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Optimizing a mineral supply chain under uncertainty with long-term sales contracts

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Abstract: A two-stage stochastic mixed integer non-linear program is formulated for a mining complex to optimize strategic and tactical plans. The objective is to find the near optimal decisions for a mineral supply chain in the context with uncertainties in both ore supply and the commodity market (price and demand). The endogenous spot price in the commodity market and long-term sales contracts are considered in the formulation of the mining complex's optimization model and an ad hoc heuristic is developed to deal with the throughput- and head-grade-dependent recovery rate in processing plants. Numerical results indicate that the proposed heuristic is effective and efficient in numerical tests. Based on the proposed model and heuristic, a long-term contract design strategy is proposed for making decisions on the contract price and strategic investments. A shadow price based method is also proposed to evaluate the existing mining schedule.

1 Introduction

A mining supply chain is an end-to-end supply chain including all value-added production operations from the procurement of raw materials to the delivery of final products (or commodity). A typical mining supply chain in the contex of a “mining complex”, consists of mines, waste dumps, ore stockpiles, processing plants, product warehouses and fleet vehicles for product transportation. When mining complexes are considered, integrated mining supply chains are formed to provide comprehensive mine-to-port supply of materials/commodities, including mining and processing ore, generating products (gold bullion, copper concentrate, iron supply, and so on) and transportation. In this work, stochastic modelling and solving techniques are used to find the near optimal decisions for a mining complex to maintain its profitability in the context with uncertainties in both ore supply and the commodity market.

The mineral supply chain optimisation model proposed herein, as shown in Figure 1, uses the data obtained from three program modules, i.e., mine scheduler, market analyser and contract manager, which are assumed already available. The spot market and contracted customers are considered in the proposed model. The dynamic recovery rate that depends on the head grade of the feeding material and the throughput of a processing plant is also considered. A sales contract designing strategy is proposed using the proposed model and heuristic.

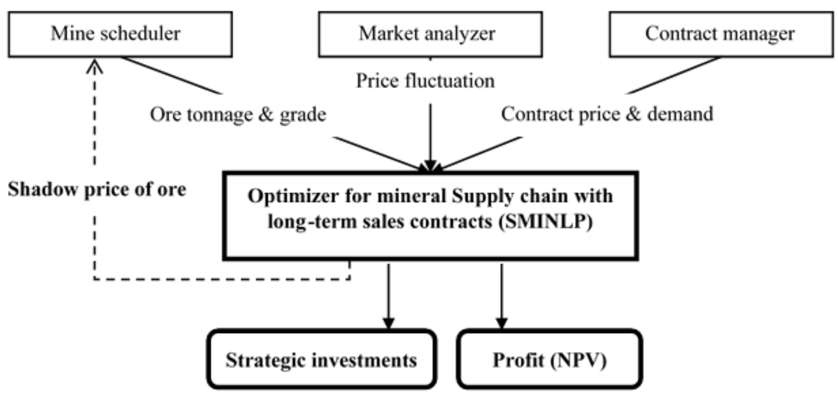

Figure 1: The decision support system for mineral supply chain optimization with long-term sales contracts.

The remainder of this paper is organized as follows. In Section 2, the notation and assumptions for the optimization model are provided. In Section 3, a stochastic mixed integer nonlinear program is formulated and the corresponding solving heuristic is developed. A series of numerical tests are conducted in Section 3 to show the accuracy and the efficiency of the proposed heuristic. In Section 4, a long-term sales contract design strategy based on the proposed model and heuristic is proposed. Section 5 contains the remarks of conclusions and future work.

2 Notation and planning assumptions

The mining complex's planning horizon is T time periods, and $t \in \{1, \ldots, T\}$ is the period index. The uncertainty of the mining complex as considered here includes metal (geological) uncertainty and commodity price (market) uncertainty represented by scenarios. Each scenario combines both sources of uncertainty. Let S be the total number of scenarios that account for the uncertainties of both mineral deposits and commodity markets, and $s \in \{1, \ldots, S\}$ is the scenario index. The mining complex includes a number of mines, stockpiles, processing plants and fleet vehicles for transporting material and products (as shown in Figure 2). The detailed assumptions for each facility in a mining complex are described below.

Mines: A mining complex consists of I mines each of which is indexed by $i \in \{1, \ldots, I\}$. For each mine, the mining schedule is predetermined and hence the tonnage and the grade of the extracted material are treated as exogenous. Because of the geological uncertainty, the tonnage and the grade of ore extracted in

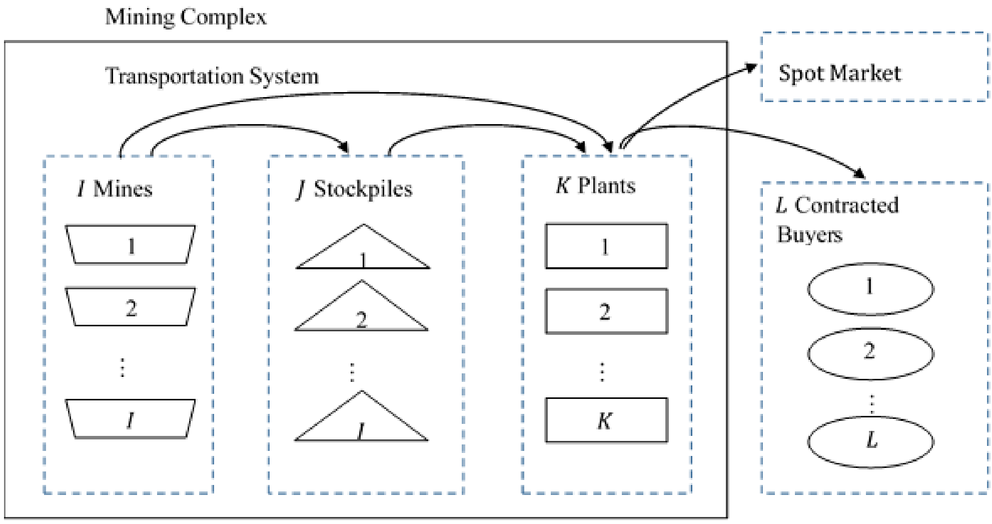

Figure 2: The structure of the mineral supply chain studied in this work.

each period are stochastic. Because the mine production schedule is assumed to be predetermined in this work, the tonnage and the grade of the ore extracted in each period can be simulated. The tonnage and the grade of ore extracted in each period are denoted by $o_{its} \in [0, +\infty)$ and $g^M_{its} \in (0, 1)$, respectively, where the superscript M indicates that the symbol is associated with a "Mine". After the material is extracted from a mine, it can be sent to either a processing plant or a stockpile.

Stockpiles: The mining complex has J stockpiles each of which is indexed by $j \in \{1, \ldots, J\}$. A stockpile only accepts the material of a particular type which is determined by certain parameters such as grade, hardness, composition, etc. A waste pile can also be treated as a special stockpile if the waste is treated special ore of which the grade is below the cutoff grade for waste. For clarity, a binary parameter, denoted by $a_{ijts} \in \{0, 1\}$ is used to indicate if stockpile j accepts material extracted from mine i in scenario s and period t. Because a material stockpile is not homogeneous and it is usually the case that only the surface is accessible for grade test (Holmes 2004), the grade of the material from a stockpile can only be tested when it is being moved. Thus, the grade of the material from a stockpile is treated as exogeneous and stochastic. Let $g^H_{jts} \in (0, 1)$ be the grade of material from stockpile j, where the superscript H indicates that the symbol is associated with a "Stockpile". The tonnage of material stocked in stockpile j is denoted by $v^H_{jts} \in [0, +\infty)$. The flow from mine i to stockpile j is denoted by $x^{MH}_{ijts} \in [0, +\infty)$, and the cost of transporting a unit tonnage material is denoted by $c^{MH}_{ij} \in [0, +\infty)$. Because c^{MH}_{ij} can also capture other costs incurred at stockpile j, we do not use an additional parameter to denote the rehandling cost.

Processing plants: The mining complex has K processing plants each of which is indexed by $k \in \{1, \ldots, K\}$. For any plant k, the head grade of the feeding material is denoted by $g^P_{kts} \in (0, 1)$, where the superscript P indicates that the symbol is associated with a "Processing plant". The head grade should satisfy $g^P_{kts} \in [\underline{g}^P_{kts}, \overline{g}^P_{kts}]$ to meet the requirements of the processing method employed by plant k. The unit processing cost for plant k is denoted by $c^P_{kts} \in [0, +\infty)$. The throughput in plant k, denoted by $v^P_{kts} \in [0, +\infty)$, is constrained by plant k's processing capacity $\overline{v}^P_{kts}$. The recovery rate in a processing plant is dynamic depending on the throughput and the head grade. Plant k's recovery rate function is denoted by $f^P_k(v^P_{kts}, g^P_{kts})$ which is decreasing in v^P_{kts} and increasing in g^P_{kts} based on the observations by Hadler et al. (2010) and Splaine et al. (1982). The material flow from mine i to plant k is denoted by $x^{MP}_{ikts} \in [0, +\infty)$, and $c^{MP}_{ik} \in [0, +\infty)$ is the unit transportation cost. Similarly, the material flow from stockpile j to plant k is denoted by $x^{HP}_{jkts} \in [0, +\infty)$, and $c^{MH}_{ik} \in [0, +\infty)$ is the unit transportation cost.

The spot market and the contracted buyers: The mining complex sells its commodity to contracted buyers at contract prices or to "uncontracted" buyers in the spot market at the *spot price*. The spot price fluctuates and is influenced by the mining complex's commodity supply. Let x^U_{kts} be plant k's commodity supply to the spot market, and c^U the expected transaction cost of selling a unit tonnage of commodity to the spot market, where the superscript U indicates that the symbol is associated with the "uncontracted" spot market. The spot price is obtained as $p^U_{ts} - \eta_{ts}\sum_{k=1}^{K} x^U_{kts}$ where p^U_{ts} and η_{ts} are random parameters reflecting market status. η_{ts} is the spot price's sensitivity to the mining complex's commodity supply and depends on the mining complex's market share. In the literature, similar assumptions can be found in the articles reviewed by Yano and Gilbert (2004). The mining complex has L contracted buyers each of which is indexed by $l \in \{1,\dots,L\}$. The contract price and demand for buyer l are denoted by $p^C_l \in [0,+\infty)$ and $d^C_{lt} \in [0,+\infty)$, respectively, where the superscript C indicates that the symbol is associated with a "contract" buyer. If buyer l's contract demand is not fulfilled by the mining complex, a penalty of $\overline{p}^U_{ts} - p^C_l$ is incurred for each unit of shortage to ensure the contracted buyer to find an alternative supply. The quantity of the unfulfilled demand is denoted by $d^-_{lts} \in [0,+\infty)$ for buyer l. The commodity flow from plant k to buyer l is denoted by $x^{PB}_{klts} \in [0,+\infty)$, and the unit transportation cost is denoted by $c^{PB}_{kl} \in [0,+\infty)$.

Transportation system: Without loss of generality, the mining complex's transportation system is simplified to two subsystems: *internal* and *outbound* subsystems. The internal subsystem transports material between mines, stockpiles and processing plants, and the outbound subsystem transport commodity from processing plants to buyers. The internal and outbound transportation capacities are determined by the number of trucks $\overline{y}^{INT} \in \{0,1,\dots\}$ and $\overline{y}^{OUT} \in \{0,1,\dots\}$ denote the numbers of existing internal and outbound trucks, respectively. $u \in [0,+\infty)$ is the transportation capacity of each truck. The mining complex can expand the capacity of its internal and/or outbound transportation system by adding $y^{INT} \in \{0,1,\dots\}$ and/or $y^{OUT} \in \{0,1,\dots\}$ trucks at a cost of $\tau \in R_{[0,+\infty)}$ for each.

The profit of a mining complex is evaluated by its net present value (NPV) that accounts for the time value of the cash flows in all planning periods. The rate of return for each planning period is denoted by γ. For clarity, the notation defined in this section is listed in Table 1.

3 Model and heuristic

In this section, the problem is formulated as a 2-stage stochastic mixed integer nonlinear program (SMINLP) and a constructive heuristic is developed following the model.

3.1 Model

The mining complex's objective function is to maximize its expected profit over the planning horizon as

$$
\begin{aligned}
Maximize\ \frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\frac{1}{[1+\gamma]^t}\Bigg[& \underbrace{\left[p^U_{ts} - \eta_{ts}\sum_{k=1}^{K} x^U_{kts} - c^U\right]\sum_{k=1}^{K} x^U_{kts} + \sum_{l=1}^{L}\sum_{k=1}^{K} p^C_l x^{PB}_{klts}}_{(i)} \\
& - \Bigg[\underbrace{\sum_{j=1}^{J}\sum_{i=1}^{I} c^{MH}_{ij} x^{MH}_{ijts} + \sum_{k=1}^{K}\sum_{i=1}^{I} c^{MP}_{ik} x^{MP}_{ikts} + \sum_{k=1}^{K}\sum_{j=1}^{J} c^{HP}_{jk} x^{HP}_{jkts} + \sum_{l=1}^{L}\sum_{k=1}^{K} c^{PB}_{kl} x^{PB}_{klts}}_{(ii)}\Bigg] \\
& \underbrace{-\sum_{k=1}^{K} c^P_k v^P_{kts}}_{(iii)} \underbrace{-\sum_{l=1}^{L}\left[p^U_t - p^C_l\right] d^-_{lts}}_{(iv)}\Bigg] - \underbrace{t^{INT} y^{INT} - t^{OUT} y^{OUT}}_{(v)}. \qquad (1)
\end{aligned}
$$

In the objective function (1), (i) computes the gross profit from the spot market and the contracted customers, (ii) computes the total transportation cost, (iii) computes the total processing cost incurred in processing

Table 1: List of notation (subscripts t and s, representing period and scenario, are omitted.

Symbol	Description
Indices:	
$s \in \{1, \ldots, S\}$	Scenario index.
$t \in \{1, \ldots, T\}$	Period index.
$i \in \{1, \ldots, I\}$	Mine index.
$j \in \{1, \ldots, J\}$	Stockpile index.
$k \in \{1, \ldots, K\}$	Plant index.
$l \in \{1, \ldots, L\}$	Contracted buyer index.
Parameters:	
$\gamma \in R_{[0,+\infty)}$	The mining complex's rate of return.
$o_{its} \in R_{[0,+\infty)}$	Ore tonnage extracted from mine i.
$g_{its}^{M} \in R_{(0,1)}$	Ore grade extracted from mine i.
$a_{ijts} \in \{0, 1\}$	Indicate if mine i's material is acceptable to stockpile j.
$g_{its}^{H} \in R_{(0,1)}$	Material grade obtained from stockpile j.
$c_{ij}^{MH} \in R_{[0,+\infty)}$	Unit transportation cost from mine i to stockpile j.
$\underline{g}_k^P, \overline{g}_k^P \in R_{[0,+\infty)}$	Plant k's lower and upper limits for acceptable head grade.
$c_k^P \in R_{[0,+\infty)}$	Plant k's unit processing cost.
$\overline{v}_k^P \in R_{[0,+\infty)}$	Plant k's processing capacity.
$\alpha_k \in R_{(0,1)}, \beta_k^V, \beta_k^G \in R_{[0,+\infty)}$	Parameters of plant k's recovery function.
$c_{ik}^{MP} \in R_{[0,+\infty)}$	Transportation cost from mine i to plant k.
$c_{jk}^{HP} \in R_{[0,+\infty)}$	Transportation cost from stockpile j to plant k.
$p_{ts}^{U} \in R_{[0,+\infty)}$	Spot price without the mining complex's commodity supply.
$\eta_{ts} \in R_{[0,+\infty)}$	Spot price's sensitivity to the mining complex's total commodity supply.
$c^U \in R_{[0,+\infty)}$	The expected unit transaction cost of selling in the spot market.
$p_l^C \in R_{[0,+\infty)}$	Buyer l's contracted price.
$d_{lt}^C \in R_{[0,+\infty)}$	Buyer l's contracted demand.
$u^{INT}, u^{OUT} \in R_{[0,+\infty)}$	Capacities of an equipment for internal and outbound transportation.
$\overline{y}^{INT}, \overline{y}^{OUT} \in Z_{[0,+\infty)}$	Quantities of the existing internal and outbound trucks.
$\tau^{INT}, \tau^{OUT} \in R_{[0,+\infty)}$	Cost of a unit internal and outbound trucks.
Strategic variables:	
$y^{INT}, y^{OUT} \in Z_{[0,+\infty)}$	Quantities of new internal and outbound trucks.
Tactical variables:	
$x_{ijts}^{MH} \in R_{[0,+\infty)}$	Material flow from mine i to stockpile j.
$x_{ikts}^{MP} \in R_{[0,+\infty)}$	Material flow from mine i to plant k.
$x_{jkts}^{HP} \in R_{[0,+\infty)}$	Material flow from stockpile j to plant k.
$x_{klts}^{PB} \in R_{[0,+\infty)}$	Commodity flow from plant k to buyer l.
$x_k^U \in R_{[0,+\infty)}$	Plant k's supply to the spot market.
Intermediate variables:	
$v_{jts}^{H} \in R_{[0,+\infty)}$	Stockpile j's tonnage.
$v_{kts}^{P} \in R_{[0,+\infty)}$	Plant k's throughput.
$g_{kts}^{P} \in R_{[0,+\infty)}$	Plant k's head grade.
$d_{lts}^{-} \in R_{[0,+\infty)}$	Buyer l's unfulfilled demand.
Function:	
$f_k\left(v_{kts}^P, g_{kts}^P\right)$	Compute plant k's recovery rate from its throughput and head grade.

plants, (iv) computes the total penalty incurred for not fulfilling contract demands, and (v) computes the total strategic investment considered in this work. As noted earlier, any scenario s contains a combination of simulated parameters that account for the uncertainties in both geology and commodity market, and the expected profit is obtained by averaging the profit obtained in each scenario.

The constraints of the model are formulated as follows.

Mine constraint:

$$o_{its} = \sum_{j=1}^{J} x^{MH}_{ijts} + \sum_{k=1}^{K} x^{MP}_{ikts}, \qquad \forall i,t,s. \tag{2}$$

(2) constrains that at any mine j, the total amount of flows to stockpiles and processing plants equals to mine j's yield.

Stockpile constraints:

$$x^{MH}_{ijts} \leq a_{ijts} M \qquad \forall i,j,t,s. \tag{3}$$

$$v^{H}_{jts} = v^{H}_{j,t-1,s} + \sum_{i=1}^{I} x^{MH}_{ijts} - \sum_{k=1}^{K} x^{HP}_{jkts}, \qquad \forall j,t,s. \tag{4}$$

(3), where M is a large constant, determines whether the material extracted from mine i is acceptable to stockpile j. (4) computes the stock level in stockpile j in each period.

Plant constraints:

$$v^{P}_{kts} = \sum_{i=1}^{I} x^{MP}_{ikts} + \sum_{j=1}^{J} x^{HP}_{jkts}, \qquad \forall k,t,s. \tag{5}$$

$$g^{P}_{kts} = \frac{\sum_{i=1}^{I} g^{M}_{its} x^{MP}_{ikts} + \sum_{j=1}^{J} g^{H}_{jts} x^{HP}_{jkts}}{v^{P}_{kts}}, \qquad \forall k,t,s. \tag{6}$$

$$\underline{g}^{P}_{k} v^{P}_{kts} \leq \sum_{i=1}^{I} g^{M}_{its} x^{MP}_{ikts} + \sum_{j=1}^{J} g^{H}_{jts} x^{HP}_{jkts} \leq \overline{g}^{P}_{k} v^{P}_{kts}, \qquad \forall k,t,s. \tag{7}$$

$$\left[\sum_{i=1}^{I} g^{M}_{its} x^{MP}_{ikts} + \sum_{j=1}^{J} g^{H}_{jts} x^{HP}_{jkts} \right] f_k \left(v^{P}_{kts}, g^{P}_{kts} \right) = x^{U}_{kts} + \sum_{l=1}^{L} x^{PB}_{klts}, \qquad \forall k,t,s. \tag{8}$$

$$v^{P}_{kts} \leq \overline{v}^{P}_{k}. \qquad \forall k,t,s. \tag{9}$$

(5) and (6) compute the throughput and the head grade of plant k, respectively. (7) constrains the material fed to plant k to meet plant k's requirements on head grade. (8) computes the plant k's commodity output based on its material input and recovery rate function, and the commodity is sent to contracted customers and the spot market. (9) constrains plant k's throughput to be within its processing capacity.

Transportation constraints:

$$\sum_{j=1}^{J}\sum_{i=1}^{I} x^{MH}_{ijts} + \sum_{k=1}^{K}\sum_{i=1}^{I} x^{MP}_{ikts} + \sum_{k=1}^{K}\sum_{j=1}^{J} x^{HP}_{jkts} \leq u \left[\overline{y}^{INT} + y^{INT} \right], \qquad \forall t,s. \tag{10}$$

$$\sum_{l=1}^{L}\sum_{k=1}^{K} x^{PB}_{klts} \leq u \left[\overline{y}^{OUT} + y^{OUT} \right], \qquad \forall t,s. \tag{11}$$

(10) and (11) constrain the transportation to be within an expandable capacity. Because the transport distance is usually in proportion to the unity transportation cost, here the unit transportation cost is used as the transport distance without loss of generality.

Penalty constraint:

$$\sum_{k=1}^{K} x^{PB}_{klts} + d^{-}_{lts} = d^{C}_{lt}, \qquad \forall l,t,s. \tag{12}$$

(12) computes the unfulfilled demand for each contracted buyer in any period t and scenario s.

3.2 Solution heuristic

From the model above, it can be observed that because the recovery rate function in a processing plant, in (8), depends on throughput and head grade, the mining complex has to solve a nonconvex nonlinear program, which is difficult to solve using general methods. Hence, a heuristic is developed to solve the proposed program.

Because our heuristic is iteration-based, for clarity, we use a hat mark, "ˆ", to label a constant parameter equal to optimal solution of the corresponding variable obtained in the previous iteration. The main idea for modifying the program is removing the non-convexity by fixing the inputs of the recovery rate function. Thus, in each iteration, constraints (6), (8) and (9) are substituted with

$$F_k\left(0, \hat{g}^P_{kts}\right) + v^P_{kts} F'_k\left(0, \hat{g}^P_{kts}\right) + \kappa^{G1}_{kts} z^G_{kts} \geq x^U_{kts} + \sum_{l=1}^{L} x^{PB}_{klts}, \qquad \forall k,t,s. \tag{13}$$

$$F_k\left(v'^P_{kts}, \hat{g}^P_{kts}\right) + \left[v^P_{kts} - v'^P_{kts}\right] F'_k\left(v'^P_{kts}, \hat{g}^P_{kts}\right) + \kappa^{G2}_{kts} z^G_{kts} \geq x^U_{kts} + \sum_{l=1}^{L} x^{PB}_{klts}, \qquad \forall k,t,s. \tag{14}$$

$$z^G_{kts} = \sum_{i=1}^{I} g^M_{its} x^{MP}_{ikts} + \sum_{j=1}^{J} g^H_{jts} x^{HP}_{jkts} - \hat{g}^P_{kts} v^P_{kts}, \qquad \forall k,t,s. \tag{15}$$

$$\Delta^G v^P_{kts} \leq z^G_{kts} \leq \Delta^G v^P_{kts}, \qquad \forall k,t,s. \tag{16}$$

$$v^P_{kts} \leq v'^P_{kts}, \qquad \forall k,t,s. \tag{17}$$

In the modified constraints above, $F_k\left(v, \hat{g}^P_{kts}\right)$ can be obtained as

$$F_k\left(v, \hat{g}^P_{kts}\right) = \hat{g}^P_{kts} \times v \times f_k\left(v, \hat{g}^P_{kts}\right),$$

and $F'_k\left(v, \hat{g}^P_{kts}\right)$ is the first-order derivative of $F_k\left(v, \hat{g}^P_{kts}\right)$ with respect to v.

The throughput in the recovery function is set to 0 and v'^P_{kts} in (13) and (14), respectively, for outer linearization. Additional terms, $\kappa^{G1}_{kts} z^G_{kts}$ and $\kappa^{G2}_{kts} z^G_{kts}$, are introduced in the left-hand side of (8) to correct the error incurred by fixing head grade.

Because for any k, t and s, the optimal setting of v'^P_{kts} in the recovery function always makes (17) bind, in our heuristic, v'^P_{kts} for all k, t and s are gradually reduced from its maximum value, $\overline{v}^P_k$, until the slack of constraint (17) is small enough for any k, t and s.

3.3 Numerical test

The proposed heuristic is tested through a series of numerical experiments. The parameters of the mining complex's optimization problem are simulated as in Table 2. To test the accuracy of the proposed heuristic, the proposed heuristic is first compared with Lingo Global Solver, which is capable of solving general nonlinear program, through a small-scale problem with a planning horizon of 2 periods. For Lingo global solver, the limit of computation time is set to 2 hours. The objective value obtained by Lingo global solver and the proposed heuristic are displayed in Figure 3. It can be observed that within reasonable computation time, the solutions found by Lingo global solver are worse than the ones found by the proposed heuristic, and the proposed heuristic requires much less computation time to solve the test problems at different scales.

The efficiency of the proposed heuristic is tested for larger problem with 24 periods and the 100 scenarios. Because the Lingo Global Solver cannot solve the modified program at this scale, the heuristic is programed

Table 2: Parameters settings in the hypothetical case.

Category	Parameter settings
Mines	$I = 2,\ o_{its} \sim U\,(2500, 3000),\ g^M_{its} \sim U\,(0,1)$
Stockpiles	$J = 2,\ \underline{g}^H_j = [j-1]\,/2,\ \overline{g}^H_j = j/2,\ g^H_{jts} \sim U\left(\underline{g}^H_j, \overline{g}^H_j\right)$
Processing plants	$K = 2,\ \underline{g}^P_k = U\,(0.1, 0.5),\ \overline{g}^P_k = \underline{g}^P_k + 0.5,\ c^P_k \sim U(0.5,1),\ \overline{v}^P_k = 3000,\ f_k\left(v^P_{kts}, g^P_{kts}\right) = \beta^0_k - \beta^1_k v^P_{kts} + \beta^2_k g^P_{kts}$ where $\beta^0_k \sim U(0.6, 0.7),\ \beta^1_k \sim U(0.00005, 0.0001)$ and $\beta^2_k \sim U(0.2, 0.3)$
Contracted buyers	$L = 3,\ p^C_l \sim U\,(8,10)\,,\ d^C_{lt} \sim U(500, 1000)$
Spot market	$\overline{p}^U_{ts} \sim U(15, 25),\ \eta_{ts} \sim U(0,\ 0.001),\ c^U = 1$
Transportation	$c^{MH}_{ij} \sim U(0.5,1),\ c^{HP}_{jk} \sim U(0.5,1),\ c^{MP}_{ik} = min_j\left[c^{MH}_{ij} + c^{HP}_{jk}\right] - 0.1,\ c^{PB}_{kl} \sim U(0.5,1),\ \overline{y}^{INT} = \overline{y}^{OUT} = 0,\ u^{INT} = u^{OUT} = 50,\ \tau^{INT} = \tau^{OUT} = 100$
Other	$\gamma = 0.01$

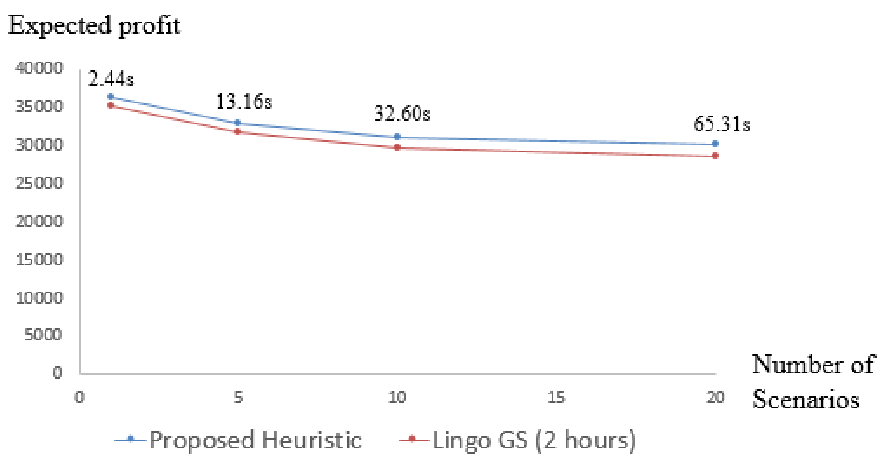

Figure 3: Results of small-case test.

using CPLEX OPL script. It takes 1498.74 seconds for the proposed heuristic to solve the problem. Figure 4(a) and Figure 4(b) shows the averages of $v'^P_{kts} - v^P_{kts}$ and $\left|g^P_{kts} - \hat{g}^P_{kts}\right|$ for all k, t and s, and Figure 4(c) shows the change of the obtained objective value as the heuristic progresses.

3.4 Designing long-term sales contracts

A long-term sales contract specifies the price and the purchase quantity of the commodity produced by the mining complex. For clarity, we use “principal buyer” ($l = 1$) to refer to the buyer that is currently signing a long-term sales contract with the mining complex. In the contract negotiating phase, a common assumption is made that the principle buyer's preference on the contracted demand is price-sensitive, which reflects a practice that the contract demand increases with quantity discount. A good review of the operations literature on quantity discounts can be found in Viswanthan and Wang (2003). Without loss of generality, the relationship between the price quote and the principal customer's contracted demand is simplified to be linear as

$$d^C_{1t}\left(p^C_1\right) = \overline{d}_{1t} - \alpha_{1t} p^C_1$$

where $\overline{d}_{1t}$ and α_{1t} represents the principal buyer's willingness to buy and demand-price sensitivity, respectively. The mining complex's maximum expected profit can be estimated using the proposed model and heuristic if the contract is signed at any contract price p^C_1. The worst-case profit, which is the lowest possible profit that can be obtained from the scenarios considered in the optimization model, is also evaluated as a

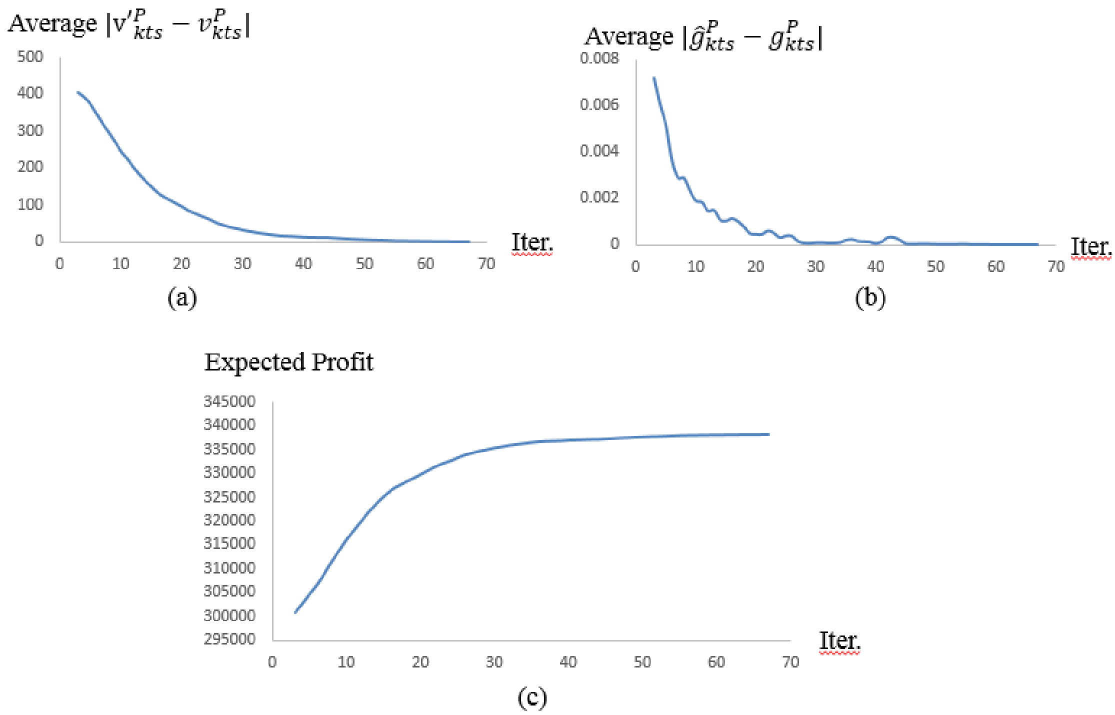

Figure 4: Convergence curve for solving a large-scale problem ($T = 24,\ S = 100$).

consideration factor to decide if a contract offer should be accepted. The worst-case scenario usually includes parameters with low yield of ore tonnage and grade, and low market price and demand. We demonstrate the implementation of the proposed contract design strategy by a hypothetical case with the identical settings as in Section 3. Without loss of generality, the principal buyer's parameters are set to $\overline{d}_{1t} = 2200$ and $\alpha_{1t} = 100$ for all $t \in \{1, \ldots, 24\}$. Figure 5 shows the mining complex's expected profit and the worst-case profit at different contract prices. The dashed line shows the mining complex's expected profit and the worst-case profit if the contracted demand is 0, which is equivalent to the case without the present contract. It can be observed that when the contract price is over 19.5, the contract brings positive profit to the mining complex, and the mining complex's expected profit is maximized if the contract can be signed at $p_1^C = 21$. When the contract price is lower than 20 but higher than 18.5, the risk-averse mining complex still has an incentive for signing the contract because the worst-case profit with the present contract is higher than the one obtained without the present contract.

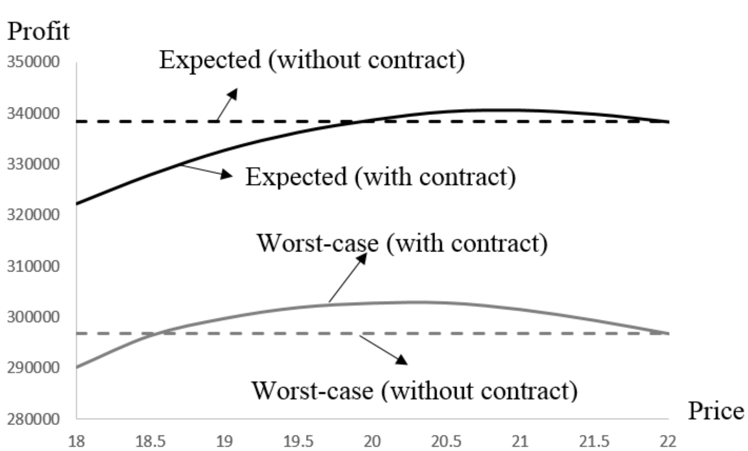

Figure 5: The mining complex's expected profit and worst-case profit at different contract prices.

Figure 6 shows the mining complex's optimal strategic investments, which are the expansions of internal and outbound transportation capacities, at different contract prices, and the dashed lines are the mining complex strategic investment without the present contract. It can be observed that the requirement for internal transport capacity is not significantly changed as the contracted demand is increasing because the mining complex reduces the sales quantity to spot market to honour contracted demand. When the contracted demand increases to a certain level, the production capacity in the processing plant becomes the bottleneck of the supply chain so that there is no need to continue to increase the outbound transportation capacity and the penalty for undelivered demand is increased to reduce the mining complex's expected profit.

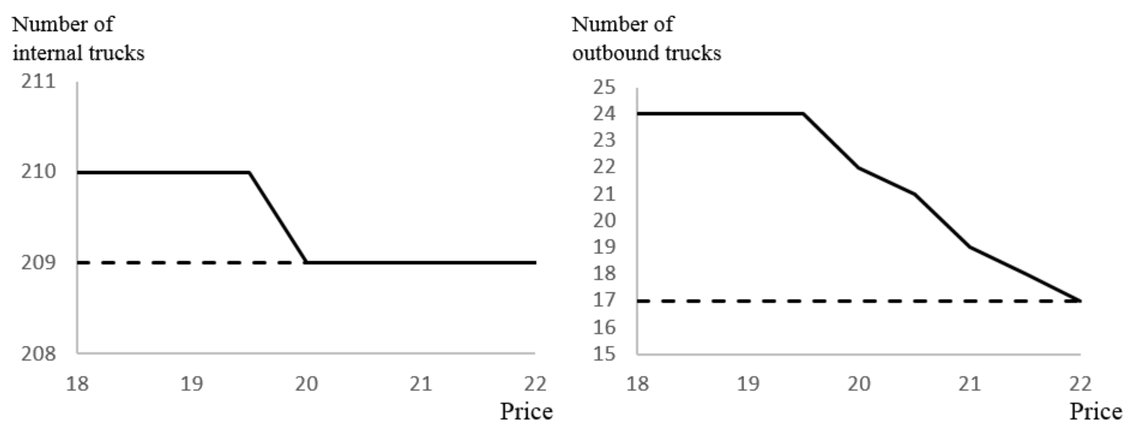

Figure 6: Optimal strategic investment at different contract prices.

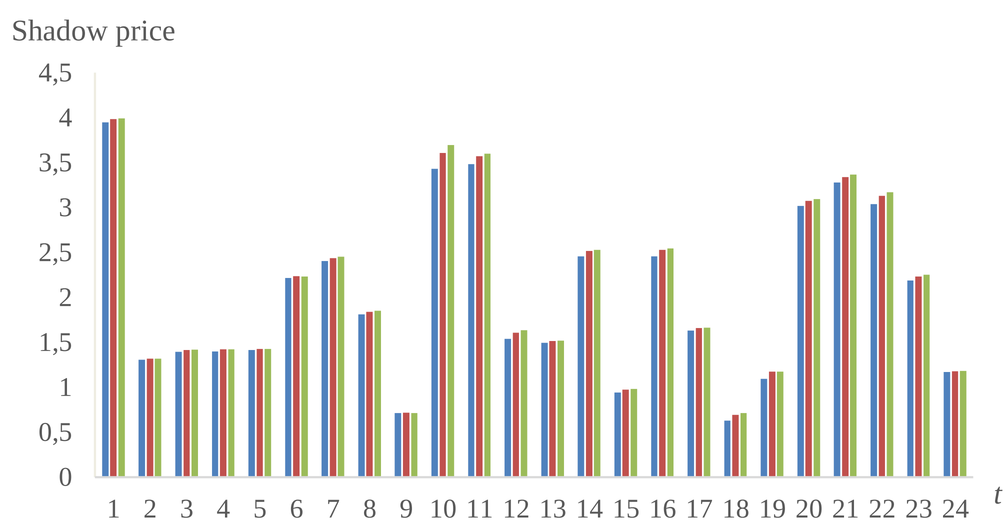

Figure 7: The shadow price of ore extracted at different periods.

4 Conclusions

For a mining supply chain, the uncertainties in mineral deposits (material types, ore grade and tonnage) and the commodity market (price and demand) should be considered in making strategic decisions to maximize the profit of the entire supply chain. In this work, a stochastic nonlinear mixed integer programming model is proposed to help a mining complex to make decisions in signing a long-term sales contract to deal with the uncertainties existing in both ends of a mineral supply chain. Due to the complexity in solving the proposed nonconvex and nonlinear model, a heuristic is developed and tested by a number of numerical experiments. The result shows that a near optimal solution can be found efficiently.

As a suggestion for the future research, methodology in mining scheduling based on ore shadow prices (or dual price) will be studied, and the shadow-price-based mine production scheduling method will account for the cost, the profit and the uncertainty incurred in mineral resource supply chain regardless of the complex structure of the supply chain.

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