthly Amihud measure is defined by taking the median of the daily measures.

Amihud risk is simply the standard deviation of the daily Amihud measure over one month.

Feldhütter (2012) proposes an alternative measure of transaction costs based on the so-called Imputed Roundtrip Trades (IRT). Often, a corporate bond is traded two or three times within a very short period of time; this is likely to occur because a dealer matches a buyer and a seller and collects the bid-ask spread as a fee. If two or three trades in a given bond with the same trade size take place on the same day, and there are no other trades with the same size on that day, it is assumed that the transactions are part of an IRT. The imputed roundtrip cost is then defined as

$$\text{IRC} = \frac{P_{\max} - P_{\min}}{P_{\max}} \tag{E5}$$

where $P_{\max}$ is the largest price in the IRT and $P_{\min}$ is the smallest one. A daily estimate of roundtrip costs is the average of roundtrip costs on that day. In addition, the monthly IRC is simply the average of daily roundtrip costs.

IRC risk is the standard deviation of the daily IRC measure over one month.

Roll (1984) suggests that, under certain assumptions, the percentage bid-ask spread equals two times the square root of minus the covariance between two consecutive returns:

$$\mathrm{Roll}_t = 2\sqrt{-\mathrm{cov}(r_i, r_{i-1})} \tag{E6}$$

where t is the time period for which the measure is calculated. If the covariance is negative, this measure is not well-defined; the negative values are therefore discarded. A daily Roll measure is calculated on days with at least one transaction using a rolling window of 28 days. The monthly Roll measure is computed by taking the median within the month. The intuition behind Roll (1984)'s measure is that the bond price bounces back and forth between the bid and the ask prices and higher bid-ask spread shall lead to a higher negative correlation.

The monthly turnover rate of bonds is defined by the ratio of the total trading volume during the month divided by the amount outstanding:

$$\mathrm{Turnover}_t = \frac{\text{Total trading volume}_t}{\text{Amount outstanding}} \tag{E7}$$

where t is the considered month.

The proportion of zero-trading days is the percentage of days during a month where the bond did not trade.

Finally, Dick-Nielsen et al. (2012)'s λ measure is a composite factor that loads evenly on Amihud measure, Amihud risk, IRC measure, and IRC risk. Precisely, each measure L_t^j is normalized (i.e. $\tilde{L}_t^j = (L_t^j - \mu^j)/\sigma^j$) and then added up to obtain

$$\lambda_t = \sum_{j=1}^{4} \tilde{L}_t^j. \tag{E8}$$

This is also done on a monthly basis.

Panel A of Table E4 shows a summary of the distribution of the considered liquidity proxies. The composite λ measure has a zero mean by construction; however, its distribution seems highly asymmetrical. The first percentile is around -3.6 and the 99-th percentile is at 13.0. The same asymmetric behavior is true for the four constituents of the λ measure.

The median Amihud measure is 0.158. This means that a trade of 300,000 dollars implies a median move in the bond price of 4.7%. This value is somewhat larger than the one obtained by Dick-Nielsen et al. (2012): they found roughly 0.13%. However, they consider different firms and a different period; they also remove small trades. Han and Zhou (2008) find that a trade of 300,000 dollars moves the price by 10.2% on average. The median roundtrip cost in percentage of the price is 0.5% according to the IRC measure. The roundtrip costs for the 5% most liquid bonds is 0.03%, which is coherent with the results of Dick-Nielsen et al. (2012). The average number of bond zero-trading days is 71.2%. This is consistent with popular thinking: corporate bond market is an illiquid market. Moreover, Dick-Nielsen et al. (2012) find similar figures for the bond zero-trading days proxy. The average monthly turnover rate is 3.2%, meaning that for the average bond in our sample it takes between 2 and 3 years to turn over once.

In Panel B of Table E4, a correlation matrix for the different liquidity proxies and the theoretical YTM spread is presented. Some of the liquidity proxies do not seem to be highly correlated one another. Nonetheless, some others are mildly correlated. Note that the λ measure is a construction based on the Amihud measure, the Amihud risk, the IRC measure, and the IRC risk, therefore it is natural that the λ measure is moderately correlated with the other four. The theoretical YTM spread is somehow related with some of our measures: for instance, the correlation between the Roll measure and the YTM spread is 30%. The liquidity measure are construction made from bond prices; thus, it is very intuitive to have moderate correlation between the liquidity measures and the yield spread.

Table E4: Descriptive statistics for the liquidity proxies.

Panel A: Summary statistics for liquidity proxies and the theoretical yield spread.

	λ	Amihud	A. risk	IRC	IRC risk	Roll	Turnover	Bond zero	Spread
Mean	0.011	0.483	0.713	0.007	0.005	0.015	0.032	0.712	1.785
SD	3.250	1.756	2.362	0.007	0.006	0.018	0.101	0.283	2.660
1%	-3.560	0.000	0.000	0.000	0.000	0.001	0.000	0.043	0.080
5%	-3.158	0.002	0.000	0.000	0.000	0.003	0.000	0.091	0.134
25%	-1.915	0.043	0.039	0.003	0.001	0.007	0.001	0.545	0.423
50%	-0.767	0.158	0.225	0.005	0.004	0.012	0.012	0.818	1.066
75%	0.902	0.403	0.627	0.009	0.007	0.019	0.035	0.952	1.917
95%	5.744	1.733	2.812	0.018	0.014	0.037	0.111	1.000	6.403
99%	13.010	5.566	7.785	0.031	0.026	0.069	0.299	1.000	12.361

Panel B: Correlation matrix for liquidity proxies and the theoretical yield spread.

	λ	Amihud	A. risk	IRC	IRC risk	Roll	Turnover	Bond zero	Spread
λ	1.000								
Amihud	0.317	1.000							
A. risk	0.539	0.425	1.000						
IRC	0.418	0.094	0.118	1.000					
IRC risk	0.528	0.032	0.196	0.589	1.000				
Roll	0.275	0.204	0.206	0.391	0.357	1.000			
Turnover	-0.022	-0.058	-0.026	0.007	0.008	-0.026	1.000		
Bond zero	0.013	0.169	0.016	-0.024	-0.168	0.027	-0.132	1.000	
Spread	0.161	0.059	0.154	0.291	0.281	0.300	0.057	-0.207	1.000

[1] This table shows statistics for corporate bond liquidity proxies. The proxies are calculated monthly for each bond from January 2005 to December 2012. Panel A shows quantiles for the proxies. Panel B reports correlation among the proxies.

[2] Amihud means Amihud measure, A. risk stands for Amihud risk, IRC corresponds to IRC measure, Roll means Roll measure, Bond zero is for Bond zero-trading days, and Spread stands for Theoretical YTM spread in percentage.

Appendix F Derivative pricing (Additional material)

Figure F2 shows the different branches to be considered to use this numerical scheme when $K = 2$. Note that even if the figure contains two different trees, these trees represent the same lattice: only the weights change across the different regimes.

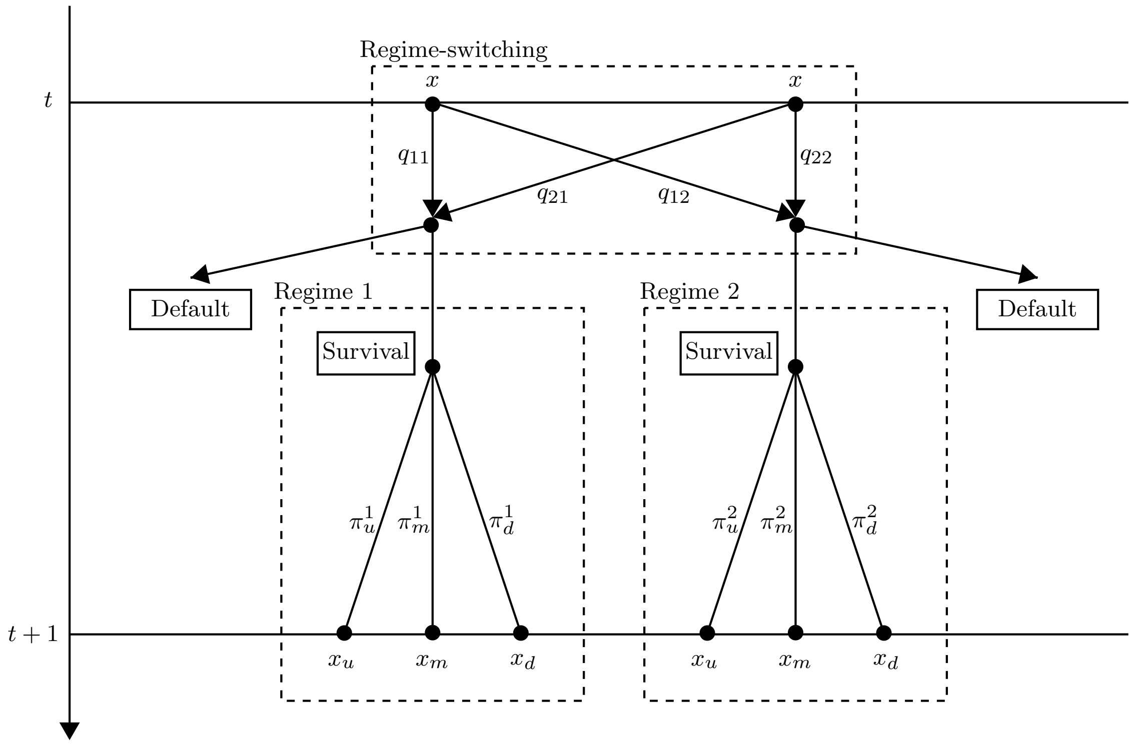

Figure F2: The branching to default at a typical node in the tree when the number of regimes is set to two.

References

Acharya, V. V., S. T. Bharath, and A. Srinivasan. 2007. Does industry-wide distress affect defaulted firms? Evidence from creditor recoveries. Journal of Financial Economics 85:787–821.

Alexander, C., and A. Kaeck. 2008. Regime dependent determinants of credit default swap spreads. Journal of Banking & Finance 32:1008–1021.

Altman, E. I., B. Brady, A. Resti, and A. Sironi. 2005. The link between default and recovery rates: Theory, empirical evidence, and implications. Journal of Business 78:2203–2228.

Amihud, Y. 2002. Illiquidity and stock returns: cross-section and time-series effects. Journal of Financial Markets 5:31–56.

Arora, N., P. Gandhi, and F. A. Longstaff. 2012. Counterparty credit risk and the credit default swap market. Journal of Financial Economics 103:280–293.

Augustin, P. 2012. The term structure of CDS spreads and sovereign credit risk. Working paper pp. 1–61.

Bade, B., D. Rösch, and H. Scheule. 2011. Default and recovery risk dependencies in a simple credit risk model. European Financial Management 17:120–144.

Bai, J., and P. Collin-Dufresne. 2011. The determinants of the CDS-bond basis during the financial crisis of 2007-2009. Working paper pp. 1–58.

Bakshi, G., D. Madan, and F. X. Zhang. 2006a. Investigating the Role of Systematic and Firm-Specific Factors in Default Risk: lessons from Empirically Evaluating Credit Risk Models. Journal of Business 79:1955–1987.

Bakshi, G., D. Madan, and F. X. Zhang. 2006b. Understanding the role of recovery in default risk models: empirical comparisons and implied recovery rates. Working paper pp. 1–33.

Black, F., and J. C. Cox. 1976. Valuing corporate securities: Some effects of bond indenture provisions. The Journal of Finance 31:351–367.

Bollen, N. P. 1998. Valuing options in regime-switching models. Journal of Derivatives 6:38–49.

Boudreault, M., G. Gauthier, and T. Thomassin. 2013. Recovery rate risk and credit spreads in a hybrid credit risk model. Journal of Credit Risk 9:3–39.

Brigo, D., M. Predescu, A. Capponi, T. R. Bielecki, and F. Patras. 2011. Liquidity modeling for credit default swaps: an overview. Credit Risk Frontiers: Subprime Crisis, Pricing and Hedging, CVA, MBS, Ratings, and Liquidity pp. 585–617.

Bruche, M., and C. González-Aguado. 2010. Recovery rates, default probabilities, and the credit cycle. Journal of Banking & Finance 34:754–764.

Bühler, W., and M. Trapp. 2009a. Explaining the bond-CDS basis: the role of credit risk and liquidity. Working paper pp. 1–35.

Bühler, W., and M. Trapp. 2009b. Time-varying credit risk and liquidity premia in bond and CDS markets. Working paper pp. 1–44.

Carr, P., and L. Wu. 2010. Stock options and credit default swaps: a joint framework for valuation and estimation. Journal of Financial Econometrics 8:409–449.

Christoffersen, P., C. Dorion, K. Jacobs, and L. Karoui. 2014. Nonlinear Kalman filtering in affine term structure models. Management Science 60:2248–2268.

Creal, D. 2012. A survey of sequential Monte Carlo methods for economics and finance. Econometric Reviews 31:245–296.

Dick-Nielsen, J. 2009. Liquidity biases in TRACE. Journal of Fixed Income 19:43–55.

Dick-Nielsen, J., P. Feldhütter, and D. Lando. 2012. Corporate bond liquidity before and after the onset of the subprime crisis. Journal of Financial Economics 103:471–492.

Dionne, G., and O. Maalaoui Chun. 2013. Default and liquidity regimes in the bond market during the 2002–2012 period. Canadian Journal of Economics 46:1160–1195.

Doshi, H., J. Ericsson, K. Jacobs, and S. M. Turnbull. 2013. Pricing credit default swaps with observable covariates. Review of Financial Studies 26:2049–2094.

Duan, J.-C., and A. Fulop. 2009. Estimating the structural credit risk model when equity prices are contaminated by trading noises. Journal of Econometrics 150:288–296.

Duffie, D., and D. Lando. 2001. Term structures of credit spreads with incomplete accounting information. Econometrica 69:633–664.

Duffie, D., and K. J. Singleton. 1999. Modeling term structures of defaultable bonds. Review of Financial Studies 12:687–720.

Dumas, B., J. Fleming, and R. E. Whaley. 1998. Implied volatility functions: empirical tests. Journal of Finance 53:2059–2106.

Elizalde, A., S. Doctor, and Y. Saltuk. 2009. Bond-CDS basis handbook. Tech. rep., JP Morgan European Credit Derivatives Research.

Elliott, R. J., L. Aggoun, and J. B. Moore. 1995. Hidden Markov models: Estimation and control, vol. 29. Springer Science & Business Media.

Fabozzi, F. J., and S. V. Mann. 2012. The handbook of fixed income securities, chap. 6. McGraw Hill Professional.

Fearnhead, P., and P. Clifford. 2003. On-line inference for hidden Markov models via particle filters. Journal of the Royal Statistical Society 65:887–899.

Feldhütter, P. 2012. The same bond at different prices: identifying search frictions and selling pressures. Review of Financial Studies 25:1155–1206.

Fontana, A. 2010. The negative CDS-bond basis and convergence trading during the 2007/09 financial crisis. Working paper pp. 1–63.

Garzarelli, F. 2009. The 2007-09 Credit Crisis and its Aftermath. Working paper pp. 1–37.

Guarin, A., X. Liu, and W. L. Ng. 2014. Recovering default risk from CDS spreads with a nonlinear filter. Journal of Economic Dynamics and Control 38:87–104.

Han, S., and H. Zhou. 2008. Effects of liquidity on the nondefault component of corporate yield spreads: evidence from intraday transactions data. Working paper pp. 1–49.

Huang, A. Y., and W.-C. Hu. 2012. Regime-switching dynamics in credit default swaps: evidence from smooth transition autoregressive model. Physica A: Statistical Mechanics and its Applications 391:1497–1508.

Huang, S. J., and J. Yu. 2010. Bayesian analysis of structural credit risk models with microstructure noises. Journal of Economic Dynamics and Control 34:2259–2272.

Jarrow, R., and P. Protter. 2004. Structural versus reduced form models: a new information based perspective. Journal of Investment Management 2:1–10.

Julier, S., and J. Uhlmann. 1997. A new extension of the Kalman Filter to nonlinear systems. In SPIE proceedings series, pp. 182–193. Society of Photo-Optical Instrumentation Engineers.

Kalman, R. E. 1960. A new approach to linear filtering and prediction problems. Journal of Basic Engineering 82:35–45.

Kim, C.-J. 1994. Dynamic linear models with Markov-switching. Journal of Econometrics 60:1–22.

Kwon, T. Y., and Y. Lee. 2015. Estimating structural credit risk models when market prices are contaminated with noise. Applied Stochastic Models in Business and Industry .

Kyle, A. S. 1985. Continuous Auctions and Insider Trading. Econometrica 53:1315–1336.

Longstaff, F. A., S. Mithal, and E. Neis. 2005. Corporate yield spreads: default risk or liquidity? New evidence from the credit default swap market. Journal of Finance 60:2213–2253.

Maalaoui Chun, O., G. Dionne, and P. François. 2014. Detecting regime shifts in credit spreads. Journal of Financial and Quantitative Analysis 49:1339–1364.

Madan, D., and H. Unal. 2000. A two-factor hazard rate model for pricing risky debt and the term structure of credit spreads. Journal of Financial and Quantitative Analysis 35:43–66.

Mahanti, S., A. Nashikkar, M. Subrahmanyam, G. Chacko, and G. Mallik. 2008. Latent liquidity: a new measure of liquidity, with an application to corporate bonds. Journal of Financial Economics 88:272–298.

Merton, R. C. 1974. On the pricing of corporate debt: the risk structure of interest rates. Journal of Finance 29:449–470.

Moody's. 2009. Corporate Default and Recovery Rates, 1920–2008. http://v2.moodys.com/cust/content/content.ashx?source=StaticContent/Free%20Pages/Credit%20Policy%20Research/documents/current/2007400000578875.pdf.

Mueller, P. 2008. Credit spreads and real activity. Working paper pp. 1–72.

NAIC. 2015. Update on the Insurance Industry's Use of Derivatives and Exposure Trends. Tech. rep., National Association of Insurance Commissionners. http://www.naic.org/capital_markets_archive/150807.htm.

Qiu, J., and F. Yu. 2012. Endogenous liquidity in credit derivatives. Journal of Financial Economics 103:611–631.

Roll, R. 1984. A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market. Journal of Finance 39:1127–1139.

Saldías, M. 2013. Systemic risk analysis using forward-looking distance-to-default series. Journal of Financial Stability 9:498–517.

Saunders, A., and L. Allen. 2010. Credit risk management in and out of the financial crisis: new approaches to value at risk and other paradigms. John Wiley & Sons.

Schönbucher, P. J. 2002. A tree implementation of a credit spread model for credit derivatives. Journal of Computational Finance 6:1–38.

Tang, D., and H. Yan. 2007. Liquidity and credit default swap spreads. Working paper pp. 1–65.

Tugnait, J. K. 1982. Detection and estimation for abruptly changing systems. Automatica 18:607–615.

Van Trees, H. L. 1968. Detection, estimation and linear modulation theory, Part 1. John Wiley & Sons.

Vasicek, O. 1977. An equilibrium characterization of the term structure. Journal of Financial Economics 5:177–188.

Viterbi, A. J. 1967. Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. IEEE Transactions on Information Theory 13:260–269.

Yuen, F. L., and H. Yang. 2010. Option pricing with regime-switching by trinomial tree method. Journal of Computational and Applied Mathematics 233:1821–1833.

Zhang, B. Y., H. Zhou, and H. Zhu. 2009. Explaining credit default swap spreads with the equity volatility and jump risks of individual firms. Review of Financial Studies 22:5099–5131.