CURRICULUM

# Mathematics 514

## Secondary School

Québec

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Direction de la formation générale des jeunes

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PAULINE MAROIS
Minister of Education

## DEVELOPMENT TEAM


**Coordination and Design**

*Mihran Djiknavorian*
Mathematics Coordinator
Direction de la formation générale des jeunes
Ministère de l'Éducation

**Design and Development**

*Françoise Boulanger*
Commission scolaire Baldwin-Cartier

*Jacques Lagacé*
Commission scolaire de Charlesbourg

**Direction de la formation générale des jeunes**

*Alain Mercier*
Acting Director
Direction de la formation générale des jeunes

**ENGLISH VERSION**

**Translation**

Services à la communauté anglophone
Direction de la production en langue anglaise
Ministère de l'Éducation



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# Introduction

*Mathematics 514* is a program designed for Secondary V students attending schools in Québec.

To prepare young Quebecers for the demanding world of the twenty-first century, schools must focus on the students' cognitive growth and the development of basic skills (i.e. communication and problem-solving skills as well as the ability to work with technology).

As an important component of a solid basic education, this mathematics program provides fertile ground for the development of the skills that students will require in the future. As Resnick and Kloffer have noted, "Graduates must not only be literate; they must also be competent thinkers."[1]

*Mathematics 514* is part of the basic secondary school curriculum and is designed for students who have successfully completed the Secondary IV mathematics course. It completes the sequence of secondary school mathematics courses and provides the students with the skills required by every citizen to function productively in society.

Because society is changing so rapidly, and owing to developments in the field of mathematics education, it is important to stress the interconnection of knowledge, skills and attitudes in the teaching of this program.

## Three Major Guiding Principles

Current knowledge of the learning process and the focus of student learning have led to an emphasis on three principles intended to guide teachers in their work with students. These principles are as follows: to encourage the students to participate actively in the learning process, to encourage them to use a problem-solving approach at each stage of the learning process and to encourage them to use the appropriate technology for each task.

### Encouraging Students to Take an Active Part in Their Own Learning Process

A great many research studies have shown that students should play a central role in their own learning process. In short, they should be ultimately responsible for their education:

1. L. B. Resnick and L. E. Kloffer, "Toward the Thinking Curriculum: An Overview," in *Toward the Thinking Curriculum: Current Cognitive Research, 1989 Yearbook of the Association for Supervision and Curriculum Development*, ed. Lauren B. Resnick and Leopold E. Kloffer (Alexandria, Va.: Association for Supervision and Curriculum Development, 1989), 1.

The construction of a given concept is a complex process that depends first and foremost on the student. Concepts are not directly transmitted from a knowledgeable person to a student who supposedly knows nothing in a given field. Before they tackle new subject matter, students have already developed their own conceptions, which are well organized, practical and sometimes fairly resistant to the changes targeted in a course of study.

Thus, teaching involves creating situations in which students draw on their own knowledge. Teaching involves structuring the learning process around their strategies and thinking in order to try to get them to make progress in the construction of a given concept.[2]

To help students acquire the knowledge and skills targeted by this program, it is important to design learning situations that call upon their powers of observation and dexterity and that involve manipulations, exploration, construction and simulations. Through these activities, the students analyze hypotheses, actively look for solutions, discuss their approaches, analyze concepts or theories from their own point of view while taking into account other points of view, actively question the meaning and consequences of the procedures they use and relate the knowledge they have acquired to their own experience. These situations encourage the students to reflect, act, react and establish connections with what they have already learned.

Another way teachers can encourage students to participate in their learning process is by developing a suitable teaching approach. Teachers will do more to help young people build their knowledge by asking them questions than by giving them the answers.

Any question that helps students get on the right track or answer their own questions encourages them to participate in their own learning.

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2. Nadine Bednarz, "L'enseignement des mathématiques et le Québec de l'an 2000," excerpted from Richard Pallascio, ed., *Mathématiquement vôtre! Défis et perspectives pour l'enseignement des mathématiques* (Montréal: Les éditions Agence d'ARC inc., 1990), 69 (Free translation).

**Encouraging Students to Use a Problem-Solving Approach at Every Stage in the Learning Process**

Problem solving is an essential teaching and learning tool in several general education programs (e.g. pure sciences, social studies) and is an integral part of any mathematical activity. Problem solving is not a separate theme, but rather a process that should be applied throughout the program and that provides a suitable context for learning concepts and acquiring skills.

> Problem solving is both a basic skill that students should develop and an effective teaching approach that promotes the development of mathematical knowledge, thinking skills, socio-affective attitudes and problem-solving strategies.[3]

Learning through problem-solving calls for the active involvement of the students and the use of questions. It is important that the teacher ask the students questions and that they in turn ask questions of each other and the teacher.

Some problem-solving tasks may be more difficult than others and the problems themselves can be quite varied. For instance, the students may encounter the following:

> problems with solutions requiring students to choose an appropriate combination of knowledge and skills from among several possible combinations seen in the past.[4]

They may even encounter the following:

> problems requiring students to create a new combination of knowledge and skills, exercise a great deal of intellectual independence and use plausible reasoning in order to solve them.[5]

Problem solving is a very effective means of developing knowledge and skills. The quality of learning depends on the variety of the problems assigned and on their level of difficulty. In a learning context, the students can even be presented with very challenging problems. By solving these problems, the students can discover such things as properties, relationships and strategies by themselves. A wide variety of problems allows students to conceptualize their knowledge and develop numerous problem-solving strategies. Problem solving is a way of learning and a way of teaching.

3. Québec, ministère de l'Éducation, *Mathematics Curriculum Guide, Elementary School, Booklet K, Problem Solving*, Code 16-2300-11A (Québec: ministère de l'Éducation, 1989), 47-51.

4. Ibid, p. 15.

5. Ibid, p. 15.

The problems can be related to the students' environment and used at various stages in the learning process. Problem solving can help students learn new concepts and develop skills or help them expand their knowledge and reinforce what they have learned.

Thus, problems provide an opportunity to:

- apply and integrate mathematical knowledge (e.g. concepts, properties, algorithms, techniques, procedures);
- develop intellectual skills (e.g. organizing, structuring, abstracting, analyzing, synthesizing, estimating, generalizing, deducing, justifying);
- develop positive attitudes (e.g. becoming aware of one's potential, respecting the opinions of others, and being imaginative and creative, rigorous and precise);
- use different problem-solving strategies (e.g. looking for patterns, representing a problem by means of a figure or a graph, constructing a table, referring to a known model, using a formula, formulating an equation, working backwards).

The emphasis on problem solving does not mean that exercises have no part in the teaching or learning of mathematics. Exercises play a different role, but one that is complementary to that of problem solving. For instance, exercises can help students consolidate skills and habits that they have already begun to develop. They can also help the students practise applying definitions and properties that they have learned in class. Exercises cannot replace problems and, conversely, problems cannot replace exercises.

By using a problem-solving approach, the students become accustomed to referring to a known mathematical model and are thereby more likely to attain the terminal objectives. The teacher should also assist the students in using a procedure that will enable them to acquire more knowledge and generate other models. This will help them attain the global objectives in accordance with the first guiding principle, namely, to encourage the students' active participation.

Students must have the opportunity to analyze their work methods and organize their thinking. In short, they must be able to learn how to learn.

**Encouraging Students to Use the Appropriate Technology for Each Task**

New technology is being used in all human activities and is bringing about a veritable revolution. It has had a noticeable impact on employment and has often changed the nature of work itself. The ability to work with electronic devices has become an essential skill because they will form an integral part of the students' everyday life.

In school, this technology can influence the teaching of mathematics and the way students learn. In addition to facilitating calculations, graphing, and data management, it enables students to deal with more complex problems. Technology complements the problem-solving tools that students already have. Students will have to learn when to use these different tools and to appreciate their advantages.

It is important to take the guiding principles into account when integrating new technology into the learning process and the evaluation of learning.

## Connection with Previous Programs

With continuity in learning, students can review topics they have already studied and further develop their conceptions and representations. This mathematics program enables students to build on the knowledge acquired in elementary school and in the first four years of secondary school.

This learning process will be dynamic if the learning activities allow the students to use their previously acquired knowledge and skills in new situations and help them to become more proficient at applying what they have learned.

As they acquire new knowledge, the students will review the following skills and concepts acquired in previous programs:

- number sense and facility with operations;
- the habit of estimating;
- proportionality;
- the concept of a variable;
- translation from one mode of representation to another;
- the relationships between variables;
- systems of linear relations;
- justifying the steps in the solution of a problem by using definitions, properties, theorems or corollaries related to different geometric concepts;
- spatial relationships;
- the ability to organize and process statistical data;
- simulation of random events and the concept of probability.

## Evaluation of Learning

### Orientations and Practices Relating to the Evaluation of Learning

> The evaluation of student learning has come in for a great deal of discussion in the Québec education system over the last decade and it is surely no exaggeration to say that this field has been and to some extent remains a subject of scrutiny. Teachers today are more knowledgeable about the evaluation of student learning than they were in the past. . . .[6]

It is important to draw on all the available expertise in evaluation and ensure that evaluation practices increasingly tie in with the essential learning pursued in the programs of study. Thus, the aim should be to establish greater consistency between the spirit of these programs and evaluation practices.

### Procedures for Evaluating Learning

When evaluating student learning, teachers should keep in mind the purpose of evaluation. Whether the goal is to give immediate educational feedback (formative evaluation) or to determine whether one or more terminal objectives have been attained (summative evaluation), evaluation provides individual students with useful information about their learning progress. It also helps teachers to assess the organization of program content and the effectiveness of teaching methods. Since the program is aimed at helping students acquire a solid basic education and the skills that will enable them to adapt to a constantly changing society,

> the evaluation of learning should take into account the various components of human development and the complex nature of education, [and] be consistent with the learning activities carried out in the classroom.[7]

In this program, the students not only acquire knowledge, but also learn how to investigate, communicate, represent, reason and use a variety of approaches in order to solve problems. They also acquire other skills and attitudes.

Because the students' knowledge, skills and attitudes are constantly evolving, it is necessary to create situations which will yield information that, after criterion- or norm-referenced interpretation, is likely to provide a reliable indication of each student's or group's knowledge.

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6. Conseil supérieur de l'éducation, *Évaluer les apprentissages au primaire : un équilibre à trouver* (Québec: Direction des communications du CSE, 1992), 1 (Free translation).

7. Ibid., 2 (Free translation).

Since "paper-and-pencil" evaluation may not be appropriate to every aspect of this program, a certain amount of adaptation will be necessary. Depending on the specific goals and in keeping with a spirit of diversification, the following means of evaluation could be appropriate:

- Log
- Oral presentation of a solution or a mathematical subject
- Quiz
- Class discussion
- Group project
- Interview
- Comprehensive examination comprising a number of sections
- Evaluation during computer-assisted learning activities
- Observation checklist
- Self-evaluation
- other

The different types of evaluation must also take into account the variety of learning activities:

- Manipulation activity
- Communication activity (oral or written, individual or group)
- Estimation activity
- Activity using a calculator
- Activity using a computer
- other

When planning education evaluation, it is important to vary the means of evaluation. However, this does not mean that only one type of evaluation (i.e. diagnostic, formative and summative) should be used. Choices must be made in this regard.

The evaluation of learning, be it formative or summative, is essentially aimed at improving both learning and teaching.

As Esther Paradis notes in *L'évaluation des apprentissages : valoriser sa mission pédagogique*, "Isn't it essentially a matter of rediscovering the educational merit of evaluation?"[8]

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8. Esther Paradis, *L'évaluation des apprentissages : valoriser sa mission pédagogique* (Québec: Fédération des enseignantes et des enseignants de commissions scolaires, Centrale de l'enseignement du Québec, 1992), 26 (Free translation).

## Relative Importance of the General Objectives

The following table shows the relative importance of each general objective.

| General Objectives | % |
|---|---|
| 1. To help students learn to apply optimization techniques. | 50 |
| 2. To help students develop their ability to analyze statistical data and data related to probabilities. | 30 |
| 3. To have the students analyze geometric situations. | 20 |

# PROGRAM CONTENT

## Program Structure

This program is made up of global, general, terminal and intermediate objectives. These objectives should reflect the aims of mathematics education and the guiding principles mentioned previously.

### Global Objectives

Objectives that summarize the *role* that mathematics plays in providing students with the basic education they need to integrate into our changing society. These global objectives remain the same throughout the five years of secondary school and form the nucleus around which the objectives for each level are structured.

### General Objectives

Objectives that specify the context in which the global objectives will be pursued and that describe in general terms the *expected educational outcomes* associated with each program theme. General objectives can be broken down into a set of terminal objectives.

### Terminal Objectives

Objectives that clarify the general objectives and describe the *anticipated results*. Each objective is described in three paragraphs:

- The first paragraph indicates what the students have learned from their previous studies.
- The second paragraph provides criteria for determining whether the students have attained the terminal objective.
- The third paragraph outlines activities that are consistent with the general objective, the global objectives and the guiding principles. In this way, it reflects the spirit of the program.

The terminal objective is attained when the students are able to establish a link between a situation and acquired knowledge. This ability is directly related to attainment of the terminal objective and not to attainment of each of the underlying intermediate objectives, a complex object of knowledge being more than the sum of its parts. Hence, the primary goal is to have the students achieve the terminal objectives of the program. The degree to which the terminal objectives of the program are attained is directly related to the appropriateness of the measurement instruments, which must take into account the scope of the intermediate objectives and the context outlined by the general objective and the global objectives.

### Intermediate Objectives

Objectives that specify the scope of a terminal objective, intermediate objectives might also be described as "reference objectives." They are not intended as a series of steps to be completed one after the other. Such a process would give a very fragmented picture of teaching and learning. Rather, intermediate objectives are:

- aspects of a theme that have been chosen for the program;
- clarifications to ensure that the terminal objective is clearly understood;
- guidelines that indicate the connection between the terminal objective and student learning;
- prerequisites for attaining a terminal objective.

## Program Objectives

### Global Objectives

**Establishing Connections**

Increasing the students' ability to establish connections between the knowledge they are acquiring and the knowledge they already have in mathematics and other disciplines, and encouraging them to view their knowledge as a tool that can be useful to them in everyday life.

**Problem Solving**

Increasing the students' ability to analyze the data associated with a problem and use appropriate strategies to arrive at a solution that they will be able to verify, interpret and generalize.

**Communicating**

Increasing the students' ability to grasp and transmit information and to express their thoughts clearly, using mathematical language.

**Reasoning**

Increasing the students' ability to formulate hypotheses and verify them using an inductive or a deductive method.

## GENERAL OBJECTIVE 1

**To help the students learn to apply optimization techniques**

Mathematical and scientific models for solving management problems provide useful decision-making tools.

Management problems are sometimes very complex, and it is often advisable to disregard certain aspects of the problem in order to create a simplified model of a situation. Such a model can help us better understand reality and is used to represent the fundamental properties of a given phenomenon. A model makes it possible to simulate a real-life situation in order that we may better understand and analyze it. If the model is to be comprehensive, it should reflect the essential elements of the phenomenon under study as accurately as possible.

To this end, it is important for the students to devise optimization tools, using models such as graphs and linear programming, which have many applications. The students will become familiar with useful decision-making tools and will discover how effective models can be in the analysis of complex situations. In short, the students will acquire knowledge they will need to become well-informed citizens.

It is important to choose situations that show how mathematics facilitates the decision-making process, using simplified models if necessary. Each topic should be introduced with very concrete examples and the level of formalism must be geared to the capability of the students for whom this course was designed.

It is important to emphasize problem solving, the understanding of concepts, the use of mathematical models, teamwork and communication. In this way, the students learn to appreciate the value of mathematics and become confident in their own abilities.

## Terminal Objective 1.1

### To solve problems using a graph

Since the beginning of secondary school, the students have learned how to represent situations in different ways and to translate from one mode of representation to another. Certain special graphs, called tree diagrams, were used in Secondary II to solve counting problems.

Students who have attained Terminal Objective 1.1 of this program will be able to use graphs to represent certain situations when solving problems. The students will have to analyze communcations networks and diagrams representing circuits, tournaments and production schedules that can be easily modelled by means of graphs. Graphs not only describe phenomena, but also have mathematical properties which can be used to solve problems and thereby facilitate decision making. After solving problems using fundamental concepts (graph, edge, vertex, path, circuit), the students will work with directed graphs (digraphs). Problems will then be related to situations represented by weighted graphs that may be directed or undirected. The students will complete their study of graphs by examining tree diagrams that may or may not be weighted.

A variety of learning situations that call for problem solving, logical reasoning, teamwork and communication are consistent with the global objectives, General Objective 1 and the guiding principles. The students will learn that the basic concepts of graph theory are simple and effective and can be used to solve real-world problems which, at first, seem difficult to understand.

## 1.1

## Intermediate Objectives

- To represent a situation by a graph, a directed graph (digraph) or a weighted graph.
- To distinguish between a path and a circuit.
- To use Euler's path or circuit, Hamilton's path or circuit, or a weighted tree diagram to determine an optimum solution.
- To interpret a graph.
- To justify a statement made in solving a problem that involves using a graph.

## Terminal Objective 1.2

### To solve problems using a system of linear inequalities

In Secondary IV, the students used different modes of representation to solve problems related to systems of linear relations. Only situations that could be represented by straight lines were studied.

Students who have attained Terminal Objective 1.2 of this program will be able to determine the values of the decision variables that maximize (or minimize) a function subject to a set of constraints. These constraints usually take the form of limitations on such things as raw materials, production capacity, the number of employees and capital requirements. The students will use linear programming to construct a model that will facilitate decision making. They may have to choose from among a number of optimum solutions and justify their choice. They can occasionally use an algebraic approach to determine the coordinates of a vertex or the vertices of the polygon of constraints. The following table indicates the different types of translations from one mode of representation to another; the shaded boxes indicate the translations covered by this terminal objective. Note that the students have used this approach to study different types of relations since Secondary II. The Roman numerals indicate the level(s) at which these translations were studied.

Translations from one mode of representation to another

| from / to | words or drawing | table of values | graph | rule equation or inequality |
|---|---|---|---|---|
| words of drawing | II, III and IV | II, III and IV | II, III and IV | II, III and IV |
| table of values | II, III and IV | | III and IV | III |
| graph | II, III and IV | | | III |
| rule equation or inequality | II, III and IV | III and IV | IV | II, III and IV |

A variety of learning situations in which the students can acquire new mathematical knowledge and skills and use what they have already learned are consistent with the global objectives, General Objective 1, and the guiding principles.

## 1.2

## Intermediate Objectives

- To represent a situation using a system of linear inequalities.
- To graph a system of linear inequalities.
- To formulate an algebraic expression that will represent the function to be optimized.
- To determine the best solution(s) for a particular situation, given a number of different possibilities.
- To justify the choice of values that optimize the function.

## GENERAL OBJECTIVE 2

**To help the students develop their ability to analyze statistical data or data related to probabilities**

In this era of rapid communication, we encounter a great deal of qualitative and quantitative data. Indeed, raw data, graphs, rates, percentages, probabilities, averages, predictions and tendencies have become a part of our everyday life. They influence decisions related to health care, the family, citizenship, employment, finance, sports and many other things. To obtain the information we need as citizens or to work productively in today's world, we must deal with data and be able to make intelligent decisions with ease. It is therefore essential to make students aware of the importance of statistics and probability in their daily lives.

With their knowledge of statistics, the students will be able to summarize the information gathered during a study, poll or random experiment, using different types of graphs. The students will also be able to describe a set of data, using certain numerical summaries. In addition, they will be able to analyze certain phenomena by evaluating the probability of a given event or outcome, for example. Models based on statistical and probability theory will enable students to simulate real-life situations in order to better understand and analyze them.

Modern technology provides excellent ways of examining data, creating models or performing simulations. The situations studied should be realistic, but they should also be simple and of interest to the students.

## Terminal Objective 2.1

**To solve problems using the concept of correlation**

In Secondary II, the students analyzed statistical data using measures of central tendency (mean, median, mode). In Secondary IV, they continued this analysis using measures of position. Over the last two years of secondary school, the students have also become familiar with the concept of dispersion of data by studying the concept of range as well as box-and-whisker plots.

When doing a statistical study, students who have attained Terminal Objective 2.1 of this program will be able to determine if two given variables are related and will also be able to describe that relationship, where applicable. The students will discover that a population is characterized not only by the distribution of variables, but also by the relationship between those variables. A scatter plot is a useful way of representing the relationship between the variables. It tells you whether there is a correlation between the variables and allows you to describe some of its characteristics. The students will be asked to describe the correlation as positive, negative or zero, high or low, or perfect or imperfect. Calculations will be limited to estimating the correlation coefficient by means of a graphical method.

Activities in which the students must analyze and interpret data are consistent with the global objectives, General Objective 2 and the guiding principles. Emphasis should be placed on the analysis and presentation of information rather than on calculations. The students should not be required to use sophisticated techniques when common sense will suffice.

## 2.1

### Intermediate Objectives

- To construct a two-variable distribution table.
- To construct a scatter plot.
- To describe the correlation between two variables in one's own words.
- To estimate the correlation coefficient.
- To interpret the correlation between two variables.

## Terminal Objective 2.2

### To solve problems using probabilities

Using grids, tree diagrams and networks, Secondary II students determined the total number of possible outcomes in various situations involving chance. As needed, they used concepts such as complementary, mutually exclusive and non-mutually exclusive events to determine the probability of an event. The students studied random experiments involving one or more steps (e.g. drawing two objects in succession, with or without replacement).

Students who have attained Terminal Objective 2.2 of this program will be able to choose a suitable model (table, grid, tree diagram, area model, enumeration, fundamental counting principle) and assign a probability to an event when solving a problem. In certain cases, they will simulate the situation in order to estimate the probability of an event. While consolidating what they have already learned, the students will have to interpret the following ratios:

- the probability of an event (number of favourable outcomes/ total number of possible outcomes);
- the "odds for" an event taking place (number of favourable outcomes : number of unfavorable outcomes);
- the "odds against" an event taking place (number of unfavourable outcomes : number of favorable outcomes).

Furthermore, in calculating the probabilities of compound events, the students will examine conditional probability when the sample space has been restricted. Lastly, the students are introduced to the concept of mathematical expectation, which will simply be used to determine the fairness of a game or the possibility of winning or losing.

Random experiments aimed at developing the students' ability to assess given hypotheses and make predictions and better decisions are consistent with the global objectives, General Objective 2 and the guiding principles. The students should be encouraged to solve problems in different ways and to back up their conclusions.

## 2.2

## Intermediate Objectives

- To distinguish between odds (for or against) and probability.
- To evaluate the probability that an event will occur during a random experiment, knowing that another event has occurred during that experiment.
- To calculate the mathematical expectation of a random variable.
- To interpret the mathematical expectation of a random variable.

## GENERAL OBJECTIVE 3

**To have the students analyze geometric situations**

In everyday life, we do our work using all the knowledge we have accumulated. Mathematical work is often no different in this regard. Instead of focusing on new concepts, students in Mathematics 514 will develop or perfect relevant strategies and use all their acquired knowledge to improve their decision-making ability in new situations.

From the beginning of secondary school, the students have progressed through a hierarchy of levels in developing their geometric thinking skills. They first learned to recognize shapes and then analyzed the properties of these shapes before making deductions by establishing relationships between these properties. When studying the concepts of congruence and similarity in Secondary IV, the students discovered that the reasoning used to solve a problem is similar to the sound, structured argumentation needed to present a proof.

In Secondary V, the students will be able to consolidate their knowledge and skills by applying them to a variety of problems, while continuing to justify their reasoning. They will examine geometric situations in which they must use the concept of distance along with other concepts they have studied. In addition, the students will be required to solve problems using their knowledge of probability in a geometric context.

Through activities involving active exploration and observation, Secondary V students will be able to use their imagination and creativity.

## Terminal Objective 3.1

**To solve problems using the concept of distance**

From Secondary I to Secondary IV, the students built up a system of concepts and relationships pertaining to two- and three-dimensional figures. They also examined these figures using isometries and dilatations. In addition, they solved problems involving the concept of proportion or the Pythagorean theorem.

Students who have attained Terminal Objective 3.1 of this program will be able to solve problems involving the concept of distance as well as other geometric concepts and relationships they have studied. They will be able to explore situations represented in a Cartesian plane and other situations involving two- or three-dimensional figures without precise coordinates. Every case should involve calculating a distance, but this task should not be the focus of the problem.

All-encompassing activities in which the students can use their considerable knowledge of geometry are consistent with the global objectives, General Objective 3 and the guiding principles. The students will not only be able to apply concepts, but will also realize the importance of choosing the appropriate problem-solving strategy.

## 3.1

### Intermediate Objectives

- To calculate the distance between two points.
- To determine the coordinates of the point on a line segment which divides that segment in a given ratio.
- To compare distances.
- To justify a statement in the solution of a problem.[1]

1. See Appendix.

## Terminal Objective 3.2

**To solve problems using the concept of probability in a geometric context**

From Secondary I to Secondary IV, the students established relationships between the dimensions of different figures and between their perimeters or their areas. In Secondary II and in Terminal Objective 2.2 of this program, the students calculated the probability of an event during a random experiment.

Students who have attained Terminal Objective 3.2 of this program will be able to solve problems where an event is a set of points in a region of a figure. The students will be able to determine probabilities by, for example, comparing lengths or areas, using the graph of a system of inequalities or applying concepts related to circles (e.g. central angles, measures of sectors). In this case, the outcomes are associated with points chosen at random in one- or two-dimensional geometric regions which represent the sample space (set of possible outcomes).

Activities in which the students can establish connections between the concepts they have studied in geometry and probability theory are consistent with the global objectives, General Objective 3 and the guiding principles. Problems that are sometimes complex can become more accessible when they are represented by geometric models. Modern technology provides excellent ways of creating such models or simulating situations.

## 3.2

### Intermediate Objectives

- To estimate the probability of an event in a geometric context.
- To calculate the probability of an event in a geometric context.
- To justify a statement in the solution of a problem.

# Appendix

In the Secondary I, II, III and IV mathematics programs, the students gradually built up a system of axioms. Using the principles listed below, the students can deduce certain measurements and justify certain steps involved in solving problems. They will thus be able to structure an argument and present simple proofs.

### *Secondary I Program*

1. Adjacent angles whose external sides are in a straight line are supplementary.
2. Vertically opposite angles are congruent.
3. The sum of the measures of the interior angles of a triangle is 180º.
4. In any triangle, the length of any side is less than the sum of the lengths of the other two sides.
5. In any triangle, the length of any side is greater than the difference of the lengths of the other two sides.
6. In any triangle, the longest side is opposite the largest angle.
7. In any isosceles triangle, the angles opposite the congruent sides are congruent.
8. In any equilateral triangle, each angle measures 60º.
9. In any right triangle, the acute angles are complementary.
10. In any isosceles right triangle, each acute angle measures 45º.
11. The axis of symmetry of an isosceles triangle contains a median, a perpendicular bisector, an angle bisector and an altitude of the triangle.
12. The axes of symmetry of an equilateral triangle contain the medians, perpendicular bisectors, angle bisectors and altitudes of the triangle.
13. The opposite angles of a parallelogram are congruent.
14. The opposite sides of a parallelogram are congruent.
15. The diagonals of a parallelogram bisect each other.
16. The diagonals of a rectangle are congruent.
17. The diagonals of a rhombus are perpendicular to each other.

## *Secondary II Program*

1. The diagonals from one vertex of a convex polygon form n — 2 triangles, where n is the number of sides in that polygon.

2. In a convex polygon, the sum of the measures of the exterior angles, one at each vertex, is 360º.

3. The sum of the measures of the interior angles of a polygon is 180º (n — 2), where n is the number of sides in the polygon.

4. Three non-collinear points determine one and only one circle.

5. All the perpendicular bisectors of the chords of a circle meet at the centre of that circle.

6. All the diameters of a circle are congruent.

7. In a circle, the measure of the radius is half the measure of the diameter.

8. The axes of symmetry of a circle contain its centre.

9. The ratio of the circumference of a circle to its diameter is a constant known as $\pi$ .

10. In a circle, the measure of the central angle is equal to the measure of its intercepted arc.

11. In a circle, the ratio of the measures of two central angles is equal to the ratio of the measures of their intercepted arcs.

*Secondary III Program*

1. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

2. A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other two sides.

3. In any convex polyhedron, the sum of the number of vertices and the number of faces is equal to the number of edges plus two.

4. Any translation and any dilatation will transform a straight line into another line parallel to it.

5. Isometries or dilatations have one or more of the following properties :

   - they preserve collinearity;
   - they preserve parallelism;
   - they preserve the order of points;
   - they preserve the orientation of the plane;
   - they preserve distances and measures of angles.

## *Secondary IV Program*

1. If a transversal intersects two parallel lines, then:

   • the alternate interior angles are congruent;
   • the alternate exterior angles are congruent;
   • the corresponding angles are congruent.

2. If two corresponding (or alternate interior or alternate exterior) angles are congruent, then they are formed by two parallel lines and a transversal.

3. The angles and sides of isometric figures* are equal in measure.

4. Figures are isometric if and only if there is an isometry or a composite of isometries that makes one figure coincide with the other.

5. Two triangles whose corresponding sides are congruent must be isometric.

6. If two sides and the contained angle of one triangle are congruent to two sides and the contained angle of another triangle, then the triangles must be isometric.

7. If two angles and the contained side of one triangle are congruent to two angles and the contained side of another triangle, then the triangles must be isometric.

8. Transversals intersected by parallel lines are divided into segments of proportional lengths.

9. Any straight line that intersects two sides of a triangle and is parallel to a third side forms a smaller triangle similar to the larger triangle.

10. The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.

11. Similar figures have congruent corresponding angles and proportional corresponding sides.

12. Two figures are similar if and only if there is a dilatation or a composite of transformations that preserves the order of points, the measures of the corresponding angles and the ratio of the corresponding sides.

13. If two angles of one triangle are congruent to two angles of another triangle, then the triangles must be similar.

14. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles must be similar.

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* The term “figure” designates a plane figure or a solid.

15 - If the lengths of two sides of one triangle are proportional to the lengths of two sides of another triangle and the contained angles are congruent, then the triangles must be similar.

16 - In similar polygons:

- the ratio between the measures of corresponding angles is 1;
- the ratio between the lengths of corresponding elements is equal to the ratio between the lengths of the corresponding sides;
- the ratio of the areas is equal to the square of the ratio between the lengths of the corresponding sides.

17 - Figures with a scale factor of 1 are isometric.

18 - In similar solids, the ratio of the volumes is equal to the cube of the ratio between the lengths of the corresponding sides.

19 - The length of the side opposite the 30º angle in a right triangle is half the length of the hypotenuse.

20 - **Hero's (or Heron's) Formula**

The area $S$ of a triangle whose sides measure *a, b and c* is:

$$S = \sqrt{p(p - a)(p - b)(p - c)}$$

where $p = \frac{1}{2}(a + b + c)$ (i.e. half the perimeter of the triangle).

21 - **The Law of Sines**

The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides. This can be expressed as follows:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

# Bibliography

**Alalouf, Serge, Denis Labelle and Jean Ménard.** *Introduction à la statistique appliquée.* Les Éditions Addison-Wesley, 1985.

**Amyotte, Luc.** *Méthodes quantitatives : applications à la recherche en sciences humaines.* Saint-Laurent: Éditions du Renouveau pédagogique, 1996.

**Baillargeon, Gérald.** *Programmation linéaire : outils d'optimisation et d'aide à la décision.* Trois-Rivières: Les Éditions SMG, 1994.

**Baillargeon, Gérald.** *Introduction au calcul des probabilités.* Trois-Rivières: Les Éditions SMG, 1981.

**Bednarz. Nadine.** "L'enseignement des mathématiques et le Québec de l'an 2000." In *Mathématiquement vôtre! Défis et perspectives pour l'enseignement des mathématiques,* edited by Richard Pallascio. Montréal: Les éditions Agence d'ARC, 1990.

**Bergeron, Anne, and Jacques Bordier.** *Enseignement des probabilités et des statistiques au secondaire, PMM 5029.* Québec: Télé-université, 1982.

**Bertrand, Richard, Claude Castonguay and Denis Paradis.** *Initiation à la théorie des graphes, PMM 3018.* Québec: Télé-université, 1976.

**Bordier, Jacques, et al.** *La mathématique et l'activité humaine : rencontre avec Pascal C.* Québec: Télé-université, 1979.

**Burril, Gail, et al.** *Data Analysis and Statistics across the Curriculum, Curriculum and Evaluation Standards for School Mathematics, Addenda Series, Grades 9-12.* Reston, Va.: The Council, 1992.

**Burril, Gail F.** *Guidelines for the Teaching of Statistics K-12 Mathematics Curriculum.* Alexandria, Va.: Center for Statistical Education, American Association, March 1991.

**Conseil supérieur de l'éducation.** *Évaluer les apprentissages au primaire : un équilibre à trouver.* Québec: Direction des communications, 1992.

**COMAP.** *For All Practical Purposes: Introduction to Contemporary Mathematics*. New York: W.H. Freeman and Company, 1991.

**Copes, Wayne, et al.** *Graph Theory: Euler's Rich Legacy.* Providence, RI: Janson Publications, 1987.

**Dahlke, Richard, and Robert Fakler.** "Geometrical Probability." In *Teaching Statistics and Probability, 1981 Yearbook of the National Council of Teachers of Mathematics,* edited by Albert P. Schulte and James R. Smart. Reston, Va.: The Council, 1981.

**Dossey, John A.** "Discrete Mathematics: The Math for Our Time." In *Discrete Mathematics across the Curriculum K-12, 1991 Yearbook of the National Council of Teachers of Mathematics,* edited by Margaret J. Kenney and Christian R. Hirsch. Reston, Va.: The Council, 1991.

**Henry, Michel.** *L'enseignement des probabilités, perspectives historiques, épistémologiques et didactiques.* Irem de Besançon, October 1994.

**Janvier, Claude, Catherine Girardon and Jean-Charles Morand.** "Mathematical Symbols and Representations." In *Research Ideas for the Classroom: High School Mathematics,* edited by P. S. Wilson. New York: Macmillan, 1993.

**Ledoux, Antoine.** "Comment maximiser l'enseignement de l'optimisation." In *Bulletin de l'AMQ,* Octobre 1995.

**National Council of Teachers of Mathematics.** *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: The Council, 1989.

**———.** *Geometric Probability, New Topics for Secondary School Mathematics,* Department of Mathematics and Computer Science North Carolina School of Science and Mathematics. Reston, Va.: The Council, 1988.

**Paradis, Denis.** *Théorie des graphes 2, PMM 3024.* Québec: Télé-université, 1976.

**Paradis, Esther.** *L'évaluation des apprentissages : valoriser sa mission pédagogique.* Québec: Fédération des enseignantes et des enseignants de commissions scolaires, Centrale de l'enseignement du Québec, 1992.

**Québec, Ministère de l'Éducation.** *Mathematics Curriculum Guide Elementary School, Booklet K, Problem Solving.* Code 16-2300-11A. Québec: ministère de l'Éducation, 1989.

**Resnick, L. B., and L. E. Kloffer.** "Toward the Thinking Curriculum: An Overview." In *Toward the Thinking Curriculum: Current Cognitive Research, 1989 Yearbook of the Association for Supervision and Curriculum Development,* edited by Lauren B. Resnick and Leopold E. Kloffer. Alexandria, Va.: Association for Supervision and Curriculum Development, 1989.

**Trudel, Robert, and Rachad Antonius.** *Méthodes quantitatives appliquées aux sciences humaines.* Montréal: Centre éducatif et culturel, 1991.

**Ulep, Soledad A.** "An Intuitive Approach in Teaching Linear Programming in High School." In *Mathematics Teacher,* January 1990.

**Zawojewski, Judith S., et al.** *Dealing With Data and Chance, Curriculum and Evaluation Standards for School Mathematics, Addenda Series, Grade 5-8.* Reston, Va.: The Council, 1991.

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