Optimization of a Class of Decentralized Hedging Policies in a Stochastic Two-Machine Flow Shop

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Abstract

This paper deals with the optimal production control problem in a stochastic two-machine flow shop. Our aim is to develop approximation techniques for deriving close to optimal production policies within a suboptimal class of *decentralized hedging policies*, whereby each machine strives to maintain a fixed critical level of parts to be determined. For that purpose, using two approximate decoupling principles, a *machines decoupling approximation* and a so-called *demand averaging principle*, both upstream and downstream production costs required to maintain a given *availability coefficient of the work-in-process* (as well as the associated critical levels of parts) are derived analytically. The quality of the demand averaging principle as an approximation is theoretically assessed, and both approximations are numerically tested. Subsequently, one searches over all admissible availability coefficients for one that minimizes the overall production cost. The proposed optimization scheme appears to be very competitive for real-time use since it runs almost instantaneously when implemented using Matlab on a personal computer. In addition, it provides good performance when tested against Monte Carlo based optimization over the same class of control laws. Results emerging from a wide set of sample systems are presented, and the decoupling principles initiated appear to be generalizable for an N-machine flow shop, where $N>2$.

Résumé

Cette recherche porte sur le problème de commande optimale du flux de production dans un atelier de fabrication, constitué de deux machines non fiables en tandem. Notre objectif est de développer des techniques approximatives permettant de caractériser les règles optimales, à l'intérieur de la classe sous-optimale des politiques de production à seuils critiques, associant à chaque machine un seuil critique fixe à déterminer. Au moyen de deux principes de découplage approximatifs dont celui de *découplage approximatif des machines* et celui de *moyennage de la demande*, les coûts de production en amont et en aval requis pour le maintien d'un coefficient de disponibilité du processus des encours (ou des niveaux de pièces associés) sont calculés analytiquement. Ensuite, on cherche dans l'espace admissible des coefficients de disponibilité, le coefficient qui minimise le coût global de production. Le schéma d'optimisation proposé s'avère très compétitif pour une utilisation en temps réel, car la version Matlab implantée dans un ordinateur personnel opère presque instantanément. Par ailleurs, il offre de bonnes performances comparé aux résultats de tests effectués sur un simulateur de Monte Carlo opérant sous la même classe de lois de commandes. Les résultats obtenus sur un grand échantillon de systèmes sont présentés, et les principes approximatifs sur lesquels notre démarche est fondée, s'avèrent généralisables au cas de N machines non fiables organisées en tandem, où $N>2$.

1 Introduction

Since the pioneering works of Olsder and Suri [1] and Kimemia and Gershwin [2], the problem of optimally controlling the production rate in failure prone manufacturing systems has received increasing attention among both practitioners and researchers. Surveys regarding the general topic have been reported by many authors including Panwalkar and Iskander [3], Graves [4], Dupont-Gathermand [5], Gershwin, Hildebrant, Suri and Mitter [6], Buzacott and Shanthikumar [7], Maison and Gershwin [8], Rodammer and White [9] and Reisman, Kumar, Motwani and Cheng [10]. See also Conway, Maxwell, McClain and Thomas [11], for an overview regarding the role of work-in-process and inventory in tandem manufacturing systems. Generally speaking, the optimal production control problem could become quite complex, even when a single machine is involved.

For an unreliable manufacturing system with a single machine, one part type, a constant demand and subjected to Markovian breakdowns and repairs, the optimal production rule for a piecewise linear cost function is available analytically and belongs to the class of hedging policies (see Akella and Kumar [12], Bielecki and Kumar [13], Hu, Vakili and Yu [14]). The hedging production policy concept was initiated earlier in a wider context by Kimemia and Gershwin [2]. This was followed by improvements presented by Akella, Choong and Gershwin [15], Gershwin, Akella and Choong [16], and Gershwin [17]. The idea behind other subsequent works was to relax a few assumptions while maintaining the use of hedging production policies as a suboptimal or asymptotically optimal class of policies in the resulting manufacturing system. For example, Hu and Xian [18, 19], Krichagina, Lou, Sethi and Taksar [20] and Glasserman [21] have given some insights into the case of non Markovian machines. Algoet [22], Sharifnia [23], Malhamé and Boukas [24, 25] and Malhamé [26, 27] have extended in different ways the basic characterization of the hedging production policy, in multiple-mode manufacturing systems with Markovian machines. Boukas and Haurie [28], Boukas [29], Boukas and Yang [30] and Kenné, Gharbi and Boukas [31] analyzed the case of machines where operating modes are age-dependent. Furthermore, the case of a single machine producing multiple part-types has been investigated by El-Férik, Malhamé and Boukas [32], and Perkins and Srikant [33, 34], while Lehoczky, Sethi, Soner and Taksar [35], Caramanis, Liberopoulos [36], Liberopoulos and Camaranis [37] have been interested in multiple part-types manufacturing systems with parallel machines. On the other hand, Postner, Berg and Zhao [38], Berg, Postner and Zhao [39], Srinivassan and Lee [40] have taken into consideration the stochastic nature of the demand process, while Sethi and Zhang [41] have developed a hierarchical, singular perturbation theory based methodology, for production control in manufacturing systems.

For manufacturing systems involving several unreliable machines connected in a tandem configuration, the exact computation of optimal production policies based on the dynamic programming equations is a formidable task either analytically or numerically. Because of the complexity involved, a set of approximate suboptimal production rules have already been proposed, including *numerically optimized* controls (NOC), *asymptotic hierarchical* controls (AHC), *two-boundary* controls (TBC), *pure surplus* control (PSC) and *Work-in-process* control (WIPC). The NOC combine a suitable discretization technique (see Ryzin, Lou and Gershwin [42], Veatch and Wein [43]), with the policy iteration approach, but unfortunately the whole numerical procedure requires a massive computational effort. AHC has been developed by Fong and Zhou [44], Sethi and Zhang [41], Presman, Sethi and Zhang [45], and Samaratunga, Sethi and Zhou [46]. The idea behind AHC is to construct numerically approximate optimal production rules, based on the production decisions for an approximately defined limit system obtained based on a separation of time scales assumption. However, we conjecture that the adaptation of AHC for solving an average cost optimization problem might be computationally hard because of the discounted rate problem (see Colonius [47] and Wirth [48]). The TBC consisting of three parameters, has been initiated by Lou and Kager [49] for a VLSI wafer fabrication, and has been investigated later by Ryzin, Lou and Gershwin [42]. However, its numerical implementation remains a very demanding task. The PSC of a tandem two-machine system is in fact a simplified version of the TBC with only two parameters for a tandem two-machine system. Yan, Yin, and Lou [50] have proposed a stochastic optimization method for the numerical computation of these parameters in the case of a two-machine flow shop. A heuristic-type method for the *surplus* (cumulative difference between production and final product demand) control has been reported by Bai and Gershwin [51, 52] for single part and multiple parts manufacturing flow shops. Finally, the WIPC involves a given set of thresholds for each machine. For example, Yan, Yin and Lou [50] have considered one hedging level of WIP per machine in the case of a tandem two-machine system. They have numerically computed the optimal thresholds using an infinitesimal perturbation technique. Alternatively, Hu [53] has proposed a decomposition approach (based on the so-called starvation cost) for the determination of hedging levels associated with a tandem two-machine system. Hu's approach differs from the one investigated herein both in the structure of the control policy considered which is not decentralized, and in the decomposition approach which tends to ignore starvation phenomena as they affect the demand backlogs at the downstream machine. Rather, the downstream machine is considered as having a perfectly reliable supply, and instead a starvation cost is associated with a zero buffer in the upstream machine. In fact, the optimal hedging level of the downstream machine is calculated independently from any knowledge of upstream machine storage costs. The starvation cost for the upstream machine is computed using an analytical cost expression due to Hu which is applied although valid neither under variable demand, nor under a random starvation cost. Clever as they may be, the quality of the approximations remains untested. Furthermore, the modeling approach seems to move away from the physics of the processes involved. Finally, it appears that the complexity structure of the control laws, and the associated computations could rapidly explode in the case of N machine systems, $N \geq 2$. Thus, despite this rich literature, a thoroughly satisfactory methodology for the flow control of stochastic tandem machines appear to be missing.

In the aforementioned literature, one can observe that except for Yan, Yin and Lou [50], little attention has been devoted to direct extensions of hedging production policies for single machine systems. However, one could intuitively argue for preserving, even in the more complicated cases, the simplicity and decentralized behavior of hedging production policies for single machine facilities. The proposed production class of production policies will be called *decentralized hedging production (DHP*) policies. It consists in associating to each machine throughout the transfer line, a set of parts critical levels (control law design parameters) to be optimized and maintained whenever possible (one for each feasible mode of that machine). When adequately optimized, each critical level could be thought of as a form of optimal insurance policy against production resource failures. Subsequently, the choice of production level at any given workstation, would depend only on local information regarding the operating state of the workstation and the levels of immediate upstream and downstream buffers. We submit that DHP policies could be competitive with good performance for real time use in tandem manufacturing plants. In fact, they mimic in a fluid model context the kanban control structure [62, 63].

In this paper, DHP policies for stochastic flow shops are investigated. We propose a decomposition methodology over the class of DHP policies which reduces the complex optimization problem for a tandem two-machine system to a collection of single optimization subproblems with known analytical solutions, by relying on two (approximate) decoupling principles: a so-called *machines decoupling approximation* which helps modeling the upstream machine as viewed from downstream (stochastic supply), and a so-*called demand averaging principle* which helps modeling the downstream machine as viewed from upstream (stochastic demand). The nature of approximations and conditions for their validity are discussed. Quantitative arguments are presented to help explain the quality of the demand averaging principle approximation, and numerical evidence is presented for both approximations. Subsequently, fictitious isolated machine equivalents are obtained using first passage-time analysis, and whenever necessary Markovianization via a Padé approximants technique. The remainder of the paper is organized as follows. Section 2 is devoted to the description of the production system of interest, and to the formulation of the optimal production control problem. In Section 3, our optimization approach is developed within the suboptimal class of *decentralized hedging production policies*. Diverse sample systems are investigated in Section 4 so as to challenge the proposed optimization scheme. Finally, conclusions are drawn in Section 5.

2 System description and problem formulation

In this section, the two-machine flow shop considered is described and the optimal flow control problem is formulated for that system within the suboptimal class of DHP policies.

2.1 System description

We consider throughout the paper a single product manufacturing system consisting of two unreliable machines organized in tandem (see Figure 1). The raw material processed by the upstream machine M_1 is transferred into an internal buffer, and finished goods produced by the downstream machine M_2 are stored in a warehouse buffer. Each machine and its associated buffer represent an aggregated cell or workstation. Such an aggregated structure could be associated with a numerical control machine, a conveyor, a robot, a communication network, and a programmable logic controller or a computer. Both upstream machine M_1 and downstream machine M_2 are subject to random breakdown and repair over an infinite time horizon. It is assumed that the upstream machine M_1 is never starved nor blocked, and the downstream machine M_2 is never blocked. Such assumptions hold well in practice for the class of manufacturing systems with a reliable source of raw material and with large buffers. The production capacity of each machine is zero when the machine is down or eventually starved, and is a constant otherwise. The breakdown and repair statistics of machines captured by the technical staff, are assumed to be Markovian. The demand rate emerging from marketing data is a constant d, and backlog (or unsatisfied demand for finished goods) is allowed although it is associated with costs. In addition, neither reprocessing nor discarding of defective products are considered. Following Olsder and suri [1], Kimemia and Gershwin [2], Akella and Kumar [12] and Bielecki and Kumar [13], we use further a fluid hybrid state model of the production flow.

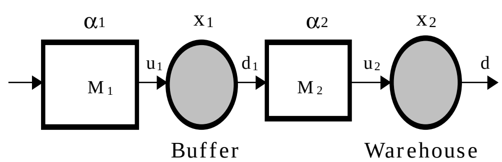

Figure 1: Unreliable tandem two-machine system

2.2 Notations and problem formulation

Here, we first develop notation for the relevant dynamic quantities in the two-machine system. Subsequently, given a piecewise linear cost function, an infinite horizon average cost optimization problem over the defined class of DHP policies is formulated.

The notation used further is as follows: i=1,2 (index of machine M_i), X_i (buffer level - continuous state variable), u_i (production rate - decision or control variable), k_i (maximum production rate), α_i(discrete state -1 if M_i is up, and 0 otherwise), r_i and p_i (repair and failure rates respectively), π (arbitrary production policy), π_h^* (an optimal production policy, if it exists, within the class of DHP policies), J_π (overall cost associated with production policy π), Z_i(critical hedging level of parts), and the production cost per unit time $g(X)=c_1X_1+c_2^+max(0,X_2)+c_2^-max(0,-X_2)$ being piecewise linear, with no explicit starvation cost (x=0) as in [53] since unlike for the analysis in [53], we conjecture that starvation effects (X_1=0, α_2=0) will ultimately materialize as a decrease in the surplus x_2 of finished products relative to where it would have stood at had there been no starvation. This should eventually lead to an increase in the resulting average backlog costs.

With the help of these notations, the optimal production control problem can now be stated. Find an admissible DHP policy (if it exists) which achieves the long term average cost J^* defined below,

$$J^* = \inf_{U(X,\ \alpha)\ \subset\ \pi_h^{ad}} \left[\lim_{T \to \infty} \left\{ E_\alpha \left(\frac{1}{T} \int_{t_0}^{T} g(X)\, dt \right) / X_0, \alpha_0 \right\} \right] \tag{1}$$

where π_h^{ad} is the set of admissible DHP controls, under the following state equation and associated constraints:

$$\begin{cases} \dot{X}_1 = u_1 - d \\ \dot{X}_2 = u_2 - d \end{cases} \qquad \begin{cases} 0 \le u_1 \le \alpha_1 k_1 \\ 0 \le u_2 \le \alpha_2 k_2 \end{cases} \qquad \begin{cases} 0 \le X_1 < \infty \\ -\infty < X_2 < \infty \end{cases} \tag{2}$$

Throughout the analysis analysis, it is assumed that $k_1 > k_2$. Note that the class of DHP policies is parameterized in this case by arbitrary non negative hedging levels, Z1 and Z_2 and the control structure is more precisely defined by the following set of equations:

$$\begin{cases} u_1(x,\alpha) = \begin{cases} \mathrm{k}_1\ \alpha_1(\mathrm{t}) & \text{if } \quad 0 \le x_1(t) < Z_1 \\ u_2(t)\ \ \alpha_1(\mathrm{t}) & \text{if } \quad \mathrm{x}_1(t) = Z_1 \\ 0 & \text{otherwise} \end{cases} \\ u_2(x,\alpha) = \begin{cases} \mathrm{k}_2\ \alpha_2(\mathrm{t}) & \text{if } \begin{cases} x_2(t) < Z_2 \quad \text{and} \\ \mathrm{x}_1(\mathrm{t}) > 0 \ or \ \alpha_1(t) = 1 \end{cases} \\ d\ \ \alpha_2(\mathrm{t}) & \text{if } \begin{cases} x_2(t) = Z_2 \quad \text{and} \\ \mathrm{x}_1(\mathrm{t}) > 0 \ or \ \alpha_1(t) = 1 \end{cases} \\ 0 & \text{otherwise} \end{cases} \end{cases} \tag{3}$$

Furthermore, note from (3) that: a) each control law u_i (i=1, 2) *strives to maintain as long as possible* its marginal hedging level Z_i, b) only *immediate upstream and downstream information* is needed to achieve that goal (decentralized control structure). Finally, the admissible range of hedging level Z_1 is defined to be the one which guarantees an ergodic system (long term feasibility of the constant demand). Thus, in the very spirit of Bielecki and Kumar [13], we work with a *parameter optimization* problem rather than an optimal control problem. Indeed, our aim is to find within the above defined suboptimal class of DHP policies, the critical hedging points Z_1^* and Z_2^* which minimize the total average production cost.

DHP policies for a two-machine system have been suggested and studied by Yan, Yin and Lou [50] as a good class for suboptimal control. However, unlike the perturbation analysis technique proposed in [50] which might require intensive computations for the optimal parameters search, the optimization methodology initiated in the next section attempts to build upon a number of analytical results relating to single machine systems, via appropriate complexity reducing decoupling principles.

3 Approximate analysis of DHP policies

In this section, we present the foundations of an approximate analysis of DHP policies. The proposed analysis crucially depends on the application of a *machines decoupling approximation step*, followed by the application of an *averaging principle* hereon referred to as the *demand averaging principle (DAP)*. We now discuss these steps in further details.

3.1 Machines decoupling approximation step (MDA)

A two-machine flow shop shown in Figure 1 can be decomposed intuitively into two isolated single machines as in Figure 2. While individual machines can influence the *statistical parameters* associated with either supply reliability for M_2 or demand stochasticity of machine M_1, once this influence is accounted for, the two dynamics under control are viewed approximately as *independent stochastic processes*. It is clear that, in reality, there is a measure of correlation between the supply reliability on-off process and the state of M_2(for example x_1 cannot go to zero unless M_2 is on). We conjecture that if the mean first return time to buffer emptiness say $\mu_1(Z_1)$ is much larger than the mean on-off cycle, $1/r_2+1/p_2$, of machine M_2, then the $I(x_1>0)$ binary (supply availability) process where $I(.)$ denotes the characteristic function of a set, "sees" the average behavior of M_2 in the "on" part of its cycle (and thus becomes virtually independent of the instantaneous fluctuations of M_2), while in the "off" part, due to the $k_1>k_2$ assumption, starvation effects depend strictly on the behavior of M_1. Furthermore, note that both feasibility of the demand and optimization requirements dictate that Z_1 (and thus $\mu_1(Z_1)$) remain sufficiently high. Once the machines are decoupled as two stochastically independent subsystems, the link between the two being established through the parameters associated with the statistics of the corresponding processes, the analysis of the isolated upstream subsystem could be greatly facilitated following an application of the *demand averaging principle* which we now discuss.

3.2 Demand averaging principle (DAP)

Since the downstream machine M_2 (source of stochastic demand $d_1(t)$; see Fig. 1) operates under a hedging control policy, the stochastic demand process $\{d_1(t)\}$ for machine M_1 takes values in the set $\{k_2, d, 0\}$. Although the characterization of the $\{d_1(t)\}$ stochastic process as viewed from M_1 may in general be a complex task, it so happens (under the ergodicity assumption) that the upstream production cost

$$J_{1\pi} = \lim_{t\to\infty} \frac{1}{t}\int_0^t c_1 X_1(\tau)d\tau = c_1 \lim_{t\to\infty} \frac{1}{t}\int_0^t c_1 X_1(\tau)d\tau \tag{4}$$

is influenced mostly by $\lim_{t\to\infty} \frac{1}{t}\int_0^t d_1(\tau)d\tau = d$ (since a necessary condition for ergodicity is the stability of the $x_1(t)$ and $x_2(t)$ processes, and thus the average of $d_1(t)$ must be the same as the constant rate of demand for finished parts), as though machine M_1 was subjected to a constant demand rate d of finished parts. This is the so-called *demand averaging principle* (DAP). Strictly speaking, this principle is an approximation. However it is a very high quality approximation as sample numerical evidence reported in Appendix 3 shows. Also, in Appendix 3, we develop qualitative arguments that justify the use of this approximation.

The above principle is in effect a *virtual transparency principle* indicating that everything happens as though machine M_2 was absent and M_1 was responding directly to customer demand. Summarizing in this context, the DAP states that as viewed from M_1, the cost of maintaining a given critical level of work-in-process is the same that would be incurred by a single machine with completely reliable supply, the failure characteristics of machine M_1, and a *constant rate of demand* for finished parts d.

3.3 Application of the DAP to the analysis of the upstream subsystem

Three useful results deriving from an application of the DAP idea are investigated in this section. The first result is that for a given Z_1, the upstream cost is available analytically. The second result is that the DAP idea permits the computation of moments of the first return random variable associated with excursions of WIP away from and back to zero. This in turn permits the construction of a multi-mode approximate Markovian model of the downstream Machine.

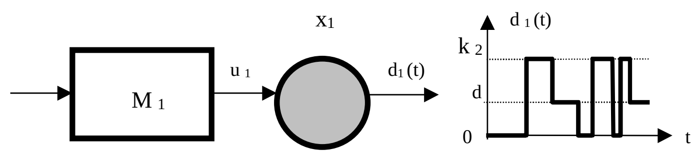

Fig. 2-1: Machine M_1 with a stochastic demand $d_1(t)$

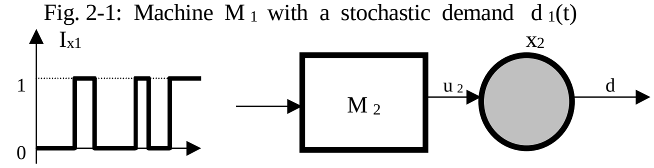

Fig. 2-2: Machine M_2 with an unreliable supply process I_{x1}

Figure 2: Decomposition scheme of a two-machine flow shop

3.3.1 Exact computation of the upstream cost

Using the DAP idea, it is possible to view the upstream machine as a single *HU type of machine* (two-state, no backlog permitted, constant demand). The analytical expression of the upstream cost is then given by (see Hu [55]):

$$J_1(a) = \frac{c_1 k_1}{(p_1 + r_1)(1-\rho)} - \frac{c_1 k_1 (1 - a(Z_1))}{p_1(1-\rho)} - \frac{\rho}{r_1(1-\rho)}\left(c_1 - c_1 \frac{(r_1 + p_1)(1-a(Z_1))}{p_1(1-\rho)} \right) \text{Log}\left(\frac{1}{\rho} - \frac{p_1(1-\rho)}{\rho(r_1 + p_1)(1 - a(Z_1))} \right) \tag{5}$$

where

$$\rho = \frac{r_1(k_1 - d)}{p_1 d} > 1, \quad \lambda = -(1-\rho)\frac{p_1}{k_1 - d} > 0, \quad a(Z_1) = 1 - \frac{p_1}{p_1 + r_1}\,\frac{1-\rho}{1-\rho e^{\lambda Z_1}} \tag{6}$$

Thus, the long term time fraction for which the buffer is empty will be given by:

$$P_{Z_0} = \frac{p_1}{(p_1 + r_1)}\,\frac{(1-\rho)}{(1-\rho e^{\lambda Z_1})} = 1 - a(Z_1) \tag{7}$$

and the idea behind the choice of $a(Z_1)$, the so-called availability coefficient, as a new design parameter will become apparent later. Note that at this stage, we are not interested in the marginal minimization of $J_1(Z_1)$ as was the case in the decomposition approach suggested by Hu [53] for a tandem two-machine. Again, unlike in this analysis, we do not associate a direct cost with the starvation of machine M_2 given that we consider that any cost accruing from starvation effects will show up at machine M_2 as an increase in average backlog. We view the role of Z_1 only as a means to bolster the effective reliability of machine M_2. Let us now examine the second useful result deriving from an application of DAP to the upstream machine.

3.4 Generation of the moments of the first return-time to buffer emptiness

The first, second, and higher order moments (respectively denoted μ_1, μ_2, …) of the first return-time to buffer emptiness $(X_1(t)=0)$ could be generated by adapting Lemma 1 in El-Férik and Malhamé [54] to the current situation where backlog is not allowed $(X_1 \geq 0)$. Details regarding the generation of the μ_i's $(i=1, 2, \dots)$ are presented in Appendix 2.

Exact expression of the first moments

The analytical expressions of the first two moments obtained using the algorithm in Appendix 1 (with $v_1=k_1-d$ and $v_2=-d$) are as follows:

$$\mu_1(Z_1) = \left(\frac{p_1 + r_1}{p_1 r_1}\right)\frac{\left(1 - \rho\, e^{\lambda Z_1}\right)}{(1 - \rho)} - \frac{1}{r_1} = \frac{-\,p_1(v_1 - v_2) + (r_1 + p_1)v_1 e^{\lambda Z_1}}{p_1(p_1 v_2 + r_1 v_1)} \tag{8}$$

$$\mu_2(Z_1) = \left(2\,\frac{p_1 v_2 (v_1 - v_2)^2}{(v_2 p_1 + r_1 v_1)^3} + 2\,\frac{r_1 v_1^3 (r_1 + p_1)^2}{p_1^2 (v_2 p_1 + r_1 v_1)3}\, e^{2\lambda Z_1} + \frac{\left(\begin{array}{l} 4 v_1 v_2 (v_2^2 p_1^2 - v_1^2 r_1^2)\, e^{-\lambda Z_1} \\ +\, 2 v_1 v_2 (3 r_1 v_2 - v_1 p_1)(v_2 p_1 + r_1 v_1)\, e^{\lambda Z_1} \\ +\, 2(r_1 + p_1)(v_2^2 p_1^2 - v_1^2 r_1^2)(v_1 - v_2)\, Z_1(a)\, e^{\lambda Z_1} \end{array}\right)}{v_2 p_1 (v_2 p_1 + r_1 v_1)^3} \right)$$

The analytical expressions of higher order moments μ_i $(i \geq 3)$ have also been computed in all generality, but are too complex to report here. The above expressions were obtained using MAPLE V (see Darren-Redferm, [56]).

Availability coefficient of the work-in-process

Using the key renewal theorem (see Ross [57]), the availability coefficient (a) of X_1 can be obtained as follows:

$$a = \frac{\mu_1(Z_1)}{\mu_1(Z_1) + \frac{1}{r_1}}, \quad \text{so that} \quad \mu_1(Z_1) = \frac{a}{r_1\,(1 - a)} \tag{9}$$

This a is denoted $a(Z_1)$ to emphasize its dependence on Z_1 in (7). Clearly, a is a normalized physical variable $(0 \le a < 1)$ which could independently replace Z_1 throughout this paper, since recalling (8) and (9), they are connected by:

$$\begin{cases} Z_1 = \dfrac{1}{\lambda} \operatorname{Log}\left(\dfrac{1}{\rho} - \dfrac{p_1(1-\rho)}{\rho\,(r_1+p_1)} \dfrac{1}{(1-a)} \right) \\ a = 1 - \dfrac{p_1}{(p_1+r_1)} \dfrac{(1-\rho)}{(1-\rho\, e^{\lambda Z_1})} \end{cases} \tag{10}$$

Since $Z_1 \ge 0$ and $\lambda > 0$ to be able to meet demand, (6), (10) imply that:

$$\frac{1}{1+\dfrac{p_1}{r_1}} \le a < 1 \tag{11}$$

From (11), we conclude that $0.5 < a < 1$ since in practice r_1 will be in general greater than p_1. Thus, assuming $r_2 > p_2$, the starvation coefficient $a_s = (1-a)\, r_2/(r_2+p_2)$ of machine M_2 is such that $0 < a_s < 1/4$. This conclusion is consistent with the very low values of starvation coefficients reported by Bai and Gershwin [51] based on heuristics. Indeed, a maximum starvation coefficient $a_{smax} = 0.167$ arises from 24 sample two-machine systems investigated in [51].

Coefficient of variation of the return time to buffer emptiness

The *coefficient of variation* of a random variable is defined as the ratio of its standard deviation to its mean (see Griffiths, [58]). Thus, the coefficient of variation of the *first return- time* to the upstream buffer emptiness is given by:

$$c^2(a) = \frac{\mu_2(Z_1(a) - (\mu_1(Z_1(a)))^2}{(\mu_1(Z_1(a)))^2} \tag{12}$$

Recall that an arbitrary distribution can be viewed as quasi-exponential if its coefficient of variation is close to one. This remark will help us later in Section 4 for the validation of a forthcoming approximate Markovianized representation of machine M_2.

3.5 Approximate Markovian representation of machine M_2

3.5.1 Foundations of the approximate Markovianization

When viewed from the downstream machine, the *upstream supply variable* $I_{X1}(t)$ behaves as a binary random process ($X_1 > 0$ and $X_1 = 0$ respectively). Under the decoupling approximation it evolves *independently* from the downstream machine. At this stage, an *approximate Markovianization* of the process $I_{X1}(t)$ will be helpful. A p^{th} order Markovianized representation of the first return-time to buffer emptiness, could be computed from a sufficient number of moments μ_i*'s* (i=1, 2, …) using the Padé approximants approach presented in El-Férik and

Malhamé [54]). In other to keep the level of complexity under control, we rely on the simplest Markovian approximation of the first return random variable. It depends strictly on the first moment (expected return time μ_1). Thus, the supply process I_{X1} could be approximated by means of a two-state Markov chain with mean time to OFF equal to μ_1 and mean time to ON equal to $1/r_1$ (recall that $k_1>k_2$ by assumption). Notice also from (9) that $\mu_1=a/r_1(1-a)$, in which case the transition rate of the process I_{X1} from ON to OFF could be written down as follows:

$$p_0(a) = \frac{1}{\mu_1(Z_1(a))} = r_1\frac{(1-a))}{a} \tag{13}$$

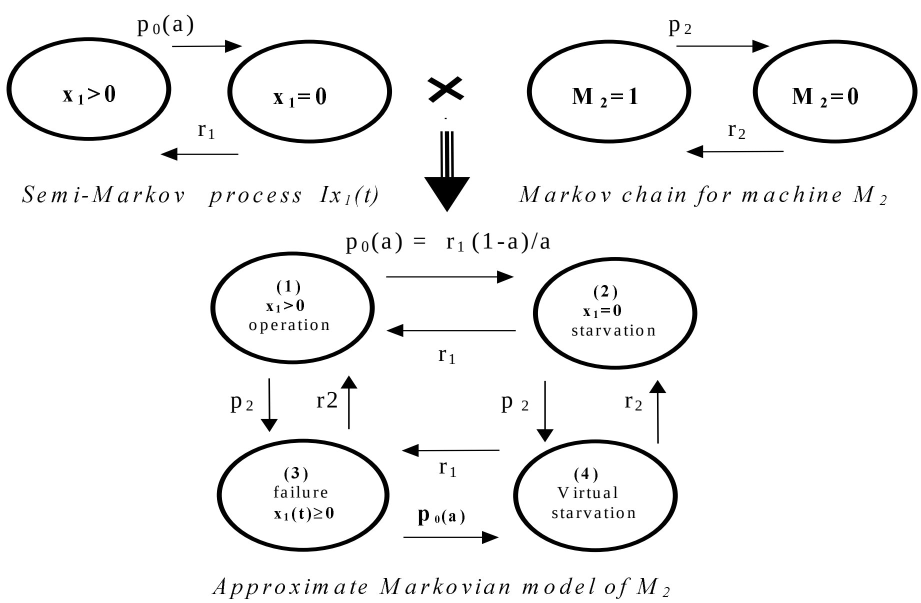

Figure 3: Aproximate four-state Markovian Model of M_2

Such a highly simplified representation of the I_{X1} process leads to good results as will become clear in Section 4 where numerical evidence over a number of test cases suggests that the associated coefficient of variation associated with the ON time of I_{X1} variable remains close to 1. Given the approximate two-state Markov supply variable I_{X1}, and the independence assumption relative to M_2 , the *approximate Markovian model for the downstream subsystem* will arise from the *Cartesian product* of both state spaces related respectively to: a) the approximate two-state Markov supply, b) the operational two-state Markov associated with machine M_2. As shown in Figure 3, the resulting approximate model machine M_2 is a continuous time, four-state Markov process. The states obtained are numbered **1**, **2**, **3** and **4**. The operating state **1** is feasible (demand can be met) and the remaining states **2**, **3** and **4** are deficit states. In addition, except in general for state **1**, sojourn times in all other states are originally exponential. With the approximate Markovian representation of M_2 thus obtained, the path is cleared for an optimization of hedging point Z_2 using standard single-part multi-mode machine theory [23].

3.5.2 Optimal dowstream cost and hedging point $Z_2^*(a)$

In the following, a condition on the parameters of the equivalent machine of Fig. 3 is first derived to ensure the ergodicity of the resulting production process. Subsequently, a Sharifnia type of calculation is applied to the multi-mode machine of Fig. 3 to compute for a given coefficient *a*, the optimal parameterized optimal hedging level $Z_2^*(a)$ and cost $J_2^*(a)$. We first present additional notations needed further: a^* (optimal availability coefficient *a* assuming it exists), $Z_1^*(a)$ (optimal hedging level associated with machine M_2), $J_1(a)$ (average cost of maintaining critical buffer level $Z_1(a)$), $J_2(a, Z_2)$ (average cost of maintaining critical inventory level Z_2 under availability coefficient *a*), $J^*_2(a)$ (downstream production cost for the optimal critical inventory level), $J^*=J_1(a^*)+J_2^*(a^*)$ (overall optimal cost).

Admissible bounds on the availability coefficient *a*

Given the production downstream production subsystem of Figure 3 with velocity matrix:

$$V_2=Diag(k_2-d, -d, -d, -d) \tag{14}$$

the intensity matrix of M_{2eq} being,

$$\Lambda_2(a) = \begin{pmatrix} -(p_2+p_0(a)) & p_0(a) & p_1 & 0 \\ r_1 & -(r_1+p_2) & 0 & p_2 \\ r_2 & 0 & -(r_2+p_0(a)) & p_0(a) \\ 0 & r_2 & r_1 & -(r_1+r_2) \end{pmatrix}, \tag{15}$$

a standard ergodicity criterion requires that the average capacity of the system must be greater than the demand (Malhamé [27]). Let the vector $\pi(a)=[\ \pi_1(a)\ \pi_2(a)\ \pi_3(a)\ \pi_4(a)]$ denote the unique stationary distribution of the irreductible Markov chain M_{2eq}. It is straightforward to show that:

$$\pi(a) = \left[\frac{a\ r_2}{r_2+p_2},\ \frac{(1-a)r_2}{r_2+p_2},\ \frac{a\ p_2}{r_2+p_2},\ \frac{(1-a)\ p_2}{r_2+p_2}\right] \tag{16}$$

Given that *one* is the only feasible mode, the ergodicity condition $\pi_1(a)\ k_2 > d$ is equivalent to $(d/k_2)(r_2+p_2)/r_2<a<1$. Together with (11), the global ergodicity constraint can be written as follows:

$$\max\left(\frac{r_1}{r_1+p_1}, \frac{(r_2+p_2)}{r_2}\ \frac{d}{k_2}\right) < a < 1 \tag{17}$$

Analysis of the downstream cost

An alternative and equivalent ergodicity criterion (see Malhamé [26, 27]) reveals that a matrix such as $A_2(a) = (V_2)^{-1}\Lambda_2^T(a)$ with dimension n=4 and m=1 feasible mode must have exactly n-m (=3 in this case) eigenvalues with positive real part , *one* zero eigenvalue γ_1=0, and exactly m-1 (=0 in this case) eigenvalues with negative real part. Under the assumption of this ergodicity criterion for V_2 *and* $\Lambda_2(a)$ given by (13) and (14) respectively, the hybrid probability density functions $f_i(a,x_2)$ i=1,…, 4, and the marginal probability density function $f(a,X_2)$ would take the form (Algoet, [22], Sharifnia [23], Malhamé [26, 27]):

$$\begin{cases} f_i(a,Z_2,X_2) = \sum_{j=2}^{4} C_j(a)\, w_j(a,i)\, e^{\gamma_j(a)(X_2 - Z_2(a))} \quad \text{for } i=1,\dots,4 \\ f(a,Z_2,X_2) = \sum_{j=2}^{4}\left(\left(C_j(a)\sum_{i=1}^{4} w_j(a,i)\right) e^{\gamma_j(a)(X_2 - Z_2(a))}\right) \end{cases} \tag{18}$$

where $w_j(\boldsymbol{a},i)$ is the i^{th} component of the j^{th} eigenvector associated with the j^{th} eigenvalue $\gamma_j(a)$ for j= 2, 3 and 4. In (18), $f_i(a,Z_2,x_2)dx_2 = P_r[(x_2 \le X_2 \le x_2 + dx_2);\ M_{2eq}$ in state $i]$, i=1, …, 4, and $f(a,Z_2,x_2)\, dx_2 = P_r[(x_2 \le X_2 \le x_2 + dx_2)$. Furthermore, the constants $C_j(a)$ could be obtained as:

$$\begin{pmatrix} P_{Z_2}(a) \\ C_2(a) \\ C_3(a) \\ C_4(a) \end{pmatrix} = \begin{pmatrix} -\dfrac{(p_2 + p_0(a))}{k_2 - d} & w_2(a,1) & w_3(a,1) & w_4(a,1) \\ -\dfrac{p_0(a)}{d} & w_2(a,2) & w_3(a,2) & w_4(a,2) \\ -\dfrac{p_2}{d} & w_2(a,3) & w_3(a,3) & w_4(a,3) \\ 1 & \dfrac{\sum_{i=1}^{4} w_2(a,i)}{\gamma_2(a)} & \dfrac{\sum_{i=1}^{4} w_3(a,i)}{\gamma_3(a)} & \dfrac{\sum_{i=1}^{4} w_4(a,i)}{\gamma_4(a)} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{19}$$

with $P_{Z2}(\boldsymbol{a})$ denoting the steady-state probability mass at Z_2. Thus, the associated downstream cost:

$$J_2(a,Z_2) = c_2^+ . Z_2 . P_{Z_2} + \int_{-\infty}^{Z_2}\left(c_2^+ X_2^+ + c_2^- X_2^-\right) \mathrm{f}(\mathrm{a}, Z_2, x_2)\, dx_2 \tag{20}$$

takes the closed form:

$$\begin{aligned} J_2(a,Z_2) = {} & c_2^+\left(P_{Z_2}(a) + \sum_{j=2}^{4}\left(\frac{C_j(a)}{\gamma_j(a)}\sum_{i=1}^{4} w_j(a,i)\right)\right) Z_2 - c_2^+ \sum_{j=2}^{4}\left(\frac{C_j(a)}{\gamma_j^2(a)}\sum_{i=1}^{4} w_j(a,i)\right) \\ & + \left(c_2^+ + c_2^-\right)\sum_{j=2}^{4}\left(\frac{C_j(a)}{\gamma_j^2(a)}\, e^{-\gamma_j(a)\, Z_2} \sum_{i=1}^{4} w_j(a,i)\right) \end{aligned} \tag{21}$$

Clearly, for any admissible *a* (or equivalently Z_1), the downstream optimal hedging point $Z_2^*(a)$ could be obtained from (21) by solving $\partial J_2(a,Z_2)/\partial Z_2=0$ for Z_2. Thus, the optimal $Z_2^*(a)$ satisfies the following non linear equation:

$$\sum_{j=2}^{4}\left(\frac{\mathrm{C_j(a)}}{\gamma_{\mathrm{j}}(a)}\; e^{-\gamma_j(a)\,Z_2^*(a)}\sum_{i=1}^{4} w_j(a,i)\right)=\frac{c_2^+}{\left(c_2^+ + c_2^-\right)}\left(P_{Z_2}(a)+\sum_{j=2}^{4}\left(\frac{C_j(a)}{\gamma_j(a)}\sum_{i=1}^{4} w_j(a,i)\right)\right) \qquad (22)$$

Numerical experimentation with MATLAB Software [59] (see *fmin.m*, *fmins.m* and *fzeros.m*) or MAPLE V (see *minimize.mws, minimize.mws*, *solve.mws and fsolve.mws* in Darren-Redferm [56]) suggests that it is easier to numerically solve (22) than it is to optimize (21).

3.6 Optimal total cost and hedging points ($Z_1^*=Z_1(a^*)$ and $Z_2^{**}=Z_2^*(a^*)$)

By summing (5) and (21) after it has been optimized with respect to Z_2 for a given *a* (or equivalently Z_1), the optimal upstream hedging point is obtained from the following one-dimensional closed form optimization problem:

$$J^{**} = \inf_{\mathrm{a}}\left(\begin{array}{l} \dfrac{c_1 k_1}{(p_1+r_1)(1-\rho)} - \dfrac{c_1 k_1 (1-a)}{p_1(1-\rho)} \\ \quad - \dfrac{\rho}{r_1(1-\rho)}\left(c_1 - c_1\dfrac{(\mathrm{r}_1+p_1)(1-a)}{p_1(1-\rho)}\right)\log\left(\dfrac{1}{\rho}-\dfrac{p_1(1-\rho)}{\rho(r_1+p_1)(1-a)}\right) \\ \quad + c_2^+\left(P_{Z_2^*}(a) + \displaystyle\sum_{j=2}^{4}\left(\dfrac{C_j(a)}{\gamma_j(a)}\sum_{i=1}^{4} w_j(a,i)\right)\right)\mathrm{Z}_2^*(a) \\ \quad - \mathrm{c}_2^+\displaystyle\sum_{j=2}^{4}\left(\dfrac{C_j(a)}{\gamma_j^2(a)}\sum_{i=1}^{4} w_j(a,i)\right) \\ \quad + \left(c_2^+ + c_2^-\right)\displaystyle\sum_{\mathrm{j}=2}^{4}\left(\dfrac{C_j(a)}{\gamma_j^2(a)}\; e^{-\gamma_j(a)\,Z_2^*(a)}\sum_{i=1}^{4} w_j(a,i)\right) \end{array}\right) \qquad (23)$$

and the unknown optimal hedging points are computed from the solution a^* if it exists from:

$$Z_1^* = \frac{1}{\lambda}\log\left(\frac{1}{\rho}-\frac{p_1(1-\rho)}{\rho(r_1+p_1)}\frac{1}{(1\text{-}\mathrm{a}^*)}\right), \qquad Z_2^{**} = Z_2(a^*) = \mathit{solution}\text{ of } (22) \qquad (24)$$

Finally, in order to numerically solve (22) and (23), an initial condition a_0 is required for *a*.

3.7 Initializing the proposed optimization scheme

A suboptimal initial condition a_0* could be derived from the analysis of a further simplified two-state aggregated machine version of machine M_{2eq} (see Appendix 2 for more details). An appproximate solution based on the first order Taylor approximation of the first derivatives of production costs is obtained from this highly simplified analysis is as follows:

$$a_0^* = \frac{\left(\tilde{J}_1''(a_1) + \tilde{J}_2''(a_1)\right)a_1 - \left(\tilde{J}_1'(a_1) + \tilde{J}_2'(a_1)\right)}{\tilde{J}_1''(a_1) + \tilde{J}_2''(a_1)} \tag{25}$$

where a_1 is a value of a chosen in the left neighborhood of a_{max} =1, whereas $(\tilde{J}_1'(a_1), \tilde{J}_1''(a_1))$ and $(\tilde{J}_2'(a_1), \tilde{J}_2''(a_1))$ denote first and second derivatives of the corresponding costs (see Appendix 2). Now for the sake of clarity, let us organize the whole optimization algorithm developed above into a hierarchical computational structure.

3.8 Hierarchical optimization scheme

The whole procedure presented in Section 3 is organized into a hierarchical optimization scheme (Fig. 4). At the lower level, an optimization *subproblem* consisting of a single equation (22) is numerically solved to obtain $Z_2^*(a)$ for any value of the availability coefficient ***a***. Then, a second *optimization superproblem* is solved at higher level (23) until the optimum a^* is reached (if it exists), and the globally optimal hedging points Z_1^* and Z_2^{**} are then obtained from (24). Note that each run of this optimization methodology can be carried out almost instantly, given the analytical nature of (22) and (23).

4 Numerical sample systems

In this section, optimization is carried out for the class of DHP policies using the theory and algorithms developed in Section 3 for a wide set of sample two-machine systems listed in Table 1. Note that S_1 is the nominal parameter set. S_2-S_3 involve a *decrease* in the backlog cost coefficient, while S_4-S_5 involve an *increase* in the storage cost coefficient. S_6-S_7 deal with an *increase* in the inventory cost coefficient, while S_8-S_9 illustrate the effect of a *decrease* in the repair rate of both machines for a balanced line. For each sample two-machine system, the corresponding approximate optimal hedging points (Z_1*, Z_2**) and costs (J_1*, J_2**) are computed following the optimization scheme presented in Figure 4. Furthermore, using a discretization step of 0.01 for the availability coefficient ***a*** candidates within the suboptimal class of DHP policies, are scanned and their theoretically computed performance is obtained almost (instantaneously) and validated against (very lengthy) Monte Carlo simulations.

4.1 Application of the DHP policies

In the following, we investigate the application of DHP policies in detail for the particular case of the sample two-machine system S_1. However, results obtained for all S_i(i=1,…,10) are summarized in subsection 4.3.

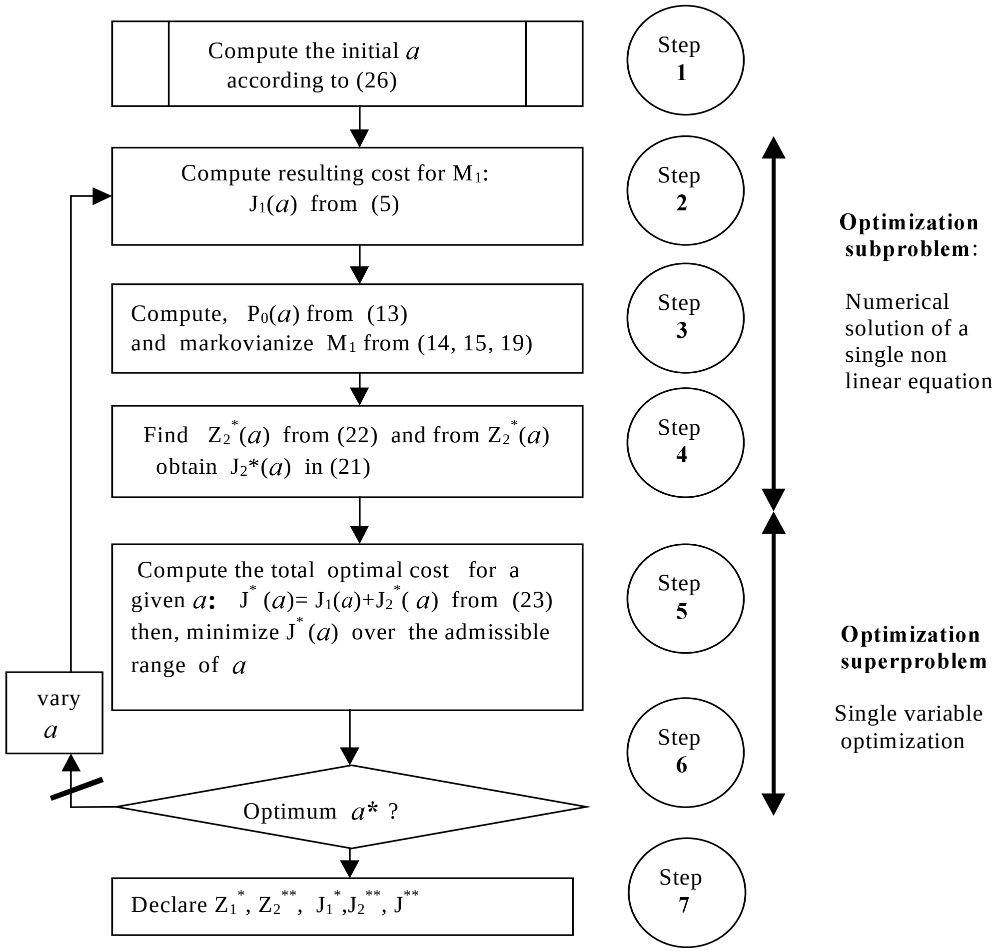

Figure 4: Hierarchical optimization scheme

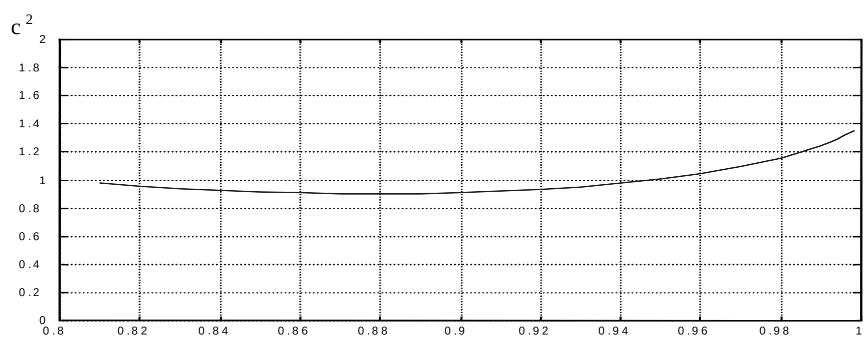

Figure 5: Coefficient of variation of availability periods (Nominal System S_1)

4.1.1 Coefficient of variation of availability periods

From Equations (8), (12), the following quantities are calculated immediately in the case of S_1:

$$\mu_1 = -5 + 15e^{\frac{1}{3}Z_1}, \quad \mu_2 = -10 - 330e^{\frac{1}{3}Z_1} + 540e^{\frac{2}{3}Z_1} - 70Z_1e^{\frac{1}{3}Z_1}$$
$$c^2 = -\frac{1}{5}\frac{\left(7 + 14Z_1e^{\frac{1}{3}Z_1} - 63e^{\frac{2}{3}Z_1} + 36e^{\frac{1}{3}Z_1}\right)}{\left(3e^{\frac{1}{3}Z_1} - 1\right)^2} \tag{26}$$

Using (26) and recalling that Z_1 and a are connected by (10), the graph of the coefficient of variation $c^2(a)$ can be obtained and is shown in Fig. 5. Since $c^2(a)$ remains in a reasonable neighborhood of 1 (perfect exponential distribution), it is quite reasonable to approximate the first return-time to buffer emptiness by an exponential distribution as in Section 3.3.

4.1.2 Predicted performances

Using an optimization tolerance da=0.01, we have initialized Eq. (23) with a_0^*=0.92 given by (25) which is very close to the obtained optimum a*=0.95 (see Figure 6 for the summary of results). Despite the fact that global optimality was attained very quickly, we have intentionally extended the computation over the whole admissible range of a because some interesting findings could be pointed out from the complete shape of computed quantities. These findings are summarized as follows:

- The suboptimal initial parameter $\mathbf{a_0^*}$=0.92 appears to be very close to the optimum a*=0.95.
- As a approaches 1, the upstream cost $J_1(a)$ goes to infinity while the downstream cost $J_2^*(0.9999)$=11.4693 converges to the Bielecki-Kumar limiting cost J_{2BK}=11.4642
- The overall cost appears to be strictly convex, with a unique minimum (Fig. 6).
- An increase in a gives rise to a decrease in the downstream cost $J_2(\mathbf{a})$ and to an increase in $J_1(\mathbf{a})$. Both results are expected since a rise in a comes at the expense of a higher hedging level for the WIP, while because of the resulting higher supply availability, the requirements for a safety stock of finished parts as a hedge against uncertainty is reduced. The latter monotonicity result was mathematically established by Liberopoulos and Hu [60], and Sethi and Zhang [41].
- At a*=0.95, the sum of the slopes of costs (upstream and downstream) is exactly zero.
- In Fig. 5, at the optimum a*=0.95, the coefficient of variation is c^2=1.01.

In the next section, the theoretical results for S_1 are tested against Monte Carlo simulations under the same class of decentralized hedging production policies.

4.2 Monte Carlo costs for the same class of DHP policies

A Markov failure generator and a Monte Carlo simulation model have been built , tested and used as a validation basis for the theoretical performance computations associated with the proposed hierarchical DHP optimizations scheme.

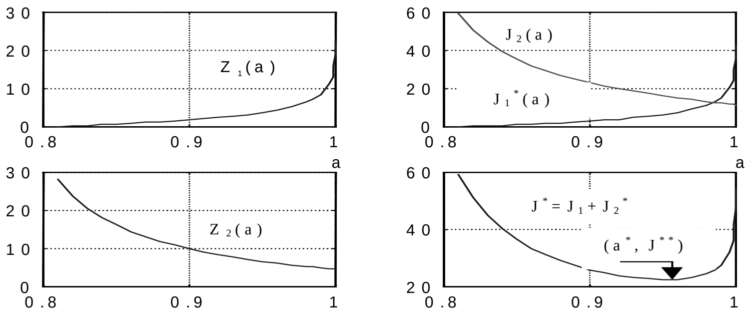

Figure 6: Predicted hedging points Z_1(a), Z_2^*(a) and associated costs for system S_1

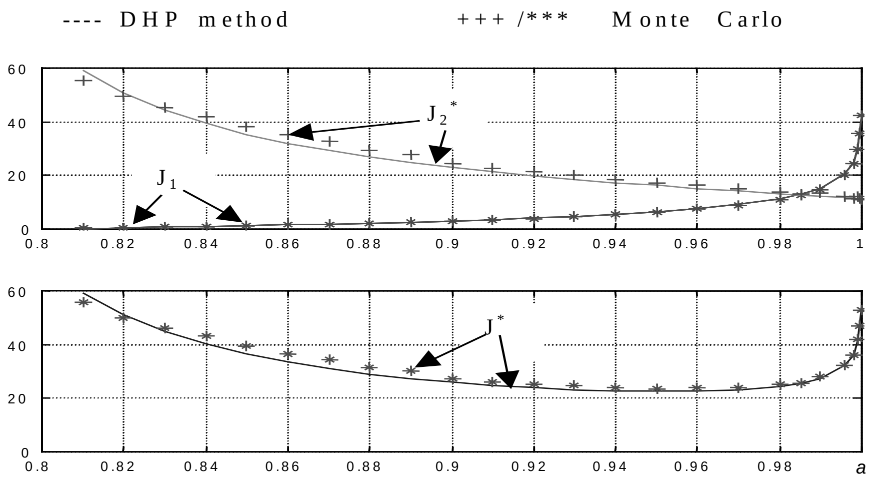

Figure 7: Comparison of the DHP method and the Monte Carlo simulations

The Markov failure generator is designed by relying on well tested algorithms (see Sethi and Zhang [41] , and Naylor et al. [61]). For any sample two-machine system S_i (i=1,..,10), given $Z_1(a)$ and $Z_2^*(a)$ delivered in each case by our theoretical DHP computations, the corresponding Monte Carlo costs have been computed over the whole admissible range of ***a***. From the comparison of results shown in Figure 7 for the case of system S_1, additional remarks can be made.

- Monte Carlo costs are very close to their predicted values
- The upstream costs emerging from the Monte Carlo simulations are in fact similar (thus strengthening the case for DAP) to the predicted shape over the whole admissible range of *a*.
- The observations reported in the case of system S_1, could in fact be reasonably well extended to the remaining systems S_2-S_{10}. Notice however, as anticipated in the discussions on the quality of the machine decoupling approximation in Section 3.1,

whenever $Z_1(a^*)$ decreases because of an increase in WIP storage costs as in case S_6, the error on cost prediction appears to increase. However, the cost estimates remain quite acceptable.

4.3 Summary of the main results

The same types of simulations presented in Section 4.1 and 4.2 in the case of the sample two-machine system S_1, have been performed for all sample systems S_1-S_{10} listed in Table 1. The corresponding results are summarized in Table 2.

The first three sub-colons attest to the fact that the initial condition a_0^* suggested in (25) with a_1=0.95 is generally located very near the optimum a*. From the fourth subcolon, we observe that values of c^2 are in general close to 1, thus the exponential assumption made in Subsection 3.3 is quite acceptable. Z_1^*, Z_2^*, $J_{1\ DH}^*$, $J_{2\ DH}^*$ are predicted hedging points and costs respectively, while $J_{1\ MC}^*$, $J_{2\ MC}^*$ are Monte-Carlo costs computed according to (29). The last two quantities represent in principle the real performances recorded from the implementation of the DHP policies on the original tandem two-machine system. Note that for all cases, $J_{1\ DH}^*$ and $J_{1\ MC}^*$ are quite similar due to the DAP property. Note also that the approximated Markovian model adopted for the downstream machine works well since the values of $J_{2\ DH}^*$ and $J_{2\ MC}^*$ remain reasonably close, even when the coefficient of variation deviates by as much as 10 % from unity. The worst case computation is accurate within 15 %. The mean relative error is 7.4 %.

5 Conclusion

Although structurally simple, stochastic tandem manufacturing systems are notoriously complex to analyze from a control point of view as witnessed by an abundant and intricate related literature. The hierarchical optimization scheme proposed in this paper permits to construct, almost instantaneously, approximately optimal policies within the class of DHP policies for an unreliable two-machine flow shop. It is founded on a mix of approximations and analytical results yielding minimal computations, and the resulting accuracy of computations is quite acceptable when validated against numerous Monte Carlo runs. This makes it a good candidate either for real time production control of two-machine flow shops, or for the optimal design of the class of tandem production systems investigated. Numerical evidence suggests that convexity of the overall cost of DHP policies as a function of $\boldsymbol{a}$ appears to hold, but it remains mathematically unproven. This particular issue could be further investigated. The key ideas in this paper point at ways in which DHP policies could be optimized in an arbitrary *N*-machine unreliable tandem system. This will be the subject of future research.

Acknowledgement

The authors wish to thank the anonymous reviewers for throwing doubt in their minds as to the mathematical exactness of the demand averaging principle, thus leading them to a better assessment of its nature, and an understanding of some of the reasons why it works so well.

Sample systems	Data									
	k_1	K_2	D	r_1	r_2	p_1	p_2	c_1	c_2^+	c_2^-
S_1	**2.5**	**2**	**1**	**0.4**	**0.6**	**0.1**	**0.3**	**2**	**2**	**10**
S_2	2.5	2	1	0.4	0.6	0.1	0.3	2	2	**8**
S_3	2.5	2	1	0.4	0.6	0.1	0.3	2	2	**6**
S_4	2.5	2	1	0.4	0.6	0.1	0.3	2	**6**	10
S_5	2.5	2	1	0.4	0.6	0.1	0.3	2	**8**	10
S_6	2.5	2	1	0.4	0.6	0.1	0.3	**6**	2	10
S_7	2.5	2	1	0.4	0.6	0.1	0.3	**8**	2	10
S_8	2	2	1	**0.6**	**0.6**	0.1	0.1	1	2	10
S_9	2	2	1	**0.5**	**0.5**	0.1	0.1	2	2	10
S_{10}	2	2	1	**0.4**	**0.4**	0.1	0.1	2	2	10

Table 1: Numerical sample systems (S_1 is the nominal system and changes in the off-nominal systems are highlighted)

S	**Parameters** of the *Decentralized Hedging Production Control (DHP)*						Production **costs**					
							DHP method (Predicted costs)			Monte-Carlo simulation (Costs)		
	a_{min}	a_0^*	*a**	C_V^*	Z_1^*	Z_2^*	J_1^*	J_2^{**}	J^{**}	J_1^*	J_2^{**}	J^{**}
S_1	0.80	0.94	0.95	1.01	3.76	6.71	6.39	16.19	22.58	6.23	17.16	23.39
S_2	0.80	0.94	0.95	1.01	3.76	5.86	6.39	14.48	20.87	6.22	15.87	22.09
S_3	0.80	0.94	0.94	0.98	3.24	5.23	5.47	13.26	18.73	5.35	16.77	22.12
S_4	0.80	0.96	0.97	1.09	5.23	2.49	9.08	22.44	31.52	8.99	23.58	32.57
S_5	0.80	0.96	0.97	1.09	5.23	1.81	9.08	24.38	33.47	8.91	25.25	34.16
S_6	0.80	0.92	0.91	0.92	2.12	9.20	10.48	21.35	31.84	10.36	25.77	36.12
S_7	0.80	0.92	0.90	0.91	1.82	10.00	12.00	22.98	34.98	11.73	25.62	37.34
S_8	0.86	0.94	0.95	0.99	1.87	1.82	1.63	6.56	8.19	1.43	7.41	8.84
S_9	0.75	0.93	0.95	1.03	2.63	2.63	2.23	8.68	10.92	2.00	10.03	12.03
S_{10}	0.80	0.95	0.95	1.09	3.93	4.12	3.26	12.33	15.59	2.95	14.54	17.49

Table 2: Comparison of the DHP method and the Monte Carlo simulation: summary of results

Appendix 1

Moments of the *first return-time (FPT) to buffer emptiness*

In this appendix, the moments of the first return-time to buffer emptiness (X_1=0) are generated using a version of Lemma 1 in El-Férik and Malhamé [54] specially adapted to our context.

Probability density $g_{cycle}(t)$ of the first return-time T_{cycle} to buffer emptiness

Under the adopted hedging production policies, the upstream subsystem operates as a two feasible state machine with two hedging points Z_1 (for the operating mode) and Z_0=0 (for the failure mode), and can be viewed as subjected to a constant demand rate *d*, given the DAP. The shape of a particular upstream buffer trajectory is illustrated in Figure A.1.

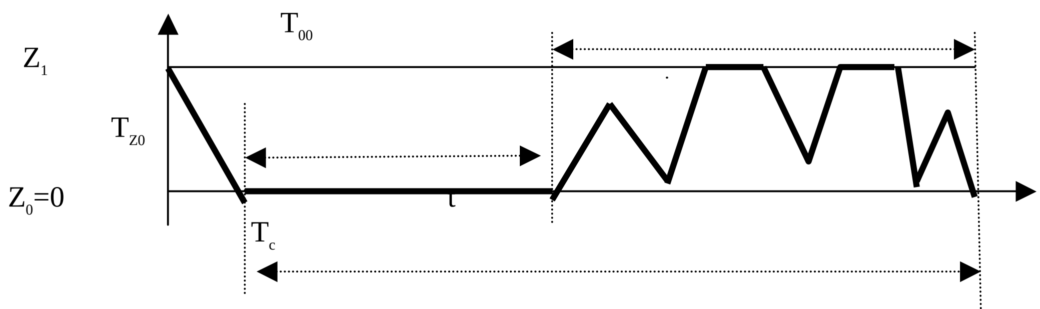

Figure A.1: The upstream trajectory of the hybrid process $\left(X_1(t);\alpha_1(t)\right)$

In order to compute the probability density of the return time to buffer emptiness, the same types of notations as in El-Férik and Malhamé [32] will be used here. Thus define: $f_i(X_1)dX_1=P_r([x_1<X_1(t)\leq x_1+dx_1,\ \alpha_i(t)=1])$ for i=1, 2 and $f(X_1,t)=(f_1(X_1,t)\ f_2(X_1,t))^T$; $P_{Z1}(t)$= probability mass at Z_1 at time t; $P_{Z0}(t)$= probability mass at Z_0=0 at time t; T_c= time of first return to buffer emptiness; T_{Z0}= Portion of T_c during which the buffer remains empty; T_{00}≅Portion of T_c for which supply is available; $g_{cycle}(t)$= probability density function of random time T_c and $g_{00}(t)$=probability density of return time to buffer emptiness not including sojourn at Z_0=0. Furthermore, define the velocity matrix V_1 and the intensity matrix Λ_1 as:

$$V_1 = \begin{pmatrix} v_1 = k_1 - d & 0 \\ 0 & v_2 = -d \end{pmatrix};\quad \Lambda_1 = \begin{pmatrix} -p_1 & p_1 \\ r_1 & -r_1 \end{pmatrix}$$

Then according to Lemma 1 in El-Férik and Malhamé [54], it is possible to obtain $g_{cycle}(t)$ using

$$g_{cycle}(t) = -v_2 f_2(Z_0^+, t) \tag{A.1.1}$$

where $f_2(Z_0^+,t)$ is computed by solving the following Kolmogorov equations :

$$\frac{\partial f(X_1,t)}{\partial t} = -V_1 \frac{\partial f(X_1,t)}{\partial X_1} + \Lambda_1^T f(X_1,t) \tag{A.1.2}$$

with boundary conditions,

$$\begin{cases}\lambda_{12}P_{Z_1}(t)+v_2f_2(Z_1^-,t)=0\\ \lambda_{21}P_{Z_0}(t)-v_1f_1(Z_0^+,t)=0\end{cases}\quad \begin{cases}\dot{P}_{Z_1}=\lambda_{11}P_{Z_1}(t)+v_1f_1(Z_1^-,t)\\ \dot{P}_{Z_0}=\lambda_{22}P_{Z_0}(t)-v_2f_2(Z_0^-,t)\end{cases} \tag{A.1.3}$$

Constraints and initial conditions are given by:

$$\begin{cases}f(X_1,t)=0\ \ \forall\ \ X_1>Z_1\\ f(X_1,t)=0\ \ \forall\ \ X_1<0\end{cases}\quad \begin{cases}P_{Z_0}(0)=1,\quad P_{Z_1}(0)=0\\ f(X_1,0)=0,\ \text{for}\ \ 0<x_1<Z\end{cases} \tag{A.1.4}$$

A straightforward manipulation of the previous expressions rewritten in the Laplace transform domain leads to the following set of algebraic equations:

$$\begin{cases}F(Z_0^+,s)=e^{V_1^{-1}(\Lambda_1^T-sI)(Z_0-Z_1)}\begin{pmatrix}F_1(Z_1^-,s)\\ -\dfrac{v_1}{v_2}\dfrac{\lambda_{12}}{(s-\lambda 11)}F_1(Z_1^-,s)\end{pmatrix}\\ F_2(Z_0^+,s)=\dfrac{1}{v_2}-\dfrac{v_1}{v_2}\dfrac{(s-\lambda_{22})}{\lambda_{21}}F_1(Z_0^+,s)\end{cases} \tag{A.1.5}$$

with three unknowns $F_1(Z_0^{+},s)$, $F_2(Z_0^+,s)$ and $F_1(Z_1^-,s)$ and where capitalization of variables indicates Laplace transforms. Once $F_2(Z_0^+,s)$ is determined, the Laplace transform of the time to first return to buffer emptiness is obtained from (A.1.1) as:

$$g_{cycle}(s)=-v_2\,\mathrm{f}_2(Z_0,s) \tag{A.1.6}$$

Since $T_{cycle}=T_{Z0}+T_{00}$ and the latter two random variables are independent, the convolution theorem yields to $G_{cycle}(s)=G_{Z0}(s)\ G_{00}(s)$. Note that if the upstream capacity k_1 of M_1 is greater than the downstream capacity k_2 of M_2, then $G_{Z0}(t)=r_1\ e^{-r1\ t}$ because in this case T_{Z0} is exponentially distributed with a rate r_1 imposed by the time to repair of the upstream machine M_1. Thus, the Laplace transform of the supply availability of Tc is given by:

$$G_{00}(s)=G_{cycle}(s)\left(\frac{s+r_1}{r_1}\right) \tag{A.1.7}$$

and the moments of $g_{00}(t)$ can now be generated by differentiation of $G_{00}(s)$ as in El-Férik and Malhamé [54]. That is:

$$\mu_i=(-1)^i\frac{d^iG_{00}(s)}{ds^i},\ \ \mathrm{i}=1,\ 2,\ \ \ldots \tag{A.1.8}$$

A symbolic computation using MAPLE V leads to the generic moment expressions given by (8).

Appendix 2

Initial condition for the proposed optimization scheme

This appendix is devoted to the determination of the suboptimal initial condition a_0^*. Using the Taylor approximation of the derivative of costs (denoted $\frac{d\tilde{J}_1(a)}{da}$ and $\frac{d\tilde{J}_2(a)}{da}$ at $\boldsymbol{a}=\boldsymbol{a}_1$ some arbitrary value close to 1), the optimal initial condition a_0^* is obtained by solving the equation:

$$\left.\frac{d\tilde{J}_1(a)}{da}\right|_{a_1}+\left.\frac{d\tilde{J}_2(a)}{da}\right|_{a_1}=0\cdot$$

Upstream cost (expression, derivative, and Taylor approximation)
From (5), if we consider the following new notations :

$$\rho=\frac{r_1(k_1-d)}{p_1 d},\ \psi=\frac{c_1(r_1+p_1)\rho\ d}{\mathrm{p}_1 r_1(1-\rho)^2},\ \pi=\frac{\mathrm{p}_1(1-\rho)}{\rho(r_1+p_1)},\ \ \phi=\mathrm{c}_1-c_1\frac{(r_1+p_1)}{p_1(1-\rho)}\frac{\rho}{r_1(1-\rho}) \quad \text{(A.2.1)}$$

then, straightforward computations lead to:

$$\begin{aligned}
J_1(a)&=\frac{c_1k_1}{(r_1+p_1)(1-\rho)}-\frac{c_1k_1(1-a)}{p_1(1-\rho)}-(\phi+\psi\ \mathrm{a})\log_n\left(\frac{1}{\rho}-\frac{\pi}{(1-a)}\right)\\
\frac{dJ_1(a)}{da}&=\frac{c_1k_1(1-a)}{p_1(1-\rho)}-\psi\log_n\left(\frac{1}{\rho}-\frac{\pi}{(1-a)}\right)+\frac{(\phi+\psi\ \mathrm{a})\pi}{(1-a)^2\left(\frac{1}{\rho}-\frac{\pi}{(1-a)}\right)}\\
\frac{dJ_1(a)}{da}&=J_1^{'}(a_1)+J_1^{''}(a_1)(a-a_1)\ +\mathrm{O}(\mathrm{a}-\mathrm{a}_1)^2
\end{aligned} \quad \text{(A.2.2)}$$

where $O(a-a_1)^2$ indicate remainder terms of the order of $(a-a_1)^2$.

Downstream cost (expression, derivative, and Taylor approximation)
Consider a two-state *aggregated version* of the downstream machine model already shown in Figure 3, whereby all three unproductive modes **1**, **2**, and **3** have been lumped into one supermode. Then, in view of the limiting probability expression $\pi_1(a)$ in mode one (see (16)), the failure rate $P(a)$ and the repair rate $R(a)$ of the two-state aggregated version of M_2 must satisfy:

$$\begin{cases} P(a) = p_2 + r_1\dfrac{1-a}{a} \\ R(a) = \dfrac{r_2\left(r_1+(p_2-r_1)a\right)}{\mathrm{p}_2+(1-a)\mathrm{r}_2} \end{cases}$$

and the optimal cost for a given $\boldsymbol{a}$ can then be computed using the closed form results of Bielecki and Kumar [13]. This leads to:

$$\tilde{J}_2(a) = c_2^+ \frac{\log\left(\frac{k_2 P(a)(c_2^+ + c_2^-)}{(k_2 - d)(P(a) + R(a))c_2^+}\right)}{\frac{R(a)}{d} - \frac{P(a)}{k_2 - d}} + \frac{c_2^+ d}{P(a) + R(a)} \qquad \text{(A.2.3)}$$

From (2.9), if we now introduce the following notations:

$$\sigma_2 = \frac{r_2}{r_2 + p_2}, \quad \beta_2 = \frac{(c_2^+ + c_2^-)k_2}{c_2^+(-k_2 + d)}, \quad \delta_2 = \frac{r_2 k_2}{(r_2 + p_2)d}, \quad \mathrm{v}_2 = k_2 - d \qquad \text{(A.2.4)}$$

one has :

$$\begin{aligned} &\tilde{J}_2(a) = -\frac{c_2^+(\sigma_2 a - 1)v_2\, a \ln((\sigma_2 a - 1)\beta_2)}{((-p_2 + r_1)a - r_1)(1 - \delta_2 a)} + \frac{c_2^+ d(\sigma_2 a - 1)a}{(-p_2 + r_1)a - r_1} \\ &J_2^{'}(a) = dJ_2(a)/da, \quad J_2^{''}(a) = dJ_2^{'}(a)/da \\ &\frac{d\tilde{J}_2(a)}{da} = \tilde{J}_2^{'}(a_1) + \tilde{J}_2^{''}(a_1)(a - a_1) + \mathrm{O}(\mathrm{a} - \mathrm{a}_1) \end{aligned} \qquad \text{(A.2.5)}$$

Suboptimal initial condition: Now, a suboptimal condition a_0^* could be obtained from: $\tilde{J}_1^{'}(a_1) + \tilde{J}_1^{''}(a_1)(a - a_1) + \tilde{J}_2^{'}(a_1) + \tilde{J}_2^{''}(a_1)(a - a_1) \approx 0$ or equivalently, recalling (2.2) and (2.5):

$$\mathrm{a}_0^* = \frac{(\tilde{J}_1^{''}(a_1) + \tilde{J}_2^{''}(a_1))a_1 - (\tilde{J}_1^{'}(a_1) + \tilde{J}_2^{'}(a_1))}{\tilde{J}_1^{''}(a_1) + \tilde{J}_2^{''}(a_1)} \qquad \text{(A.2.6)}$$

Appendix 3

Demand Averaging Principle (DAP)

Consider the following single part production systems and their associated performance criteria under an *ergodic assumption*, and a hedging control policy with common critical level Z_1:

$$S:\begin{cases} J_1 = \lim\limits_{T\to+\infty}\frac{1}{T}\int_{t_0}^{T} c_1 x_1(t)dt = c_1 \lim\limits_{T\to+\infty} E(x_1(t)) \\ \dot{x}_1(t) = u_1(t) - d_1(t),\ x_1(0) = x_0 \\ x_1(t) \geq 0 \\ 0 \leq u_1(t) \leq \alpha_1(t)\, k_1 \end{cases} \quad ; \quad \tilde{S}:\begin{cases} \tilde{J}_1 = \lim\limits_{T\to+\infty}\frac{1}{T}\int_{t_0}^{T} c_1 \tilde{x}_1(t)dt = c_1 \lim\limits_{T\to+\infty} E(\tilde{x}_1(t)) \\ \dot{\tilde{x}}_1(t) = \tilde{u}_1(t) - d,\ \tilde{x}_1(0) = x_0 \\ \tilde{x}_1(t) \geq 0 \\ 0 \leq \tilde{u}_1(t) \leq \alpha_1(t) k_1 \end{cases} \tag{A.3.1}$$

Notice that system S is essentially the upstream subsystem for a two-machine flow shop under DHPC policies, whereas system $\tilde{S}$ corresponds to a Hu type of machine. Notice also that $d_1(t)$ is a stochastic demand process with long term average d.

The essence of the *demand averaging principle* is in the possibility of approximating the cost J_1 associated with S, with the cost $\tilde{J}_1$ associated with $\tilde{S}$. The goal of this appendix is twofold; first, we develop qualitative arguments explaining the role of the boundaries at zero and Z_1 in insuring that the two buffer processes remain very close in an average sense; this in turn insures that if a piecewise linear cost structure is adopted, the corresponding costs are close to each other. Secondly, we present a summary table of extensive numerical Monte-Carlo testing of the quality of the approximation.

We now present our qualitative arguments. We first note that away from boundaries 0 and Z_1, whenever the buffer processes are in what we shall term as *synchrony*, their trajectories will coalesce during sojourn times of surplus processes x_2 on its hedging level Z_2 (at which point machine M_2 will consume d parts per unit time), and they will part whenever x_2 leaves hedging level Z_2. *This is because, away from boundaries, M_1 and $\tilde{M}_1$ have exactly the same production rate*. During separation intervals (which correspond to first return times of process x_2 to hedging level Z_2), x_1 will dominate (see Fig. A.3.1), and the average excess of x_1 relative to $\tilde{x}_1$ can be precisely computed (see [32], Lemma 3).

Upon the return of x_2 to Z_2, x_1 and $\tilde{x}_1$ merge again. If the situation persisted undisturbed, in the long run, x_1 would dominate $\tilde{x}_1$ on average. However, the boundary at zero destroys any synchrony that may have developed between x_1 and $\tilde{x}_1$. Indeed the boundary at zero plays totally asymetric roles for x_1 and $\tilde{x}_1$. In the case of x_1, it can be viewed as merely a *demand postponement mechanism*, but machine M_1 remains unproductive. Thus, the pent up demand will reemerge as soon as M_1 becomes operational again. In the case of $\tilde{x}_1$, a sojourn at the zero boundary can be viewed as *productive*, with the production rate of machine $\tilde{M}_1$ set at d. Thus assuming that x_1 and $\tilde{x}_1$ merge at zero, respectively spending times T_{off} and $\tilde{T}_{off}$ on the zero boundary, when they both reemerge together from zero, machine $\tilde{M}_1$ would have a cumulative production lead of $d\tilde{T}_{off}$ relative to machine M_1. Furthermore, process x_1 will reemerge more slowly from zero than x_1', because it is subjected to a pent-up cumulative demand $d\ T_{off}$ above the normal demand. Thus, after a transient whereby $\tilde{x}_1$ dominates x_1, the two processes will meet again at times t such that the cumulative demand of x_1 lags behind the cumulative demand of $\tilde{x}_1$, $d\ t$, by an amount $d\ T_{off}$. Thus they can *never meet* at a point such that x_2 is on its hedging level Z_2 (because if x_2 starts from Z_2 at t=0, it can only come back on Z_2 at times such that the cumulative production=cumulative demand= $d\ t$). As a result, x_1 and $\tilde{x}_1$ will keep on meeting and parting away, with in general no clear winner (since starting from a $d\ T_{off}$ offset, cumulative demand $D_1(t)$ will oscillate *evenly* around the line $d\ t\text{-}dT_{off}$), except for a small transient if x_1 has also sojourned at zero. This transient will give a slight (and periodic) advantage to $\tilde{x}_1$ early on. The only way x_1 can ever catchup with the cumulative production of $\tilde{x}_1$ is if the two processes sojourn sufficiently long on Z_1 that x_2 can reach its hedging level. If this does not happen before one of the two leaves Z_1, they will keep crossing each other with no clear winner. However if they stay long enough that x_2 reaches Z_2, then when they depart, their trajectories are common, until x_2 leaves Z_2 again, in which case x_1 *will dominate* $\tilde{x}_1$. This domination will persist until $\tilde{x}_1$ hits the zero level or until x_1 prematurely returns to the Z_1 level in which case M_1 loses some production relative to $\tilde{M}_1$. Thus, summarizing, the zero boundary clearly favors $\tilde{x}_1$ over x_1 upon leaving, but not for long, whereas the Z_1 boundary will tend to favor x_1 upon leaving if synchrony has been achieved. Both boundaries will tend to limit dominance of one trajectory with respect to the other. Furthermore, Z_1 acts dominantly as a *cohesiveness factor* between the instantaneous trajectories, while the zero boundary acts as a *dispersion factor*. This all makes for a high degree of solidarity between $x_1(t)$ and $\tilde{x}_1(t)$ and thus the corresponding costs.

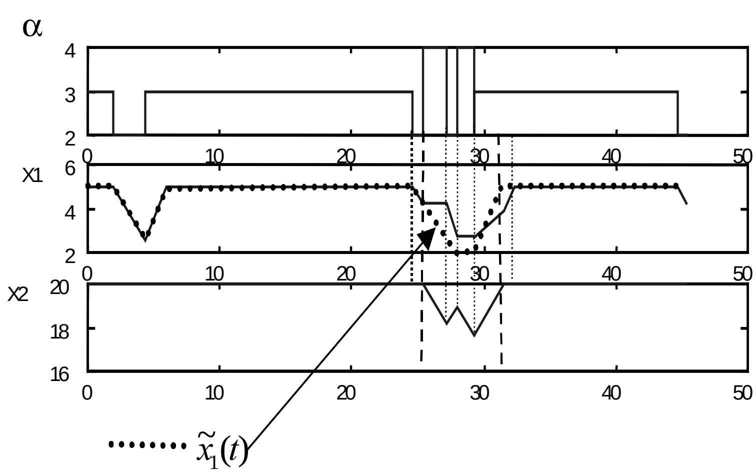

Fig. A.3.1: Domination of the x_1(t) process over separation intervals corresponding to first return times of process x_2(t) to hedging level Z_2

Below, we present a typical Monte-Carlo trajectory of x_1 and $\tilde{x}_1$ parameters, showing the contribution of boundaries to the minimization of the gap between both processes x_1 and $\tilde{x}_1$ (see Fig. A.3.2), as well as a summary table of Monte-Carlo cost comparisons for a diverse range of demand rates as well as for *arbitrary hedging levels* Z_1 *and* Z_2 (see Table A.3.1). The numerical evidence reinforces the conclusion of the qualitative arguments that the demand averaging principle will in general be an excellent approximation.

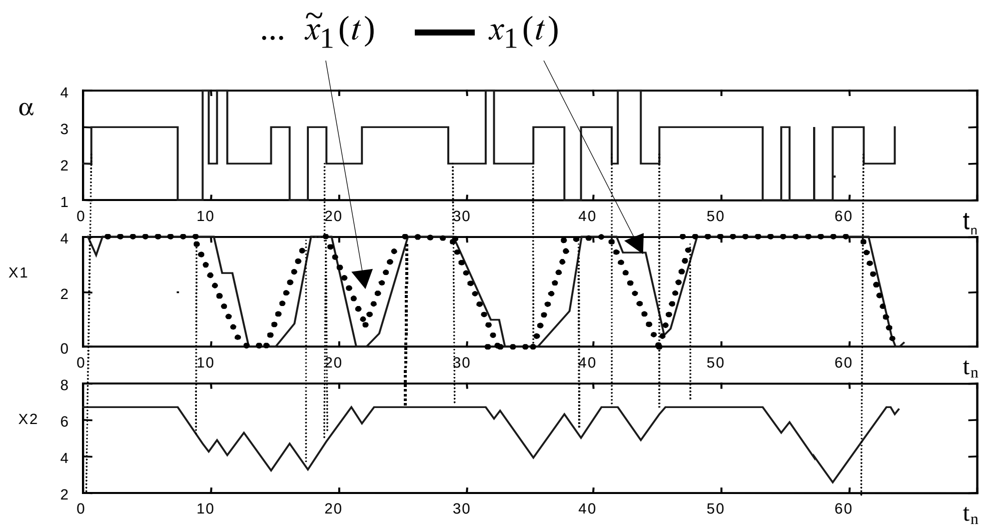

Figure A.3.2: A given realization of successive (alternating type) sequences

In table A.3.1, $P(Z_1)$=probability of being at Z_1 (associated with x_1), $P(0)$=probability of being at zero (associated with x_1), $\tilde{P}(Z_1)$ = probability of being at Z_1 (associated with $\tilde{x}_1$), $\tilde{P}(0)$ = probability of being at zero (associated with $\tilde{x}_1$), and the parameters used are: k_1=2.5, k_2=2, r_1=0.3, r_2=0.6, p_1=p_2=0.1, c_1=2, c_2^+=2, c_2^-=20.

Z_1 and Z_2	d	$P(0)$	$\tilde{P}(0)$	$P(Z_1)$	$\tilde{P}(Z_1)$	J_1	$\tilde{J}_1$
Z_1=5 Z_2=5	1.2	0.0979	0.0837	0.5524	0.5965	7.33	7.79
	1.1	0.0861	0.0746	0.5759	0.6122	7.55	7.94
	1	0.0799	0.0651	0.5961	0.6267	7.72	8.00
	0.7	0.0405	0.0340	0.6607	0.6662	8.44	8.60
	0.5	0.0161	0.0147	0.6986	0.6912	8.94	8.98
	0.2	0.003	0.0002	0.7356	0.7283	9.62	9.63
Z_1=50 Z_2=5	1.2	0	0.00003	0.5230	0.5193	94.32	94.45
	1.1	0	0.000007	0.5566	0.5536	95.37	95.56
	1	0	0.000002	0.5929	0.5833	96.34	96.43
	0.7	0	0.0	0.6622	0.6528	98.13	98.14
	0.5	0	0	0.6937	0.6875	98.84	98.86
	0.2	0	0	0.7363	0.7283	99.61	99.63
Z_1=0.5 Z_2=5	1.2	0.2182	0.2211	0.7048	0.7233	0.74	0.74
	1.1	0.2155	0.2186	0.7048	0.7254	0.74	0.75
	1	0.2101	0.2157	0.7094	0.7271	0.75	0.76
	0.7	0.2011	0.2023	0.7128	0.7314	0.75	0.76
	0.5	0.1846	0.1858	0.7198	0.7340	0.77	0.77
	0.2	0.1174	0.1188	0.7446	0.7386	0.82	0.82

Table A.3.1: Summary of results for a diverse range of demand rates as well as for arbitrary hedging level Z_1 and Z_2

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