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# **A cost-effectiveness differential game model for climate agreements**

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**Abstract:** In this paper, we propose a differential game model with a coupled constraint to represent the possible effects of climate agreements between industrialized, emerging and developing countries. Each group of countries is represented by an economic growth model where two different types of economies, called respectively ‘low-carbon’ and ‘carbon’ can co-exist, each of which having different productivities of capital and of emissions due to energy use. We assume that each group of countries participating in the negotiations has identified a damage function, which determines a loss of GDP due to warming and has also a possibility to invest in a capital permitting adaptation to climate changes. The climate agreements we consider have two main components: (i) they define a global emission budget for a commitment period and impose it as a limit on cumulative emissions during that period; (ii) they distribute this global budget among the different coalitions of countries taking part in the agreement. This implies that the game has now a coupled constraint for all participants in the negotiations. The outcome of the agreement is therefore obtained as a generalized or ‘Rosen’ equilibrium which can be selected among a whole manifold of such solutions. We show that the family of Nash equilibria in the games obtained through a distribution of the total budget among the different parties corresponds to the manifold of normalized equilibria. We then propose an equity criterion to determine a fair division of this total emission budget, or equivalently to select a proper weighting for a normalized equilibrium.

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# 1 Introduction

In this paper, we model the outcome of an international agreement on climate change as an equilibrium in an open-loop dynamic game with a coupled constraint on cumulative emissions over a specific negotiated commitment period. Our objective is to analyze the global climate change issue as a competition among different growing economies, which are linked by the damage caused by climate changes due to atmospheric accumulation of their GHG emissions. The model emphasizes also the role played by adaptation in the design of optimal response by competing groups of countries to climate change. The numerical simulations presented here consider climate change as a very long-term issue for which a cost-benefit approach based on the use of damage functions by the negotiating countries leads to equilibrium solutions with delayed action to reduce emissions, a not very attractive policy from an ecological point of view. However if we use a cost-effectiveness approach, by imposing a constraint in the form of a global emission budget over the negotiated commitment period that will be compatible with maintaining temperature increase below 2°C, immediate transition toward a low-carbon economy is favored.

We represent three main regions in the World, corresponding grossly to industrialized (OECD), emerging (BRIC) and developing or rest of the world (ROW) countries, and we associate an economic growth model à la Ramsey (1928) coupled with a climate module as in Nordhaus & Boyer (2000), with a dual economy ('carbon' vs. 'low-carbon') as in Bahn & Haurie (2008) and with an adaptation capital permitting each region to partially alleviate damages caused by the temperature increase as in Bahn et al. (2012). We neglect the trade effects and we consider only the interdependence caused by the influence of each group of countries on the global climate change and therefore on the damage caused to the three groups. We introduce a differential game formulation of this interdependence, using the open-loop information structure. We adopt an open-loop information structure as we view the climate negotiations as the search for an optimal trade-off between different and possibly conflicting economic growth paths. Several integrated models have been proposed permitting a search for an equilibrium under an open-loop information structure, in particular RICE (Nordhaus & Yang, 1996), WITCH (Bosetti et al., 2007), REMIND-R (Leimbach et al., 2010) and CWS (Eyckmans & Tulken, 2006). The originality of the approach in our paper lies in the interpretation of an international agreement on climate as the introduction of a coupled constraint in a differential game. Such games have been initially proposed in Haurie (1995) and Haurie & Zaccour (1995) and further studied in Carlson & Haurie (2000). This concept has also been used for an environmental model in Krawczyk (2005). The effect or impact of the agreement must then be evaluated as the outcome of a normalized equilibrium as defined in Rosen (1965). It is well known that a game with coupled constraint admits a manifold of normalized equilibria. In the particular situation of this climate model it can be shown (Bahn & Haurie, 2008) that each normalized equilibrium corresponds to a particular sharing of the global budget among the players (groups of countries). Therefore, an agreement would consist in allocating a specific emission budget for the commitment period to each player and letting them play a Nash equilibrium (Nash, 1950) under these decoupled constraints. The particular sharing of the global budget could be done in order to attain some fairness objective. This interpretation of climate negotiations and agreements has been exploited recently to design meta-games based on statistical emulation of a general equilibrium model (Haurie et al., 2013; Babonneau et al., 2013). These models captured well the macroeconomic effects, including the modifications in the terms of trade, for several coalitions of countries (7 or 11). However the issue of long-term economic growth was not tackled in these models. The present study, based on infinite horizon differential game models is focused on the analysis of these long-term effects.

The climate change issue is a very long-term concern, so we model economic growth on a very long time interval, which can be taken as infinite ( $\infty$ ). However the negotiations on climate are concerned essentially with the emission situation on shorter commitment periods (e.g., 2010–2050). So we represent an international agreement on climate as the introduction of a constraint on cumulative emissions over a given commitment period, corresponding to the definition of a global emission budget, and the distribution of this budget among the three groups of countries, considered now as three players. This introduces a cost-effectiveness structure in the differential game.

We compare the different outcomes corresponding in particular to the following situations:

1. BAU, a counterfactual baseline where no damage is caused by temperature increase;
2. Pareto, where a weighted sum of the welfares is maximized;
3. Nash, where the three players define growth paths corresponding to a Nash equilibrium;
4. Rosen, where the three players define growth paths corresponding to a Nash equilibrium with a coupled constraint, as indicated above.

The paper is organized as follows: in Section 2, we give a brief description of the integrated model we use. In Section 3, we define the differential game with coupled constraint and we show how the manifold of normalized equilibria corresponds to a family of Nash equilibria with specific sharing of the budget constraint. In Section 4, we present the result of a set of numerical simulations corresponding to the different equilibrium solutions discussed. And finally, in Section 5, we conclude with a discussion of the policy implications of these simulations.

## 2 The integrated economic growth models

This section extends the one-region Ada-BaHaMa model proposed in Bahn et al. (2012) to a version where the world is comprised of  $m$  independent regions. We do not consider trade among the regions. We use a continuous time formulation. For a more detailed justification of the functional forms used to describe economics and climate change dynamics, we refer to Bahn et al. (2012).

### 2.1 Variables

The model has the following variables, where  $j = 1, \dots, m$  is the index of each of the  $n$  regions and  $t$  the model running time:

$AD(j, t)$ :	reduction of damages due to adaptation measures in region $j$ at time $t$ , in %;
$C(j, t) \geq 0$ :	total consumption in region $j$ at time $t$ , in trillions ( $10^{12}$ ) of dollars;
$c(j, t) \geq 0$ :	per capita consumption in region $j$ at time $t$ , $c(j, t) = \frac{C(j, t)}{L(j, t)}$ ;
$E_1(j, t) \geq 0$ :	yearly emissions of GHG (in Gt– $10^9$ tons–carbon equivalent) in the carbon economy of region $j$ at time $t$ ;
$E_2(j, t) \geq 0$ :	yearly emissions of GHG in the low-carbon economy of region $j$ at time $t$ , in GtC;
$ELF(j, t)$ :	economic loss factor in region $j$ , which is the share of output remaining after deducting climate change damages at time $t$ , in %;
$I_i(j, t) \geq 0$ :	investment in capital $K_i$ ( $i = 1, 2, 3$ ) in region $j$ at time $t$ , in trillions of dollars;
$K_1(j, t) \geq 0$ :	physical stock of productive capital in the carbon economy of region $j$ at time $t$ , in trillions of dollars;
$K_2(j, t) \geq 0$ :	physical stock of productive capital in the low-carbon economy of region $j$ at time $t$ , in trillions of dollars;
$K_3(j, t) \geq 0$ :	physical stock of adaptation capital in region $j$ at time $t$ , in trillions of dollars;
$K_{3\max}(j, t) \geq 0$ :	maximal stock of adaptation capital in region $j$ at time $t$ , in trillions of dollars;
$L_1(j, t) \geq 0$ :	part of the (exogenously defined) labor force $L(j, t)$ of region $j$ allocated at time $t$ to the carbon economy, in millions ( $10^6$ ) of persons;
$L_2(j, t) \geq 0$ :	part of the labor force of region $j$ allocated at time $t$ to the low-carbon economy, in millions of persons;
$M(t) \geq 0$ :	atmospheric concentration of GHG at time $t$ , in GtC equivalent;
$WRG(j)$ :	discounted welfare of region $j$ ;
$Y(j, t) \geq 0$ :	economic output of region $j$ at time $t$ , in trillions of dollars.

## 2.2 Economic modeling

In each region  $j = 1, \dots, m$ , a social planner is assumed to maximize social welfare (WRG), given by the integral over the model horizon ( $T$ ) of a discounted utility from per capita consumption ( $c$ ) with a pure time preference rate  $\rho$ :

$$\text{WRG}(j) = \int_0^T e^{-\rho t} L(j, t) \log[c(j, t)] dt. \quad (1)$$

Total labor ( $L$ ) is divided between labor allocated to the carbon economy ( $L_1$ ) and labor allocated to the low-carbon economy ( $L_2$ ):

$$L(j, t) = L_1(j, t) + L_2(j, t). \quad (2)$$

Consumption comes from an optimized share of production ( $Y$ ), the remaining being used to invest in the production capitals (carbon-intensive— $K_1$ —and/or low-carbon— $K_2$ ), in the adaptation capital ( $K_3$ ) and to pay for energy costs (energy being measured through emission levels  $E_i$ ). The presence of damages (defined by the ELF factor) reduces the available production such that:

$$C(j, t) = \text{ELF}(j, t) Y(j, t) - I_1(j, t) - I_2(j, t) - I_3(j, t) - p_{E_1}(j, t) \phi_1(j, t) E_1(j, t) - p_{E_2}(j, t) \phi_2(j, t) E_2(j, t), \quad (3)$$

where  $p_{E_i}$  are energy prices and  $\phi_i$  energy conversion factors for emissions  $E_i$ . Capital stock evolves according to the choice of investment ( $I_i$ ) and a depreciation rate  $\delta_{K_i}$  through a standard relationship:

$$\dot{K}_i(j, t) = I_i(j, t) - \delta_{K_i} K_i(j, t) \quad i = 1, 2, 3. \quad (4)$$

Economic output ( $Y$ ) occurs in the two economies according to an extended Cobb-Douglas production function in three inputs, capital ( $K$ ), labor ( $L$ ) and energy (measured through emission levels  $E$ ):

$$Y(j, t) = A_1(j, t) K_1(j, t)^{\alpha_1(j)} (\phi_1(j, t) E_1(j, t))^{\theta_1(j, t)} L_1(j, t)^{1-\alpha_1(j)-\theta_1(j, t)} + A_2(j, t) K_2(j, t)^{\alpha_2(j)} (\phi_2(j, t) E_2(j, t))^{\theta_2(j, t)} L_2(j, t)^{1-\alpha_2(j)-\theta_2(j, t)} \quad (5)$$

where  $A_i$  is the total factor productivity in the carbon (resp. low-carbon) economy (when  $i = 1$ , resp.  $i = 2$ ),  $\alpha_i$  the elasticity of output with respect to capital  $K_i$  and  $\theta_i$  the elasticity of output with respect to emissions  $E_i$ .

## 2.3 Damages and adaptation

Climate change dynamics are from the DICE model (Nordhaus, 2008). First, stocks of GHGs accumulates in three reservoirs, an atmospheric reservoir ( $M_{AT}$ ), a quickly mixing reservoir in the upper oceans and the biosphere ( $M_{UP}$ ), and a slowly mixing deep-ocean reservoir ( $M_{LO}$ ) which acts as a long-term sink:

$$\dot{M}_{AT}(t) = \sum_{j=1}^n (E_1(j, t) + E_2(j, t)) + \psi_{11} M_{AT}(t) + \psi_{21} M_{UP}(t) \quad (6)$$

$$\dot{M}_{UP}(t) = \psi_{12} M_{AT}(t) + \psi_{22} M_{UP}(t) + \psi_{32} M_{LO}(t) \quad (7)$$

$$\dot{M}_{LO}(t) = \psi_{23} M_{UP}(t) + \psi_{33} M_{LO}(t), \quad (8)$$

where  $\psi_{i,j}$  are calibration parameters. Second, accumulation of GHGs increases the earth radiative forcing  $F$ :

$$F(t) = \eta \log_2 \left( \frac{M_{AT}(t)}{M_{AT}(1750)} \right) + F_{EX}(t), \quad (9)$$

where  $F_{EX}$  is an exogenous radiative forcing term. And third, a stronger radiative forcing yields an increase in the earth's mean surface temperature  $T_{AT}$  and more gradually in the mean (deep) ocean temperature ( $T_{LO}$ ):

$$\begin{aligned}\dot{T}_{AT}(t) &= T_{AT}(t) + \xi_1 [F(t+1) - \xi_2 T_{AT}(t) - \xi_3 (T_{AT}(t) - T_{LO}(t))] \\ \dot{T}_{LO}(t) &= \xi_4 (T_{AT}(t) - T_{LO}(t)),\end{aligned}\tag{10}$$

where  $\xi_i$  and  $\eta$  calibration parameters for an assumed climate sensitivity of 3°C that corresponds to the best estimate given by the IPCC (2013).

Increasing temperature triggers climate changes that yield economic losses affecting regional production (see again Eq. (3)). These (net) regional damages take into account the effects of adaptation (AD):

$$\text{ELF}(j, t) = 1 - \text{AD}(j, t) \left( \frac{T_{AT}(t) - T_d(j)}{\text{cat}_T(j) - T_d(j)} \right)^2,\tag{11}$$

where  $T_d(j)$  is the temperature deviation (from pre-industrial level) at which damages start to occur in region  $j = 1, \dots, n$  and  $\text{cat}_T(j)$  is the ‘catastrophic’ temperature level that depends on the assumed climate sensitivity and at which the entire production of region  $j$  would be wiped out. To reduce the damaging effects of climate change, regions can invest in an adaptation stock. Adaptation dynamics in each region  $j$  is then modelled as follows:

$$\text{AD}(j, t) = 1 - \alpha_{\text{AD}}(j) \frac{K_3(j, t)}{K_{3\text{max}}(j, t)},\tag{12}$$

where  $\alpha_{\text{AD}}$  is the maximal adaptation effectiveness and  $K_{3\text{max}}$  the maximal amount of adaptation capital that would ensure the optimal effectiveness of the adaptation measures. The latter is modelled as an increasing function of temperature level to capture the fact that adaptation costs should increase whenever temperature (and therefore damages) increases:

$$K_{3\text{max}}(j, t) = \beta_{\text{AD}}(j) \left( \frac{T_{AT}(t)}{T_d(j)} \right)^{\gamma_{\text{AD}}(j)},\tag{13}$$

where  $\beta_{\text{AD}}$  and  $\gamma_{\text{AD}}$  are calibration parameters.

The calibration of the damage function in Eq. (11) and of the adaptation effectiveness in Eq. (12) is crucial for a cost-benefit analysis. Indeed, the parameters involved are uncertain and, in particular, the damages caused by high temperature increases are probably underestimated. There is a consensus among climate scientists that a temperature increase above 2°C is dangerous and should be avoided. In our calibration, which is consistent with the AD-DICE model (see Appendix A), the economic loss is not prohibitive at 2°C. The suggestion that a temperature increase above 2°C is dangerous implicitly assumes that there are additional (non-market) damages that should be taken into account.

### 3 A differential game with coupled constraint

In this section, we relate the Ada-BaHaMa model to the general class of differential games with coupled constraints and we characterize the manifold of normalized equilibria. For that purpose, it will be convenient to temporarily switch notations and adopt the usual control theory formalism.

#### 3.1 A coupled isoperimetric constraint

We model the competition of different countries to tackle climate change as a differential game having the following structure. Eqs. (14)–(15) are the state equations and initial conditions for the decoupled variables and controls of the players  $j = 1 \dots, m$ . More precisely, the decoupled state variables  $x_j(t)$  refer to  $(K_1(j, t), K_2(j, t), K_3(j, t))$  and the control variables  $u_j(t)$  refer to  $(I_1(j, t), I_2(j, t), I_3(j, t), E_1(j, t), E_2(j, t))$  for all  $j$  and  $t$ . Eq. (16) corresponds to the decoupled state and control constraints for each of the players  $j$ . Eqs. (17)–(18) are the state equations and initial conditions for the coupling state variable, in our case they are the climate state variables:

$$\dot{x}_j(t) = f_j(t, x_j(t), u_j(t)), \quad j = 1 \dots, m \quad (14)$$

$$x_j(0) = x_j^o, \quad j = 1 \dots, m \quad (15)$$

$$0 \geq h_j(t, u_j(t), x_j(t)), \quad j = 1 \dots, m \quad (16)$$

$$\dot{y}(t) = g(t, \bar{x}(t), y(t), u(t)) \quad (17)$$

$$y(0) = y^o. \quad (18)$$

The payoff to player  $j$  is given in Eq. (19). Note that the variable  $C(j, t)$  is a hidden variable which serves in the definition of the reward  $L_j$ .

$$\int_0^\infty e^{-\rho t} L_j(t, x_j(t), u_j(t), y(t)) dt, \quad j = 1 \dots, m. \quad (19)$$

These variables and functions have the following dimensions:

$$x_j \in \mathbb{R}^{n_j}, u_j \in \mathbb{R}^{p_j}, j = 1 \dots, m, \quad y \in \mathbb{R}^q.$$

$$f_j : \mathbb{R} \times \mathbb{R}^{n_j} \times \mathbb{R}^{p_j} \rightarrow \mathbb{R}^{n_j}, \quad h_j : \mathbb{R} \times \mathbb{R}^{n_j} \times \mathbb{R}^{p_j} \rightarrow \mathbb{R}^{p_j}, j = 1 \dots, m.$$

$$g : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^q \rightarrow \mathbb{R}^q, \quad n = \sum_j n_j, \quad L_j : \mathbb{R} \times \mathbb{R}^{n_j} \times \mathbb{R}^{p_j} \times \mathbb{R}^q \rightarrow \mathbb{R}.$$

The climate agreement is summarized by the definition of a global budget (BUD) for the sum of emissions from all players defined by the functions  $\sum_j \ell_j(u_j(t))$ ,  $\ell_j : \mathbb{R}^{p_j} \rightarrow \mathbb{R}$ , over a planning period  $[t_0, t_1]$ . This is written:

$$\int_{t_0}^{t_1} \sum_j \ell_j(u_j(t)) dt \leq \text{BUD}, \quad (20)$$

where  $\ell_j : \mathbb{R}^{p_j} \rightarrow \mathbb{R}$ . This is a coupled constraint imposed on the differential game. We will show that finding a normalized equilibrium for the game with coupled constraints is equivalent to computing a Nash equilibrium with the following decoupled constraints:

$$\int_{t_0}^{t_1} \ell_j(u_j(t)) dt \leq \theta_j \text{BUD} \quad j = 1 \dots, m, \quad (21)$$

with:

$$0 \leq \theta_j, \quad j = 1 \dots, m, \quad \sum_j \theta_j = 1. \quad (22)$$

Varying the shares  $\theta_j$  of the global emission budget going to each player  $j = 1 \dots, m$ , one generates the whole manifold of normalized equilibria. So a second part of the agreement will be the choice of a particular set of  $\theta_j$ 's under some fairness criteria.

### 3.2 The manifold of normalized equilibria

Given the initial state, the payoff functions of each player can be written  $\phi_j(\underline{u}_{M-j}(\cdot), u_j(\cdot))$ , where  $\underline{u}_{M-j}(\cdot)$  denote the controls of the players other than  $j$  and  $u_j(\cdot)$  is the control of player  $j$ , over the whole time interval  $[0, \infty)$ . The coupled constraint is denoted  $\underline{u}(\cdot) \in \mathcal{U}$ . Let  $\mathbf{r}$  be a vector of positive weights  $r_j > 0$ ,  $\underline{u}(\cdot)$  and  $\underline{v}(\cdot)$  two  $m$  control vectors. We define the response function with weighting  $\mathbf{r}$  as:

$$\Psi(\underline{u}(\cdot), \underline{v}(\cdot); \mathbf{r}) = \sum_j r_j \phi_j(\underline{u}_{M-j}(\cdot), v_j(\cdot)) \quad (23)$$

and the optimal response map as the set valued mapping:

$$\Upsilon(\underline{u}(\cdot); \mathbf{r}) = \left\{ \underline{v}^o(\cdot) = \operatorname{argmax}_{\underline{v}(\cdot) \in \mathcal{U}} \Psi(\underline{u}(\cdot), \underline{v}(\cdot); \mathbf{r}) \right\}. \quad (24)$$

An  $m$ -control vector  $\underline{u}^*(\cdot)$  is an equilibrium under the coupled constraint  $\underline{u}(\cdot) \in \mathcal{U}$ , if it is a fixed point of the optimal response mapping, i.e, if the following holds:

$$\underline{u}^*(\cdot) \in \Upsilon(\underline{u}^*(\cdot); \mathbf{r}). \quad (25)$$

In our particular case the constraint set  $\mathcal{U}$  is defined by the inequality:

$$\int_{t_0}^{t_1} \sum_j \ell_j(u_j(t)) dt - \text{BUD} \leq 0 \quad (26)$$

Assuming enough regularity, the optimization in (24) implies the existence of a multiplier  $\mu$  verifying:

$$0 = \mu \left( \int_{t_0}^{t_1} \sum_j \ell_j(u_j(t)) dt - \text{BUD} \right) \quad (27)$$

$$0 \leq \mu. \quad (28)$$

Then the necessary optimality conditions at equilibrium are the same as those for a Nash equilibrium in the differential game with modified payoffs given by:

$$\begin{aligned} J_j &= \int_0^{t_0} e^{-\rho t} L_j(t, x_j(t), u_j(t), y(t)) dt \\ &+ \int_{t_0}^{t_1} \left( e^{-\rho t} L_j(t, x_j(t), u_j(t), y(t)) dt - \frac{\mu}{r_j} \sum_{i=1}^m \ell_i(u_i(t)) \right) dt \\ &+ \int_{t_1}^{\infty} e^{-\rho t} L_j(t, x_j(t), u_j(t), y(t)) dt \quad j = 1 \dots, m. \end{aligned} \quad (29)$$

Now it is easy to verify that these optimality conditions are equivalent to those of a Nash equilibrium with decoupled constraints:

$$\int_{t_0}^{t_1} \ell_j(u_j(t)) dt \leq \theta_j \text{BUD} \quad j = 1 \dots, m. \quad (30)$$

Indeed, at a Nash equilibrium there will exist multipliers  $\mu_j$  satisfying:

$$0 = \mu_j \left( \int_{t_0}^{t_1} \ell_j(u_j(t)) dt - \theta_j \text{BUD} \right). \quad (31)$$

$$0 \leq \mu_j, \quad j = 1, \dots, m. \quad (32)$$

Since the constraint is scalar (one dimensional) so is also each  $\mu_j$ . If we define  $\mu = \sum_j \mu_j$  and  $r_j = \frac{\mu}{\mu_j}$  we can write these optimality conditions as in Eqs. (27)–(28). Therefore there is a one-to-one correspondence between the manifold of normalized equilibria, indexed over the weighting  $\mathbf{r}$  and the set of Nash equilibria with decoupled constraints (30), indexed over the possible sharings of the global budget defined by:

$$\begin{aligned} 0 &\leq \theta_j, \quad j = 1 \dots, m \\ 1 &= \sum_j \theta_j. \end{aligned} \quad (33)$$

### 3.3 Economic interpretation

The introduction of a coupled constraint in the equilibrium game, based on a global emission budget over a given commitment period, can be interpreted as equivalent to the introduction of a tax on emissions. In the model emissions are represented by the function  $\ell_j(u_j)$ <sup>1</sup> during the period  $[t_0, t_1]$ , and depending on the weighting  $\mathbf{r}$  the tax  $\frac{\mu}{r_j}$  is different in each group of countries. We notice that the tax is increasing in current value, since the direct sum of emissions is considered and not their discounted sum. This has the interesting effect of preventing a large increase of emissions at the end of the  $[t_0, t_1]$  period.

<sup>1</sup>In the model we use  $\ell_j(u_j)$  is the projection of the control vector on the emission component.

## 4 Numerical illustration

In this section we solve numerically a differential game having the structure described in Section 3 and where the players dynamics correspond to the integrated economic growth model of Section 2. There are three players, corresponding roughly to industrialized (OECD), emerging (BRIC) and developing or rest of the world (ROW) countries. Each player  $j$  controls a decoupled economic system through the accumulation of three sorts of capital ( $K_i(j, t)$ ,  $i = 1, \dots, 3$ ) and has an influence, through the accumulation of its GHG emissions, on the climate dynamics. The temperature increase causes a loss of economic output in each economy and this creates an interdependence between the players that leads to the formulation of the fundamental game structure of the climate change issue.

Assuming that each player develops its economic systems in order to optimize the discounted sum of utility derived from consumption, over an infinite time horizon, we will first compute the solution of the Pareto and Nash equilibrium solutions of the open-loop differential game, and compare them with the BAU case, a counterfactual baseline obtained when we assume that there is no economic damage caused by temperature increase. Notice that, in this case, the players control totally decoupled systems and the game situation disappears.

In a second part, we introduce a constraint in the Pareto solution, which consists in keeping the temperature increase below  $2^\circ\text{C}$ , forever, as proposed in the Copenhagen Accord (United Nations, 2009). We use the emission budget over the period 2010–2050 or 2010–2070 obtained in this Pareto solution to introduce a coupled constraint in the non-cooperative game. We next compute the normalized equilibrium corresponding to a sharing of the budget that is the same as in the Pareto solution. We compare then the normalized equilibrium solution with the simple Nash equilibrium obtained earlier.

Finally, in a third part, we explore the effect of modifying the shares of the global emission budget on a fairness criterion designed around the discounted sum of consumption of the population concerned during the commitment period.

All the calculations are made using a simple discretization of the dynamics, leading to difference equations. A mathematical programming approach is then used to solve the optimization problems, using GAMS with CONOPT. The equilibrium solutions are obtained by solving a fixed point problem via a cobweb approach.<sup>2</sup> Please also refer to the Appendices at the end of the manuscript.

### 4.1 Cooperative vs. non-cooperative solutions

In this section we first compare the results obtained in the baseline (BAU case), Pareto and Nash equilibrium solutions.

#### 4.1.1 GHG emissions and temperature

In Table 1, we show first temperature deviation paths from preindustrial levels. In the BAU scenario, temperature reaches around  $3.6^\circ\text{C}$  by 2100. This corresponds approximately to the *Representative Concentration Pathways* RCP8.5 scenario of the IPCC (2013) where the mean concentration reaches  $3.7^\circ\text{C}$  by 2100. When taking into account climate damages, and assuming that the three regions are collaborating to reach a Pareto equilibrium,<sup>3</sup> temperature reaches a much lower level ( $2.7^\circ\text{C}$  by 2100), but still above the safe limit of  $2^\circ\text{C}$  set by the Copenhagen Accord. In the Nash equilibrium setting, as each player optimizes its own welfare without consideration of the other players' welfare, the environmental situation deteriorates with temperature reaching  $3.4^\circ\text{C}$  by 2100, close to the BAU level. Table 2 shows next GHG emission levels for the three players.

In the Pareto scenario, the reduction effort is until 2050 more important for Player 1 (OECD countries) than for the other two players. But after 2050, all players have about the same reduction effort (in % from

<sup>2</sup>Even though it is well known that cobweb does not always converge, we never had such an occurrence in our numerical experiments.

<sup>3</sup>By optimizing a weighted sum of their social welfare, with each weight set to  $1/3$ .

Table 1: Temperature deviation paths (in °C)

	BAU	Pareto	Nash
2010	0.9	0.9	0.9
2020	1.2	1.1	1.2
2030	1.4	1.4	1.4
2040	1.7	1.6	1.7
2050	2.0	1.8	1.9
2060	2.3	2.1	2.2
2070	2.6	2.2	2.5
2080	2.9	2.4	2.8
2090	3.3	2.6	3.1
2100	3.6	2.7	3.4

Table 2: Emission paths (in GtC)

	BAU			Pareto			Nash		
	1	2	3	1	2	3	1	2	3
2010	1.9	3.5	3.2	1.5	3.1	2.9	1.8	3.3	3.1
2020	2.0	4.3	4.1	1.6	3.7	3.5	1.9	4.0	3.8
2030	2.2	5.1	4.9	1.8	4.3	4.1	2.1	4.7	4.5
2040	2.4	6.0	5.8	1.9	4.9	4.7	2.3	5.5	5.3
2050	2.8	7.0	6.8	2.1	5.5	5.3	2.6	6.3	6.1
2060	3.2	8.2	7.9	1.6	4.1	3.9	2.9	7.1	6.9
2070	3.6	9.4	9.0	1.5	3.8	3.7	3.3	8.0	7.8
2080	4.1	10.7	10.4	1.5	4.0	3.8	3.7	9.0	8.7
2090	4.7	12.2	11.8	1.6	4.2	4.1	4.2	10.0	9.7
2100	5.3	13.8	13.3	1.8	4.6	4.4	4.7	11.0	10.7

the baseline). By contrast, in the Nash equilibrium scenario, where each player acts selfishly, they all emit more than in the Pareto scenario, especially Player 1.

#### 4.1.2 Capital accumulation paths

Low-carbon capital accumulation levels are given in Table 3.

Table 3: Low-carbon capital  $K_2$  accumulation paths (in trillion USD)

	BAU			Pareto			Nash		
	1	2	3	1	2	3	1	2	3
2010	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2020	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2030	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2040	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2050	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2060	0.0	0.0	0.0	24.5	62.3	60.0	0.0	0.0	0.0
2070	0.0	0.0	0.0	37.0	95.0	91.5	0.0	0.0	0.0
2080	0.0	0.0	0.0	45.6	118.0	113.7	0.0	0.0	0.0
2090	0.0	0.0	0.0	53.2	138.2	133.3	0.0	0.0	0.0
2100	0.0	0.0	0.0	60.6	158.3	152.7	0.0	0.0	0.0

The accumulation of low-carbon capital  $K_2$  and thus the use of clean energy sources are directly related to the emission reduction efforts carried out by the players. Neither in the baseline, where there is no climate change related damages, nor in the Nash case, where each player is more concerned about his own (rather short-term<sup>4</sup>) welfare, does low-carbon capital accumulate. By contrast, in the Pareto scenario, all regions start accumulating low-carbon capital by 2060, at a time when they more significantly curb their GHG

<sup>4</sup>As reflected by the pure time preference discount rate  $\rho$  in Eq (1).

emissions (compared to baseline levels). Table 4 shows the accumulation path of carbon-intensive capital  $K_1$ . Both in the baseline and in the Nash scenarios, the stock of carbon-intensive capital keeps increasing, as there is in these settings a lack of (serious) effort to abate GHG emissions. Conversely, in the Pareto scenario, carbon-intensive capital accumulates steadily only until one starts to invest in low-carbon capital. Afterwards, the carbon economy co-exists with the low-carbon one, but at reduced levels (compared to the baseline).

Table 4: Carbon-intensive capital  $K_1$  accumulation paths (in trillion USD)

	BAU			Pareto			Nash		
	1	2	3	1	2	3	1	2	3
2010	47.3	41.5	36.4	47.1	41.4	36.2	47.3	41.5	36.3
2020	45.4	66.1	60.7	45.0	65.8	60.4	45.3	66.0	60.6
2030	48.6	92.9	87.4	48.0	92.3	86.8	48.5	92.8	87.3
2040	54.9	121.6	115.7	54.2	119.9	114.1	54.8	120.6	114.8
2050	63.7	152.2	145.8	62.7	150.0	143.7	63.3	150.8	144.5
2060	74.3	185.2	178.0	48.7	120.3	115.5	73.8	183.1	176.0
2070	86.7	220.9	212.7	47.8	120.9	116.4	86.0	217.7	209.7
2080	100.6	259.5	250.1	52.2	133.2	128.4	100.1	254.7	245.6
2090	116.0	301.2	290.5	56.9	150.6	145.3	112.6	294.2	283.8
2100	132.8	346.2	333.9	65.0	170.8	164.8	129.2	336.1	324.4

Table 5 reports finally on the accumulation of adaptation capital  $K_3$ . Note first that adaptation capital does not accumulate in the baseline, where it is not needed. By contrast, adaptation capital starts accumulating in the Pareto and Nash scenarios, as early as 2040 for Player 2 and 3, but only toward the end of the century for Player 1. For the latter player, adaptation acts as a complement to the mitigation efforts started earlier (especially in the Pareto setting). For the former players, there is during some initial periods (especially in the Pareto setting) a clear substitution between adaptation and mitigation efforts. These differences highlight the different trade-off between costs of adaptation and reduction of (some of) climate damages for the players. Note also that in the Nash scenario, the required amount of adaptation capital for a maximal effectiveness is higher (compared to the Pareto scenario), as temperature reaches higher levels.

Table 5: Adaptation capital  $K_3$  accumulation paths (in trillion USD)

	BAU			Pareto			Nash		
	1	2	3	1	2	3	1	2	3
2010	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2020	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2030	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2040	0.0	0.0	0.0	0.0	1.1	1.1	0.0	1.1	1.1
2050	0.0	0.0	0.0	0.0	1.4	1.4	0.0	1.5	1.5
2060	0.0	0.0	0.0	0.0	1.8	1.8	0.0	2.0	2.0
2070	0.0	0.0	0.0	0.0	2.1	2.1	0.0	2.6	2.6
2080	0.0	0.0	0.0	0.0	2.4	2.4	0.0	3.2	3.2
2090	0.0	0.0	0.0	2.7	2.7	2.7	4.0	4.0	4.0
2100	0.0	0.0	0.0	3.0	3.0	3.0	4.8	4.8	4.8

## 4.2 Equilibria with coupled constraints

This section explores the results obtained when imposing a coupled constraint on global cumulative emissions over a period of 50 to 70 years consistent with an objective of maintaining the temperature increase below  $2^\circ\text{C}$ . We note that all the solutions reported in the previous section exceed this temperature increase level. This is in particular the case of the Pareto solution, where temperature reaches  $2.7^\circ\text{C}$  by 2100. Imposing a limit of  $2^\circ\text{C}$  to temperature increase implicitly assumes that there are additional (non-market) damages not taken into account by the economic loss factor.

We call ‘ParetoLT’ the scenario corresponding to a Pareto solution when imposing a 2°C maximum temperature increase. To obtain this Pareto solution we give to each player an equal weight (1/3) and we optimize the weighted sum of payoffs. Then we compute for each player the cumulative emissions over the negotiation period (2010–2050, or 2010–2070) and we impose the global emission budget as a coupled constraint in the computation of a normalized equilibrium. When doing so over the period 2010–2050 we call ‘Rosen5P’ the normalized equilibrium solution. When the constraint is imposed over the 2010–2070 period, we call ‘Rosen7P’ the normalized equilibrium solution. As shown in Section 3, the manifold of normalized equilibria can be also be obtained as the set of Nash equilibria with decoupled constraints corresponding to different ways of sharing the global cumulative emission budget. When referring to Rosen5P and Rosen7P, we consider a sharing which corresponds to the emission budget of each player in the ParetoLT solution. Then, in the next Section 4.3, the sharing will be modified in order to explore alternative normalized equilibria.

#### 4.2.1 GHG emissions and temperature

Table 6 reports first on temperature deviation paths from preindustrial levels. In the ParetoLT scenario, temperature reaches the limit by the end of the century (and remains at this level afterwards). Under a non-cooperative setting, temperature is kept in check only during the commitment period, where it follows the same path as in the cooperative setting. Afterwards, temperature quickly exceeds the 2°C temperature limit, as players return to a purely selfish (Nash) behavior. Table 7 reports on GHG emission levels for the three players. In the ParetoLT scenario, as in the unconstrained Pareto case, the reduction effort is initially more important for Player 1 than for the other two players. But here, to avoid temperature exceeding the 2°C limit, the other players join with similar effort (in % from the baseline) as early as 2030. In the Rosen setting, players make similar reduction efforts as for the ParetoLT scenario during the commitment period. Afterwards, players return to their Nash emission pattern and emit much more GHGs, but especially (again) Player 1.

Table 6: Temperature deviation paths (in °C)

	ParetoLT	Rosen5P	Rosen7P
2010	0.9	0.9	0.9
2020	1.1	1.1	1.1
2030	1.3	1.3	1.3
2040	1.5	1.5	1.5
2050	1.6	1.6	1.6
2060	1.7	1.8	1.7
2070	1.8	2.0	1.8
2080	1.9	2.3	2.0
2090	1.9	2.7	2.2
2100	2.0	3.0	2.6

Table 7: Emission paths (in Gt C)

	ParetoLT			Rosen5P			Rosen7P		
	1	2	3	1	2	3	1	2	3
2010	1.4	3.0	2.8	1.5	3.1	2.9	1.5	3.1	2.9
2020	1.0	3.4	3.3	1.3	3.6	3.4	1.3	3.6	3.4
2030	0.9	2.4	2.3	1.0	2.5	2.4	1.0	2.5	2.4
2040	0.8	2.2	2.1	0.9	2.2	2.2	0.9	2.2	2.1
2050	0.8	2.1	2.1	0.8	2.1	2.1	0.8	2.1	2.1
2060	0.8	2.0	2.0	2.5	6.2	6.0	0.7	2.0	1.9
2070	0.7	1.9	1.8	3.2	7.7	7.5	0.7	1.8	1.8
2080	0.6	1.6	1.6	3.7	8.9	8.7	3.2	7.7	7.5
2090	0.5	1.4	1.3	4.2	10.0	9.7	4.0	9.6	9.4
2100	0.4	1.1	1.1	4.7	11.1	10.8	4.7	11.0	10.7

### 4.2.2 Capital accumulation paths

Low-carbon capital accumulation levels are given in Table 8. In the ParetoLT case, low-carbon capital  $K_2$  accumulates throughout the century, as more clean energy sources are used. These accumulations are directly related to the emission reduction efforts carried out by the players. By contrast, in the Rosen setting, low-carbon capital only accumulates during the commitment period, when players make a substantial effort to abate emissions. Afterwards, low-carbon capital is phased out, as players are basically no longer concerned with emission control for the common good.

Table 8: Low-carbon capital  $K_2$  accumulation paths (in trillion USD)

	ParetoLT			Rosen5P			Rosen7P		
	1	2	3	1	2	3	1	2	3
2010	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2020	13.4	0.0	0.0	7.8	0.0	0.0	7.4	0.0	0.0
2030	20.0	33.4	31.7	18.0	33.5	31.8	17.9	33.4	31.8
2040	24.7	53.9	51.5	24.1	54.0	51.6	24.0	54.0	51.6
2050	29.3	70.3	67.5	29.1	70.4	67.6	29.1	70.4	67.5
2060	34.3	85.8	82.5	10.2	24.6	23.6	34.2	85.8	82.6
2070	39.6	101.4	97.6	3.5	8.6	8.2	39.6	101.4	97.7
2080	45.4	117.4	113.2	1.2	3.0	2.9	13.8	35.4	34.1
2090	51.5	133.9	129.1	0.4	1.0	1.0	4.8	12.3	11.9
2100	57.8	150.7	145.4	0.2	0.4	0.3	1.7	4.3	4.1

Table 9 gives next accumulation levels of carbon-intensive capital  $K_1$ . The development of the carbon economy is again (inversely) proportional to the rise of the low-carbon economy and emission reduction efforts. More precisely, the carbon economy only flourishes in the Rosen setting when the commitment period is over. Otherwise, the carbon economy co-exists with the low-carbon one, but at reduced levels (compared to the baseline).

Table 9: Carbon-intensive capital  $K_1$  accumulation paths (in trillion USD)

	ParetoLT			Rosen5P			Rosen7P		
	1	2	3	1	2	3	1	2	3
2010	47.1	41.3	36.2	47.2	41.4	36.2	47.2	41.4	36.2
2020	31.6	65.6	60.3	37.2	65.8	60.4	37.6	65.8	60.4
2030	27.8	58.8	55.0	29.9	59.0	55.2	30.0	59.0	55.1
2040	28.6	64.6	61.3	29.5	64.9	61.6	29.4	64.7	61.4
2050	31.5	75.7	72.4	32.2	76.7	73.6	31.8	75.8	72.6
2060	35.4	88.5	85.0	61.7	154.4	148.7	35.6	88.8	85.4
2070	39.7	101.6	97.8	81.2	206.6	199.1	40.7	103.8	100.2
2080	44.3	114.4	110.3	98.3	250.8	241.9	80.9	207.9	200.7
2090	48.6	126.4	121.8	113.0	293.8	283.5	106.0	275.8	266.2
2100	51.7	136.9	132.0	130.1	337.7	325.9	127.3	330.5	319.0

Table 10 reports finally on the accumulation of adaptation capital  $K_3$ . In the ParetoLT scenario, adaptation acts as a complement to the mitigation efforts started earlier by all players. The same happens in the coupled equilibrium (Rosen) setting during the commitment period. But here, when the commitment period is over, adaptation acts more and more as a substitute to the mitigation efforts with the progressive phase out of the low-carbon capital. Note also that when the commitment period is over, the required amount of adaptation capital for a maximal effectiveness is higher (compared to the ParetoLT scenario), as temperature reaches again higher levels.

Table 10: Adaptation capital  $K_3$  accumulation paths (in trillion USD)

	ParetoLT			Rosen5P			Rosen7P		
	1	2	3	1	2	3	1	2	3
2010	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2020	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2030	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2040	0.0	0.9	0.9	0.0	0.9	0.9	0.0	0.9	0.9
2050	0.0	1.0	1.0	0.0	1.1	1.1	0.0	1.1	1.1
2060	0.0	1.2	1.2	0.0	1.3	1.3	0.0	1.2	1.2
2070	0.0	1.3	1.3	0.0	1.7	1.7	0.0	1.4	1.4
2080	0.0	1.4	1.4	0.0	2.3	2.3	0.0	1.6	1.6
2090	0.3	1.5	1.5	3.0	3.0	3.0	2.1	2.1	2.1
2100	1.6	1.6	1.6	3.8	3.8	3.8	2.8	2.8	2.8

### 4.3 Consumption losses in the different constrained solutions

Tables 11 and 12 report first on the relative variations (losses) of the discounted sum of consumption with respect to the BAU scenario for different (commitment) periods.

Table 11: Consumption losses over the period 2010–2050

	1	2	3
ParetoLT	1.41%	1.27%	1.27%
Rosen5P	1.53%	1.44%	1.46%

Table 12: Consumption losses over the period 2010–2070

	1	2	3
ParetoLT	1.91%	1.74%	1.74%
Rosen7P	2.15%	2.00%	2.01%

These relative losses could be used as a criterion of fairness among the groups of countries participating in the climate negotiations. It appears that the losses are relatively balanced, although the first group, Player 1, loses slightly more than the two other players during the commitment periods. We have computed Rosen solutions for different sharing  $\theta_j$ . Sharing has again been obtained from the Pareto solution when considering a limit of 2°C on temperature increase (ParetoLT scenario). Let us first recall that the first sharing (labelled ‘equal weights’) considered so far as been computed with equal weights assigned to the players. We have also computed weights on a per capita basis (proportionally to initial population levels—sharing labelled ‘per capita’), and on a ‘grand-fathering’ basis (proportionally to initial emission levels—sharing labelled ‘grand-fathering’). This corresponds to an allocation of the total emission budget among the three players as shown in Tables 13 and 14.

Results, in terms of income losses, are given in Tables 15 and 16.

As expected, compared to the first sharing where losses appear relatively similar between players, the second (per capita) sharing favors Players 2 and 3 where population levels are higher than in Player 1. Conversely, the third (grand-fathering) sharing favors Players 1 and 2 where initial GHG emission levels are higher than in Player 3.

Table 13:  $\theta_j$  for the period 2010–2050

Players	Equal weights	Per capita	Grand-fathering
1 (OECD)	17%	14%	21%
2 (BRIC)	43%	44%	45%
3 (ROW)	41%	42%	34%
Total	100%	100%	100%

Table 14:  $\theta_j$  for the period 2010–2070

Players	Equal weights	Per capita	Grand-fathering
1 (OECD)	17%	17%	21%
2 (BRIC)	43%	27%	45%
3 (ROW)	41%	56%	34%
Total	100%	100%	100%

Table 15: Consumption losses over the period 2010–2050

	1	2	3
Rosen5P, equal weights sharing	1.53%	1.44%	1.46%
Rosen5P, per capita sharing	1.80%	1.40%	1.42%
Rosen5P, grand-fathering sharing	1.30%	1.41%	1.73%

Table 16: Consumption losses over the period 2010–2070

	1	2	3
Rosen7P, equal weights sharing	2.15%	2.00%	2.01%
Rosen7P, per capita sharing	2.65%	1.91%	1.95%
Rosen7P, grand-fathering sharing	1.85%	1.93%	2.37%

## 5 Conclusion

In this paper, we have represented the climate negotiations and the resulting agreement as the imposition of a coupled constraint in a dynamic economic growth game, with a climate module and economic loss factor induced by temperature change. The economic growth models have two particular features. The players, i.e. the group of countries involved in the negotiations, can invest in a carbon and in a low-carbon economy. They can also invest in an adaptation capital which tends to alleviate the impact of climate change on economic production. We have shown that the normalized equilibria obtained under the coupled constraint, which consists in the definition of a global emission budget for the negotiated commitment period that has to be satisfied collectively by all the players, correspond to standard Nash equilibria for a differential game with decoupled constraints, where each player receives a share of the global budget and has to comply with this limit.

Using a 3-player model, corresponding to the division of the world economy into industrialized (OECD), emerging (BRIC) and developing (ROW) countries, and calibrated as in Bahn et al. (2012), we have simulated different possible solutions to the resulting differential game. The following two remarks can be made based on the obtained results:

1. The impact of climate change on the economy is important in the very long term. This implies that, in a cost-benefit approach, very little is done in early commitment periods (e.g., 2010–2050), even in a Pareto optimal solution. In order to obtain a sensible drive toward the use of low-carbon technologies, one must adopt a cost-effectiveness approach and impose a 2°C constraint on temperature increase in

the Pareto optimal solution. Otherwise, even in the long-term, investment in adaptation capital is the favored (climate) policy.

2. A negotiation and climate agreement, based on this 2°C maximum temperature increase objective, is represented by the introduction of a coupled constraint for the three players, consisting in limiting the total emission budget over a commitment period to the emissions that would result from a Pareto optimal solution with the temperature constraint. We show that the normalized equilibrium solutions corresponding to a Nash equilibrium under a particular sharing of the global budget can yield to moderate losses in consumption over the commitment period.

We also notice that the normalized equilibria tend to reach a high temperature increase after the period of negotiation. A natural way to deal with that phenomenon would be to repeat the game with coupled constraint at the end of each negotiation period.

Our numerical illustration is based on a rather simple economic model, with a limited number of players. But our approach can be extended to more encompassing models, that would include for instance trade effects, as in REMIND-R or in WITCH. This modeling, that has put a cost-effectiveness structure in the climate economic game, has nonetheless shown that differential game models including a coupled constraint can shed some light on the assessment of possible climate negotiation outcomes.

## Appendices

### A Model calibration

The calibration of the 3-player Ada-BaHaMa model follows the approach detailed in Bahn (2010) for a 2-player model. It is done for the Pareto scenario.

In short, the different economic and climate parameters (Eq. (1) to (10)) are mostly from the DICE model (version 2007,<sup>5</sup> thereafter referred to as DICE2007). Compared to the carbon economy, production in the low-carbon economy has higher energy costs but a better energy efficiency. As a result, the overall production of the 3-player Ada-BaHaMa reproduces the economic output of DICE2007.

In addition, some regional parameter values have been adapted in the spirit of the RICE model. In particular, the three players have different population levels and initial values for capital accumulation in the carbon economy:

$L(j, 0)$ : initial value for population level of player  $j$ , in millions of persons;  $L(1, 0) = 1043.9$ ;  $L(2, 0) = 2731.5$ ;  
 $L(3, 0) = 2635.5$ ;

$K_1(j, 0)$ : initial value for carbon intensive capital of player  $j$ , in trillions USD;  $K_1(1, 0) = 60.2$ ;  $K_1(2, 0) = 20.6$ ;  $K_1(3, 0) = 16.6$ .

Damages and adaptation parameters (Eq. (11) to (13)) are from the AD-DICE model (de Bruin et al., 2009) and the World Bank (Margulis & Narain, 2009). Note that the maximal adaptation effectiveness is assumed to be 0.33 in all three regions. As a result, Ada-BaHaMa reproduces the overall magnitude of climate change damages estimated by DICE2007 and AD-DICE.

### B GAMS code

The different GAMS codes used to perform our numerical experiments are available from <http://www.ordcsys.com>. We provide in particular:

<sup>5</sup>Nordhaus, W. “Notes on how to run the DICE model”. In Yale University. [On line]. <http://nordhaus.econ.yale.edu/DICE2007.htm> (Website accessed on October 13, 2010).

- **Ada\_Bahama-3pBAU.gms**: the code to run our baseline (BAU) scenario;
- **Ada\_Bahama-3pPareto.gms**: the code to run our Pareto scenario;
- **Ada\_Bahama-3pNash.gms**: the code to run our Nash scenario;
- **Ada\_Bahama-3pRosen.gms**: the code to run our Rosen scenarios.

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