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A multivariate destination policy for geometallurgical variables in mineral value chains using coalition-formation clusterin

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Abstract: Complex polymetallic mining projects with multiple processing streams tend to require tight blending constraints, with different operational and processing targets. These blending requirements are generally not focused solely on metal grade, but rather on a set of geometallurgical variables that affect the performance of the operation and its ability to meet targets and maximize project value. Because of this, a multivariate destination policy is developed here, based on coalition formation clustering (a line of study of cooperative game theory), which avoids the use of cut-off grades and defines where material is sent by accounting for the value and relation of groups of blocks being processed together. This allows improving investment decisions as a result of optimizing project performance, because the variables that affect blending and processing requirements are actively accounted for in the optimization process.

A case study on a copper-gold mine with six destinations is presented, where the method proposed shows significant improvements in meeting processing requirements and increases the expected net present value by 5.6% when compared to a traditional method. This shows that complex processing requirements can be accounted for and respected without any loss of project value.

1 Introduction

In mining, orebodies define the design and the value of a project. In other words, the rock attributes and operational features of the project will model the processing streams used along the life of mine (LOM), and the range of profit produced by the deposit. However, a common oversimplification done in traditional optimization of mining projects is to assume that a block of material has an intrinsic dollar value, and given a certain cut-off grade, it will be defined as ore (to be processed for profit) or waste. But there are various other parameters that affect the value of a block, and they should be considered when choosing the destination for processing. The parameters can be, for example, the presence of deleterious elements (such as arsenic), different material types (sulphides, oxides, etc.), rock hardness, spatial location of a block (which will define when the block can be extracted), discounting monetary flows (time value of money), and the current commodity price of the metal once the block is processed to be sold.

This issue takes special importance in the increasingly complex deposits being developed nowadays, where it is necessary to consider from an early stage not only grade uncertainty, but also all the variability of relevant geometallurgical characteristics (such as rock hardness, material types, deleterious elements, etc.) that affect the different processing streams (such as energy consumption, use of additives, metallurgical recovery, etc.), in order to correctly evaluate and fully maximize project value. This paper proposes to do this by developing an optimal destination policy mechanism which accounts for geometallurgical variables and complex blending requirements of polymetallic deposits in the decision process.

Within a mining complex, processing plants and refineries are designed for specific capacities and tasks, with certain inputs and limited space for flexibility. Because of this, their performance depends greatly on how the different requirements for these tasks are met (for example, blending constraints must be met in order to maximize metallurgical recovery). These hard constraints force the project to be optimized around them, so that requirements are met and overall project value is maximized.

However, because of the high costs associated with exploration, the limited information obtained from composites and flawed sampling systems, the geology of a deposit is highly uncertain, being one of the main sources of risk in a mining operation (Godoy and Dimitrakopoulos, 2004). Thus, many efforts have been put in developing methods that allow the integration of this uncertainty into the design and evaluation of the project at acceptable time limits, such as conditional simulation which, to date, has been successfully implemented in various mining projects (Goodfellow, 2014; Montiel, 2014).

This geological uncertainty translates into uncertainty in the processing streams' supply, so the optimization process must make sure that once the geological uncertainty is revealed (after material is mined), the extracted material is reordered and distributed between the available destinations in such a way that the different constraints are met. This reordering and delivering process, referred to as *destination policy*, is especially important in poly-metallic mines with multiple processing streams. Optimizing a destination policy has an iterative effect that affects the whole mining complex. For example, if the available sulphur content is not enough to reach blending constraints in the plant, then lower grade material with higher sulphur content should be sent to the processor from areas of the deposit with higher sulphur content. In other words, an optimal destination policy affects the mining schedule in order to select the material needed to be processed to maximize the overall project's performance.

Traditional mine planning models define destination policies solely based on the material's concentration with respect to different cut-off grades of some commodity and treat it as waste if its (assumed) value is negative. This assumption is misleading, as blocks have different attributes and concentration of elements which must be extracted, transported, blended, processed, and sold in order to obtain a financial gain from them. All these activities are also strongly affected by the geological uncertainty present in the deposit, which will ultimately define the performance of the mining system over the processing decisions. Thus, the actual value of a block depends on the period when it is extracted, on the quality of the elements contained in it, on the current price for each of these elements, as well as on the destination where the block is processed, which entails the blending requirements, processing costs, recovery curve of the metallurgical process, etc. (most of which are non-linear aspects which are avoided in most of these traditional optimization models). In other words, the actual "block value" cannot be calculated individually. The revenue of each block, and the global mine sequence, depends on the global optimization of the mining complex and the destination policy must be considered as a dynamic part of this optimization.

The problem addressed in this paper is to optimize what material should be processed where annually, in order to maximize processing performance, project value, and meet processing targets. In other words, defining where the extracted material is sent to, under geological uncertainty, to maximize the profit obtained from the mineral value chain. To do this, it is not only important to extract the highest grade material and fill the plant, but also to meet blending constraints in the different processing streams, so that full advantage is obtained from the limited resources. The destination policy optimization proposed here is developed based on a multidisciplinary implementation, combining mine planning with coalition formation theory using the "Shapley Value," which is a line of study of cooperative game theory.

2 Destination policy

Thus far, the decision of defining where each block is sent after extraction is mainly based on two aspects: Defining certain ranges of grades accepted at the different destinations, commonly referred to as cut-off grades (Lane, 1988; Rendu, 2013) or based on the general revenues expected from sending a block to each of the possible processing streams. However, these policies are based on a serious oversimplification in mine planning: to assume that a block has an inherent dollar value (Lerchs and Grossman, 1965; Tolwinski and Underwood, 1996; Ramazan, 2007; Meagher et al., 2010). This results in severe deviations from expected project revenues and performance and clear suboptimal results (Wharton, 2004). Based on this oversimplification of a "dollar value," most mine planning optimization formulations assume a priori when a block will be extracted (i.e. the mining sequence), and what material is ore and what is waste (thus, where it should be sent), before any optimization has been done, bypassing the actual destination policy decision (Leite and Dimitrakopoulos, 2007; Ramazan and Dimitrakopoulos, 2004, 2007, 2013; Lamghari and Dimitrakopoulos, 2012).

Some work has been done in designing dynamic policies, such as Meagher et al. (2010), where the destination decisions are updated on a yearly basis according to new information that becomes available once a block is extracted. The possibility to re-optimize is considered as valuable flexibility which is added to the block's value. In this paper, the authors account for geological uncertainties, market uncertainties, and the time value of money in calculating the value of a block at its period of extraction. However, the formulation proposed by the authors grows exponentially if multiple elements, deposits, and processing streams are considered; the focus is still placed on assigning an individual dollar value to each block instead of optimizing the mining complex as a whole. Asad and Dimitrakopoulos (2013) propose a heuristic approach to select an annual cut-off grade under geological uncertainty which maximizes the net present value (NPV) of the entire mining complex and satisfies production constraints. Continuing on this line, Meagher et al. (2014) develop a dynamic cut-off grade policy to define block destination under geological uncertainty. Here, the optimal cut-off grade is defined on a yearly basis in order to optimize the pushback design and maximize project value. However, the model only considers one element with one processing facility and the optimization is done greedily by sequentially maximizing the NPV of each pushback, instead of optimizing the whole deposit simultaneously.

To account for the destination policy dynamically into the optimization and at the same time develop a global mine plan optimization, few methods have been presented in the literature. The multistage optimization process developed by Boland et al. (2008) presents a destination policy mechanism that optimizes each geological scenario independently once the scenarios have "differentiated enough" along the LOM. Other studies have developed robust destination policies, such as Montiel (2014), where the destination of a block is the same for every geological scenario and, as such, the destination policy is a first stage variable of the stochastic optimization model. Here, the author considers the optimization of the whole mining complex under geological uncertainty, with multiple material types and processing streams. The method presented is able to develop a mining schedule that defines when each block is mined and where it is sent in order to maximize project value while meeting production constraints; avoiding the need for pre-defined cut-off grades. However, the actual destination policy is only optimized if the material type of a block is the same for all the simulations. In other words, the model might produce misclassification errors where oxides are sent to processing streams that only accept sulfides (therefore introducing infeasibility). Menabde et al. (2007) also define a robust destination policy but based on cut-off grade optimization. In their study, the authors account for geological uncertainty and present a MIP formulation where the destination policy is defined so that blocks with similar grades are sent to the same destination, and by doing so, they are able to avoid the misclassification problems seen in the previous case. However, their formulation only accounts for a single mine with one element and a single processing stream without considering problems that arise with blending requirements.

As the complexity of mining projects increase (in terms of number of deposits, processing streams, and number of elements), traditional destination policies lack in the ability to consider the multidimensional aspect of the mining optimization problem. Recent works on destination policy have extended from cut-off grade optimization to integrate multivariate distributions which makes them more adept for complex mining projects. Goodfellow and Dimtrakopoulos (2014) develop a stochastic optimization of a mining complex which accounts for geological uncertainty, and considers a multidimensional destination policy. To do this, the authors use k-means++ clustering (Lloyd, 1982; Gan et al., 2007; Arthur and Vassilvitskii, 2007) to classify the blocks of all simulations in a deposit by similarity of attributes and define the destination policy as common for all the blocks in a cluster, subject to geological uncertainty. This way, as in Menabde et al. (2007), blocks with similar attributes are grouped together in order to be sent to the same destination, no matter which geological scenario is encountered. The K-means++ clustering algorithm is easy to implement and it allows analyzing all the orebody simulations simultaneously. This makes it possible to account for the geological uncertainty of the deposit and obtain a guide for additional simulations or newly obtained data.

An example of this clustering process is presented in Figure 1, where each point in the graph represents the concentration of “x” and “y” that is present in a block.

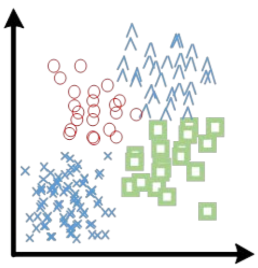

Figure 1 - Creation of 4 clusters according to “x” and “y” concentration.
From: Goodfellow and Dimitrakopoulos (2014)

However, in Goodfellow and Dimtrakopoulos (2014), even though the destination policy is decided over each cluster, the decision of where a given cluster is sent does not directly take into consideration the relation of aggregates of material being processed together which must meet the complex blending constraints. For example, if particularly tight metallurgical constraints are required by a process, such as a silica-magnesium ratio, if one block has a high magnesium concentration and high copper but low silica, and another block has a high silica concentration though a low copper grade, then it might be preferable to “cluster” and process the two blocks together (even if their attributes are not similar).

The method proposed here aims at creating clusters of blocks with two objectives. On one hand, to reduce the computational cost inherent in mining optimization, caused mainly by the large amount of blocks encountered in real size mining projects; on the other hand, to take into account not only metal grade and recovery, but a wider range of geometallurgical variables. The latter is to obtain more reliable values of the aggregates of blocks in the different processing streams. This happens because the final recovery and metallurgical performance will depend on the total material being processed together and not on the individual properties of each block. As, once material arrives to the plant, there is no longer any perception of a “block” but rather a blend of extracted material.

2.1 Geometallurgical variables

Geometallurgical variables involve any rock property that has a positive or negative effect on the business’s ultimate value (Coward et al., 2009; Dunham et al., 2011). Some of the more critical (and known ones) are recovery, grindability, throughput, power consumption, mineralogy, and content of deleterious materials. As such, geometallurgy is a cross-discipline that combines geology, metallurgy, and mine planning to design a processing stream fit to the actual characteristics of the resource (Dunham et al., 2011). It is known that a mining project’s value and performance depends not only on the ore grade and the plant’s recovery, but also on other variables such as the mineral composition, energy consumption, additives needed, penalties involved, mineability of the deposit, etc. (Van den Boogaart et al., 2011). However, most of these variables are omitted from traditional mine planning methods, not to mention the variability and uncertainty related to them. Because of this, in order to obtain a reliable mine plan and a better representation of a project's value, more detail on the rock properties of the extracted material need to be considered in the optimization.

Some work has been done incorporating these variables into the mine design and planning steps, such as Coward et al. (2009) who classify as "primary" and "response" geometallurgical variables. Primary variables are defined to be additive or easily manipulated to be linear; response variables generally present complex distributions which cannot be easily combined. The authors note that because most of the rock properties are non-linear, traditional estimation methods for orebody modeling, such as kriging, are unable to reproduce them without serious biases in the results. Conversely, conditionally simulated orebody models have the advantage of presenting the rocks' actual spatial variability and avoiding estimation mechanisms. This allows the integration of complex non-linear variables into the model. Coward et al. (2013) apply this framework to a mining operation, and aim at generating a value chain model by evaluating geometallurgical recovery factors.

Van den Boogaart et al. (2011) focus on optimising the mineral processing stage by generating "adaptive processing" using a geomathematical model developed from conditional expectations and regression models which adapt the mill's grinding size to the material being processed (if the benefit obtained exceeds the investment required for having a flexible process). But note that care must be taken when planning for adaptive processing, as it is a recurrent mistake to develop models assuming perfect information from limited samples. Here, they define a simplified model to calculate the value generated by different types of ore, depending on their mass, grade, the liberation per grinding size selected, and the corresponding milling energy required. However, the authors neither consider the inherent uncertainty of these variables nor the geometallurgical effect over the whole mining complex., Rather the authors solely consider the processing stage which has been proven to be suboptimal in terms of maximizing project value.

Dunham et al. (2011) comment on the impact that geometallurgy can have over the value and viability of a mining project, transforming particularly the scheduling of the deposit compared to traditional evaluations. This, extending to the stockpiling and blending strategies, can simultaneously affect the processing strategies chosen if non-grade properties of the rock are taken into account. The authors note that integrating geometallurgy into the modeling and optimization of the problem provides a spatial context to study the distribution of deleterious elements, which can drastically change the project's design and improve its performance.

With respect to the accurate representation of geometallurgical variables, Van den Boogaart et al. (2014) present a simulation method for representing discrete and continuous geometallurgical parameters. The authors state that "conditional geostatistical simulation of geometallurgical parameters enables the construction of a processing model for computing recovery, equipment usage, processing costs and other relevant parameters and thus the monetary value for mining and processing a block with certain parameters." Together with this, they also note that traditional geostatistical techniques cannot be directly applied for conditional simulation of geometallurgical parameters for two main reasons, first, many variables have non-Euclidean statistical scales (such as geometry, mineralogy, etc.) producing, in some cases, unfeasible values in the simulation, and second, because processing material entails nonlinear transformations of the rock's properties, and as such, the conditional distribution of the variables is relevant for the simulation, and not only their mean and variance (such as done in Gaussian simulations). Because of this, the simulation needs to reproduce the joint conditional distribution of all the relevant geometallurgical variables being considered. Thus, they propose an adaptation to the traditional conditional simulating procedure by using a joint multipoint conditional simulation framework. In their approach, the authors adapt the single normal equation simulation (SNESIM) proposed by Strebelle (2002) to simulate categorical variables by estimating the conditional probability distribution functions of a training image via multinomial logistic regression (i.e. it is assumed that un-sampled locations follow a multinomial distribution), and extend its application to consider information provided by different scale layers (i.e. other type of variables, not necessarily categorical) in conditioning locations.

Deutsch et al. (2015) adapt different geostatistical and numerical techniques to generate high-resolution simulations of mixed, continuous, and categorical geometallurgical variables, accounting for their non-linearity and for the correlation existing between different variables being jointly simulated. In their study, the authors focus on the grinding index and on the mill's Bond Work index (BWi) to maximize the throughput and metallurgical recovery of an operation. However, they fail to provide a simulation method able to simultaneously simulate geometallurgical variables sampled in different scales, which is often the case with regionalized variables, disabling the possibility of accounting for the existing correlation between these variables. Together with this, there is a lack of detail provided on the actual application of these simulations in the optimization process to maximize throughput.

It is known that geometallurgical variables of the deposit directly affect the performance of the downstream processes of a mining complex. Because of this effect, the following study includes these variables directly into the destination policy mechanism by implementing a method that selects which blocks are processed together at a given place, given their combined multivariable attributes. This is done by implementing coalition formation algorithms (which are an extension of cooperative game theory), to consider not only the main elements' grade in a block, but rather a set of properties of the rock which have an effect on the downstream processes and group them according to their processing preferences. This new clustering algorithm will better maximize project value, achieving production targets while taking into account the complex blending requirements.

A global overview of game theory and in particular coalition formations is presented next, together with its relation to the destination policy of a mining project.

2.2 Proposed destination policy method

Game theory can be seen as the study of strategic interactions between decision makers (Schelling, 1980). Formally, Myerson (1991) defines game theory as the study of mathematical models of conflict and cooperation between rational decision makers, or "players." In order to maximize its utility, a player must make decisions while strategically predicting what the other players will do, as his payoff depends on his own actions as well as the other players' actions. This way, game theory provides the techniques for analyzing situations in which two or more agents make decisions that will influence one another's welfare (Aumann, 1976). In particular, cooperative game theory focuses on studying games where players have the opportunity to communicate with each other and form coalitions in order to increase their utility (Osborne and Rubinstein, 1994). This increase of value is obtained given the non-linearity of their utility function. For example, if two agents decide to team up, then their compound value must be higher or equal than the sum of their individual initial values. The individual contribution of each player to the coalition is usually different and a "solution concept" for a coalitional game is a revenue and/or information sharing mechanism (von Neumann and Morgenstern, 1947; Brandenburger and Nalebuff, 2002).

There are multiple ways to divide revenues, but the most recurrent form studied is such that the value allocation is "fair" (Leyton-Brown and Shoham, 2008), making sure that the coalition remains "stable." "Stable" means that none of the players wish to leave the group to form another group as their value is maximized in their current state (Aumann and Dreze, 1974). To solve the problem of "fair value allocation," the most studied revenue sharing mechanism is based on the *Shapley Value* (Shapley, 1953), which is the mathematical evaluation of a player's gain in a game. The Shapley Value is defined using "characteristic functions," which is the mathematical representation of the value generated by a subset (or coalition) of players in the game (von Neumann and Morgenstern, 1947; Brandenburger, 2007). By definition, this characteristic function must satisfy three axioms in an N-player game. Given $V(S_i)$, defined as the characteristic function of subset S_i of the N players ($S_i \subseteq N$):

1. $V(\varnothing) = 0$
2. Given two coalitions S_1 and S_2, where $S_1 \subseteq S_2$; $S_1, S_2 \subseteq N$, then $V(S_1) \leq V(S_2)$
3. Given two disjoint coalitions S_1, S_2 where $S_1, S_2 \subseteq N$; $V(S_1) + V(S_2) \leq V(S_1 + S_2)$

Translating these three axioms into mathematical form we obtain the Shapley Value formula, which is a unique payoff allocation that divides the full profit of the grand coalition among the players (Osborne and Rubinstein, 1994) and is calculated in function of their marginal contributions to all possible coalitions (Roth, 1988; Gulf, 1989; Brandenburger, 2007; Branzei et al., 2008). This allocation system defines the revenue (or satisfaction degree) obtained by player i, $i = 1 \ldots N$ in a game of N players, for all possible coalitions C, where $C \subseteq N$. The mathematical formulation is presented in Equation 2.1.

$$SH_V(i, N) = \underbrace{\sum_{\forall C} \frac{(|C| - 1)!(N - |C|)!}{N!}}_{\text{Part 1}} \underbrace{\left[V(C) - V(C - \{i\})\right]}_{\text{Part 2}} \tag{2.1}$$

Which is referred to as "the Shapley Value of player 'i' "; here, the value is calculated over all possible coalitions ($\forall C$), where $V(C)$ and $V(C-\{i\})$ represent the characteristic function of a coalition with and without player $\{i\}$, respectively. This way, "Part 1" corresponds to the summation of all possible permutations of coalitions that can be formed in an N-player game, and "Part 2" corresponds to the marginal contribution of player i to each of this coalitions. Also note that $\sum_{\forall i} SH_V(N,i) = V(N)$, i.e. all the value generated in a game is divided between its players.

However, the main drawback of the Shapley Value is that it cannot be calculated in polynomial time, as the calculation of all permutations of players in a game is computationally very expensive and unmanageable as the number of players increase. Because of this, Liu et al. (2011) present a cooperative game theory approach for multi-objective categorization where the Shappley Value is approximated by "priority groups" which are computed in $O(n^3)$ time, where n corresponds to the number of players in the game. In their paper, the authors present a simple example of three families who must decide where to go on vacation and the value of the different families (players) is represented in a characteristic function. In this case, the function depends on the number of families joining (as economies of scale allow the cost of the trip to be lower for more players), on their holiday destination preference, and on the relative preference of each family to go with each other. Here, the satisfaction of each family is then measured by their Shapley Values and the vacation groups are defined.

The basic idea behind these *priority groups* is that players which present favorable coalitions will still remain together as the coalition gets bigger. This means that the ultimate optimal priority groups can be obtained by recursively combining the latest larger Shapley Values. The authors present the following example: suppose there are three players (A, B and C), and two available destinations (L and P). All the options are stated and as a first step, a pairwise combination of the players is done. However, only the combinations that actually create value are kept (as coalitions are formed only if $V(A) + V(B) \leq V(AB)$). These combinations are recursively created, level by level, and unfavorable groups are eliminated in order to reduce the computational cost. This process can be seen in Figure 2, where two example priority groups are created (PG1 and PG2), and is further explained in Liu et al. (2011).

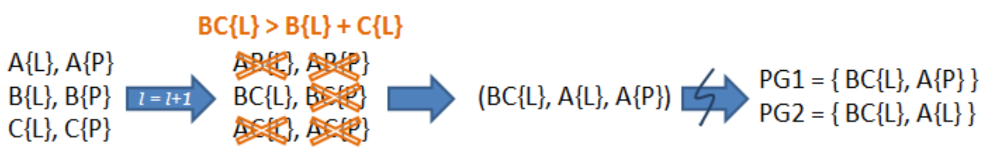

Figure 2- Priority group creation process. Adapted from Liu et al. (2011)

In the case of mining, the goal is to define the destinations of blocks being mined on a given year. So, parallel with Liu et al.'s (2011) paper, each family may be represented by each block being extracted on the same period and each destination can be seen as a block's possible processing destination, such as a stockpile, waste dam, mill, leach pad, etc. This leads to a large problem, considering that mining projects usually entail millions of blocks coming from multiple mines. However, it is possible to pre-process the deposit by clustering similar blocks using a traditional clustering method such as k-means++, where blocks being extracted in the same period which belong to the same initial cluster can be treated as families of blocks and optimized together, as presented in Figure 3. However, the appropriate clustering method must be chosen depending on the type of data available and are usually based on grouping data based on their density and distance in a standardized grid. Clustering algorithms can be classified as centroid-based (such as k-means or affinity propagation clustering), hierarchical (such as spectral clustering), or neighbourhood growers (such as DBSCAN and Agglomerative clustering), where each one has its advantages depending on the type and amount of data being analyzed. K-means++ clustering is implemented here (Goodfellow and Dimitrakopoulos, 2014) which allows the development of robust destination policies that account for multiple attributes and material types, as well as geological uncertainty (as all scenarios are clustered together). However, there are multiple other methods that could be implemented.

Another pre-processing mechanism that can be considered is to remove from the destination policy optimization blocks which are clearly waste (defined by low concentrations of any or all of the valuable elements encountered) and can be sent directly to the waste dump. However, there are two points that might hinder this removal. First, due to geological uncertainty, a block may appear as waste in some simulations but not in others, adding ambiguity to the robust definition of a block as "waste." Second, even though a block may appear to be waste, as different geometallurgical characteristics are considered, the block can still contain other valuable elements needed for meeting blending constraints.

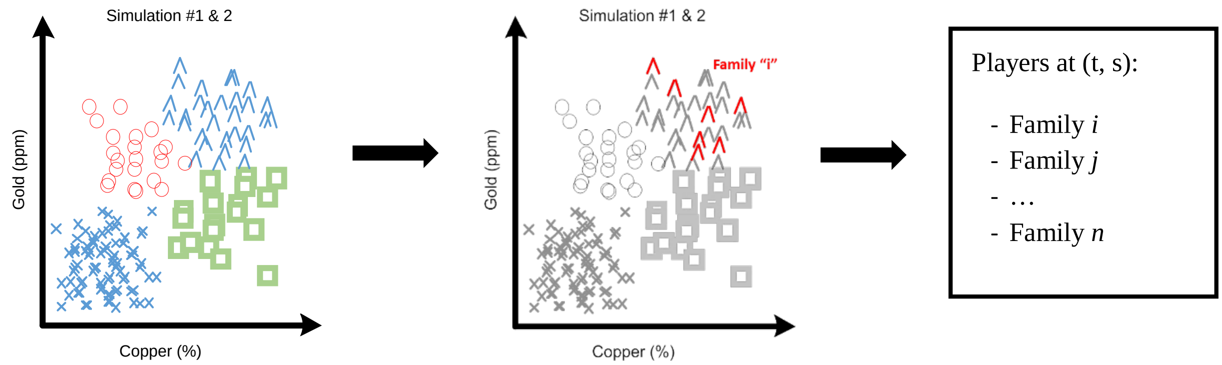

Figure 3 - Definition of families to pre-process data as input to priority group generation

Together with the clustering mechanism, as an initial measure to reduce the complexity of the formulation (as the methodology is being developed), in the following case study the mine production schedule will be assumed fixed. This way, it will be known which blocks are to be extracted in each period. The focus of the optimization process will be then to determine the different coalitions involved on a period and where a coalition represents the blocks sent to each destination (as all blocks scheduled in one period must be sent somewhere).

Here, multiple simulated realizations are considered to account for geological uncertainty. Each realization has multiple attributes. This process allows us to respect the nonlinearity of the recovery of the different processing streams, as well as the optimization of the different blending constraints.

2.3 Definition of characteristic function

When applying coalition formation onto the mining complex problem, the different possible targets of the cooperative game correspond to the possible processing destinations of the system, where the characteristic function of a coalition can be considered as the willingness to pay of a given destination for the set of blocks contained in the coalition. This means that each destination may have a different characteristic function, specified to meet the individual processing constraints, so the characteristic function V_d is such that $V_d : 2^N \to R$, where *N* is the number of players and *d* the destination to which the characteristic function is defined for.

In general, the characteristic function is a linear combination of a set of *j* different parameters (*j=1,…, J* such as preference, cost, targets, etc.), for each available destination $d \in D$ $(p_{j,d})$, which depend on each block (b_i) and/or on the coalition (C) being analyzed; which are weighted ($w_j(b_i)$) according to their level of importance in the overall value definition. These parameters can be a function of the whole coalition *C* (such as processing costs and recovery), or defined by a specific player b_i's characteristics (such as metal content). As presented in Equation 2.2.

$$V_{d\in D}(C = b_0 b_1 \dots b_k) = \sum_{b_i \in C}\left[\sum_{j\in J} w_j(b_i)\, p_{j,d}(b_i, C)\right], \qquad V_d : 2^N \to R \tag{2.2}$$

Given this definition, a simple and direct definition of the characteristic function could be to calculate the discounted revenue of the cluster in a given destination. This formulation is presented in the following equation:

$$V_{d_t}(C = b_1 b_2 \dots b_N) = \frac{1}{(1+d)^t}\cdot[m(\bar{C})\cdot r_d(\bar{C})\cdot(price_t - RC(\bar{C})) - (MC_d(\bar{C}) + PC(\bar{C}))] \tag{2.3}$$

Where *m(C)* is the tonnage of cluster *C*, *r(C)* is the recovery, $price_t$ is the commodity price at time *t*, *RC* corresponds to refinery costs, and *MC(C)* and *PC(C)* correspond to the mining and processing costs respectively. However, the transformation of all variables into dollar value through the previous equation causes a loss of information and tractability of the geometallurgical variables that are needed to be controlled. Given that costs, price, recovery, etc. are not constant, using the previous characteristic function should be avoided.

A possible alternative is to generate independent characteristic functions for each set of comparable attributes of a block, obtaining a characteristic function *vector* for each coalition. This will improve the tractability of variables of interest, having more flexibility to manage their effect over the coalition formation process. Particularly the relational characteristics which are heavily affected by the global material processed together, such as processing costs, blending constraints and recovery.

3 Case study

3.1 Overview of the mining complex

The following case study corresponds to a copper-gold deposit, extracted as an open pit with a mining capacity of 25Mtpa and 6 different processing streams. The deposit, together with gold and copper, contains arsenic and sulphur sulphide concentrations which must be measured for mill performance. Together with the deleterious elements, the lithology of the deposit presents 6 different material types, which correspond to high and low grade of oxides, sulphides and transition material, with different hardness. These material types affect where a given block can be processed.

Fifteen different geological simulations where generated using direct block simulation (DBSim) (Godoy, 2003) to assess the geological uncertainty of the project. Geological uncertainty presents as different grades as well as different material types with variable tonnages per block. Table 1 presents the main mining and economical parameters, scaled by the mining cost for confidentiality purposes.

A diagram of the system is also provided (Figure 4), showing the 6 different destinations available, and what do they produce (in brackets), as well as the different material types accepted in each case (shown by the numbers 1 to 6 in Figure 4). The sulphide mill (SM) is the only processing stream which produces both copper and gold, and it has a stockpile available of 1Mt. The sulphide heap leach (SHL) and dump leach (SDL) both produce copper; the transition heap leach (THL)and the oxide heap leach (OHL) produce only gold.

Table 1 – Mining and economical parameters of the copper/gold mine

Mining Parameters	
Mining Cost	$\$1.00 \cdot x$
Mining Capacity	25 Mt
Processing Costs	
Sulphide Mill	$\$11.30 \cdot x$
Sulphide Heap Leach	$\$2.98 \cdot x$
Sulphide Dump Leach	$\$1.87 \cdot x$
Transition Heap Leach	$\$2.15 \cdot x$
Oxide Heap Leach	$\$2.06 \cdot x$
Economic Parameters	
Copper Price	$2.9/lb
Gold Price	$1050/oz
Discount rate	10%

Each of these processing streams has a variable recovery curve dependent on the head grade fed to the destination. These curves are presented in Figure 5, scaled by the maximum recovery (of the SM in this case). Together with this, the two main processing streams (being the SM and the SHL) present a set of processing constraints and blending requirements.

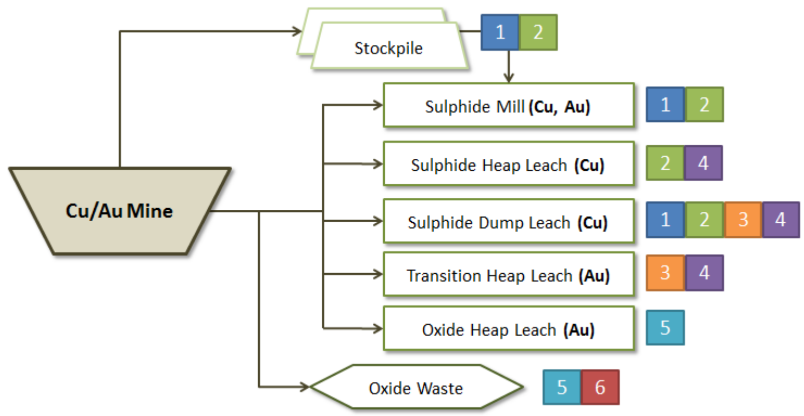

Figure 4 - Diagram of the different processing streams available and the material types accepted

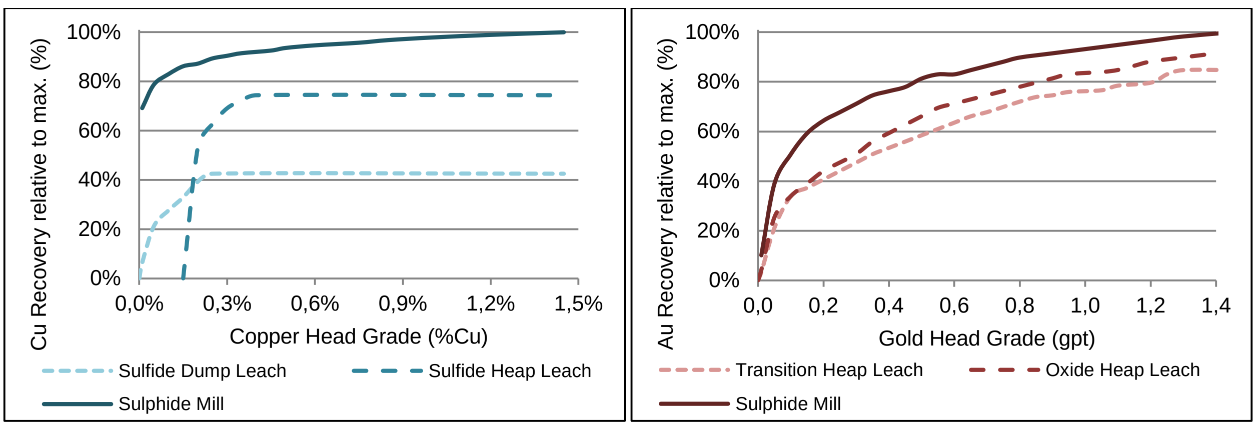

Figure 5 - Recovery curves for (a) copper and (b) for the different processing destinations, with respect to maximum recovery (mill)

For the SM:

- Accepts material types 1 and 2.
- Processing capacity of 3Mtpa, plus a stockpile of 1Mt capacity.
- Sulphur sulphide concentration must be within 6.5 and 8.2%.
- Arsenic content must be below 0.2 to maximize recovery.
- Processing cost of material type 2 is 10% more expensive to process than material type 1 due to rock hardness.

On the other hand, the SHL:

- Accepts material types 2 and 4.
- Processing capacity of 8Mtpa.
- Copper concentration must be over 0.2% at all times.

It must be recalled that these requirements must be met simultaneously, where the value generated will proportionally correspond to the quality of material being sent to be processes together in the different destinations. To track the different requirements, a set of characteristic functions will be defined according to the requirements of each main destination which will help define the most valuable coalitions to process together.

3.2 Set of characteristic functions

Based on the previous mining complex and the different requirements to maximize processing performance, the following characteristic functions have been defined. In this case, these functions are divided between:

(i) Global functions applied over all destinations, which correspond to:
- Maximize revenue (function of recovery and metal content of material processed together)
- Minimize deviation from production targets

(ii) Sulphide mill functions:
- Minimize deviation from sulphur sulphide concentration limits (6.5 - 8.2%)
- Minimize deviations from arsenic maximum concentration (< 0.2%)
- Minimize processing costs (function of material types of rock being processed)

(iii) Sulphide heap leach
- Minimize deviations from copper's minimum concentration (> 0.2%)

3.3 Pre-processing priority groups and coalition generation

Each of these characteristic functions is applied over all the pairwise combinations of players. However, due to the computational intensity of performing all these combinations, two pre-processing steps are applied over the data to reduce the computing time of the algorithm.

First, all scenarios of the whole deposit are clustered together using k-means++ to group blocks into families, making the material type is the same for all blocks within a family. Together with k-means++, material-type/destination connections between families and the different destinations are kept to avoid performing calculations with blocks that are not allowed to be processed in a particular destination due to their material type. This relationship is depicted in Figure 6.

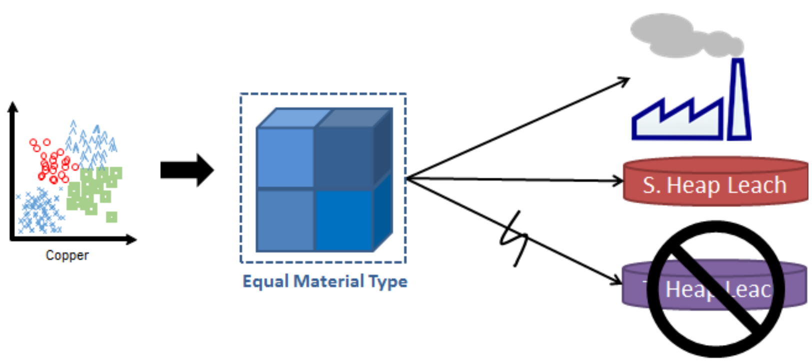

Figure 6 – Rock/destination linkage pre-processing

The initial application of the method, the deposit is assumed to have a fixed schedule. Then the extracted material of each period is optimized into coalitions to be sent to the best available destination given the system constraints and the maximization of revenue.

3.4 Numerical results

To compare the algorithm proposed with current practices, a base case is developed using the traditional method where blocks are sent to a certain destination given their particular attributes (copper and gold grades in this case). Figure 7 presents (a) the mill tonnage feed per period, and (b) the SHL feed. The orange line shows the expected tonnage feed given the estimated orebody model (base case) The grey lines beneath show the risk analysis of this base case (BC) representing the performance of the proposed schedule and destination policy for the 15 different geological scenarios.

It can be seen that in the case of the mill, there is a 20% shortfall in tonnage along the LOM, showing that the base case is not really realistic when faced to geological uncertainty. In the case of the SHL the shortfall is less, but there is still difficulty to meet production targets. The other destinations are not presented, as their capacities where unlimited and no geometallurgical constraints where applied over them.

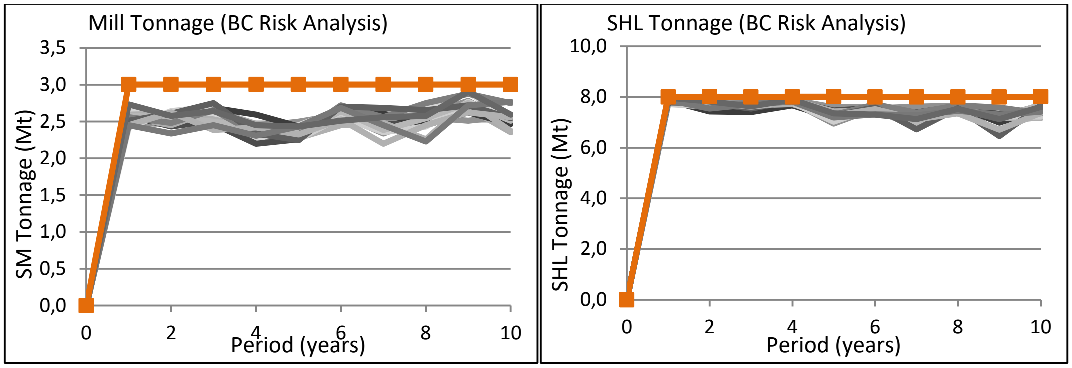

Figure 7 - (a) Sulphide Mill feed and (b) Sulphide Heap Leach, for the deterministic case (orange) and the 15 geological scenarios (gray)

The main constraints considered in this case study where

i) the sulphur sulphide blending limits in the mill,
ii) arsenic's maximum concentration in the mill, and
iii) copper's minimum grade in the SHL.

The comparison between the base case and the proposed method for these three constraints are presented next, where (a) presents the base case's performance (left side figure) and (b) presents the results obtained by optimizing the destination policy with the proposed coalition formation algorithm (right side figure). Figure 8a presents sulphur sulphide (SS) grade for the deterministic case in orange, where the blending constraints are barely met until year 8 when the fed SS grade passes the maximum concentration. However, when geological uncertainty is considered (gray lines), it can be seen that SS exceeds the maximum limit in almost every year. On the other hand, Figure 8b shows that the priority group coalition method proposed (PG Risk Analysis) manages to reduce SS grade up to the acceptable limits considerably better. There can still be seen big deviations in period 5 and between periods 7 and 9, however this is mostly due to the fact that the schedule is assumed fixed for this case study, so the material extracted in that period has considerably high SS grade. If the algorithm was able to adapt the schedule, then it would be possible to manage the processed material in order to meet blending constraints by delaying low SS grade material from the initial periods to reduce de feed grade of later periods. This is proposed as future research.

In the case of arsenic (As), the mill requires a concentration lower than 0.2% in order to maximize metal recovery and obtain optimum processing performance. The base case presented in Figure 9a shows that the material fed to the mill exceeds the maximum concentration in almost every case up until period 7 and again at period 10. On the other hand, the proposed method improves considerably this processing requirement and is able to keep arsenic concentration below the limit in almost every case, except in period 3 for some of the geological scenarios.

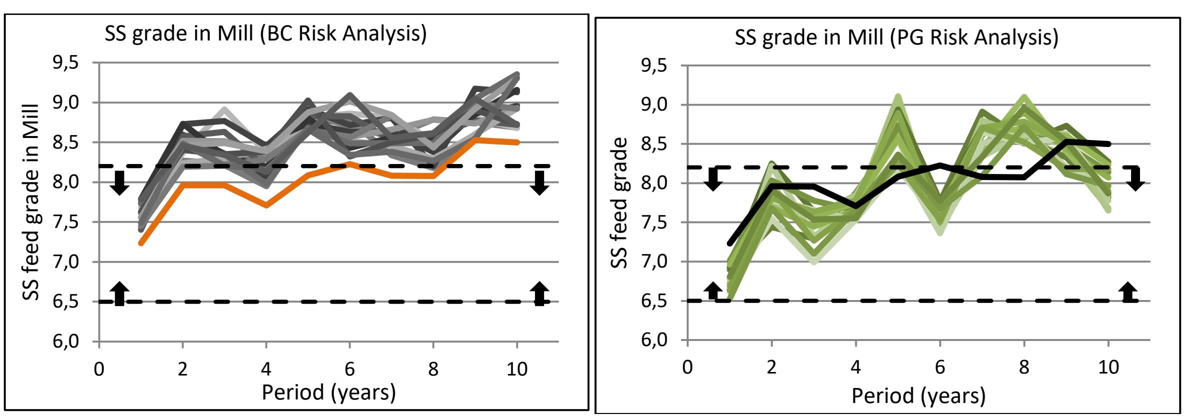

Figure 8 - Risk profiles for the SS grade fed to the Mill in the (a) base case, for the deterministic case (orange) and the 15 geological scenarios (gray) and (b) the optimized destination policy

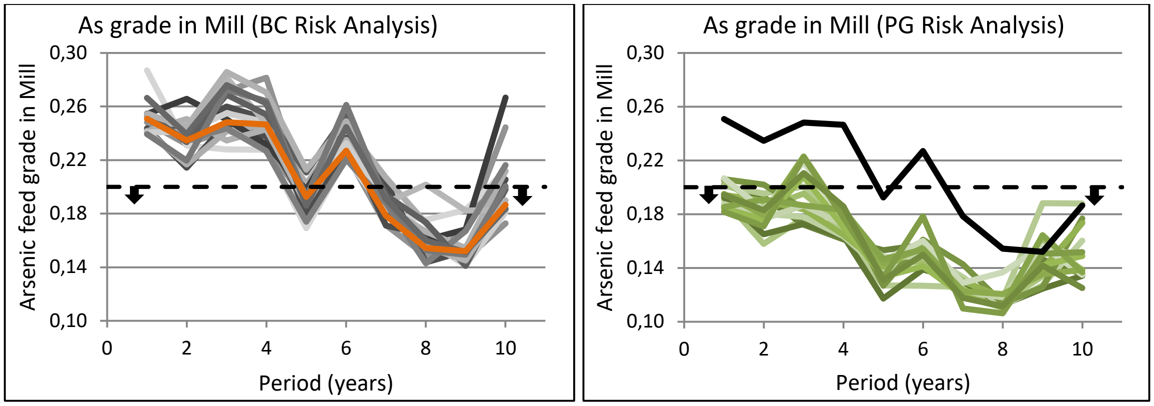

Figure 9 - Risk profiles for the As grade fed to the Mill in the (a) base case, for the deterministic case (orange) and the 15 geological scenarios (gray) and (b) the optimized destination policy

Finally, Figure 10 shows that copper (Cu) concentration of the material fed to the SHL is above the required minimum in every scenario for both the base case, and the proposed method. This shows that the coalition formation algorithm manages to improve blending requirements without punishing the grade of valuable metals.

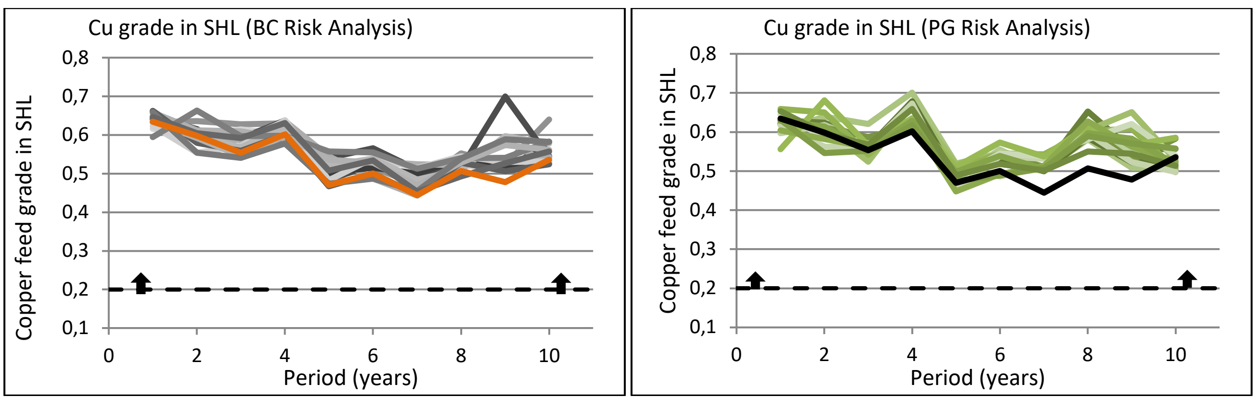

Figure 10 - Risk profile of Cu grade fed to the SHL in the (a) base case, for the deterministic case (orange) and the 15 geological scenarios (gray) and (b) the optimized destination policy

Considering a discount rate of 10% over the 10 year LOM, Figure 11 presents the cumulative discounted cash flow for the deterministic base case (in red) which, for confidentiality reasons, is presented as the 100% reference value; for the different geological scenarios over the base case (in gray, which show a 4.8% lower NPV than the expected by the deterministic model) and the optimized priority group coalition scenarios (in dotted black), increasing the NPV in an average of 5.6% over the deterministic base case.

4 Conclusions

This previous study presents a destination policy mechanism developed through coalition formation, by using game theory techniques to account for the value generated by groups of blocks being processed together, and at the same time, considering complex geometallurgical constraints that are often left out of the mining complex optimization. This mechanism develops characteristic functions that describe the value of coalitions of blocks being processed together on a same destination, and define the optimal destinations by calculating the Shapley Value of each block or cluster of blocks (i.e. the players of this cooperative game).

A case study of a copper-gold deposit with six material types and six possible destinations showed that the proposed PG method is able to account for the value generated from extracted material with multiple categorical and continuous characteristics, and optimize its processing destination so that not only all processing and blending constraints are met, but also project value is maximized. This allows having a more realistic representation of the project value. Results from the case study showed that the proposed algorithm was able to reduce As concentrations and improve SS ranges in the mill feed material, without reducing Cu grades nor final revenue, as the PG destination policy delivered a project with an NPV 5.6% higher than the base case (developed by traditional methods). These results were obtained just by redistributing the extracted material, as the schedule was assumed constant for both BC and PG cases.

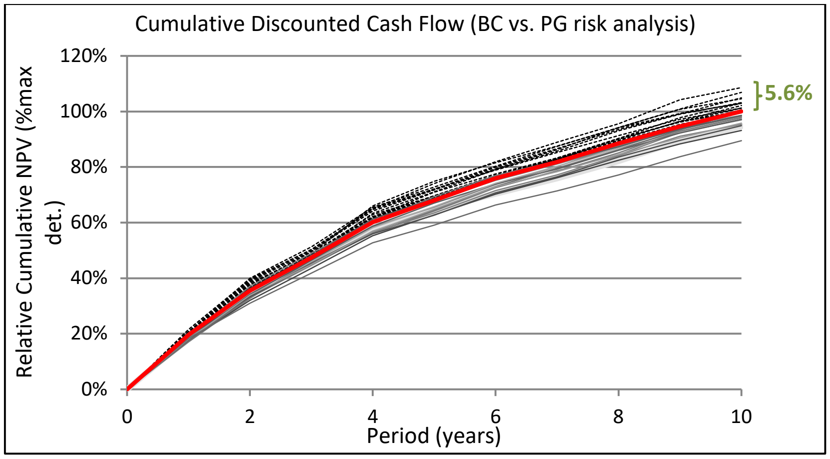

Figure 11 - Cumulative Discounted Cash Flow (CDF) of the BC (red), and the risk analysis over the BC (gray) and the PG (dotted)

Future research will focus at extending the current formulation to the scheduling problem; i.e. to be able to select which material is extracted in every period so that blending constraints are met while maximizing project value. Together with this, we will look at directly integrating geological and geometallurgical uncertainty into the coalition formation process by using a stochastic Shapley Value (Kargin, 2005) represented by a set of scenarios. Here, the coalition formation process can aim at maximising the expected Shapley Value of a group and minimize its standard deviation (i.e. the risk of not obtaining that satisfaction level). This will provide a scenario independent destination policy, which facilitates the operational applicability of the method, as scenarios do not correspond to reality the actual material extracted will deviate from its simulations, making it necessary to classify material and decide its destination based on this classification. However, a crucial problem with this approach that must be studied further is the misclassification errors, as the simulated material type of a block can vary from one scenario to another making a block infeasible to process in a given location on one scenario, but not in others.

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