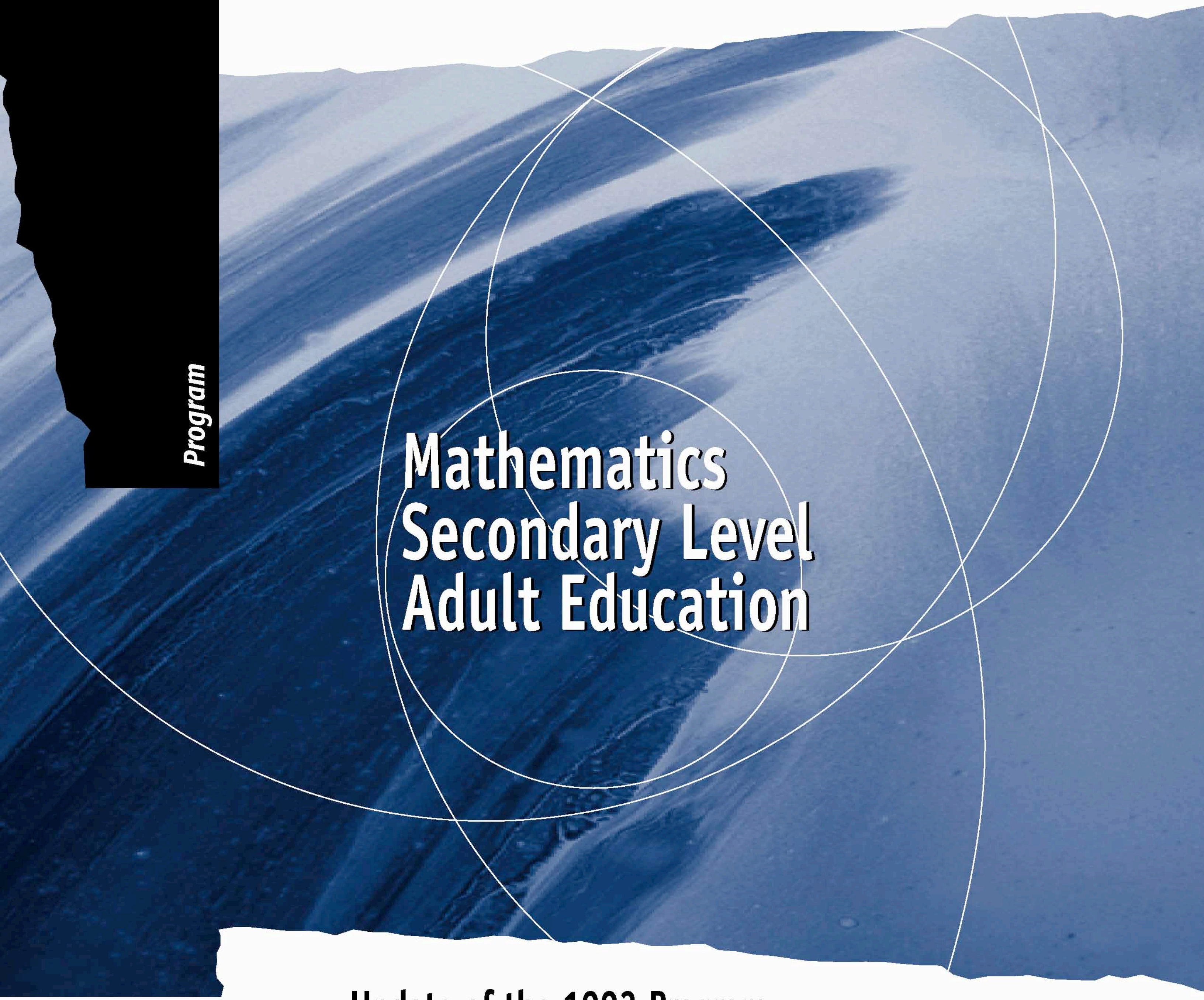
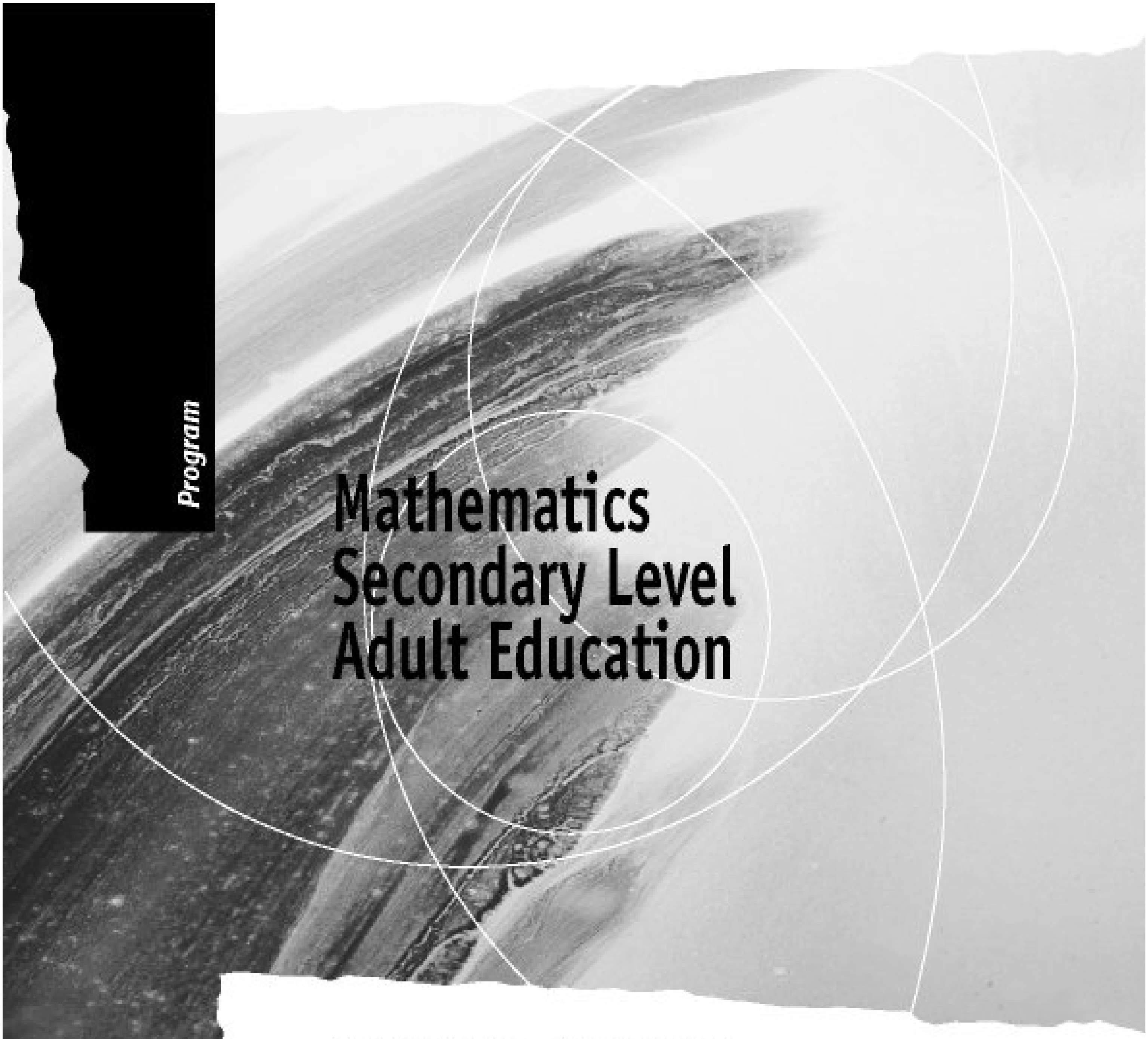


Program

Mathematics Secondary Level Adult Education

Update of the 1993 Program

Québec



Program

Mathematics Secondary Level Adult Education

Update of the 1993 Program

Formation professionnelle et technique
et formation continue

Direction de la formation générale
des adultes

ISBN 2-550-43274-6

Legal deposit — Bibliothèque nationale du Québec, 2004

The following people contributed to the revision and modification of this program:

Drafting committee	Louis-Marie Gaulin, teacher Commission scolaire des Premières-Seigneuries
	Jacqueline Robillard, teacher Commission scolaire Marguerite-Bourgeoys
	Daniel Taillon, teacher Commission scolaire Marguerite-Bourgeoys
Validation committee	Dominique Boucher, teacher Commission scolaire du Chemin-du-Roy
	Marie-Reine Couillard, education consultant Commission scolaire des Sommets André Cormier, education consultant Commission scolaire des Grandes-Seigneuries
	Régina Lavoie, education consultant Commission scolaire des Affluents
	Normand Marsolais, education consultant Commission scolaire des Portages-de-l'Outaouais
	Denise Martin, evaluation consultant Société GRICS
	Édith Paradis, evaluation consultant Société GRICS
	Paul Pompa, teacher Lester-B.-Pearson School Board
Program coordinators	Pierrette Marcotte and Pierrette Guay Coordinators Direction de la formation générale des adultes
Management	Alain Mercier, director Direction de la formation générale des adultes
English version	Direction de la production en langue anglaise Services à la communauté anglophone Ministère de l'Éducation
Word processing	Louise Vallières, secretary Commission scolaire des Premières-Seigneuries
	Monique Boyce, secretary
	Agathe Chamberland, secretary Direction de la formation générale des adultes

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INTRODUCTION

The *Mathematics, Secondary Level, Adult Education* program was revised so it could be brought more into line with the program offered in the youth sector. For several years, the youth sector program has offered three different paths (i.e. regular, intermediate and enriched) leading to college-level studies. The program used in the adult sector since 1993 does not include courses that reflect these three paths, which prepare students for college-level studies.

The revision of the program primarily involved reorganizing the Cycle Two courses and updating their content. This was done to make it easier for young adults coming from the youth sector to continue their education and for adults to go on to CEGEP.

The layout of the program has been modified (both for Cycle One and Cycle Two courses) to make it easier to consult the terminal objectives of each course.

1. GUIDING PRINCIPLES

These courses were developed on the basis of three guiding principles:

- To provide adults with the mathematical knowledge they need to perform everyday activities, to train for and practise an occupation or to pursue postsecondary level studies
- To provide access to recognized, high-quality mathematical training
- To consider adults to be the prime architects of their own education, particularly by encouraging them to actively participate in their own education and by enabling them to learn at their own pace

2. GOALS

The three guiding principles above are the foundation for the following goals:

- To prepare adults to use mathematics in their role as a citizen in order to organize information, structure their ideas and develop their critical judgment
- To encourage adults to develop positive attitudes toward mathematics and its applications
- To increase adults' confidence in their ability to solve mathematical problems
- To enable adults who will be pursuing postsecondary studies in mathematics, science or certain technical fields to master the mathematical concepts and tools necessary for such studies

3. OBJECTIVES

This program requires adult students to achieve the following objectives:

- the general objectives of the entire program
- the terminal and intermediate objectives of each course in the program

3.1. General Objectives

The general objectives of this program are:

- To master mathematical concepts essential to developing a better understanding of one's environment, particularly by making connections between these concepts
- To acquire mathematical knowledge that can be used in concrete everyday situations, in scientific and technical work or in trades
- To process information by applying mathematical models and appropriate strategies to solve a problem
- To communicate information clearly using mathematical language
- To acquire a rigorous work method, in particular by increasing one's ability to deal with a problem using systematic mathematical reasoning
- To master the use of appropriate technological tools (calculator or computer) when performing a mathematical task

3.2. Objectives of Each Course

Depending on the branch of mathematics it covers, each course has one or more terminal objectives subdivided into one or more intermediate objectives, which are linked to different learning objectives.

In each course, the terminal objectives are presented in numbered boxes, the intermediate objectives are numbered and in bold, and the learning objectives follow the related intermediate objective.

4. LIST OF COURSES

CODE	COURSE TITLE
MTH-1005-2	Operations on Integers
MTH-1006-2	The Four Operations on Fractions
MTH-1007-2	Decimals and Percent
MTH-2006-2	Equations and Inequalities I
MTH-2007-2	Geometry I (Straight Lines, Angles and Pythagorean Theorem)
MTH-2008-2	Statistics and Probability I
MTH-3001-2	Operations on Polynomials
MTH-3002-2	Geometry II (Perimeter, Area and Volume)
MTH-3003-2	Straight Lines I (Equations, Graphs and Slope)
MTH-4101-2	Equations and Inequalities II (First-Degree Systems)
MTH-4102-1	Geometry III (Isometry and Similarity)
MTH-4103-1	Trigonometry I (Right Triangles and Any Given Triangle)
MTH-4104-2	Statistics II (Measures and Data Gathering)
MTH-4105-1	Exponents and Radicals
MTH-4106-1	Factoring and Algebraic Fractions
MTH-4107-1	Straight Lines II (Parallel and Perpendicular Lines and Distance)
MTH-4108-1	Quadratic Functions
MTH-4109-1	Sets, Relations and Functions
MTH-4110-1	Operations on Algebraic Fractions
MTH-4111-2	Complement and Synthesis I
MTH-5101-1	Optimization I (Linear Programming)
MTH-5102-1	Statistics III (Correlation)
MTH-5103-1	Probability II
MTH-5104-1	Optimization II (Graphs)
MTH-5105-1	Conics
MTH-5106-1	Real Functions and Equations
MTH-5107-2	Exponential and Logarithmic Functions and Equations
MTH-5108-2	Trigonometric Functions and Equations
MTH-5109-1	Geometry IV (Circles and Right Triangles)
MTH-5110-1	Introduction to Vectors
MTH-5111-2	Complement and Synthesis II
MTH-5112-1 (Optional course)	Logic

5. CHANGES MADE TO THE 1993 PROGRAM

5.1. Secondary Cycle One Courses

No major changes have been made to the Secondary Cycle One courses. However, MTH-3003-2 has been added to the two preexisting Secondary III courses.

This makes the requirements for admission to Secondary IV in the adult sector consistent with those in the youth sector. The Straight Lines I course (GSM 241 in the 1993 program) therefore becomes MTH-3003-2.

5.2. Secondary Cycle Two Courses

The Secondary IV and V courses in the 1993 program were significantly modified in order to bring the adult education program more into line with the “568” series of mathematics courses offered in the youth sector (i.e. 568-416 and 568-514; 568-426 and 568-436; 568-526 and 568-536). New courses that did not appear in the 1993 program were also added. In some cases, these courses cover, in whole or in part, new subject matter now found in the “568” series of mathematics courses offered in the youth sector. In other cases, these courses cover topics that used to be taught in the mathematics program for adults, but that were left out of the 1993 program.

Below is an overview of additions and modifications made at the Secondary IV level:

- the same statistics course for all Secondary IV students
- introduction to functions that illustrates their connection to set theory and relations
- courses on exponents and radicals
- study of quadratic functions
- solving systems of equations where one equation is of degree 0 or 1 and the other is of degree 2
- operations on polynomial functions of a degree less than 3
- deductive reasoning in analytic geometry and Euclidean geometry (isometries, similarity transformations and equivalences)

n.b. Table 1 on page 7 provides a more detailed description of these changes.

Below is an overview of additions and modifications made at the Secondary V level:

– the same statistics course for all Secondary V students
– the same optimization course (linear programming) for all Secondary V students
– new content relating to probability (odds of an event occurring, mathematical expectation, calculating probability in a geometric context)
– introduction to graph theory
– analyzing and solving problems using functions
– exponential, logarithmic and trigonometric equations
– operations on functions
– introduction to vectors
– completing proofs in Euclidean geometry (more emphasis on this theme)

n.b. Table 2 on page 8 provides a more detailed description of these changes.

TABLE 1: Changes to Secondary IV Courses

CURRENT PROGRAM	1993 PROGRAM
MTH-4101-2 Equations and Inequalities II (First-Degree Systems)	Course identical to GSM 243 (MTH-4067-2)
MTH-4102-1 Geometry III (Isometry and Similarity)	Course identical to GSM 242 (MTH-4066-1)
MTH-4103-1 Trigonometry I (Right Triangles and Any Given Triangle)	Course identical to GSM 244 (MTH-4068-1)
MTH-4104-2 Statistics II (Measures and Data Gathering)	New course that covers part of objectives 01 to 05 and 08 to 10 of GSM 252 (MTH-5084-2), as well as new content
MTH-4105-1 Exponents and Radicals	New course
MTH-4106-1 Factoring and Algebraic Fractions	Course equivalent to GMO 241 (MTH-4058-1), but objective 06 has been simplified, as well as objectives 01 to 04 of GMO 242 (MTH-4059-1)
MTH-4107-1 Straight Lines II (Parallel and Perpendicular Lines and Distance)	Course identical to GMO 243 (MTH-4060-1), but includes new content on problem-solving
MTH-4108-1 Quadratic Functions	Course identical to GMO 251 (MTH-5076-1)
MTH-4109-1 Sets, Relations and Functions	Course corresponding to GMO 244 (MTH-4061-2), but the “Sets and Relations” section has been simplified and contains an introduction to the study of functions
MTH-4110-1 Operations on Algebraic Fractions	Course identical to GMO 242 (MTH-4059-1)
MTH-4111-2 Complement and Synthesis I	New course that entails a more in-depth study of the content of previous Secondary IV courses (functions, systems of equations, analytic geometry, isometries, similarity transformations and equivalences), and emphasizes the synthesis of concepts studied

TABLE 2: Changes to Secondary V Courses

CURRENT PROGRAM	1993 PROGRAM
MTH-5101-1 Optimization I (Linear Programming)	Course identical to GSM 251 (MTH-5083-1)
MTH-5102-1 Statistics III (Correlation)	New statistics course including the material covered in objectives 06 and 07 of GSM 252 (MTH-5084-2)
MTH-5103-1 Probability II	New probability course including a new approach to the material covered in objectives 11 to 13 of GSM 252 (MTH-5084-2)
MTH-5104-1 Optimization II (Graphs)	New course
MTH-5105-1 Conics	Course identical to GMO 254 (MTH-5079-1) with a new section on problem-solving
MTH-5106-1 Real Functions and Equations	Course equivalent to GMO 252 and GMO 253 (MTH-5077-1 and MTH-5078-1), but with significant changes to the content
MTH-5107-2 Exponential and Logarithmic Functions and Equations	Course equivalent to GMO 255 (MTH-5080-1), but with significant changes to the content
MTH-5108-2 Trigonometric Functions and Equations	Course equivalent to GMO 256 (MTH-5081-2), but with significant changes to the content
MTH-5109-1 Geometry IV (Circles and Right Triangles)	Course identical to GSM 253 (MTH-5085-1)
MTH-5110-1 Introduction to Vectors	New course
MTH-5111-2 Complement and Synthesis II	New course that entails a more in-depth study of the content of previous Secondary V courses (operations on functions, composition of functions, inequalities, relationships governing measurements in circles and right triangles), and emphasizes the synthesis of concepts studied

6. DISTRIBUTION OF SECONDARY IV AND V COURSES

The tables on pages 10 and 11 illustrate which courses make up the different possible paths for Secondary Cycle Two mathematics.

The regular path corresponds to the 568-416 Secondary IV and the 568-514 Secondary V courses in the youth sector.

The intermediate path corresponds to the 568-426 Secondary IV and the 568-526 Secondary V courses in the youth sector.

The enriched path corresponds to the 568-436 Secondary IV and the 568-536 Secondary V programs in the youth sector.

Table 3 on page 10 shows the Secondary IV courses corresponding to the different paths, while table 4 on page 11 shows the Secondary V courses associated with each path.

Table 5 on page 12 indicates the Secondary IV mathematics courses in the adult sector that correspond to those in the youth sector, while table 6 on page 13 provides this information for Secondary V mathematics courses.

TABLE 3: Secondary IV Paths and Courses

Path	Code	Title
Regular Path	MTH-4101-2	Equations and Inequalities II (First-Degree Systems)
	MTH-4102-1	Geometry III (Isometry and Similarity)
	MTH-4103-1	Trigonometry I (Right Triangles and Any Given Triangle)
	MTH-4104-2	Statistics II (Measures and Data Gathering)
Intermediate Path	MTH-4101-2 MTH-4102-1 MTH-4103-1 MTH-4104-2	
	MTH-4105-1	Exponents and Radicals
	MTH-4106-1	Factoring and Algebraic Fractions
	MTH-4107-1	Straight Lines II (Parallel and Perpendicular Lines and Distance)
	MTH-4108-1	Quadratic Functions
	MTH-4109-1	Sets, Relations and Functions
Enriched Path	MTH-4101-2 MTH-4102-1 MTH-4103-1 MTH-4104-2 MTH-4105-1 MTH-4106-1 MTH-4107-1 MTH-4108-1 MTH-4109-1	
	MTH-4110-1	Operations on Algebraic Fractions
	MTH-4111-2	Complement and Synthesis I

TABLE 4: Secondary V Paths and Courses

Path	Code	Title
Regular Path	MTH-5101-1	Optimization I (Linear Programming)
	MTH-5102-1	Statistics III (Correlation)
	MTH-5103-1	Probability II
	MTH-5104-1	Optimization II (Graphs)
Intermediate Path	MTH-5101-1	
	MTH-5102-1	
	MTH-5105-1	Conics
	MTH-5106-1	Real Functions and Equations
	MTH-5107-2	Exponential and Logarithmic Functions and Equations
	MTH-5108-2	Trigonometric Functions and Equations
	MTH-5109-1	Geometry IV (Circles and Right Triangles)
Enriched Path	MTH-5101-1	
	MTH-5102-1	
	MTH-5105-1	
	MTH-5106-1	
	MTH-5107-2	
	MTH-5108-2	
	MTH-5109-1	
	MTH-5110-1	Introduction to Vectors
	MTH-5111-2	Complement and Synthesis II

TABLE 5: Adult Education Courses Corresponding to the 568-416, 568-426 and 568-436 Courses (Youth Sector, Secondary IV)

MTH-4101-2 to MTH-4104-2: Courses corresponding to 568-416 (Prerequisites*: MTH-3001-2 to MTH-3003-2 or 568-314)	**MTH-4101-2 to MTH-4109-1: Courses corresponding to 568-426** (Prerequisites*: MTH-3001-2 to MTH-3003-2 or 568-314)	**MTH-4101-2 to MTH-4111-2: Courses corresponding to 568-436** (Prerequisites*: MTH-3001-2 to MTH-3003-2 or 568-314)
MTH-4101-2 Equations and Inequalities II MTH-4102-1 Geometry III MTH-4103-1 Trigonometry I MTH-4104-2 Statistics II	MTH-4101-2 Equations and Inequalities II MTH-4102-1 Geometry III MTH-4103-1 Trigonometry I MTH-4104-2 Statistics II MTH-4105-1 Exponents and Radicals MTH-4106-1 Factoring and Algebraic Fractions MTH-4107-1 Straight Lines II MTH-4108-1 Quadratic Functions MTH-4109-1 Sets, Relations and Functions **MTH-4105-1 to MTH-4109-1: Bridge between 568-416 and 568-426** (Prerequisites*: MTH-4101-2 to MTH-4104-2 or 568-416 and MTH-4101-2)	MTH-4101-2 Equations and Inequalities II MTH-4102-1 Geometry III MTH-4103-1 Trigonometry I MTH-4104-2 Statistics II MTH-4105-1 Exponents and Radicals MTH-4106-1 Factoring and Algebraic Fractions MTH-4107-1 Straight Lines II MTH-4108-1 Quadratic Functions MTH-4109-1 Sets, Relations and Functions MTH-4110-1 Operations on Algebraic Fractions MTH-4111-2 Complement and Synthesis I **MTH-4110-1 and MTH-4111-2: Bridge between 568-426 and 568-436** (Prerequisites*: MTH-4101-2 to MTH-4109-1 or 568-426)

* Equivalent or more advanced prerequisites can replace the prerequisites mentioned in this table.

TABLE 6: Adult Education Courses Corresponding to the 568-514, 568-526 and 568-536 Courses (Youth Sector, Secondary V)

MTH-5101-1 to MTH-5104-1: **Courses corresponding to 568-514** (Prerequisites[*]: MTH-4101-2 to MTH- 4104-2 or 568-416) MTH-5101-1 Optimization I MTH-5102-1 Statistics III MTH-5103-1 Probability II MTH-5104-1 Optimization II	**MTH-5101-1 and MTH-5102-1 and** **MTH-5105-1 to MTH-5109-1:** **Courses corresponding to 568-526** (Prerequisites[*]: MTH-4101-2 to MTH-4109-1 or 568-426) MTH-5101-1 Optimization I MTH-5102-1 Statistics III + MTH-5105-1 Conics MTH-5106-1 Real Functions and Equations MTH-5107-2 Exponential and Logarithmic Functions and Equations MTH-5108-2 Trigonometric Functions and Equations MTH-5109-1 Geometry IV **MTH-5105-1 to MTH-5109-1:** **Bridge between 568-514 and 568-526** (Prerequisites[*]: MTH-4101-2 to MTH-4109-1 or 568-426 and MTH-5101-1, MTH-5102-1 or 568-514)	**MTH-5101-1 and MTH-5102-1 and** **MTH-5105-1 to MTH-5111-2:** **Courses corresponding to 568-536** (Prerequisites[*]: MTH-4101-2 to MTH-4111-2 or 568-436) MTH-5101-1 Optimization I MTH-5102-1 Statistics III + MTH-5105-1 Conics MTH-5106-1 Real Functions and Equations MTH-5107-2 Exponential and Logarithmic Functions and Equations MTH-5108-2 Trigonometric Functions and Equations MTH-5109-1 Geometry IV MTH-5110-1 Introduction to Vectors MTH-5111-2 Complement and Synthesis II **MTH-5110-1 and MTH-5111-2:** **Bridge between 568-526 and 568-536** (Prerequisites[*]: MTH-4101-2 to MTH-4111-2 or 568-436 and MTH-5101-1, MTH-5102-1 and MTH-5105-1 to MTH-5109-1 or 568-526).

[*] Equivalent or more advanced prerequisites can replace the prerequisites mentioned in this table.

7. LEARNING MATERIALS

7.1. Basic Materials

Below are a few remarks concerning the learning materials that should be used for the mathematics courses in this program.

The materials should meet the following requirements:

- be organized in the same way as the courses in the program
- present all the subject matter of a course in one coherent whole
- be composed of learning modules arranged in such a way that students can take what they learned in one module and apply it in the next module
- make it possible to determine whether students have acquired the prerequisites for a course
- present the course content and objectives to the students
- be designed so that learning activities can be organized in a logical and progressive manner
- offer different types of learning activities that vary in level of difficulty
- offer activities that introduce the students to the use of technological tools or that require the use of such tools
- provide learning activities that stimulate the students' interests
- provide appropriate illustrations
- include formative evaluation activities
- encourage the students to learn and use mathematical vocabulary

The sequence in which the objectives are presented in the learning materials may differ from that of the program.

7.2. Calculators

Calculators are useful for many of the learning activities in this program. The calculators* that are recommended for each course are identified below:

<table>
<tr><th colspan="2">TYPE</th><th>COURSE</th></tr>
<tr><td colspan="2">Basic scientific calculator</td><td>MTH-1005-2 to MTH-4103-1
MTH-4105-1 to MTH-4107-1
MTH-4110-1
MTH-5101-1
MTH-5103-1 and MTH-5104-1
MTH-5109-1 and MTH-5110-1</td></tr>
<tr><td rowspan="2">Scientific calculator</td><td>with one-variable statistical functions</td><td>MTH-4104-2</td></tr>
<tr><td>with one- and two-variable statistical functions</td><td>MTH-5102-1</td></tr>
<tr><td colspan="2">Graphing calculator</td><td>MTH-4109-1
MTH-4111-2
MTH-5105-1 to MTH-5108-2
MTH-5111-2</td></tr>
</table>

8. EVALUATION OF LEARNING

Evaluation is designed to help students in their learning and to gather information necessary for the certification of studies.

Formative evaluation is part of teaching and learning. It supports and guides decision making with respect to learning situations, instructional materials and teaching approaches. Evaluation is the teacher's responsibility, and schools have established an evaluation policy. The Ministère de l'Éducation (MEQ) can, if necessary, propose conceptual frameworks or examples of formative evaluation instruments.

Summative evaluation takes place at the end of the learning process. To that end, the MEQ provides *Definitions of the Domain for Summative Evaluation* for each course in this program. The instructions in these documents must be followed in the development of summative examinations.

Information on evaluation of learning and certification of studies can be found in official documents other than the program itself, mainly in *Definitions of the Domain for Summative Evaluation* and the *Administrative Manual for the Certification of Studies.* These documents should be consulted to obtain specific information on these topics.

* An equivalent technological tool may also be used (e.g. computer with appropriate software).

9. TERMINAL OBJECTIVES OF EACH COURSE

The following pages provide a list of the terminal objectives for each course in this program.

SECONDARY I COURSES

MTH-1005-2 Operations on Integers

1. To apply the order of operations rule to integers
2. To solve problems related to everyday situations

MTH-1006-2 The Four Operations on Fractions

1. To locate fractions on a number line
2. To apply the order of operations rule to fractions
3. To solve problems related to everyday situations

MTH-1007-2 Decimals and Percent

1. To apply the order of operations rule to decimals and mixed numbers
2. To solve problems related to everyday situations

SECONDARY II COURSES

MTH-2006-2 Equations and Inequalities I

1. To solve first-degree equations and inequalities in one variable
2. To solve problems related to everyday situations

MTH-2007-2 Geometry I (Straight Lines, Angles and Pythagorean Theorem)

1. To construct a given angle
2. To determine the measure of certain angles
3. To apply the Pythagorean Theorem

MTH-2008-2 Statistics and Probability I

1. To derive information from a variety of graphs containing statistical data
2. To calculate the probability of a particular event or outcome

SECONDARY III COURSES

MTH-3001-2 Operations on Polynomials

1. To apply the order of operations rule to calculations involving polynomials

MTH-3002-2 Geometry II (Perimeter, Area and Volume)

1. To solve problems involving triangles and quadrilaterals
2. To solve problems involving circles
3. To solve problems involving solids

MTH-3003-2 Straight Lines I (Equations, Graphs and Slope)

1. To graph a line in a Cartesian plane
2. To determine the slope of a line and its equation
3. To solve problems involving lines

SECONDARY IV COURSES – REGULAR PATH

MTH-4101-2 Equations and Inequalities II (First-Degree Systems)

1. To solve a system of first-degree equations
2. To graphically solve a system of first-degree inequalities in two variables

MTH-4102-1 Geometry III (Isometry and Similarity)

1. To identify certain geometric transformations
2. To apply the properties of congruence and similarity in a variety of situations
3. To solve problems requiring the application of the properties of congruent and similar figures

MTH-4103-1 Trigonometry I (Right Triangles and Any Given Triangle)

1. To solve problems requiring the application of trigonometric ratios to right triangles
2. To solve problems requiring the application of the law of sines and the law of cosines to any given triangle

MTH-4104-2 Statistics II (Measures and Data Gathering)

1. To identify the characteristics of different methods of gathering data for a statistical survey
2. To interpret measures of central tendency, measures of dispersion and measures of position relating to a concrete situation
3. To analyze graphic representations of distributions

SECONDARY IV COURSES – INTERMEDIATE PATH

MTH-4105-1 Exponents and Radicals

1. To apply the laws of exponents
2. To convert an expression containing a radical into exponential form
3. To simplify expressions containing radicals
4. To compare algebraic or numerical expressions that are in exponential form or that contain radicals

MTH-4106-1 Factoring and Algebraic Fractions

1. To apply factoring techniques to polynomials
2. To simplify algebraic fractions
3. To compare algebraic expressions containing two algebraic fractions that are added or subtracted

MTH-4107-1 Straight Lines II (Parallel and Perpendicular Lines and Distance)

1. To find the equation of a line that is parallel or perpendicular to another line
2. To determine the distance between two points or the coordinates of the point of division of a segment
3. To solve problems related to analytic geometry

MTH-4108-1 Quadratic Functions

1. To formulate a second-degree equation representing a situation illustrated in a table
2. To solve a second-degree equation algebraically
3. To graph a second-degree equation of the form $y = ax^2 + bx + c$
4. To solve a problem that involves determining a second-degree equation

MTH-4109-1 Sets, Relations and Functions

1. To perform set operations on sets of real numbers
2. To study relations, the different ways of representing them and their characteristics
3. To describe the characteristics of various functions
4. To solve problems related to real functions

SECONDARY IV COURSES – ENRICHED PATH

MTH-4110-1 Operations on Algebraic Fractions

1. To simplify algebraic expressions
2. To apply the order of operations rule to algebraic fractions

MTH-4111-2 Complement and Synthesis I

1. To solve a system of equations in two variables that consists of an equation of degree 0 or 1 and a second-degree equation
2. To recognize, on the basis of its characteristics, a graph representing the result of an operation involving functions
3. To apply concepts of analytic geometry in a geometric context
4. To solve problems related to isometry, similarity and equivalence

SECONDARY V COURSES – REGULAR PATH

MTH-5101-1 Optimization I (Linear Programming)

1. To translate an optimization problem into mathematical language
2. To draw a polygon of constraints
3. To solve optimization problems

MTH-5102-1 Statistics III (Correlation)

1. To interpret measures of dispersion and measures of positions in one-variable statistical distributions
2. To analyze two-variable statistical distributions

MTH-5103-1 Probability II

1. To solve problems related to probabilities or the odds of an event occurring
2. To calculate the probability of an event in a random experiment consisting of several steps
3. To solve problems involving mathematical expectation

MTH-5104-1 Optimization II (Graphs)

1. To identify certain types of graphs and their characteristics
2. To use graphs to determine the optimal value in a given situation
3. To solve optimization problems using graphs

SECONDARY V COURSES – INTERMEDIATE PATH

MTH-5105-1 Conics

1. To study a circle as a locus, its graph and its distinctive features
2. To study a parabola as a locus, its graph and its distinctive features
3. To study an ellipse as a locus, its graph and its distinctive features
4. To study a hyperbola as a locus, its graph and its distinctive features
5. To find the inequality or the equation associated with a conic
6. To solve problems involving conics

MTH-5106-1 Real Functions and Equations

1. To recognize functions and determine their characteristics given their rule or graph
2. To solve the equation of certain real functions or to determine the inverse of these functions
3. To solve problems related to real functions

MTH-5107-2 Exponential and Logarithmic Functions and Equations

1. To study the graphs and characteristics of exponential functions
2. To find the rule of a given exponential function or its inverse
3. To apply the properties of logarithms to simplify logarithmic expressions
4. To solve logarithmic equations or exponential equations
5. To study the graphs and characteristics of logarithmic functions
6. To find the rule of a logarithmic function or its inverse
7. To solve problems involving exponential and logarithmic functions

MTH-5108-2 Trigonometric Functions and Equations

1. To study the unit circle and wrapping function
2. To study trigonometric functions
3. To prove trigonometric identities and to solve simple trigonometric equations
4. To study the graphs, characteristics and rules of sinusoidal functions
5. To solve problems involving sinusoidal functions

MTH-5109-1 Geometry IV (Circles and Right Triangles)

1. To solve problems pertaining to relationships governing measurements within the same circle or involving two circles
2. To solve problems pertaining to relationships governing measurements in right triangles

SECONDARY V COURSES – ENRICHED PATH

MTH-5110-1 Introduction to Vectors

1. To know the definitions, notations and symbols relating to vectors
2. To add vectors
3. To give a linear combination of vectors
4. To perform a scalar multiplication of vectors
5. To prove statements using vectors
6. To solve problems involving vectors

MTH-5111-2 Complement and Synthesis II

1. To study the composition of real functions and the four operations on real functions
2. To solve inequalities in one real variable and problems related to these inequalities
3. To solve problems involving circles and right triangles

10. DETAILED COURSE DESCRIPTIONS

The following pages provide detailed descriptions of the various courses. The courses have been divided into six sections for easy reference, as follows:

10.1. Secondary I Courses

10.2. Secondary II Courses

10.3. Secondary III Courses

10.4. Secondary IV Courses

10.5. Secondary V Courses

10.6. Optional course

10.1. Secondary I Courses

MTH-1005-2

MTH-1006-2

MTH-1007-2

1. To apply the order of operations rule to integers

1.1. Evaluate an arithmetic expression containing integers by performing the appropriate operations and by following the order of operations. (The arithmetic expression contains up to five sets of parentheses or brackets.)

- Compare two integers by locating them on the number line and by using the appropriate symbol: is greater than (>), is less than (<) or is equal to (=). The numbers to be compared are quantities used in everyday situations.
- Add two integers that are greater than −30 and less than +30.
- Subtract two integers that are greater than −30 and less than +30.
- Multiply two integers that are greater than −30 and less than +30.
- Divide two integers that are greater than −30 and less than +30.

2. To solve problems related to everyday situations

2.1. Perform the appropriate operations required to solve problems related to everyday situations that can be represented by arithmetic expressions containing integers only.

* Equivalent to GSM 211 in the 1993 program.

1. To locate fractions on a number line

1.1. Locate a set of positive or negative fractions, improper fractions or mixed numbers on a number line. (The interval on the number line ranges from –2 to +2, the subdivisions are equal to the value or the common denominator and the denominators are less than or equal to 12.)

- Express the ratio of two quantities as a fraction, a mixed number or an improper fraction, as the case may be. These quantities are found in statements or diagrams relating to everyday situations.
- Apply the method for converting a fraction to an equivalent fraction in order to solve ratio and proportion problems relating to everyday situations.
- Simplify fractions, reducing them to equivalent fractions in lowest terms. The numerator and denominator of each given fraction must be less than 225.
- Convert improper fractions to mixed numbers and mixed numbers to improper fractions. In each case, the result must be reduced to its lowest terms.

2. To apply the order of operations rule to fractions

2.1. Evaluate arithmetic expressions containing up to six positive or negative fractions, improper fractions or mixed numbers, by performing the appropriate operations, by following the order of operations and by applying the law of signs. (The arithmetic expression should contain no more than three sets of parentheses or brackets, no more than three types of operations and denominators less than or equal to 12.)

- Compare two fractions, mixed numbers or improper fractions using the appropriate symbol (>, < or =).
- Multiply three fractions, improper fractions or mixed numbers and reduce the product to its lowest terms. The problems are presented in words or as arithmetic expressions.

* Equivalent to GSM 212 in the 1993 program.

- Divide two fractions, improper fractions or mixed numbers and reduce the quotient to its lowest terms. The problems are presented in words or as arithmetic expressions.

- Add three fractions, improper fractions or mixed numbers and reduce the sum to its lowest terms. The problems are presented in words or as arithmetic expressions.

- Subtract different fractions, improper fractions or mixed numbers and reduce the difference to its lowest terms. The problems are presented in words or as arithmetic expressions containing two or three terms.

3. To solve problems related to everyday situations

3.1. Solve word problems that can be written as arithmetic expressions containing fractions, improper fractions or mixed numbers, by following the order of operations and by applying the law of signs. (Evaluating the arithmetic expression involves performing up to five operations and no more than two types of operations—i.e. any two of the following: addition, subtraction, multiplication and division.)

1. To apply the order of operations rule to decimals and mixed numbers

1.1. Evaluate an arithmetic expression containing up to four decimals and two mixed numbers, whether positive or negative, by performing the appropriate operations, following the order of operations and applying the law of signs. (The arithmetic expression should contain up to three sets of parentheses or brackets and no more than three types of operations—i.e. any three of the following: addition, subtraction, multiplication and division. The decimals are limited to thousandths and the mixed numbers have a denominator less than 13.)

- Convert a decimal (limited to thousandths) to a fraction or mixed number reduced to its lowest terms, and convert a fraction or mixed number to a decimal limited to thousandths. When the resulting decimal is a repeating or periodic decimal, standard notation is required.

- Round a decimal limited to ten thousandths to the nearest tenth, hundredth or thousandth.

- Multiply and divide a decimal (the decimal is limited to ten thousandths) by 10, by 100, by 1 000 and by 10 000.

- Add two positive or negative decimals limited to thousandths. The problems are presented as statements or arithmetic expressions.

- Subtract two positive or negative decimals limited to thousandths. The problems are presented in words or as arithmetic expressions.

- Multiply two positive or negative decimals limited to thousandths. The product must be rounded to the nearest thousandth if the decimal part contains more than three digits. These decimals represent quantities used in everyday situations.

- Divide two positive or negative decimals limited to thousandths. The quotient must be rounded to the nearest thousandth if the decimal part contains more than three digits. These decimals represent quantities used in everyday situations.

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* Equivalent to GSM 213 in the 1993 program.

2. To solve problems related to everyday situations

2.1. **Solve word problems that can be written as arithmetic expressions containing decimals and mixed numbers, by following the order of operations and applying the law of signs**. (The decimals are limited to thousandths and the mixed numbers have a denominator less than 13. Evaluating the arithmetic expression involves performing up to five operations and no more than three types of operations—i.e. any three of the following: addition, subtraction, multiplication and division.)

2.2. **Solve word problems that can be written as arithmetic expressions containing integers, decimals, mixed numbers and percentages.** (Solving the problem involves calculating part of the whole when the ratio is given in percentage form or calculating the ratio of a part to the whole and expressing it in percentage form, given the value of the part and the whole. Solving the problem involves performing up to five operations and no more than three types of operations. The decimals are limited to thousandth.)

- Convert a given percentage to a ratio with a denominator of 100. The given percentage is limited to tenths of a percent. The problems deal with everyday situations.

- Convert a percentage to a decimal (the percentage is limited to tenths of a percent) and convert a decimal to a percentage (the decimal is limited to thousandths). The problems deal with everyday situations.

- Convert a percentage to a fraction reduced to its lowest terms and convert a fraction to a percentage. The percentage is usually limited to tenths of a percent. The problems deal with everyday situations.

10.2. Secondary II Courses

MTH-2006-2

MTH-2007-2

MTH-2008-2

1. To solve first-degree equations and inequalities in one variable

1.1. Solve a first-degree equation in one variable. The equation can be expressed in the form $ax + b = 0$, where *a* and *b* are rational numbers and *x* represents the variable. (The initial equation contains up to six terms and is defined within a given universe [$\mathbb{N}$, $\mathbb{Z}$ or $\mathbb{Q}$].)

- Simplify an algebraic expression containing similar terms and addition or subtraction operations. The initial algebraic expression consists of up to five terms containing no more than three first-degree variables. It contains no parentheses, brackets or braces.

- Apply the distributive property of multiplication over addition or over subtraction to an algebraic expression of the form $a(bx + cy)$, where a, b and c are rational numbers and x and y represent variables. The initial algebraic expression contains up to four terms.

1.2. Solve a first-degree inequality in one variable. The inequality can be converted to one of the following forms:

- $\boldsymbol{ax + b \geq 0}$
- $\boldsymbol{ax + b \leq 0}$
- $\boldsymbol{ax + b > 0}$
- $\boldsymbol{ax + b < 0}$

(The constants a and b are rational numbers and x represents the variable. The initial equation contains up to six terms and is defined within a given universe [$\mathbb{N}$, $\mathbb{Z}$ or $\mathbb{R}$]. The solution set and its graphical representation on the number line must be clearly indicated.)

1.3. Solve a first-degree equation in one variable by applying the fundamental property of proportions: the product of the extremes is equal to the product of the means. The given equation is stated as a proportion. (The exercises consist of mathematical expressions containing up to six terms or word problems related to everyday situations.)

[*] Equivalent to GSM 221 in the 1993 program.

1.4. **Solve an equation representing a given scientific formula, which can be expressed as an equation containing an unknown variable.** (The formula and the value of the other variables are provided.)

2. To solve problems related to everyday situations

2.1. **Solve word problems that can be written as first-degree equations in one variable.** (The problems can involve up to three unknowns.)

MTH-2007-2* **Geometry I**
(Straight Lines, Angles and Pythagorean Theorem)

1. To construct a given angle

1.1. Using a protractor, construct an angle of *n* degrees to the nearest 2°. (The measure of the angle to be drawn is an integer between 0° and 180°.)

- Distinguish between the following geometric figures:
  - line
  - ray
  - line segment
  - angle
  - acute angle
  - obtuse angle
  - right angle
  - straight angle
- Using a protractor, measure a given angle between 0°and 180° to the nearest 2°.

2. To determine the measure of certain angles

2.1. Determine the measure of one or more angles in a geometric figure containing two parallel lines cut by a transversal, given the measure of one of the angles of that figure. (The measure should be justified by applying the properties of pairs of angles.)

- Distinguish between the following pairs of lines:
  - parallel lines
  - perpendicular intersecting lines
  - non-perpendicular intersecting lines
- Identify and distinguish between the following pairs of angles and their properties:
  - complementary angles
  - supplementary angles
  - adjacent angles
  - vertically opposite angles
  - alternate interior angles
  - alternate exterior angles
  - corresponding angles

* Equivalent to GSM 222 in the 1993 program.

2.2. Determine the measures of angles and sides in a geometric figure containing any of the following polygons: equilateral triangle, isosceles triangle, right triangle, isosceles right triangle, scalene triangle, parallelogram, rhombus, square, rectangle and trapezoid. (The measures should be justified by applying the properties of polygons.)

- Identify the properties of the angles, sides and diagonals of each of the following polygons:
  - triangles
  - equilateral triangles
  - isosceles triangles
  - right triangles
  - isosceles right triangles
  - scalene triangles
  - quadrilaterals
  - parallelograms
  - rhombuses
  - squares
  - rectangles
  - trapezoids
- Identify the different polygons by the properties of their angles, sides and diagonals.
- Using a protractor and a ruler, identify these polygons among a set of geometric figures.

3. To apply the Pythagorean Theorem

3.1. Given the measures of two sides of a right triangle, calculate the measure of the third side by applying the Pythagorean Theorem. (These triangles illustrate everyday situations.)

3.2. Given the measure of one side of a right triangle in which one of the angles measures 30° or 45°, calculate the measure of one of the other two sides by applying the Pythagorean Theorem. (These triangles illustrate everyday situations.)

1. To derive information from a variety of graphs containing statistical data

1.1. Derive information from a broken-line graph, a vertical bar graph, a horizontal bar graph or a pictograph.

- Find the following information in a data table: title, quantified objects, data. Find the following information in a frequency distribution table: title, data, frequencies. If applicable, each numerical value must be stated in the corresponding unit of measure. Also, calculate the range of a given distribution.

- Identify the characteristics of the following graphs:

  - broken-line graph
  - vertical bar graph
  - horizontal bar graph
  - pictograph

- Given a data table or a frequency distribution table, construct a broken-line graph, a vertical bar graph, a horizontal bar graph or a pictograph. The table contains between three and eight data values or frequencies. The instructions concern the length of the axes, the creation of a *scale break*, the width of the bars in a bar graph and the numerical value of each symbol in a pictograph. The graph must be given a title, and each axis must be properly identified and graduated.

1.2. Collect information from a circle graph.

- Identify the characteristics of a circle graph.

- Given a data table or a frequency distribution table, construct a circle graph by using a compass, a protractor and a ruler. The given table contains between three and eight data values or frequencies. The graph must be given a title, and each sector must be given a subheading and assigned an appropriate percentage value.

* Equivalent to GSM 223 in the 1993 program.

1.3. Derive information from a histogram.

- Identify the characteristics of a histogram.
- Using a frequency distribution table in which the data is grouped into classes, construct a histogram, following any instructions regarding the creation of a *scale break.* The table contains between three and eight classes along with their respective frequencies. The histogram must be given a title, and each axis must be properly identified and graduated.

2. To calculate the probability of a particular event or outcome

2.1. Given a tree diagram, indicate the following information:

- **a particular outcome to be written as an ordered pair or ordered triple**
- **the universe of possible outcomes (usually denoted by U) to be written as a set of ordered pairs or ordered triples**
- **an event (usually denoted by E) to be written as a set of ordered pairs or ordered triples**

(Set theory notation is used. The problems deal with everyday situations and involve a series of two or three random experiments leading to no more than 36 possible outcomes.)

2.2. Calculate the probability of a particular event or outcome, given a description of a series of two or three random experiments. (Probability can be expressed in one of two ways: as a decimal number between 0 and 1, rounded to the nearest thousandth, or as a percentage whose value ranges from 0% to 100%. The problems involve a series of random experiments leading to no more than 36 possible outcomes.)

10.3. Secondary III Courses

MTH-3001-2

MTH-3002-2

MTH-3003-2

1. To apply the order of operations rule to calculations involving polynomials

1.1. Evaluate an algebraic expression by following the order of operations. (The algebraic expression contains no more than three sets of parentheses, one set of brackets and ten terms. Each term contains up to two variables. The numerical coefficients are rational numbers and the exponents are natural numbers. The exponents assigned to the variables in the simplified form (solution) are all positive. The resulting polynomial must be in simplified form and its term must be ordered.)

- Determine the value of the numerical coefficient of a monomial as well as the value of the exponent assigned to each of the variables (bases) in the monomial. This monomial contains up to three variables. Identify similar monomials.

- Select the monomials, binomials, trinomials and polynomials from a list containing up to ten algebraic expressions and order a polynomial according to the increasing or decreasing powers of a given variable.

- Add or subtract two polynomials, each containing up to four terms. The terms of these polynomials contain no more than three variables. The numerical coefficients are rational numbers and the exponents are natural numbers. The resulting polynomial must be in simplified form and its terms must be ordered.

- Multiply two algebraic expressions: either a monomial by a monomial, a binomial by a binomial or a monomial by a polynomial containing up to three terms. Each term contains up to three variables. The numerical coefficients are rational numbers and the exponents are natural numbers. If the resulting product is a polynomial, its terms must be ordered.

- Divide two algebraic expressions: either a monomial by a monomial, a binomial by a monomial, a trinomial by a monomial or a trinomial by a binomial. Each term contains up to three variables. The numerical coefficients are rational numbers and the exponents (of the divisor, the dividend and the quotient) are natural numbers. If the resulting quotient is a polynomial, its terms must be ordered.

[*] Equivalent to GSM 231 in the 1993 program.

MTH-3002-2* **Geometry II (Perimeter, Area and Volume)**

1. To solve problems involving triangles and quadrilaterals

1.1. Using a ruler, a set-square and a protractor, construct triangles and quadrilaterals.

- Using a ruler, set-square and protractor, construct the following quadrilaterals:
  - a square, given the measure of one of its sides
  - a rectangle, given its length and width
  - a parallelogram, for which the given measures may be:
    - the long side, the short side and the altitude
    - the long side, the short side and one of the angles
    - the altitude and one of the angles
- Using a ruler, set-square and protractor, construct the following triangles:
  - an equilateral triangle, given the measure of a side
  - an isosceles triangle, given the measure of a side and the base or given the measure of the base and one of the angles
  - a right triangle, given the measure of the base and the altitude
- Indicate the altitudes, perpendicular bisectors, medians and bisectors in a triangle.
- Using a ruler, set-square and protractor, construct the following geometric figures:
  - a rhombus, given the lengths of the long and short diagonals
  - an isosceles trapezoid, given the lengths of the long base, the short base and one of the non-parallel sides
  - a right trapezoid, given the lengths of the long base, the short base and one of the non-parallel sides (the altitude)

* Equivalent to GSM 232 in the 1993 program.

1.2. **Solve problems that involve calculating the perimeter and area of any polygon by breaking it up into simpler figures and by applying the appropriate formulas.** (The students are assigned word problems related to everyday situations and accompanied by illustrations.)

- Using an appropriate formula, solve word problems involving the calculation of the perimeter or area of the following polygons:
  - squares
  - rectangles
  - parallelograms
  - triangles
  - rhombuses
  - trapezoids

2. To solve problems involving circles

2.1. **Given the radius, construct a circle using a ruler and compass.**

2.2. **Solve problems that involve calculating the circumference and area of a circle.** (The students are asked to solve word problems dealing with everyday situations.)

3. To solve problems involving solids

3.1. **Solve word problems that involve calculating the lateral and total area of the following solids: cubes, right prisms, cones and cylinders.** (A ruler is required. The problems deal with everyday situations.)

3.2. **Solve word problems that involve calculating the volume and capacity of the following solids: cubes, right prisms, cones and cylinders**. (A ruler is required. The problems deal with everyday situations. A conversion table showing units of volume and corresponding units of capacity is provided.)

MTH-3003-2*

Straight Lines I (Equations, Graphs and Slope)

1. To graph a line in a Cartesian plane

1.1. Graph an equation of the form $y = mx + b$ after completing a table of values containing at least five pairs of coordinates. (These equations represent everyday situations. The Cartesian plane used to graph the equation is already drawn; the axes are labelled and their graduations are shown.)

- Given the coordinates of a point, plot it in a graduated Cartesian plane. Then, identify the coordinates of a point located in a graduated Cartesian plane. The coordinates are rational numbers, but only the most common fractions and decimals are used.

1.2. In a Cartesian plane, graph an equation of the form $Ax + By + C = 0$. (Three points, including the x- and y-intercepts if any, must be plotted on the graph. The numbers used are rational numbers.)

2. To determine the slope of a line and its equation

2.1. Given the equation of a line, calculate its slope by converting the equation to the form $y = mx + b$ or $Ax + By + C = 0$.

- Calculate the slope of a line illustrating an everyday situation, given two points on that line.
- Calculate the slope of a line that passes through points (x_1, y_1) and (x_2, y_2), by applying the formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

  Indicate whether this slope is positive, negative, equal to zero, or undefined.
- Graph a line in the Cartesian plane, given the slope of the line and one of its points.

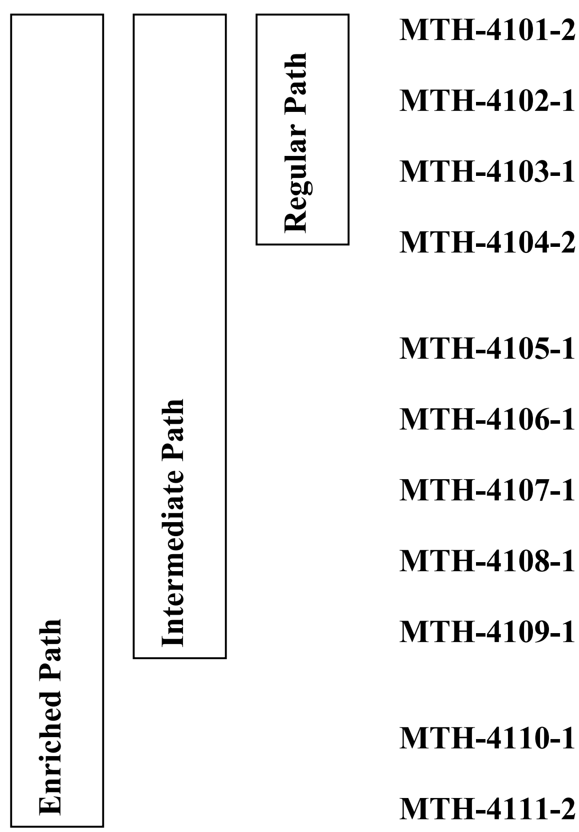
* Equivalent to GSM 241 in the 1993 program.

2.2. Determine the equation of a line, given two of its points or its slope and one of its points. The resulting equation must be expressed in one of the following forms: $y = mx + b$ or $Ax + By + C = 0$.

3. To solve problems involving lines

3.1. Solve word problems that involve drawing a graph in a Cartesian plane to determine one of the coordinates of the third point on a line. (The coordinates of two points on the line and one of the coordinates of the third point are given. These problems deal with everyday situations. The given numbers are rational numbers, but only the most common fractions and decimals are used.)

10.4. Secondary IV Courses



MTH-4101-2* **Equations and Inequalities II (First-Degree Systems)**

1. To solve a system of first-degree equations

1.1. Graphically solve a system of two first-degree equations in two variables. The equations are of the form $Ax + By + C = 0$, where the coefficients *A*, *B* and *C* are rational numbers.

1.2. Solve a system of two first-degree equations in two variables by applying one of the following methods:

- **graphing method**
- **substitution method**
- **comparison method**
- **elimination method by addition**

The equations are of the form $Ax + By + C = 0$, where the coefficients *A*, *B* and *C* are rational numbers.

- Solve a system of two first-degree equations in two variables by applying the comparison method. The equations are of the form $Ax + By + C = 0$, where the coefficients *A*, *B* and *C* are rational numbers.

- Solve a system of two first-degree equations in two variables by applying the substitution method. The equations are of the form $Ax + By + C = 0$, where the coefficients *A*, *B* and *C* are rational numbers.

- Solve a system of two first-degree equations in two variables by applying the elimination method. The equations are of the form $Ax + By + C = 0$, where the coefficients *A*, *B* and *C* are rational numbers.

1.3. Solve word problems that can be written as a system of two first-degree equations in two variables and that involve solving this system of equations. (These problems deal with everyday situations. The numbers used are rational numbers.)

* Equivalent to GSM 243 in the 1993 program.

2. To graphically solve a system of first-degree inequalities in two variables

2.1. Graphically solve a system of two first-degree inequalities in two variables. These inequalities can be of any of the following forms:

- $Ax + By + C < 0$
- $Ax + By + C > 0$
- $Ax + By + C \leq 0$
- $Ax + By + C \geq 0$

(The coefficients *A*, *B* and *C* are rational numbers.)

MTH-4102-1* **Geometry III (Isometry and Similarity)**

1. To identify certain geometric transformations

1.1. Determine the characteristics of isometric transformations.

- Given a set of diagrams representing isometric transformations of geometric figures, identify illustrations that represent a translation, illustrations that represent a rotation and illustrations that represent a reflection.

- Using a ruler, a set-square, a compass or a protractor, draw the images of simple geometric figures under the following transformations:

  - translation *t*, given the size and the direction of displacement
  - rotation *r*, given the location of the centre of rotation and the measure of the angle of rotation
  - reflection *s*, given the location of the axis of reflection

1.2. Determine the characteristics of a dilation.

- Using a ruler and a set-square, draw the image of a geometric figure under a dilatation *h*, given the location of the dilatation centre *o* and the scale factor *k*. The value of *k* can be either positive or negative.

- Given diagrams of geometric transformations, indicate those that represent a dilatation *h*.

2. To apply the properties of congruence and similarity in a variety of situations

2.1. Given the measures of some of the angles and some of the sides of two triangles, determine whether these triangles are congruent or similar by applying the properties of congruent and similar triangles. Give reasons for each conclusion.

- Using a ruler, a protractor and a compass, construct the triangle defined by one of the following groups of measures:

  - the measure of an angle and the lengths of the two sides that form this angle
  - the measures of two angles and the length of the side contained between these angles
  - the measures of the three sides

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* Equivalent to GSM 242 in the 1993 program.

2.2. Calculate the length of one or more sides in one of two similar triangles, given the length of one or more corresponding sides of the other triangle, the scale factor *k* or the lengths required to calculate *k*.

2.3. Calculate the length of one or more sides in one of two similar polygons, given the length of one or more corresponding sides of the other polygon, the scale factor *k* or the lengths required to calculate *k*. (The polygons have a maximum of eight sides.)

3. To solve problems requiring the application of the properties of congruent and similar figures

3.1. Given the measures on a plan, apply the properties of congruent and similar figures to solve problems that involve calculating actual dimensions and measures of angles. Given actual dimensions, apply these same properties to solve problems that involve calculating measures to be used in a scale diagram. (A ruler, a set-square and a protractor are required. Drawings illustrating various real-life situations will be provided.)

- Given a scale diagram representing an everyday situation, apply the properties of similar figures to solve word problems that involve calculating actual dimensions.

- Given a drawing that illustrates an everyday situation and using a ruler and a set-square, apply the properties of similar figures to solve problems that involve drawing a diagram to a given scale. Only 90° angles will be represented in the original drawing.

1. To solve problems requiring the application of trigonometric ratios to right triangles

1.1. Determine the unknown dimensions of a right triangle by using the definitions of trigonometric ratios, the Pythagorean theorem and the measures of certain angles in this triangle, as the case may be. There are two possible situations:

- **The lengths of two sides of the triangle are given.**
- **The length of one side and the measure of one acute angle are given.**

(A trigonometric table or a scientific calculator must be used.)

- Determine the measures of the angles and the sides of a right triangle, using the Pythagorean theorem and given the measures of some of its angles. There are two possible cases:
  - The measure of one acute angle and the lengths of two sides are given.
  - An angle that measures either 30° or 45° and the length of one side are given.

  The situations are presented in the form of word problems dealing with everyday situations.

- Given the lengths of the three sides of right triangle *ABC* (m < *C* = 90°), evaluate one of the three trigonometric ratios (sine, cosine and tangent) of angle A.

- Given the lengths of two of the sides of a right triangle, determine the measures of its acute angles by applying the definitions of trigonometric ratios and by using a trigonometric table or scientific calculator.

1.2. Solve word problems that involve determining the measures of angles and sides of right triangles by applying the definitions of the sine, cosine and tangent ratios. (The problems deal with everyday situations and involve determining the measures of angles, sides, or both angles and sides in a right triangle that is given or is to be drawn. A trigonometric table or a scientific calculator must be used.)

* Equivalent to GSM 244 in the 1993 program.

2. To solve problems requiring the application of the law of sines and the law of cosines to any given triangle

2.1. Solve word problems by applying one of the two laws given below:

- **Law of sines:** $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- **Law of cosines:** $a^2 = b^2 + c^2 - 2bc \bullet \cos A$

(The problems deal with everyday situations and involve determining the measures of angles, sides, or both angles and sides in any type of triangle that is given or to be drawn. A trigonometric table or a scientific calculator must be used.)

1. To identify the characteristics of different methods of gathering data for a statistical survey

1.1. Justify the choice of method used (census, sample survey, study) to collect the information needed to conduct a statistical survey.

- Distinguish between the concept of sample and population when gathering data for a statistical survey.

- Describe the characteristics specific to the following types of statistical surveys: a census, a sample survey or a study.

1.2. Demonstrate whether a sample is representative in the case of a sample survey whose characteristics (target population, expected reliability, etc.) are described.

- For a sample survey, determine the criteria that must be met in order for a sample to be representative of a given population:

  - the size of the sample depending on the size of the population studied and the expected reliability of the survey results
  - the sampling method to be used according to the situation (i.e. random sampling, stratified sampling or systematic sampling)

1.3. Interpret the results of a sample survey taking into account the margin of error and the distribution of the undecided respondents.

- Given the size of a sample, calculate the margin of error of a survey (and vice versa).

- Calculate the results of a survey in percentage terms, taking into account the margin of error and the distribution of the undecided respondents.

1.4. Identify sources of bias that may affect the conclusions of a statistical survey.

- Describe elements that can introduce bias:
  - the choice of a sample, which may or may not be representative of the entire population
  - the method of gathering data (telephone interviews, face-to-face interviews, written questionnaires, etc.), which can introduce subjective or other types of bias
  - the processing and analysis of data, where technical or other errors can bias the presentation of the results and the conclusions drawn

2. To interpret measures of central tendency, measures of dispersion and measures of position relating to a concrete situation

2.1. Solve word problems involving measures of central tendency (mean, median, mode) and of dispersion (range).

(The proposed situations can involve one or more one-variable distributions.)

- Describe the concept of a statistical variable.
- Distinguish between quantitative statistical variables and qualitative statistical variables.
- When a variable is quantitative, determine whether it is discrete or continuous.
- Construct various types of tables involving one statistical variable: tally sheet; frequency table (possible values of the variable with frequencies, relative frequencies or cumulative frequencies), in which the possible values may or may not be grouped into classes.
- Given a distribution, determine the measures of central tendency (mean, median, mode) and of dispersion (range). The data presented must consist of rational numbers that are related to a clearly defined concrete situation. The data are presented in a frequency table and may or may not be grouped into classes.
- Given data from a statistical survey involving one variable, determine the most appropriate measure of central tendency (mean, median, mode) in the context described. The distribution must be related to a concrete situation that is clearly defined.

2.2. In a distribution related to a concrete situation that is clearly defined, assign a quintile rank or a percentile rank to a particular data value. Determine the data value that corresponds to a given percentile rank in a distribution.

- Describe the concept of measure of position.

- Illustrate the concrete significance, in a distribution, of the most frequently used measures of position: quartiles, quintiles, deciles, percentiles.

- Distinguish between the role of measures of position and the respective roles of measures of central tendency and measures of dispersion (range).

2.3. Given a distribution that corresponds to a clearly defined concrete situation, construct a box-and-whisker plot and determine the various statistical measures that can be derived from it.

- Calculate the quartiles in a distribution.

- Construct a box-and-whisker plot representing a distribution.

- Construct a stem-and-leaf table corresponding to a distribution.

- Illustrate the complementary role of the stem-and-leaf table in relation to the box-and-whisker plot.

- Define the concept of interquartile range.

- Given a box-and-whisker plot, calculate the interquartile range.

2.4. Interpret the box-and-whisker plot for a distribution corresponding to a concrete situation that is clearly defined.

- Describe what information may be derived from a box-and-whisker plot.

- Using the interquartile range, determine if a distribution includes an outlier.

3. To analyze graphic representations of distributions

3.1. Compare distributions represented by graphs. (The distributions to be compared should be represented by box-and-whisker plots or by a box-and-whisker plot and another type of graph.)

- Analyze information obtained from the principal graphs that can represent a one-variable statistical distribution: bar graphs, circle graphs, broken-line graphs, histograms, pictographs, stem-and-leaf tables and box-and-whisker plots.

1. To apply the laws of exponents

1.1. Simplify an algebraic or numerical expression written in exponential form by applying the laws of exponents. (The simplification process involves the application of no more than four laws.)

- By applying the laws of exponents, transform the power of a natural number or of an algebraic expression into an exponential expression with the lowest possible base. The exponents can be positive or negative. The laws to be applied are the following:

  - $a^m \times a^n = a^{(m+n)}$
  - $\dfrac{a^m}{a^n} = a^{(m-n)}$
  - $a^0 = 1$
  - $\left(a^m\right)^n = a^{m \bullet n}$
  - $a^{-m} = \dfrac{1}{a^m}$
  - $\left(abc\right)^m = a^m b^m c^m$
  - $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

  In these expressions, *a, b* and *c* are rational numbers or variables and *m* and *n* are rational numbers.

- Describe the sign and the value of a^m, where *m* is an integer, given:

  - the sign of *a* (+ or −)
  - a description of the value of *a* ($|a| > 1$ or $0 < |a| < 1$)
  - a description of exponent *m* (i.e. sign of *m*, *m* is an even or odd number)

- Transform a number expressed in decimal notation into a number expressed in scientific notation, and vice versa.

1.2. Perform the four operations on algebraic or numeral expressions written in exponential form, then simplify the results, using positive exponents.

2. To convert an expression containing a radical into exponential form

2.1. Convert a numerical or algebraic expression containing a radical into a simplified expression in exponential form. (The given expression is of the form $a^m \bullet \sqrt[n]{b^p}$, where a and b are variables or positive rational numbers that are powers with the same base; n and p are natural numbers; m is a rational number.)

- Transform an algebraic expression or a numerical expression containing a radical into a simplified exponential expression. Transform an algebraic expression or a numerical expression given in exponential form into a simplified expression containing a radical: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$.

3. To simplify expressions containing radicals

3.1. Perform operations on a numerical expression containing a maximum of two polynomials and three square roots, then simplify it.

- Simplify an expression containing a numerical radical. The index of the root is two.

- Calculate the sum, difference, product or quotient of a numerical expression containing up to three square roots. In the case of a quotient, the denominator is at most a binomial containing just one simplified radical, which must be rationalized if necessary.

4. To compare algebraic or numerical expressions that are in exponential form or that contain radicals

4.1. Find the equivalent expressions for algebraic expressions that can be written in the form $a^m \sqrt[n]{a^p}$, where *a* is the base; *m* is a rational number; *n* is a natural number; *p* is an integer.

4.2. Find the equivalent expressions for numerical expressions that can be written in the form $a\sqrt{b}$ or $\left(ab^n\right)^m$, where *a, m* and *n* are rational numbers and *b* is a natural number.

4.3. Find the equivalent expressions for numerical expressions. (Each expression contains a maximum of two binomials of the form $\left(a\sqrt{b}+c\right)$ or no more than four terms of the form $a\sqrt{b}$. In the expression, *a* and *c* are integers and *b* is a natural number.)

1. To apply factoring techniques to polynomials

1.1. Factor a polynomial containing up to six terms as the product of no more than three prime factors by removing the common factor and applying one other appropriate factoring method selected from the list below:

- **factoring by grouping**
- **factoring trinomials of the form $x^2 + bx + c$ or $x^2 + bxy + cy^2$**
- **factoring trinomials of the form $ax^2 + bx + c$ or $ax^2 + bxy + cy^2$**
- **factoring a difference of squares**

▪ Find the common factor of all the terms of a polynomial containing up to six terms linked by + or - signs. The result must be expressed as the product of a monomial and a polynomial, which is placed in parentheses. The numerical coefficients of the terms of the polynomial are rational numbers, and the exponents of the variables are natural numbers.

▪ Factor a polynomial of up to six terms linked by + or – signs by applying the method of grouping. The result must be expressed as the product of two binomials or as the product of a binomial and a trinomial. The terms of the polynomial may have to be rearranged before being grouped and factored. The numerical coefficients of the terms of the polynomial are rational numbers, and the exponents of the variables are natural numbers.

▪ Factor a trinomial of the form $x^2 + bx + c$ or $x^2 + bxy + cy^2$, where b and c are integers. For trinomials of the form $x^2 + bx + c$, the result must be expressed as the product of two binomials of the form $(x + d) (x + e)$. For trinomials of the form $x^2 + bxy + cy^2$, the result must be expressed as the product of two binomials of the form $(x + dy) (x + ey)$. In both cases, d and e are integers.

▪ Factor a trinomial of the form $ax^2 + bx + c$ or $ax^2 + bxy + cy^2$, where a, b and c are integers. For trinomials of the form $ax^2 + bx + c$, the result must be expressed as the product of two binomials of the form $(kx + d) (x + e)$. For trinomials of the form $ax^2 + bxy + cy^2$, the result must be expressed as the product of two binomials of the form $(kx + ly) (mx + ny)$. In both cases, k, l, m and n are integers.

- Factor the difference of two squares as the product of two binomials consisting of the sum and by the difference of the square roots of each term of the initial algebraic expression. The difference of squares is of the form ax^{2n} - by^{2m}, where a and b are squares of rational numbers, x and y are variables, and n and m are natural numbers equal to or greater than 1 and less than or equal to 4.

2. To simplify algebraic fractions

2.1. Reduce a rational algebraic fraction to its lowest terms. The numerator and the denominator are factorable polynomials that contain up to three terms each. (Each term contains no more than two variables. The entire operation should involve no more than four factorizations and no more than two for each polynomial. If two factorizations are required for the same polynomial, one of these must involve finding the common factor.)

2.2. Reduce the product of two rational algebraic fractions to its lowest terms. (The polynomials in the numerators and denominators are factorable and contain up to three terms. Each term contains no more than two variables. The solution should require no more than four factorizations and no more than two for each polynomial. If two factorizations are required for the same polynomial, one of these must involve finding the common factor.)

2.3. Reduce the quotient of two rational algebraic fractions to its lowest terms. (The polynomials in the numerators and denominators are factorable and contain up to three terms. Each term contains no more than two variables. The solution should require no more than four factorizations and no more than two for each polynomial. If two factorizations are required for the same polynomial, one of these must involve finding the common factor.)

2.4. Simplify an algebraic expression containing two rational algebraic fractions that are added or subtracted. (The numerators and denominators are polynomials that contain up to three terms each and each term contains no more than two variables. These polynomials may or may not be factorable. If two factorizations are required for the same polynomial, one of these must involve finding the common factor. The common denominator must be made up of no more than two binomials and one monomial.)

3. To compare algebraic expressions containing two algebraic fractions that are added or subtracted

3.1. **Find the equivalent expressions for algebraic expressions by simplifying them.** (The expressions consist of the sum or difference of two algebraic fractions. The numerators and denominators are polynomials that contain up to three terms. Each term contains no more than two variables.)

MTH-4107-1* — Straight Lines II (Parallel and Perpendicular Lines and Distance)

1. To find the equation of a line that is parallel or perpendicular to another line

1.1. Determine the equation of a line, given one of the two following items of information:

- **the coordinates of one of its points and the equation of a line parallel to it**
- **the coordinates of one of its points and the equation of a line perpendicular to it**

(The coefficients of these linear equations and the coordinates of the points are rational numbers. The resulting equation must be of the form $y = mx + b$ or $Ax + By + C = 0$.)

- **Determine the equation of a line, given one of the three following items of information**:
  - the slope and the y-intercept of the line
  - the slope of the line and the coordinates of one of its points
  - the coordinates of two points on the line
- **Analyze the connections between equations representing lines.** (Lines can be parallel, perpendicular, coincident or convergent.)

2. To determine the distance between two points or the coordinates of the point of division of a segment

2.1. Determine the distance between two given points in the Cartesian plane. (The coordinates of these points and the distance are rational numbers, and the problems deal with everyday situations.)

2.2. Determine the point that divides a line segment in a particular ratio (including the midpoint). (The coordinates of the endpoints of the line segment are given. The coordinates and the ratio are rational numbers, and the problems deal with everyday situations. The ratio in which the segment is divided must be derived from the information given in the problem.)

* Equivalent to GMO 243 in the 1993 program.

3. To solve problems related to analytic geometry

3.1. Solve problems that involve applying the following concepts: calculating the distance between two points, determining the coordinates of the point of division of a segment and finding the equation of a line. (Solving the problem may involve all or some of these concepts.)

1. To formulate a second-degree equation representing a situation illustrated in a table

1.1. Using a partially completed table of values, formulate a second-degree equation for a word problem dealing with an everyday situation. The equation must be of the form $y = ax^2 + bx + c$, where *a*, *b* and *c* are rational numbers and $a \neq 0$.

1.2. Determine the value of the variables *x* and *y* that corresponds to a required maximum quantity (maximum output, maximum profit, maximum height), given a word problem dealing with an everyday situation that can be expressed as an equation of the form $y = ax^2 + bx + c$, where *a*, *b* and *c* are rational numbers and $a \neq 0$. (This must involve filling in a partially completed table of values or substituting different values for *x* in a second-degree equation when the equation and the values of *x* are given. The given values of *x* are usually natural numbers. The answer must be written as an ordered pair (*x*, *y*).)

2. To solve a second-degree equation algebraically

1.1 **2.1. Find the value of the discriminant $\Delta = b^2 - 4ac$ to determine the number of roots (none, one or two) of a second-degree equation of the form $ax^2 + bx + c = 0$, where *a*, *b* and *c* are rational numbers and $a \neq 0$. If necessary, solve this equation using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.** (The resulting values are real numbers.)

2.2. Using the appropriate factoring method (removing the common factor, factoring by grouping, factoring a trinomial of the form $ax^2 + bx + c$, factoring a difference of squares) as well as the zero product property, solve a second-degree equation of the form $ax^2 + bx + c = 0$, where *a*, *b* and *c* are rational numbers and $a \neq 0$.

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* Equivalent to GMO 251 in the 1993 program.

3. To graph a second-degree equation of the form $y = ax^2 + bx + c$

3.1. Graph a second-degree equation of the form $y = ax^2 + c$, where *a*, *b* and *c* are rational numbers and $a \neq 0$. (The result should be the graph of a parabola clearly indicating the following information: the coordinates of the vertex, the axis of symmetry and its equation, the *y*-intercept, the coordinates of the point symmetric with the *y*-intercept and, if necessary, the coordinates corresponding to the zeros of this equation.)

- Graph a second-degree equation of the form $y = ax^2$, where *a* is a rational number between –5 and 5 ($a \neq 0$). The result should be the graph of a parabola with the coordinates of the vertex, the axis of symmetry and the equation of the axis of symmetry clearly indicated.

- Graph a second-degree equation of the form $y = ax^2 + c$, where *a* is a rational number between –5 and 5 ($a \neq 0$) and where *c* is a rational number. The result should be the graph of a parabola clearly indicating the coordinates of the vertex, the axis of symmetry and the equation of the axis of symmetry. Whether the vertex of the parabola is a maximum or a minimum should also be indicated.

3.2. Given a graph of a second-degree equation of the form $y = ax^2 + bx + c$, determine the number of zeros in this equation and indicate the coordinates of the points that correspond to these zeros.

4. To solve a problem that involves determining a second-degree equation

4.1. Determine the abscissa and the ordinate of the maximum point or minimum point of a parabola, given a second-degree equation of the form $y = ax^2 + bx + c$, where *a, b* and *c* are rational numbers and $a \neq 0$. (The problems focus on situations related to science or business.)

4.2. Using the factoring method or the quadratic formula, solve a word problem that can be written as a second-degree equation of the form $ax^2 + bx + c = 0$, where a, *b* and *c* are rational numbers and $a \neq 0$. (Solving the problem involves rejecting inappropriate values and finding a maximum of two values. The problems deal with computation, geometry or everyday situations.)

1. To perform set operations on sets of real numbers

1.1. Perform a sequence of two set operations $(\cup, \cap, \setminus, ')$ on a maximum of three sets of real numbers that are graphed on a number line or described by means of interval or set-builder notation.

- Describe a set of integers by listing its elements, drawing a Venn diagram or using set-builder notation.
- Describe a set of real numbers by drawing a graph or by using set-builder or interval notation.
- Determine whether an element belongs to a set of numbers.
- Determine whether there is a relation of inclusion or equality between two sets.
- Perform set operations on sets whose elements are listed.

2. To study relations, the different ways of representing them and their characteristics

2.1. Draw a Cartesian graph of a relation defined by means of set-builder notation in a subset of $\mathbb{R} \times \mathbb{R}$ or given the graph of a relation, define it using set-builder notation. Determine the domain and the range of the relation. (The rule of correspondence must be expressed as a first-degree equation or inequality in one or two variables in $\mathbb{R} \times \mathbb{R}$.)

- Determine the Cartesian product of two sets.
- Distinguish between the source set and the target set.
- Describe a relation as a subset of a Cartesian product.
- Determine the source set, target set, domain and range of a relation, then specify the relation of inclusion or equality between the sets.

- In the Cartesian plane, graph the region associated with a first-degree inequality in one or two variables.

- Determine the first-degree inequality corresponding to a region of the plane.

3. To describe the characteristics of various functions

3.1. Describe the characteristics of various functions.

- For a concrete situation expressed in the form of a written statement, a graph, a table of values or its rule, determine whether the relation is a function.

- In a functional situation, determine the dependent variable and the independent variable.

- Describe a function using functional notation.

- Given a functional situation represented by a Cartesian graph, describe the characteristics of the function:

  - increasing or decreasing
  - sign
  - rate and type of change
  - axes of symmetry, if any
  - minimums or maximums, if any
  - *x*-intercept(s) (zeros)
  - *y*-intercept
  - domain and range
  - image of certain values of the domain
  - elements of the domain that are associated with a given image

3.2. Given a written statement providing relevant information about a functional situation, find the equation of the function corresponding to this situation, then write the equation in functional notation. (The situation should cover only those cases set out in the table below.)

- Draw a parabola given an equation of the form $y = ax^2 + bx + c$ or $y = (ax - h)^2 + k$ and determine the relationship between the standard form and the general form of the equation.

- Determine the standard or general form of the equation of a parabola, given the vertex and another one of its points.

- Determine the general form of the equation of a parabola, given its zeros and another point.

3.3. Given a functional situation described in the form of a written statement, a table of values or a rule, draw its corresponding Cartesian graph and describe the characteristics of the function. (Refer to the information provided in the table below.)

- Determine or estimate certain values that make up the domain or the range in a functional situation described in the form of a written statement, a Cartesian graph or a rule. The situation may be described by a combination of two of more functions over consecutive intervals.

- Translate from one mode of representation of a function to another according to the possibilities listed in the table below.

TO / FROM	Words (Written Statement)	Table of Values	Graph	Rule or Equation
Words (Written Statement)		YES	YES	YES[1]
Table of Values	NO		YES	YES[1]
Graph	NO	YES		YES[1]
Rule or Equation	NO	YES	YES[2]	

1. When translating a written statement, a graph or a table of values into a rule or an equation, students are limited to the following cases:

   – a first-degree polynomial function, given two points, or the slope and a point

   – a second-degree polynomial function, given the zeros and a point, or the vertex and a point

2. When translating a rule or an equation into a graph, various situations are possible:

   – polynomial functions, inverse variation functions, rational functions, square root functions, greatest integer functions, absolute value functions, exponential functions, etc. A technological tool may be used in the case of a function that has not been covered in class or in previous courses

4. To solve problems related to real functions

4.1. Solve problems by performing a comparative analysis of similar functional situations. (Each situation must be described as a function presented in the form of a written statement, a table of values, a rule or a graph.)

4.2. Solve problems related to second-degree polynomial functions. (The written information provided in the problem is generally illustrated in a diagram. The given information may be the zeros and a point or the vertex and a point. Solving the problem may require finding the rule of the function, the values of the domain or the range or the distance between two points.)

MTH-4110-1[*] Operations on Algebraic Fractions

1. To simplify algebraic expressions

1.1. Reduce, to its lowest terms, an algebraic expression containing up to four rational algebraic fractions that are multiplied and divided. (The numerators and denominators are factorable polynomials that contain up to four terms each, and each term contains no more than two variables.)

- Reduce a rational algebraic fraction to its lowest terms. The numerator and the denominator are factorable polynomials that contain up to four terms each, and each term contains no more than two variables.

- Reduce the product of three rational algebraic fractions to its lowest terms and divide two rational algebraic fractions. The polynomials in the numerators and denominators are factorable and contain up to four terms. Each term contains no more than two variables.

1.2. Reduce, to its lowest terms, an algebraic expression containing up to four rational algebraic fractions that are added and subtracted. (The numerators and denominators are factorable polynomials that contain up to three terms each, and each term contains no more than two variables.)

2. To apply the order of operations rule to algebraic fractions

2.1. Reduce, to its lowest terms, an algebraic expression containing up to four rational algebraic fractions by performing the appropriate operations and by following the order of operations. (This algebraic expression contains no more than two sets of parentheses. The numerators and denominators are factorable polynomials that contain up to four terms each, and each term contains no more than two variables.)

[*] Equivalent to GMO 242 in the 1993 program.

1. To solve a system of equations in two variables that consists of an equation of degree 0 or 1 and a second-degree equation

1.1. By comparison or substitution, solve algebraically a system of two equations in two variables. One of these equations is of degree 0 or 1 and the other is of degree 2.

- Translate a situation into a system of two equations in two variables. One of these equations is of degree 0 or 1 and the other is of degree 2.

- Graphically solve a system of two equations in two variables. One of these equations is of degree 0 or 1 and the other is of degree 2.

1.2. Solve problems that are represented by a system of two equations in two variables. One of these equations is of degree 0 or 1 and the other is of degree 2.

- Determine the number of solutions for a system of two equations in two variables according to the type of graph representing this system. One of the equations is of degree 0 or 1 and the other is of degree 2. The system may have no solutions (disjoint curves), one solution (tangent curves) or two solutions (intersecting curves).

2. To recognize a graph representing the result of an operation involving functions on the basis of its characterisics

2.1. Given graphs or equations of two polynomial functions of a degree less than 3, graph the sum, difference or product of these functions. (The solution must be of a degree less than 3. The equations may be parametric and the graphs may be sketched on paper without any underlying grid.)

- Find the sum, difference or product of two polynomial functions of a degree less than 3 and expressing the result in one or more of the following ways: as an equation, a graph or a table of values. The given polynomials may be presented as equations, graphs or tables of values.

- Recognize, in the original functions, certain important points (vertex, zeros, etc.) to be considered when performing an operation involving these functions.

- Describe certain characteristics of graphs representing the sum, difference or product of two polynomial functions of a degree less than 3.

3. To apply concepts of analytic geometry in a geometric context

3.1. Solve problems using the concepts of analytic geometry.

- Calculate the distance between a point and a line using a structured algebraic approach.
- Calculate the distance between a point and a line using the appropriate formula.

3.2. Determine the length of one or more sides, the equation of one or more altitudes or the perimeter or area of a polygon, given the coordinates of its vertices or the equations of its sides.

3.3. Given the coordinates of the vertices of a triangle or a quadrilateral, find the equation of its significant lines (e.g. altitudes, medians, perpendicular bisectors).

3.4. Given the coordinates of the vertices of a triangle or a quadrilateral, use the characteristics of the figure and analytic geometry to prove that it belongs to a specific category.

3.5. Prove simple statements using analytic geometry.

- Determine the important steps in presenting a proof:
  - identifying the given information and formulating hypotheses
  - identifying the required conclusion
  - organizing the steps in a line of deductive reasoning
  - justifying each step of the proof
- Complete a proof in which certain steps or justifications are missing.

4. To solve problems related to isometry, similarity and equivalence

4.1. Using the concepts of isometry, similarity or equivalence, solve problems by applying a rigorous approach that involves deductive reasoning.

- Using the properties of geometric transformations (translation, rotation, reflection, glide reflection, dilatation), describe congruent or similar figures (congruent corresponding angles, congruent or proportional corresponding sides or segments).

- Determine the properties of congruent or similar plane figures and solids: measures of the corresponding sides and angles, perimeters, areas, volumes, measures of all corresponding segments (altitudes, medians, etc.).

- Recognize equivalent plane figures as being figures that have the same area.

- Recognize equivalent solids as being solids that have the same volume.

- For congruent, similar or equivalent figures, where certain measures or ratios are given, calculate the missing measures or ratios. The given figures may be either plane figures or solids.

4.2. Prove geometric statements involving plane figures.

- State the minimum conditions under which two triangles can be considered congruent or similar.

- Complete a proof in which certain steps or justifications are missing, using the concepts of isometry and similarity.

APPENDIX

USEFUL PRINCIPLES OF GEOMETRY FOR MTH-4111-2

PART ONE: PRINCIPLES LEARNED IN PREVIOUS COURSES (numbers 1 to 35)

ANGLES

1. Adjacent angles whose external sides are in a straight line are supplementary.
2. Vertically opposite angles are congruent.
3. If a transversal intersects two parallel lines, then:
   a) the alternate interior angles are congruent
   b) the alternate exterior angles are congruent
   c) the corresponding angles are congruent
4. If two corresponding (or alternate interior or alternate exterior) angles are congruent, then they are formed by two parallel lines and a transversal.

TRIANGLES

5. The sum of the measures of the interior angles of a triangle is 180°.
6. In any triangle, the longest side is opposite the largest angle.
7. In any isosceles triangle, the angles opposite the congruent sides are congruent.
8. In any equilateral triangle, each angle measures 60°.
9. In any isosceles triangle, the perpendicular bisector of the side adjacent to the congruent angles is the bisector of the angle opposite this side as well as the median and altitude to this side.
10. In any right triangle, the acute angles are complementary.
11. In any isosceles right triangle, each acute angle measures 45°.
12. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides (Pythagorean theorem).
13. A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other two sides.
14. In a right triangle, the length of the side opposite a 30° angle is equal to half the length of the hypotenuse.
15. Two triangles whose corresponding sides are congruent must be congruent.
16. If two sides and the contained angle of one triangle are congruent to the corresponding sides and contained angle of another triangle, then the triangles must be congruent.
17. If two angles and the contained side of one triangle are congruent to the corresponding angles and contained side of another triangle, then the triangles must be congruent.
18. If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles must be similar.
19. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles must be similar.
20. If the lengths of two sides of one triangle are proportional to the lengths of the two corresponding sides of another triangle and the contained angles are congruent, then the triangles must be similar.

21. In a right triangle, the sine of an acute angle is equal to the ratio obtained by dividing the length of the side opposite this angle by the length of the hypotenuse.

$\sin A = \frac{a}{c}$, where a is the length of the side opposite angle A and c is the length of the hypotenuse.

22. In a right triangle, the cosine of an acute angle is equal to the ratio obtained by dividing the length of the side adjacent to this angle by the length of the hypotenuse.

$\cos A = \frac{b}{c}$, where b is the length of the side adjacent to angle A and c is the length of the hypotenuse.

23. In a right triangle, the tangent of an acute angle is equal to the ratio obtained by dividing the length of the side opposite this angle by the length of the side adjacent to it.

$\tan A = \frac{a}{b}$, where a is the length of the side opposite angle A and b is the length of the side adjacent to angle A.

24. The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides (law of sines):

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

25. The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle (law of cosines):

$$a^2 = b^2 + c^2 - 2bc \bullet \cos A$$

$$b^2 = a^2 + c^2 - 2ac \bullet \cos B$$

$$c^2 = a^2 + b^2 - 2ab \bullet \cos C$$

QUADRILATERALS

26. The opposite angles of a parallelogram are congruent.
27. The opposite sides of a parallelogram are congruent.
28. The diagonals of a parallelogram bisect each other.
29. The diagonals of a rectangle are congruent.
30. The diagonals of a rhombus are perpendicular to each other.

CIRCLES

31. All the diameters of a circle are congruent.
32. In a circle, the measure of a diameter is equal to twice the measure of the radius.
33. The axes of symmetry of a circle contain its centre.
34. The ratio of the circumference of a circle to its diameter is a constant known as π: $C = \pi d$ or $C = 2\pi r$, where C is the circumference, d is the diameter and r is the radius.
35. The area of a circle is equal to πr^2: $A = \pi r^2$, where A is the area and r is the radius.

PART TWO: PRINCIPLES OF GEOMETRY LEARNED IN THIS COURSE (Numbers 36 to 55)

ISOMETRIES AND CONGRUENT FIGURES

36. An isometry preserves collinearity, parallelism, the order of points, distances and the measures of angles. In addition, translations and rotations preserve the orientation of the plane.
37. Any translation will transform a straight line into another line parallel to it.
38. Plane figures or solids are congruent if and only if there is an isometry that maps one figure onto the other.
39. In congruent plane figures or solids, the measures of the following elements are equal:
    a) the corresponding segments and angles
    b) the perimeters
    c) the areas
    d) the volumes
40. Any point on the perpendicular bisector of a segment is equidistant from the two endpoints of this segment.
41. Any point on the bisector of an angle is equidistant from the sides of this angle.
42. In any right triangle, the length of the median to the hypotenuse is equal to half the length of the hypotenuse.*
43. The three perpendicular bisectors of the sides of a triangle are concurrent in a point that is equidistant from the three vertices.
44. The diagonals from one vertex of a convex polygon form $n - 2$ triangles, where n is the number of sides in that polygon.
45. The sum of the measures of the interior angles of a polygon is $180° (n - 2)$, where n is the number of sides in the polygon.
46. In a convex polygon, the sum of the measures of the exterior angles, one at each vertex, is 360°.

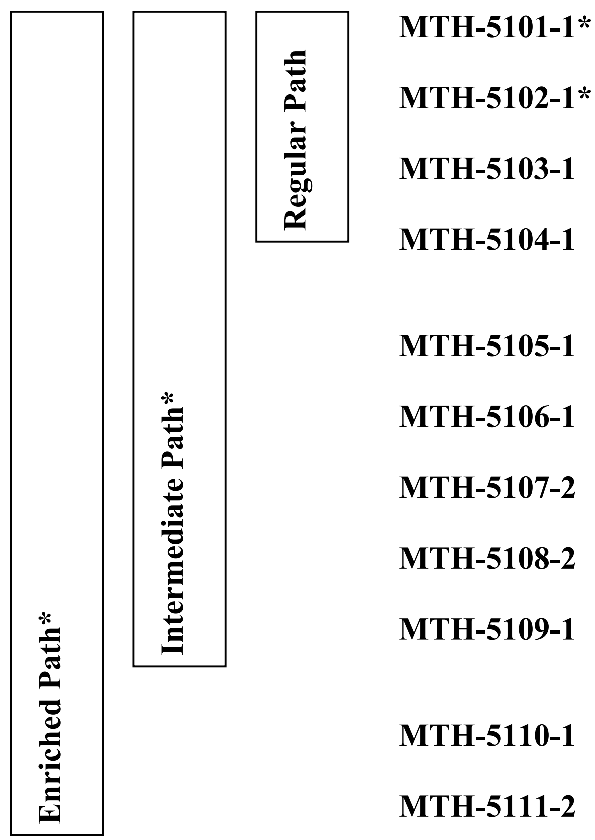
SIMILARITY TRANSFORMATIONS AND SIMILAR FIGURES

47. Any similarity transformation preserves collinearity, parallelism, the order of points, the orientation of the plane, the measures of angles and the ratio of the distances.
48. Any dilatation will transform a straight line into another line parallel to it.
49. Plane figures or solids are similar if and only if there is a similarity transformation that maps one figure onto the other.
50. In similar plane figures or solids:
    a) the ratio of the lengths of the corresponding segments is equal to the scale factor
    b) the ratio of the measures of the corresponding angles is 1
    c) the ratio of the areas is equal to the square of the scale factor
    d) the ratio of the volumes is equal to the cube of the scale factor
51. Plane figures or solids with a scale factor of 1 are congruent.
52. Any straight line that intersects two sides of a triangle and is parallel to the third side forms a smaller triangle similar to the larger triangle.

* Proving this statement also involves applying the principles associated with similar triangles.

53. Transversals intersected by parallel lines are divided into segments of proportional lengths.
54. The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.
55. The three medians of a triangle are concurrent in a point that is two-thirds the distance from each vertex to the midpoint of the opposite side.

10.5. Secondary V Courses



* MTH-5101-1 and MTH-5102-1 are prerequisites.

MTH-5101-1* **Optimization I (Linear Programming)**

1. To translate an optimization problem into mathematical language

1.1. Given the required elements of an optimization word problem, express the function to be optimized (i.e. the objective function) as an equation of the form $Ax + By + C = Z$, where A, B and C are rational numbers. In addition, write the inequalities that correspond to the constraints imposed on the variables x and y of the function to be optimized. Mathematize the constraints that affect each variable individually and the constraints that affect all the variables simultaneously. (Each problem should be no more than 150 words long.)

- For an optimization word problem, indicate the elements that make it possible to define the function to be optimized, or the objective function (i.e. the function that makes it possible to determine the required values). In addition, indicate the elements that make it possible to define the constraints imposed on the variables of the function to be optimized. Each problem should be no more than 150 words long.

2. To draw a polygon of constraints

2.1. Given a system of inequalities representing all the constraints imposed on the variables of a function to be optimized, graph all the given constraints in one Cartesian plane, draw the polygon of constraints defined by this system of inequalities and find the coordinates of the vertices of that polygon.

- Given a system of inequalities representing all the constraints imposed on the variables of a function to be optimized, perform algebraic operations to verify whether a given point belongs to the polygon of constraints defined by that system of inequalities.

* Equivalent to GSM 251 in the 1993 program.

3. To solve optimization problems

3.1. For a given optimization word problem, calculate the value of the objective function (function to be optimized) at each vertex of the polygon of constraints and determine the optimal solution. (Each problem should be no more than 150 words long.)

1. To interpret measures of dispersion and measures of positions in one-variable statistical distributions

1.1. Compare data from one or more one-variable statistical distributions by interpreting the standard score of each data value.

- Calculate the mean deviation of a one-variable statistical distribution.
- Calculate the variance and the standard deviation of a one-variable statistical distribution.
- Given a one-variable statistical distribution, compare information provided by different measures of dispersion: range, semi-interquartile range, mean deviation, and standard deviation.
- Determine the standard score (Z-score) of a data value in a one-variable statistical distribution.

1.2. Compare measures of dispersion and measures of position in one-variable distributions.

1.3. Solve problems involving one or more one-variable statistical distributions. (Solving the problem requires an analysis of the standard deviation, the standard score or the mean of the values in question.)

2. To analyze two-variable statistical distributions

2.1. In a given context, estimate or calculate and then interpret the coefficient of linear correlation between the two variables of a statistical distribution.

- Construct a joint distribution table corresponding to a two-variable statistical distribution.
- Draw a scatter plot corresponding to a two-variable statistical distribution.

- Given a scatter plot representing a two-variable statistical distribution, describe the linear correlation between these two variables:

  - given the orientation and distribution of the points, determine the direction of the correlation (positive or negative) as well as its degree (strong, moderate, weak or zero)
  - associate an approximate correlation coefficient value with the scatter plot

- By drawing a rectangle or an ellipse, estimate the coefficient of linear correlation between the two variables of a statistical distribution presented in the form of a data table or a scatter plot.

- Given data representing a two-variable statistical distribution, calculate the coefficient of linear correlation between these two variables using a numerical method.

2.2. Using the regression line, predict the data values corresponding to an ordered pair that is not represented on a scatter plot or in a table outlining a two-variable statistical distribution.

- Given a scatter plot representing a two-variable statistical distribution, estimate the position of the regression line.

- Apply a numerical method to determine the approximate equation of the regression line.

- Find the equation of the regression line and calculate the correlation coefficient using a technological tool.

2.3. Solve problems concerning situations that involve one or more two-variable statistical distributions. (Solving the problem may involve calculating and analyzing the correlation or the regression line. The distribution contains approximately 15 ordered pairs.)

1. To solve problems related to probabilities or the odds of an event occurring

1.1. By comparing the lengths or the areas of plane figures, calculate the probability or the odds of an event occurring in a geometric context.

- Calculate the probability of a simple event in a random experiment that involves only one step.
- Compare probabilities expressed in different forms.
- Given the sample space of a random experiment, define an impossible event, a certainty and a complementary event. Determine the probability of these events.
- In a random experiment, determine the odds of an event occurring ("odds for" and "odds against").
- Interpret an outcome expressed as "odds for" and "odds against."
- Describe an event from a random experiment in a geometric context: a set of points chosen at random in one- or two-dimensional geometric regions that represent a sample space.
- By comparing the lengths or the areas of plane figures, estimate the probability of an event in a geometric context.

1.2. Distinguish between the probability of an event, the odds of it occurring and the odds against it occurring. The event may occur in a geometric or non-geometric context.

1.3. Solve problems using probabilities or the odds of an event occurring in a two-dimensional geometric context.

- In a two-dimensional geometric context, determine the probability of an event by calculating the areas of the regions or the figures and then establishing the ratio of these areas.
- Justify a statement made in solving a problem.

2. To calculate the probability of an event in a random experiment consisting of several steps

2.1. In a random experiment consisting of no more than three steps, calculate the probability of an event or of its complementary event using an appropriate model, the rule of multiplication and, if required, the rule of addition.

- Depict a random experiment consisting of several steps by using an appropriate model:
  - tree diagram or list of all the possible outcomes
  - table or contingency table
  - probability tree diagram or area model
- Given a written description of a random experiment consisting of no more than three steps, use the rule of multiplication to calculate the following:
  - the total number of possible outcomes of a random experiment
  - the number of favourable outcomes of an event, which are described correctly
  - the probability of these events

The description of the random experiment may or may not include an appropriate model.

2.2. In a random experiment consisting of two steps, calculate the conditional probability of an event, given that another event in this experiment has occurred. (The experiment is described in writing and illustrated by a probability tree diagram or a contingency table.)

- Given a written description of a two-step random experiment, which includes a complete or incomplete contingency table, determine the following:
  - any data values missing from the tables
  - the probability of events using the various data values in the table
- When calculating the probability of compound events, identify situations where the conditional probability of an event must be determined, given that another event has occurred. The situations are described in writing and illustrated by a contingency table or a probability tree diagram.

3. To solve problems involving mathematical expectation

3.1. Given a description of various winning or losing situations based on the outcomes of a random game, show that a game situation is fair.

- Calculate the mathematical expectation of a random variable, determine whether it is positive, zero or negative and interpret this outcome.

3.2. Solve problems involving mathematical expectation, which may or may not be presented in a geometric context.

1. To identify certain types of graphs and their characteristics

1.1. Identify the characteristics of a graph or the type of graph in question, which may or may not represent a concrete situation.

- Draw a graph representing a situation.
- Describe the components of a graph and certain types of graphs:
  - vertices, edges, loops, degree of a vertex
  - paths, circuits, simple paths, simple circuits
  - connected graphs, complete graphs
- In a graph, determine whether the sum of the degrees of the vertices is equal to twice the number of edges.

1.2. Determine whether a given situation involves a route that passes along every edge or through every vertex of a graph only once. (The solution may involve drawing the graph corresponding to a diagram for the given situation.)

- In a graph, identify a Euler path or a Euler circuit.
- In a graph, identify a Hamiltonian path or a Hamiltonian circuit.

2. To use graphs to determine the optimal value in a given situation

2.1. In a weighted graph illustrating a concrete situation, determine the optimal path or circuit corresponding to a given route. (The problem may involve finding an optimal path or circuit that passes through every vertex once and only once.)

- Draw a weighted graph representing a situation (i.e. a graph in which a numerical value is assigned to each edge).
- In a weighted graph, calculate the value of a path or a circuit.
- In a weighted graph, calculate the value of the shortest path between two vertices.

2.2. Given a weighted graph illustrating a concrete situation that involves linking together various points in a network, construct a tree of minimum value that optimizes this network.

- Determine whether a graph is a tree (i.e. a connected graph that has no simple circuit).
- In a weighted graph, calculate the value of different trees corresponding to a given situation.
- In a weighted graph, identify the edges that constitute a tree of minimum value.

2.3. Using a weighted and directed graph illustrating a concrete situation, find the optimal path or circuit corresponding to a given route.

- Draw a directed graph representing a situation (i.e. a graph in which the edges are directed).
- Describe a directed graph using the terms *arc*, *path* and *circuit*.
- Interpret information provided in a directed graph.
- Draw a weighted and directed graph representing a situation (i.e. a graph in which each edge has a numerical value and a direction).
- In a weighted and directed graph, calculate the weight of different paths or circuits.

2.4. Use a weighted and directed graph to find the critical path of a project. (The solution may require drawing a graph illustrating the project.)

- Draw a weighted and directed graph that represents a project consisting of a certain number of steps. These steps should be described in such a way that the following may be determined:
    - the duration of each step
    - the steps that must be performed in sequence
    - the steps that may be performed simultaneously
- Using a weighted and directed graph representing a project, indicate the critical path of the project, i.e. the sequence of steps that cannot be delayed without delaying completion of the whole project.
- Using a critical path, calculate the minimum duration of a project.

3. To solve optimization problems using graphs

3.1. **Solve optimization problems using graphs.** (The solution to each problem may require drawing a graph illustrating the given situation.)

3.2. **Solve problems that involve finding the critical path and the minimum duration of a project.** (The project involves ten or so steps.)

1. To study a circle as a locus, its graph and its distinctive features

1.1. Graph the region determined by a relation defining a circle. In addition, find the domain and range of this relation and indicate them using interval or set-builder notation.

(The relation can be written in one of the following forms:

- $x^2 + y^2 + Dx + Ey + F = 0$ or $(x-h)^2 + (y-k)^2 = r^2$
- $x^2 + y^2 + Dx + Ey + F < 0$ or $(x-h)^2 + (y-k)^2 < r^2$
- $x^2 + y^2 + Dx + Ey + F \leq 0$ or $(x-h)^2 + (y-k)^2 \leq r^2$
- $x^2 + y^2 + Dx + Ey + F > 0$ or $(x-h)^2 + (y-k)^2 > r^2$
- $x^2 + y^2 + Dx + Ey + F \geq 0$ or $(x-h)^2 + (y-k)^2 \geq r^2$

The parameters D, E, F, h, k and r are rational numbers. The centre of the circle and its radius must be clearly indicated on the graph.)

1.2. Find the general form of the equation of a circle (i.e. $x^2 + y^2 + Dx + Ey + F = 0$), given its centre ($h, k$) and its radius r. Conversely, find the centre (h, k) and the radius r of a circle, given its equation in general form. (The parameters D, E, F, h, k and r are rational numbers and are usually integers.)

- Find and graph the equation of the circle obtained after a given circle centred at the origin (i.e. $x^2 + y^2 = r^2$) has been translated so that its centre becomes (h, k). The parameters h, k and r are integers.

1.3. Find the equation of a line tangent to a circle, given the point of tangency (x_1, y_1) and the equation of the circle in standard form [i.e. $(x-h)^2 + (y-k)^2 = r^2$] or in general form (i.e. $x^2 + y^2 + Dx + Ey + F = 0$). (The parameters x_1, y_1, h, k, r, D, E and F are integers.)

2. To study a parabola as a locus, its graph and its distinctive features

2.1. Graph the region determined by a second-degree relation corresponding to a parabola. In addition, find the domain and range of this relation and indicate them using interval or set-builder notation.

(The relation can be written in one of the following forms:

- $(y-k)^2 = \pm 4a(x-h)$ or $(x-h)^2 = \pm 4a(y-k)$
- $(y-k)^2 < 4a(x-h)$ or $(x-h)^2 < \pm 4a(y-k)$
- $(y-k)^2 \leq \pm 4a(x-h)$ or $(x-h)^2 \leq \pm 4a(y-k)$
- $(y-k)^2 > 4a(x-h)$ or $(x-h)^2 > \pm 4a(y-k)$
- $(y-k)^2 \geq \pm 4a(x-h)$ or $(x-h)^2 \geq \pm 4a(y-k)$

The parameter a is a natural number and $a \neq 0$, whereas h and k are rational numbers that are usually integers. Clearly indicate the following on the graph: the vertex, the focus, the axis of symmetry and the directrix of the parabola.)

2.2. Find the standard form of the equation of a parabola [i.e. $(y-k)^2 = \pm 4a(x-h)$ or $(x-h)^2 = \pm 4a(y-k)$], given the coordinates (h, k) of its vertex and the coordinates (x_1, y_1) of its focus. (The parameter a is a natural number other than 0 and h, k, x_1 and y_1 are integers.)

- Find and graph the equation of a parabola obtained after a given parabola with vertex at the origin (i.e. $y^2 = \pm 4ax$ or $x^2 = \pm 4ay$) has been translated so that its vertex becomes (h, k). The constants h and k are integers, and a is a natural number other than 0.

3. To study an ellipse as a locus, its graph and its distinctive features

3.1. Graph the region determined by a relation defining an ellipse centred at the origin. Indicate the axes and the two foci of the ellipse clearly on the graph. In addition, find the domain and the range of this relation and indicate them using interval or set-builder notation.

(The relation can be written in one of the following forms:

- $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
- $\frac{x^2}{a^2}+\frac{y^2}{b^2}<1$
- $\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1$
- $\frac{x^2}{a^2}+\frac{y^2}{b^2}>1$
- $\frac{x^2}{a^2}+\frac{y^2}{b^2}\ge 1$

The parameters *a* and *b* are natural numbers other than 0.)

4. To study a hyperbola as a locus, its graph and its distinctive features

4.1. Graph the region determined by a relation defining a hyperbola centred at the origin. Indicate the asymptotes, vertices and foci of the hyperbola clearly on the graph. In addition, find the domain and range of this relation and indicate them using interval or set-builder notation.

(The relation can be written in one of the following forms:

- $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
- $\frac{x^2}{a^2}-\frac{y^2}{b^2}<1$
- $\frac{x^2}{a^2}-\frac{y^2}{b^2}\le 1$
- $\frac{x^2}{a^2}-\frac{y^2}{b^2}>1$
- $\frac{x^2}{a^2}-\frac{y^2}{b^2}\ge 1$
- $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
- $\frac{y^2}{b^2}-\frac{x^2}{a^2}<1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}>-1$
- $\frac{y^2}{b^2}-\frac{x^2}{a^2}\le 1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}\ge -1$
- $\frac{y^2}{b^2}-\frac{x^2}{a^2}>1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}<-1$
- $\frac{y^2}{b^2}-\frac{x^2}{a^2}\ge 1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}\le -1$

The parameters *a* and *b* are natural numbers other than 0.)

5. To find the inequality or the equation associated with a conic

5.1. Find the equation or inequality associated with the graph of one of the following conics: a circle, a parabola, an ellipse centred at the origin or a hyperbola centred at the origin. The graph may or may not contain a shaded region, depending on the situation. (The distinctive features of each curve [i.e. the radius, the centre, one or more vertices, one or more foci, one or more axes of symmetry, the directrix, the asymptotes, as the case may be] are clearly indicated on the graph. The equation or inequality must be written in standard form.)

5.2. Find the equation of a conic given a description of another conic. The equation or certain features of this other conic are known. These conics and their respective features can be any of the following: a circle (centre and radius), a parabola (vertex, focus and directrix), an ellipse and a hyperbola centred at the origin (vertices, foci, equations of the asymptotes). (The features may be described indirectly.)

5.3. Given their definition as a locus, find the equation of one of the following conics: a circle, a parabola, an ellipse centred at the origin and a hyperbola centred at the origin.

6. To solve problems involving conics

6.1. Solve problems that involve applying concepts related to the following conics: a circle, a parabola, an ellipse centred at the origin or a hyperbola centred at the origin. (The solution may require finding an equation describing a relation, drawing a graph, determining the coordinates of certain points and calculating the distance between certain points. The equations should be given in standard form only, with the exception of those associated with a circle, which can be defined in general form.)

1. To recognize functions and determine their characteristics given their rule or graph

1.1. Determine the connections between the change in a parameter of the rule of a real function and the transformation of the corresponding Cartesian graph for various functions.**

(The functions may be:

- linear function $f(x) = ax + b$
- quadratic function $f(x) = a(b(x - h))^2 + k$
- absolute value function $f(x) = a|b(x - h)| + k$
- greatest integer function $f(x) = a[b(x - h)] + k$
- square root function $f(x) = a\sqrt{b(x - h)} + k$
- rational function $f(x) = \dfrac{a}{b(x - h)} + k$)

▪ Distinguish a real function from a relation by referring to its characteristics.

▪ Determine whether a given relation is a function. The elements of the relation are listed or defined by means of set-builder notation or a Cartesian graph.

1.2. Given the rule of a real function, draw a Cartesian graph representing this function.

(The source and target sets are subsets of $\mathbb{R}$, given in the form of continuous intervals.)

* For this course, the use of an appropriate technological tool (graphing calculator or computer with appropriate software) is recommended as it helps students learn how to analyze these functions, while reducing the time devoted to finding the equation and drawing the graph.

** For the quadratic function, the absolute value function and the rational function, parameter *b* should be considered only in terms of its specific effect on the graph. For the remainder of the concepts studied in this course, parameters *a* and *b* will be combined. The rules used will be of the form $f(x) = a(x - h)^2 + k$ or $f(x) = a|x - h| + k$.

1.3. Given the rule or the Cartesian graph of a real function, identify the function and determine its characteristics.

(The following characteristics are studied:

- domain and range
- image of certain values of the domain
- elements of the domain that are associated with a given image
- coordinates of the vertex
- zeros
- *y*-intercept
- slope or rate of change
- equations of the asymptotes
- equation of the axis of symmetry
- maximum or minimum
- intervals over which the function is increasing or decreasing, with an explanation
- sign with an explanation in the form of a graph)

2. To solve the equation of certain real functions or to determine the inverse of these functions

2.1. Solve the equation of a real function (with the exception of rational and greatest integer functions).

2.2. Determine the inverse of a linear function, a quadratic function or a square root function defined by means of set-builder notation.

2.3. Determine whether the inverse of a real function is a function.

- Graphically determine the inverse of a real function defined by its graph or by means of set-builder notation.

3. To solve problems related to real functions

3.1. Determine the rule of a real function defined by relevant information or by a Cartesian graph (with the exception of rational and greatest integer functions).

3.2. Solve problems that involve applying concepts related to real functions. (The solution may involve describing certain characteristics of a function, graphing a function, calculating the distance between certain points on the graph, determining the rule to model a situation, determining the inverse of a function or comparing certain characteristics of various functions over a given interval.)

MTH-5107-2 Exponential and Logarithmic Functions and Equations*

1. To study the graphs and characteristics of exponential functions

1.1. Graph an exponential function of the form $f(x) = a \bullet c^{b(x-h)} + k$ and determine the connections between the change in a parameter of the rule and the transformation of the corresponding Cartesian graph.

- Identify the base, exponent and power in an exponential expression.
- Multiply or divide algebraic expressions using the following laws of exponents:
  - $a^m \times a^n = a^{(m+n)}$ $\qquad$ $\dfrac{a^m}{a^n} = a^{(m-n)}$
  - $a^0 = 1$ $\qquad$ $(a^m)^n = a^{m \bullet n}$
  - $a^{-m} = \dfrac{1}{a^m}$ $\qquad$ $(abc)^m = a^m b^m c^m$
  - $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$ $\qquad$ $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

1.2. Given the rule or the Cartesian graph of an exponential function of the form $f(x) = a \bullet c^{b(x-h)} + k$, determine the characteristics of the function.

(The following characteristics are studied:

- equation of the asymptote
- domain and range
- zeros
- *y*-intercept
- intervals over which the function is increasing or decreasing, with an explanation
- sign with an explanation

* For this course, the use of an appropriate technological tool (graphing calculator or computer with appropriate software) is recommended as it helps students learn how to analyze these functions, while reducing the time devoted to finding the equation and drawing the graph.

- image of certain values of the domain
- the element of the domain associated with a given image)

1.3. Compare the characteristics of two exponential functions given their rule or their graph.

2. To find the rule of a given exponential function or its inverse

2.1. Find the rule of an exponential function of the form $f(x) = \pm c^x + k$, given relevant information or its graph. (The relevant information or the graph must include the equation of the asymptote or the coordinates of a point whose x-coordinate is not 0.)

2.2. Find the inverse of an exponential function of the form $f(x) = \pm c^x + k$. The inverse of the function must be written in set-builder notation.

3. To apply the properties of logarithms to simplify logarithmic expressions

3.1. Simplify a logarithmic expression by applying the properties of logarithms. (The given expression should contain no more than three algebraic or numerical terms, each of which is a logarithm or a number to be expressed in logarithmic form. Simplifying the expression may involve the simple factorization of polynomials. The result should be a simplified logarithmic expression or a numerical value.)

- Convert an expression from exponential form into logarithmic form or vice versa. The exponential expressions are of the form $y = c^x$ and the logarithmic expressions are of the form $y = \log_c x$.
- Calculate the value of a logarithm in base 10, in base *e* and in any other base.

- Prove and apply the following properties of logarithms:

$\log_c 1 = 0$	$\log_c c = 1$
$\log_c c^n = n$	$\log_c M^n = n \bullet \log_c M$
$\log_{\frac{1}{c}} M = -\log_c M$	$\log_c M = \dfrac{\log_a M}{\log_a c}$
$\log_c (M \bullet N) = \log_c M + \log_c N$	$\log_c \left(\dfrac{M}{N}\right) = \log_c M - \log_c N$

where $M, N, c \in \mathbb{R}_+^*$ and $c \neq 1$.

4. To solve logarithmic equations or exponential equations

4.1. Solve a logarithmic equation in which the two members of the equation can be expressed in the same base.

4.2. Using the properties of logarithms, solve a logarithmic equation in which each member of the equation can be simplified so that the resulting expression contains only one logarithm. (The given equation should consist of no more than three terms. One of the members of the equation may contain an algebraic expression of degree 2.)

4.3. Solve an exponential equation where the two members of the equation are not powers of the same base. (The exponents of the bases are numbers or algebraic expressions of degree 1.)

5. To study the graphs and characteristics of logarithmic functions

5.1. Graph a logarithmic function of the form $f(x) = a \bullet log_c(b(x-h)) + k$ and determine the connections between the change in a parameter of the rule and the transformation of the corresponding Cartesian graph.

5.2. Given the rule of a logarithmic function of the form $f(x) = a \bullet log_c(b(x-h)) + k$, determine the characteristics of the function.

(The following characteristics are studied:

- equation of the asymptote
- domain and range
- zeros
- *y*-intercept
- intervals over which the function is increasing or decreasing, with an explanation
- sign with explanation
- image of certain values of the domain
- the element of the domain associated with a given image)

5.3. Compare the characteristics of two logarithmic functions given their rule or their graph.

5.4 Compare the characteristics of a logarithmic function and those of an exponential function given their rule or their graph.

6. To find the rule of a logarithmic function or its inverse

6.1. Find the rule of a logarithmic function of the form $f(x) = log_c \pm (x-h)$ given relevant information or its graph. (The relevant information or the graph must include the equation of the asymptote or the coordinates of a point whose *y*-coordinate is not 0.)

6.2. Find the inverse of a logarithmic function of the form $f(x) = log_c \pm (x-h)$. The inverse of the function must be written in set-builder notation.

7. To solve problems involving exponential and logarithmic functions

7.1. Solve problems that involve applying concepts related to exponential or logarithmic functions. (Depending on the context, the solution may involve finding a rule, drawing a graph, determining certain characteristics of the function and deducing certain information. For logarithmic functions, the rule will be provided.)

1. To study the unit circle and wrapping function

1.1. Identify the image of a trigonometric angle *t* under the wrapping function. Determine the reference angle t' ($0 \leq t' \leq 2\pi$) corresponding to angle *t*. (Angle *t* is expressed in radians and is of the form $n\pi$, $\frac{n\pi}{2}$, $\frac{n\pi}{3}$, $\frac{n\pi}{4}$ or $\frac{n\pi}{6}$, and *n* is an integer.)

- Determine the measure of an angle in degrees or radians.
- Given the central angles of a circle, convert angular measures from degrees into radians and vice versa.
- Using the unit circle and the wrapping function, determine the coordinates of the trigonometric points.

1.2. Determine the measure of a trigonometric angle in radians over a designated interval, given the coordinates of a trigonometric point. (The interval is of the form $[n\pi, n\pi + 2\pi]$, where *n* is an integer.)

2. To study trigonometric functions

2.1. Find the value of the image of a trigonometric function associated with a trigonometric angle. (The angle is expressed in radians and is of the form $n\pi$, $\frac{n\pi}{2}$, $\frac{n\pi}{3}$, $\frac{n\pi}{4}$ or $\frac{n\pi}{6}$ and *n* is an integer.)

- Define the trigonometric *sine*, *cosine*, *tangent*, *cotangent*, *secant* and *cosecant* functions in the context of a unit circle and the wrapping function.

* For this course, the use of an appropriate technological tool (graphing calculator or computer with appropriate software) is recommended as it helps students learn how to analyze these functions, while reducing the time devoted to finding the equation and drawing the graph.

2.2. Given the rule or graph of the *sine*, *cosine* and *tangent* functions, determine the characteristics of the functions:

(The following characteristics are studied:

- domain and range
- image of an element of the domain
- element or elements of the domain associated with a given image
- maximum and minimum
- zeros
- period
- *y*-intercept
- intervals over which the function is increasing or decreasing
- sign of the function
- equations of the asymptotes)

▪ Graph the *sine*, *cosine* and *tangent* functions over a designated interval.

2.3. Compare the characteristics of the *sine*, *cosine* and *tangent* functions over a designated interval.

3. To prove trigonometric identities and to solve simple trigonometric equations

3.1. Given the value of a trigonometric ratio at one point within a designated interval, determine the value of the other trigonometric ratios at this point using the fundamental identities. (The interval corresponds to an arc of no more than π radians and its limits are multiples of $\frac{\pi}{2}$.)

▪ Prove the fundamental trigonometric identities:

- $\sin^2 x + \cos^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $\cot^2 x + 1 = \csc^2 x$

▪ Apply the fundamental identities and the definitions of trigonometric ratios to the transformation of simple trigonometric expressions.

3.2. Prove a simple trigonometric identity. (The expression should consist of no more than two terms on each side of the equality. Each term should contain no more than two trigonometric ratios. The definitions of trigonometric ratios and the fundamental identities will not be provided during examinations.)

- Perform the four operations on trigonometric expressions, simplify trigonometric expressions and factor trigonometric expressions.

3.3. Using the unit circle or a calculator, solve a simple first- or second-degree trigonometric equation over a given interval or in $\mathbb{R}$. (The solution may require simple factorization. The limits of the interval must be multiples of π.)

- For a given trigonometric function, find the image of an angle given in radian measure, using a calculator. Given the value of a trigonometric function expressed as a real number, determine the corresponding trigonometric angle over a given interval or in $\mathbb{R}$. The limits of the interval are multiples of π.

3.4. Using the trigonometric identities related to the sum or the difference of real numbers or to twice the value of a real number, simplify a trigonometric expression. (The expression should consist of no more than two terms on each side of the equality and the expression should consist of no more than four trigonometric functions in all.)

- Using simple examples, verify the trigonometric identities related to a sum or a difference of real numbers or to twice the value of a real number:
  - $\sin(A+B) = \sin A \cos B + \cos A \sin B$
  - $\sin(A-B) = \sin A \cos B - \cos A \sin B$
  - $\cos(A+B) = \cos A \cos B - \sin A \sin B$
  - $\cos(A-B) = \cos A \cos B + \sin A \sin B$
  - $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$, where $1 - \tan A \tan B \neq 0$
  - $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$, where $1 + \tan A \tan B \neq 0$
  - $\sin 2A = 2 \sin A \cos A$
  - $\cos 2A = \cos^2 A - \sin^2 A$
  - $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$, where $1 - \tan^2 A \neq 0$

- Using the trigonometric identities related to a sum or a difference of real numbers, prove the cofunction identities, the odd-even identities, the double-angle identities or a reduction formula.

  - When the proof involves identities related to the *sine* or *cosine* functions, A or B is a multiple of $\frac{\pi}{2}$ or a variable.
  - When the proof involves identities related to the *tangent* function, A or B is a multiple of $\frac{\pi}{4}$ or a variable.

  (The formulas will be provided during examinations.)

4. To study the graphs, characteristics and rules of sinusoidal functions

4.1. Using the rule or graph of a sinusoidal function, determine the characteristics of the function.

(The following characteristics are studied:

- maximum and minimum
- amplitude
- period
- frequency
- domain and range
- sign of the function
- y-intercept
- zeros
- intervals over which the function is increasing or decreasing
- image of an element of the domain
- element or elements of the domain associated with a given image
- phase shift
- vertical shift)

- Graph a sinusoidal function of the form $f(x) = a\sin b(x - h) + k$ or $f(x) = a\cos b(x - h) + k$ and determine the connections between the change in the parameters of the rule and the transformation of the corresponding graph.

4.2. Determine the rule of a sinusoidal function given relevant information or the graph of the function.

4.3. Compare the characteristics of two sinusoidal functions given their graph.

5. To solve problems involving sinusoidal functions

5.1. Solve problems that involve applying concepts related to sinusoidal functions. (The solution may involve finding the rule of a sinusoidal function, describing certain characteristics of a sinusoidal function, determining the connections between the change in the parameters of the rule and the transformation of the corresponding graph, or comparing certain characteristics of different sinusoidal functions over a given interval.)

1. To solve problems pertaining to relationships governing measurements within the same circle or involving two circles

1.1. Solve problems related to a variety of human activities (e.g. carpentry, land surveying, architecture and technical drawing). (Relevant conclusions can be drawn from a list of theorems and corollaries pertaining to the relationships governing measures in a circle [see appendix, principles 1 to 17] and by referring to diagrams of one or two circles, which indicates the measures of angles, segments or arcs.)

- On diagrams of circles in which several elements are represented and labelled with upper case letters, find the following: a radius, a diameter, a chord, an arc, a secant, a tangent, a point of tangency, a central angle, an inscribed angle, an interior angle and an exterior angle.

- Given a list of theorems and corollaries (see appendix, principles 1 to 13) as well as a diagram of one or two circles, which indicates the measures (either numerical or literal) required to draw relevant conclusions, find a measure (radius, diameter, chord, arc or tangent segment, circumference, area) and indicate the theorems or corollaries used to support each step in the solution.

- Given a list of theorems and corollaries (see appendix, principles 14 to 17) as well as a diagram of a circle, which indicates the measures (either numerical or literal) required to draw relevant conclusions, determine the measure of a central angle, an inscribed angle, an interior angle, an exterior angle or the measure of an arc in degrees, indicating the theorems or corollaries used to support each step in the solution.

1.2. Verify the relationship between the elements or the measures of one or two circles. (The measures can be numerical or literal.)

* Equivalent to GSM 253 in the 1993 program.

2. To solve problems pertaining to relationships governing measurements in right triangles

2.1. Solve problems related to a variety of human activities (e.g. carpentry, land surveying, architecture and technical drawing). (Relevant conclusions can be drawn from a list of theorems and corollaries pertaining to the relationships governing measures in right triangles [see appendix, principles 18 to 24] and by referring to a diagram of a right triangle, which indicates the measures of angles and segments.)

- Given a list of theorems and corollaries (see appendix, principles 18 to 24) as well as a diagram of a right triangle, which indicates the measures (either numerical or literal) required to draw relevant conclusions, determine a measure (the measure of an angle, a side, a median, an altitude, the hypotenuse, the perimeter or the area) of a right triangle, indicating the theorems or corollaries used to support each step in the solution.

2.2. Verify the relationship between the elements or the measures of one or two right triangles. (The measures can be numerical or literal.)

APPENDIX

PRINCIPLES OF GEOMETRY FOR MTH-5109-1

RELATIONSHIPS GOVERNING MEASUREMENTS WITHIN A CIRCLE

1. Any perpendicular bisector of a chord is a diameter of a circle.
2. The longest chord of a circle is a diameter.
3. In a circle, any radius perpendicular to a chord divides that chord into two congruent segments.
4. In a circle, any radius perpendicular to a chord divides the subtended arc into two congruent arcs.
5. In a circle, arcs located between two parallel chords are congruent.
6. Two chords are congruent if they are equidistant from the centre of the circle.
7. In a circle, congruent chords subtend congruent arcs and, conversely, congruent arcs are subtended by congruent chords.
8. Any line tangent to a circle is perpendicular to the radius that shares the point of tangency.
9. For any circle, two tangent segments originating from the same exterior point are congruent. (The segments are measured from that exterior point to their respective points of tangency.)
10. Two parallel lines, be they tangents or secants, intercept congruent arcs of a circle.

RELATIONSHIPS GOVERNING MEASUREMENTS IN TWO CIRCLES

11. The circumferences of two circles have the same ratio as their radii.
12. The areas of two circles have the same ratio as the squares of their radii.
13. The measures of similar arcs of two circles have the same ratio as their radii.

RELATIONSHIPS GOVERNING ANGULAR MEASUREMENTS WITHIN A CIRCLE

14. In a circle, the measure of a central angle is equal to the measure of its intercepted arc.
15. In a circle, the measure of an inscribed angle is one-half the measure of its intercepted arc.
16. The measure of an angle formed by two chords intersecting in the interior of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
17. The measure of an angle formed by two tangents, a tangent and a secant, or two secants is one-half the difference of the measures of the intercepted arcs.

RELATIONSHIPS GOVERNING MEASUREMENTS IN RIGHT TRIANGLES

18. The hypotenuse of a right triangle inscribed in a circle is always a diameter of that circle.
19. In a right triangle, the length of the median to the hypotenuse is one-half the length of the hypotenuse.
20. In a right triangle with a 30° angle, the length of the side opposite this angle is one-half the length of the hypotenuse.
21. A right triangle and the altitude to its hypotenuse form two right triangles that are similar to the given triangle and to each other.
22. The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse.
23. The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse.
24. The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse.
25. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides (Pythagorean theorem).

1. To know the definitions, notations and symbols relating to vectors

1.1. Determine whether a vector is a zero vector or a unit vector, or whether the two vectors are collinear or noncollinear, orthogonal (perpendicular), equipollent (equivalent) or opposite.

- Describe a geometric vector as a directed arrow on a plane: a quantity that has a norm (magnitude) and a direction.
- Distinguish between the concepts of vector and scalar.
- Use different concepts related to vectors:
  - distinctive symbol $\vec{v}$
  - tail A and head B of vector $\overrightarrow{AB}$
  - norm $\|\vec{v}\|$
  - direction: angle of direction, angle in relation to cardinal points, angle in relation to vertical or horizontal line
- Draw an algebraic vector in a Cartesian plane:
  - tail (X_1, Y_1) and head (X_2, Y_2)
- Describe an algebraic vector using the notation for vector components:
  - $\vec{v} = (\overrightarrow{a, b})$, where $a = X_2 - X_1$ and $b = Y_2 - Y_1$
- Given the components of an algebraic vector, calculate:
  - the norm of the vector
  - the angle of direction that describes the vector's direction

2. To add vectors

2.1. Given the description of two vectors, determine:

- **their sum (resultant): norm and direction**
- **the angle between the resultant and each original vector**

- Add two vectors using:
  - the geometric method (the triangle method and the parallelogram method)
  - the algebraic method (adding their components)
- State the addition properties of vectors:
  - commutativity: $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
  - associativity: $\left(\vec{u} + \vec{v}\right) + \vec{w} = \vec{u} + \left(\vec{v} + \vec{w}\right)$
  - existence of an identity element: $\vec{u} + \vec{0} = \vec{u}$
  - existence of an opposite: $\vec{u} + \left(-\vec{u}\right) = \vec{0}$
- Given the norms of two vectors and the angle formed by these vectors when their tails coincide, calculate the norm of the sum (resultant) of the two vectors.
- Given the norms of two vectors and their resultant, calculate the angle formed by the two vectors when their tails coincide.

3. To give a linear combination of vectors

3.1. Perform a linear combination of two vectors described by their components or by their norm and direction. Determine the characteristics of the result: components, or norm and direction.

- Multiply a vector $\vec{v}$ by a scalar *k*.
- Describe the characteristics of product $k\vec{v}$:
  - $k\vec{v}$ is a vector with the same direction as $\vec{v}$ (collinear with $\vec{v}$)
  - the norm of $k\vec{v}\left(\left\|k\vec{v}\right\|\right)$ is equal to $|k|\left\|k\vec{v}\right\|$
  - $k\vec{v}$ has the same direction as $\vec{v}$ if $k > 0$, or the opposite direction if $k < 0$
  - if $\vec{v} = \left(\overrightarrow{a, b}\right)$, then $\vec{k} = \left(\overrightarrow{ka, kb}\right)$
- State the properties of the multiplication of a vector by a scalar:
  - associativity: $k_1\left(k_2\vec{u}\right) = \left(k_1 k_2\right)\vec{u}$
  - existence of a scalar acting as an identity scalar: $1\vec{u} = \vec{u}$
  - existence of a zero scalar and a zero element: $\left(k\,\vec{u} = \vec{0}\right) \Leftrightarrow \left(k = 0 \text{ or } \vec{u} = \vec{0}\right)$
  - distributivity over vector addition: $k\left(\vec{u} + \vec{v}\right) = k\,\vec{u} + k\,\vec{v}$
  - distributivity over scalar addition: $\left(k_1 + k_2\right)\vec{u} = k_1\vec{u} + k_2\vec{u}$
- Define the concept of linear combination of vectors.
- Describe a vector basis on the plane (a set of two linearly independent vectors that can generate any coplanar vector by a linear combination).
- Describe the particular case of an orthonormal vector basis: a basis formed by vectors $\vec{i} = (1, 0)$ and $\vec{j} = (0, 1)$.

3.2. Given three vectors described by their components and knowing that one of these vectors is the result of a linear combination of the other two, determine the coefficients of this linear combination.

4. To perform a scalar multiplication of vectors

4.1. Calculate the scalar product of two vectors given the norm and angle of direction of each vector or given the norm of each vector and the angle between the two vectors.

- Perform a scalar multiplication of two vectors using algebra (i.e. using their components): $\left(\overrightarrow{a,b}\right) \bullet \left(\overrightarrow{c,d}\right) = ac + bd$.

- Using geometry, perform a scalar multiplication of two vectors, given the angle between the two vectors and their norms:

  $\vec{u} \bullet \vec{v} = \left\| \vec{u} \right\| \left\| \vec{v} \right\| \cos\theta$, where θ is the measure of the angle between the two vectors.

- State the properties of scalar multiplication:

  - commutativity: $\vec{u} \bullet \vec{v} = \vec{v} \bullet \vec{u}$
  - distributivity over vector addition: $\vec{u} \bullet \left(\vec{v} + \vec{w}\right) = \vec{u} \bullet \vec{v} + \vec{u} \bullet \vec{w}$
  - associativity of the scalars: $k_1 \, \vec{u} \bullet k_2 \, \vec{v} = k_1 \, k_2 \left(\vec{u} \bullet \vec{v}\right)$

4.2. Determine the measure of the angle between the two vectors, using their components or their norms and their scalar product.

5. To prove statements using vectors

5.1. Verify a statement containing vectors by using the properties of vectors, the Chasles relation or the properties of operations involving vectors.

- State the Chasles relation and apply it to verify statements using vectors.

- Show an understanding of a proof constructed using vectors.

- Use vectors to complete a proof relating to a property of vectors or to a principle of geometry.

- Prove the properties of vectors using vectors.

- Prove principles of geometry using vectors.

6. To solve problems involving vectors

6.1. Solve problems involving vectors.

1. To study the composition of real functions and the four operations on real functions

1.1. Given the rules of two real functions, find:

- **the rule of a function representing the sum, difference, product or quotient of these functions**
- **the image of any element associated with the resulting function**

(The result of an operation must correspond or relate to a function previously studied.)

1.2. Compare the characteristics of the function resulting from an operation to the characteristics of the original functions that were used to perform the operation. (The rules of functions, relevant principles or graphs may be used to compare the functions.)

- Given the rules of two real functions, graph the sum, difference, product or quotient of these functions and determine the characteristics of the result.

  The characteristics are:

  - type of function
  - domain and range
  - maximum and minimum
  - intervals over which the function is increasing and decreasing

- Define functions representing the sum, difference, product or quotient of two real functions *f* and *g*:

  - $f + g : x \mapsto (f + g)(x) = f(x) + g(x)$
  - $f - g : x \mapsto (f - g)(x) = f(x) - g(x)$
  - $f \bullet g : x \mapsto (f \bullet g)(x) = f(x) \bullet g(x)$
  - $\frac{f}{g} : x \mapsto \left(\frac{f}{g}\right)(x) = f(x) \div g(x)$ with $g(x) \neq 0$

* For this course, the use of an appropriate technological tool (graphing calculator or computer with appropriate software) is recommended as it helps students learn how to analyze these functions, while reducing the time devoted to finding the equation and drawing the graph.

1.3. Given the rules of two real functions *f* and *g*, find:

- **the rule of the real function representing the composition $f \circ g$ or $g \circ f$ of these functions**
- **the image of any element associated with this composite function**

(The result of a composition must correspond or relate to a function previously studied.)

- Define the functions representing the composition of two real functions *f* and *g*:
  - $f \circ g : x \mapsto (f \circ g)(x) = f(g(x))$
  - $g \circ f : x \mapsto (g \circ f)(x) = g(f(x))$

1.4. Compare the characteristics of the function resulting from a composition to the characteristics of the original functions that were the basis for the composition. (The composition can be performed using the rules of functions and relevant principles.)

- Given the rules of two real functions *f* and *g*, graph the composition $f \circ g$ or $g \circ f$ of these functions and determine the characteristics of the result.

  The characteristics are:

  - type of function
  - domain and range
  - maximum and minimum
  - intervals over which the function is increasing and decreasing

2. To solve inequalities in one real variable and problems related to these inequalities

2.1. Algebraically solve a second-degree inequality in one real variable.

- Algebraically solve a first-degree inequality in one real variable.
- Graphically solve a second-degree inequality in one real variable.

2.2. Algebraically solve an inequality in one real variable. The inequality contains an absolute value. (The expression inside the absolute value is a first-degree expression.)

- Graphically solve an inequality in one real variable. The inequality contains an absolute value. The expression inside the absolute value is a first-degree expression.

2.3. Algebraically solve an inequality in one real variable. The inequality contains a square root. (The expression under the square root is a first-degree expression.)

- Graphically solve an inequality in one real variable. The inequality contains an square root. The expression under the square root is a first-degree expression.

2.4. Solve problems that involve a second-degree inequality in one real variable, an inequality containing an absolute value or an inequality containing a square root. (The problems must involve determining an inequality given the rule of a function.)

3. To solve problems involving circles and right triangles

3.1. Find the following measures and justify the steps in your reasoning by referring to the appropriate principle:

- **in any given triangle, the measure of a side or of one of the segments formed when the bisector of an interior angle divides the side opposite this angle into two segments** (See appendix, under “Circles,” principle *80.)
- **in a circle, the measure of one of the segments that is determined by either two chords, two secants or a tangent and a secant intersecting at a given point** (See appendix, under “Circles,” principles 81 to 83.)

- In a circle, find the measure of an angle, an arc, a chord, the diameter (or the radius), or a segment situated on a chord or on a tangent. Justify the steps in your reasoning by referring to the appropriate principles. (See appendix, under “Circles,” principles *68 to 79.)

- In a right triangle, find the measure of an angle, a side of the right angle, the hypotenuse, the median or the altitude to the hypotenuse or a segment determined on the hypotenuse by this median or altitude. Justify the steps in your reasoning by referring to the appropriate principles. (See appendix, principles 89 to 91; principles 10 to 14 and principle 42.)

3.2. **Verify the relationships that exist between the elements of a circle or those of a right triangle.** (The given measures of the figures may be numerical or literal.)

3.3. **Prove geometric principles involving circles and right triangles.** (See appendix, under "Circles" and "Right Triangles," or related principles).

- Determine the important steps in presenting a geometric proof:
  - identifying the information given and formulating hypotheses
  - identifying the required conclusion
  - organizing the steps in a line of deductive reasoning
  - justifying each step of the proof
- In a geometric proof, distinguish between:
  - the types of statements used (definitions, axioms, theorems, corollaries)
  - the statement of a theorem and its inverse
  - a direct proof and an indirect proof
- Complete the proof of a geometric principle, in which certain steps or justifications are missing.
- Demonstrate an understanding of the proofs of certain geometric principles involving circles and right triangles. (See principles in the appendix under "Circles" and "Right Triangles," or related principles.)

3.4. **Solve problems that involve applying relationships governing measurements in circles and right triangles.** (See appendix, under "Circles" and "Right Triangles") **and justify each step in the solution.**

(The solution may involve applying:

- knowledge of geometry acquired in previous courses [See appendix, principles 1 to 55.]
- equations, functions, trigonometric ratios or analytic geometry formulas)

APPENDIX

PRINCIPLES OF GEOMETRY FOR MTH-5111-2

PART ONE: PRINCIPLES FROM MTH-4111-2 (Numbers 1 to 55)

ANGLES

1. Adjacent angles whose external sides are in a straight line are supplementary.
2. Vertically opposite angles are congruent.
3. If a transversal intersects two parallel lines, then:
   a) the alternate interior angles are congruent
   b) the alternate exterior angles are congruent
   c) the corresponding angles are congruent
4. If two corresponding (or alternate interior or alternate exterior) angles are congruent, then they are formed by two parallel lines and a transversal.

TRIANGLES

5. The sum of the measures of the interior angles of a triangle is 180°.
6. In any triangle, the longest side is opposite the largest angle.
7. In any isosceles triangle, the angles opposite the congruent sides are congruent.
8. In any equilateral triangle, each angle measures 60°.
9. In any isosceles triangle, the perpendicular bisector of the side adjacent to the congruent angles is the bisector of the angle opposite this side as well as the median and altitude to this side.
10. In any right triangle, the acute angles are complementary.
11. In any isosceles right triangle, each acute angle measures 45°.
12. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides (Pythagorean theorem).
13. A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other two sides.
14. In a right triangle, the length of the side opposite a 30° angle is equal to half the length of the hypotenuse.
15. Two triangles whose corresponding sides are congruent must be congruent.
16. If two sides and the contained angle of one triangle are congruent to the corresponding sides and contained angle of another triangle, then the triangles must be congruent.
17. If two angles and the contained side of one triangle are congruent to the corresponding angles and contained side of another triangle, then the triangles must be congruent.
18. If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles must be similar.
19. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles must be similar.
20. If the lengths of two sides of one triangle are proportional to the lengths of the two corresponding sides of another triangle and the contained angles are congruent, then the triangles must be similar.

21. In a right triangle, the sine of an acute angle is equal to the ratio obtained by dividing the length of the side opposite this angle by the length of the hypotenuse.

$\sin A = \frac{a}{c}$, where a is the length of the side opposite angle A and c is the length of the hypotenuse.

22. In a right triangle, the cosine of an acute angle is equal to the ratio obtained by dividing the length of the side adjacent to this angle by the length of the hypotenuse.

$\cos A = \frac{b}{c}$, where b is the length of the side adjacent to angle A and c is the length of the hypotenuse.

23. In a right triangle, the tangent of an acute angle is equal to the ratio obtained by dividing the length of the side opposite this angle by the length of the side adjacent to it.

$\tan A = \frac{a}{b}$, where a is the length of the side opposite angle A and b is the length of the side adjacent to angle A.

24. The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides (law of sines):

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

25. The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle (law of cosines):

$$a^2 = b^2 + c^2 - 2bc\bullet\cos A$$

$$b^2 = a^2 + c^2 - 2ac\bullet\cos B$$

$$c^2 = a^2 + b^2 - 2ab\bullet\cos C$$

QUADRILATERALS

26. The opposite angles of a parallelogram are congruent.
27. The opposite sides of a parallelogram are congruent.
28. The diagonals of a parallelogram bisect each other.
29. The diagonals of a rectangle are congruent.
30. The diagonals of a rhombus are perpendicular to each other.

CIRCLES

31. All the diameters of a circle are congruent.
32. In a circle, the measure of a diameter is equal to twice the measure of the radius.
33. The axes of symmetry of a circle contain its centre.
34. The ratio of the circumference of a circle to its diameter is a constant known as π: $C = \pi d$ or $C = 2\pi r$, where C is the circumference, d is the diameter and r is the radius.
35. The area of a circle is equal to πr^2: $A = \pi r^2$, where A is the area and r is the radius.

ISOMETRIES AND CONGRUENT FIGURES

36. An isometry preserves collinearity, parallelism, the order of points, distances and the measures of angles. In addition, translations and rotations preserve the orientation of the plane.
37. Any translation will transform a straight line into another line parallel to it.
38. Plane figures or solids are congruent if and only if there is an isometry that maps one figure onto the other.
39. In congruent plane figures or solids, the measures of the following elements are equal:
    a) the corresponding segments and angles
    b) the perimeters
    c) the areas
    d) the volumes
40. Any point on the perpendicular bisector of a segment is equidistant from the two endpoints of this segment.
41. Any point on the bisector of an angle is equidistant from the sides of this angle.
42. In any right triangle, the length of the median to the hypotenuse is equal to half the length of the hypotenuse.
43. The three perpendicular bisectors of the sides of a triangle are concurrent in a point that is equidistant from the three vertices.
44. The diagonals from one vertex of a convex polygon form n - 2 triangles, where n is the number of sides in that polygon.
45. The sum of the measures of the interior angles of a polygon is 180° (n - 2), where n is the number of sides in the polygon.
46. In a convex polygon, the sum of the measures of the exterior angles, one at each vertex, is 360°.

SIMILARITY TRANSFORMATIONS AND SIMILAR FIGURES

47. Any similarity transformation preserves collinearity, parallelism, the order of points, the orientation of the plane, the measures of angles and the ratio of the distances.
48. Any dilatation will transform a straight line into another line parallel to it.
49. Plane figures or solids are similar if and only if there is a similarity transformation that maps one figure onto the other.
50. In similar plane figures or solids:
    a) the ratio of the lengths of the corresponding segments is equal to the scale factor
    b) the ratio of the measures of the corresponding angles is 1
    c) the ratio of the areas is equal to the square of the scale factor
    d) the ratio of the volumes is equal to the cube of the scale factor
51. Plane figures or solids with a scale factor of 1 are congruent.
52. Any straight line that intersects two sides of a triangle and is parallel to the third side forms a smaller triangle similar to the larger triangle.
53. Transversals intersected by parallel lines are divided into segments of proportional lengths.
54. The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.
55. The three medians of a triangle are concurrent in a point that is two-thirds the distance from each vertex to the midpoint of the opposite side.

PART TWO: PRINCIPLES SPECIFIC TO THIS COURSE (Numbers 56 to 91)

FUNDAMENTAL PRINCIPLES

56. Two points are in exactly one line.
57. If two lines intersect, they share exactly one point.
58. There is exactly one line that is parallel to a given line and that contains a given point not in the given line.
59. If two lines are parallel to a third line, then they are parallel to each other.
60. If two lines are perpendicular to the same third line, then they are parallel to each other.
61. If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.
62. For any line and point in a plane, there is exactly one line in the plane that contains the point and is perpendicular to the line.
63. In any triangle, the length of any side is less than the sum of the lengths of the other two sides.
64. In any triangle, the length of any side is greater than the difference of the lengths of the other two sides.
65. If an acute angle and a leg of one right triangle are congruent to an acute angle and the corresponding leg of another right triangle, then the triangles are congruent.[**]
66. If the legs of one right triangle are congruent to the legs of another right triangle, then the triangles are congruent.

CIRCLES

67. Three non-collinear points determine one and only one circle.

*68. The diameter is the longest chord of a circle.[*]

*69. Any diameter divides a circle into two congruent parts.

*70. In a circle or in congruent circles, congruent arcs are subtended by congruent chords, and vice versa.

*71. Any diameter perpendicular to a chord divides that chord and each of the arcs that it subtends into two congruent parts. Conversely, any diameter that divides a chord (and each arc that it subtends) into two congruent parts is perpendicular to that chord.

*72. In a circle or in two congruent circles, two congruent chords are equidistant from the centre and vice versa.

*73. If a line is perpendicular to a radius of a circle at the endpoint of the radius in the circle, the line is tangent to the circle. The converse is also true.

*74. Two parallel lines, be they secants or tangents, intercept congruent arcs of a circle.

*75. If point P is located outside circle O, and if segments PA and PB are tangents to that circle at points A and B respectively, then OP bisects the angle APB and $\overline{PA} \cong \overline{PB}$.

*76. In a circle, the measure of the central angle is equal to the degree measure of its intercepted arc.

** Congruent right triangles have not been specifically covered in previous courses. These particular examples of congruent triangles have been included because they are used specifically to prove certain principles involving circles.

* This course focuses on the statements preceded by an asterisk. The other statements mentioned and those found in Part One may also be used to solve problems.

*77. The measure of an inscribed angle is one-half the measure of its intercepted arc.

*78. The measure of an angle formed by two chords intersecting in the interior of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

*79. The measure of an angle formed by two tangents, a tangent and a secant, or two secants is one-half the difference of the measures of the intercepted arcs.

80. In any triangle, the bisector of an angle divides the opposite side into two segments whose lengths are proportional to those of the adjacent sides.

*81. If two chords of a circle intersect in its interior, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

*82. If secants PAB and PCD of a circle have the same external endpoint P, then $m\overline{\mathrm{PA}} \times m\overline{\mathrm{PB}} = m\overline{\mathrm{PC}} \times m\overline{\mathrm{PD}}$.

*83. If tangent PA and secant PBC of a circle have the same external endpoint P, then $(m\overline{\mathrm{PA}})^2 = m\overline{\mathrm{PB}} \times m\overline{\mathrm{PC}}$.

84. In a circle, the ratio of the measures of two central angles is equal to the ratio of the measures of their intercepted arcs.

85. In a circle, the ratio of the areas of two sectors is equal to the ratio of the measures of their central angles.

86. The circumferences of two circles have the same ratio as their radii.

87. The areas of two circles have the same ratio as the square of their radii.

88. The measures of the similar arcs of two circles have the same ratio as their radii.

RIGHT TRIANGLES

*89. The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse.

*90. The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse.

*91. In a right triangle, the length of the hypotenuse multiplied by the length of the altitude to the hypotenuse is equal to the product of the lengths of the sides of the right angle.

* This principle does not relate to circles, but may be proved by inscribing the triangle in a circle.

10.6. Optional Course

MTH-5112-1

MTH-5112-1 Logic*

1. To understand the concepts of simple and compound propositions, logical operations and truth tables

1.1. Given the truth table for each type of proposition (negation, conjunction, disjunction, conditional statement and biconditional statement), determine the truth value of a proposition consisting of up to three simple propositions by following the order of logical operations. The truth value of each simple proposition is known. (The given compound proposition must be written symbolically and should contain no more than three logical connectives.)

- Identify the propositions in a list of five to ten simple verbal and mathematical sentences.

- Given a proposition expressed as a verbal or mathematical sentence, determine whether it is a negative, a conjunction, a disjunction (inclusive or exclusive), a conditional statement or a biconditional statement on the basis of its logical connective. Then, transcribe this proposition, using one of the following symbols to represent the logical connective:

  - $\neg$ for a negation (*not*)
  - $\wedge$ for a conjunction (*and*)
  - $\vee$ for a disjunction (*or*)
  - $\rightarrow$ for a conditional statement (*if... then*)
  - $\leftrightarrow$ for a biconditional statement (*if and only if*)

  Simple statements must be selected.

* MTH-4101-1 to MTH-4104-1 or 568-416 are prerequisites for MTH-5112-1.

2. To understand the concepts of tautology and contradiction, implications and logical equivalences as well as the negation of a compound proposition

2.1. Given two compound propositions linked together to form a biconditional statement, set up a truth table and determine if that biconditional statement is always true whatever the truth value of each of its simple statements. If such is the case, connect the two compound propositions using the symbol for a logical equivalence (⇔). (These given compound propositions must be written symbolically and each should contain no more than three simple propositions and three logical connectives.)

- Set up a truth table for a proposition consisting of up to three simple propositions and three logical connectives in order to determine if that proposition is a tautology (i.e. it is always true whatever the truth value of its simple propositions) or a contradiction (i.e. it is always false whatever the truth value of each of its simple propositions). The given compound proposition must be written symbolically and all possibilities must be included in the truth table. The steps in the solution must be shown.

- Given two compound propositions linked together to form a conditional statement, set up a truth table and determine if that conditional proposition is always true whatever the truth value of each of its simple propositions. If such is the case, connect the two compound propositions using the symbol for a logical implication (⇒). These given compound propositions must be written symbolically and each should contain no more than three simple propositions and three logical connectives. All possibilities must be included in the truth table and the steps in the solution must be shown.

2.2. Determine the negation of a compound proposition written symbolically. The negation is determined by rewriting the given proposition so that only its simple propositions bear the negation symbol. (Each compound proposition should contain no more than three simple propositions and five logical connectives.)

3. To describe simple propositional forms and compound propositional forms

3.1. Given a universe containing five to ten elements, list the elements of the solution set of a simple propositional form or the elements of the solution set of a propositional form containing two simple propositional forms linked together by a logical connective. (In the latter case, the answer must also include the solution set of each simple propositional form. The propositional forms must be expressed mathematically.)

- Identify the propositions and the propositional forms in a list of five to ten verbal and mathematical sentences, some of which contain variables.

4. To determine the truth table of a quantified compound propositional form

4.1. Given a universe containing five to ten elements, determine the truth value of a quantified compound propositional form by following the order of logical operations. (This compound propositional form should contain no more than three simple propositional forms expressed mathematically and three logical connectives.)

- Given a list of five to ten quantified verbal sentences, indicate those that contain an existential quantifier and those that contain a universal quantifier. Then, transcribe these statements, using one of the following symbols to represent the quantifier:
  - $\exists$ for the existential quantifier (*There is at least one...*)
  - $\exists!$ for the unique existential quantifier (*There is only one...*)
  - $\forall$ for the universal quantifier (*For all...*)

- Determine the negation of a quantified compound propositional form expressed either verbally, mathematically or symbolically. The negation is determined by rewriting the given proposition so that only its simple statements bear the negation symbol. This compound propositional form should contain no more than three simple propositional forms and three logical connectives.

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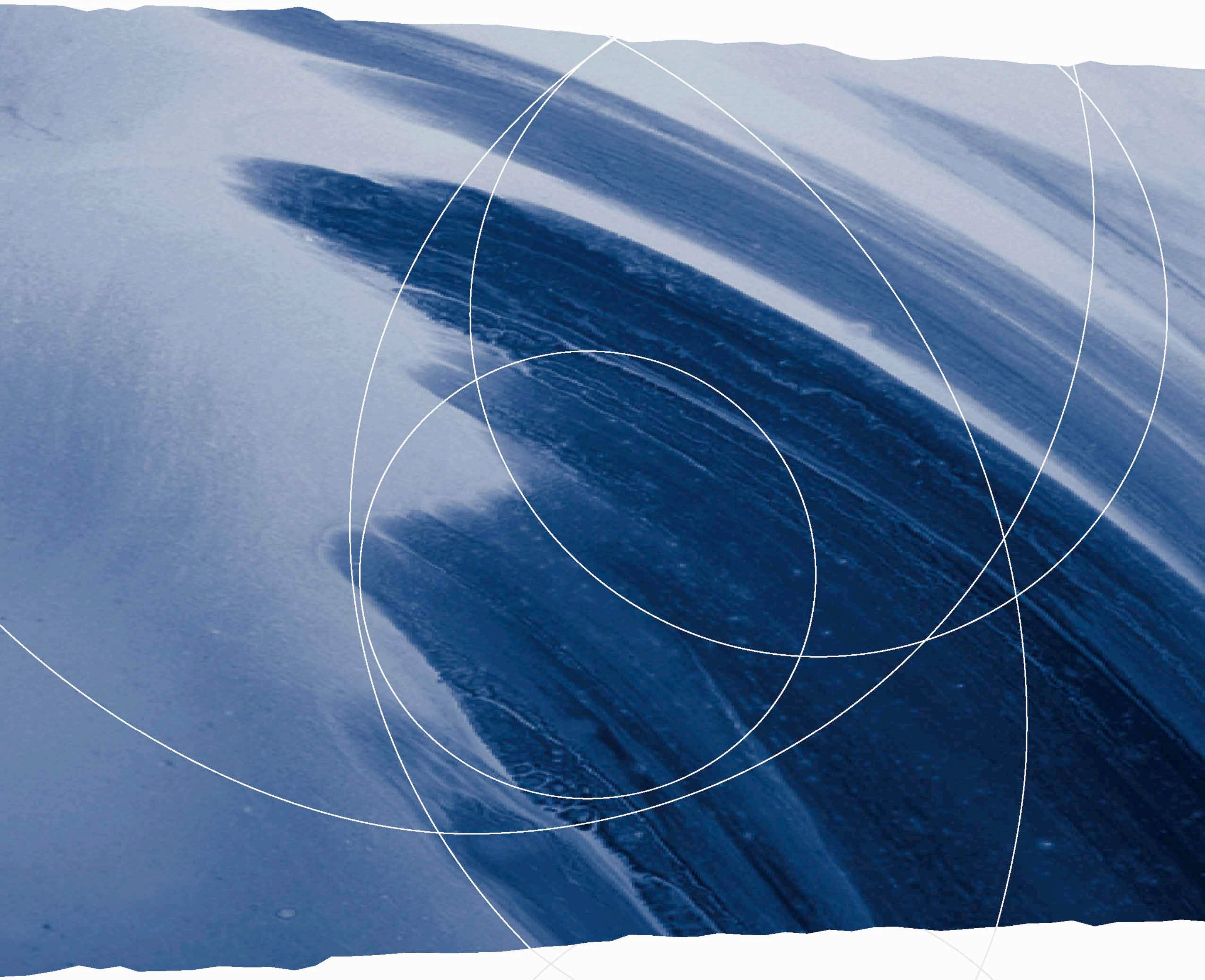

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