Les Cahiers du GERAD ISSN: 0711–2440

Incentive mechanisms for price and advertising coordination in dynamic marketing channels

S. Taboubi

G–2016–50

June 2016

GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Incentive mechanisms for price and advertising coordination in dynamic marketing channels

Sihem Taboubi

[a] *GERAD & Marketing Department, HEC Montréal, Montréal (Québec) Canada H3T 2A7*

sihem.taboubi@hec.ca

June 2016

Les Cahiers du GERAD

G–2016–50

Abstract: This paper examines the issue of price and advertising coordination in bilateral monopolies from a dynamic perspective. Its main objectives are to design incentive mechanisms that the manufacturer can use in order to reduce double marginalization, and to investigate whether these mechanisms can be implemented by the manufacturer and accepted by the retailer. To answer these questions, we consider a differential game where a manufacturer controls its wholesale price and its national advertising effort. The carry-over effects of national advertising build up brand goodwill (i.e., equity or image), which represents the model's state variable. The retailer controls the product's retail price and the local advertising effort, which affect consumer demand, in conjunction with the brand goodwill. We compute strategies and outcomes under three scenarios: (i) a fully coordinated scenario where channel members act as if the channel was vertically integrated, (ii) a non coordinated scenario, considered as the status quo, where the channel is decentralized, and (iii) a scenario where incentive mechanisms are offered by the manufacturer. The main results indicate that two incentive mechanisms, namely, a wholesale-price reduction and an advertising allowance offered by the manufacturer to the retailer, mitigate the double-marginalization problem since they push both channel members to decrease their margins relative to the status-quo scenario. Furthermore, with these incentives, the manufacturer invests more in national advertising than under the status quo. Interestingly, this level is different (i.e., lower) than the fully coordinating level of national advertising. This means that with the incentives, the manufacturer can induce the retailer to fix its control variables at their fully coordinating levels without committing to implementing the fully coordinating level of national advertising. Demand and goodwill are higher with the incentives, and both channel members benefit from an increase in their individual profits (i.e., relative to the status quo).

Keywords: Game theory, differential games, advertising, pricing, coordination mechanisms, marketing channels

Acknowledgments: This research was supported by the Research and Knowledge Transfer Office at HEC Montréal. (CCS 32 153 302 27 G119A).

1 Introduction

An established result in the channels literature states that lack of coordination in marketing channels leads to inefficiencies. Indeed, channels involve interactions between at least two independent firms[1] (e.g., manufacturers, retailers, etc.) controlling interdependent decision variables (eg., prices at the wholesale and retail levels, local and national advertising efforts, shelf space, etc.). When the channel is decentralized, these firms act in a non-cooperative (i.e., conflicting or uncoordinated) manner. Consequently, they set their margins above those of a channel whose members follow a collective objective (i.e., cooperation or coordination). This is known as the problem of double marginalization (Spengler, 1950). Hence, decentralized channels are inefficient if no coordinating mechanisms are designed, because double marginalization leads to higher retail prices, lower demand, lower total profits for the channel, and lower consumer welfare. Another consequence of decentralization is an underinvestment in non-price marketing-mix variables (e.g., advertising and promotional efforts, service, etc.). Unambiguously, the highest level of efficiency is obtained under full coordination,[2] that is, a situation of joint ownership (i.e., vertical integration),[3] since double marginalization is avoided when only one agent controls all the decisions. However, it is often complicated, and in many cases illegal, to vertically integrate the channel structure (Jeuland and Shugan, 1983).

Furthermore, in decentralized channels, the cooperative solution cannot be imposed by one channel member onto its partners. Hence, many studies have investigated whether this optimal solution can be reached as an equilibrium in a decentralized channel, through the implementation of incentive mechanisms. Such mechansims align all of the channel member's strategies in a way that allows them to implement the cooperative solution, or any other desired solution, as a best reply to the other channel members' strategies. Other forms of incentives align individual channel profits with the total channel profit, allowing channel members to maximize the joint outcome while maximizing their individual outcomes. Profit sharing, quantity discounts, and two-part tariffs (Jeuland and Shugan, 1983; McGuire and Staelin, 1983; Moorthy, 1987, 1988; etc.) are examples of such mechanisms.

Channel efficiency can also be improved under a less constrained setting (compared to joint ownership). We label this case partial coordination (or simply, coordination). This case covers all situations where channel members remain independent institutions but implement coordinating mechanisms that reduce the inefficiencies of a decentralized channel. With partial coordination, the total channel profit is improved relative to its level in the absence of coordination, but it does not necessarily coincide with the total profit of a vertically integrated channel. Cooperative advertising (Berger, 1972; Bergen and John, 1997), pull promotions (Gerstner and Hess, 1995) and customer satisfaction assistance (Chu and Desai, 1995) allow for such coordination.

This paper contributes to the dynamic literature investigating the use of coordination mechanisms in channels. Our objective is twofold. First, we design incentives that the manufacturer, as a channel leader, offers to the retailer in order to induce him to fix desired levels of local advertising and retail price. These levels correspond to the channel-coordinating values. The second objective is to examine whether these mechanisms can be implemented in the channel and if they are Pareto-improving. The existing dynamic literature has not investigated this issue by considering the simultaneous coordination of both marketing decision variables. Retail prices have often been considered constant or exogenous in these studies.

The paper is organized as follows: In the next section, we provide a literature review on coordination mechanisms in distribution channels. Section 3 describes the model and the scenarios investigated in the study. In Sections 4 and 5, we compute the channel members' strategies and outcomes under decentralization and vertical integration respectively. Section 6 is devoted to the scenario where incentive mechanisms are implemented in the channel. We compute and compare the results under this scenario to those obtained in Sections 4 and 5 and investigate whether the incentive mechanisms can be implemented in the channel. Section 7 concludes.

[1]Except the particular case of vertically integrated channels, were all the decisions are controlled by a single decision maker.
[2]Or "perfect" coordination, as denoted by Jeuland and Shugan (1983).
[3]Or a situation where channel members remain independent institutions, but act as if the channel was vertically integrated.

2 Literature review

There is no doubt that coordinated channels perform better than decentralized one, but answering the question on how independent decision makers can achieve coordination without vertical integration was not clear before the Jeuland and Shugan (1983) study. This paper is one of the first to investigate this issue and to suggest a coordinating mechanism that eliminates double marginalization in a decentralized channel. The authors demonstrate that a pricing schedule based on a quantity discount enables a decentralized channel to replicate the performance of a vertically integrated one. A significant game-theoretic literature has extended these findings by examining different channel structures, demand specifications, leadership roles, etc., and by designing the appropriate coordinating mechanisms. However, this literature is mainly based on static models that do not account for the long-term effects of marketing decisions.[4] Extensions to a dynamic setting are sparse, and most are devoted to the study of non-price marketing variables, and more specifically, to advertising and cooperative advertising programs as coordinating mechanisms (Jørgensen and Zaccour, 2004; Zaccour, 2008).[5] Chintagunta and Jain (1992) is the first study to examine the channel-coordination issue in a dynamic setting. The authors use the Nerlove and Arrow (1962) model to capture the accumulation of goodwill through the manufacturer's and retailer's investments in national and local advertising respectively. Three remarks on this study should be raised at this stage. First, this study extends the results on the efficiency of channel coordination to a dynamic setting, but does not suggest solutions on how to reach this efficiency when the channel is decentralized. Second, the authors assume that prices are fixed. Hence, the decentralized scenario is investigated by considering that channel members share the total channel revenues according to an exogenously fixed rate. Finally, the authors solve this problem by considering that channel members use open-loop information structures. This is equivalent to saying that channel members precommit to playing static strategies at the beginning of the planning horizon. This information structure is often criticized in the channel literature for its lack of realism. Most of the studies suggest the use of a feedback (i.e., Markovian) information structure where channel members use state-dependent strategies, and hence, adjust their actions with respect to the state of the dynamic system under which they evolve (e.g., the goodwill stock).

To overcome these last two limitations, Jørgensen and Zaccour (1999) examined the problem by adding prices as control variables and by considering that channel members implement feedback information structures. Hence, they obtain state-dependent advertising strategies but constant prices, since the latter do not have an impact on the state equation. As in Chintagunta and Jain (1992), Jørgensen and Zaccour (1999) point out the inefficiency of channel decentralization, but do not provide a solution that would allow a decentralized channel to reach the optimal outcome of a centralized one. Jørgensen et al. (2000, 2001a,b, 2003), Jørgensen and Zaccour (2003a), and Zaccour (2008) focus on the design of coordinating mechanisms in dynamic marketing channels. As mentioned above, these studies are based mainly on advertising models. They demonstrate that cooperative advertising programs and manufacturer leadership improve channel efficiency. In Zaccour (2008), the author demonstrates that two-part tariffs, as suggested by Jeuland and Shugan (1983) in a static setting, fail to coordinate the channel when non-price marketing variables and their long-term effects are taken into account. The same conclusion holds even when the manufacturer, as channel leader, commits to playing its part of the coordinating solution.

Jørgensen et al. (2000) compare three cooperative advertising programs and find that the full-support program, a mechanism whereby the manufacturer subsidizes the retailer's short-term and long-term advertising expenditures, is Pareto-improving, in addition to being the most efficient.[6] Jørgensen et al. (2001a) examine a similar issue but use a different game structure in order to obtain non-degenerate advertising strategies, a more realistic result compared to the static strategies obtained in Jørgensen et al. (2000). Jørgensen et al. (2003) and Jørgensen and Zaccour (2003a) extend the literature on the use of cooperative advertising programs as coordinating mechanisms to the situation where retailer's promotional activities harm the brand goodwill.

[4] For an extensive review of the static literature on the coordination of non-competitive channel structures, see Ingene, Taboubi and Zaccour (2012).

[5] Dynamic studies on channel coordination and on the use of cooperative advertising programs as coordinating mechanisms are surveyed in Jørgensen and Zaccour (2004, 2014).

[6] With respect to partial-support programs where only one type of advertising is subsidized.

In Jørgensen, Taboubi and Zaccour (2006), the authors design coordination mechanisms based on promotional allowance paid by the manufacturer for each unit of promotional activity carried out by the retailer to promote the brand. The authors consider two scenarios related to the manufacturer's objectives when implementing the incentive. In the first scenario, the manufacturer wants to reach the efficiency of a vertically integrated channel. He commits to offering the incentive and to implementing the coordinating level of national advertising in order to induce the retailer to react by fixing promotions at their coordinating level. In the second scenario, the manufacturer wants to induce the retailer to implement the promotional level that allows the manufacturer to maximize its own payoff, but commits only to offering the promotional incentive. The main result in Jørgensen, Taboubi and Zaccour (2006) states that both the manufacturer and the retailer make more advertising and promotional efforts under the first scenario. The promotional support rate given by the manufacturer is also higher under the first scenario.

Channel efficiency can also be enhanced when the choice of a channel leader is made endogenously. Jørgensen et al. (2001a) find that a channel where the manufacturer is the leader is the most efficient, and that it is better not to have a leader than to have the retailer as the channel captain.

Note that all the studies on channel coordination through mechanism design cited above[7] prove that, with the implementation of the mechanism, the channel remains decentralized but performs better than without the mechanism. However, this performance does not reproduce the performance of a vertically integrated channel. Hence, these mechanisms lead to partial rather than full coordination. Only two-sided incentive equilibria, introduced by Jørgensen and Zaccour (2003b) fully coordinate the channel. To prove this result, the authors consider that the manufacturer and the retailer simultaneously control their advertising strategies, denoted by $s_M(t)$ and $s_R(t)$ and want to implement the coordinating levels for these instruments. To do so, each channel member offers an incentive, in a way that its strategy depends on the other channel member's choice, and none of the channel members can improve his payoff when deviating from a desired level for the instruments. The authors use linear strategies given by

$$\begin{aligned}\gamma_M(s_R)(t) &= \max\left\{0, s_M^{VI}(t) + \mu_M(t)\left[s_R(t) - s_R^{VI}(t)\right]\right\}\\ \gamma_R(s_M)(t) &= \max\left\{0, s_R^{VI}(t) + \mu_R(t)\left[s_M(t) - s_M^{VI}(t)\right]\right\},\end{aligned}$$

where the superscript VI stands for Vertical Integration and denotes the desired levels for the marketing instruments . $\mu_M(t)$ and $\mu_R(t)$ are weights applied to the deviations from these desired levels. Besides the coordination of advertising efforts, Jørgensen and Zaccour (2003b) consider that channel members also coordinate their prices. However, their incentive mechanism does not allow them to reach this objective by following the same methodology since the manufacturer's incentive strategy for prices involves the computation of a coordinating wholesale price, and the latter is "immaterial" for a vertically integrated channel. The authors proceed by separating the advertising and the pricing problems and the identify a linear quantity-discount that coordinates pricing decisions in the channel.

Jørgensen and Zaccour (2003b) and Jørgensen, Taboubi and Zaccour (2006) are the closest studies to the problem we investigate in this article. There are four main differences between this study and Jørgensen and Zaccour (2003b). First, their incentive is a full coordinating mechanism since its main objective is to push both channel members to implement the coordinating solution as an equilibrium. In the present study, we design incentive mechanisms leading to partial coordination with the objective of reducing the channel inefficiency resulting from decentralization. Second, in Jørgensen and Zaccour (2003b), the authors solve the price-coordination problem by separating it from the advertising-coordination issue, while we solve both problems simultaneously. Third, Jørgensen and Zaccour (2003b) identify incentives for a symmetric game where channel members take their actions simultaneously (i.e., Nash equilibria are computed). In the channel setting we are investigating in this study, the channel has a leader, the manufacturer, which announces its strategies first, and then the retailer takes his actions by reacting to these strategies. Finally, we consider a feedback information structure where channel members' strategies are state dependent. Jørgensen, Taboubi and Zaccour (2006) also examine a feedback Stackelberg game; but while they do not consider prices in the model, we do exactly that in the present study.

[7] Except for one of the scenarios examined in Jørgensen, Taboubi and Zaccour (2006).

3 Model and scenarios

We study a channel structure where a monopolist manufacturer M distributes its brand via a single retailer R. A standard assumption in the channels literature is to consider that channel members play à la Stackelberg, where the manufacturer is the leader. The variables under his control are the wholesale price and the level of national advertising effort, denoted by $w(t)$ and $a_M(t)$ respectively. National advertising has a carry-over effect that contributes to building the brand's goodwill (i.e., brand equity or image), denoted by $G(t)$, which evolves according to Nerlove and Arrow (1962) capital-accumulation dynamics:

$$\begin{aligned} \dot{G}(t) &= \alpha a_M - \delta G(t), \\ G(0) &= G_0 \geq 0 \end{aligned} \tag{1}$$

where α and δ are positive parameters capturing the impact of national advertising on the brand goodwill and the decay (i.e., depreciation) rate of goodwill, respectively.

The retailer controls the product's retail price $p(t)$ and the level of local advertising $a_R(t)$. Local advertising represents all the short-term promotional activities designed to increase demand.

We assume that the manufacturer has a production cost denoted by $c \geq 0$ and that both channel members have quadratic advertising costs given by the following expressions:

$$C(a_i) = \frac{(a_i)^2}{2},\ i \in \{M, R\}.$$

As in Jørgensen et al. (2001a), we consider that product demand, denoted by $D(t)$, is given by the following expression:

$$D(t) = (\lambda - \beta p(t))\, a_R(t) \sqrt{G(t)}, \tag{2}$$

where λ and β are positive parameters. λ can be interpreted as the market potential for the brand, and β denotes the effects of price on demand. The latter is also affected by the retailer's local advertising and by brand goodwill in a multiplicative manner. This functional form captures the fact that the non-price marketing-mix variables have a higher effect on demand when prices are lower. The square root indicates that the brand goodwill has a marginal decreasing effect on demand. Two remarks should be added regarding this demand specification. First, note that this function does not explicitly include the effects of the manufacturer's national advertising. National advertising activities have an indirect effect, since they build the brand goodwill, which in turn affects demand. A second remark on equation (2) involves the sign of the expression $(\lambda - \beta p(t))$. We have to add a constraint on the parameters' values and verify ex-post that this expression is always positive in order to have a positive demand. This constraint is given by $(\lambda - \beta c) \geq 0$ and can be rewritten as $c \leq \frac{\lambda}{\beta}$, which means that the unit production cost must be lower than or equal to the consumers' willingness-to-pay when $D(t) = 0$.[8]

In the following sections, we compute channel members' strategies and outcomes under three scenarios: (i) a decentralized channel without incentives that we consider the status quo scenario, (ii) a vertically integrated channel, where we compute the desired levels of local advertising and retail price that maximize channel efficiency, and (iii) a decentralized channel where the manufacturer designs incentive mechanisms to induce the retailer to choose the desired levels of local advertising and retail price computed under scenario (ii). We use the superscripts D, VI, and I to denote scenarios (i), (ii) and (iii) respectively.

We suppose that channel members interact in an infinite time horizon and seek stationary feedback equilibria (i.e., their decisions depend only on goodwill). The next sections characterize the results under all the scenarios described above.

[8] A similar constraint is obtained in Zaccour (2008). It suffices to compute the inverse demand function as $p = \frac{D}{\beta a_R \sqrt{G}} - \frac{\lambda}{\beta}$ and to set $D = 0$.

4 Channel decentralization

Channel decentralization corresponds to the situation where each channel member maximizes its own objective functional. Let J_M and J_R respectively denote the manufacturer's and retailer's objective functionals. We consider that channel members discount their stream of profits by using the same discount rate $\rho \geq 0$.

$$J_M = \int_0^\infty e^{-\rho t} \left[(w(t) - c) (\lambda - \beta p(t)) \, a_R(t) \sqrt{G(t)} - \frac{(a_M(t))^2}{2} \right] dt, \tag{3}$$

$$J_R = \int_0^\infty e^{-\rho t} \left[(p(t) - w(t)) (\lambda - \beta p(t)) \, a_R(t) \sqrt{G(t)} - \frac{(a_R(t))^2}{2} \right] dt. \tag{4}$$

subject to to the state dynamics given by (1).

The following proposition identifies equilibrium strategies, demand, channel members' value functions, and the goodwill trajectory under this scenario.

Proposition 1 *Assuming an interior solution, the equilibrium strategies under decentralization are given by the following expressions:*

$$\begin{aligned}
w^D(G) &= \frac{\lambda + c\beta}{2\beta}, \\
a_M^D(G) &= \frac{\alpha (\lambda - c\beta)^4}{64\beta^2 (2\delta + \rho)}, \\
p^D(G) &= \frac{3\lambda + c\beta}{4\beta}, \\
a_R^D(G) &= \frac{(\lambda - c\beta)^2}{16\beta} \sqrt{G}.
\end{aligned}$$

Demand is given by

$$D^D(G) = \frac{(\lambda - c\beta)^3}{64\beta} G$$

The channel members' value functions are

$$V_M^D(G) = M_1 G + M_2 \tag{5}$$

$$V_R^D(G) = \frac{(\lambda - c\beta)^4}{512\beta^2} G \tag{6}$$

where M_1,and M_2 are positive parameters. Their expressions are given in Appendix A.

The goodwill trajectory

$$G^D(t) = (G_0 - G_{SS}^D) e^{-\delta t} + G_{SS}^D$$

converges to its steady state given by $G_{SS}^D = \frac{\alpha^2 (\lambda - \beta c)^4}{64\beta^2 \delta (2\delta + \rho)}$.

Proof. See Appendix A. □

The results in proposition 1 state that national advertising, wholesale and retail prices are degenerate, that is, they are constant, whereas the retailer's local advertising is state dependent. Its expression indicates that local advertising is subject to decreasing marginal effects of goodwill, meaning that an increase in goodwill contributes to an increase in the local advertising effort, but at a decreasing rate. Demand is also increasing in the goodwill stock.

5 Channel coordination

Under this scenario, the channel is coordinated. Channel members agree to maximize a joint outcome as if the channel was controlled by a single decision maker. The latter solves the following optimization problem:

$$\begin{aligned} J_C &= J_M + J_R \\ &= \int_0^{\infty} e^{-\rho t}\left[(p(t)-c)\left(\lambda-\beta p(t)\right)a_R\left(t\right)\sqrt{G\left(t\right)}-\frac{\left(a_R\right)^2+\left(a_M\right)^2}{2}\right]dt, \end{aligned} \tag{7}$$

subject to (1).

The following proposition characterizes the optimal strategies, demand, channel value function, and goodwill trajectory under this scenario.

Proposition 2 *Optimal strategies under coordination are given by the following expressions:*

$$\begin{aligned} p^{VI} &= \frac{\lambda+c\beta}{2\beta}, \\ a_M^{VI} &= \frac{\alpha\left(\lambda-c\beta\right)^4}{32\beta^2\left(\delta+\rho\right)}, \\ a_R^{VI}(G) &= \frac{\left(\lambda-c\beta\right)^2}{4\beta}\sqrt{G\left(t\right)}. \end{aligned}$$

Demand is given by

$$D^{VI}(G) = \frac{\left(\lambda-c\beta\right)^3}{8\beta}G$$

and channel's value function is

$$V^{VI}(G) = K_1 G + K_2\ , \tag{8}$$

where K_1, K_2 are positive parameters. Their expressions are given in Appendix B.

The goodwill trajectory

$$G^{VI}(t) = (G_0 - G_{SS}^{VI})e^{-\delta t} + G_{SS}^{VI}$$

converges to its steady state given by $G_{SS}^{VI} = \frac{\alpha^2(\lambda-\beta c)^4}{32\beta^2\delta(\delta+\rho)}$.

Proof. See Appendix B. □

These results are qualitatively similar to those obtained under decentralization. By comparing the expressions in this proposition to their respective expressions in proposition 1, we replicate the results in

Jørgensen et al. (2001a), which are in line with previous results in the channels literature. Indeed, as expected, retail prices, national and local advertising levels, demands, and total channel outcomes under the coordinated and the decentralized channels compare as follows:

$$\begin{aligned} p^{VI}(G) &< p^D(G),\ a_M^{VI}(G) > a_M^D(G),\ a_R^{VI}(G) > a_R^D(G), \\ D^{VI}(G) &> D^D(G),\ \text{and } V^{VI}(G) > V_M^D(G) + V_R^D(G). \end{aligned}$$

As mentioned in the previous section, we study the scenario of vertical integration because it allows us to compute the desired levels of the control variables, while the levels of these variables under the decentralization scenario allow us to obtain the status quo, which serves as a benchmark to assess the impact of the incentive mechanisms. Since these results are expected and do not constitute the contribution of this study, we move directly to the computations and interpretations of the third scenario.

6 Channel coordination with incentive mechanisms

This section is devoted to the design of coordinating mechanisms that the manufacturer, as channel captain, implements in order to induce the retailer to act in a "desired" way. Under this scenario, we consider that the manufacturer offers the retailer a wholesale-price reduction and an advertising allowance with the aim of pushing the retailer to act as if the channel was vertically integrated, that is, have the retailer set its retail price and the local advertising at their levels p^{VI} and $a_R^{VI}(G)$ respectively.

Three main questions should be investigated under this scenario:

- Do these incentives allow the manufacturer to increase its outcome? Incentives designed by the manufacturer will be implemented only if they contribute to increasing the manufacturer's individual profit (i.e., $V_M^I(G)$) with respect to the status quo scenario, when the channel is decentralized and no incentives are offered (i.e., $V_M^D(G)$). To provide an answer to this question, we need to compare $V_M^I(G)$ and $V_M^D(G)$.
- What should be the level of national advertising effort that the manufacturer sets when he offers the incentives? Does he need to implement the channel-coordinating level? And how does this national advertising level compare with its level under the status quo scenario? Computing a_M^I and comparing its level to the levels of a_M^{VI} and a_M^D allow us to answer this second question,
- Will the retailer accept the implementation of these incentives? This condition corresponds to retailer's participation constraint. Indeed, although the retailer, as a follower, reacts to the leader's announcements, the retailer could reject these incentives and stop distributing the manufacturer's product if these incentives reduce her profits. Hence, the incentive mechanisms should be Pareto-improving (i.e., contribute to the increase of both channel members' profits). This question can be answered by comparing $V_R^I(G)$ and $V_R^D(G)$.

Let $w^I(p)$ and $A(a_R)$ denote the incentives designed by the manufacturer to coordinate the retail price and the local advertising respectively. We consider that these incentives have the following expressions:

$$w^I(p) = w^D - \psi\left(p^D - p\right) \tag{9}$$

$$A(a_R) = \eta a_R\ , \tag{10}$$

where ψ and η are positive parameters that have to be fixed by the manufacturer.

The incentive in equation (9) is designed to reduce double marginalization by pushing the retailer to fix p at its coordinating level p^{VI}. It indicates that the manufacturer offers the retailer a wholesale-price discount whenever the retailer fixes the retail price p at a lower level with respect to the decentralized scenario. Indeed, if $p < p^D$, the retailer benefits from a lower wholesale price than in the wholesale price that he would have paid without the incentive. The wholesale-price reduction corresponds in this case to the amount given by $\psi\left(p^D - p^{VI}\right)$. If the retailer fixes its retail price at the non-coordinating level, or at a higher level, then the manufacturer “punishes” the retailer by increasing its wholesale price: The manufacturer fixes the wholesale price at its level under decentralization w^D whenever the retailer fixes the retail price at its non-coordinating level p^D. But if the retail price is higher than p^D, then the retailer's cost increases by an amount equal to $\psi\left(p-p^D\right)$ per unit of product purchased from the manufacturer.

Equation (10) represents the second incentive, which takes the form of an allowance given by the manufacturer to the retailer for each unit of local advertising effort. This coordinating mechanism subsidizes the retailer's advertising expenditures, and hence, corresponds to a cooperative advertising program.

By giving this allowance, the manufacturer wants the retailer to implement the coordinating level of national advertising, that is, a_R^{VI}. The parameter η is then chosen by the manufacturer by setting $a_R = a_R^{VI}$. Note that this specification allows the manufacturer to offer the retailer an advertising allowance only when the latter invests in local advertising effort.

We follow the same steps as in the previous scenarios to find the value functions $V_M^I(G)$ and $V_R^I(G)$, which for all $G \geq 0$, satisfy the HJB equations for the manufacturer and the retailer. We start by computing the retailer's best response. Its HJB equation is given by the following expression:

$$\begin{aligned}
\rho V_R^I &= \max_{p,a_R\geq 0}\left[\left(p-w^I(p)\right)(\lambda-\beta p)\,a_R\sqrt{G}+\eta\,(a_R)^2-\frac{(a_R)^2}{2}\right] \\
&= \max_{p,a_R\geq 0}\left[\left(p-w^I(p)\right)(\lambda-\beta p)\,a_R\sqrt{G}+\left(\eta-\frac{1}{2}\right)(a_R)^2\right],
\end{aligned} \tag{11}$$

subject to equations (9) and (10).

Maximization on the right-hand side of equation (11) with respect to p and a_R and solving the system of equations obtained after setting the derivatives equal to 0 gives the following reaction functions:[9]

$$\begin{aligned}
p &= \left\{\begin{array}{c} \frac{c\beta(\psi-2)+\lambda(7\psi-6)}{8\beta(\psi-1)} \text{ if } p>0 \\ 0 \quad \text{otherwise} \end{array}\right\}, \\
a_R(G) &= \left\{\begin{array}{c} \frac{(\lambda-\beta c)^2(\psi-2)^2}{64\beta(\psi-1)(2\eta-1)}\sqrt{G} \text{ if } (\psi-1)(2\eta-1)>0 \\ 0 \quad \text{otherwise} \end{array}\right\}.
\end{aligned}$$

The values of the parameters ψ and η are obtained by setting $p = p^{VI}$ and $a_R(G) = a_R^{VI}(G)$ and solving the system. The solution is given by

$$\left\{\begin{array}{c} \psi = \frac{2}{3} \\ \eta = \frac{1}{3} \end{array}\right\}.$$

The following proposition gives the incentive mechanisms that the manufacturer could implement to induce the retailer to act in a cooperative manner.

Proposition 3 *Manufacturer's incentive mechanisms for the retail price and local advertising are given by*

$$w^I(p) = w^D - \frac{2}{3}\left(p^D - p\right), \tag{12}$$

$$A(a_R) = \frac{1}{3}a_R. \tag{13}$$

[9] The system gives three solutions, two of which give $u_R = 0$. These solutions lack interest because they lead to $D = 0$. We concentrate the analysis only on the solution where $u_R > 0$.

Proof. It suffices to replace ψ and η by their values in the incentive expressions given by equations (9) and (10). $\square$

Replacing w^D, p^D, and p^{VI} by their expressions from propositions 1 and 2 gives the manufacturer's wholesale price when the incentive is implemented and when the retailer reacts by fixing its retail price at the coordinating level. This wholesale price is given by

$$w^I = \frac{\lambda + 2c\beta}{3\beta}.$$

Hence, by implementing this retail-price-coordinating incentive, the manufacturer fixes the wholesale price at a lower level with respect to the wholesale price of the decentralized channel, given by $w^D(G) = \frac{\lambda+c\beta}{2\beta}$. This means that the manufacturer sacrifices its unit margin by $\frac{\lambda-c\beta}{6\beta}$ in order to push the retailer to implement the coordinating retail price. Note also that the wholesale-price reduction resulting form the incentive pushes the retailer to fix the retail price at its coordination level $p^{VI} = \frac{\lambda+c\beta}{2\beta}$, and that the retailer's cost corresponds to the wholesale price with the incentive, that is, $w^I = \frac{\lambda+2c\beta}{3\beta}$. Hence, retailer's unit margin with the incentive is given by $p^{VI} - w^I = \frac{\lambda-c\beta}{6\beta}$. This margin corresponds to the margin sacrified by the manufacturer when implementing the retail-price-coordinating incentive. Without the incentive, the retailer's margin corresponds to $p^D - w^D = \frac{\lambda-c\beta}{4\beta}$. This result indicates that the retailer also sacrifices its margin by an amount corresponding to $\left(\frac{\lambda-c\beta}{4\beta} - \frac{\lambda-c\beta}{6\beta}\right) = \frac{\lambda-c\beta}{12\beta}$ per unit of product purchased from the manufacturer when the latter implements the incentive.

Note that the wholesale price under this scenario is constant. This result is expected, since its expression in (12) indicates that it depends on w^D and p^D, and both strategies are found to be degenerate in the previous sections.

Furthermore, when the advertising allowance is implemented, the retailer is called to fix its local advertising effort at its coordinating level. Hence, the incentive allowance can be obtained by substituting a_R by its level a_R^{VI} given in proposition 2. The expression in equation (13) indicates that the manufacturer has to pay 1/3 of the retailer's total expenditures on local advertising in order to push the latter to fix her advertising effort at its channel-coordinating level. Hence, the total allowance given by the manufacturer to the retailer is equal to

$$A(G) * a_R(G) = \frac{1}{3}\left(a_R^{VI}(G)\right)^2 = \frac{(\lambda - c\beta)^4}{48\beta^2} G$$

and indicates that the manufacturer's advertising allowance increases with the brand's goodwill.

The second step in the resolution of this Stackelberg game is to solve the manufacturer's problem by taking into account the retailer's reaction functions. We start by computing the manufacturer's HJB equation under this scenario. Its expression is given by the following equation:

$$\rho V_M^I = \max_{a_M \geq 0} \left[\left(w^I - c\right)\left(\lambda - \beta p^I\right) a_R^I \sqrt{G} - \frac{(a_M)^2}{2} - \frac{\left(a_R^I\right)^2}{3} + \frac{dV_M^I}{dG}(\alpha a_M - \delta G) \right]. \tag{14}$$

In order to compute the manufacturer's value function, we substitute p and a_R by their coordinating levels p^{VI} and a_R^{VI}, given in proposition 2. Note that the manufacturer commits only to the implementation of the incentive strategies given by (12) and (13). He does not commit to implementing the coordinating level for national advertising.

Computing the derivative with respect to a_M on the right-hand side of equation (14), setting it equal to 0, and solving for a_M gives

$$a_M(G) = \alpha \frac{dV_M^I}{dG}.$$

We substitute $a_M(G)$ by its expression in (14) and conjecture that $V_M^I(G)$ is a linear value function given by

$$V_M^I(G) = L_1 G + L_2.$$

The value functions' parameters $L_1$and L_2 are found by identification. Their expressions are given by

$$L_1 = \frac{(\beta c - \lambda)^4}{48\beta^2(\delta + \rho)},$$

$$L_2 = \frac{\alpha^2 (\beta c - \lambda)^8}{4608\beta^4 \rho (\delta + \rho)^2}\ .$$

Since $\frac{dV_M^I}{dG} = L_1$, we can compute the national advertising level at the equilibrium under the incentive scenario. The latter is given by the following expression:

$$a_M^I = \frac{\alpha (\beta c - \lambda)^4}{48\beta^2(\delta + \rho)}. \tag{15}$$

This first result indicates that, with the implementation of the incentives, the manufacturer does not fix its national advertising level at its channel-coordinating level. Indeed, by comparing the expression of a_M^I with its counterpart in the vertical integration scenario, we have

$$\begin{aligned} a_M^I - a_M^{VI} &= \frac{\alpha (\beta c - \lambda)^4}{48\beta^2(\delta + \rho)} - \frac{\alpha (\lambda - c\beta)^4}{32\beta^2(\delta + \rho)} \\ &= -\frac{\alpha (\beta c - \lambda)^4}{96\beta^2(\delta + \rho)} < 0, \end{aligned}$$

and comparing the national advertising level at the equilibrium under the incentive scenario with its level under the status quo gives

$$\begin{aligned} a_M^I - a_M^D &= \frac{\alpha (\beta c - \lambda)^4}{48\beta^2(\delta + \rho)} - \frac{\alpha (\lambda - c\beta)^4}{64\beta^2(2\delta + \rho)} \\ &= \frac{\alpha (\beta c - \lambda)^4 (5\delta + \rho)}{192\beta^2(\delta + \rho)(2\delta + \rho)} > 0. \end{aligned}$$

Hence, these results compare as follows:

$$a_M^{VI} > a_M^I > a_M^D\ ,$$

meaning that the highest level of national advertising is reached under vertical integration, and that, with the incentive, the manufacturer invests more effort in advertising than in the scenario where the channel is decentralized and no incentives are offered to the retailer.

Demand under the three scenarios can be computed after substitution of a_R and p by their expressions in the corresponding scenarios. Since demand is also affected by the goodwill level, we can compute and compare the sales trajectories under the different scenarios. It suffices to substitute $G(t)$ by its expression given in the corresponding scenario. Note that, in the scenario with the incentive, the goodwill at the steady state can be computed after substitution of a_M by $a_M^I = \frac{\alpha(\beta c - \lambda)^4}{48\beta^2(\delta+\rho)}$ in the state dynamics given by equation (1) and solving for $\dot{G}(t) = 0$. The solution is given by the following expression:

$$G_{SS}^{I}=\frac{\alpha^{2}\left(\lambda-\beta c\right)^{4}}{48\beta^{2}\delta\left(\delta+\rho\right)},$$

and the goodwill trajectory is given by

$$G^{I}(t)=(G_{0}-G_{SS}^{I})e^{-\delta t}+G_{SS}^{I}.$$

Since the goodwill steady states under the three scenarios compare as follows:

$$G_{SS}^{VI}\geq G_{SS}^{I}\geq G_{SS}^{D}\ ,$$

we can conclude that, from the initial time, where the goodwill is equal to G_0, till the steady-state, sales trajectories compare as follows

$$D^{VI}(t)\geq D^{I}(t)\geq D^{D}(t).$$

These results can be summarized in the next proposition.

Proposition 4 *The manufacturer's incentive mechanisms lead to a decrease in both channel members' unit margins and an increase in local and national advertising efforts (with respect to the status quo). Consequently, demand and goodwill levels are higher with the incentive mechanism.*

Furthermore, by comparing the manufacturer's value functions under the scenario with the incentives and the status quo,[10] we can easily prove that the manufacturer is better off when he implements the incentives. These results allow us to state the following propositions.

Proposition 5 *It is in the manufacturer's best interest to implement the incentive mechanisms since his individual profit is higher when he offers these incentives to the retailer (with respect to the status quo scenario).*

Proof. It suffices to compare the expressions of L_1 and L_2 to M_1 and M_2. We can clearly see that $L_1 > M_1$ and $L_2 > M_2$. □

Moving to the retailer's problem, we proceed in a similar manner: Substituting p and a_R by their coordinating levels p^{VI} and a_R^{VI}, given in proposition 2, and w by its level under the scenario with incentives, given by equation (12), into the retailer's HJB equation (11) gives the retailer's value function with the incentive

$$V_{R}^{I}\left(G\right)=\frac{\left(\lambda-c\beta\right)^{4}}{96\beta^{2}}G. \tag{16}$$

Comparing this expression to its counterpart in the status quo scenario given by the expression in (6) indicates clearly that the retailer's value function is higher when the incentives are implemented, as compared to the status quo scenario. Combining this result to the result announced in the previous proposition allows us to state the next proposition.

Proposition 6 *The incentives are Pareto-improving since both channel members increase their profits when these incentives are implemented, relative to the status quo scenario.*

Results in propositions 4 and 6 allow us to conclude with the following proposition:

Proposition 7 *The wholesale-price incentive mitigates the inefficiency resulting from double marginalization in decentralized channels since it induces both channel members to fix lower margins than under a decentralized channel without incentives.*

[10] This result can also be anounced for the intial level of the goodwill state G_0 and for the whole state trajectory.

7 Conclusion

In this study, we examine a differential game where a manufacturer, as a channel captain, offers incentive mechanisms to its retailer in order to induce him to play in a "desired" manner. We compute the channel members' strategies and profits for the desired solution, which corresponds to the channel-coordinating levels (i.e., vertical integration), and for a status quo scenario. The latter corresponds to the case where the channel is decentralized and the manufacturer does not offer incentives. Then, we design two incentive mechanisms based on a wholesale-price reduction and an advertising allowance, and investigate whether these mechanisms can be implemented by the manufacturer and if the retailer will accept these incentives. We demonstrate that a wholesale-price reduction corresponding to $2/3$ of the difference between the channel-coordinating retail price and the status quo retail price, and an advertising allowance where the manufacturer subsidizes $1/3$ of the retailer's local advertising expenditures pushes the retailer to fix its retail price and the local advertising effort at their channel-coordinating levels. When offering these incentives, both the manufacturer and the retailer sacrifice a portion of their unit profit margin, which mitigates the double-marginalization problem resulting from decentralization.

Furthermore, by offering these incentives, the manufacturer has to increase its national advertising effort with respect to the status-quo scenario. By doing so, he contributes to an increase in goodwill, which in conjunction with the decrease in prices and the increase in the retailer's local advertising, contributes to the increase in demand. This demand increase compensates the decrease in the channel members' profit margins, allowing them both to increase their profits. As a result it is in the interest of the manufacturer and the retailer to implement and to participate in, respectively, the incentive program.

With the design of these incentive mechanisms, we contribute by adding new coordinating mechanisms to the literature on channel coordination, where only a few studies have been able to examine the issue in a dynamic setting by including price and non-price-marketing variables.

Future extensions of this study should examine if similar incentive mechanisms could be implemented in channels where the retailer plays the role of the channel captain, and investigate whether these incentives can be used when the manufacturer distributes its product via multiple retailers. Furthermore, in this study, we examined a particular form of coordinating mechanisms, where the incentive depends on the channel members' control variables. One could think of state-dependent incentives, where the state could be the goodwill level, as it is in this study, or the level of sales for example.

Appendix A

By equations (3), (4), and (1), we define an infinite-horizon differential game with two players, one state variable, $G(t)$, two control variables for the manufacturer (w and a_M), and two control variables for the retailer (p and a_R). The time argument will be omitted from now on when no confusion can arise.

We use the sufficiency theorem to solve this differential game with a feedback information structure. Hence, we need to find two bounded and continuously differentiable functions $V_M^D(G)$ and $V_R^D(G)$ that for all $G \geq 0$, satisfy the following Hamilton-Jacobi-Bellman (HJB) equations:

$$\rho V_M^D = \max_{w, a_M \geq 0} \left[(w-c)(\lambda - \beta p)\, a_R \sqrt{G} - \frac{(a_M)^2}{2} + \frac{dV_M^D}{dG}(\alpha a_M - \delta G) \right], \tag{17}$$

$$\rho V_R^D = \max_{p, a_R \geq 0} \left[(p-c)(\lambda - \beta p)\, a_R \sqrt{G} - \frac{(a_R)^2}{2} + \frac{dV_M^D}{dG}(\alpha a_M - \delta G) \right]. \tag{18}$$

Since the game is played à la Stackelberg, with the manufacturer as a channel leader, we start by computing the retailer's best response derived from her HJB equation. Performing the maximization on the right-hand side of equation (18) w.r.t. the control variables p and a_R and solving the resulting system of equations gives retailer's reaction functions. Note that this system gives three solutions, two of which lead to a demand equal to 0. Hence, we consider only the solution given by the following expressions:

$$p(w) = \frac{\beta w + \lambda}{2\beta}, \tag{19}$$

$$a_R(w, G) = \frac{(\lambda - \beta w)^2}{4\beta}\sqrt{G}. \tag{20}$$

Substituting the retailer's reaction functions into the manufacturer's HJB equation and performing its maximization w.r.t. its control variables w and a_M yields

$$\begin{aligned} w^D &= \frac{\beta c + \lambda}{2\beta}, \\ a_M(G) &= \alpha \frac{dV_M^D}{dG}. \end{aligned} \tag{21}$$

Then we insert these expressions into equation (18) and conjecture that $V_M^D(G)$ is a linear value function given by the expression in equation (5). Substituting $\frac{dV_M^D}{dG}$ into the equation (21) and inserting all the expressions into retailer's HJB equation allows us to conjecture that the retailer's value function is also linear in $G(t)$ and given by equation (6) in proposition 1.

Solution at the equilibrium in proposition 1 is obtained after substitution of w and $a_M(G)$ into the retailer's reaction functions given by (19) and (20) respectively.

The value functions' parameters M_1,and M_2 are found by identification after rearranging all the terms corresponding to the coefficient of G and the constants in the HJB equations obtained after substitution and solving the resulting system of Riccati equations.

Their expressions are given by

$$M_1 = \frac{(\beta c - \lambda)^4}{64\beta^2(2\delta + \rho)},$$

$$M_2 = \frac{\alpha^2(\beta c - \lambda)^8}{8192\beta^4\rho(2\delta + \rho)^2},$$

Demand under this scenario is obtained after substitution of p and a_R by their expressions in proposition 1 in the demand function given by equation (2). Note that this expression is always positive because of the constraint $c \leq \frac{\lambda}{\beta}$.

The goodwill at the steady state is computed after substituting a_M by its expression in proposition 1 in the state dynamics given by equation (1) and solving for $\dot{G}\,(t) = 0$.

Appendix B

We follow the same approach as the one described in Appendix A. The HJB equation under vertical integration is given by

$$\rho V^{VI} = \max_{p,a_R,a_M \geq 0} \left[(p-c)(\lambda - \beta p)\, a_R \sqrt{G} - \frac{(a_M)^2}{2} + \frac{dV_M^D}{dG} (\alpha a_M - \delta G) \right].$$

We maximize the right-hand side of this equation with respect to the control variables p, a_R, and a_M and solve a system of three equations to obtain the expressions for these variables. We substitute these expressions in the HJB equation and conjecture that the value function is linear in the state. The parameters of the value function are obtained by identification after rearranging all the terms in order to correspond to the form given in the proposition. The parameters of this value function are given by

$$K_1 = \frac{(\beta c - \lambda)^4}{32\beta^2 (2\delta + \rho)},$$

$$K_2 = \frac{\alpha^2 (\beta c - \lambda)^8}{2048\beta^4 \rho (2\delta + \rho)^2}.$$

References

[1] Berger, P.D. (1972), "Vertical Cooperative Advertising Ventures", Journal of Marketing Research, 9(3) 309–312.

[2] Bergen, M. and John, G. (1997), "Understanding Cooperative Advertising Participation Rates in Conventional Channels", Journal of Marketing Research, 34(3) 357–369.

[3] Chintagunta, P.K., and Jain, D. (1992), "A Dynamic Model of Channel Member Strategies for Marketing Expenditures", Marketing Science, 11(2) 168–188.

[4] Chu, W. and Desai, P.S. (1995), "Channel Coordination Mechanisms for Customer Satisfaction", Marketing Science, 14(4) 343–359.

[5] Gerstner, E. and J.D. Hess (1995), "Pull Promotions and Channel Coordination", Marketing Science, 14(1) 43–60.

[6] Jeuland, A.P. and S.M. Shugan (1983), "Managing Channel profits", Marketing Science, 2(3) 239–272.

[7] Jørgensen, S., Sigué, S. P., and Zaccour, G. (2000), "Dynamic Cooperative Advertising in a Channel", Journal of retailing, 76(1) 71–92.

[8] Jørgensen, S., Sigué, S. P., and Zaccour, G. (2001a), "Stackelberg Leadership in a Marketing Channel", International Game Theory Review, 3(1) 13–26.

[9] Jørgensen, S., Taboubi, S., and Zaccour, G. (2001b), "Cooperative Advertising in a Marketing Channel", Journal of Optimization Theory and Applications, 110(1) 145–158.

[10] Jørgensen, S., Taboubi, S. and Zaccour, G. (2003), "Retail Promotions with Negative Brand Image Effects: Is Cooperation Possible?", European Journal of Operational Research, 150(2) 395–405.

[11] Jørgensen, S., Taboubi, S. and Zaccour, G. (2006), "Incentives for Retailer Promotion in a Marketing Channel", Annals of the International Society of Dynamic Games, vol.8, 365–378.

[12] Jørgensen, S. and Zaccour, G. (1999), "Equilibrium pricing and Advertising Strategies in a Marketing Channel", Journal of Optimization Theory and Applications, 102(1) 111–125.

[13] Jørgensen, S. and Zaccour, G. (2002), "Time Consistency in Cooperative Differential Games" In Zaccour, G. (ed.), Decision and Control in Management Science: Essays in Honor of Alain Haurie. Kluwer, Boston.

[14] Jørgensen, S. and Zaccour, G. (2003a), "A Differential Game of Retailer Promotions", Automatica. 39(7) 1145–1155.

[15] Jørgensen, S. and Zaccour, G. (2003b), "Channel Coordination over Time: Incentive Equilibria and Credibility", Journal of Economic Dynamics and Control, Vol.27, 801–822.

[16] Jørgensen, S. and Zaccour, G. (2004), "Differential Games in Marketing", International Series in Quantitative Marketing, Kluwer Academic Publishers, Boston.

[17] Jørgensen, S. and Zaccour, G. (2014), "A Survey of Game-Theoretic Models of Cooperative Advertising", European Journal of Operational Research, Vol.237, 1–14.

[18] McGuire T. and Staelin, R. (1983), "An Industry Equilibrium Analysis of Downstream Vertical Integration", Marketing Science , 2(2) 161–192.

[19] Moorthy S. (1987), "Managing Channel Profits: Comments", Marketing Science , 6(4) 375–379.

[20] Moorthy S. (1988), "Strategic Decentralization in Channels", Marketing Science , 7(4) 335–355.

[21] Nerlove, M. and Arrow, K.J. (1962), "Optimal Advertising Policy Under Dynamic Conditions ", Economica, 39(114) 129–142.

[22] Spengler, J.J. (1950), "Vertical Integration and Antitrust Policy", Journal of Political Economy, Vol.58, 347–352.

[23] Taboubi, S. and Zaccour, G. (2002), "Impact of Retailer's Myopia on Channel's Strategies", in Optimal Control and Differential Games: Essays in Honor of Steffen Jørgensen, G. Zaccour ed., Kluwer Academic Publishers 179–192.

[24] Zaccour, G. (2008), "On the Coordination of Dynamic Marketing Channels and Two-part Tariffs", Automatica. Vol.44, 1233–1239.