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A. Akgunduz, B. Jaumard,
G. Moeini

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**GERAD** HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

**Tél. : 514 340-6053**
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

# Conflict free air traffic flow management with variable speeds

**Ali Akgunduz** [a]

**Brigitte Jaumard** [b]

**Golbarg Moeini** [a]

[a] *Mechanical and Industrial Engineering, Concordia University, Montréal (Québec) Canada H3G 1M8*

[b] *GERAD & Computer Science and Software Engineering, Concordia University, Montréal (Québec) Canada H3G 1M8*

ali.akgunduz@concordia.ca
bjaumard@cse.concordia.ca

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**Abstract:** This paper discusses a unique formulation of en-route flight planning problem in a constrained airspace with the objective of minimizing the total cost while all safety rules are respected. A mixed integer programming (MIP) model that considers mid-air collision avoidance, separation distance between aircrafts, speed changes, speed-dependent fuel consumption rate and exact traveling times has been developed using a non-time indexed modeling approach. A 3-dimensional (3D) mesh network is used to provide alternative routing options for aircrafts. The MIP solution provides a en-route flight plan as a list of consecutive nodes to be visited by an aircraft, time of arrival at each node and average speed on each link. Consequently, the cost incurred from delays, earliness and fuel usage is minimized. Fuel consumption is modeled as a function of aircraft speed without loosing the linearity of the mathematical model. The proposed non-time indexed model guarantees separation distances between aircrafts and ensures the mid-air collision avoidance during flight, particularly around airports. The MIP model is NP-hard. While only small to medium size problems can be solved using CPLEX on a personal computer, a real-time decision making capability is achieved through decentralized and hybrid solution strategies.

**Key Words:** Air Traffic Control Management, Aircraft Routing, 3D Mesh Network, Exact Arrival Times, Exact Speed, Speed-dependent Fuel Consumption.

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# 1 Introduction

Airline industry has been facing various economical and managerial challenges since the beginning of the millennium. On the other hand, despite the negative impact of September 11 terrorist attacks and ongoing financial turmoil globally, demand for airline services has been steadily increasing. International Air Transportation Association (IATA) has estimated 125 billion dollars increase in the Revenue Passenger Kilometers (RPKs) and 150 billion dollars rise in Available Seat Kilometers (AKSs) from 2002 to 2012 for international scheduled passenger traffic. In most major markets including Europe, North America and large part of Asia, since early 90s, both seat capacity and number of airline companies has increased significantly. While long-haul routes are still being dominated by major carriers such as United Airlines, Lufthansa or Singapore Airlines, relatively smaller airline companies have become a major player in short flight services (within country in North America or in China or short international flights particularly in Europe). The competition brought by smaller carriers to the airline industry made the airline traveling a necessity, rather than a luxury.

The new market conditions brought many challenges along with its benefits for the industry. Crowded airport terminals, frequent delays, overcrowded airspaces around the airports particularly in North America and crowded airspaces between airports in Europe are some of the challenges for the airline industry as well as for the transportation authorities to tackle. Furthermore, volatile fuel prices in recent years, increasing labor costs and unpredictable weather conditions in most parts of the world are putting many airline companies in the extreme financial turmoils.

According to 2012 Federal Aviation Administration (FAA) report, air traffic has increased by 3.5% in 2012. FAA also projects 2 to 3 percent activity increase in civil aviation industry during next 20 years. Moreover, it is expected that the number of passengers carried will increase from 731 million passengers in 2011 to1.2 billion in 2032 [2]. While the traffic increase has positive impact on airlines (most major airlines reported profit in 2011), increasing traffic is causing significant congestion on airports and in various air-sectors. A report prepared by Airlines for America (A4A) estimates 103 million system delay minutes for 2011 which has resulted to $7.7 billion in direct aircraft operating costs for scheduled U.S. passenger airlines.

Growing airline industry puts serious pressure on Air Traffic Controllers (ATC). From taxing to navigating in open skies, ATC plays a crucial role in delivering on-time service and ensuring the safety of aircrafts and passengers at all times. During a flight's entire journey, its speed and precise route are planned by the airline company, and approved and monitored by ATC. Pilot's discretion in en-route flight planning is rarely an options (under emergency situations pilot can make real-time decisions). FAA estimates an increase of 56.9% on control tower operations and over 100% on en-route (high altitude flights) traffic-control operations by 2030 [1]. It is clear that, increasing air traffic is becoming unmanageable for ATC personal to effectively determine a flight plan for each aircraft. Long working hours, highly stressful working conditions and continuously increasing air traffic conditions are frequently causing ATC personnel to make poor decisions. Each year, several airports in USA report that mid-air collisions are frequently avoided through sophisticated Traffic-alert and Collision Avoidance System (TCAS) such as Mid-air Collision Avoidance System (MIDCAS) imbedded in airplanes [24]. Furthermore, due to high air traffic volume, ATC personal frequently ignores economic and service objectives of airlines; rather they mainly focus on safety. Hence, fuel consumption cost, delays and early arrivals are frequently ignored. Free Flight Concept (FFC) on the other hand aims at transferring the en-route flight planning task to individual aircrafts. This paper proposes a mathematical model that in theory, can assist pilots to determine the en-route flight plan in real-time with an objective to optimize the overall performance of aircrafts in a given airspace while ensuring all safety rules even when the airspace is heavily congested.

The developed mixed-integer programming (MIP) model takes all meaningful airport and flight characteristics into consideration to provide a real sense of the situation. Flights schedules, speed changes, speed-dependent fuel consumption rate, separation distances and conflict avoidance are some of the issues that are precisely incorporated in the mathematical model. A non-time indexed formulation determines a flight plan for each aircraft by identifying the sequence of nodes to be visited and determines the aircrafts' exact arrival and departures times to these node, average speed between two consecutive nodes (on the link) and fuel consumption rates on that speed while ensuring the safety conditions such as no two aircraft can be in a proximity that violates the predefined horizontal and vertical separation distances. Although the developed model is capable of planning the entire flight from airport to airport, a hypothetical case around Montreal Trudeau Airport is studied for verification. The case problem is solved under various traffic conditions through utilizing three different solution strategies in CPLEX. In the first solution strategy a centralized management approach is considered. It is assumed that the air-traffic flow management (ATFM) authority determines en-route flight plans for all incoming and departing aircrafts at the beginning of a planning horizon. The mathematical model is known to be NP-Hard hence as the air space becomes larger and the number of aircrafts in consideration increases, the scalability problem raises. The proposed second methodology addresses the scalability issue through utilizing a sequential aircraft management strategy which conforms very well with the objectives of the NASA'S free-flight concept which aims at giving the autonomy to the pilots in en-route plight planning [21]. The decentralized (or sequential) en-route flight planning method allows each airplane to determine its flight plan with respect to the given current traffic conditions in which the flight plans for all other aircrafts are known (previously determined by the same model). Finally, we introduce the third solution strategy that combines both centralized and decentralized approaches. The proposed hybrid solution methodology, overcomes the scalability issue of the centralized approach and provides much better results than the decentralized approach.

The remainder of the paper is organized as follows. In Section 2, a brief literature review, in Section 3 model, and in Section 4 solution and results are introduced. Finally in Section 5, the conclusions and future works are summarized.

## 2 Literature review

In this section, we discuss the relevant literature on the Air Traffic Flow Management (ATFM) problem. First we briefly discussed the history of ATFM problem. Later the literature closely related to the proposed en-route flight planning model is discussed. In this regard, studies on airport and airspace congestion, dynamic speed control and finally the speed dependent fuel consumption problems are reviewed.

### 2.1 A brief history of ATFM problem

Earlier works on ATFM focused on Air Traffic Control (ATC) and Airport Congestion problems. Airport congestion problems are further categorized as aircraft landing and airport take-off problems [7]. Air traffic controllers are responsible of managing air-traffic within their air segments namely sectors. Most research relevant to ATC focused on technical innovations. A number of mathematical models have been introduced to determine the size of air-sectors in order to improve the overall ATC performances. Particularly near airports, air-sectors are divided into 3 categories: Ground control, Landing-departure control and Higher altitude control (3000 ft. above ground near airport) [31]. Due to high volume of air-traffic, each sector is controlled by a different team of air traffic controller. Reducing the size of air-sectors improves the working conditions of air traffic controllers; however smaller air-sectors creates burdens for the pilots since they frequently reports to a different ATC tower. Li [25] developed mathematical models to measure the efficiency of air-sectors and their theoretical capacities. Krozel [23] discussed future of national airspace systems particularly from the weather perspective. In his PhD thesis, Zou [39] discussed methods for air-sector capacity estimation with weather constraints. Earlier works on aircraft landing and airport takeoff problems focused on deterministic ground holding and airborne delay strategies. Large body of these works have considered airspace and airport capacities in their models since congestion and delays are result of dismissed available capacities in the system. Odoni [33] is one of the first researchers to formally define the ground holding problem for a single airport setting. Same year Andreatta and Romanin-Jacur [6] and later Wang [38] and Mukherjee and Hansen [30] studied the dynamic ground holding problem in a single airport. Vranas, Bertsimas and Odoni [37] was the first to introduce a mathematical mode to handle multi-airport setting in a dynamic environment. Lindsay *et al.* [26] has designed a binary integer programming model to minimize the total delay (ground and airborne) with respect to airport and airspace capacity constraint for individual flights. Agustin et al. [3] have studied the Air Traffic Management problem by considering the uncertainty in airport and airspace capacity due to weather condition. Flener et al. [18] introduce a mathematical model to minimize the air traffic complexity within each air sector. Air sector traffic is affected by the number of flights traveling within in and near its boarders. Therefore, they have considered different factors to manage this complexity such as changing the flights take off time and their arrival time to that specific air sector, optimizing their speed and re arranging their traveling point altitudes within the air sector. In their review paper, Navazio and Romanin-Jacur [32] summarized the works on multi-airport ground holding problems.

In recent years, research focus has shifted to the routing problems. Rather than focusing only on the airport congestion, researchers have started investigating air traffic flow management (ATFM) problem including congestion in air-sectors. Bertsimas and Patterson [10] was the first to study the ATFM problem with routing option. While most work on ATFM focused on determining flight times given that aircrafts follow a fixed route, Dell'Olmo and Lulli [16] proposed a model that assumes aircrafts are free to select their routes in a network, it is also known as free-flight concept. Later on, researchers included rerouting into the ATFM problem. One of the earliest rerouting algorithms was proposed by Helm [20] who presented a multi-commodity minimum cost flow on a time space network. Later Leal de Matos [14], Ma [28] and Bertsimas [8] proposed mathematical models to tackle rerouting problem in ATFM.

Each study has proposed a different design for the airspace in their network. Helm [20], unlike other works, had defined a spatial network of containing nodes and links; they designed a time-space network of vortex and links. Dell'Olmo and Lulli [16] have proposed a mixed integer heuristic model with a free flight path scenario that considers a network with no fix routes for European air traffic network. Churchill et al. [11] have introduced an airspace volume containing airspace and airports. They have defined set of possibilities for each flight as entry and exit points for each volume. Similarly, in this work, we also define the airspace as a 3-D mesh network in which nodes and links are used to generate alternative routes for aircrafts.

## 2.2 Literature pertinent to the proposed flight planning model

The work presented in this paper share several commonalities with the literature reviewed above. Due to the example problem discussed in the paper, a single airport with ground and airborne delay options may best represents the flight planning model discussed in this paper. On the other hand, the proposed model is general and it has the potential to study the ATFM problem with multiple airports. Yet, the proposed model differs significantly from the previous literature due to modeling technique and also usage of varying speed and most importantly the speed dependent fuel consumption constraints. Hence in this section, we briefly cover the literature on ATFM models with varying speed and fuel consumption characteristics.

One of the earliest works on ATFM with varying speed was proposed in Bertsimas and Patterson [9]. They have covered all phases of flight, take off, cruise and landing and corporate all the management options like ground holding, airborne holding, speed changes and most important rerouting. In a more recent work, Lulli and Odoni [27] assumes all aircrafts in an air-sector fly with the same speed which is also not a practical in real-life cases. Delgado and Prats [15] proposed varying speed reduction strategies for airborne aircrafts to better coordinate the traffic between airports.

Fuel consumption problem in ATFM is relatively new. Most researches have focused on developing technologies to build more fuel efficient aircrafts. Regardless of the technology used, it is known that aircraft fuel consumption rate varies depending on the speed. Fuel consumption is not only a cost issue but also a significant contributor to the air pollution. Particularly around airports, aircraft emissions has been a major contributor to the local air quality. However, due to the complex nature of aircraft flight planning problem, the aircraft fuel consumption problem has not been studied adequately. The relationship between the fuel consumption and speed of an aircraft is studied in Clarke *et al.* [13]. More recently, Vela et al. [36] proposed a model for conflict resolution while ensuring the optimal fuel consumption rate.

Airspace is not large enough to ignore possible collision between aircrafts. In any given moment, thousands of aircrafts fill most regions in the world. Particularly during take off and landing, aircrafts fly in low altitude and share the airspace with several other aircrafts. In order to ensure the safety of aircrafts, aerospace industry developed several sophisticated devices to eliminate mid-air collision. FAA requires all aircrafts flying in US airspace to install a traffic alert and collision avoidance system. Radar systems such as collision avoidance radar able to discriminate objects (COLARADO) and millimeter-wave (MMW) are able to detect objects in front and below of the moving aircrafts to provide early warnings for pilots [29]. In recent years, collision avoidance is studied from operational perspective where automated collision free path planning tools have been modeled [4], [34] and [19]. Recently Alonso-Ayuso, Escudero and Martin-Campo [5] studied the speed-change policy within the collision avoidance perspective through computing the trajectory of an aircraft using geometric transformation. They have designed a mixed integer linear model which adjusts flights speed and altitude for their best performance by avoiding the air collision. Ariano et al. [17] have focused on the flight conflict detection in the airport terminal maneuvering area. They have studied safe airborne decisions in congested airports by considering rerouting and scheduling the flights. Later, they have added some realistic factors like speed changing and airspace capacities as well as conflict constraints to their work. They have attempted to optimize the use of runways and airways between flights and balance the flights delays. In their optimized airport taxiway routing study, Clare et al. [12] proposed a mixed-integer linear programming model to determine collision free taxing planning for aircrafts using Heathrow Airport. Similar to the most other works in the literature, route planning model proposed in [12] uses a time indexed (periodical) modeling approach. On the other hand, the model discussed in this paper introduces time as a decision variable. Moreover, the model has the flexibility to change flight speed based on the necessity at every link in the network. Another unique feature of the presented mathematical model is its capability of defining fuel consumption rate as a function of the speed. Most previous works modeled the fuel consumption cost as a function of delay. The variation of the speed which inherently moves the fuel usage has not been taken in to the consideration. Finally, the mid-air collision problem is explicitly incorporated through a set of conflict resolution constraints in the proposed MIP model.

# 3 Formulation of aircraft routing problem

In this section, the details of the mathematical model are provided. In the modeling of the en-route flight planning problem, it is assumed that all aircrafts enter a 3D mesh network (see Fig. 1 for illustration) from one of the available two dummy nodes ($v_1^{\mathrm{D}}$ and $v_2^{\mathrm{D}}$). While aircrafts enter to or exit from the airspace through given two points of entries, aircrafts travel inside of the airspace, by visiting nodes ($v$) through traveling on links ($\ell$). The objective of an aircraft is to reach to opposite dummy node within shortest time, with minimum cost and without violating any safety rules.

The mathematical formulation determines a flight plan ($R^f$) for each aircraft on the 3D mesh network. Aircraft's flight plan includes set of links ($x_\ell^f = 1$), arrival ($a_\ell^f$) and departure ($d_\ell^f$) times to links, average speed ($s_\ell^f$) and the fuel consumption rate on the link ($\mathrm{FCR}_\ell^f$). Hence the en-route flight plan is defined as $R^f = (x_\ell^f,\, a_\ell^f,\, d_\ell^f,\, t_\ell^f,\, s_\ell^f,\, \mathrm{FCR}_\ell^f\colon\, \ell \in L)$. While the primary objective of the mathematical model is to determine $R^f$ for all $f$ for the given objective (cost or time in our case), all safety concerns such as separation distance between aircrafts and mid-air collision avoidance are precisely incorporated in the formulation. One of the unique characteristics of the model is the non-time indexed formulation which enables us to determine exact arrival and departure times to each node (and the duration on each link). Knowledge of exact arrival and departure times of aircrafts on 3D mesh network is crucial for ensuring the safety in real-time applications. The most literature on the topic formulates the aircraft routing problem by using discrete time periods (time-indexed strategies) where $t$ is a period and arrival and departure of an aircraft to a node (or an airspace in some literature) can only occur when the time is equal to one of the predefined periods in $t = \{0, 1, \ldots, T\}$. Since the aircraft's speed in the air is significantly high, small variations on the flight times may result with mid-air conflicts. Non-time indexed formulation eliminates such possibilities since the speed and flight time on each link is in real-time. Another unique feature of the given MIP model is its ability to formulate the fuel consumption rate as a function of aircraft-speed.

The details of the model is discussed in three sub-sections. First, we provide the list of parameters and decision variables in Section 3.1. Next, in Section 3.2, assumptions made in the modeling is discussed. Finally, the problem formulation is provided in Section 3.3.

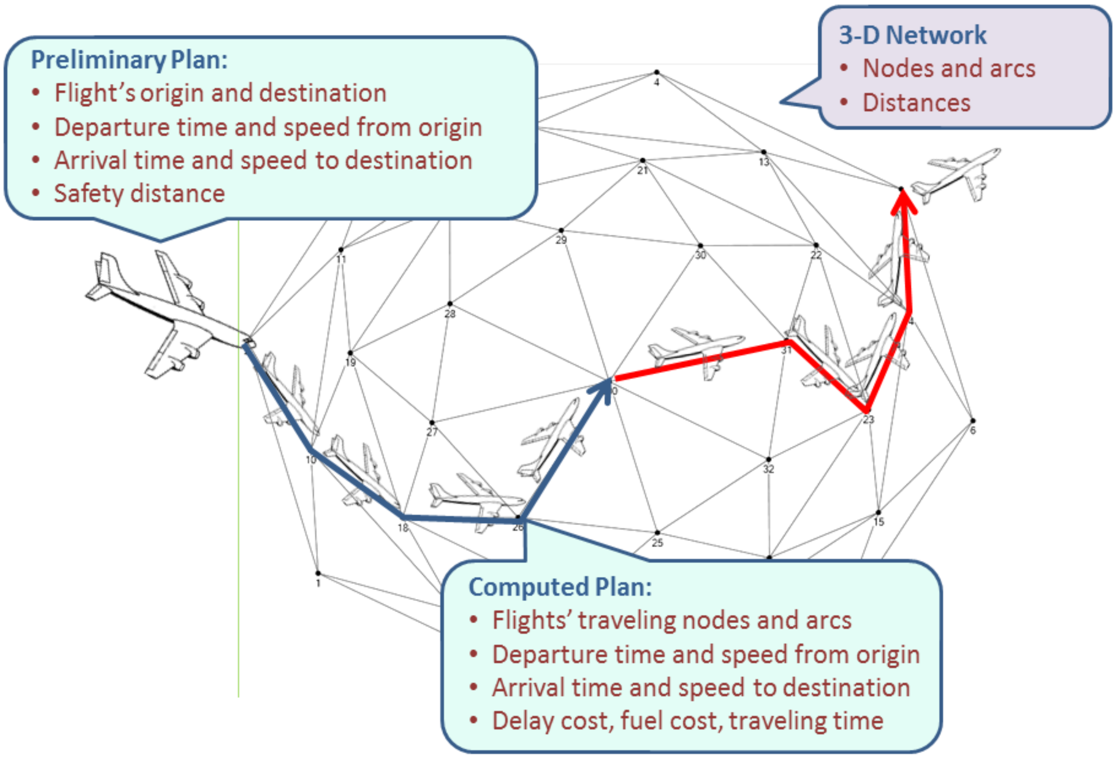



Figure 1: Illustration of 3D mesh used in the formulation

## 3.1 Description of model parameters and decision variables

### 3.1.1 Parameters

| | |
|---|---|
| $F$ | Set of flights, indexed by $f$ |
| $V$ | Set of nodes, indexed by $v$ |
| $L$ | Set of links, indexed by $\ell$ |
| $S$ | Set of possible speeds (discretized), indexed by $s$ |
| $\Delta$ | Predefined set of speed changes between two consecutive links, indexed by $\delta$ |
| $T$ | Set of possible traveling times when speed is discrete, indexed by $t$ |
| $\text{FUEL\_D}$ | Set of possible fuel consumption rates depends on the discrete set of speeds, indexed by $\text{FCR\_D}$ |
| $v_f^{\text{IN}}$ | Entry point of flight $f, (v_f^{\text{IN}} \in \{v_1^{\text{D}}, \ldots, v_n^{\text{D}}\})$ |
| $v_f^{\text{OUT}}$ | Exit point of flight $f, (v_f^{\text{OUT}} \in \{v_1^{\text{D}}, \ldots, v_n^{\text{D}}\})$ |
| $\text{EARLY}_f$ | Earliest possible departure time of flight $f$ at its destination point |
| $\text{LATE}_f$ | Latest possible arrival time of flight $f$ at its destination point |
| $\text{PENAL}_f^{\text{EARLY}}$ | Penalty cost of flight $f$ for arriving or departing early |
| $\text{PENAL}_f^{\text{LATE}}$ | Penalty cost of flight $f$ for arriving or departing late |
| $\text{COST\_U}^{\text{FUEL}}$ | Cost of per gallon of aircraft fuel |
| $\text{LENGTH}_\ell$ | Length of link $\ell$ |
| $\omega^-(v)$ | Set of incoming nodes that can route aircrafts to node $v$ |
| $\omega^+(v)$ | Set of outgoing nodes that node $v$ can route aircrafts to. |
| $\omega_\ell^-(v)$ | Set of allowed incoming links for a flight leaving $v$ through link $\ell$ |
| $\omega_\ell^+(v)$ | Set of allowed outgoing links for a flight entering $v$ through link $\ell$ |
| $w^{\min}, w^{\max}$ | Weights (smoothing coefficients) used to determine speed bounds $w^{\max} = \{\underline{w}^{\max}, \overline{w}^{\max}\}$ and $w^{\min} = \{\underline{w}^{\min}, \overline{w}^{\min}\}$ |

| | |
|---|---|
| $t_f^{\text{OUT}}$ | Schedule time for leaving the airspace (either landing or departing) |
| $t_f^{\text{IN}}$ | Scheduled time for entering the airspace (departure time or entry time to the system |
| $t_{ff'}$ | Required separation distance (expressed in time units) for flight $f'$ following departure of flight $f$ within the same nodes |
| $\tau^{\min}$ | Time to travel a unit distance with a minimum possible speed |
| $\tau^{\max}$ | Time to travel a unit distance with a maximum possible speed |

#### 3.1.2 Decision variables

| | |
|---|---|
| $a_\ell^f \in R^+$ | Arrival time for flight $f$ at node $v$ from link $\ell \in \omega^-(v)$ |
| $d_\ell^f \in R^+$ | Departure time for flight $f$ from node $v$ on link $\ell \in \omega^+(v)$ |
| $x_\ell^f$ | $= \begin{cases} 1 & \text{if flight } f \text{ travels on link } \ell \\ 0 & \text{Otherwise} \end{cases}$ |
| $\beta_\ell^{ff'}$ | $= \begin{cases} 1 & \text{flight } f' \text{ follows flight } f \text{ on link } \ell \\ 0 & \text{Otherwise} \end{cases}$ |
| $\theta_v^{ff'}$ | $= \begin{cases} 1 & \text{flight } f \text{ leaves node } v \text{ before flight } f' \\ 0 & \text{Otherwise} \end{cases}$ |
| $\alpha_v^{ff'}$ | $= \begin{cases} 1 & \text{flight } f \text{ enters a link } \ell \text{ connecting to node } v \text{ before flight } f' \\ & \quad \text{enters the same link from opposite direction} \\ 0 & \text{Otherwise} \end{cases}$ |
| $\text{FCR}_\ell^f$ | Fuel consumption rate for per unit of flight time for flight $f$, on link $\ell$ |
| $\text{COST}_\ell^f$ | Fuel consumption cost in link $\ell$ |
| $\text{EARLY}^f$ | Earliness of flight $f$ |
| $\text{LATE}^f$ | Lateness of flight $f$ |
| $x_{\ell s}^f$ | $= \begin{cases} 1 & \text{if flight } f \text{ travels on link } \ell \text{ with speed } s \\ 0 & \text{Otherwise} \end{cases}$ |
| $y_{\ell\delta}^f$ | $= \begin{cases} 1 & \text{if the speed of flight } f \text{ is increased by } \delta \text{ on link } \ell \\ 0 & \text{Otherwise} \end{cases}$ |
| $s_\ell^f$ | Speed of flight $f$ at $\ell$ |
| $t_\ell^f$ | Travel time for flight $f$ on link $\ell$ |
| $\tau_\ell^f$ | Time to travel a unit distance for flight $f$ on link $\ell$. |

## 3.2 Assumptions

We made the following assumptions in order to model the aircraft routing problem:

- An aircraft can visit a node only once
- Average speed change of an aircraft from link $\ell$ to the consecutive link $\ell'$ is bounded based on aircrafts' speed on link $\ell$.

## 3.3 Problem formulation

In order to solve a large scale optimization problem, it is important to obtain a strong formulation. The proposed formulation avoids non-linearity under all circumstances, yet still archives all its objectives. Precise control of speed on each link enables us to determine exact traveling times so the midair collision is avoided. The proposed mathematical model considers the minimization of total cost that is incurred from delays, earliness and speed-dependent fuel consumption. Other strategies can easily be integrated into the objective function. Constraints of the model are categorized in 4 groups: routing; timing; speed and fuel consumption; and the safety and conflict.

### 3.3.1 Objective function

$$\min \quad \sum_{f \in F} \left( \text{PENAL}_f^{\text{EARLY}} \, \text{EARLY}_f + \text{PENAL}_f^{\text{LATE}} \, \text{LATE}_f + \sum_{\ell \in L} \text{COST}_{f\ell}^{\text{FUEL}} \right) \tag{1}$$

Delay cost is a penalty for being early or late based on the primary schedule of the aircraft. The fuel consumption cost is written as a function of traveling distances and the fuel consumption coefficient for the given speed (speed and fuel consumption relationship is discussed in details later in the section).

### 3.3.2 Flight routing constraints

Flight routing constraints are written in order to determine the flight plan as a set of visited nodes including arrival and departure times on each connecting link.

$$\sum_{\ell \in \omega^+(v_f^{\text{IN}})} x_\ell^f = 1 \qquad f \in F \tag{2}$$

$$\sum_{\ell \in \omega^-(v_f^{\text{OUT}})} x_\ell^f = 1 \qquad f \in F \tag{3}$$

$$\sum_{\ell \in \omega^-(v)} x_\ell^f = \sum_{\ell \in \omega^+(v)} x_\ell^f \qquad v \in V, f \in F \tag{4}$$

$$x_\ell^f + \sum_{\ell' \in \omega_\ell^+(v)} x_{\ell'}^f \leq 1 \qquad v = \text{DST}(\ell), \ell \in L, f \in F \tag{5}$$

$$\sum_{\ell \in \omega^-(v)} x_\ell^f \leq 1 \qquad v \in V, f \in F \tag{6}$$

$$\sum_{\ell \in \omega^+(v)} x_\ell^f \leq 1 \qquad v \in V, f \in F. \tag{7}$$

Constraint (2) ensures all flights to enter the network from the entry node using a single link. Similarly, constraint (3) ensures all flights to reach their destinations (i.e., exit point) through a single link. Conservation constraint (4) forces all flights entering an interior node to leave the node. Constraint (5) enforces airplanes not to make sharp turns at nodes. Inequalities (6) and (7) ensure that no aircraft can travel through the same node and link more than once. The reason constraints (6) and (7) are imbedded in to the models is to reduce the computational complexity. In order to allow a flight to travel through the same node or link multiple times, either additional indices (visiting index) to differentiate each visit, or multiple links between node pairs (i.e., use of a multi-graph) to allow alternatives should be introduced to the model.

Note that constraints (2) and (3) are easy to extend for systems with several entrance/exit points:

$$\sum_{\ell \in \omega^+(v_f^{\text{IN}})} x_\ell^f = 1 \qquad f \in F, v_f^{\text{IN}} \in V^{\text{IN}} \tag{2'}$$

$$\sum_{\ell \in \omega^-(v_f^{\text{OUT}})} x_\ell^f = 1 \qquad f \in F, v_f^{\text{OUT}} \in V^{\text{OUT}}, \tag{3'}$$

where $V^{\text{IN}}$ and $V^{\text{OUT}}$ are the sets of entry and exit points, respectively.

### 3.3.3 Timing constraints

We next describe the constraints setting the relationship between arrival and departure times on nodes and links.

$$
\begin{aligned}
& d_\ell^f \ge x_\ell^f t_f^{\text{IN}} && \ell \in \omega^+(v_f^{\text{IN}}), f \in F && (8)\\
& \text{EARLY}_f \le a_\ell^f \le \text{LATE}_f && \ell \in \omega^-(v_f^{\text{OUT}}), f \in F && (9)\\
& a_\ell^f \le M x_\ell^f && v \in V \setminus \{v_f^{\text{IN}}, v_f^{\text{OUT}}\}, \ell \in \omega(v), f \in F && (10)\\
& d_\ell^f \le M x_\ell^f && v \in V, \ell \in \omega(v), f \in F && (11)\\
& \sum_{\ell \in \omega^-(v)} a_\ell^f = \sum_{\ell \in \omega^+(v)} d_\ell^f && v \in V \setminus \{v_f^{\text{IN}}, v_f^{\text{OUT}}\}, f \in F && (12)\\
& \sum_{\ell \in \omega^-(v_f^{\text{OUT}})} a_\ell^f = t_f^{\text{OUT}} + \text{LATE}_f - \text{EARLY}_f && f \in F. && (13)
\end{aligned}
$$

Inequality (8) ensures that when an aircraft enters the airspace from the entry node $v_f^{\text{IN}}$ through link $\ell$, then the link is added into the flight plan of the aircraft ($x_\ell^f = 1$) and consecutively a departure time from the entry node is assigned ($d_\ell^f \ge 0$). Inequality (9) controls the earliest arrival and latest departure times if desired. Constraints (10) and (11) ensure that when an aircraft is not assigned to a link, then the arrival or departure time to that link is forced to be zero in order to sustain the accurate network flow without creating non-linearity. In the following constraint (12), it is assured that aircrafts are not delayed on intermediate nodes. Finally, in equation (13) , exact earliness or lateness are determined.

### 3.3.4 Speed control

In the proposed mathematical model, flight time between two consecutive nodes is determined based on the flight-speed. The distance between two consecutive nodes ($\text{LENGTH}_\ell$) is known. Therefore the time to travel link $\ell$ with an average speed of $s_\ell^f$ is

$$t_\ell^f = \text{LENGTH}_\ell / s_\ell^f \qquad \ell \in L, f \in F \tag{14}$$

which is a nonlinear term. Two different speed control policies are proposed in order to overcome the non-linearity: *(i)* Discrete speed control; and *(ii)* Continuous speed control in which the speed is defined as the *time to travel a unit distance*. The proposed mathematical model also handles speed changes between two consecutive links. It is assumed that an aircraft can change its average speed from one link to the next by a percentage ($p$) of its current speed:

$$(1-p)s_{\ell'}^f \quad \le \quad s_\ell^f \quad \le \quad (1+p)s_{\ell'}^f \qquad \ell \in L, \ell' \in \omega_\ell^-(v), f \in F \tag{15}$$

Particularly in North American airspaces around airports and in European airspace between airports (average flight distance is small and number of aircrafts is high), aircrafts' speed-changes are gradual (speed on the next link is dependent on the current speed).

- Discrete speed control constraints

In the discrete speed control approach, it is assumed that the average speed of an aircraft is modified by a predefined discrete-amount $\delta \in \Delta$ as:

$$s_{\ell'}^f = s_\ell^f + \sum_{\delta \in \Delta} \delta \, y_{\ell\delta}^f \qquad \ell \in L, f \in F, \delta \in \Delta \tag{16}$$

where $\Delta = \{-\delta_{\max}, \ldots, -\delta_1, 0, \delta_1, \ldots, \delta_{\max}\}$ and $y_{\ell\delta}^f = 1$ if the speed is changed by $\delta$ amount in link $\ell$. Since the speed change is discrete and finite, and the link-length ($\text{LENGTH}_\ell$) is constant, then the traveling time on a given link ($t_{\ell s}$) is discrete and finite. Similarly, there exists a unique fuel consumption rate $\text{FCR\_D}_s \in \text{FUEL\_D}$ for every $s \in S$.

Let us now introduce the necessary constraints to determine the traveling times on links with respect to traveling speeds.

$$\sum_{\ell \in \omega^+(v)} \left( \sum_{s \in S} s\, x^f_{\ell s} + \sum_{\delta \in \Delta} \delta y^f_{\ell \delta} \right) = \sum_{\ell' \in \omega^-(v)} \sum_{s \in S} s\, x^f_{\ell' s} \qquad v \in V, f \in F \tag{17}$$

$$\sum_{s \in S} x^f_{\ell s} = x^f_\ell \qquad v \in V, \ell \in \omega(v), f \in F \tag{18}$$

$$\sum_{\delta \in \Delta} y^f_{\ell \delta} = x^f_\ell \qquad v \in V, \ell \in \omega(v), f \in F \tag{19}$$

$$a^f_\ell = d^f_\ell + \sum_{s \in S} x^f_{\ell s}\, t_{\ell s} \qquad v \in V, \ell \in \ell_i, f \in F \tag{20}$$

$$x^f_\ell\, s^{\text{IN}}_f = \sum_{s \in S} x^f_{\ell s} \qquad \ell \in \omega^-(v^{\text{IN}}_f), f \in F \tag{21}$$

$$x^f_\ell\, s^{\text{OUT}}_f = \sum_{s \in S} x^f_{\ell s} \qquad \ell \in \omega^-(v^{\text{OUT}}_f), f \in F \tag{22}$$

$$\text{COST}^{\text{FUEL}}_{f\ell} = \text{COST\_U}^{\text{FUEL}}_f \times \text{LENGTH}_\ell \times \sum_{s \in S} x^f_{\ell s} \text{FCR}_s \qquad \ell \in \omega^-(v^{\text{OUT}}_f), f \in F \tag{23}$$

The speed adjustment from a link $\ell' \in \omega^+(v)$ to the consecutive link $\ell \in \omega^-(v)$ is controlled in equation (17). Equations (18) and (19) ensures that for $x^f_\ell = 1$, one of the speeds $(x^f_{\ell s})$ and a speed change $(y^f_{\ell\delta})$ are assigned to an aircraft. In equation (20), the arrival time to the destination node is determined. In the model, it is assumed that the initial and final desired speeds of the aircraft are known $(s^{\text{IN}}_f, s^{\text{OUT}}_f)$. Equations (21) and (22) determine the index of initial and final speeds from speed set $S$ where $x^f_{\ell s}$ is the binary decision variable used for identifying speed. Finally, Equation (23) determines the total fuel consumption cost in link $\ell$ at speed $s$.

- Continuous speed control constraints

The discrete speed control policy is easier to model. However, it introduces two new binary decision variables ($x^f_{\ell s}$ and $y^f_{\ell\delta}$). In order to overcome the complexity that these two decision variables add to the model, a continuous speed control policy is proposed. Let $\tau_\ell$ be the required time to fly a unit distance with a given speed on link $\ell$ where $\tau_\ell = 1 \,/\, s_\ell$. Describing speed in terms of time to travel a unit distance enables us to determine the traveling time on a link using a linear expression as:

$$t^f_\ell = \tau^f_\ell \text{LENGTH}_\ell \qquad \ell \in L, f \in F \tag{24}$$

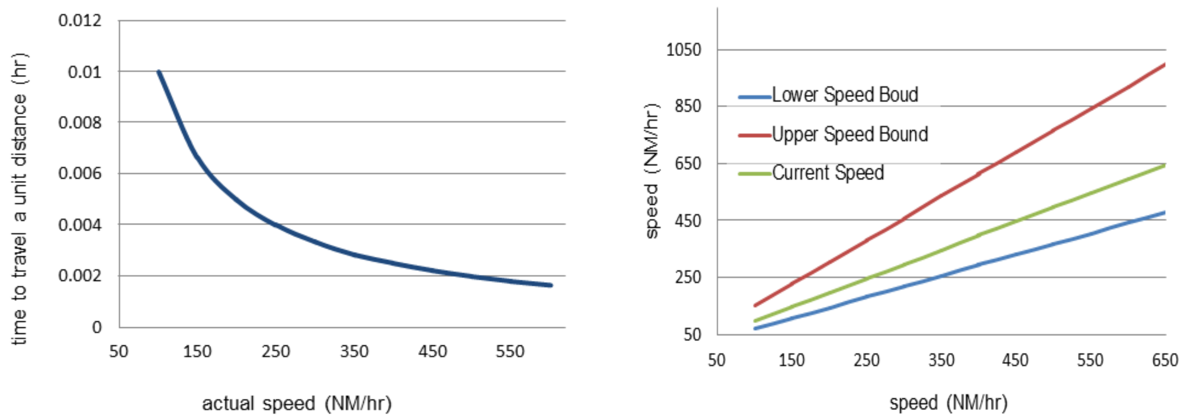
Consecutively, the (20) is modified as:

$$a^f_\ell = d^f_\ell + \tau^f_\ell \text{LENGTH}_\ell \qquad v \in V, \ell \in \omega(v), f \in F \tag{25}$$

The main challenge of working with $\tau^f_\ell$ is in defining the speed bounds between two consecutive links. If the assumption is that the actual flight-speed in a given link is bounded by the percentage ($p$) of flight's speed on the previous link as described earlier in equation (15), a similar bound should be defined on $\tau^f_\ell$. Let $\tau^f_\ell$ for a given flight be bounded as shown in (26).

$$(1 - p_1)\tau^f_{\ell'} \le \tau^f_\ell \le (1 + p_2)\tau^f_{\ell'} \qquad \ell \in L, \ell' \in \omega^-_\ell(v), f \in F \tag{26}$$

As shown in Fig. 2(a), when plotted against the actual speed, $\tau$ resembles an exponential behavior. Consequently, when speed is bounded by a fixed percentage, the magnitude of impact on speed increase and decrease is sensitive to the current speed. As depicted in Fig. 2(b), a constant speed change rate on $\tau$ leads to smaller bound when the current speed is lower and larger bound when the current speed is higher. In order to overcome the problem depicted in Fig. 2(b) for speed bound, we introduced a dynamic speed bound control logic. A numerical study revealed that, Equation (27) along with Equations (28) and (29) would




(a) Trend of $\tau$ with respect to $s$

(b) Impact of fixed $p$ on actual speed-bound

Figure 2: Trend of $\tau$ and impact of fixed $p$ on speed-bounds (speed is expressed in Nautical Mile – NM per hour)

imitate the desired speed change policies as shown in Fig. 3(a). By calibrating smoothing parameters $w^{\min}$ and $w^{\max}$, variety of speed bounds can be generated.

$$\tau_\ell - \tau_\ell^- \leq \tau_{\ell'} \leq \tau_\ell + \tau_\ell^+ \qquad \ell \in L, f \in F \tag{27}$$

In (27), speed increase $\tau_\ell^+$ and speed decrease $\tau_\ell^-$ limits are determined as:

$$\tau_\ell^- = \underline{w}^{\min}(\tau^{\min} - \tau_\ell) + \underline{w}^{\max}(\tau_\ell - \tau^{\max}) \tag{28}$$

$$\tau_\ell^+ = \overline{w}^{\min}(\tau^{\min} - \tau_\ell) + \overline{w}^{\max}(\tau_\ell - \tau^{\max}). \tag{29}$$

In Fig. 3(a), we plot the speed $s_\ell^f$ on a unit distance. It is then bounded as follows:

$$\underline{s}_\ell^f = 1/(\tau_\ell + \tau_\ell^+) \quad \leq \quad s_\ell^f \quad \leq \quad \overline{s}_\ell^f = 1/(\tau_\ell - \tau_\ell^-).$$

Computing $\tau_\ell^-$ and $\tau_\ell^+$ with relations (28) and (29), and the following parameter values:
Speed Increase: $[\underline{w}^{\min}, \underline{w}^{\max}] = [0.01, 0.44]$
Speed Decrease: $[\overline{w}^{\min}, \overline{w}^{\max}] = [0, 0.85]$
For both cases: $[\tau^{\min}, \tau^{\max}] = [0.01, 0.0009]$,
we deduce the lower and upper bound curves as plotted in Fig. 3(a). As we observe a near-linear relationship for the bound on $\tau$ vs. the bound on actual speed $s$ (see Fig. 3(b) for illustration), we modified the model and replaced constraints (17)–(22) by constraints (25), (27), (28) and (29).

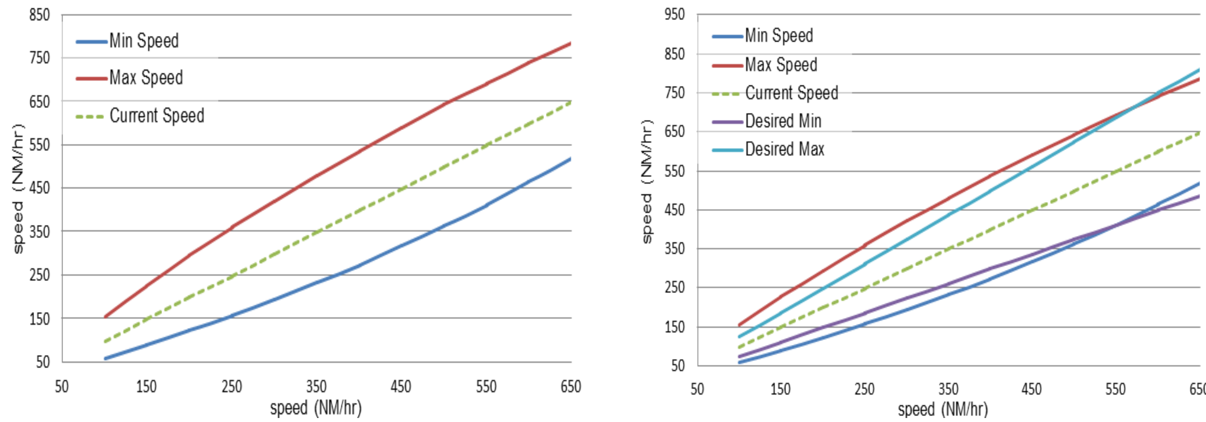
### 3.3.5 Fuel consumption constraints

Several factors including aircraft type, weather condition, flight altitude, load and speed impact on the fuel consumption rate of an aircraft. In the problem addressed in this paper, controllable factors are speed and altitude. In this paper, we modeled the fuel consumption rate as a function of speed only.

Let $\text{FCR}_\ell^f$ be the amount of fuel required to fly an aircraft for per nautical mile with a given speed $s$. Then, the cost of traveling entire link is:

$$\text{COST}_{f\ell}^{\text{FUEL}} = \text{COST\_U}_f^{\text{FUEL}} \times \text{LENGTH}_\ell \times \text{FCR}_\ell^f \tag{30}$$

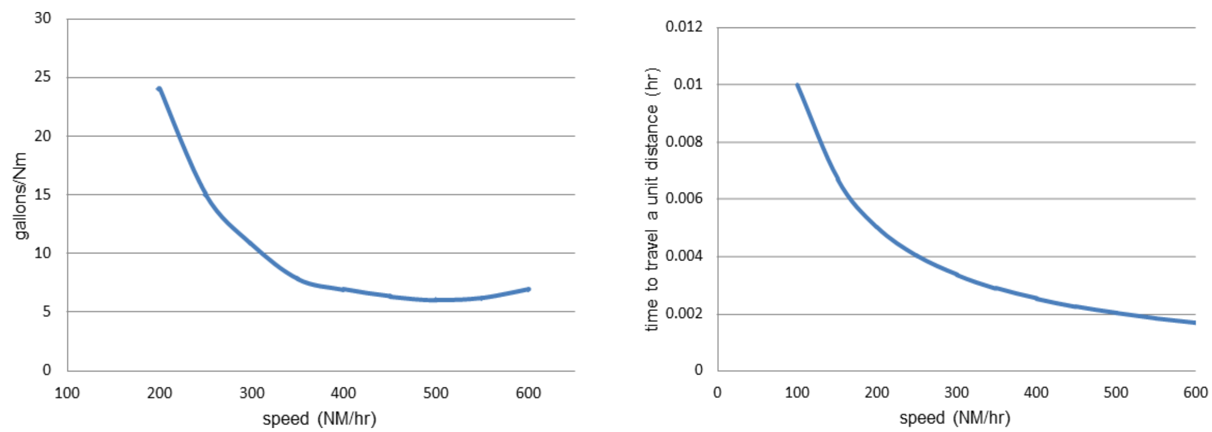
where $\text{COST\_U}_f^{\text{FUEL}}$ is the unit cost of aircraft fuel and $\text{LENGTH}_\ell$ is the length of the given link. In Clarke *et al.* [13], a relationship between speed and fuel consumption is established from industry data for various aircraft types similar to the trend illustrated in Fig. 4(a).




(a) Projected speed limits ($\tau_{\ell'} - \tau^{-} \leq t_\ell \pi \leq t_{\ell'} - \tau^{+}$

(b) Comparisons with the linear speed limit control: desired and estimate speed limits

Figure 3: Speed control between consecutive links (speed is expressed in Nautical Mile – NM per hour)




(a) Fuel consumption-speed relationship: industry data

(b) Speed vs. time to travel

Figure 4: Comparison of fuel consumption rate vs. speed; and time to travel a unit distance vs. speed relationships

Although the fuel consumption rate is different for each aircraft, a similar speed-fuel consumption rate relationship as illustrated in Fig. 4(a) can be established for all types of aircrafts. In this study, we used the fuel consumption rate for Boeing 777-200LR as a reference. In order to define the fuel consumption rate as a function of speed, we used decision variable $\tau$. As shown in Fig. 4(b), when plotted $\tau$ against actual speed $s$, we observe a strong correlation with the trend given in Fig. 4(a) for the fuel consumption rate of Boeing 777-200LR. For $s \geq s^\star$, an inverse relationship is observed ($s^\star$ is the speed leads to optimal fuel consumption rate). Consequently, we scaled the $\tau$ and $s$ relationship through scaling parameters $k^1$ and $k^2$ and obtained the following expression as the speed-dependent fuel consumption rate:

$$\text{FCR}_\ell^f = \begin{cases} \text{FCR}_\ell^{f\star}\left(k^1 x_\ell^f + k^2\left(\tau_\ell^f - \dfrac{x_\ell^f}{s^\star}\right)\right) & \text{if } \tau_\ell^f \geq \dfrac{x_\ell^f}{s^\star} \\ \text{FCR}_\ell^{f\star}\left(k^1 x_\ell^f + k^2\left(\dfrac{x_\ell^f}{s^\star} - \tau_\ell^f\right)\right) & \text{otherwise.} \end{cases} \tag{31}$$

where $\text{FCR}_\ell^{f\star}$ is the fuel consumption rate at the optimum speed $s^\star$. For scaling parameters $k^1 = 0.8$ and $k^2 = 1,000$, optimum speed $s^\star = 500$ NM/hr and $\text{FCR}_{s^\star}^f = 6$ gallons/NM, the fuel consumption-speed relationship given in Fig. 5 is obtained. It is clear from Fig. 5 that equation (31) corresponds very well with

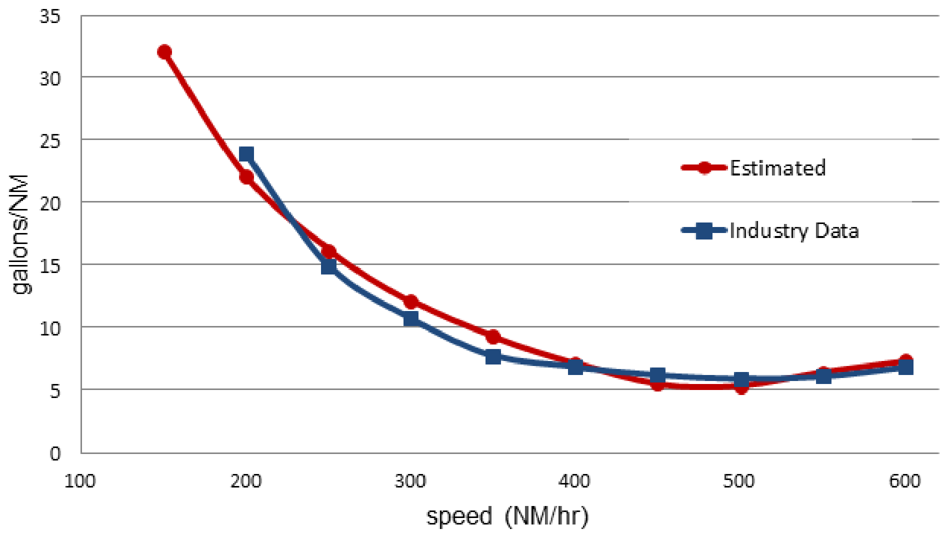



Figure 5: Estimating fuel consumption rate for given flight speed

the industry data. Consequently, constraints (30) and (31) enable us to incorporate speed-dependent fuel consumption cost into the objective function.

#### 3.3.6 Safety and conflict constraints

Let us now introduce a set of constraints to ensure a minimum separation between aircrafts traveling on the same direction and to guarantee collision avoidance between aircrafts that are traveling on opposite directions.

For all $v \in V, \ell \in \omega^+(v), f, f' \in F : f < f'$

$$d_\ell^{f'} - d_\ell^f \geq t_{ff'} - M(1 - \beta_\ell^{ff'}) - M(2 - x_\ell^f - x_\ell^{f'}) \tag{32}$$

$$d_\ell^f - d_\ell^{f'} \geq t_{ff'} - M\beta_\ell^{ff'} - M(2 - x_\ell^f - x_\ell^{f'}) \tag{33}$$

For all $v \in V, \ell \in \omega^-(v),\ f, f' \in F : f < f'$

$$a_\ell^{f'} - a_\ell^f \geq t_{ff'} - M(1 - \beta_\ell^{ff'}) - M(2 - x_\ell^f - x_\ell^{f'}) \tag{34}$$

$$a_\ell^f - a_\ell^{f'} \geq t_{ff'} - M\beta_\ell^{ff'} - M(2 - x_\ell^f - x_\ell^{f'}) \tag{35}$$

Inequalities (32) and (33) ensure that when two aircraft are traveling on the same direction using the same link, a minimum separation distance of $t_{ff'}$ is guaranteed at the point when they enter the link from node $v$. Through inequalities (34) and (35), when such two aircrafts arrive to the next node, the minimum separation distance is sustained in order to avoid for aircrafts to pass each other while they are traveling on $\ell$. Binary decision variable $\beta_\ell^{ff'} = 1$ implies that flight $f$ is the leader on link $\ell$. The situation is illustrated on Fig. 6.

The purpose of the next set of inequalities is to ensure that no two aircrafts fly on the same link from opposite directions at the same time. The binary decision variable $\alpha_\ell^{ff'} = 1$ if flight $f$ occupies the link earlier than $f'$. Consequently, the model enforces flight $f'$ to keep least $t_{ff'}$ distance from flight $f$ on node $v$, see Fig. 7 for an illustration.

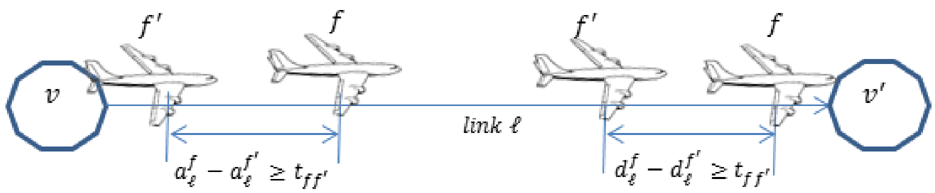



Figure 6: Separation distance between two consecutive flights

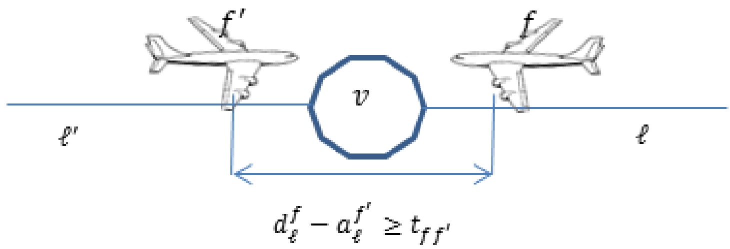



Figure 7: No two flights can travel on the same link at the same time

For all $\ell \in L,\ f, f' \in F : f < f'$ where $\textsc{opp}(\ell)$ is the link flow opposite to $\ell$:

$$d^{f'}_{\textsc{opp}(\ell)} - a^f_\ell \geq t_{ff'} - M(1 - \alpha^{ff'}_\ell) - M(2 - x^f_\ell - x^{f'}_{\textsc{opp}(\ell)}) \tag{36}$$

$$d^f_\ell - a^{f'}_{\textsc{opp}(\ell)} \geq t_{ff'} - M(\alpha^{ff'}_\ell) - M(2 - x^f_\ell - x^{f'}_{\textsc{opp}(\ell)}). \tag{37}$$

Finally, the next set of inequalities ensures the separation of aircrafts by a given time $t_{ff'}$ at any node. The binary decision variable $\theta^{ff'}_v = 1$ if aircraft $f$ passes through node $v$ before aircraft $f'$. Consequently, a safe separation distance (time) between aircraft pairs is imposed. The cross-passing on a node is illustrated in Fig. 8.

For all $v \in V,\ f, f' \in F : f < f'$

$$\sum_{\ell \in \omega^-(v)} a^{f'}_\ell - \sum_{\ell \in \omega^-(v)} a^f_\ell \geq t_{ff'} - M(1 - \theta^{ff'}_v) - M(2 - \sum_{\ell \in \omega^-(v)} x^f_\ell - \sum_{\ell \in \omega^-(v)} x^{f'}_\ell) \tag{38}$$

$$\sum_{\ell \in \omega^-(v)} a^f_\ell - \sum_{\ell \in \omega^-(v)} a^{f'}_\ell \geq t_{ff'} - M\theta^{ff'}_v - M(2 - \sum_{\ell \in \omega^-(v)} x^f_\ell - \sum_{\ell \in \omega^-(v)} x^{f'}_\ell). \tag{39}$$

## 4 Solutions and results

The en-route flight planning model discussed in Section 3 is designed to serve for both current ATC centered and FFC based air traffic management philosophies. The ATC centered flight planning strategy can be viewed as the centralized and the FFC based management as the decentralized solution methodologies. In the centralized solution methodology, we assume that the flight plan for all approaching aircrafts are optimally determined through solving the proposed mathematical model for $N$ aircrafts. On the other hand, the decentralized model determines the flight plan for a single aircraft assuming that the flight plans for all other aircrafts that are projected to enter to the airspace before the arrival of the newly approaching aircraft are known. The experimental studies with the decentralized solution methodology showed low utilization of airspace capacity. Consequently, a hybrid solution methodology that combines centralized and decentralized methods in which the flight plans of a small number of newly approaching aircrafts are determined with respect to the existing traffic in the airspace prior to their arrivals. The hybrid solution strategy produced flight plans with much higher airspace utilization and more homogenous cost allocation among all aircrafts.

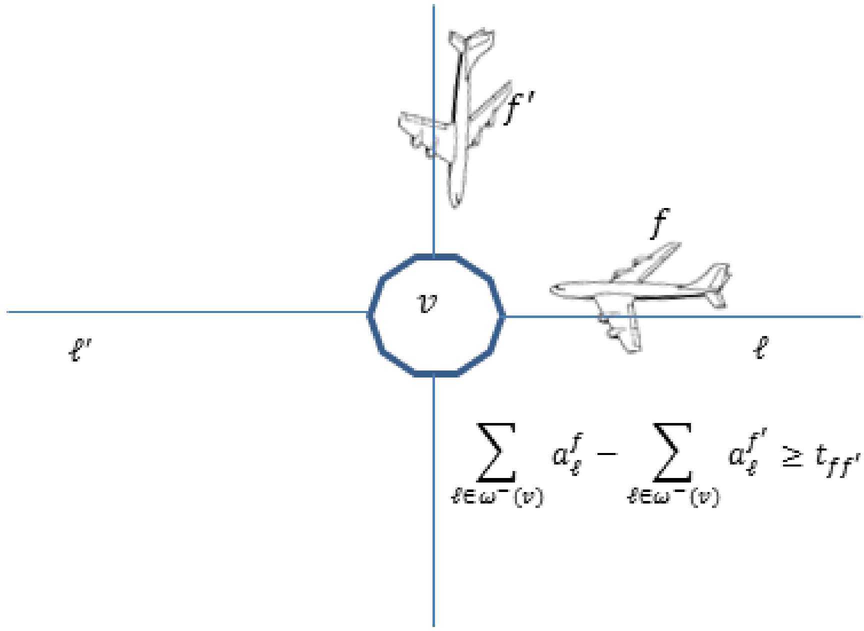



Figure 8: Collision avoidance at nodes

The proposed en-route flight planning model is tested on all three solution approaches. For all cases, the objective is to determine conflict-free en-route flight plans for all aircrafts with minimum delay, earliness and fuel consumption costs.

## 4.1 Data instances

We design an airspace around an airport as follows. A 3D mesh that consists of 15 nodes and 70 links is used to model the airspace. Aircrafts enter (or exit from) airspace through two dummy nodes ($v_1^{\mathrm{D}}$ and $v_2^{\mathrm{D}}$). All aircrafts are forced to use a single runway which is a bi-directional arc connected to the internal dummy node ($v_2^{\mathrm{D}}$). The external dummy node ($v_1^{\mathrm{D}}$) is connected to four transition nodes for aircrafts to enter (or exit from) the airspace. Aircrafts enter the airspace for either departure or landing purpose. Time of entry to the airspace and the purpose of the flight (arrival or departure) are randomly generated. It is assumed that 50% of the flights are arriving flights. Time Between Arrivals (TBA) are exponentially distributed. In Table 1, a sample for input data is provided. Length of each link is determined based on their locations in the airspace. Links near the runway are shorter. For aircrafts approaching the airspace from outside, entry speeds are assumed to be 300 NM/hr. Minimum speed on the ground is set to 150 NM/hr. Between two consecutive links, aircrafts are allowed to change their speeds by approximately 50% at lower speeds and up to 20% at higher speeds. Aircrafts' traveling speeds vary in [150−550] NM/hr. The traveling time on a link is determined based on the average speed and the link length. The cost of a gallon of aircraft fuel is estimated to be \$3. Finally aircrafts are separated from each other by a Separation Distance (SD) which is measured in time ($t_{ff'}$). In order to test the capabilities of the proposed flight planning model, various traffic conditions were generated by changing SD and average TBA. The impact of SD and TBA on the given objectives (average flight time in airspace, average cost and program execution times) are studied for all three scenarios.

All three solution schemes were tested on various traffic conditions. Corresponding mathematical models were solved with IBM ILOG CPLEX Optimization Studio 12.2, using Optimization Programming Language (OPL) on a personnel computer with 64 bit operating system, 3.40 GHz Intel Core i7-2600 CPU and 16.0 GB RAM.

Table 1: Sample flight data

| Flight number | Arrival node | Departure node | Expected entry time | Expected exit time | Entry speed (NM/hr) | Exit speed (NM/hr) |
|---|---|---|---|---|---|---|
| 1 | $v_1^{\mathrm{D}}$ | $v_2^{\mathrm{D}}$ | 0.00 | 2.00 | 300 | 150 |
| 2 | $v_2^{\mathrm{D}}$ | $v_1^{\mathrm{D}}$ | 0.23 | 2.23 | 150 | 300 |
| 3 | $v_2^{\mathrm{D}}$ | $v_1^{\mathrm{D}}$ | 0.46 | 2.46 | 150 | 300 |
| 4 | $v_1^{\mathrm{D}}$ | $v_2^{\mathrm{D}}$ | 0.82 | 2.82 | 300 | 150 |
| 5 | $v_2^{\mathrm{D}}$ | $v_1^{\mathrm{D}}$ | 1.21 | 3.21 | 150 | 300 |
| 6 | $v_1^{\mathrm{D}}$ | $v_2^{\mathrm{D}}$ | 1.44 | 3.44 | 300 | 150 |
| 7 | $v_2^{\mathrm{D}}$ | $v_1^{\mathrm{D}}$ | 1.87 | 3.87 | 150 | 300 |
| 8 | $v_1^{\mathrm{D}}$ | $v_2^{\mathrm{D}}$ | 2.14 | 4.14 | 300 | 150 |
| 9 | $v_1^{\mathrm{D}}$ | $v_2^{\mathrm{D}}$ | 2.37 | 4.37 | 300 | 150 |
| 10 | $v_2^{\mathrm{D}}$ | $v_1^{\mathrm{D}}$ | 2.60 | 4.60 | 150 | 300 |

## 4.2 Centralized solution strategy

In the centralized solution strategy, we solve the proposed mathematical model for all aircrafts that use the airspace during a planning horizon. Hence, a non-time indexed network flow problem with $F$ flights, $V$ nodes and $L$ links is solved to determine the en-route flight plans $R^f = (x_\ell^f,\ a_\ell^f,\ d_\ell^f,\ t_\ell^f,\ s_\ell^f,\ \mathrm{FCR}_\ell^f\colon\ \ell \in L)$ for all aircrafts ($f \in F$) in the airspace with a given objective (either the minimum cost incurred from delays, earliness and fuel consumption, or average flight time in the airspace).

We conduct experiments on various sizes of instances (number of aircrafts ranges from 4 to 20). Impact of TBA and SD on the average flight time in airspace and traveling cost are investigated. Results provided in Table 2 show that the required computation time increases significantly for the instances that include more than 15 aircrafts in the airspace. For all other cases, the optimal result is obtained in less than 1 minute on a personal computer. As expected, there is a strong relationship between time between arrival to the airspace and the imposed separation distance between consecutive aircrafts. Smaller TBA and larger SD leads to more congested airspace. As seen in Table 2, the proposed mathematical model requires longer time to converge to an optimal solution when airspace is congested. Keeping in mind that O'Hare International Airport handles over 2500 flights per day and there are more than 5000 aircrafts flying over USA skies at any given time, a mathematical model should be capable of handling realistic problem sizes in order to be considered as a solution tool for ATFM. In order to overcome the scalability issues of the proposed mathematical model,

Table 2: Impact of TBA and SD on flight time in airspace and delay cost

| Number of Flights | TBA (Seconds | SD (Seconds) | Execution Time (Seconds) | Average Flight Time in Airspace ($) | Delay Cost ($) |
|---|---|---|---|---|---|
| 4 | 30 | 30 | 0.33 | 6.4245 | 38.5 |
| 8 | 30 | 30 | 1.63 | 6.4245 | 38.5 |
| 12 | 30 | 30 | 4.44 | 6.4869 | 39.8 |
| 16 | 30 | 30 | 13.43 | 6.7113 | 44.2 |
| 20 | 30 | 30 | 413.93 | 7.1043 | 52.1 |
| 4 | 30 | 60 | 0.35 | 6.4250 | 38.5 |
| 8 | 30 | 60 | 2.04 | 6.4245 | 38.5 |
| 12 | 30 | 60 | 6.88 | 7.0740 | 51.5 |
| 16 | 30 | 60 | 273.60 | 8.1496 | 73.0 |
| 20 | 30 | 60 | Out of Memory | | |

| Number of Flights | TBA (Seconds | SD (Seconds) | Execution Time (Seconds) | Average Flight Time in Airspace ($) | Delay Cost ($) |
|---|---|---|---|---|---|
| 4 | 15 | 30 | 0.25 | 6.4250 | 19.3 |
| 8 | 15 | 30 | 1.46 | 6.4245 | 38.5 |
| 12 | 15 | 30 | 4.92 | 6.4248 | 38.5 |
| 16 | 15 | 30 | 13.24 | 6.4250 | 38.5 |
| 20 | 15 | 30 | 60.45 | 6.6994 | 44.0 |
| 4 | 15 | 60 | 0.37 | 6.6748 | 43.5 |
| 8 | 15 | 60 | 2.64 | 7.1650 | 53.4 |
| 12 | 15 | 60 | 144.85 | 8.4490 | 79.0 |
| 16 | 15 | 60 | Out of Memory | | |
| 20 | 15 | 60 | Out of Memory | | |

two solution strategies, namely decentralized and hybrid solution strategies, are presented in the following sub-sections.

## 4.3 Decentralized solution strategy

In the decentralized solution strategy, we modeled and solved the air traffic management problem according to the principles of Free Flight Concept. Aircrafts depart or approach to an airport independent from each other based on their schedules. The objective is to determine the best flight plan for the next approaching/departing aircraft with respect to the current traffic conditions. Hence the problem is solved for a single aircraft given that $F$ aircrafts are currently in the airspace. When the flight $f$ arrives to the airspace through one of the dummy nodes ($v_1^{\text{D}}$ or $v_2^{\text{D}}$), flight plans $R^{f'}$ ($f' \in F$) are transmitted to the approaching aircraft $f$ (see Fig. 9 for illustration). The objective is to determine a flight plan for the incoming aircraft, i.e., values of $x_\ell^f, a_\ell^f$ and $d_\ell^f$, $\ell \in L$, for the given objective (either minimum cost or minimum flight time in the airspace in our case) while all safety rules are satisfied. Keeping in mind that an aircraft can fly within the speed range of [150−550] NM/hr, the flight plan of an aircraft, approaching to a new airspace (an airport), should be planned within seconds in order to avoid any mid-air or ground conflicts. We studied the effectiveness of the model under various traffic conditions (different rates of TBA and SD are used to generate traffic conditions).

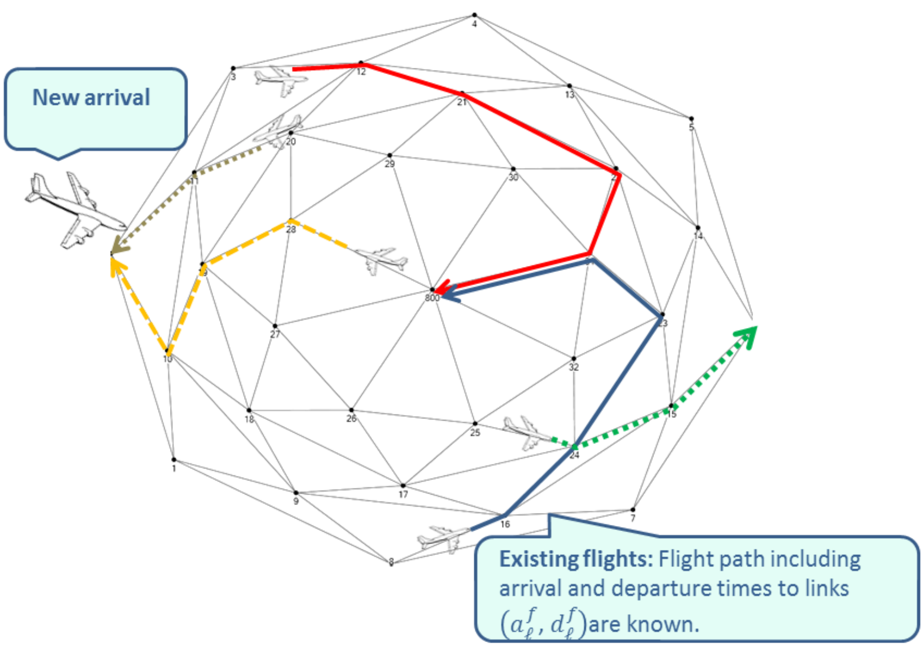



Figure 9: Traffic conditions at the time of new arrival

The decentralized model can be solved in less than one minute for most cases. Therefore, significantly larger traffic conditions as well as a larger network (34 nodes and 192 links) are used for experiments. Aircrafts arrive in the airspace in sequential order. At the outset of the planning horizon, the airspace is empty in our experiments, consequently the performance measures (cost and flight time in airspace) for earlier flights are smaller. As the time goes, the traffic condition changes until the system reaches its steady-state. The duration of transition period varies depending on the selected TBA and SD. In Fig. 10, the impact of TBA on flight time in airspace is illustrated for SD = 0.3. For each TBA, a total of 100 flights were generated. When the airspace is congested (0.15 min. $\leq$ TBA $\leq$ 0.35 min.), as expected, aircrafts' flight-times in the airspace are higher and a steady-state flight condition cannot be reached. When the capacity of airspace is larger than the requirement (TBA $\geq$ 0.35 min.), the transition period is either short or does not exist. Existence of a steady state in 0.35 $\leq$ TBA $\leq$ 0.4 minutes arrival rate indicates the maximum capacity of the airspace for the given SD (0.3 min. for the scenario illustrated in Fig. 10).

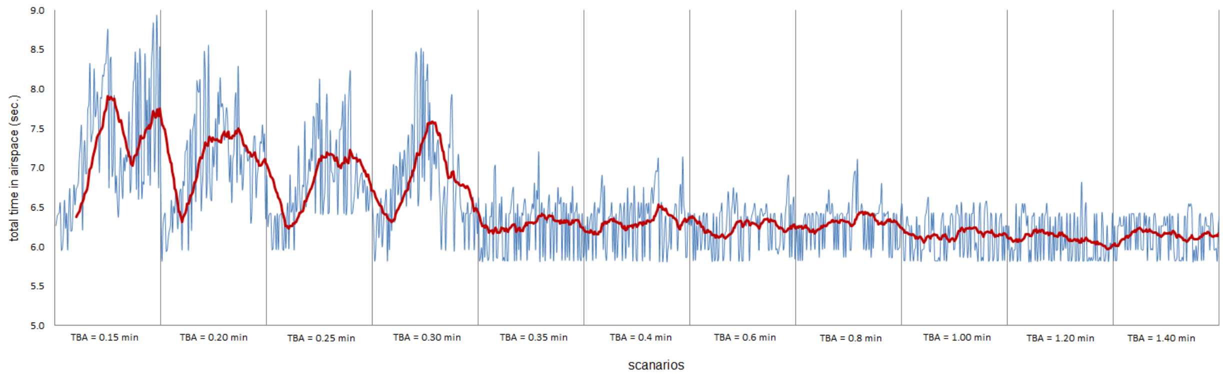



Figure 10: Impact of TBA on flight time in airspace

### 4.4 Hybrid solution methodology

In spite of the good computational performance of the decentralized solution strategy, determining the flight plan for a single aircraft at a time leads to significant deviation from the global optimization solution, particularly when the airspace is heavily congested. Iterative solution methodology assigns the best available route to a single aircraft at a time. Since future aircrafts are not considered, they may be significantly penalized. In order to improve the results of the decentralized solution strategy, we introduce a hybrid strategy in which the en-route flight planning model is solved for the next $N'$ arriving or departing flights taking into account the current traffic conditions of the $N$ earlier flights. In our case study, we introduced 10 new aircrafts in the airspace at a time. Hybrid solution strategy provided superior results than decentralized solution strategy and much faster computation times than the centralized solution strategy. As shown in Fig. 11, for TBA = 0.3 and SD = 0.31, while the decentralized model fails to reach a steady state condition, the hybrid model provides flight plans with significantly less total costs with a smaller variation. In Fig. 12, the performance of both solution strategies on less congested traffic conditions (TBA = 0.75 and SD = 0.3) were compared. Therein, in spite of reaching a steady state condition, the decentralized solution strategy provides larger variances on the performance of individual aircrafts. The variance on the total flight cost is significantly less for the hybrid solution strategy.

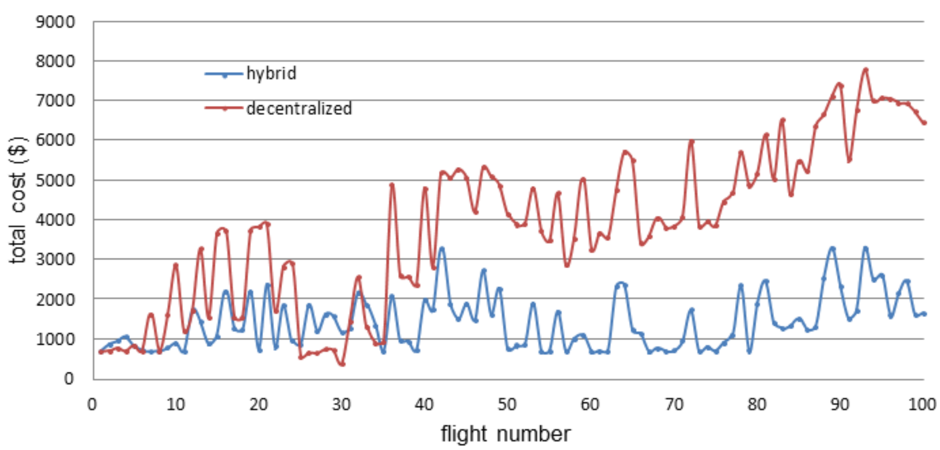



Figure 11: Performance comparison-hybrid vs decentralized for 100 flights: TBA = 0.3 and SD = 0.31

### 4.5 Mid-air-collision avoidance and other features

Safety is the most important consideration in air transportation. We now demonstrate the capability of the proposed mathematical model for ensuring the mid-air-collision avoidance for all aircrafts at all times

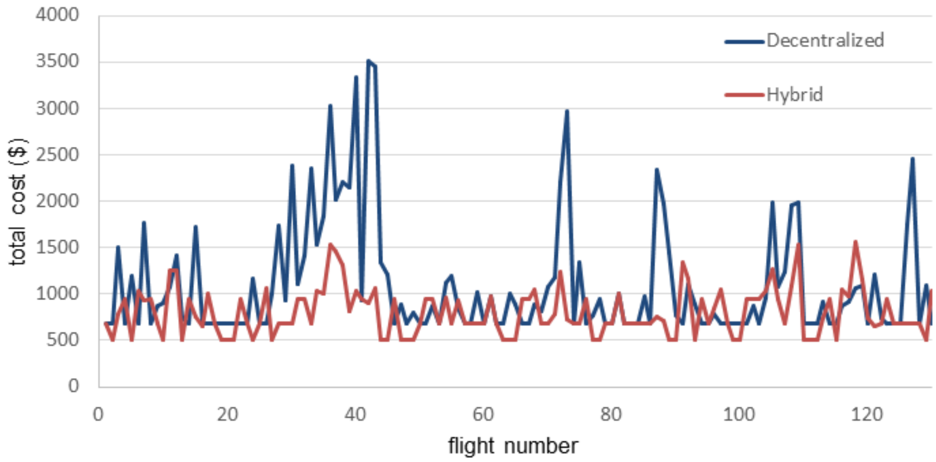



Figure 12: Performance comparison-hybrid vs decentralized for 100 flights: TBA = 0.75 and SD = 0.3

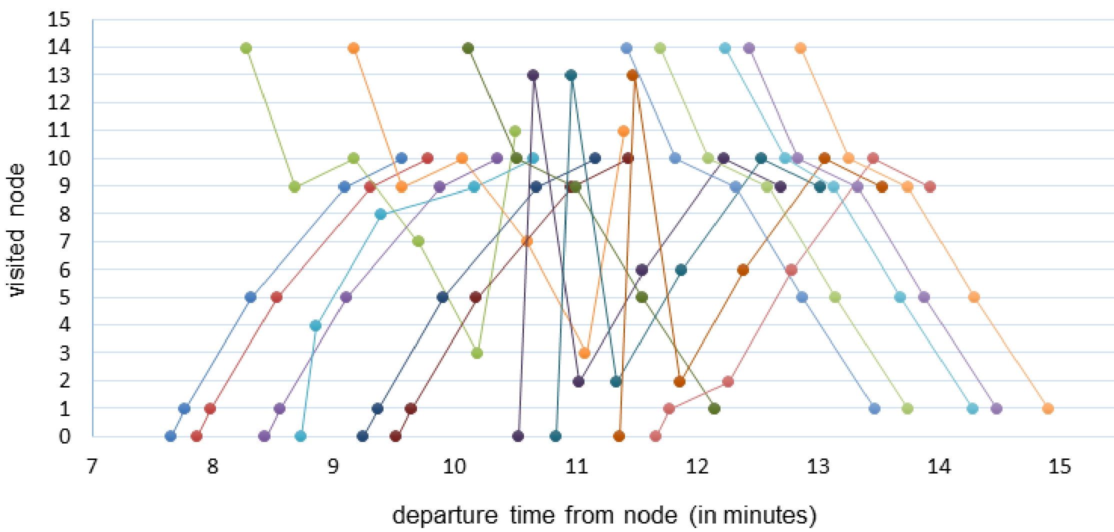



Figure 13: Illustration of aircraft locations in a time-space diagram

through relative locations of aircrafts on a time-space diagram. As shown in the Fig. 13, aircrafts keep the required minimum separation distances between each other at all times.

Next, we studied to measure the impact of SD and TBA on flight plans. In Fig. 14, the impact of separation distance on flight route is demonstrated. When SD is increased from 0.2 min. to 0.25 min., flight plan for some of the aircrafts are changed. Due to increasing demand on air traveling, lower TBA is unavoidable. In order to improve the congestion around airports, either the infrastructures need to be improved or SD should be reduced. Reducing SD would increase the capacity of an airspace. In general ATC and pilots are required to keep a 5 nautical miles SD between two aircrafts that are flying at the same altitude. Speijker [35] studied the possibility of reducing current SD levels in order to improve the congestion in airports. Their findings suggest that SDs can be reduced without putting aircrafts in danger. The proposed flight planing model can further help decision makers to determine the optimal SD levels without jeopardizing the safety.

Finally, in Fig. 15, we show how the mathematical model assigns speeds to individual aircrafts (a sample for 5 aircrafts is illustrated in the figure) in order to minimize the fuel consumption and delay/earliness costs.

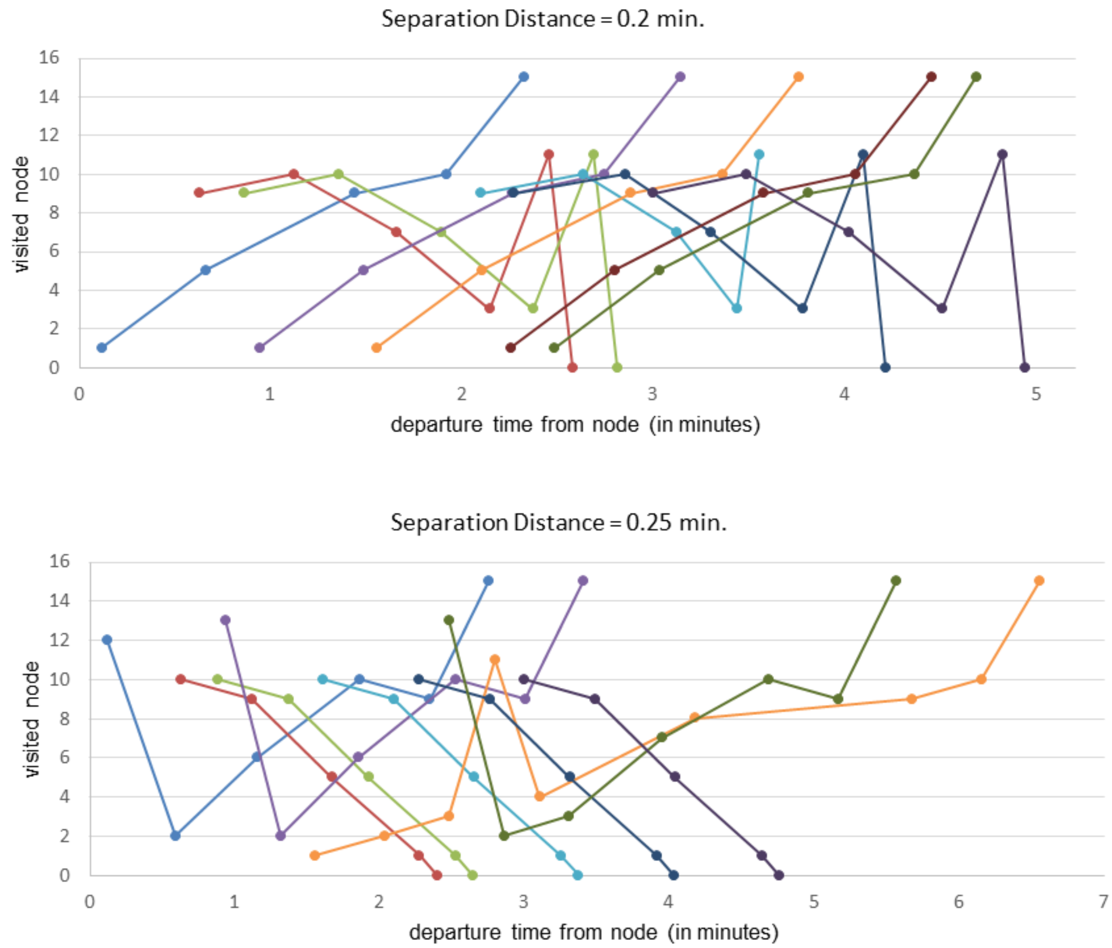



Figure 14: Impact of SD on flight plans: a) Flight plan for SD = 0.2 min; b) Flight plan for SD = 0.25 min

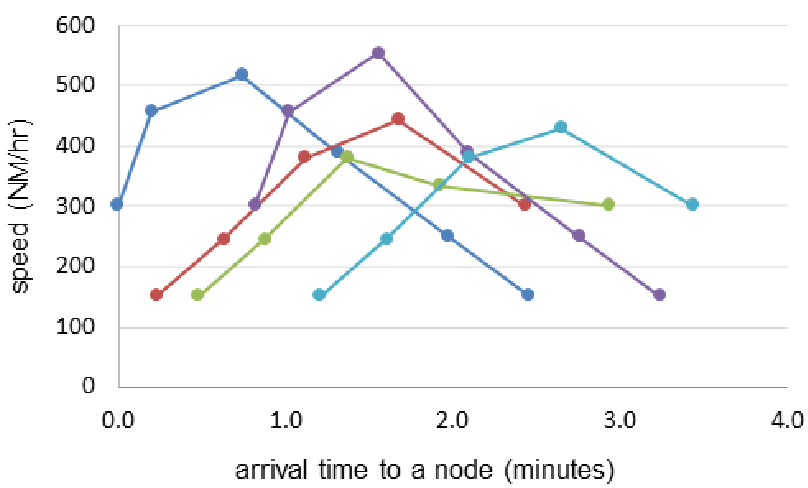



Figure 15: Speed changes during flight

### 4.6 Comparison of solution methodologies

Scheduling problems are known to be NP-Hard. As the problem size increases (number of aircrafts and size of the network – 3D mesh in our case), computation times for the centralized solution strategy increases significantly. However, adding a solution strategy as in [22] with a cutting-plane strategy could help to significantly reduce the computational times, and consequently could allow the solution of much larger data instances. In order to reduce the computation time, in this paper, we utilized a decentralized solution strategy (capable of converging to a solution in less than 20 seconds in most cases). The decentralized approach complies very well with the free flight philosophy [21], which is considered to be the future of the ATFM. However, our case study clearly demonstrated that optimizing the airspace for a single aircraft at a time leads to unfair conditions for the future arriving/departing aircrafts. A hybrid solution strategy that overcomes the computational complexity of centralized approach and provides a more efficient airspace utilization in comparison to the decentralized model can be used to assist decision makers to manage the air transportation system effectively.

## 5 Conclusions and future works

An optimization model based on continuous time bases is developed to determine en-route flight plans for aircrafts entering an airspace with a minimum cost. A 3-D mesh network is used to represent the airspace. The cost in this work is introduced as a function of speed dependent fuel consumption rate and flight delays and earliness. Indeed, the model assigns best timing, routes and speed maneuvers to flights in their landing or take-off phases. Additionally, the mid-air collision avoidance is explicitly modelled within a three-level conflict and safety constraints.

Non-time indexed modelling enables the determination of exact arrival and departure times at each node; consequently safety is guaranteed at all times. Moreover, it allows a denser usage of the airspace with respect to the safety criterion which helps to overcome the congestion. The solution provided is practical enough whether in a context where air traffic controllers want to handle the entire traffic stream or in a new generation of traffic controlling systems that allow pilots to be in charge. In short, the following advantages are achieved: *(i)* collision avoidance is guaranteed; *(ii)* airspace is more effectively used by allowing large number of aircrafts in an airport region; *(iii)* the fuel consumption cost is formulated as a function of speed; *(iv)* finally, computational time of the model is improved. One of the immediate areas in which the proposed model can be applied to is the airport operations in which both taxing and runway usage can be explicitly modeled in order to increase the utilization of airports.

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