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B. Vardar,
G. Zaccour

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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

The strategic impact of adaptation in a transboundary pollution dynamic game

Baris Vardar [a]
Georges Zaccour [b]

[a] *GERAD, HEC Montréal*

[b] *Chair in Game Theory and Management & GERAD, HEC Montréal*

`baris.vardar@gerad.ca`
`georges.zaccour@gerad.ca`

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Abstract: This work studies the strategic impact of a region's investment in adaptation measures on the equilibrium outcomes of a transboundary pollution dynamic game played in finite horizon. We incorporate adaptation as a region-specific capital stock that decreases local damages and study the feedback (subgame perfect) equilibrium of the non-cooperative game between two regions. In order to discern the impact of adaptation, we compare the equilibrium solutions of three scenarios, which differ in the regions' ability to invest in adaptation measures. The results show that investing in adaptation gives regions an incentive to increase their emissions, which causes an inverse strategic response in the other region. The anticipation of a rise in pollution makes the other region respond by cutting its emissions and investing more in adaptation. The equilibrium trajectories of the stocks of pollution and adaptation capital follow the highest path over time when both regions adapt. When there is an asymmetry between regions in their adaptation capabilities, the region that does not (or cannot) adapt becomes worse off due to lower emissions and higher damages, while the adapting region finishes the game better off than the no-adaptation case.

Keywords: Adaptation, transboundary pollution, dynamic game, non-cooperative solution, Feedback-Nash equilibrium.

Résumé : Cet article analyse l'effet stratégique d'investir en adaptation dans un jeu dynamique de pollution transfrontalière à horizon fini. Nous intégrons l'adaptation comme un investissement dans un stock de capital spécifique à chaque région et qui vise à réduire les coûts de dommage, et étudions l'équilibre de Nash parfait en sous-jeu entre deux régions. Afin de discerner l'impact de l'adaptation, nous comparons trois scénarios différant en termes de capacité des régions à investir en mesures d'adaptation. Les résultats montrent que la possibilité d'investir dans l'adaptation incite les régions à augmenter leurs émissions, provoquant une réponse stratégique inverse dans l'autre région. Anticipant une augmentation du stock de pollution, l'autre région réduit ses émissions et investit davantage en capital d'adaptation. Les trajectoires d'équilibre des stocks de pollution et de capital d'adaptation suivent le sentier le plus élevé lorsque les deux régions s'adaptent. Quand il y a une asymétrie entre les régions dans leurs capacités d'adaptation, la région qui ne pas (ou ne peut pas) s'adapter souffre de dommages plus élevés, alors que la région qui s'adapte fait mieux que le jeu sans adaptation.

Mots clés : Adaptation, pollution transfrontalière, jeu dynamique, solution non-coopérative, l'équilibre de Nash parfait en sous-jeu.

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1 Introduction

Adaptation is increasingly considered a policy response to climate change, alongside the mitigation of greenhouse gas emissions. Adaptation measures consist of the set of actions that prevent or decrease the adverse effects of accumulated pollution. Whereas mitigation efforts have positive effects on all, protective measures usually provide only local benefits and thus are typically seen as private goods. Moreover, most of these measures involve large investments in capital and infrastructure, and influence not only current environmental damage costs and decisions but also future ones.

This work aims at characterizing the strategic and intertemporal impacts of adaptation policies on mitigation efforts and the outcomes for the players (countries, regions, etc.) behaving non-cooperatively when making their decisions. The literature in this area is sparse and of relatively recent vintage. For example, Buob and Stephan (2011) consider a two-stage decision-making-process and show that regions are more willing to engage in adaptation activities when they act non-cooperatively. More recently, Bréchet et al. (2016) present a model that includes capital accumulation, pollution mitigation, and environmental adaptation decisions in a multi-country setting and investigate the long-run open-loop Nash equilibrium; they obtain that non-cooperation between regions leads to an increase in production, consumption, pollution, and adaptation. In another related work, Benchekroun and Taherkhani (2014) study the allocation of pollution reduction costs within an n-player transboundary pollution game, and show that reducing the environmental damage cost of pollution in one region can harm the other regions. Ingham et al. (2007) analyze a two-period transboundary pollution game that includes uncertainty and learning with adaptation and mitigation strategies, and discuss in Ingham et al. (2013) whether these strategies are complements or substitutes. The effects of adaptation policy on the formation and stability of international environmental agreements (IEAs) are studied in Benchekroun et al. (2017), Breton and Sbragia (2017a,b), and Masoudi and Zaccour (2016, 2017). In an earlier work, Zehaie (2009) examines the timing and strategic role of self-protection in a two-stage game in both non-cooperative and cooperative frameworks, and shows that self-protection, when timed before abatement, has strategic effects in the other region. Finally, we note that Ebert and Welsch (2011, 2012) and Heuson et al. (2015) are among the recent studies that consider adaptation policy in static and multi-stage game-theoretical frameworks.

There is also a growing literature that investigates various aspects related to adaptation policy. Recent works include Hritonenko and Yatsenko (2016), who study different static and dynamic problems involving the combination of mitigation and adaptation decisions in representative agent frameworks, by using optimal control methods. Zemel (2015) adopts similar methods to investigate the role of damage uncertainty in adaptation decisions, and Bréchet et al. (2013) study the long-run equilibrium of a social planner problem where adaptation and abatement investments are separate decision variables. Another modeling approach is presented by Habla and Roeder (2017), where adaptation and mitigation measures are studied in an overlapping generations framework. In another stream of the literature (see, e.g., De Bruin et al. (2009) and Agrawala et al. (2011a,b)) the intertemporal impacts of adaptation are measured using an integrated assessment modeling approach. A related work in this stream is Felgenhauer and Webster (2014), who consider short-term, temporary (flow), and long-lived, capital-intensive (stock) adaptation measures together with a mitigation policy, and demonstrate the trade-offs between them.

Our framework is a transboundary pollution dynamic game initially proposed by van der Ploeg and de Zeeuw (1992) and Dockner and Long (1993). This framework has been extended in different directions by many authors. We refrain from reviewing the very dense literature in this area and instead refer the interested reader to the surveys in Jørgensen et al. (2010) and Long (2011, 2012) for extensive reviews of the earlier literature, to Calvo and Rubio (2012) and Benchekroun and Long (2012) for the research focusing mostly on applications to the international environmental agreements, and to de Zeeuw (2016) for a recent, selective review of the literature in this area.

More specifically, we consider a non-cooperative, two-player, finite-horizon transboundary pollution differential game. The regions gain benefits by generating emissions, which are interpreted as a by-product of their economic activities. Emissions accumulate in the common pollution stock and cause environmental damage in both regions. We focus our analysis on the feedback (subgame perfect) equilibrium of the game in which the regions' strategies are strongly time consistent. We consider three scenarios that differ in terms of the regions' ability to invest in adaptation measures to reduce the environmental damage caused by the pollution stock. In the first scenario, both regions can invest in adaptation capital, whereas in the second, only one region has this option. In the third (benchmark) scenario, no region invests in adaptation, that is, the two regions play the classical transboundary pollution dynamic game. By comparing the outcomes of these three scenarios, we aim to address the following questions:

1. What is the impact of one region investing in adaptation while the other does not (or cannot) on the equilibrium levels of the regions' emissions, on the level of pollution, and on the total payoffs for the regions?
2. How do the regions' emissions and investment in adaptation strategies respond to a change in the level of the region's own stock of adaptation capital and in that of the other region?
3. How does the length of the planning horizon affect decisions on emissions, pollution, and adaptation?

The results show that the regions' ability to invest in adaptation measures and to protect themselves from the damage from the common pollution stock gives them an incentive to increase their production activities (emissions). The incentive to increase emissions in the adapting region causes an inverse strategic effect in the other region. More specifically, when a region increases its stock of adaptation capital, the other region anticipates that this will be accompanied by an increase in emissions and thus an increase in the level of pollution, and thus, it responds by decreasing its emissions and increasing its investment in its own stock of adaptation capital.

The equilibrium trajectories of the pollution stock and adaptation capital stocks are highest when both regions invest in adaptation. In the case of asymmetry in the regions' ability to invest in adaptation, the adapting region becomes better-off while the not-adapting region emits less and faces higher damage over the whole duration of the game.

The rest of the article is structured as follows: Section 2 introduces the model and describes the functions that define the regions' payoffs, Section 3 analyzes the subgame perfect equilibrium solutions of the three selected scenarios. Section 4 presents the equilibrium strategies and trajectories, and Section 5 concludes.

2 The model

Consider two a priori symmetric regions indexed by $i = 1, 2$. Whereas symmetry is often imposed for analytical tractability, here, however, we assume it to guarantee that variations in the equilibrium results between the different scenarios are due exclusively to the behavior of regions, that is, whether or not they adapt, and not to differences in parameter values. Time t runs continuously, and the planning horizon is finite and denoted by T. Production activities by each region yield utility U_i and, as a by-product, pollutant emissions, e.g., CO_2. Denote by Y_i region i's production of goods and services and by e_i the resulting emissions, with $e_i = h_i\left(Y_i\right)$, where $h_i\left(\cdot\right)$ is an increasing function satisfying $h_i\left(0\right) = 0$. Assuming a monotone increasing relationship between production and utility, we can express the utility directly as a function of emissions, that is, $U_i\left(e_i\right)$. The utility function is assumed to have a linear-quadratic form given by

$$U_i(e_i) = e_i\left(\alpha_i - \beta_i\frac{e_i}{2}\right), \tag{1}$$

where α_i and β_i are strictly positive parameters. This functional form is used in many studies (see the survey in Jørgensen et al. (2010)) and it implies a diminishing marginal utility of emissions in the interior solution. The emissions generated in each region accumulate in the stock of common pollution $P(t)$ whose evolution is governed by the following differential equation:

$$\frac{dP}{dt}(t) = e_1(t) + e_2(t) - kP(t). \tag{2}$$

At each instant of time, each region's emissions are added to the stock of pollution, which decays at the natural sequestration rate denoted by $k \in (0, 1)$. The accumulated stock of pollution causes damage in each region, which can be reduced locally by investing in specific adaptation capital denoted by A_i. Similarly to Benchekroun et al. (2017) and Masoudi and Zaccour (2016, 2017), we assume that the damage cost depends on the levels of pollution stock and adaptation capital, and retain the following functional form:

$$D_i(P, A_i) = \frac{s_i P^2}{2} - \gamma_i A_i P, \tag{3}$$

where s_i and γ_i, $i = 1, 2$ are strictly positive parameters. The first element of the damage function is by now standard in the literature (see, e.g., Long (1992), van der Ploeg and de Zeeuw (1992), and Dockner and Long (1993)). The second term includes the region's stock of adaptation capital as a variable that decreases the damage. The assumption of this functional form implies that the reduction in the marginal damage by a marginal increase in adaptation capital is a given constant $\gamma_i \geq 0$ $\left(\frac{\partial^2 D}{\partial P \partial A_i} = -\gamma_i\right)$. Therefore, the parameter γ_i can be interpreted as the efficiency of adaptation capital in Region i. The case $\gamma_i = 0$ represents a context where adaptation is not possible, which is one of the scenarios considered in the paper.

Since adaptation measures mostly take the form of large investments in capital and infrastructure, and since their impacts are intertemporal, we incorporate adaptation as a stock variable that is private for each region. Each region owns an initial stock of adaptation capital, denoted by $A_i(0)$, and must make an investment in order to increase its stock or to maintain it. The stock of adaptation capital in each region evolves as follows:

$$\frac{dA_1}{dt}(t) = I_1(t) - \psi_1 A_1(t), \tag{4}$$

$$\frac{dA_2}{dt}(t) = I_2(t) - \psi_2 A_2(t), \tag{5}$$

where $\psi_i > 0$ is the depreciation rate of adaptation capital in Region i, and $I_i(t)$ denotes the investment in adaptation capital in Region i at date t. The cost of investment in each region is assumed to be strictly increasing and convex and given by

$$H_i(I_i) = \frac{\phi_i I_i^2}{2} \text{ for } i = 1, 2, \tag{6}$$

where $\phi_i > 0$ denotes the coefficient of the marginal cost of investment in adaptation capital in Region i.

Using the functional forms given above, the instantaneous payoff is written as

$$J_i(t) = U_i(e_i(t)) - D_i(P(t), A_i(t)) - H_i(I_i(t)), \tag{7}$$

which means that, at each date t, Region i obtains a benefit from its emissions, receives damage that depends on the accumulated common pollution stock and on its own stock of adaptation capital, and pays the cost of investment in adaptation capital at that date.

3 Three scenarios

In this section, we investigate how the consideration of investing in adaptation influences the regions' strategies in the transboundary pollution game. As mentioned in the introduction, we study three scenarios that differ in terms of the regions' ability to invest in adaptation measures. In all these scenarios, the two regions play a non-cooperative game using a feedback information structure, that is, emissions and investment in adaptation strategies are state-dependent, with the state being defined as a four-dimensional vector (t, P, A_1, A_2). By comparing the solutions of these scenarios, we aim to gain insights into the impact of adaptation on the equilibrium outcomes.

Remark 1 *The reason for having non-stationary (time-varying) strategies is our assumption that the planning horizon is finite. The dynamic game literature has almost always considered an infinite planning horizon, either for tractability or because there is no valid conceptual reason for a player (country, region, etc.) to retain a short planning horizon. In our setting, it does not make much intuitive sense to rule out adaptation forever, while it is realistic to assume that such an option may not be available in the short term.*

3.1 Both regions invest in adaptation

In this scenario, both regions consider investing in adaptation capital in their decision-making. Technically, the problem consists of a linear-quadratic differential game that includes two control variables for each region, that is, e_i and I_i, and four state variables, namely, t, P, A_1, and A_2. Each region's objective is to maximize its sum of discounted instantaneous payoffs given by the following program:

$$\max_{e_i(t),I_i(t)} \int_{t=0}^{T} e^{-\rho_i t}\Big(U_i(e_i(t)) - D_i(P(t), A_i(t)) - H_i(I_i(t))\Big)dt + e^{-\rho_i T} S_i(P(T), A_i(T)) \tag{8}$$

$$\text{subject to (2), (4) and (5),}$$

$$\text{with } P(0), A_1(0), A_2(0) \geq 0 \text{ given,}$$

where ρ_i is the discount rate and $S_i(P(T), A_i(T))$ represents the salvage value of Region i. We assume that the function S_i is linear in its arguments and is decreasing in pollution and non-decreasing in the stock of adaptation capital. Formally,

$$S_i(P(T), A_i(T)) = \mu_i P(T) + \lambda_i A_i(T), \quad \mu_i < 0, \quad \lambda_i \geq 0. \tag{9}$$

Together with the parameter γ_i in the damage function, the parameter λ_i in the salvage value function also determines the benefit of accumulating adaptation capital. When it is positive, there is a reward for arriving at the terminal time with a higher stock of adaptation capital, which gives regions another incentive to invest in it.

Denote by $V_i(t, P, A_1, A_2)$ the value function of player i. Recall that the value function measures the gain that a player can get from any position of the game, that is, $(t, P(t), A_1(t), A_2(t))$. The Hamilton-Jacobi-Bellman (HJB) equation associated with Region i's optimization problem is as follows:

$$\begin{aligned} \rho_i V_i(t, P, A_1, A_2) - \frac{\partial V_i}{\partial t} = \max_{e_i, I_i} \Big\{ & U_i(e_i) - D_i(P, A_i) - H_i(I_i) + \frac{\partial V_i}{\partial P}\Big(e_1 + e_2 - kP\Big) \\ & + \frac{\partial V_i}{\partial A_1}\Big(I_1 - \psi_1 A_1\Big) + \frac{\partial V_i}{\partial A_2}\Big(I_2 - \psi_2 A_2\Big)\Big\} \text{ for } i = 1, 2, \end{aligned} \tag{10}$$

with

$$V_i(T, P(T), A_1(T), A_2(T)) = S_i(P(T), A_i(T)), \tag{11}$$

where (11) is the transversality condition. The first-order conditions for the maximization program in the right-hand side yield the following feedback strategies for the two regions:

$$e_i^*(t, P, A_1, A_2) = \frac{1}{\beta_i}\left(\alpha_i + \frac{\partial V_i}{\partial P}(t, P, A_1, A_2)\right) \text{ for } i = 1, 2, \tag{12}$$

$$I_i^*(t, P, A_1, A_2) = \frac{1}{\phi_i}\frac{\partial V_i}{\partial A_i}(t, P, A_1, A_2) \text{ for } i = 1, 2. \tag{13}$$

The emissions strategies in (12) state that, in equilibrium, the marginal benefits of emissions must be equal to their marginal costs. The marginal cost of emitting pollution is measured as the impact of a marginal increase in pollution on the total payoff, which corresponds to the shadow value of pollution in Region i, given by the derivative of the value function with respect to pollution ($\frac{\partial V_i}{\partial P}$). Similarly, the investment strategies in (13) equalize the marginal cost of investment in adaptation capital to its marginal benefit. This benefit is measured as the impact of an additional unit of adaptation capital on the total payoff. Indeed, since we consider adaptation a stock variable, investing in an additional unit yields intertemporal benefits because it will decrease current and future damage in the region. This effect is represented by the shadow value of adaptation capital in Region i, given by $\frac{\partial V_i}{\partial A_i}$. Note that if we had considered adaptation a flow variable, i.e., a type of measure that decreases only the instantaneous damage, then the investment strategy would result from a static optimization problem, which would not include the adaptation measures' intertemporal effects.

The following proposition presents the characterization of the feedback-Nash equilibrium in this scenario:

Proposition 1 *If there exist value functions $V_i(t, P, A_1, A_2)$ for $i = \{1, 2\}$ and the feedback strategies $(e_i^*(t, P, A_1, A_2), I_i^*(t, P, A_1, A_2))$ satisfy the equilibrium conditions given in (12) and (13) for all $t \in [0, T]$, then the pairs (e_i^*, I_i^*) for $i = \{1, 2\}$ constitute a feedback-Nash equilibrium. Moreover, the value function V_i represents the equilibrium total payoff of Region i for the game starting at point (t, P, A_1, A_2).*

Inserting the equilibrium strategies given in (12–13) into (10) and rearranging the terms, the HJB equations become[1]

$$\begin{aligned}\rho_i V_i(t, P, A_1, A_2) - V_{it} = \kappa_{i1} + \kappa_{i2} P^2 + \kappa_{i3} V_{iP} + \kappa_{i4} V_{iP}^2 + \kappa_{i5} V_{iP} P + \kappa_{i6} V_{iP} V_{jP} \\ + \kappa_{i7} V_{iA_i}^2 + \kappa_{i8} V_{iA_i} A_i + \kappa_{i9} A_i P + \kappa_{i10} V_{iA_j} V_{jA_j} + \kappa_{i11} V_{iA_j} A_j, \end{aligned} \tag{14}$$

where the coefficients of the value function of Region i are given by

$$\begin{aligned}&\kappa_{i1} = \frac{\alpha_i^2}{2\beta_i}, \quad \kappa_{i2} = \frac{-s_i}{2}, \quad \kappa_{i3} = \frac{\alpha_i}{\beta_i} + \frac{\alpha_j}{\beta_j}, \quad \kappa_{i4} = \frac{1}{2\beta_i}, \quad \kappa_{i5} = -k, \quad \kappa_{i6} = \frac{1}{\beta_j}, \\ &\kappa_{i7} = \frac{1}{2\phi_i}, \quad \kappa_{i8} = -\psi_i, \quad \kappa_{i9} = \gamma_i, \quad \kappa_{i10} = \frac{1}{\phi_j}, \quad \kappa_{i11} = -\psi_j, \end{aligned} \tag{15}$$

for $i = \{1, 2\}$ and $j \neq i$. Since the game has a linear-quadratic structure, we make the informed guess that the value function of each region is a polynomial of degree 2, with 10 undetermined coefficients, each of them time dependent. The value function of each region can be written in the following form:

$$\begin{aligned}V_i(t, P, A_1, A_2) = c_{i0}(t) + c_{i1}(t)P + c_{i2}(t)P^2 + c_{i3}(t)A_1 + c_{i4}(t){A_1}^2 + c_{i5}(t)A_1 P \\ + c_{i6}(t)A_2 + c_{i7}(t){A_2}^2 + c_{i8}(t)A_2 P + c_{i9}(t)A_1 A_2, \end{aligned} \tag{16}$$

for $i = \{1, 2\}$. Replacing these forms into the rearranged HJB equations in (14) results in a polynomial of degree 2 in three variables for each region. Furthermore, we apply the undetermined coefficients method (see Haurie et al. (2012)) to compute the coefficients associated to the value functions. This method imposes that the polynomials obtained by rewriting the HJB equations with the form in (16) hold true for all values of $\{t, P, A_1, A_2\}$. The solution of the resulting differential equation system gives us the trajectories of the value function's coefficients for each region.

Remark 2 *Let C_i denote the vector of the coefficients of V_i and $C = \{C_1, C_2\}$. The resulting Ricatti differential equation (RDE) system can be written as follows:*

$$\rho_i c_{il} - \dot{c}_{il} = A_{il} + B_{il} C + C^T M_{il} C, \tag{17}$$

for $i = \{1, 2\}$ and $l = \{0, 1, ..., 9\}$, where $A_{il} \in \mathbb{R}$ is a constant, $B_{il} \in \mathbb{R}^{20}$ a vector for linear terms and $M_{il} \in \mathbb{R}^{20\times 20}$ a matrix for cross and quadratic terms. The system is written explicitly in Appendix A.1.

The solution to this differential equation system must be found backwards, with the terminal values of the coefficients being exogenously given by the salvage value function in (9). Accordingly, in order to satisfy (11), we must have

$$c_{i1}(T) = \mu_i \text{ for } i = \{1, 2\}, \qquad c_{13}(T) = \lambda_1, \qquad c_{26}(T) = \lambda_2, \tag{18}$$

with the rest of the coefficients being equal to zero at the terminal time T. Unfortunately, it is not possible to obtain an analytical solution to the differential equation system; nevertheless, we can determine the conditions for studying some characteristics of the regions' strategies. We focus on the case in which both regions make their decisions using linear feedback strategies. Rewriting the strategies given in (12–13) with the form of the value functions in (16) results in linear functions of the state variables given by

$$e_i^*(t, P, A_1, A_2) = \left(\alpha_i + c_{i1}(t) + 2c_{i2}(t)P + c_{i5}(t)A_1 + c_{i8}(t)A_2\right)/\beta_i \text{ for } i = \{1, 2\}, \tag{19}$$

---

[1] V_{ix} denotes the partial derivative of Region i's value function with respect to the variable x ($\frac{\partial V_i}{\partial x}$).

$$I_1^*(t,P,A_1,A_2) = (c_{13}(t) + 2c_{14}(t)A_1 + c_{15}(t)P + c_{19}(t)A_2)/\phi_1, \tag{20}$$

$$I_2^*(t,P,A_1,A_2) = (c_{26}(t) + 2c_{27}(t)A_2 + c_{28}(t)P + c_{29}(t)A_1)/\phi_2. \tag{21}$$

The feedback response functions depend on the values of the state variables as well as on time through the value function coefficients. This means that, for a given set of values of pollution and adaptation capital in each region, the emissions and investment strategies differ over time due to the finite planning horizon. In addition, applying the conditions in (18), we see that the strategies at the terminal time are determined by the exogenously given salvage value parameters and are the same for all games that start in any initial state.

One of the main questions of this study is how a region's emissions and choices regarding adaptation investment respond to an increase in the level of pollution, and to its own and the other region's adaptation capital. To address this point, we look at the partial derivatives of (19), (20), and (21) with respect to each state variable:

$$\begin{aligned}
&\frac{\partial e_i^*}{\partial P} = 2c_{i2}(t)/\beta_i, \quad \frac{\partial e_i^*}{\partial A_1} = c_{i5}(t)/\beta_i, \quad \frac{\partial e_i^*}{\partial A_2} = c_{i8}(t)/\beta_i \text{ for } i = \{1,2\}\,,\\
&\frac{\partial I_1^*}{\partial P} = c_{15}(t)/\phi_1, \quad \frac{\partial I_1^*}{\partial A_1} = 2c_{14}(t)/\phi_1, \quad \frac{\partial I_1^*}{\partial A_2} = c_{19}(t)/\phi_1,\\
&\frac{\partial I_2^*}{\partial P} = c_{28}(t)/\phi_2, \quad \frac{\partial I_2^*}{\partial A_1} = c_{29}(t)/\phi_2, \quad \frac{\partial I_2^*}{\partial A_2} = 2c_{27}(t)/\phi_2.
\end{aligned} \tag{22}$$

Accordingly, the signs of these response functions can be identified by studying the signs of the value functions' coefficients. In Section 4, we shall look into these signs in detail with various calibration settings to see whether their signs constitute a regular pattern among all solutions and are thus independent of the values of the state variables and time.

3.2 Only one region invests in adaptation

In this second scenario, we consider that there is an asymmetry between regions in their ability to decrease the damage of pollution by investing in adaptation. There may be several reasons for a region not having the possibility to invest in adaptation measures. For instance, if the nature of the damage faced by the region does not allow for protective measures, or if domestic funds are insufficient and adaptation measures too costly, then the region cannot choose to invest in adaptation. As a representative case, we consider that only Region 1 invests in its own adaptation capital, whereas Region 2 does not have the option to invest, hence $A_2(t) = 0$ and $I_2(t) = 0$ for all $t \in [0,T]$. (In fact, it suffices to set $\gamma_2 = 0$ and $\lambda_2 = 0$ to have $A_2(t) = I_2(t) = 0$ at equilibrium, as there would be no reason to incur the investment cost in adaptation.) The problem then consists of three state variables, namely, t, P, and A_1.

Each region maximizes its discounted sum of instantaneous payoffs given by the following programs:

Region 1:

$$\max_{e_1(t),I_1(t)} \int_{t=0}^{T} e^{-\rho_1 t}\Big(U_1(e_1(t)) - D_1(P(t),A_1(t)) - H_1(I_1(t))\Big)dt + e^{-\rho_1 T}S_1(P(T),A_1(T))$$

subject to (2) and (4),
with $P(0), A_1(0) \geq 0$ given,

Region 2:

$$\max_{e_2(t)} \int_{t=0}^{T} e^{-\rho_2 t}\Big(U_2(e_2(t)) - D_2(P(t),0)\Big)dt + e^{-\rho_2 T}S_2(P(T),0)$$

subject to (2) and (4),
with $P(0), A_1(0) \geq 0$ given.

As the maximization programs show, even though Region 2 does not invest in adaptation, it takes into account the fact that Region 1 is investing in adaptation to decrease its damage cost. The HJB equations associated to the two regions' problems are written as

$$\rho_1 \widetilde{V}_1(t,P,A_1) - \frac{\partial \widetilde{V}_1}{\partial t} = \max_{e_1, I_1} \Big\{ U_1(e_1) - D_1(P,A_1) - H_1(I_1) + \frac{\partial \widetilde{V}_1}{\partial P}(e_1+e_2-kP) + \frac{\partial \widetilde{V}_1}{\partial A_1}(I_1 - \psi_1 A_1) \Big\}, \tag{23}$$

and

$$\rho_2 \widetilde{V}_2(t,P,A_1) - \frac{\partial \widetilde{V}_2}{\partial t} = \max_{e_2} \Big\{ U_2(e_2) - D_2(P,0) + \frac{\partial \widetilde{V}_2}{\partial P}(e_1+e_2-kP) + \frac{\partial \widetilde{V}_2}{\partial A_1}(I_1 - \psi_1 A_1) \Big\}. \tag{24}$$

Maximization of the right-hand sides gives the following rules for the feedback strategies in this scenario:

$$\tilde{e}_i(t,P,A_1) = \frac{1}{\beta_i}\left(\alpha_i + \frac{\partial \widetilde{V}_i}{\partial P}(t,P,A_1)\right) \text{ for } i = \{1,2\}, \tag{25}$$

$$\tilde{I}_1(t,P,A_1) = \frac{1}{\phi_1}\frac{\partial \widetilde{V}_1}{\partial A_1}(t,P,A_1). \tag{26}$$

The comments we made on the feedback strategies of the previous scenario (12–13) can be made for this scenario as well, except that Region 2 has no adaptation stock in this scenario. The following proposition gives the characterization of the feedback-Nash equilibrium of this scenario:

Proposition 2 *If there exist value functions $\widetilde{V}_i(t,P,A_1)$ for $i = \{1,2\}$ and the feedback strategies $(\tilde{e}_1^*(t,P,A_1), \tilde{e}_2^*(t,P,A_1), \tilde{I}_1^*(t,P,A_1))$ satisfy the equilibrium conditions given in (25) and (26) for all $t \in [0,T]$, then the strategies $(\tilde{e}_1^*, \tilde{e}_2^*, \tilde{I}_1^*)$ constitute a feedback-Nash equilibrium. Moreover, the value function $\widetilde{V}_i$ represents the equilibrium total payoff of Region i for the game starting at point (t,P,A_1).*

Following the same methodology as that outlined in the previous subsection, we replace the equilibrium strategies into the HJB equations and rearrange them, which yields a similar equation to (14) with the following coefficients:

$$\begin{aligned}
&\kappa_{i1} = \frac{\alpha_i^2}{2\beta_i}, \quad \kappa_{i2} = \frac{-s_i}{2}, \quad \kappa_{i3} = \frac{\alpha_i}{\beta_i} + \frac{\alpha_j}{\beta_j}, \quad \kappa_{i4} = \frac{1}{2\beta_i}, \quad \kappa_{i5} = -k, \quad \kappa_{i6} = \frac{1}{\beta_j},\\
&\kappa_{17} = \frac{1}{2\phi_1}, \quad \kappa_{18} = -\psi_1, \quad \kappa_{19} = \gamma_1, \quad \kappa_{110} = 0, \quad \kappa_{111} = 0,\\
&\kappa_{27} = 0, \quad \kappa_{28} = 0, \quad \kappa_{29} = 0, \quad \kappa_{210} = \frac{1}{\phi_1}, \quad \kappa_{211} = -\psi_1,
\end{aligned} \tag{27}$$

for $i = \{1,2\}$ and $j \neq i$. We observe that the maximized HJB equations in the first and second scenarios are equivalent when $\gamma_2 = \psi_2 = 1/\phi_2 = 0$. This is an intuitive result saying that when adaptation has no benefit and its marginal cost of investment is infinite in Region 2, the solutions of the first and present scenarios are the same. Below we will see that the same result appears when we compare the differential equation systems associated to the first and the second scenarios.

The value function of each region is a polynomial of degree 2 in the two state variables P and A_1, with time-dependent coefficients, and can be written as follows:

$$\widetilde{V}_i(t,P,A_1) = \tilde{c}_{i0}(t) + \tilde{c}_{i1}(t)P + \tilde{c}_{i2}(t)P^2 + \tilde{c}_{i3}(t)A_1 + \tilde{c}_{i4}(t){A_1}^2 + \tilde{c}_{i5}(t)A_1 P, \tag{28}$$

for $i = \{1,2\}$. Using the same methodology as in the first scenario, we insert the form in (28) into the HJB equations in (14) with (27) and find the coefficients of the value functions by the method of undetermined coefficients. We obtain a system of 12 differential equations, which has to be solved backward by using the terminal condition given by the salvage value function. The following remark states the relation of this scenario's differential equation system to that of the first one:

Remark 3 *The Ricatti differential equation system that corresponds to the solution of this scenario is nested within the system of the first scenario. The system in Appendix A.2 is equivalent to the one in Appendix A.1 when $\kappa_{110} = \kappa_{111} = \kappa_{27} = \kappa_{28} = \kappa_{29} = 0$.*

The statement in Remark 3 is consistent with our observation on the comparison of the HJB equations of the first and the present scenario in (27). After replacing the forms of value functions in (28) into the strategies given in (25–26), we write the equilibrium strategies that are linear in state variables as follows:

$$\tilde{e}_i(t, P, A_1) = \left(\alpha_i + \tilde{c}_{i1}(t) + 2\tilde{c}_{i2}(t)P + \tilde{c}_{i5}(t)A_1\right)/\beta_i \text{ for } i = \{1, 2\}, \tag{29}$$

$$\tilde{I}_1(t, P, A_1) = (\tilde{c}_{13}(t) + 2\tilde{c}_{14}(t)A_i + \tilde{c}_{15}(t)P)/\phi_1. \tag{30}$$

These strategies have the same form as the ones in the first scenario with $A_2 = 0$. However, the coefficient trajectories in the two scenarios are different, which implies that the strategies differ as well. To see how each strategy responds to a change in a state variable, we look at the following partial derivatives:

$$\frac{\partial \tilde{e}_i}{\partial P} = 2\tilde{c}_{i2}(t)/\beta_i, \quad \frac{\partial \tilde{e}_i}{\partial A_1} = \tilde{c}_{i5}(t)/\beta_i, \quad \frac{\partial \tilde{I}_1}{\partial P} = \tilde{c}_{15}(t)/\phi_1, \quad \frac{\partial \tilde{I}_1}{\partial A_1} = 2\tilde{c}_{14}(t)/\phi_1.$$

The sign of each derivative is given by the sign of the corresponding coefficient. All these signs will be characterized numerically in the next section.

3.3 No adaptation

In this scenario, none of the regions invest in adaptation, hence $A_i(t) = 0$ and $I_i(t) = 0$ for all $t \in [0, T]$ for $i = \{1, 2\}$. This is the transboundary pollution dynamic game in van der Ploeg and de Zeeuw (1992) and Dockner and Long (1993), the only difference being that, here, the planning horizon is finite. To recapitulate, the objective of each region is given by

$$\max_{e_i(t)} \int_{t=0}^{T} e^{-\rho_i t}\Big(U_i(e_i(t)) - D_i(P(t), 0)\Big)dt + e^{-\rho_i T} S_i(P(T), 0)$$

subject to (2),

with $P(0) \geq 0$ given.

As the above maximization programs show, each region decides only on its emissions, taking into account the dynamics of the pollution stock. The HJB equations associated to the regions' problems in this scenario are the following:

$$\rho_i \widehat{V}_i(t, P) - \frac{\partial \widehat{V}_i}{\partial t} = \max_{e_i} \left\{ U_i(e_i) - D_i(P, 0) + \frac{\partial \widehat{V}_i}{\partial P}(e_1 + e_2 - kP) \right\} \text{ for } i = 1, 2. \tag{31}$$

The maximization on the right-hand side yields the following strategies:

$$\hat{e}_i(t, P) = \frac{1}{\beta_i}\left(\alpha_i + \frac{\partial \widehat{V}_i}{\partial P}(t, P)\right) \text{ for } i = 1, 2. \tag{32}$$

The following proposition characterizes the feedback-Nash equilibrium in this scenario:

Proposition 3 *If there exist value functions $\widehat{V}_i(t, P)$ for $i = \{1, 2\}$ and the feedback strategies $(\hat{e}_i^*(t, P))$ satisfy the equilibrium conditions given in (32) for all $t \in [0, T]$, then the pair $(\hat{e}_1^*, \hat{e}_2^*)$ constitutes a feedback-Nash equilibrium. Moreover, the value function $\widehat{V}_i$ represents the equilibrium total payoff of Region i for the game starting at point (t, P).*

Replacing the equilibrium strategies given in (32) into the right-hand side of (31), we can write the maximized HJB equation as follows:

$$\rho_i \widehat{V}_i(t, P) - \widehat{V}_{it} = \hat{\kappa}_{i1} + \hat{\kappa}_{i2}P^2 + \hat{\kappa}_{i3}\widehat{V}_{iP} + \hat{\kappa}_{i4}\widehat{V}_{iP}^2 + \hat{\kappa}_{i5}\widehat{V}_{iP}P + \hat{\kappa}_{i6}\widehat{V}_{1P}\widehat{V}_{2P} \text{ for } i = 1, 2, \tag{33}$$

where the $\hat{\kappa}$ coefficients are given by

$$\hat{\kappa}_{i1} = \frac{\alpha_i^2}{2\beta_i}, \quad \hat{\kappa}_{i2} = \frac{-s_i}{2}, \quad \hat{\kappa}_{i3} = \frac{\alpha_i}{\beta_i} + \frac{\alpha_j}{\beta_j}, \quad \hat{\kappa}_{i4} = \frac{1}{2\beta_i}, \quad \hat{\kappa}_{i5} = -k, \quad \hat{\kappa}_{i6} = \frac{1}{\beta_j},$$

for $i = \{1, 2\}$ and $j \neq i$. The value function of each region is a polynomial of degree 2, with only three terms, that is,

$$\widehat{V}_i(t, P) = \hat{c}_{i0}(t) + \hat{c}_{i1}(t)P + \hat{c}_{i2}(t)P^2 \text{ for } i = 1, 2. \tag{34}$$

By using the same method as in the previous scenarios, we determine the differential equation system to be solved. In this scenario, the system has 6 equations and is given in explicit form in Appendix A.3. Finally, we establish that the system of differential equations associated to this scenario is nested within those of first and second scenarios.

Remark 4 *The Ricatti differential equation system of this scenario, given in Appendix A.3, is equivalent to the one in Appendices A.1 and A.2 when $\kappa_{i7} = ... = \kappa_{i11} = 0$ for $i = \{1, 2\}$.*

In this scenario, rewriting the feedback emissions strategies in (32) with the form of the value function given in (34), we obtain

$$\hat{e}_i(t, P, A_1) = (\alpha_i + \hat{c}_{i1}(t) + 2\hat{c}_{i2}(t)P) / \beta_i \text{ for } i = \{1, 2\}\,. \tag{35}$$

The derivative of $\hat{e}_i(t, P, A_1)$ with respect to P is given by

$$\frac{\partial \hat{e}_i}{\partial P} = 2\hat{c}_{i2}(t)/\beta_i \text{ for } i = \{1, 2\}\,,$$

and therefore, the sign of how emissions vary with the pollution stock is given by the sign of $\hat{c}_{i2}(t)$, which will be established numerically in the next section.

To summarize, we characterized the feedback-Nash equilibrium in each scenario and obtained the systems of differential equations in order to calculate the corresponding value functions and thus the feedback strategies that satisfy the equilibrium conditions. These systems of equations cannot be solved analytically, and therefore, we will use numerical methods to study the equilibrium outcomes in the next section.

4 Numerical analysis

The main objectives of our numerical analysis are to find out the characteristics of regions' strategies when they invest in adaptation measures, and to determine the impact of adaptation on the equilibrium emission levels, on the pollution stock, and on the total payoffs of the regions. We start by describing the methodology used, and next present the results.

The first step is to solve the differential equation system in each scenario to obtain the value function coefficients $c_{il}(t)$ for $t \in [0, T]$, $i = \{1, 2\}$ and $l = \{0, 1, ..., 9\}$. This is a two-point boundary-value problem that can be solved using Mathematica's numerical differential equation solver (NDSolve functionality). Next, we insert the solution into the strategies given in (19) to (21) to obtain the strategies $e_i^*(t, P, A_1, A_2)$ and $I_i^*(t, P, A_1, A_2)$. After, we use equations (2), (4), and (5) to compute the trajectories of the state variables $\{P(t), A_1(t), A_2(t)\}$ for $t \in [0, T]$. Finally, we verify, by performing the following test (by numerical integration), that the obtained solution is indeed the feedback-Nash equilibrium:

$$\begin{aligned} V_i(0, P(0), A_1(0), A_2(0)) - \Bigg(\int_{t=0}^{T} e^{-\rho_i t} \Big(U_i(e_i^*(P(t), A_1(t), A_2)(t)) - D_i(P(t), A_i(t)) \\ - H_i(I_i^*(P(t), A_1(t), A_2)(t)) \Big) dt + e^{-\rho_i T} S_i(P(T), A_i(T)) \Bigg) < \epsilon \text{ for } i = \{1, 2\}\,, \end{aligned} \tag{36}$$

where we take $\epsilon = 10^{-6}$. For each region, the above calculation verifies that the value in the game starting at the initial state $(0, P(0), A_1(0), A_2(0))$ approximates the total discounted payoff that is gained by implementing the feedback strategies. As a second check, we calculate the left-hand side and the maximized right-hand side of the HJB equations given in (14) and verify that their difference is also less than 10^{-6} and robust to variations in the state variables and time.

The model includes twenty parameters, that is,

Marginal utility : α_i, β_i
Marginal pollution damage : s_i,
Adaptation efficiency : γ_i,
Marginal cost of investment in adaptation : ϕ_i,
Depreciation rate of adaptation capital : ψ_i,
Salvage value for pollution and adaptation : μ_i, λ_i,
Discount rate : ρ_i,

for $i = 1, 2$ and the common parameters are

Natural decay rate of pollution : k,
Planning horizon : T.

In our analysis, we only consider the calibration settings that yield an interior solution, that is, a solution in which all the control variables are non-negative along the whole trajectory ($e_i(t) \geq 0, I_i(t) \geq 0 \quad \forall t \in [0, T]$ for $i = \{1, 2\}$). We also limit the calibration settings to those that lead to non-negative damage trajectories ($D_i(P(t), A_i(t)) \geq 0 \quad \forall t \in [0, T]$) for $i = \{1, 2\}$. For the previously stated reason, we only focus on the symmetric case in which both regions have the same values for all parameters.[2] The following parameter values are retained:

$$\begin{aligned}
&\alpha_i = 1, \quad \beta_i = 1, \quad s_i = 0.003, \quad \gamma_i = 0.001, \quad \psi_i = 0.1, \quad \phi_i = 0.05, \\
&\mu_i = -0.9, \quad \lambda_i = 0.01, \quad \rho_i = 0.025 \text{ for } i = \{1, 2\}, \\
&\text{and } k = 0.005, \quad T = 20.
\end{aligned}$$

Taking the above setting as the base, we vary each parameter within a range of ±25% to check whether the qualitative results on the strategies and trajectories are robust to model calibration. Recall that the solution of the second scenario does not involve the parameters of Region 2 that are related to adaptation, that is, γ_2, ψ_2, ϕ_2, and λ_2. Similarly, both regions' adaptation parameters (γ_i, ψ_i, ϕ_i, and λ_i for $i = \{1, 2\}$) are dropped altogether in the solution of the third scenario.

4.1 The equilibrium strategies

In this subsection, we study the strategic responses of each region to a change in the level of pollution, the level of the region's own adaptation capital and the level of the other region in the feedback-Nash equilibrium. We only present the results of the first scenario, in which both regions invest in adaptation, but we verify that these results also hold in the second and the third scenarios.

In Section 3.1, we obtained the partial derivatives of the response functions with respect to each state variable in (22). By using the numerical methods explained above, we compute the trajectories of these partial derivatives for different calibration settings to see if their signs and trajectories constitute a regular pattern.

Figure 1 presents the results of this analysis for the parameters related to adaptation (γ, ϕ, and ψ), the marginal pollution damage (s), the decay rate of pollution (k), and the discount rate (ρ).[3] Each panel consists of six plots: three for the emissions' response to each state variable, and three for the investment strategy's responses. Since we consider a symmetric case, the response functions of each region are also symmetric; hence, we present the results in general for Region i against Region $j \neq i$. In each panel, we plot five curves for the concerned parameter corresponding to variations of -25%, -12.5%, 0%, +12.5%, and +25% with respect to the base calibration, while keeping all other parameters at their base case values.

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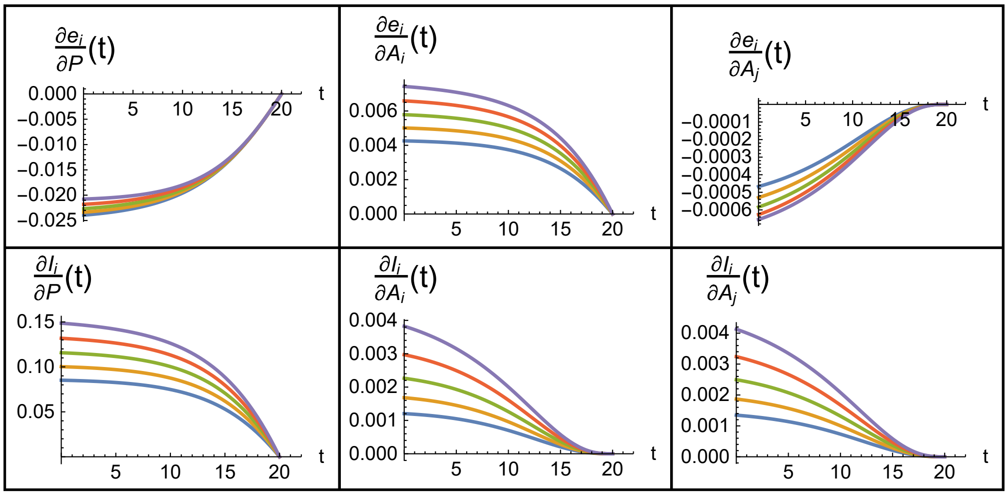
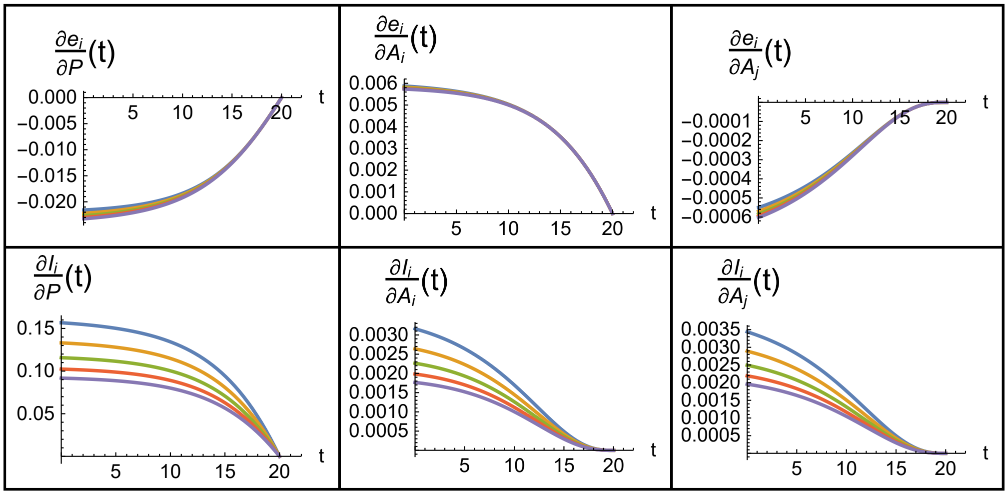
[2] The results associated to any calibration setting, including the heterogeneous ones, can be provided by the authors upon request.

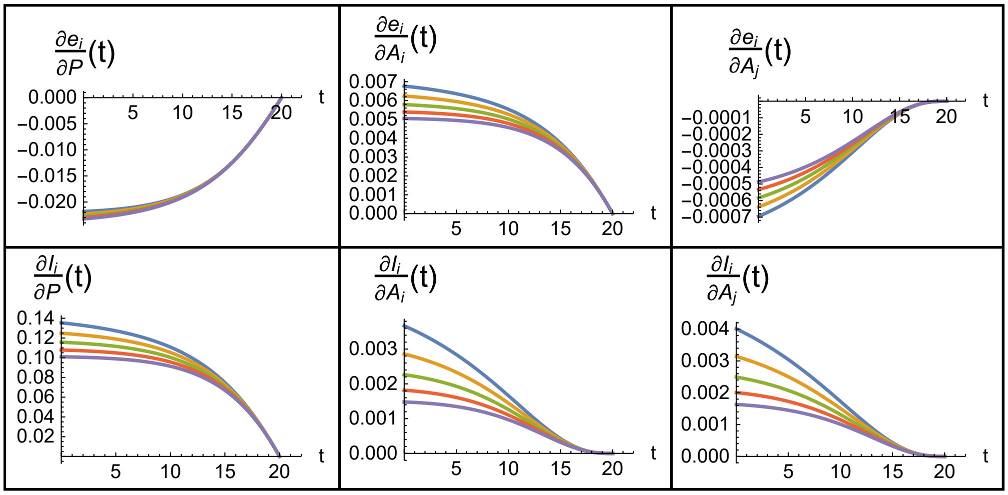
[3] We verify that the results hold for the variations in the rest of the parameters.



(a) ±%25 variation in γ



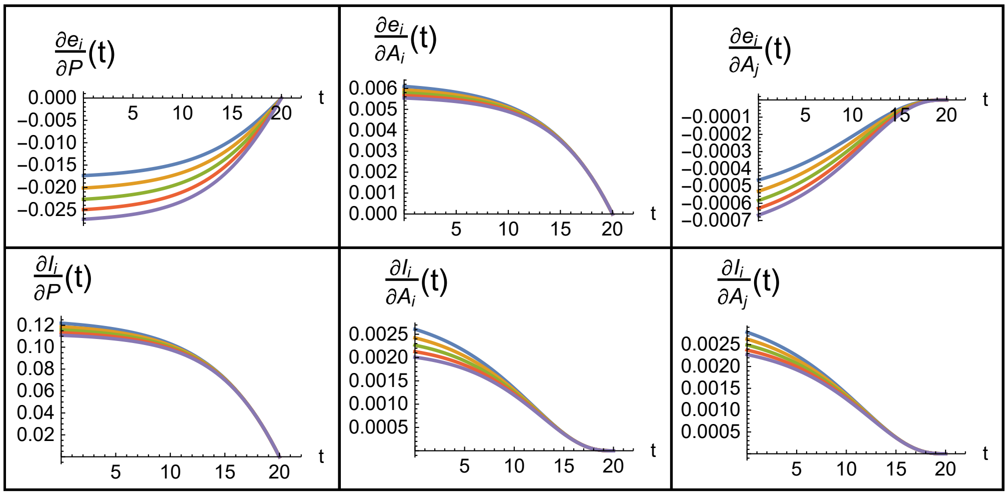
(b) ±%25 variation in ϕ



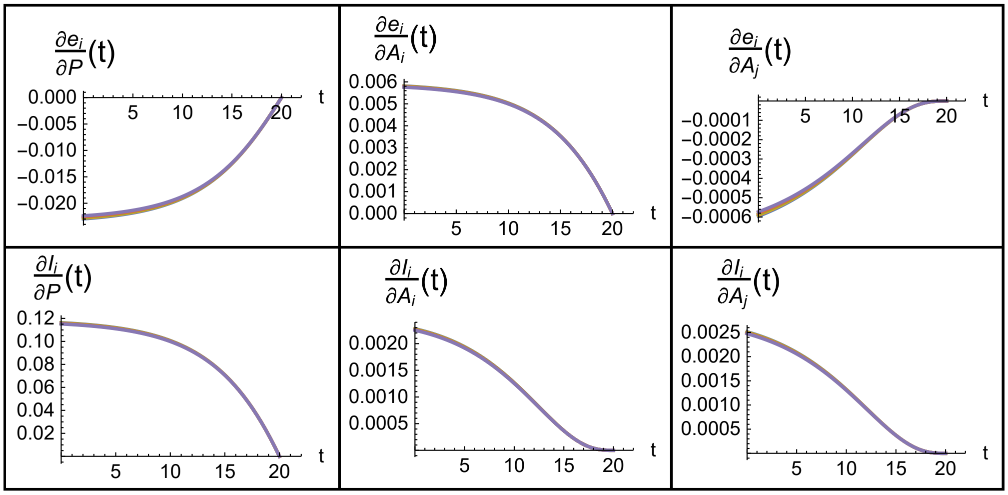
(c) ±%25 variation in ψ

— −%25 — −%12.5 — 0 — +%12.5 — +%25

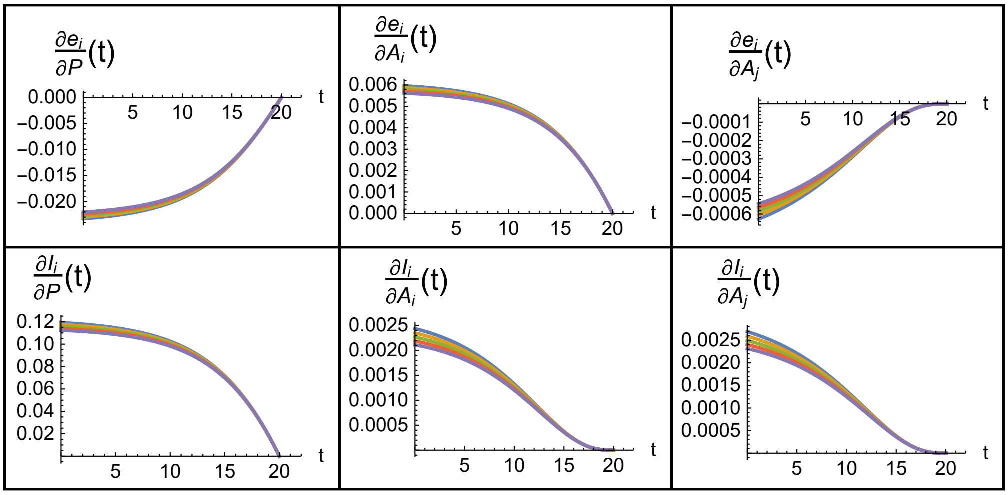
Figure 1: Sensitivity analysis of the response functions



(d) ±%25 variation in s



(e) ±%25 variation in k



(f) ±%25 variation in ρ

— −%25 — −%12.5 — 0 — +%12.5 — +%25

Figure 1 (cont.): Sensitivity analysis of the equilibrium response functions

We observe that the signs of the trajectories of the response functions remain the same throughout the game and are independent of the calibration that is used in the set of interior solutions. Accordingly, we can make the following claim on the regions' emissions strategies:

Claim 1 *For each region, emissions are decreasing in the pollution stock, increasing in own stock of adaptation capital, and decreasing in the other region's stock of adaptation capital. Formally,*

$$\frac{\partial e_i^*}{\partial P} < 0, \quad \frac{\partial e_i^*}{\partial A_i} > 0, \quad \frac{\partial e_i^*}{\partial A_j} < 0 \textit{ for } i = \{1, 2\} \textit{ and } j \neq i.$$

The results of Claim 1 are mostly due to the (natural) assumptions made on the marginal damage function (increasing in pollution) and on the marginal utility function (decreasing in emissions). Keeping constant the other state variables, a rise in the level of accumulated pollution leads to an increase in the marginal damage cost. In turn, the region responds by cutting its emissions to the level that equalizes the marginal benefit. The second part of Claim 1 states that when there is an increase in a region's adaptation capital, which induces a decrease in the current and future pollution damage, the region tends to emit more. This result is also related to the marginal damage cost of pollution. Indeed, when the stock of adaptation capital is higher, the marginal pollution damage is lower, and consequently, the region can increase its emissions in order to equalize the marginal benefit to the marginal damage. Furthermore, the strategic interaction between the regions appears in the last part of Claim 1. When region j increases its adaptation capital, region i anticipates that region j will emit more as it becomes more resilient to pollution damage. The strategic response to this anticipation is to cut down its own emissions to reduce the increase in pollution level.

Another observation from the results in Figure 1 is that the regions' emissions strategies are more robust to variations of some model parameters than others. For instance, the region's response to an increase in pollution is more robust to variations in adaptation efficiency than its response to an increase in its stock of adaptation capital (Figure (1,a)). On the other hand, the region's response to an increase in its stock of adaptation capital is more robust to variations in the marginal cost of investment than its response to an increase in pollution or in the stock of adaptation capital in the other region (Figure (1,b)). Comparing the panels also shows that the emissions strategies are more robust to variations in the pollution decay rate and the discount rate than to any other parameter (Figure (1,e,f)). In addition, we see that the regions' responses to a change in all state variables become null as the time approaches the terminal date.

We can also make some comments on how the strategies vary when we change the value of a parameter. The results in Figure 1 show that, keeping the other variables constant, as a response to an increase in pollution, a region decreases its emissions more when adaptation is less efficient, when the marginal cost of investment in adaptation capital is high, when the depreciation rate of adaptation capital is high, when the marginal pollution damage is high, when the pollution decay rate is low, and when the discount rate is low. Similar comments can also be made by using the same approach for the cases in which a region increases its emissions less as a response to an increase in its own adaptation stock, or when a region decreases its emissions less as a response to an increase in the other region's adaptation stock.

Based on the results provided in Figure 1, we can make the following claim regarding investment in adaptation:

Claim 2 *Each region's investment in adaptation is increasing in the pollution stock, and in its own as well as in the other region's adaptation capital. Formally,*

$$\frac{\partial I_i^*}{\partial P} > 0, \quad \frac{\partial I_i^*}{\partial A_i} > 0, \quad \frac{\partial I_i^*}{\partial A_j} > 0 \textit{ for } i = \{1, 2\} \textit{ and } j \neq i.$$

To interpret the above claim, we recall that the investment strategy is directly linked to the shadow price of adaptation capital, which measures how the total payoff varies when the adaptation stock is marginally increased. An additional unit of pollution increases the marginal pollution damage, which increases the benefit that could be gained by an additional unit of adaptation capital, and consequently, the region reacts by increasing its investment. The second part of Claim 2 is mostly due to considering adaptation as a stock variable. An additional unit of adaptation capital brings current and future benefits, but since it depreciates, it must be maintained with larger investments. Therefore, the region's investment is increasing in its own adaptation stock. This also means that the second derivative of the value function of Region i with respect to its own adaptation stock is positive ($\partial^2 V_i/\partial A_i^2 > 0$). The third part of Claim 2 shows the strategic interaction between regions in terms of their investments in adaptation. The higher is region j's adaptation capital, the more it can afford to emit because it is less vulnerable to pollution. Knowing this, region i's strategic reaction is to increase its adaptation stock to protect itself from the damage of increased pollution.

We next look at the robustness of the investment strategy to the variations in model parameters, as we did for the emissions strategies. Figure 1 shows that the investment response functions vary most with the variations of adaptation efficiency and the marginal investment cost, as well as with the decay rate of adaptation capital. Moreover, the investment strategy is more robust to variations in the pollution decay rate than any other parameter. Similarly to the emissions response functions, the region's investment response becomes null as the time approaches the terminal date regardless of the choice of parameters.

Lastly, we can comment on how the investment strategy responses change with the model's parameters. In Figure 1, we see that the region responds with a larger investment when adaptation is more efficient, when the cost of investment is lower, when the decay rate of adaptation capital is lower, when the marginal pollution damage is lower, when the natural decay rate of pollution is lower, and when the discount rate is lower.

4.2 The equilibrium trajectories

In this section, we compare the equilibrium control and state trajectories obtained in the three scenarios. Figure 2 exhibits these trajectories for the base case calibration. We computed these trajectories for different parameter values, and the results have always been qualitatively similar to those shown in Figure 2, and therefore, there is no need to show them.[4]

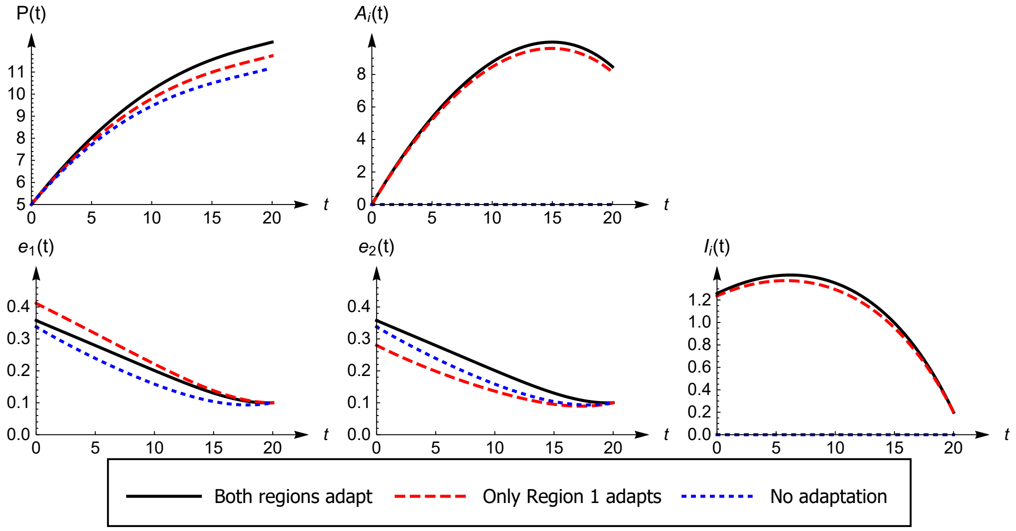


Figure 2: Sample equilibrium trajectories of the state and control variables in the three scenarios

We summarize our results in the following claims. (Recall that ˜ is for the case where only one region adapts and ˆ is for the no-adaptation scenario.)

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[4] Again, we note that the results for any constellation of parameter values can be obtained from the authors upon request.

Claim 3 *For all calibration settings, the ordering of pollution stock trajectories is as follows:*

$$P(t) > \tilde{P}(t) > \hat{P}(t) \quad \forall t \in (0, T].$$

The comparison in Claim 3 shows that, for any given initial level $P(0)$, the pollution stock is the highest at each instant of time when both regions adapt, and the lowest when no region invests in adaptation measures. As discussed in the previous subsection, this result is due to the fact that adaptation allows regions to emit more by reducing the marginal pollution damage.

In the following claim, we compare the level of adaptation capital in Region 1 under the three scenarios. Since we consider a symmetric calibration setting, the level of adaptation capital in Region 2 follows the same trajectory as that of Region 1 in the first scenario ($A_2(t) = A_1(t)$), and in the second and third scenarios, Region 2 does not invest in adaptation, hence we have $\tilde{A}_2(t) = \hat{A}_2(t) = 0$.

Claim 4 *For all calibration settings, the stock of adaptation capital in Region i is higher when Region j also invests in adaptation, that is,*

$$A_1(t) > \tilde{A}_1(t) > \hat{A}_1(t) = 0 \quad \forall t \in (0, T].$$

Region 1 accumulates a higher stock of adaptation capital over time when Region 2 also invests in adaptation, due to the anticipation that this region will tend to emit more when it adapts.

We now look at how regions' emissions trajectories differ among the three solutions:

Claim 5 *For all calibration settings, a region emits most when it is the only one that adapts, and emits least when it does not adapt but the other region does. Formally, we have*

$$\begin{aligned} \tilde{e}_1(t) > e_1(t) > \hat{e}_1(t) \quad \forall t \in [0, T), \\ e_2(t) > \hat{e}_2(t) > \tilde{e}_2(t) \quad \forall t \in [0, T). \end{aligned}$$

Claim 5 shows how the regions' adaptation decisions affect their emissions trajectories. As the game is symmetric, both regions emit the same amount when neither invests in adaptation ($\hat{e_1}(t) = \hat{e_2}(t)$). When Region 1 is the only one that invests in adaptation, it emits more and Region 2 emits less over time compared to the no-adaptation case. When Region 2 also invests in adaptation, Region 1's emissions follow a lower path over time compared to the second scenario. These results are in line with those of Claim 1 on emissions strategies, which states that regions cut their emissions when there is an increase in the other region's stock of adaptation capital. In addition, at the terminal time T, regions emit the same amount regardless of whether they invest in adaptation or not, which is a consequence of the salvage value function being linear and additively separable in pollution and adaptation stocks.

The trajectories of investment in adaptation also consistently differ for all the calibration settings we used. The following claim presents a comparison under the three scenarios:

Claim 6 *For all calibration settings, each region invests more in adaptation when the other region also invests in adaptation, that is,*

$$I(t) > \tilde{I}(t) > \hat{I}(t) = 0 \quad \forall t \in [0, T).$$

The statement of Claim 6 is again due to anticipation of increased pollution when the other region invests in adaptation measures. In response, the region invests larger quantities in its adaptation stock when the other region also does.

The above claims compared the equilibrium trajectories of the state and control variables. Another outcome of interest is how the damage trajectories in each region differ between the three scenarios. The following claim summarizes the comparative results:

Claim 7 *The damage that a region suffers is lowest when it is the only region that invests in adaptation, and the highest when it does not invest in adaptation but the other region does. Formally, we obtain*

$$\tilde{D}_1(t) < D_1(t) < \hat{D}_1(t) \quad \forall t \in (0, T],$$
$$D_2(t) < \hat{D}_2(t) < \tilde{D}_2(t) \quad \forall t \in (0, T].$$

The comparison in Claim 7, which is illustrated by the solution of the base calibration setting in Figure 3, shows that when a region does not invest in adaptation but the other does, the damage the region receives takes the highest path over the whole duration of the game. Combining these results with the ones on emissions trajectories, we see that one region adapting makes the other emit less and face higher damage over time compared to the case in which it does not adapt. Therefore, in the feedback-Nash equilibrium, adaptation in a region causes a welfare loss in the other region by means of a reduction in emissions and an increase in domestic damage.

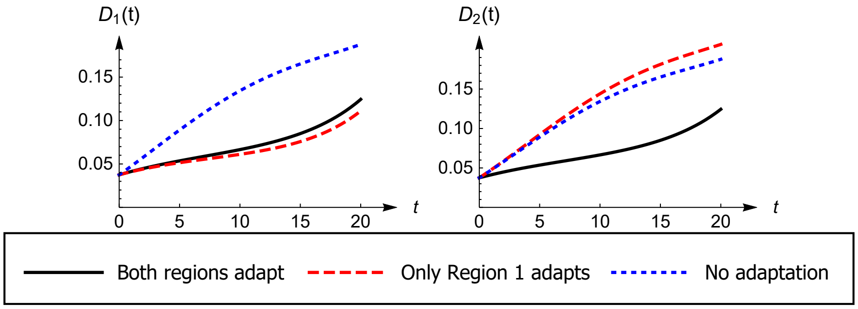

Figure 3: Sample equilibrium trajectories of the damage in regions in the three scenarios

Finally, we study how the length of the planning horizon affects the equilibrium trajectories. In our analysis, we found that when the planning horizon varies, the trajectories of the stock variables differ consistently among all calibration settings. However, this is not the case for the emissions and investment strategies. For many calibration settings, we find that emissions and investment trajectories follow a higher path over time when the planning horizon is longer. But there are interior solutions in which a region's emissions and investment trajectories at the equilibrium can intersect at a given date when we compare different planning horizons. Therefore, for a longer time horizon, a region may emit more until a date that is followed by lower emissions, as compared to a solution with a shorter planning horizon. These types of occurrences are indeed mostly related to the settings for the salvage value parameters as well as the initial state of the pollution and adaptation capital stocks. Since we cannot observe a consistent difference in the trajectories of the control variables, we present only the differences in the trajectories of the state variables. Figure 4 illustrates the trajectories of the stocks of pollution and adaptation capital in the first scenario for the base calibration setting.

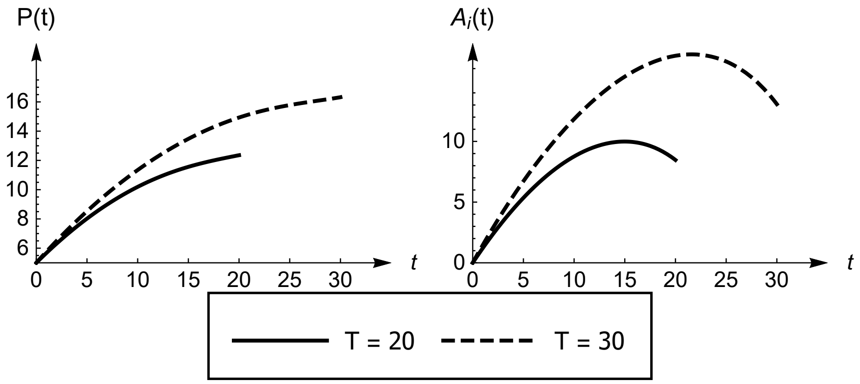

Figure 4: The effect of an extended planning horizon on the equilibrium trajectories

We observe that when the planning horizon is extended, the regions pollute more and they accumulate larger adaptation capital over time. This is mostly because the regions solve their problems backward in the feedback equilibrium. When the planning horizon is longer, the regions receive the damage of accumulated pollution for a longer time, and benefit from their emissions for a longer time as well. They face a trade-off between emitting more and arriving at the terminal date with less pollution. At the equilibrium, a longer planning horizon induces the regions to emit more and to accumulate more pollution, which also leads them to invest more in adaptation.

5 Conclusion

When regions consider adaptation measures in their decision-making processes, this has various consequences on their strategies and on the outcomes of the feedback-Nash equilibrium of the transboundary pollution dynamic game. We incorporated an adaptation policy as the region's ability to invest in its private adaptation capital stock, which reduces the damage the region receives from the accumulated common pollution stock, and conducted numerical analysis on the equilibrium strategies and the trajectories of regions' decisions.

When regions consider adaptation measures in their objectives, they tend to emit more over time because investing in adaptation capital reduces the marginal pollution damage within the region. Since adaptation measures provide only private benefits, the incentive to increase emissions in one region causes an inverse strategic response in the other, that is, one region emits less over time when the other invests in adaptation. This is due to the anticipation that increased pollution that will be caused by the rise in the adapting region's emissions. Moreover, a region's investment in its adaptation capital is also increasing in the other region's stock of adaptation because of the same anticipation. As a result, the equilibrium trajectories of the stock of pollution and the stocks of adaptation capital in each region follow the highest paths over time when both regions adapt.

An asymmetry between the regions' ability to invest in adaptation measures results in the adapting region causing a welfare loss in the other region, by means of the reduction in emissions and the increase in domestic damage. The adapting region finishes the game better off, whereas the region that does not (or cannot) adapt finishes worse off compared to the case in which no region adapts.

In our analysis, we made several assumptions on the functional forms of the utility, the damage, and the cost of investment in adaptation, as well as on the dynamics of the stock of pollution and the stocks of adaptation capital in each region. These assumptions made the problem structure linear-quadratic, which allowed us to compute the feedback equilibrium by solving the systems of differential equations associated to the value functions of the regions. This way is indeed technically more convenient than finding the feedback equilibrium of a non-linear-quadratic game, which would require finding the regions' value functions through parallel value function iteration or policy function iteration techniques for example.

Indeed, our results would differ if our assumptions were relaxed and the functions associated to the regions' payoffs were considered in more general forms. For instance, the regions' emissions and investment strategies would differ if adaptation measures had limitations and not all the damage could be reduced by applying such measures, which can be captured by a non-linear damage function in pollution and in region's stock of adaptation capital. Similarly, consideration of discontinuities in the regions' damage functions as well as the uncertainties involving pollution damage and the efficiency of adaptation measures in reducing damage are interesting research directions. The adaptation policy is modeled as a private good; therefore, regions are affected by the other regions' adaptation through indirect channels. The results would indeed differ as well if the adaptation policy had spillover effects over the regions. Further research also includes taking into account the constraints on the stock of pollution, on the adaptation measures, as well as on the emission quantities.

A Appendix

A.1 Ricatti differential equations associated to the first scenario

$$
\begin{aligned}
\rho_1 c_{10} - \dot{c}_{10} &= \kappa_{11} + c_{11}(\kappa_{13} + \kappa_{14} c_{11} + \kappa_{16} c_{21}) + \kappa_{17} c_{13}^2 + \kappa_{110} c_{16} c_{26},\\
\rho_1 c_{11} - \dot{c}_{11} &= 2(\kappa_{13} + \kappa_{16} c_{21}) c_{12} + c_{11}(\kappa_{15} + 4\kappa_{14} c_{12} + 2\kappa_{16} c_{22}) + 2\kappa_{17} c_{13} c_{15} + \kappa_{110} c_{26} c_{18} + \kappa_{110} c_{16} c_{28},\\
\rho_1 c_{12} - \dot{c}_{12} &= \kappa_{12} + 2c_{12}(\kappa_{15} + 2\kappa_{14} c_{12} + 2\kappa_{16} c_{22}) + \kappa_{17} c_{15}^2 + \kappa_{110} c_{18} c_{28},\\
\rho_1 c_{13} - \dot{c}_{13} &= c_{13}(\kappa_{18} + 4\kappa_{17} c_{14}) + (\kappa_{13} + 2\kappa_{14} c_{11} + \kappa_{16} c_{21}) c_{15} + \kappa_{16} c_{11} c_{25} + \kappa_{110} c_{26} c_{19} + \kappa_{110} c_{16} c_{29},\\
\rho_1 c_{14} - \dot{c}_{14} &= 2c_{14}(\kappa_{18} + 2\kappa_{17} c_{14}) + \kappa_{14} c_{15}^2 + \kappa_{16} c_{15} c_{25} + \kappa_{110} c_{19} c_{29},\\
\rho_1 c_{15} - \dot{c}_{15} &= \kappa_{19} + (\kappa_{15} + \kappa_{18} + 4\kappa_{14} c_{12} + 2\kappa_{16} c_{22} + 4\kappa_{17} c_{14}) c_{15} + 2\kappa_{16} c_{12} c_{25} + \kappa_{110} c_{28} c_{19} + \kappa_{110} c_{18} c_{29},\\
\rho_1 c_{16} - \dot{c}_{16} &= 2\kappa_{110} c_{26} c_{17} + c_{16}(\kappa_{111} + 2\kappa_{110} c_{27}) + (\kappa_{13} + 2\kappa_{14} c_{11} + \kappa_{16} c_{21}) c_{18} + \kappa_{16} c_{11} c_{28} + 2\kappa_{17} c_{13} c_{19},\\
\rho_1 c_{17} - \dot{c}_{17} &= 2c_{17}(\kappa_{111} + 2\kappa_{110} c_{27}) + \kappa_{14} c_{18}^2 + \kappa_{16} c_{18} c_{28} + \kappa_{17} c_{19}^2,\\
\rho_1 c_{18} - \dot{c}_{18} &= (\kappa_{111} + \kappa_{15} + 4\kappa_{14} c_{12} + 2\kappa_{16} c_{22} + 2\kappa_{110} c_{27}) c_{18} + 2(\kappa_{16} c_{12} + \kappa_{110} c_{17}) c_{28} + 2\kappa_{17} c_{15} c_{19},\\
\rho_1 c_{19} - \dot{c}_{19} &= \kappa_{16} c_{25} c_{18} + c_{15}(2\kappa_{14} c_{18} + \kappa_{16} c_{28}) + (\kappa_{111} + \kappa_{18} + 4\kappa_{17} c_{14} + 2\kappa_{110} c_{27}) c_{19} + 2\kappa_{110} c_{17} c_{29},\\
\rho_2 c_{20} - \dot{c}_{20} &= \kappa_{21} + c_{21}(\kappa_{23} + \kappa_{26} c_{11} + \kappa_{24} c_{21}) + \kappa_{210} c_{13} c_{23} + \kappa_{27} c_{26}^2,\\
\rho_2 c_{21} - \dot{c}_{21} &= 2(\kappa_{23} + \kappa_{26} c_{11}) c_{22} + c_{21}(\kappa_{25} + 2\kappa_{26} c_{12} + 4\kappa_{24} c_{22}) + \kappa_{210} c_{23} c_{15} + \kappa_{210} c_{13} c_{25} + 2\kappa_{27} c_{26} c_{28},\\
\rho_2 c_{22} - \dot{c}_{22} &= \kappa_{22} + 2c_{22}(\kappa_{25} + 2\kappa_{26} c_{12} + 2\kappa_{24} c_{22}) + \kappa_{210} c_{15} c_{25} + \kappa_{27} c_{28}^2,\\
\rho_2 c_{23} - \dot{c}_{23} &= c_{23}(\kappa_{211} + 2\kappa_{210} c_{14}) + 2\kappa_{210} c_{13} c_{24} + (\kappa_{23} + \kappa_{26} c_{11}) c_{25} + c_{21}(\kappa_{26} c_{15} + 2\kappa_{24} c_{25}) + 2\kappa_{27} c_{26} c_{29},\\
\rho_2 c_{24} - \dot{c}_{24} &= 2(\kappa_{211} + 2\kappa_{210} c_{14}) c_{24} + \kappa_{26} c_{15} c_{25} + \kappa_{24} c_{25}^2 + \kappa_{27} c_{29}^2,\\
\rho_2 c_{25} - \dot{c}_{25} &= 2\kappa_{210} c_{24} c_{15} + (\kappa_{211} + \kappa_{25} + 2\kappa_{26} c_{12} + 2\kappa_{210} c_{14}) c_{25} + 2c_{22}(\kappa_{26} c_{15} + 2\kappa_{24} c_{25}) + 2\kappa_{27} c_{28} c_{29},\\
\rho_2 c_{26} - \dot{c}_{26} &= c_{26}(\kappa_{28} + 4\kappa_{27} c_{27}) + \kappa_{23} c_{28} + \kappa_{26} c_{11} c_{28} + c_{21}(\kappa_{26} c_{18} + 2\kappa_{24} c_{28}) + \kappa_{210} c_{23} c_{19} + \kappa_{210} c_{13} c_{29},\\
\rho_2 c_{27} - \dot{c}_{27} &= 2c_{27}(\kappa_{28} + 2\kappa_{27} c_{27}) + \kappa_{26} c_{18} c_{28} + \kappa_{24} c_{28}^2 + \kappa_{210} c_{19} c_{29},\\
\rho_2 c_{28} - \dot{c}_{28} &= \kappa_{29} + (\kappa_{25} + \kappa_{28} + 2\kappa_{26} c_{12} + 4\kappa_{27} c_{27}) c_{28} + 2c_{22}(\kappa_{26} c_{18} + 2\kappa_{24} c_{28}) + \kappa_{210} c_{25} c_{19} + \kappa_{210} c_{15},\\
\rho_2 c_{29} - \dot{c}_{29} &= \kappa_{26} c_{15} c_{28} + c_{25}(\kappa_{26} c_{18} + 2\kappa_{24} c_{28}) + 2\kappa_{210} c_{24} c_{19} + (\kappa_{211} + \kappa_{28} + 2\kappa_{210} c_{14} + 4\kappa_{27} c_{27}) c_{29}
\end{aligned}
$$

A.2 Ricatti differential equations associated to the second scenario

$$
\begin{aligned}
\rho_1 c_{10} - \dot{c}_{10} &= \kappa_{11} + c_{11}(\kappa_{13} + \kappa_{14} c_{11} + \kappa_{16} c_{21}) + \kappa_{17} c_{13}^2,\\
\rho_1 c_{11} - \dot{c}_{11} &= 2(\kappa_{13} + \kappa_{16} c_{21}) c_{12} + c_{11}(\kappa_{15} + 4\kappa_{14} c_{12} + 2\kappa_{16} c_{22}) + 2\kappa_{17} c_{13} c_{15},\\
\rho_1 c_{12} - \dot{c}_{12} &= \kappa_{12} + 2c_{12}(\kappa_{15} + 2\kappa_{14} c_{12} + 2\kappa_{16} c_{22}) + \kappa_{17} c_{15}^2,\\
\rho_1 c_{13} - \dot{c}_{13} &= c_{13}(\kappa_{18} + 4\kappa_{17} c_{14}) + (\kappa_{13} + 2\kappa_{14} c_{11} + \kappa_{16} c_{21}) c_{15} + \kappa_{16} c_{11} c_{25},\\
\rho_1 c_{14} - \dot{c}_{14} &= 2c_{14}(\kappa_{18} + 2\kappa_{17} c_{14}) + c_{15}(\kappa_{14} c_{15} + \kappa_{16} c_{25}),\\
\rho_1 c_{15} - \dot{c}_{15} &= \kappa_{19} + (\kappa_{15} + \kappa_{18} + 4\kappa_{14} c_{12} + 2\kappa_{16} c_{22} + 4\kappa_{17} c_{14}) c_{15} + 2\kappa_{16} c_{12} c_{25},\\
\rho_2 c_{20} - \dot{c}_{20} &= \kappa_{21} + c_{21}(\kappa_{23} + \kappa_{26} c_{11} + \kappa_{24} c_{21}) + \kappa_{210} c_{13} c_{23},\\
\rho_2 c_{21} - \dot{c}_{21} &= 2(\kappa_{23} + \kappa_{26} c_{11}) c_{22} + c_{21}(\kappa_{25} + 2\kappa_{26} c_{12} + 4\kappa_{24} c_{22}) + \kappa_{210} c_{23} c_{15} + \kappa_{210} c_{13} c_{25},\\
\rho_2 c_{22} - \dot{c}_{22} &= \kappa_{22} + 2c_{22}(\kappa_{25} + 2\kappa_{26} c_{12} + 2\kappa_{24} c_{22}) + \kappa_{210} c_{15} c_{25},\\
\rho_2 c_{23} - \dot{c}_{23} &= c_{23}(\kappa_{211} + 2\kappa_{210} c_{14}) + 2\kappa_{210} c_{13} c_{24} + \kappa_{26} c_{21} c_{15} + (\kappa_{23} + \kappa_{26} c_{11} + 2\kappa_{24} c_{21}) c_{25},\\
\rho_2 c_{24} - \dot{c}_{24} &= 2(\kappa_{211} + 2\kappa_{210} c_{14}) c_{24} + c_{25}(\kappa_{26} c_{15} + \kappa_{24} c_{25}),\\
\rho_2 c_{25} - \dot{c}_{25} &= 2(\kappa_{26} c_{22} + \kappa_{210} c_{24}) c_{15} + (\kappa_{211} + \kappa_{25} + 2\kappa_{26} c_{12} + 4\kappa_{24} c_{22} + 2\kappa_{210} c_{14}) c_{25}
\end{aligned}
$$

A.3 Ricatti differential equations associated to the third scenario

$$
\begin{aligned}
\rho_1 c_{10} - \dot{c}_{10} &= \kappa_{11} + c_{11}(\kappa_{13} + \kappa_{14} c_{11} + \kappa_{16} c_{21}),\\
\rho_1 c_{11} - \dot{c}_{11} &= 2(\kappa_{13} + \kappa_{16} c_{21}) c_{12} + c_{11}(\kappa_{15} + 4\kappa_{14} c_{12} + 2\kappa_{16} c_{22}),\\
\rho_1 c_{12} - \dot{c}_{12} &= \kappa_{12} + 2c_{12}(\kappa_{15} + 2\kappa_{14} c_{12} + 2\kappa_{16} c_{22}),\\
\rho_2 c_{20} - \dot{c}_{20} &= \kappa_{21} + c_{21}(\kappa_{23} + \kappa_{26} c_{11} + \kappa_{24} c_{21}),\\
\rho_2 c_{21} - \dot{c}_{21} &= 2(\kappa_{23} + \kappa_{26} c_{11}) c_{22} + c_{21}(\kappa_{25} + 2\kappa_{26} c_{12} + 4\kappa_{24} c_{22}),\\
\rho_2 c_{22} - \dot{c}_{22} &= \kappa_{22} + 2c_{22}(\kappa_{25} + 2\kappa_{26} c_{12} + 2\kappa_{24} c_{22})
\end{aligned}
$$

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