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Stochastic short-term mine production schedule accounting for fleet allocation, operational considerations and blending restrictions

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Abstract: Traditional short-term production planning is carried out in two separate sequential optimizations, typically based on mixed integer programming formulations, the physical mine production schedule, and followed by the assignment of the available fleet. However, (a) the fleet availability, hauling time and mining considerations do not influence the production scheduling sequence; and (b) the fleet optimization algorithms do not consider uncertainty in their parameters or account for the variability of the local mineralization of the various mine sectors being considered. The above have adverse effects on short-term production scheduling performance and very large operating and opportunity costs, stemming from erroneous materials blending and processing destination decisions, sub-optimal mining equipment utilization and so on.

A new short-term mine production scheduling formulation is developed herein based on stochastic integer programming. The formulation simultaneously optimizes fleet and mining considerations, production constraints, uncertainty in both orebody metal quantity and quality, as well as fleet parameters and equipment availability, all leading to a well-informed sequence of mining that is expected to have realistic, as well as high performance during a mine's operation. To assess the latter performance, the proposed formulation is applied at a multi-element iron mine and the resulting monthly schedules are assessed and compared to the conventional mine scheduling approach showing: lower cost, minable patterns, efficient fleet allocation ensuring higher and lesser variable utilization of the fleet.

Key Words: Production schedule, fleet, simulations, availability, grade, uncertainty, mining considerations.

1 Introduction

Short-term mine production scheduling generates a sequence of extraction within an annual production area and may be seen as an operational guidance to match the objective of a long-term plan under current operating conditions and constraints. The short-term planning aspect outlines extraction stages in terms of months, weeks or days. The optimization of short-term scheduling is subject to the life-of-mine or long-term scheduling (Hustrulid and Kuchta, 1995). In the technical literature, several papers related to short-term production scheduling and fleet allocation were published. The first group of papers outline general concepts about short-term production scheduling, the second group of papers consider real-time fleet allocation, the third group of papers formulate deterministic short-term production schedule, which consider the total fleet capacity as the only limit on production per period, and the remaining papers are not related among them and are reviewed independently.

Early efforts in optimizing short-term mine production schedules focus on developing concepts and related formulations for deciding sequences of depletion based on mathematical programming (Wilke and Reimer, 1977; Wilke and Woehrle, 1979; Kahle and Scheafter, 1979; Gershon, 1982; Fytas and Calder, 1986; Schleifer, 1996). Accordingly, the outline of production progressions or extraction sequence on a daily, weekly, or monthly basis follows production targets set by the long-term mine production schedule. The optimization process considers the allocation of resources that match the available fleet capacity, the mine's layout and operational issues, such as mining direction. While accounting for the above, the objective function of related formulations is typically set to minimize production deviations from the yearly production plan targets; if these targets are met, then the expected long-term targets and overall mine valuation will likely be met. Key physical constrains are considered and include: the mobility of mining equipment and mineable extraction patterns, as well as quality constraints leading to blending of materials to extract so as to match quality feed targets for various ore processing streams. More recent work stays within the same context; for example, Vargas et al (2008) present a mathematical programming formulation accounting for quality and geometric constraints, mill and mine capacity. Similarly, Eivazy and Askari-Nasab (2012) account for multi-destinations, blending stockpiles and decisions on ramps, while their objective function minimizes the costs of mining, processing, waste rehabilitation, re-handling, and hauling. The latter two approaches have drawbacks, such as the use of aggregation of mining blocks prior to optimizing, leading to suboptimal solutions, as aggregation of materials ignores the practical selectivity of preferred ore types and cannot deal with the actual hauling process during the optimization process.

Typically, all the above work does not integrate a key aspect of short-term planning, namely, the management and dispatching of mining equipment/fleet. The real-time fleet allocation for short-term production planning is presented in Alarie and Gamache (2002), and Souza et al. (2010). A fleet dispatching system considers different allocation strategies given that transportation may represent 60% of operating costs. The solution strategies used in truck dispatching systems aim to improve productivity and reduce operating costs, however, the extraction patterns to be mined are assumed to be available. A shortcoming of these algorithms is that the whole tonnage of every pit is seen as a single macro block where the short-scale variability of the grade is lost and the one hour production and dynamic allocation of the fleet is only related to the dispatch system. L'Heureux et al. (2013) present a deterministic mixed integer programming model for short-term planning in open-pit mines. The sequence of mining of this model considers operational activities, such as drilling, blasting, transportation, ore processing capacity, the availability and the locations of shovels and drills. A drawback of this formulation is that the mined blocks by day are aggregated regular blocks. The definition of sectors to mine is usually linked to irregular patterns because of the local scale grade variability of the orebody and quality requirements.

Minimizing operating costs of trucks, since they represent the largest portion of the fleet in open pit mines, is presented by Topal and Ramazan (2010), and is formulated as an integer program. Maintenance costs not only are a significant proportion but also change non-linearly depending on the road conditions, truck age and truck types. The stochastic extension (Topal and Ramazan, 2012) of this model considers the uncertainty in truck maintenance costs for the available fleet when matching annual production targets. The approach provides a maintenance cost distribution of the optimized equipment schedule minimizing the cost.

However, similar to other aspects of short-term planning discussed above, this last work is done assuming a sequence of extraction.

An integrated approach to stochastic mine production scheduling is proposed herein based on stochastic integer programming. The approach presented is based on developments in long-term mine planning (Dimitrakopoulos and Ramazan, 2008; Ramazan and Dimitrakopoulos, 2013) and considers the possible fluctuation and uncertainty of the grade and ore quality, hauling time and fleet availability. It should be noted that grade and ore quality uncertainty is modelled through the generation of stochastically simulated scenarios of mineral deposit being mined (Goovaerts, 1997).

In the following sections, the proposed stochastic mathematical programming formulation for short-term mine production scheduling is described first. Then, an application at an iron ore mine presents the pertinent aspects and related intricacies of the proposed method, while also assessing its performance. Finally, conclusions and recommendations are provided.

2 Method

Short-term mine production scheduling is formulated as a stochastic integer programming model with recourse (Birge and Louveaux, 1997) and aims to minimize the total mining cost along with deviations from production targets, considers operational aspects such as mining direction and minimum width, and maximizes fleet utilization. In the formulation presented herein, the first-stage decisions are made before the uncertainty is revealed, then the second-stage decisions or recourse actions are made after uncertainty is considered. The notation used to formulate short-term scheduling follows. Note that indexes relate to the set of trucks, shovels, sectors, blocks, periods and realizations of uncertain parameters.

j: a sector or bench, where $j = 1, \ldots, J$

i: an shovel, where $i = 1, \ldots, I$

k: a block at sector, where $k = 1, \ldots, K(j)$

l: a truck model, where $l = 1, \ldots, L$

p: a period of a production schedule, where $p = 1, \ldots, P$

ε: an element grade of k block that have economical value, where $\varepsilon = 1, \ldots, E$

δ: a deleterious element grade of k block, where $\delta = 1, \ldots, D$

s: simulated grade realization or scenario, where $s = 1, \ldots, S$

α: realization of shovel mechanical availability given historical data, where $\alpha = 1, \ldots, A$

r: truck cycle trip and mechanical availability realization, where $r = 1, \ldots, R$

The parameters used at the fleet allocation, cost and penalties at objective function, production target and multi-element quality and tonnage are explained as follows:

h_{fleet}: fleet operation hours by period p

ι: maximum number of shovels allowed by sector

Q_i^{sh}: hourly production of shovel i

$\omega_i(\mu_i, \sigma_i)$: mean and standard deviation of historical mechanical availability by shovel i

$a_{ij'}^{p-1}$: binary parameter, if shovel i is or not allocated to sector j' at previous period $p-1$

$c_{j'j}^{ExcM}$: cost of moving shovel from $p-1$ allocation sector j' to new allocation sector j

$c^{prodExc-}$: penalty cost for tonnage not produced regarding to the expected productivity

Q_l^{trk}: capacity of truck l

$\phi_{jl}(\mu_{jl}, \sigma_{jl})$: mean and standard deviation of cycle time by truck l at sector j

$\psi_l(\mu_l, \sigma_l)$: mean and standard deviation of historical mechanical availability by truck l

c^{ϕ}: time cycle cost per ϕ units

c^{m-}, c^{m+}: penalty cost for shortage and surplus total mining tonnage respect to the targets

c^{o-}, c^{o+}: penalty cost for shortage and surplus ore mining tonnage respect to the targets

$c^{\varepsilon-}, c^{\varepsilon+}, c^{\delta+}, c^{\delta-}$: penalty cost for deviation from main elements and contaminants limits

$P^{\min}, P^{\max}$: minimum and maximum mining tonnage target

$O^{\min}, O^{\max}$: minimum and maximum ore tonnage target

$G^{\varepsilon-}, G^{\varepsilon+}, G^{\delta-}, G^{\delta+}$: quality or grade requirements for ore tonnage produced

$\%Tol_{o-}, \%Tol_{o+}, \%Tol_{m-}, \%Tol_{m+}, \%Tol_{\varepsilon-}, \%Tol_{\varepsilon+}, \%Tol_{\delta-}, \%Tol_{\delta+}$: allowed percentage of tonnage and grade deviation from targets

B_{jk}: block tonnage k at sector j

bh, ddh: ore control data and exploration data at mined sector A

BH, DDH: ore control data and exploration data at not mined sector B

c^m: mining cost by B_{jk} unit

$g_{jks}^{\varepsilon}, g_{jks}^{\delta}$: grade block k of main elements and deleterious in scenario s at sector j

O_{jks}: binary parameter flagging the block k at j sector for scenario s that has the minimum quality to be used at the blending process; otherwise, the block is flagged as waste

ϕ_{rjl}: truck cycle time r of truck l at sector j given cycle time distribution

θ_{rjl}: maximum number of trips of truck l at sector j for cycle hauling realization r and mechanical availability realization r

$$\theta_{jlr} = \frac{\psi_{lr} \times h_{fleet}}{\phi_{jlr}} \quad \forall r = 1, \ldots, R,\ \forall j = 1, \ldots, J,\ \forall l = 1, \ldots, L$$

$Q_{i\alpha}^{sh}$: maximum production rate of shovel i per mechanic availability realization α and each realization $\omega_{i\alpha}$ is drawn from the available mechanical availability distribution, and it is

$$Q_{i\alpha}^{sh} = \omega_{i\alpha} \times h_{fleet} \times Q_i^{sh} \quad \forall \alpha = 1, \ldots, A,\ \forall i = 1, \ldots, I$$

The decision variables used are as follows:

x_{jk}^p: binary variable, if block k at sector j is mined or not at period p

e_{ij}^p: binary variable, if shovel i is or not allocated to sector j at period p

n_{jilr}^p: number of trips of truck l to sector j, shovel i at period p for cycle time realization and mechanical availability realization r

$f_{ji\alpha}^p$: deviation of shovel i at sector j from expected shovel production $Q_{i\alpha}^{sh}$

y_{jk}^p: number of blocks that were not scheduled at period p to mine block k at sector j to match mining width requirements

d_p^{m-}, d_p^{m+}: shortage tonnage to match lower production limit and surplus tonnage to match upper production limit at period p

d_{sp}^{o-}, d_{sp}^{o+}: shortage of ore mining to match lower bound and the surplus to match upper bound at period p accounting for grade scenario s

$d_{sp}^{\varepsilon-}$, $d_{sp}^{\varepsilon+}$: deviation from ε grade targets at period p for grade scenario s

$d_{sp}^{\delta-}$, $d_{sp}^{\delta+}$: deviation from δ deleterious grade targets at period p for grade scenario s

2.1 Objective function

Decision variables x_{jk}^p, y_{jk}^p and e_{ij}^p are related with the first-stage and remaining decision variables are related with the second-stage. The first-stage decisions include minimizing the costs of extraction of materials, movement of shovels, production shortage, and lacking matching mining width. In the second-stage, these costs are minimized over a range of possibilities of a recourse cost associated with deviations from ore production and quality targets, hauling cost, and lack of mining with maximum shovel productivity. The objective function of the proposed mathematical model is:

$$
\begin{aligned}
Minimize = & \overbrace{\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{k=1}^{K(j)} c^m B_{jk} x_{jk}^p}^{1st} \\
& + \overbrace{\frac{1}{R}\sum_{p=1}^{P}\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{i=1}^{I} \phi_{jlr} c^{\phi} n_{jilr}^p}^{2nd} \\
& + \overbrace{\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{j'=1}^{J}\sum_{j=1}^{J}\left(c_{j'j}^{ExcM} e_{ij}^p a_{ij'}^{p-1}\right)}^{3rd} + \overbrace{\frac{1}{A}\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{\alpha=1}^{A}\left(c^{prodExc-} f_{ji\alpha}^p\right)}^{4th} \\
& + \overbrace{\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(c^{smoth-} y_{jk}^p\right)}^{5th} \\
& + \overbrace{\frac{1}{S}\left\{\sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{\varepsilon=1}^{E}\left(c^{\varepsilon-} d_{sp}^{\varepsilon-} + c^{\varepsilon+} d_{sp}^{\varepsilon+}\right) + \sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{\delta=1}^{D}\left(c^{\delta+} d_{sp}^{\delta+} + c^{\delta-} d_{sp}^{\delta-}\right)\right\}}^{6th} \\
& + \overbrace{\frac{1}{S}\sum_{s=1}^{S}\sum_{p=1}^{P}\left(c^{o+} d_{sp}^{o+} + c^{o-} d_{sp}^{o-}\right)}^{7th} + \overbrace{\sum_{p=1}^{P}\left(c^{m-} d_p^{m-} + c^{m+} d_p^{m+}\right)}^{8th}
\end{aligned}
\tag{1}
$$

The first component of the objective function is associated with the cost of extracting material from the mine. The second component corresponds to minimizing the hauling cost given the uncertainty in the trucks' hauling time and mechanical availability so as to ensure both optimal allocation and maximum track utilization. The third component is the minimization of cost of the shovel movements among sectors. The fourth component minimizes the lack of production per shovel given uncertainty in its mechanical availability, so as to maximize shovel utilization. The fifth term ensures that the operational considerations are respected by penalizing the lack of mining blocks that match the required mining width. The sixth, seventh and eighth components deal with the minimization of geological risk with respect to the quality and the quantity of ore production, and penalize deviations from production targets, respectively. Note that the first, second, fourth, sixth and seventh components are stochastic and contain decision variables that change, given the corresponding realizations of the fleet parameters or element quality. The deterministic solution would be generated if instead of realizations the related components consider the single corresponding average value.

The decision variables are present in the objective function and there are several constraints linking the fleet allocation decision variables with mined block decision variables. This ensures that the formulation herein delivers a short-term production schedule that account for fleet allocation and production targets.

2.1.1 Constraints for production and fleet allocation

The constraints below link the fleet allocation decision variables with mined block decision variables, to guarantee that the short-term production schedule accounts for fleet allocations and production targets.

$$\sum_{p=1}^{P} x_{jk}^{p} \leq 1, \quad \forall j = 1, \ldots, J, \; \forall k = 1, \ldots, K(j) \tag{2}$$

Constraint (2) ensures that a block of material may be mined once at any period. The block is a selective mining unit that may be mined in one period assuming that the time period may be from weeks to months.

$$\sum_{i=1}^{I} e_{ij}^{p} \leq \iota, \quad \forall p = 1, \ldots, P, \; \forall j = 1, \ldots, J \tag{3}$$

$$\sum_{j=1}^{J} e_{ij}^{p} \leq 1, \quad \forall p = 1, \ldots, P, \; \forall i = 1, \ldots, I \tag{4}$$

$$x_{jk}^{p} - \sum_{i=1}^{I} e_{ij}^{p} \leq 0, \quad \forall p = 1, \ldots, P, \; \forall j = 1, \ldots, J, \; \forall k = 1, \ldots, K(j) \tag{5}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \phi_{jlr} \times n_{jilr}^{p} \leq h_{fleet} \times \psi_{lr} \quad \forall p = 1, \ldots, P, \; \forall l = 1, \ldots, L, \; \forall r = 1, \ldots, R \tag{6}$$

$$n_{jilr}^{p} - \theta_{jlr} e_{ij}^{p} \leq 0, \quad \forall p = 1, \ldots, P, \; \forall r = 1, \ldots, R, \; \forall j = 1, \ldots, J, \; \forall l = 1, \ldots, L, \; \forall i = 1, \ldots, I \tag{7}$$

$$\sum_{l=1}^{L} \left(Q_{l}^{truck} \times n_{jilr}^{p} \right) - Q_{i\alpha}^{sh} \times e_{ij}^{p} + f_{ji\alpha}^{p} = 0 \quad \forall p = 1, \ldots, P, \; \forall j = 1, \ldots, J, \; \forall i = 1, \ldots, I, \; \forall \alpha = 1, \ldots, A, \; \forall r = 1, \ldots, R \tag{8}$$

$$\sum_{i=1}^{I} \sum_{l=1}^{L} \left(Q_{l}^{truck} \times n_{jilr}^{p} \right) - \sum_{k=1}^{K(j)} \left(B_{jk} \times x_{jk}^{p} \right) = 0, \quad \forall p = 1, \ldots, P, \; \forall j = 1, \ldots, J, \; \forall r = 1, \ldots, R \tag{9}$$

The mining equipment can be placed in a given number of locations. A possible path of the locations of each piece of equipment is provided as part of short-term plan. Shovels are allocated to available sectors or remain in the current sector previously allocated. A sector must be mined at some period and a shovel must be allocated to the sector that has a lower cost of hauling and provides the material to match quality requirements. Constraints (3) ensure that each sector is allocated with less equal than ι shovels at sector j per period p. The parameter ι is the maximum number of shovels that can be allocated in each sector. Constraint (4) ensures that each shovel i may be assigned to one sector while the cost of movement is minimized in the objective function to prevent excessive shovel movement among sectors. Inequality constraints are used for the fleet allocation because not all the available shovels or trucks are allocated in scenarios where there are more equipment than the production requires in accounting for hauling distance. Constraint (5) guarantees that a mining block in sector j is mined only if a shovel is allocated to sector j.

Variable n_{jilr}^{p} decides the optimal number of trips for truck l to sector j and shovel i per period p, thus accounting for fluctuations of truck cycle time and mechanical availability. The number of trips decision variable n_{jilr}^{p} also supports in the allocation of each truck l to shovel i to sector j for mechanical availability and hauling realization r per period p. The formulation considers that a truck can be allocated to more than one shovel at the same sector j or different sectors. Constraint (6) limits the number of trips of a truck to

its scheduled time per period as the operation progresses by extracting minerals and continuously extending the access. Indeed, the roads change dynamically. This implies uncertainty in the hauling time. The trip cycle time ϕ_{jlr} of truck l to sector j is drawn from distribution R times.

The decision variable n^p_{jilr} is also subject to the maximum number of trips that a truck l can haul from each sector j. The maximum number of trips θ_{jlr} per truck l is a preprocessed parameter because its components are not decision variables. Then, the number of total trips to each sector is restricted to a maximum number of trips times the e^p_{ij} binary decision variable. The decision variable e^p_{ij} is relevant in the constraints (7) because not all the sectors will be allocated with a shovel and a sector without a shovel cannot have number of trips. Decision variables n^p_{jilr} and e^p_{ij} are linked. The inequality constraint (7) also ensures that only an allocated sector with a shovel is assigned with trucks, and not all trucks are allocated at some scenarios. The link of truck l, shovel i and sectors j in the constraints ensure that all assignment possibilities for the trucks, shovel and sectors are taken into account.

There are capacity limits for each truck Q^{trk}_l and shovel Q^{sh}_i. The available fleet and their respective capacity are included in the formulation. The production of each shovel assigned to sector j is constrained to the maximum production of each shovel $Q^{sh}_{i\alpha}$. The e^p_{ij} binary decision variable helps to formulate the shovel capacity constraints (8) because not all of the shovels may be allocated. The lack of expected production by each shovel is stored by the decision variable $f^p_{ji\alpha}$, which is minimized at the objective function.

There are J sectors and each sector has $K(j)$ blocks to be evaluated. The tonnage of block k is B_{jk} and each block may be hauled from an in-situ location to a blending area or waste dump taking into account the fleet capacity constraints. The decision variables at operational and production constraints are linked to fleet allocation constraints. Indeed, constraint (9) links number of trips n^p_{jilr} of truck l from sector j and shovel i given mechanic availability and hauling time realization r with the mined block decision variable x^p_{jk}. The hauling tonnage by the trucks from sector j for mechanical availability and hauling time realization r must be equal to scheduled blocks tonnage at sector j.

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J} Q^{trk}_l \times n^p_{jilr} \geq M^{\min} \quad \forall p=1,\dots,P,\ \forall r=1,\dots,R \tag{10}$$

$$0 \leq d^{m-}_p \leq \%Tol_{m-} \times M^{\min} \quad \forall p=1,\dots,P \tag{11}$$

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J} Q^{trk}_l \times n^p_{jilr} - d^{m+}_p \leq M^{\max} \quad \forall p=1,\dots,P,\ \forall r=1,\dots,R \tag{12}$$

$$0 \leq d^{m+}_p \leq \%Tol_{m+} \times M^{\max} \quad \forall p=1,\dots,P \tag{13}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(O_{jks} \times B_{jk} \times x^p_{jk}\right) + d^{o-}_{sp} \geq O^{\min} \quad \forall p=1,\dots,P,\ \forall s=1,\dots,S \tag{14}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(O_{jks} \times B_{jk} \times x^p_{jk}\right) - d^{o+}_{sp} \leq O^{\max} \quad \forall p=1,\dots,P,\ \forall s=1,\dots,S \tag{15}$$

$$0 \leq d^{o-}_{sp} \leq \%Tol_{o-} \times O^{\min} \quad \forall p=1,\dots,P,\ \forall s=1,\dots,S \tag{16}$$

$$0 \leq d^{o+}_{sp} \leq \%Tol_{o+} \times O^{\max} \quad \forall p=1,\dots,P,\ \forall s=1,\dots,S \tag{17}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(B_{jk} \times \left(g^{\varepsilon}_{jks} - G^{\varepsilon-}\right) \times O_{jks} \times x^p_{jk}\right) + d^{\varepsilon-}_{ps} \geq 0 \quad \forall p=1,\dots,P,\ \forall s=1,\dots,S,\ \forall \varepsilon=1,\dots,E \tag{18}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(B_{jk} \times \left(g^{\varepsilon}_{jks} - G^{\varepsilon+}\right) \times O_{jks} \times x^p_{jk}\right) - d^{\varepsilon+}_{ps} \leq 0 \quad \forall p=1,\dots,P,\ \forall s=1,\dots,S,\ \forall \varepsilon=1,\dots,E \tag{19}$$

$$0 \leq d^{\varepsilon-}_{sp} \leq \%Tol_{\varepsilon-} \times O^{\min} \times G^{\varepsilon-} \quad \forall p=1,\dots,P,\ \forall s=1,\dots,S,\ \forall \varepsilon=1,\dots,E \tag{20}$$

$$0 \leq d^{\varepsilon+}_{sp} \leq \%Tol_{\varepsilon+} \times O^{\max} \times G^{\varepsilon+} \quad \forall p=1,\dots,P,\ \forall s=1,\dots,S,\ \forall \varepsilon=1,\dots,E \tag{21}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(B_{jk}\times\left(g_{sjk}^{\delta}-G^{\delta+}\right)\times O_{jks}\times x_{jk}^{p}\right)-d_{sp}^{\delta+}\leq 0\quad\forall p=1,\ldots,P,\ \forall s=1,\ldots,S,\ \forall\delta=1,\ldots,D \quad (22)$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(B_{jk}\times\left(g_{sjk}^{\delta}-G^{\delta-}\right)\times O_{jks}\times x_{jk}^{p}\right)+d_{sp}^{\delta-}\geq 0\quad\forall p=1,\ldots,P,\ \forall s=1,\ldots,S,\ \forall\delta=1,\ldots,D \quad (23)$$

$$0\leq d_{sp}^{\delta+}\leq \%Tol_{\delta+}\times O^{\max}\times G^{\delta+}\quad\forall p=1,\ldots,P,\ \forall s=1,\ldots,S,\ \forall\delta=1,\ldots,D \quad (24)$$

$$0\leq d_{sp}^{\delta-}\leq \%Tol_{\delta-}\times O^{\min}\times G^{\delta-}\quad\forall p=1,\ldots,P,\ \forall s=1,\ldots,S,\ \forall\delta=1,\ldots,D \quad (25)$$

Production per time period p is constrained to the production targets (10). The production includes ore tones plus the waste tones. The ore tonnage is the material that has positive economic value meanwhile the waste tonnage is the material without positive economic value that needs to be extracted to allow access to ore and ensure continuity of ore production in the following periods. The number of trip decision variables and truck capacities are used to calculate the total tonnage extracted per period. The proposed model considers strict constraints for early periods and can be relaxed for the latest periods. To relax the production constraints, the shortage d_p^{m-} with respect to the target planned is considered, along with their respective tolerance of deviation (11). Traditionally, an upper bound is not used in production formulation because the cost of mining will limit overproduction; however, at the current formulation the production must be limited because the capacity shovel constraints maximize the production by sector to increase the utilization of the shovel (12). The upper bound limits this maximization to keep close to the production targets. The deviation d_p^{m+} with respect to the upper bound total production is penalized in the objective function and their tolerance is considered (13).

As a production constraint, the ore tonnage should match the target ore production given by long-term production schedules (14,15). The shortage d_{sp}^{o-} respects to the target planned and the surplus d_{sp}^{o+}, respects the upper bound ore processing and are penalized in the objective function. The deviations are limited by a percentage of ore production $\%Tol$ (16,17). The upper bound is directly related to the ore tonnage scheduled plus the maximum capacity pile of ore next to the delivering location. The exceeding material from the upper bound may be considered as material that go to stockpile, and its tonnage are penalized by the corresponding re-handled cost.

Ore production must match certain quality constraints, that is, the expected grades or quality of the material at the end of the week or month must fit into specific ranges and this range depends on long-term production schedule specifications. To meet this demand, a block x_{jk}^{p} is mined only if their grade helps to satisfy the required quality given the available fleet. Assuming that the study case has E elements that have economic value and D elements as deleterious elements, $2(E+D)$ quality constraints are needed to meet quality conditions. The grade of the main commodity for ore tonnage should satisfy the constraints (18,19) and the quality deviations have tolerance (20,21) to ensure a production schedule with low variable average quality. Ore production cannot have more than the required limits of contaminants because this contains D deleterious elements. The constraints (22,23) ensure that the ore delivered by period given S scenarios of the grades have average grades less than $G^{\delta+}$ and more than $G^{\delta-}$ for deleterious element $\delta=1,\ldots,D$. The quality deviations related with contaminants are also constrained to tolerance (24,25) to ensure production schedule with low variable average quality.

Blending of ore from sectors is carried out based on cutoffs that define the minimum quality that a block k must have to be included in the blending process. If a block k has the chance of being used for blending $O_{jks}=1$; otherwise, the block k is allocated to the waste dump directly $O_{jks}=0$. The quality constraints are satisfied when the total ore production meets the required quality conditions set as targets.

2.1.2 Constraints for operational considerations

Operational considerations relate to the size of the equipment and accessibility restrictions that may require feasible, in a mining sense, production schedule patterns that allow the available equipment to work efficiently and streamline movements for safety reasons. The first operation consideration is the mining direction that

facilitates access to the sectors to be mined and it is:

$$x^p_{jk} - \sum_{\tau=1}^{p} x^{\tau}_{jk'} \leq 0, \quad \forall p = 1, \ldots, P, \forall j = 1, \ldots, J, \forall k = 1, \ldots, K(j), k' \in \Omega_{k'} \tag{26}$$

where $\Omega_{k'}$ is the set of indexes representing blocks that are horizontal predecessors which must be mined before block k to match the mining direction. A sector could be mined following eight directions, as shown in Figure 1.

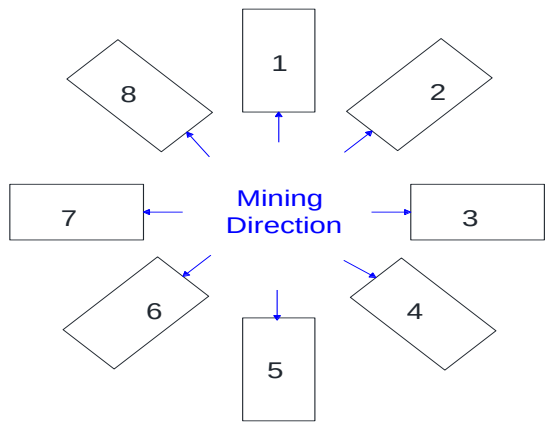

Figure 1: Eight mining directions considered by the formulation

The second operational consideration is the mining width and relates to the minimum width the patterns of a short-term schedule period has that permits fleet access to the orebody and materials that need be extracted. Production schedules without accounting for mining width may deliver schedule patterns with singular blocks of early periods surrounded by blocks from later period, as shown in Figure 2. This production scheduling cannot be implemented as the blocks scheduled for period 1, (blue squares) cannot be mined before some blocks belonging to period 2 (orange squares) are extracted.

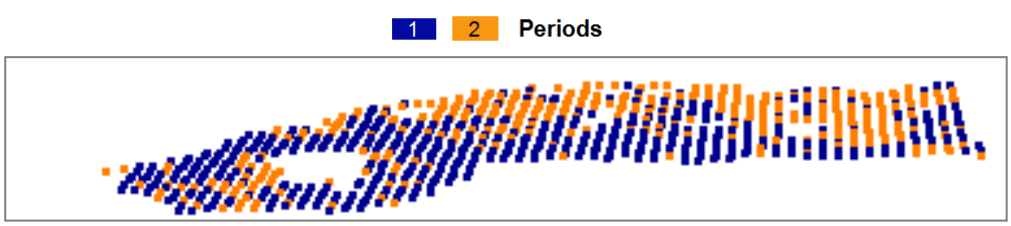

Figure 2: Production scheduling without mining width constraints

The following mining width constraints account for feasible extraction patterns and may force the mining of some blocks before a given block k as shown in Figure 2.

$$\begin{aligned} -2 \times x^p_{jk'} - x^p_{jk''} + (2 \times \upsilon + \upsilon) \times x^p_{jk} - y^p_{jk} \leq 0, \quad & \forall p = 1, \ldots, P, \forall j = 1, \ldots, J, \forall k = 1, \ldots, K(j) \\ & k^{'} \in \Psi_{k'}, k^{''} \in \Psi_{k''} \end{aligned} \tag{27}$$

The mining width is discretized into υ blocks where $\Psi_{k''}$ is its set of indexes. To mine a block k, υ blocks may be mined at the same period or have been mined at previous periods. $\Psi_{k'}$ is the set of indexes representing the adjacent blocks and priority of mining adjacent blocks ν is considered to avoid single blocks from some periods being surrounded by blocks from different periods. Indeed, the blocks ν that surround block k must be mined with twice the priority than the second term at constraints (27) to avoid infeasible mining patterns. The adjacent ν blocks belong to the inner window and the υ blocks belong to the outer

window in smooth constraints (Dimitrakopoulos and Ramazan 2004). These smooth mining constraints are linked to mining width to provide feasible mining sequences that the fleet requires to operate efficiently. It is important to remark that υ number of blocks that match mining width are variable through the sector. The blocks that are located close to the border will require less υ blocks to be moved because some blocks were already mined or are 'air' (non-physically existing) blocks.

The mining width constraints are relaxed because at some locations feasible solutions will require to mine only some υ blocks. The discrete decision variable y^p_{jk} will store the lack of mining blocks that match the mining width considerations. This decision variable is penalized and minimized at the objective function.

3 Application in an iron ore mine

The proposed stochastic short-term production schedule (SSTPS) formulation is applied at an iron deposit. Iron ore deposits are typical examples of a multi-element environment, where the main production objective is to satisfy the customer quality requirement at a lower cost by optimally blending the different sectors of a mine. More specifically, when the iron content is evaluated and must be within customer specified limits there are also specific restrictions on the content of the so-called deleterious elements, such as phosphorous (P), silica (SiO_2), alumina (Al_2O_3) and the water and organic content measured as "loss on ignition" (LOI). These deleterious elements influence the physical and chemical properties of the iron ore product, significantly varies from customer to customer and contractual agreement to be met, and the performance of the process it will be used for. For instance, phosphorous affects steel quality (added cost), high silica and high alumina affect furnace efficiency, and the LOI affect fuel use and water in a hot furnace for steel making.

As noted earlier, the stochastic long-term production scheduling (SLTPS) of a given mine provides the larger scale framework defining the targets production of the short-term production schedule. Figure 3, for example shows the long-term production schedule of the iron mine in this case study and contains five periods (years). The first year (dark blue in the figure) is used herein for short-term production scheduling which is optimized over twelve periods (months).

The quality targets and tonnes for the SSTPS and for the first year of production considered herein are given see Table 1. From the first year tonnage in Table 1; the mine must produce iron ore of about 1.16 millions of Iron tonnes each month. The average grade of the related elements per month may be in the intervals of the first year long-term ore quality given; however, the spatial variability of these grades varies when monthly increments are considered and along the mining direction and operational mining width. Ore quality intervals correspond to the upper bound and lower bound per element over the total year.

Table 1: First year production quantity and quality requirements

Period	**Ore Tonage**	**Fe_2O_3(%)**	**P(%)**	**SiO_2(%)**	**Al_2O_3(%)**	**LOI(%)**
1.00	14,000,000	57.1–59.4	0.032–0.038	4.6–5.2	0.9–1.05	9.5–11

The iron ore may be extracted from blocks of $25\times25\times12$ m^3 located at three consecutive mining benches of 12m height. For this case study, ten equally probable scenarios of iron content, phosphorous, silica, aluminum and LOI are used to quantify the joint uncertainty in the characteristics of the iron ore deposit considered and are the input to the SSTPS formulation proposed in the previous section. The simulated scenarios available were provided and generated using the stochastic simulated technique detailed in Boucher and Dimitrakopoulos (2009, 2012). The area considered is bounded by the limits of the given volume of production in the long-term first year production schedule provided. Figure 4 shows 3 scenarios of iron ore content as well as the corresponding conventional and single estimated (average) representation of iron content (Fe_2O_3%) for the upper bench. In total, 734 blocks from 3525 to 21150 tonnes, with Fe_2O_3 from 54.59% to 60.63%, P from 0.02% to 0.04%, SiO_2 from 3.10% to 8.58%, Al_2O_3 from 0.53% to 1.88% and LOI from 8.75% to 11.75% are available.

In addition to the uncertainty of the materials being extracted addressed above, the parameters related to the mining fleet available are given, so as to allocate efficiently and maximize the utilization of this fleet.

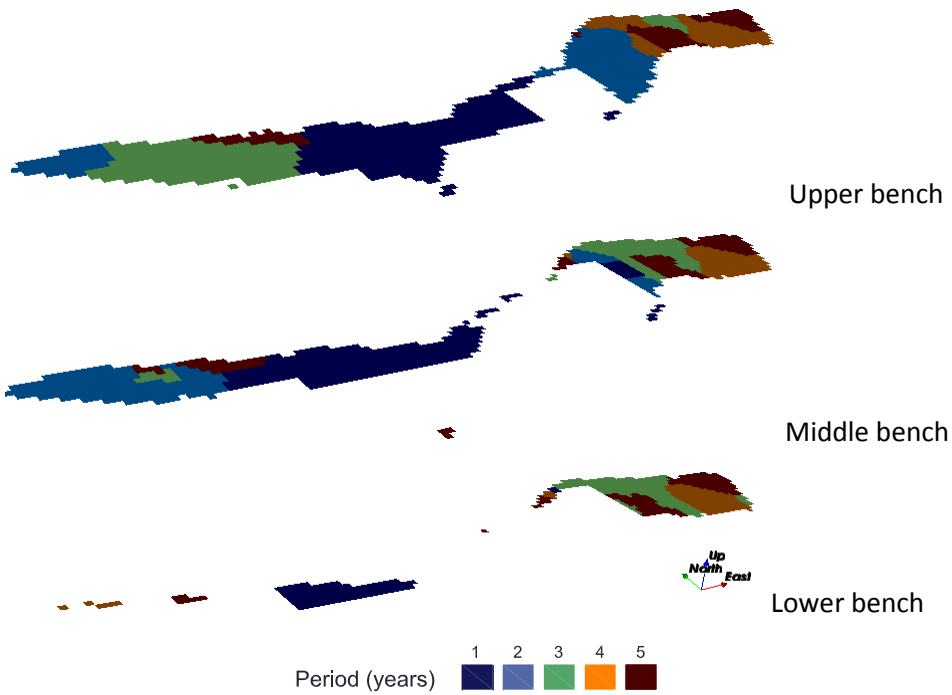

Figure 3: Stochastic long-term mine production schedule, 5 periods located at three benches (upper middle and lower), modified from (Benndorf 2005)

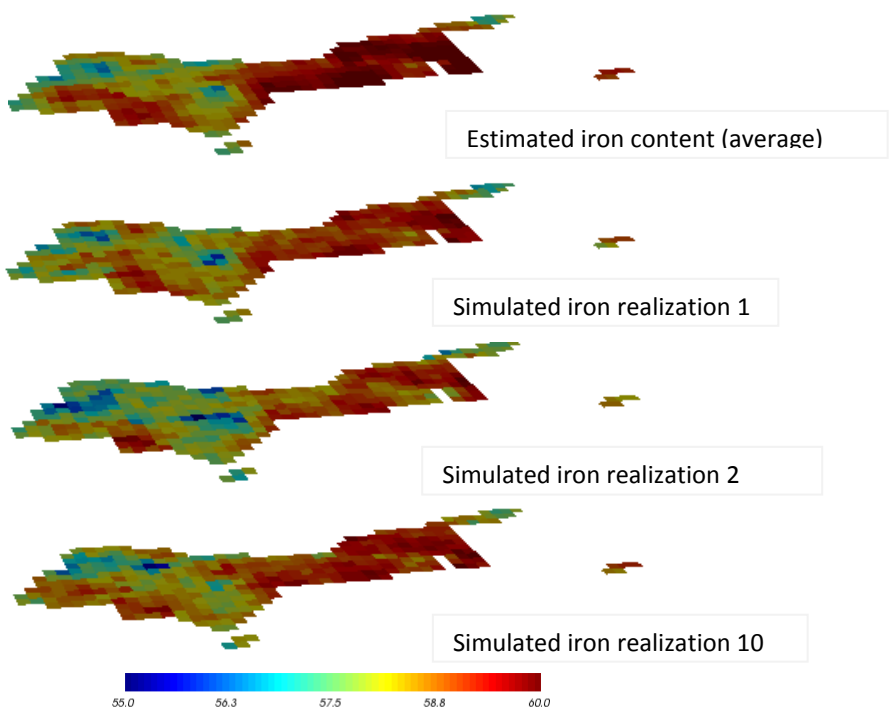

Figure 4: Iron ore content within the sector to be mined in the first year of production (Figure 3); 3 stochastically simulated realizations and the deterministic estimate for the upper bench (extraction units of $25\times25\times12$ m^3)

The fleet size, mechanical availability and hauling time from the orebody to the various destinations are parameters used to allocated shovels and trucks at the related mine sectors. For this case study two shovels and ten trucks are the available fleet. The hourly productivity of each shovel fluctuates between 1180 and 1400 tonnes. The shovel model, digging rate and mechanical availability parameter distributions are given along with the track model, capacity and mechanical availability parameter distribution per truck.

Table 2: Shovel model and mechanical availability parameter distribution

			Mechanical Availability (%)	
Model	**Shovel (i)**	**Production (Tonnes/hour)**	**Mean**	**Std.Dev.**
HS6020	1	1180	83	4.5
HS6030	2	1400	83	4

Table 3: Truck model and mechanical availability parameter distribution

			Mechanical Availability (%)	
Model	**Truck (I)**	**Tonnes**	**Mean**	**Std.Dev.**
Cat785D_501	1	136	83	5
Cat785D_502	2	136	83	4
	:	:	:	:
Cat785D_510	8	136	83	4
Cat77G_511	9	100	83	5
Cat77G_512	10	100	83	5

Short-term evaluation has the advantage of accounting for additional short-term information such as the hauling distance that is available at the short-term evaluation. This supports the allocation of trucks because the past records of speed per truck, truck hauling time per sector in a mine and blending pad location are available. Additionally, the parameter distribution of the time that spends l truck from the sector j to the destination is calculated as shown in Table 4. The cycle time ϕ_{jlr} from sector j to destination will be drawn r times from the respective distribution and the maximum trips are calculated given the mechanical availability per truck. The parameters used to implement the proposed SSTPS formulation proposed herein are given in Table 5.

The total tonnage to be mined after twelve months of production is approximately 14,400,000 iron ore tonnes, and given the ore cut-off >= 56% Fe_2O_3 almost all the material will be mined as ore. The targets of production and actual ore production are quite similar. Note that a high penalty is applied to the lack of

Table 4: Trucks cycle time and parameter distribution (ϕ_{jlr})

		Cycle time (minutes)	
Sector (j)	**Truck (I)**	**Mean**	**Std.Dev.**
1	1	32	2.8
1	:	:	:
1	10	32	3.3
2	1	25	2.6
2	:	:	:
2	10	25	3.1
3	1	20	2.5
3	:	:	:
3	10	20	3

Table 5: Target month production and parameters

Production Target	Parameter	Value	Unit	Penalty
	Max production	1,210,000	Tonnes	160
	Min production	1,100,000	Tonnes	160
	Max Ore production	1,210,000	Tonnes	16
	Min Ore production	1,000,000	Tonnes	4
	Allowed deviation tolerance	<= 10	%	
Quality Requirement	**Parameter**	**Value**	**Unit**	**Penalty**
	Iron Ore (Fe_2O_3)	57.1–59.4	%	1
	Phosphorous	0.032–0.038	%	10
	Silica	4.6–5.2	%	10
	Alumina	0.9–1.05	%	10
	Loss on ignition	9.5–11	%	1
	Allowed deviation tolerance	<= 10	%	
Ore Definition	**Parameter**	**Value**	**Unit**	
	Fe_2O_3	>= 56	%	
Economic Parameters	**Parameter**	**Value**	**Unit**	
	Mining Cost*	40	$/Tonne	
	Cycle Time Cost	120	$/hour	
	Shovel Moving Cost	1000	$/100 meters	

* Not include hauling cost

mining from the expected monthly production because all material scheduled for the twelve months must be mined to align short-term production with long-term planning targets.

3.1 Short-term scheduling under uncertainty

The uncertainty in the iron grade and deleterious elements, mechanical fleet availability and hauling time are a major source of uncertainty that is incorporated into the production schedule formulation presented in Section 2. Figure 5 (left) shows the SSTPS production schedule at the iron ore mine in this application. For reasons of comparison, Figure 5 (right) shows the corresponding deterministic schedule generated from the deterministic equivalent of the SSTPS presented in Section 2, based on the average values for all related inputs. It is important to stress that the deterministically generated schedule may not be feasible in the actual presence of uncertainty that is not accounted for but is present.

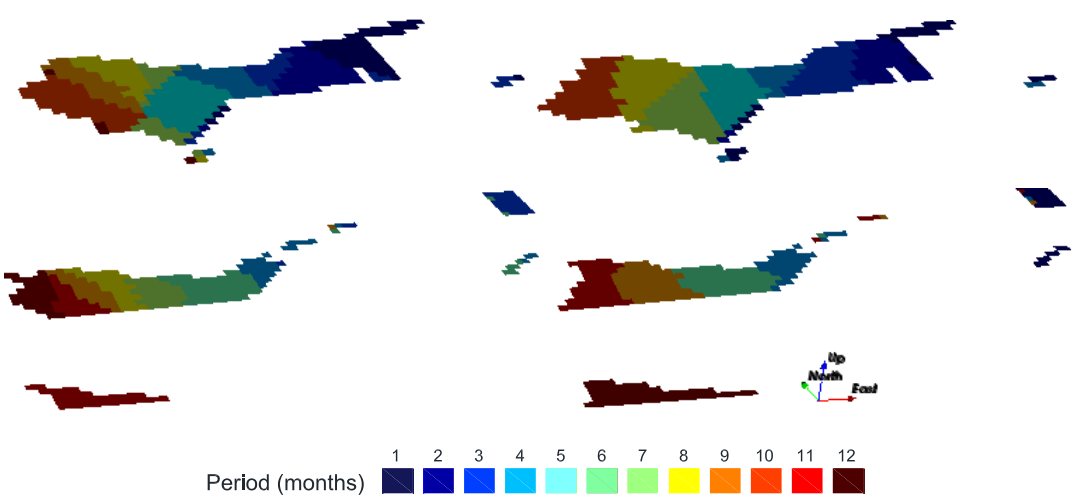

Figure 5: The stochastic short-term schedule (left) and the deterministic schedule (right)

Both production schedules in Figure 5 consider the same operational considerations and allocate similar sectors of the iron ore deposit to be mined until the 5th month of production; then the effect of uncertainty becomes evident as not enough materials are located at the upper bench to match quality requirements and the fleet is moved to lower benches

3.2 Utilization of the fleet

Maximum expected shovel production is planned given mechanical availability and scheduled time. The lack of matching this expected production is penalized to maximize the utilization of the shovels. The shovel utilization accounting for uncertainty at the SSTPS solution results in a higher and less variable than the shovel utilization of the deterministic STPS solution. From the STPS solution, the shovel with a historically high production was allocated preferentially to a sector that ensures its better utilization and the shovel with a historically low production to sectors with high production uncertainty.

The utilization of each shovel and the trucks are not exactly proportional because the trucks can be assigned to many shovels per period meanwhile the shovel is assigned to a sector and their movement between sectors are restricted by the cost associated. For example the small shovel HS6020 (Figure 6, upper) is allocated to the sector that has less available material to be mined making its utilization low at the final periods; however, the availability of the trucks are not affected because the trucks can be allocated to different sectors as shovels are allocated for each period. This ensures high utilization of the trucks, as shown in Figure 7.

In some periods not all the trucks need to be allocated to match production targets. Considering that both schedules match production targets, the SSTPS shows a more efficient allocation or high utilization than the deterministic STPS because it allocates a lower number of trucks, as shown in Figure 8, that is, the fleet allocation accounting for deterministic truck parameters is inefficient when compared to that accounting for possible fluctuations of mechanical availability and hauling time

The stochastic formulation provides a well-informed schedule because it accounts for possible fluctuations of the grades and fleet parameters. From Figure 6 and Figure 7, the utilization of the fleet is shown as less variable through the periods when the uncertainty is considered.

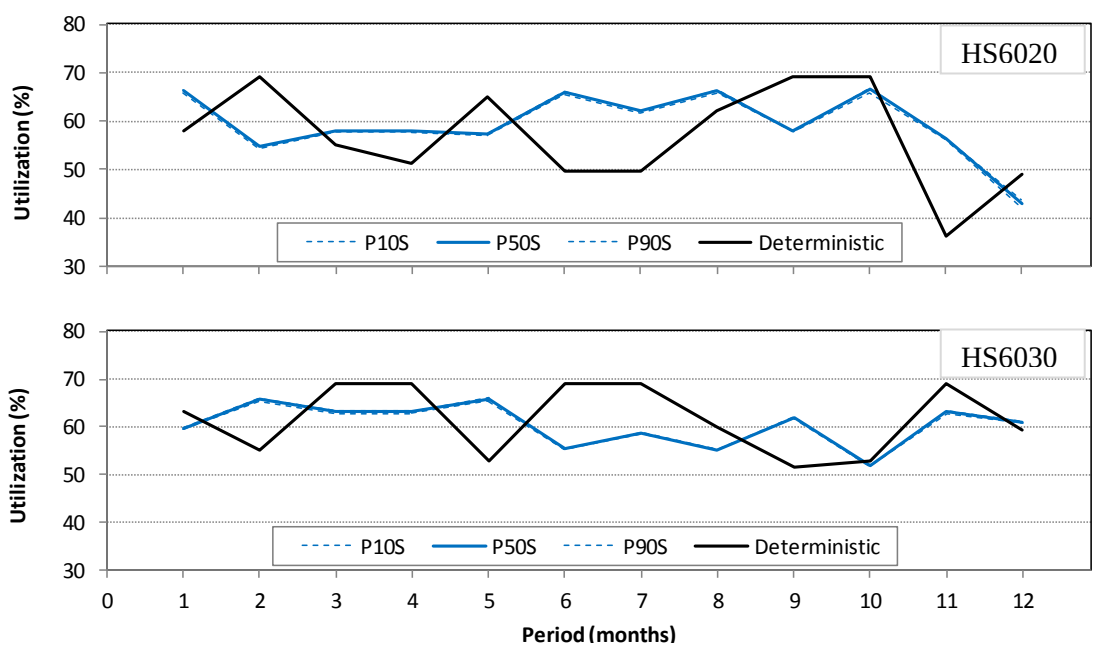

Figure 6: Utilization risk profiles of shovels for SSTPS on blue lines

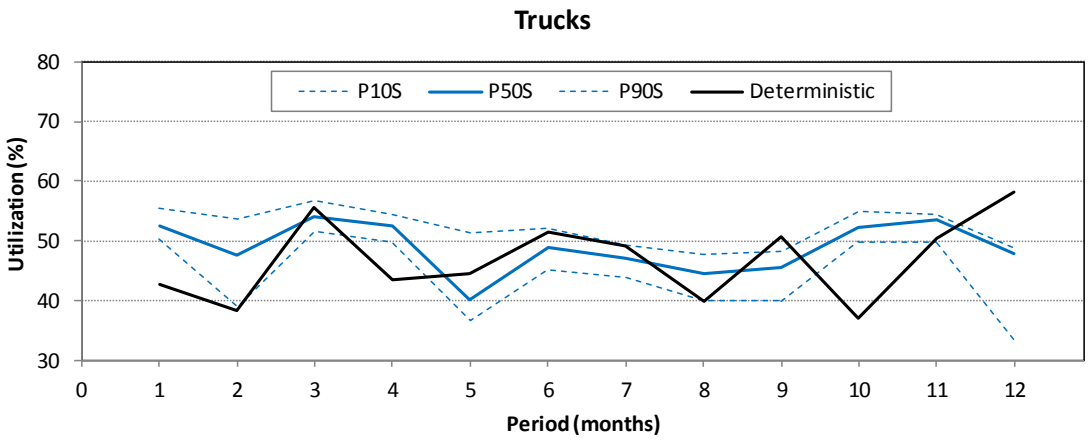

Figure 7: Utilization risk profiles of trucks for SSTPS on blue lines

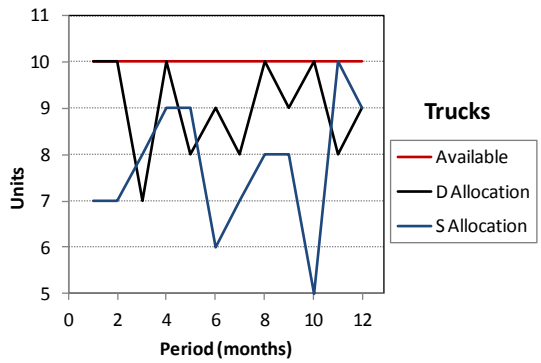

Figure 8: Available trucks in red line, number of trucks allocated accounting for four source of uncertainty on blue line and without accounting for uncertainty on black line

3.3 Cost in the objective function

The objective function consider some terms associated with operating fleet cost, mining cost, and penalty cost to penalize deviation from production target, expected fleet utilization and mining width. The penalties cause some terms to have more priority than the others because the optimization preferably minimizes the components that have high value. The stochastic short-term production schedule solution shows less cost through the terms in the objective function than the deterministic schedule solution. From the formulation, the minimized cost means that the plan guarantees the minimum deviation from the production targets, such as tonnage and quality, maximum utilization of the fleet, minimum cost of production extraction and a better match of the mining width considerations. Indeed, the best production schedule may be the one that obtains the lower minimized cost.

Figure 9 shows that the mechanical availability shovel source of uncertainty has more influence in the stochastic solution than the uncertainty in the parameters of the trucks and orebody uncertainty. Also, the proposed stochastic formulation in an iron ore deposit provides an improvement cost of about fifteen million CAD dollars less than the deterministic or production schedule that ignores parameter uncertainty. The deterministic STPS formulation cannot minimize in the same range as the stochastic STPS does because the uncertainty in the parameter is not accounted for. The uncertainties in the mechanical availability, in the orebody model and in the hauling time give more feasible solutions in the solution space to choose the best solution.

The formulation proposed for stochastic short-term production scheduling obtains the solution with the lower cost in the application at a multi-element ore iron mine; however, the robustness of this formulation is

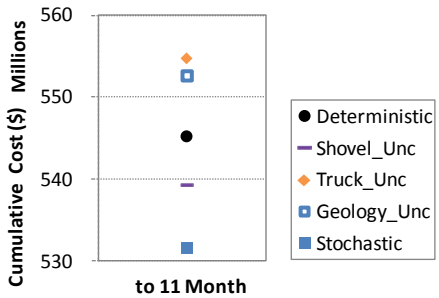

Figure 9: Cumulative minimized objective cost

based on the idea that their schedule is a well informed plan because it accounts for operations considerations, possible fluctuations of the orebody metal quality and fluctuations of the fleet parameters to decide which sector to be mined per period.

4 Conclusions

The formulation of the stochastic mixed integer programming model proposed herein for short-term mine production scheduling is developed as a single formulation where mining considerations, production constraints, uncertainty in the orebody metal quantity, as well as fleet parameters, are evaluated together to define a well-informed sequence of mining that has high performance at the mine operation. The quality of material may also influence the allocation of fleet. Indeed, the optimization allocates the fleet to sectors that ensure the accomplishment of the production target, match the quality conditions, maximize fleet utilization, respect the operational considerations and account for uncertainty in the parameters. The components of the objective function are in terms of costs where the minimized total cost implies that the plan guarantees the minimum deviation from production target, maximum utilization of the fleet, minimum cost of production extraction and better match to mining width requirements. At the time of short-term production scheduling, additional information related to the operational restrictions, such as mining width and mining directions, are available. These additional physical constraints were implemented to deliver feasible production schedule patterns that will have better performance during the operations.

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