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# **A mechanism to promote product recovery and remanufacturing**

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**Abstract:** A performance-based mechanism as a support for an environmental protection law is proposed to promote product recovery and to subsidize the pollution caused by producing brand new products (manufacturing caused pollution). We assume that the market consists of consumers with heterogeneous preferences over brand new and remanufactured products. The mechanism is flexible in structure so that it can imitate diverse taxation and reward-based laws in order to evaluate and compare them in a single general framework. We show that the regulations that are required to reduce manufacturing-caused pollution should be different from those designed for promoting product recovery. It is suggested to decrease collection target to promote product recovery, but to increase it to reduce pollution. However, increasing a target level for remanufacturing always departs the social planner from his environmental goals. Finally, it is shown that stipulating the Buy-Back, Carbon Emission Tax and Tax Subsidy regulations could help expanding the remanufacturing market while putting the quantity of brand new products in check.

**Keywords:** Remanufacturing, collecting used products, regulation, environmental protection

**Résumé:** On propose un mécanisme pour favoriser la récupération et la réutilisation de produits en fin de vie pour diminuer la pollution causée par la production de nouveaux produits. On suppose que le marché est composé de consommateurs ayant des préférences hétérogènes pour les produits neufs et remanufacturés. Le mécanisme est flexible et permet d'évaluer et de comparer diverses lois mises de l'avant pour encourager la récupération et le recyclage. On montre que les règlements qui sont nécessaires pour réduire la pollution causée par la fabrication devraient être différents de ceux conçus pour favoriser la récupération du produit.

**Mots clés:** Remanufacturing, collecte de produits, réglementation, protection environnementale

# 1 Introduction

Producing with raw (or new) materials, which requires mining and is energy intensive, and discarding the scraps into the nature are among the main sources of a variety of environmental problems including air pollution and severe damages to water and soil resources (Guide and Srivastava , 1998; Assavapokee and Wongthatsanekorn , 2012; Yenipazarli, 2016). Since production is essential to life, the way to go is to find ways to subside its negative consequences. Recycling and remanufacturing are interesting options as they reduce the needs for natural resources and energy consumption during production process and consequently air pollution and release of toxic materials into the nature.

Firms adopt a product recovery (i.e., reusing and remanufacturing) strategy whenever it is profitable. The academic and popular literature is replete with examples showing that remanufacturing is indeed profitable in some sectors and contexts, with the total benefit climbing to billions of dollars.<sup>1</sup> When firms do not implement a product recovery program and do not internalize the associated environmental cost, a regulation is justified. Government's intervention can take a variety of forms with, however, one aim, that is, protecting the environment. In the context of product recovery, *extended producer responsibility* (EPR) is a generic term used to describe governmental regulations passed to achieve product sustainability. *End-of-Life Vehicle Directives* (ELVs) and *Waste Electrical and Electronic Equipment Directive* (WEEE) are famous examples of such environmental laws. According to the *Official Journal of European Union* about WEEE Directive: "The objectives of the Union's environment policy are, in particular, to preserve, protect and improve the quality of the environment, to protect human health and to utilize natural resources prudently and rationally."<sup>2</sup>

Legislation and profitability are not the only reasons to practice product recovery process. The environmental hazards of landfilling, the detrimental impact of green-house gases to earth and species living on it, and the excess use of virgin materials and consequently harming the ecosystem have resulted in social awareness about green and sustainable production. Those consumers who consider the addressed items when deciding to buy a product are called green consumers, and are defined by Atasu et al. (2008) as those who prefer remanufactured products to new ones. (From now on, we use *new product* as a short cut to design a *good manufactured with new materials*.) Yenipazarli (2016) addresses the segment of the market that does not discount for remanufactured products as a future research topic. Thus, this relatively new concept calls for new studies in which the existence of green customers is considered. Existence of new segments of potential customers impacts the demand functions, and consequently the firms need to shift part of their capacity to remanufacturing processes to gain more market share or keep the existing customers.

Apart from the named financial and environmental reasons for product recovery, the current state of the industry should be taken into account. The social planner may intend to promote product recovery to help the firms build infrastructures to collect, recycle and reuse products. However, if the product recovery industry is well-established or pollution is at its critical level, promoting product recovery is not equivalent to reducing pollution and waste. In this case, brand new production, as the main source of pollution being addressed in this paper, also requires consideration.

Government regulations take different shapes. Minimum (or target) recovery level is one of the most general cases considered in the literature. An extended version of this regulation sets a desirable target not only for recycling, but also for reusing (remanufacturing) certain proportions of used products (Esenduran et al., 2016). In this paper, the terms reusing and remanufacturing are used interchangeably. Another form of regulation, which may be applied either as encouragement or punishment, is subsidy and taxation. While creating emissions are punished by tax (Liu et al., 2015), investing on recovery processes may enjoy subsidies (Atasu et al., 2009). This considerable diversity in environmental protection laws and different intentions of the environmental laws brings up important questions for the social planner:

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<sup>1</sup>For the US, see *Remanufactured goods: An overview of the U.S. and global industries, markets, and trades*, US International Trade Commission, USITS publication, Oct 2012. For the UK, see *Remanufacturing: Towards a resource efficient economy*, The All-Party Parliamentary Sustainable Resource Group, March 2014.

<sup>2</sup>Derivative 2012/19/EU of the European Parliament and of the Council of 4 July 2012 on waste electrical and electronic equipment (WEEE), Official Journal of European Union, 24.7.2012

- What are the impacts of penalty and reward on product recovery and pollution?
- Is it a good idea to increase the collection and remanufacturing target levels in order to achieve better environmental and product recovery performances?
- Does a penalty-based mechanism achieve a better result than a performance-based mechanism?
- Among all these environmental protection laws, which ones are preferable in which criterion? What are the advantages and disadvantages of each of them?

These questions have not been fully addressed in the literature of environmental regulations and remanufacturing. Thus, we propose a performance-based mechanism that appears as a linear cost function which directs penalties and rewards the manufacturer. It includes collection and remanufacturing with specific target levels. Moving beyond the target is rewarded per unit while not reaching it is to be penalized per unit. However, it could be manipulated in a way that it only considers penalties or rewards, or it imitates a command-and-control mechanism rather than a performance-based one.

The mechanism which is introduced in this paper provides us with sufficient tools to investigate the impact of incentives on collection and remanufacturing, either separately or collectively. Moreover, the social planner is able to see if increasing the target levels - as is the case for WEEE in Europe - will yield desirable environmental outcome. Furthermore, this research reveals if a performance-based mechanism on the basis of target levels and incentives outperforms a mechanism with minimum requirement level and penalties. In a nutshell, we propose a mechanism that: (i) Includes remanufacturing to answer the need of green customers; (ii) Employs target levels and incentives to figure out if these tools are sufficient to tackle the environmental issues; and (iv) Enables the social planner to compare and contrast different regulations within a single framework so that the advantages and disadvantages of each regulation could be easily assessed.

To justify why remanufacturing is included in the environmental regulation studied in this paper, WEEE directive revisions should be scrutinized. WEEE Directives are elaborating the Waste Management and Waste Prevention programs. In the newest version of this directive, *waste* is defined as the term for the products that are discarded, are required to be discarded or are intended to be discarded. In this sense, Waste Management programs imply all the activities concerning the right treatment of the wastes. Waste Prevention programs target all green activities from designing the product to the point just before the product is discarded. On one hand, according to Article (29) in Waste Framework Directive 2008/98/EC, the member states are required to establish appropriate Waste Prevention programs along with benchmarks, measures, targets and information channels. On the other hand, reusing is one of the most important activities within Waste Prevention context. Esenduran et al. (2016) and Yenipazarli (2016) consider reusing/remanufacturing as the targeted options in legislations.

In this research, we study a two-period market where in the first period only the new products are available. In the second period, the firm acquires a proportion of the used products and remanufactures them. The consumers who differentiate between the new and the remanufactured products on the basis of quality, green preference and price will decide to buy either the new or the remanufactured product. The reward-penalty mechanism appears as a linear cost function in the objective function of the firm, so that given the regulatory parameters (targets and penalty/reward for deviation from the targets), the firm decides on the new product prices, the acquisition price and the remanufactured product price. Two performance indicators are defined to fully embrace the social planner's public goals. The first indicator targets the quantity collected and remanufactured as the product recovery performance, and the second one considers production pollution.

## 1.1 Related studies

Closed-loop supply chains have attracted considerable attention from researchers during the last two decades or so. In this section, our objective is to position our work with respect to the papers whose objectives are directly related to what we are doing here.

To reduce the amount of energy consumed to process products, and also to decrease releasing toxic materials into the nature, the used products should be acquired and treated environmentally appropriate.

The acquired products could be used in producing the new products (Atasu and Souza , 2013; Atasu et al., 2013; De Giovanni et al., 2016), remanufactured and sold as imperfect substitutes of brand new products (Atasu et al., 2008; Bernard , 2011; Abbey et al., 2015; Esenduran et al., 2016), or recycled (Atasu et al., 2009; De Giovanni and Zaccour , 2014). The decisions about quantity to acquire, to manufacture and to remanufacture could be handled with two- or multi-period models. The reason to separate between the two classes of models is that in two-period models the remanufacturing decision is only made once, that is, in the second period, whereas in multi-period models this decision is made repeatedly over time. Two-period models are especially relevant when in the first period only new products are available, and in the second period, the firm decides on the quantity to produce of both and remanufactured products (Majumder and Groenevelt , 2001; Ferrer and Swaminathan , 2006; Ferguson and Toktay , 2006). Two-period models have the advantage of being tractable while allowing to obtain hints on the interplay between manufacturing and remanufacturing. Multi-period or infinite horizon models are of great interest when inventory or brand goodwill is included in the model (Debo et al., 2005; Ferrer and Swaminathan , 2010; De Giovanni , 2011).

In this paper, we study a two-period model where in period 1 only manufacturing new products is possible, whereas in period 2 both new and remanufactured products are made available to consumers as imperfect substitutes. The presence of customers who may prefer remanufactured products is an opportunity for the firms to gain profit. However, as the remanufactured products are imperfect substitutes of the brand new products, cannibalization happens and is considered to be a barrier to convince the OEMs to practice remanufacturing (Bulmuş et al., 2014; Abbey et al., 2015). As in Wu (2013), we capture the different preferences of consumers towards the two products by a Hotelling model.

Due to the high stake of environment in this process, several regulations are developed in different industries that target collection, recycling and remanufacturing processes. These manufacturer-directed laws act as mechanisms that transfer a variable amount of money between the firm and the social planner. These mechanisms could be performance-based (Webster and Mitra , 2007; Özdemir et al., 2012; Liu et al., 2015; Yenipazarli, 2016) or mandatory (Bernard , 2011; Assavapokee and Wongthatsanekorn , 2012; Esenduran et al., 2016). The performance-based mechanisms could be penalty-based (Webster and Mitra , 2007; Özdemir et al., 2012; Yenipazarli, 2016), reward-based (Atasu et al., 2009) or reward-penalty (Liu et al., 2015). Since the impact of incentives and penalties are intended to be addressed in this research, our paper contributes to the literature by investigating a general reward-penalty scheme. The proposed scheme gives a global insight about the nature of the performance-based mechanisms and also helps the social planners to adequately set the regulations to do their best in protecting environment and expanding remanufacturing market. Moreover, diverse penalty-based, reward-based or reward-penalty mechanisms study in the literature could be incorporated into one generic function, so that they could be evaluated and compared in a single framework.

The rest of the paper is organized as follows. Fundamental assumptions are discussed and the model is presented and solved in Section 2. Section 3 is devoted to analyzing the model in order to answer the research questions. Some specific cases for regulations are also briefly studied in Section 3.3. The model is extended and solved for a new spacial case in Section 4. The paper is concluded in Section 5 by summarizing the contributions, recapitulating the important conclusions and presenting directions for the future research.

## 2 Model

We consider a noncooperative interaction two players, namely, a manufacturer and a social planner (or regulator). The manufacturer has the capacity of producing a good with new materials as well as with old ones extracted from previously sold units. To encourage (re)manufacturing and reduce pollution, the social planner (SP) can adopt a command-and-control approach by, e.g., constraining the manufacturer to acquire back a given percentage of previously sold products, or a market-based approach by, e.g., providing a monetary incentive per unit of remanufactured product. The retained regulating policy can depend on the precise objective that the SP aims at achieving.

Table 1 summarizes the notation used in the paper.

**Table 1: Notations for parameters and decision variables**

Notation	
$U_{N1}$	Customer's utility in period 1
$U_{N2}$	Customer's utility in period 2 for brand new products
$U_R$	Customer's utility in period 2 for remanufactured products
$p_{N1}$	Selling price of the new products in period 1
$p_{N2}$	Selling price of the new products in period 2
$p_R$	Selling price of the remanufactured products in period 2
$p_A$	Acquisition price
$q_{N1}$	Demand of brand new products in period 1
$q_{N2}$	Demand of brand new products in period 2
$q_R$	Demand of remanufactured products in period 2
$q_A$	Demand of acquired used products at the end of period 1
$c_R$	Unit remanufacturing cost
$c_N$	Unit manufacturing cost
$t$	Transportation cost or competition intensity
$\rho$	Customer's willingness for remanufactured product
$\alpha$	Acquisition coefficient
$\delta$	Proportion of returnable used products in period 2

The sequence of events is as follows: (i) The SP moves first and announces its regulatory policy (stage 0). (ii) In period 1, the firm decides on the price  $p_{N1}$  of the good it produces with new materials in quantity  $q_{N1}$ . (To simplify, we shall refer from now on to such product as new product.) (iii) In Period 2, the firm acquires a quantity  $q_A$  of previously sold product ( $q_A \leq q_{N1}$ ) and decides on the prices  $p_R$  and  $p_{N2}$  of remanufactured good produced in quantity  $q_R$  and new product produced in quantity  $q_{N2}$ , respectively. (iv) The OEM pays (receives) the penalty (reward) to (from) the social planner. In this framework, the production-remanufacturing cycle is captured by introducing the two periods in (ii) and (iii), which is standard in the literature (see, e.g., Majumder and Groenevelt , 2001; Ferrer and Swaminathan , 2006; Ferguson and Toktay , 2006; De Giovanni and Zaccour , 2014). Further, we assume that the new and remanufactured products available in period 2 are partially substitutable, and that consumers are aware of the difference between the two products. This means that some degree of cannibalization will eventually take place, a factor that has been considered as a deterrent for firms to implement a recovery process (Abbey et al., 2015). Intuitively, the cannibalization intensity will depend on consumer preference and pricing strategy of the firm (Ferguson and Toktay , 2006).

We make the following two remarks. First, as the model is deterministic and there is no competition, the manufacturer's optimization problem can be solved at once for both periods, that is, there is no need to solve the model sequentially. Second, we do not specify up-front the regulator's objective but consider it ex-post, that is, given the manufacturer's reaction function to the regulator's policy, we determine the impact of the regulatory parameters on market performance.

There are four main components to our model, namely, the demand function, the acquisition incentive, the supply constraint and the reward-penalty (performance-based) mechanism.

**Demand functions:** To account for consumers' preferences for the two available products in period 2, we use a Hotelling-type model as in, e.g., Geylani et al. (2007) and Wu (2013). Consumers are uniformly distributed on the interval  $[0, 1]$ , with the new product being located at 1 and the remanufactured product at 0. Assuming full coverage of the market, which allows to capture cannibalization, we obtain the followed demand functions (see Appendix A for details):

$$q_R = \frac{\rho - p_R}{t}, \quad (1)$$

$$q_{N1} = 1 - p_{N1}, \quad (2)$$

$$q_{N2} = \frac{1 - p_{N2}}{t}, \quad (3)$$

where

$$p_{N2} = 1 + \rho - t - p_R,$$

and  $t \in (0, 1)$  measures the degree of substitutability between the two products (in period 2), and  $\rho \leq 1$  is consumer's willingness-to-pay for the remanufactured product. Note that we are assuming that the willingness-to-pay is higher for the new product than for the remanufactured one.

**Acquisition price:** To bring back the used products, the OEM needs to offer an incentive to the end-users or collection centers. This incentive, to which we shall refer as *acquisition price*, can be seen as part of a waste prevention program whose objective is recycling and reusing old products before they are disposed in nature. Considering marginal cost of collection as an increasing function of the operations scale is mentioned in Savaskan et al. (2004) as a future research topic and also addressed by Ferguson and Toktay (2006), Atasu et al. (2013) and Bulmuş et al. (2014). They argue that since acquiring more used products means higher efforts to reach more consumers, the unit acquisition cost (incentive) is volume-dependent. Denote by  $q_A$  the quantity of acquired product at the end of period 1 and by  $p_A$  the price paid. As in Bulmuş et al. (2014), we suppose that  $q_A$  is given by the linear function  $q_A = \alpha p_A$ , where  $\alpha$  is a scaling parameter. Consequently, the total acquisition cost is  $p_A q_A = \alpha p_A^2$ , which is a convex increasing function in the acquisition price.

**Supply constraint:** The supply of used products to remanufacture is limited by (a proportion of) the quantity of the new product sold to the market in the first period (Majumder and Groenevelt, 2001; Ferguson and Toktay, 2006; Atasu et al., 2008; Wu, 2013; Yenipazarli, 2016). There are three main reasons to consider this assumption. First, all of the customers who have bought the product in period one are not willing to bring their products back at any price, simply because they are still using it. Second, if part harvesting is involved in the remanufacturing (reuse) process, the OEM needs to first remove the obsolete and non-functional parts and reassemble together those that are functional to make a *remanufactured* product. Thus, even if all of the previously sold products are returned, the quantity of remanufactured products is smaller than the returns. Third, all of the returned products are not remanufacturable. Therefore, a proportion of the potential returns could be either disposed/recycled or not admitted at all. For a detailed discussion of this limitation and real examples, see Atasu and Souza (2013).

**Reward-penalty mechanism:** A variety of regulations have been considered in the literature and in practice. As in any such context, these mechanisms are either incentives, e.g., taxes and subsidies, or command-and-control rules, that is, constraints on what the firms can or cannot do. For instance, the regulator can set a minimum level for product's recovery (or collection) or set a target for remanufacturing. Since the proposed mechanism deserves detail discussion, a separated section is devoted to present and analyze the mechanism.

## 2.1 Reward-penalty mechanism

We assume that the social planner adopts the following regulatory function to promote product recovery and to reduce pollution:

$$f(q_{N1}, q_A, q_R) = \theta_A(\pi_A q_{N1} - q_A) + \theta_R(\pi_R q_A - \beta q_R),$$

where  $\pi_A$  is the target collection level;  $\pi_R$  is the target remanufacturing level;  $\theta_A$  is the unit tax/subsidy for deviating from the collection target;  $\theta_R$  is the unit tax/subsidy for deviating from the remanufacturing target; and  $\beta$  is a positive scaling parameter.

The above mechanism has four desirable properties. First, it concerns the three processes involved in the OEM's operations, namely, manufacturing, remanufacturing and collection. To see it, rewrite  $f(q_{N1}, q_A, q_R)$  as follows:

$$f(q_{N1}, q_A, q_R) = \pi_A \theta_A q_{N1} + (\pi_R \theta_R - \theta_A) q_A - \beta \theta_R q_R, \quad (4)$$

where  $\pi_A \theta_A$  represents the manufacturing tax value, and  $(\pi_R \theta_R - \theta_A)$  and  $\beta \theta_R$  represent the social planner valuation of collection and remanufacturing, respectively. This means that the SP has enough tools to impact all actions taken by the firm by providing the right incentives in the form of reward or penalty.

Second, the proposed mechanism is flexible as it allows the regulator to compare different policies. Once we have the results in the general case, it will indeed suffice to set some parameter values equal to zero to assess the impact of, e.g., not taxing new products in the first period. In this way, we can consider and contrast different scenarios related to the three aforementioned processes. Third, the mechanism is easy to communicate to the

firm, it is a simple linear rule, and easy to implement as it only requires production/collection data. Further, it is straightforward to conduct a sensitivity to assess the impact on the results, in whatever way they are measured, of varying the exogenously given collection and manufacturing targets.

The function  $f(q_{N1}, q_A, q_R)$ , which will appear with a minus sign (as a cost) in the profit-maximization problem of the OEM, has a positive side as it rewards an OEM that exceeds its target in collection ( $\pi_A q_{N1} < q_A$ ) or in remanufacturing ( $\pi_R q_A < \beta q_R$ ). In some sense, it is up to the OEM to decide if the SP is taxing or subsidizing its activities.<sup>3</sup>

Another important feature of the mechanism in (4) is that it encompasses many of the rules that have been proposed in the literature. We illustrate this statement with few examples.

Webster and Mitra (2007) study a recovery regulation where if the OEM does not buy back and take care of the products acquired by the municipality, it has to pay penalty for each unit not bought back. They present the penalizing function as  $c_d(\delta q - q_R)$  where  $c_d$  is the per-unit penalty,  $q$  is the first period production quantity,  $\delta$  is the minimum collection level and  $q_R$  is the proportion of the acquired product which are bought and remanufactured by the remanufacturer. If we set  $\pi_R \pi_A = \delta$ ,  $\theta_R = c_d$  and  $\theta_A = \theta_R \pi_R$  in (4), then we recover the penalizing function considered in Webster and Mitra (2007).

Atasu et al. (2009) study the tax subsidy for collection and recycling defined by  $\sigma r c$  where  $\sigma$  is the per unit tax subsidy,  $c$  is the proportion of the collected products, and  $r$  is the ratio of recycled products to collected products. In that paper, collection and recycling are two different activities and it is not clear what happens to the products collected but not recycled. However, the firm will receive subsidies for the proportion of the recycled products. Therefore, the same concept could be modified into a reward-penalty mechanism where acquisition is not rewarded but remanufacturing is. In this sense, function  $f$  can imitate the subsidy function by setting  $\theta_R = \sigma$  and  $\theta_A = \pi_R = 0$ , assuming that the quantity of the first period production is normalized to 1 and all produced products are acquirable.

Esenduran et al. (2016) consider two minimum mandatory levels for collection and remanufacturing and define these mandatory requirements in the constraints. Setting  $\theta_A = \theta_R = 1$  increases the marginal cost to one and consequently it is necessary to avoid. To stipulate the mandatory levels, one can define  $\theta_A(\pi_A q - q_A)^+ + \theta_R(\pi_R q_A - q_R)^+$  and further set  $\theta_A = \theta_R = 1$ ,  $\pi_A = \beta$  and  $\pi_R = \beta_R$ , where in Esenduran et al. (2016)  $\beta$  and  $\beta_R$  are the minimum collection and remanufacturing levels, respectively. This special form is excluded from the analysis in this paper, since an in-depth discussion could be found in Esenduran et al. (2016).

Yenipazarli (2016) defines the carbon emission tax on the new and remanufactured products where the remanufactured products are taxed less. The tax function  $t(x + \alpha y)$  could be transformed to reward-penalty mechanism by the following substitutions:  $t = \pi_A \theta_A$ ,  $\theta_A = \pi_R \theta_R$  and  $\alpha = -\beta/\pi_A \pi_R$ , where  $t$  is the unit tax,  $x$  is the quantity of new products,  $y$  is the quantity of remanufactured products and  $\alpha$  is the tax discount for the remanufactured products. To include the first period production, it is sufficient to replace  $c_N$  by  $c_N + T$  in  $p_R$  calculation. All of the addressed regulations are reviewed in Table 2.

**Table 2: Reward-based and penalty-based regulations in the general form**

Regulation	Original function	Regulatory Parameters
Tack-back	$c_d(\delta q_{N1} - q_R)$	$f : \pi_R \pi_A = \delta, \theta_R = c_d, \theta_A = \pi_R \theta_R$
Tax Subsidy	$\sigma r c$	$f : \theta_R = \sigma, \theta_A = \pi_R = 0, q_{N1}$ scaled to 1
Carbon Emission Tax	$T(q_{N1} + q_{N2} + \alpha q_R)$	$f : \pi_A \theta_A = t, \pi_R \theta_R = \theta_A, \alpha = -\beta/\pi_A \pi_R, p_{N2} \rightarrow p_{N2} - T$

<sup>3</sup>An alternative standard (only) taxing rule could be given by

$$g(q_{N1}, q_A, q_R) = \theta_A(\pi_A q_{N1} - q_A)^+ + \theta_R(\pi_R q_A - \beta q_R)^+.$$

We refrain from adopting this form as it lacks the positive incentive aspect of the regulation and it renders the firm's objective function non differentiable. Solving the problem becomes challenging and leads to unnecessary complications that would deviate our attention from the main qualitative insights we aim to obtain.

## 2.2 Firm's optimization problem

Accounting for the reward-penalty mechanism announced by the SP, the OEM's profit function becomes

$$\Pi = q_{N1}(p_{N1} - c_N) + q_{N2}(p_{N2} - c_N) + q_R(p_R - c_R) - q_{AP_A} - f(q_{N1}, q_A, q_R). \quad (5)$$

The constraints on acquisition and remanufacturing are as follows:

$$\text{acquisition constraint : } q_A \leq \delta q_{N1}, \quad (6)$$

$$\text{remanufacturing constraint : } q_R \leq q_A. \quad (7)$$

Substituting in (5) for the quantities by their values from (1)–(3), we obtain the following optimization problem:

$$\begin{aligned} \max_{p_{N1}, p_R, p_A} \Pi = & (1 - p_{N1})(p_{N1} - c_N) + \frac{1}{t}(1 - p_{N2})(p_{N2} - c_N) + \frac{1}{t}(\rho - p_R)(p_R - c_R) \\ & - \alpha p_A^2 - \theta_A(\pi_A(1 - p_{N1}) - \alpha p_A) - \theta_R(\pi_R \alpha p_A - \frac{\beta}{t}(\rho - p_R)), \end{aligned} \quad (8)$$

subject to

$$\text{Acquisition constraint : } \alpha p_A + \delta p_{N1} \leq \delta, \quad (9)$$

$$\text{Remanufacturing constraint : } p_R - \alpha p_A \geq \rho, \quad (10)$$

$$\text{Full market coverage : } p_{N2} = 1 + \rho - t - p_R. \quad (11)$$

Note that because of our assumption of full market coverage,  $p_{N2}$  is not a decision variable.

## 2.3 Firm's reaction function

To solve the optimization problem in (8)–(11), introduce the Lagrangian

$$\begin{aligned} L = & (1 - p_{N1})(p_{N1} - c_N) + \frac{1}{t}(-\rho + t + p_R)(1 + \rho - t - p_R - c_N) + \frac{1}{t}(\rho - p_R)(p_R - c_R) \\ & - \alpha p_A^2 - \theta_A(\pi_A(1 - p_{N1}) - \alpha p_A) - \theta_R(\pi_R \alpha p_A - \frac{\beta}{t}(\rho - p_R)) \\ & + \lambda(\delta - \alpha p_A - \delta p_{N1}) + \eta(p_R - \alpha p_A - \rho), \end{aligned} \quad (12)$$

where  $\lambda$  and  $\eta$  are the Lagrange multipliers appended to the acquisition and remanufacturing constraint, respectively. The first-order optimality conditions are given by

$$\frac{\partial L}{\partial p_{N1}} = 0 \Leftrightarrow p_{N1} = \frac{c_N + 1 + \theta_A \pi_A - \delta \lambda}{2}, \quad (13)$$

$$\frac{\partial L}{\partial p_R} = 0 \Leftrightarrow p_R = \frac{1 + 3\rho - 2t - c_N + c_R + \eta t - \beta \theta_R}{4}, \quad (14)$$

$$\frac{\partial L}{\partial p_A} = 0 \Leftrightarrow p_A = \frac{\theta_A - \pi \theta_R - \lambda - t\eta}{2}, \quad (15)$$

$$\lambda \geq 0, \quad (\delta - \alpha p_A - \delta p_{N1}) \geq 0, \quad \lambda(\delta - \alpha p_A - \delta p_{N1}) = 0, \quad (16)$$

$$\eta \geq 0, \quad (p_R - \alpha p_A - \rho) \geq 0, \quad \eta(p_R - \alpha p_A - \rho) = 0. \quad (17)$$

It is easy to verify that the objective function in (12) is strictly concave in all OEM's decision variables, that is,  $p_{N1}$ ,  $p_A$  and  $p_R$ , and that the feasibility set is convex. Consequently, the solution is unique.

As the constraints are easier to interpret when written in terms of quantities rather than in prices, we present the solution in quantities. Further, since each of the two constraints in (6)–(7) can be binding or not, the solution lies in one of the four regions described in Table 3.

**Table 3: Primary conditions for feasible solution regions**

Region	Conditions	Implications
BY	$q_R < q_A < \delta q_{N1}$	Partial acquisition and Partial remanufacturing
BX	$q_R = q_A < \delta q_{N1}$	Partial acquisition and Full remanufacturing
AY	$q_R < q_A = \delta q_{N1}$	Full acquisition and Partial remanufacturing
AX	$q_R = q_A = \delta q_{N1}$	Full acquisition and Full remanufacturing

Region AX represents the case where remanufacturing is highly beneficial and the firm acquires all available used products and remanufactures all of them. Here, both constraints are binding. In region BY, the firm does not acquire all returnables and does not remanufacture all acquired products. A variety of causes can describe this case, e.g., a high acquiring cost, low penalty for not remanufacturing, better avoiding harsh competition between the two products, etc. In region BY, none of the two constraints is binding. The two other regions, that is, AY and BX, correspond to cases where one of the constraint is binding.

Table 4 gives the optimal prices in each region and Table 5 specifies the conditions, in terms of parameter values, under which the solution is in a particular region. The computational details are presented in Appendix B.

**Table 4: Optimal solutions for all regions for the regulated market**

Region	$p_{N1}$	$p_R$	$p_A$
BY	$\frac{1+c_N+\pi_A\theta_A}{2}$	$\frac{3\rho-2t+c_R-c_N+1-\beta\theta_R}{4}$	$\frac{\theta_A-\pi_R\theta_R}{2}$
BX	$\frac{1+c_N+\pi_A\theta_A}{2}$	$\frac{\alpha t(3\rho-2t+1-c_N+c_R-\theta_A-\beta\theta_R+\pi_R\theta_R)+2\rho}{2(2\alpha t+1)}$	$\frac{1}{\alpha t}(\rho-p_R^*)$
AY	$1 + \frac{c_N+\delta\pi_R\theta_R+\theta_A(\pi_A-\delta)-1}{2(1+\delta^2/\alpha)}$	$\frac{3\rho-2t+c_R-c_N+1-\beta\theta_R}{4}$	$\frac{\delta}{\alpha}(1-p_{N1}^*)$
AX	$1 - \frac{1}{\delta t}(\rho-p_R^*)$	$\frac{\delta^2 t(\pi_R\theta_R-\beta\theta_R-c_N+c_R+3\rho-2t+1-\theta_A)+\delta t(\pi_A\theta_A+c_N-1)+2\delta^2\rho/\alpha+2\rho}{2t(2\delta^2+1+\delta^2 t)}$	$\frac{1}{\alpha t}(\rho-p_R^*)$

**Table 5: Necessary and Sufficient conditions for all regions for the regulated market**

Region	Necessary and sufficient conditions
BY	$\frac{\rho-1+2t-c_R+c_N+\beta\theta_R}{2\alpha t} + \pi_R\theta_R \leq \theta_A \leq \frac{\alpha\pi_R\theta_R+\delta(1-c_N)}{\alpha+\delta\pi_A}$
BX	$\theta_A \leq \min\left\{\frac{\delta(2\alpha t+1)(1-c_N)-\alpha(\rho+2t-1+c_N-c_R+\beta\theta_R-\pi_R\theta_R)}{\alpha+\delta\pi_A(2\alpha t+1)}, \frac{\rho-1+2t-c_R+c_N+\beta\theta_R}{2\alpha t} + \pi_R\theta_R\right\}$
AY	$\theta_A \geq \max\left\{\frac{\alpha\pi_R\theta_R+\delta(1-c_N)}{\alpha+\delta\pi_A}, \frac{(1+\delta^2/\alpha)(\rho+\beta\theta_R+c_N-c_R+2t-1)}{2\delta t(\delta-\pi_A)} + \frac{c_N-1+\delta\pi_R\theta_R}{\delta-\pi_A}\right\}$
AX	Otherwise

Unless explicitly specified, the value of scaling parameter  $\beta$  is from now on fixed at 1.

In the next section, the proposed mechanism is employed in order to gain insight into the impact of the environmental protection laws on the market. Moreover, we will discuss how the SP could use this mechanism as a toolbox to achieve the goal of efficiently protecting the environment.

### 3 Model analysis

At this point, the proposed reward-penalty mechanism, which mimics several different environmental protection laws, will enter the analysis. As alluded to it before, the objective is to gain insight into how these laws affect the market, and how the SP could move one step closer to the main goal of implementing these laws. Firstly, depending on the state of the remanufacturing industry, the performance indicators are introduced. Secondly, the model is analyzed to provide the SP with the guidelines of how to improve market performance. At the last step and to showcase how the proposed generic reward-penalty mechanism could resemble several existing environmental laws, we compare the mechanisms already discussed in Section 2.1.

As a final note, it is scarce to assume that the firm acquires all acquirable products and remanufacture all of them. In other words, a firm is constrained either by the quantity of acquirable products to collect or by the quantity of collected products to remanufacture. Therefore, it is safe to disregard the case where both  $q_A = \delta q_{N1}$  and  $q_A = q_R$  hold. Thus, for the sake of clarity and parsimony we exclude the full-acquisition and full-remanufacturing case (AX) from a large proportion of the model analysis.

### 3.1 Performance indicators

The regulator can follow two approaches to protect the environment. In the first approach, the SP can promote product recovery without considering the impact of the regulation on the new-product market. This could be the case at the primary steps of the regulation where the SP intends to spread exercising product recovery in the market. In this sense, since remanufacturing is deemed to be superior to recycling from an environmental point of view, we define the product recovery performance (PRP) function as:

$$PRP = q_R + \gamma(q_A - q_R) = (1 - \gamma)q_R + \gamma q_A, \quad (18)$$

where  $\gamma \leq 1$  denotes the lower environmental benefit of recycling compared to remanufacturing. This value addresses the (proportional) weight the SP puts on remanufacturing for any political or managerial reasons. It also could be the case where the damage caused by producing new goods could not be scaled with environmental benefit of recycling and remanufacturing.

In the second approach, the SP seeks to directly decrease landfilling and other undesirable side effects of production. This could be the case where recovery is a well-established process in the manufacturing industry. The metric to measure the environmental impact (EI) is:

$$EI = q_{N1} + q_{N2} - \omega q_R - (1 - \omega)q_A, \quad (19)$$

where  $\omega < 1$  denotes the relative environmental impact of remanufacturing versus collection and recycling, assuming that a remanufactured product has no negative environmental impact. Function (19) deserves some comments. In (19), the relative weight on manufacturing, collection and collection without remanufacturing is set to 1,  $-1$  and  $\omega$ , respectively. In other words, remanufacturing a product nullifies the negative impact of production, but collection without remanufacturing cancels  $1 - \omega$  of it. Therefore,  $EI$  addresses a combination of wastes, manufactured products, and collected-but-not-remanufactured products. To be more specific,  $EI$  considers the pollution caused by producing new products from mine extraction until disposition. Through this paper, the terms environmental performance and pollution refer to the value of  $EI$  function and will be used interchangeably. Note that whereas the SP wishes to increase the value of  $PRP$ , it aims at decreasing  $EI$ , which measures the negative environmental impact of new production. In the next section, the proposed mechanism is investigated to gain managerial insights.

### 3.2 Target-based analysis

In this section, the analysis concerns the actions that are targeted in environment protection laws. More specifically, we intend to answer the following questions:

1. What is the result of targeting collection as the only recovery action in the law?
2. What is the result if remanufacturing is directly addressed in the regulation?
3. If the current regulation targets collection (and remanufacturing), how does environmental performance vary when target levels and/or deviation incentives are modified?

Responding to these questions not only clarifies the impact of targeting specific actions in the laws, but also provides the SP with guidelines to how to improve current performance of the market. We believe that such a target-based analysis is relevant in the wake of the controversial discussions about the effect of increasing the collection target level for WEEE Directive in Europe. Moreover, since *reuse* is becoming more important in EPR context, it is also included in our analysis.

Two legislative approaches are considered for this analysis: collection-based and remanufacturing-based. The collection-based mechanism concerns the target level relative to past-production and reward/penalty for deviating from the determined target. In a similar manner, the remanufacturing-based mechanism defines a target level for the remanufacturing level in regards to the collection rate and incentivizes deviation positively or negatively. The purpose of considering these options is to gather information about the impact of targeting a specific recovery option.

### 3.2.1 Collection-based analysis

To perform this task, the parameters of the proposed mechanism are required to be set accordingly. Putting  $\theta_R = 0$  yields  $\pi_R \theta_R = 0$  and  $\beta \theta_R = 0$ , and consequently Equation (4) becomes

$$f(q_{N1}, q_A) = \pi_A \theta_A q_{N1} - \theta_A q_A. \quad (20)$$

Notice that, although we aim at addressing collection in this revised mechanism, the reference point for collection, which is the past production, also needs to be included in the mechanism. Furthermore, we could get rid of the remanufacturing quantity by defining  $\beta = 0$ . However, since  $\pi_R \theta_R$  will appear in the mechanism, one could argue that defining the target level and deviation incentive for remanufacturing in a mechanism that is not supposed to consider remanufacturing does not make sense.

Propositions 1 and 2 deal with the first and partially the third research questions.

**Proposition 1** *If the regulation only includes collection, the following conclusions hold true:*

1. *Collection target level does not affect product recovery unless the firm is restricted by the quantity of acquirable products (past production). For the latter, it impacts product recovery negatively.*
2. *Increasing deviation incentive yields more product recovery.*

**Proof.** See Appendix C. □

The first result in Proposition 1 is important and counterintuitive for the SP. As a general rule, the social planner increases the target rate to increase the recovered products. However, our analysis shows that if collection is the only targeted recovery option, increasing the target rate has no impact on product recovery performance unless collection is limited by past production. Furthermore, if the firm has already acquired as many as possible, increasing the target rate reduces product recovery. The main cause of this outcome is that increasing the collection target level when the firm is already acquiring as many as possible only creates extra cost and consequently increases the first period price. Higher price leads to lower quantity and in turn less units would be available to collect and remanufacture. In other words, increasing the collection target rate could result in lower quantity of new and remanufactured products. Therefore, the collection target level is a tool for the social planner to decrease production quantity if and only if the firm is collecting all acquirable products.

The second item in the above proposition states that although the target level may not have any impact on product recovery, the deviation reward/penalty yields higher product recovery performance. If the firm is collecting fewer products, it will be penalized by  $\theta_A$  per unit not collected, and if it collects more, then it will be rewarded by  $\theta_A$  per unit collected. Therefore, either the firm is receiving reward or paying penalty, the important decision for the social planner is the per-unit deviation incentive.

The same analysis should be conducted for the regulator who aims at reducing pollution. Proposition 2 is presented concerning the environmental impact of production.

**Proposition 2** *If the regulation only includes collection, the following conclusions hold true:*

1. *Increasing collection target level impacts the environmental performance positively.*
2. *Increasing deviation incentive affects the environment positively unless collection is limited by the quantity of the past sold products.*

**Proof.** See Appendix C. □

Unlike the product recovery performance, the SP can improve the environmental performance by increasing the target level for collection. However, increasing the deviation incentive is not always advisable as it depends on the current state of the market.

To wrap up, the main conclusion of analyzing collection-based case is obtained where Propositions 1 and 2 are seen together. On one hand, increasing collection target either has no impact or reduces product recovery. On the other hand, it also yields lower pollution. Therefore, if the regulator seeks to expand product recovery exercise, increasing the target rate is not suggested. Otherwise, if the product recovery industry (collection, recycling and remanufacturing) is advanced and mature, increasing the collection target rate means lower pollution and product landfilling.

### 3.2.2 Remanufacturing-based analysis

If the SP seeks to address remanufacturing, then it sets  $\theta_A = 0$  in (4) to get

$$f(q_A, q_R) = \pi_R \theta_R q_A - \beta \theta_R q_R. \quad (21)$$

Similar to the collection-based case, notice that the reference point for measuring the remanufacturing performance - collection amount - should also be included in the mechanism. However, by defining  $\theta_A = 0$ , the firm will be rewarded or penalized solely on the basis of its distance to the target level, defined on the basis of the collection amount. In other words, if the SP wants to target remanufacturing, the performance of the firm should be defined on the basis of the distance to the target point defined on the collection amount, that is,

$$f(q_A, q_R) = \theta_R (\pi_R q_A - \beta q_R). \quad (22)$$

Propositions 3 and 4 treat the remanufacturing-based case.

**Proposition 3** *If the regulation targets remanufacturing, then the following conclusions hold true:*

1. *Increasing deviation incentive positively affects product recovery performance.*
2. *Increasing remanufacturing target impacts the product recovery performance negatively.*

**Proof.** See Appendix C. □

If the firm is not collecting all acquirable products, increasing the deviation incentive not only promotes product recovery, but also reduces new production. However, if the firm is collecting as many as possible, increasing the incentive results in more new production in a way that it nullifies the positive impact of promoting recovery. Thus, the probable strategic response of the firm should be foreseen in advance of increasing the incentives.

Recall that the remanufacturing target concerns the proportion of the collected products to be remanufactured. Thus, the second result in Proposition 3 states the conclusion that increasing the remanufacturing target works towards less product recovery. The line of reasoning for this conclusion is the same stated for the collection-based case. Increasing the remanufacturing rate target pushes the firm to remanufacture more and consequently it loses on new products. To avoid this loss, the firm reduces the quantity of collected products, which in turn leads to fewer remanufactured products. Therefore, increasing the remanufacturing target rate is not suggested for promoting product recovery.

Proposition 4 addresses the overall environmental impact.

**Proposition 4** *If the regulator targets remanufacturing, the following conclusions hold true:*

1. *Increasing deviation incentive has positive environmental impact.*
2. *Elevating remanufacturing target yields more pollution.*

**Proof.** See Appendix C. □

Unlike collection target, remanufacturing target plays an undesirable role for environment. Comparing Propositions 3 and 4 reveals that increasing the target level for remanufacturing is never advisable. However, the incentive value impacts the environment performance positively by promoting product recovery and reducing the quantity of the brand new products in both periods.

To provide the SP with a general picture of the analysis, all propositions should be considered simultaneously. Tables 6 and 7 summarizes the propositions and conclusions of this section. Note that environmental performance is increased (and pollution is reduced) if the value of  $EI$  is decreased.

**Table 6: Impacts of target rates and deviation incentives on product recovery performance**

Parameters	Recovery Options	
	Collection	Remanufacturing
Target Level	No Impact or ↘	↘
Deviation Incentive	↗	↗

Table 6 provides a useful toolbox for the social planner whose goal is to promote product recovery. Apart from the intended recovery option, it is always advised to increase the incentives, so that the firm recovers more either to increase tax cuts or decrease the penalty. However, it is not suggested to increase the target levels as either it does not have any impacts, or these two outcomes could be observed: (i) reduction in the number of new products in the first period, or (ii) reduction in the total number of collected and remanufactured products without reducing the number of new products in the first period.

**Table 7: Impacts of target rates and deviation incentives on environmental performance**

Parameters	Recovery Options	
	Collection	Remanufacturing
Target Level	↗	↘
Deviation Incentive	↗ if $q_A < \delta q_{N1}$ ; ↘ Otherwise	↗

Table 7 provides a useful toolbox for the social planner whose goal is to reduce pollution. From this table, it could be concluded that increasing the target level for collection (as is the case for future WEEE Directives) is a reliable decision if environmental performance and pollution reduction are the ultimate goals of the SP. However, if these legal harmonizations are extended to cover remanufacturing activity, it is not advised to increase the target level.

All in all, we emphasize that the best suggestions on the target levels and incentive values depend on the main goal of the regulator. If the remanufacturing industry is in its primitive stages and the government intends to promote it, collection target needs to be kept down. However, if the industry is well-established and the social planner seeks to reduce pollution, increasing the collection target is advised. Increasing the target rate on remanufacturing is also not suggested for both cases.

### 3.3 Special cases

In this section, we revisit the special cases addressed in Section 2.1. These papers consider a variety of the general cases in real world that are worth investigating. The purpose of this section is two-fold. The first goal is to show how the general function helps generating policies under more constrained rules. The second purpose is to compare these rules to see in what ways each of them works towards promoting remanufacturing and restricting new production. For each case, the optimal values and required conditions are presented with the original notations as used in those papers.

### 3.3.1 Buy-Back

Webster and Mitra (2007) study a problem in which the municipality acquires a proportion of the used products and if the firm does not Buy-Back all of them, it has to pay penalty per unit not bought-back. In that paper, the per-unit penalty is  $c_d$ , quantity produced in the first period is denoted by  $q$ ,  $\delta$  is the minimum acquisition level and  $q_R$  is the remanufactured quantity. The equivalent parameters in our model are  $\pi_R\pi_A = \delta$ ,  $\theta_R = c_d$  and  $\theta_A = \theta_R\pi_R$ . Since no assumption is made on the basis of maximum acquirable quantity and the firm cannot buy back more than  $\delta$ , we set the maximum acquirable proportion  $\delta$  in our model to  $\delta$  in theirs. Moreover, since all of the regulatory parameters are not determined strictly, a single arbitrary set of values is selected. Let  $\pi_R = 1$ ,  $\pi_A = \delta$ ,  $\beta = 1$ , and  $\theta_A = \theta_R = c_d$ . The optimal values are presented in Table 8. By definition, collection has no value until remanufacturing process is implemented. Therefore, BY and AY do not contain the optimal values. Using the closed-form solutions in Table 4, the closed-form solutions for the buy-back regulation is obtained and presented in Table 8.

**Table 8: Optimal solutions for Buy-Back legislation**

Region	$p_{N1}$	$p_R$
BX	$\frac{1+c_N+\delta c_d}{2}$	$\frac{\alpha t(3\rho-2t+1-c_N+c_R-c_d)+2\rho}{2(2\alpha t+1)}$
AX	$1 - \frac{1}{\delta t}(\rho - p_R^*)$	$\frac{\delta^2 t(c_R - c_N + 3\rho - 2t + 1) + \delta t(c_N - 1) + 2\delta^2 \rho / \alpha + 2\rho}{2t(2\delta^2 + 1 + \delta^2 t)}$

The necessary and sufficient condition for BX to contain the optimum response is

$$c_d \leq \min \left\{ \frac{\delta(2\alpha t + 1)(1 - c_N) - \alpha(\rho + 2t - 1 + c_N - c_R)}{\alpha + \delta^2(2\alpha t + 1)}, \frac{2(\rho - 1 + 2t - c_R + c_N + c_d)}{\alpha t} + c_d \right\}.$$

This inequality is obtained by replacing the general regulatory parameters with their equivalence in Buy-Back legislation. Note that there are only two general possibilities for the solution region. At the boundary conditions, the firm either collects no products or produces nothing in the first period, which are of no interest. If  $\rho - 1 + 2t - c_R + c_N + c_d < 0$  the solutions are at the boundaries that are not analyzed in this paper. It is assumed that  $\rho - 1 + 2t - c_R + c_N > 0$  holds and consequently  $\rho - 1 + 2t - c_R + c_N + c_d > 0$  is ensured. Relaxing this assumption,  $c_d$  needs to be increased to guarantee  $\rho - 1 + 2t - c_R + c_N + c_d > 0$ . However, this increment needs to be smaller than the other upper bound for the per-unit tax:

$$c_d \leq c_d^{up} = \frac{\delta(2\alpha t + 1)(1 - c_N) - \alpha(\rho + 2t - 1 + c_N - c_R)}{\alpha + \delta^2(2\alpha t + 1)}.$$

If  $c_d$  is set between the two aforementioned values, partial acquisition happens. If this is the case, increasing the per-unit penalty increases the brand new product prices and decreases the remanufactured product prices. If  $c_d$  surpasses its upper bound, increasing it further has no impact on the firm's decisions, but decreases its profit. Since deteriorating the profit with no environmental gain is not in the SP's interest, a regulator who is seeking to promote recovery should increase the per-unit tax until it hits the upper bound. In other words,  $c_d^* = c_d^{up}$  and the firm acquires partially but remanufactures all collected products.

It is worth noting that the regulator is penalizing new production by  $-\delta c_d$  and subsidizes remanufacturing by  $c_d$ . In other words, Buy-Back legislation does not imply direct rewarding for remanufacturing, but rather reduction of manufacturing penalties.

### 3.3.2 Tax subsidy for collection and remanufacturing

Atasu et al. (2009) study a tax subsidy model where  $\sigma = \theta_R$  is the unit tax subsidy,  $\beta = 1$  and  $\theta_A = \pi_R = 0$ , and assuming that  $\delta = 1$  and  $q_{N1}$  is normalized to 1. In this case, the optimal values are those reported in Table 9. Furthermore,  $\pi_A$  could have any value which is set to be zero for the sake of simplicity. Note that since collection is not considered as a separate recovery option, the firm remanufactures all collected

products. Therefore, it is not optimal for the firm to land on BY or AY regions. Furthermore, as discussed before, the boundary conditions at which the firm either manufactures no products in the first period or collects nothing are not worth investigating. Table 9 presents the optimal prices for tax subsidy case.

**Table 9: Optimal solutions for tax subsidy legislation**

Region	$p_{N1}$	$p_R$
BX	$\frac{1+c_N}{2}$	$\frac{\alpha t(3\rho-2t+1-c_N+c_R-\sigma)+2\rho}{2(2\alpha t+1)}$
AX	$1 - \frac{1}{t}(\rho - p_R^*)$	$\frac{t(c_R-\sigma+3\rho-2t)+2\rho/\alpha+2\rho}{2t(3+t)}$

Unlike Buy-Back solutions, the regulatory parameters does not affect the price of the new products in the first period and does affect the price of the remanufactured products. However, interestingly, the unit tax subsidy for remanufacturing in this mechanism is and works the same as the unit penalty for not remanufacturing in Buy-Back mechanism. This is the case as both mechanisms incentivize remanufacturing with the same parameter:  $c_d$  and  $\sigma$ . In this mechanism, manufacturing and collection are not directly incentivized but encouraging collection is embedded in offering incentives for remanufacturing.

The necessary and sufficient condition for BX is:

$$0 \leq \min \left\{ \frac{(2\alpha t + 1)(1 - c_N) - \alpha(\rho + 2t - 1 + c_N - c_R + \sigma)}{\alpha}, \frac{2(\rho - 1 + 2t - c_R + c_N + \sigma)}{\alpha t} \right\}.$$

If  $\sigma$  is so low that  $\rho + 2t - 1 + c_N - c_R + \sigma < 0$ , the solutions are at the boundaries. The same conclusion is made if the tax subsidy is high enough to ensure  $\sigma \geq \sigma^{up} = (2\alpha t + 1)(1 - c_N)/\alpha - (\rho + 2t - 1 + c_N - c_R)$ . In region BX, increasing tax subsidy does not reduce first period production quantity but increases the quantity of the remanufactured products by the factor  $\alpha/(2(2\alpha t + 1))$ . However, if  $\sigma$  is high or low enough to violate the addressed conditions, it reduces the first period production quantity and increases the remanufactured product quantity by the factor  $1/2t(3 + t)$ . Since no assumption is made on the value of  $\alpha$ , it could not be determined if the regulator is better off increasing or decreasing  $\sigma$  to land on AX region.

### 3.3.3 Carbon emission tax

Yenipazarli (2016) studies the tax emission case where the firm has to pay tax for producing new and remanufactured products, but less for the latter. The tax function  $t(x + \alpha y)$  could be transformed to reward-penalty mechanism  $f$  by the following substitutions:  $t = \pi_A \theta_A$ ,  $\theta_A = \pi_R \theta_R$  and  $\alpha = -\beta/\pi_A \pi_R$ , where  $t$  is the unit tax,  $x$  is the quantity of the brand new products,  $y$  is the quantity of remanufactured products and  $\alpha$  is the tax discount for the remanufactured products. Since the specific case of Yenipazarli (2016) corresponds to a set of regulatory parameters in this paper, an arbitrary set of values is selected in the following way:  $\pi_A = \pi_R = 1$ ,  $\alpha = -\beta$  and  $t = \theta_R = \theta_A$ . To avoid confusion in the notations, we denote the per-unit tax by  $T$ , the per-unit tax discount by  $A$ , and the maximum acquirable proportion by  $\delta$ . Also, we are required to replace  $c_N$  with  $c_N + T$  in  $p_R^*$  to include production of the brand new products in the first and the second periods.

Similar to Buy-Back and Tax Subsidy mechanisms, the firm is better off remanufacturing all collected products to achieve the maximum profit. Therefore, the solutions for BY and AY regions are not included in Table 10. Assuming interior solutions, the optimal prices for the tax emission problem are given in Table 10.

**Table 10: Optimal solutions for Carbon tax emission**

Region	$p_{N1}$	$p_R$
BX	$\frac{1+c_N+T}{2}$	$\frac{\alpha t(3\rho-2t+1-c_N+c_R+(A-1)T)+2\rho}{2(2\alpha t+1)}$
AX	$1 - \frac{1}{\delta t}(\rho - p_R^*)$	$\frac{\delta^2 t((A-1)T - c_N + c_R + 3\rho - 2t + 1) + \delta t(T + c_N - 1) + 2\delta^2 \rho / \alpha + 2\rho}{2t(2\delta^2 + 1 + \delta^2 t)}$

The solutions are the boundaries if  $(\delta t - A)T \geq \delta t(1 - c_N) + 1 + c_R - c_N - 2t - \rho$ . Otherwise, the necessary and sufficient condition for BX could be obtained by verifying  $q_A = q_R \leq \delta q_{N1}$ :

$$T \leq T^{up} = \frac{\delta(2\alpha t + 1)(1 - c_N) - \alpha(\rho - c_R + c_N - 1 + 2t)}{\delta(2\alpha t + 1) - \alpha A},$$

for partial acquisition. If  $A < \delta(2\alpha t + 1)/\alpha$ , increasing the emission tax forces the firm to acquire and remanufacture all acquirable products. If the firm is better of acquiring partially, increasing the per-unit tax value yields to lower remanufactured products price and consequently more remanufacturing. Furthermore, it increases the new product prices in both periods that yields to fewer new products. Otherwise, if  $A$  and  $T$  are devised a way that the firm acquires all products, increasing the emission tax decreases the quantity of remanufactured products along with the quantity of brand new products. Therefore, by increasing the carbon price, the firm manufactures less and remanufactures more until there would be no product being discarded into the nature. The reason for this observation is that the quantity of new products in the first period is an upper bound for the quantity of the remanufactured products. First, the firm tries to increase the quantity of acquired products in order to remanufacture more and saturate the market with remanufactured products instead of new products. At a certain threshold, the acquired and remanufactured quantity hits the upper bound. If the regulator keeps increasing the emission tax value, decreased quantity of brand new products results in fewer remanufactured products.

Remember that the quantities of acquired and remanufactured products are considered as an environmental performance indicator as an indication of less waste and disposition. If the carbon tax value  $T$  is high enough that insignificant amount of the waste are discarded into the nature, the EPR has achieved its primary goal to protect the environment. Beyond this value, increasing the emission tax hurts the market by increasing the price and consequently decreasing consumer surplus and possibly the firm's profit. Thus, it is suggested that the carbon tax should be increased only to the point that the quantity of discarded products are low enough.

To conclude this section, we summarize the role of the remanufacturing tax discount. Since  $A$  has no impact on the new products quantity, increasing it is not interesting neither financially nor environmentally. Therefore, the regulator needs to be cautious about the market parameters and the determination of the value of taxes and discounts.

### 3.3.4 Comparing tax subsidy, carbon emission tax and Buy-Back

So far, three famous environmental laws are discussed. In this section, the main conclusions are recapitulated in order to contrast these mechanisms. Note that one of the main goals of this research is to design a mechanism that is flexible enough to imitate the behavior of several existing rules so that they could be evaluated and compared in a single framework.

**Result 1** *In all considered mechanisms, it is not beneficial for the firm to acquire but not remanufacture.*

The simple reason behind this observation is that all these mechanisms consider only one option for recovery. Translating this single option to remanufacturing and considering the acquisition cost, it is not optimal to collect but not remanufacture.

**Result 2** *The Buy-Back and Carbon Emission Tax mechanisms penalize manufacturing but discount for remanufacturing.*

The difference between these two laws is that if Buy-Back legislation is in place, increasing the penalty  $c_d$  increments remanufacturing up to a level beyond which it does not have any environmental impact. However, the carbon price in Emission Tax mechanism keeps affecting the quantity of the new and remanufactured products even for acquire-all case. Therefore, the Emission Tax mechanism could be deemed more environmentally progressive than the Buy-Back mechanism.

**Result 3** *Unlike the Carbon Emission Tax and Buy-Back legislations, the Tax Subsidy mechanism does not affect the first period production if  $\sigma$  is smaller than a certain threshold.*

If the regulator increases  $\sigma$  beyond the threshold, the quantities of the new production in the first period and the remanufactured products increases. This is the exact opposite of what could happen by increasing the emission tax above its threshold. Therefore, subsidizing remanufacturing may increase the second period production of the brand new products whereas taxing the remanufacturing could decrease this quantity.

The main difference and similarities are described in this section. The full picture of these impacts is illustrated in Table 11. The terms "Below" and "Beyond" imply the threshold that separates full acquisition from partial acquisition areas (AX and BX). Let  $T^{up}$ ,  $c_d^{up}$  and  $\sigma^{up}$  be the addressed threshold values for emission tax, buy-back penalty and unit tax subsidy, respectively.

**Table 11: Impacts of tax subsidy, emission tax and buy-back penalty on manufacturing and remanufacturing**

Production Quantities	Tax Subsidy $\sigma$		Emission Tax $T$		Buy-Back Penalty $c_d$	
	Below	Beyond	Below	Beyond	Below	Beyond
$q_R$	↗	↗	↗	↘	↗	—
$q_{N1}$	—	↗	↘	↘	↘	—

The following guidelines are extracted from Table 11. If the social planner intends to increase product recovery without affecting the market of new products, Tax Subsidy rule with a small enough (below the thresholds) unit tax subsidy is the rule that should be used. Otherwise, if reduction of the brand new products is also of interest, Emission Tax and Buy-Back rules with small enough (below the thresholds) unit tax and penalty are to be utilized. The only method that yields fewer quantities of products is to tax carbon emission where the emission tax value is high enough (above the threshold). In an expanding economy, the Tax Subsidy regulation is the best option whereas the Carbon Emission Tax regulation is the most favorable to immediately reduce the pollution by holding off the production. Thus, Tax Subsidy and Carbon Emission Tax are the two regulations to be used in different economic situations, and the Buy-Back law is solely a special case for the latter.

Some of the famous environmental legislations are evaluated and compared in this section. In the next section, the original performance-based mechanism is extended to address a specific rule that of practical importance as it is in place in a certain area, and is theoretically significant as it is an alternative to the proposed reward-penalty mechanism.

## 4 Model extension: Penalizing mechanism

In this section, we study a mechanism that corresponds to the case in Minnesota (USA) with two purposes in mind. Firstly, we show how the performance-based mechanism could be modified to tend towards a penalty-based mechanism. Secondly, we intend to compare performance-based and penalty-based mechanisms to see which one is environmentally superior.

The firm is not obligated to collect at least as the required level; it may collect fewer but it has to pay penalty (Özdemir et al., 2012). This situation could be modeled by function  $g$  where  $\theta_R = 0$ . In other words, remanufacturing is not addressed by the social planner. The objective is to maximize

$$\Pi = (1 - p_{N1})(p_{N1} - c_N) + \frac{1}{t}(1 - p_{N2})(p_{N2} - c_N) + \frac{1}{t}(\rho - p_R)(p_R - c_R) - \alpha p_A^2 - \theta_A(\pi_A(1 - p_{N1}) - \alpha p_A)^+. \quad (23)$$

which is not differentiable at  $\pi_A q_{N1} = q_A$ . We have already addressed the market coverage and supply limitation constraints, and how each of them divides the feasible region into two separated parts. To find the closed-form solutions of the model, one more distinction should be made on the basis of legislation enforcement: the minimum acquisition requirement could be enforcing or not (penalty condition). Let  $\tilde{\Pi}$  be the objective function for the unregulated market, i.e.,

$$\tilde{\Pi} = (1 - p_{N1})(p_{N1} - c_N) + \frac{1}{t}(1 - p_{N2})(p_{N2} - c_N) + \frac{1}{t}(\rho - p_R)(p_R - c_R) - \alpha p_A^2.$$

The penalty condition is enforcing (effective) if  $\pi q_{N1} \geq q_A$  holds at the maximum of  $\tilde{\Pi}$ , and it is not effective otherwise. If the penalty condition is effective, it means that the OEM has decided to pay the penalty to avoid cannibalization and to gain more profit, or it acquires as many as required to avoid penalty. Introducing the new condition creates 8 regions to deal with in the forms presented in Table 12.

**Table 12: Possible regions for the environmental regulation in Minnesota**

Subproblems	Market Coverage	Supply Limitation	Regulation Effectiveness	Paying Penalty
S1	Partial	No	No	No
S2	Partial	Yes	No	No
S3	Partial	No	Yes	Yes
S4	Partial	No	Yes	No
S5	Full	No	No	No
S6	Full	Yes	No	No
S7	Full	No	Yes	Yes
S8	Full	No	Yes	No

Solution methodology is similar to what is applied for the main model, and the optimal solutions are presented in Table 13. Note that we disregard the case where the per-unit penalty is so low that the regulation only acts as manufacturing cost. This specific case is not interesting as the regulation is not reflected in the recovery decisions. To disregard this case, it is assumed that  $\theta_A \geq \frac{1}{2\alpha t}(\rho - c_R + c_N + 2t - 1)$ . The necessary and sufficient conditions are presented in Appendix D.

**Table 13: Optimal solutions for the penalty-based mechanism**

Regions	$p_{N1}$	$p_R$	$p_{N2}$	$p_A$
S1	$\frac{1+c_N}{2}$	$\frac{\rho}{2} + \frac{\alpha t c_R + \rho}{2(\alpha t + 1)}$	$\frac{1+c_N}{2}$	$\frac{1}{\alpha t}(\rho - p_R)$
S2	$1 - \frac{1}{\delta t}(\rho - p_R)$	$\rho - \frac{\alpha \delta^2 t(\rho - c_R) + \alpha \delta t(1 - c_N)}{2\alpha \delta^2 t + 2\alpha + 2\delta^2}$	$\frac{1+c_N}{2}$	$\frac{1}{\alpha t}(\rho - p_R)$
S3	$\frac{1+c_N + \pi_A \theta_A}{2}$	$\frac{\rho + c_R}{2}$	$\frac{1+c_N}{2}$	$\frac{\theta_A}{2}$
S4	$\frac{2\pi_A^2 + \pi_A \alpha + \alpha + \alpha c_N}{2(\alpha + \pi_A^2)}$	$\frac{\rho + c_R}{2}$	$\frac{1+c_N}{2}$	$\frac{\pi_A}{\alpha}(1 - p_{N1})$
S5	$\frac{1+c_N}{2}$	$\rho + \frac{\alpha t(c_R + 1 - \rho - 2t - c_N)}{2(2\alpha t + 1)}$	$1 + \rho - t - p_R$	$\frac{1}{\alpha t}(\rho - p_R)$
S6	$1 - \frac{1}{\delta t}(\rho - p_R)$	$\rho - \frac{\alpha \delta^2 t(\rho - c_R + c_N + 2t - 1) + \alpha \delta t(1 - c_N)}{4\alpha \delta^2 t + 2\alpha + 2\delta^2}$	$1 + \rho - t - p_R$	$\frac{1}{\alpha t}(\rho - p_R)$
S7	$\frac{1+c_N + \pi_A \theta_A}{2}$	$\frac{c_R + 3\rho + 1 - 2t - c_N}{4}$	$1 + \rho - t - p_R$	$\frac{\theta_A}{2}$
S8	$\frac{2\pi_A^2 + \pi_A \alpha + \alpha + \alpha c_N}{2(\alpha + \pi_A^2)}$	$\frac{c_R + 3\rho + 1 - 2t - c_N}{4}$	$1 + \rho - t - p_R$	$\frac{\pi_A}{\alpha}(1 - p_{N1})$

In Section 3, we showed that rewarding collection and remanufacturing yields more product recovery, but has dual impacts on the environmental performance. Therefore, in this section we compare the penalty-based mechanism on the two addressed performance indicators. If the supply of used products is abundant, introducing this mechanism reduces remanufactured product price and increases first period price. The price of the brand new products may increase or stay the same. Therefore, brand new production reduction with promoted product recovery lead to less pollution. If the minimum acquisition level and the per-unit penalty are high enough, further increasing the minimum acquisition level results in less product recovery. However, if the minimum acquisition level is high enough but the per-unit penalty is relatively low, increasing the per-unit penalty means more collection and remanufacturing.

If the regulator seeks to solely promote product recovery, the minimum acquisition level has to be increased up to a level which is  $\pi_A^F = \alpha(\rho - c_R + 2t + c_N - 1)/(1 - c_N)(\alpha t + 1)$  for full market coverage and  $\pi_A^P = \alpha(\rho - c_R)/(1 - c_N)(2\alpha t + 1)$  for the partial market coverage conditions, where obviously  $\pi_A^F \geq \pi_A^P$ . Therefore, reliable information about the market coverage is important when deciding on the minimum acquisition level. At these specific levels, increasing  $\theta_A$  (per-unit penalty) up to  $(1 - c_N)/(\pi_A + \alpha)$  supports the environmental

protection, beyond which increasing the per-unit penalty has no product recovery performance but harmful for total surplus.

To conclude, assuming that the firm does not collect and remanufacture all of the acquirable products, we compare the environmental impact and product recovery performance of the performance-based mechanism with the penalty-based mechanism. BY, BX and AY are equivalent to S7, S5 and S8, respectively.

**Proposition 5** *If collection target is low enough and deviation incentive is high enough, implementing the reward-penalty mechanism could yield lower manufacturing-caused pollution and more product recovery than a penalty-based mechanism.*

**Proof.** See Appendix C. □

Proposition 5 implies that preferability of a performance-based mechanism to a penalty-based mechanism is conditional. In other words, to see if a performance-based mechanism can outperform penalty-based one, the anticipated strategic reaction of the firm should be considered. Therefore, a reward-penalty mechanism does not necessarily lead to a better environmental protection. Therefore, a penalty-based mechanism is not necessarily a dominated regulating strategy.

## 5 Conclusion

In this paper, the problem of regulation in a (re)manufacturing market is studied. Not only the general mechanism proposed in this paper allows for investigating a wide variety of current environmental regulations, but also it helps extracting guidelines about incentives and target levels to achieve the desirable results in terms environmental standards. The research is conducted from the social planner's point of view who seeks either to improve the environmental performance or to promote product recovery in the market. In this concluding section, the research questions are responded.

The impact of the target levels and incentives are also investigated in this paper. This analysis is significant as the general trend in WEEE Directives is to increase the collection target rate to achieve the main goal of the regulations, which is to protect the environment. If product recovery industry is at its early stages, it is advised to keep the collection and remanufacturing targets down while increasing the deviation incentives. The purpose of this guideline is to promote product recovery and to turn it into a well-established practice. However, if remanufacturing market is advanced and the regulator aims at reducing the negative environmental impact of production (manufacturing-caused pollution), increasing the collection target and decreasing the remanufacturing target are suggested. Furthermore, the deviation incentive on remanufacturing has negative the same impact on the environment as it has on product recovery. The effect of deviation incentive for collection on environmental performance could not be independently determined.

One of the main objectives of this paper was to have a flexible mechanism to be able to evaluate and compare a wide variety of regulations in a single framework. Three different regulations are considered for this purpose. The first one is the Buy-Back mechanism in which the firm would be penalized per unit ( $c_d$ ) not bought back from the collector. The second mechanism is Tax Subsidy which rewards remanufacturing ( $\sigma$ ). The third and last studied regulation is Carbon Emission Tax that penalizes manufacturing and remanufacturing ( $T$ ), but discounts the penalties for remanufacturing. A threshold is obtained for each of these parameters where the impact of the respective mechanism changes by surpassing the threshold. The social planner may implement one or more of these mechanisms for specific intentions. If the regulation agency wants to reduce production of the brand new products, it can implement Carbon Tax Emission or Buy-Back regulations with the per-unit penalty and emission tax values below the thresholds. It could be the case if the production is so polluting that the regulator demands immediate manufacturing reduction. In the contrary, if expanding the market is the primary intention of the regulator, the Tax Subsidy mechanism could be implemented. Ultimately, assume that the overall picture of the remanufactured products is not attractive in a society. If this is the case, familiarizing the people with the remanufactured products is one method to achieve higher willingness-to-pay for the remanufactured products, and consequently it leads to

promoting recovery in the long term. The social planner may reach his goal by expanding the market of the remanufactured products. This is possible if Tax Subsidy, Carbon Tax Emission and Buy-Back mechanisms with the per-unit penalty and emission tax values below the thresholds are implemented.

Finally, we extended the general mechanism to imitate a case which is practiced in Minnesota, US. The mechanism is penalty-based and is concentrated on used product acquisition. For this special case, we found the minimum value for the collection level to make the regulation effective. Furthermore, the best regulatory parameters to maximize product recovery performance are obtained. It is concluded that if the target level is sufficiently low and the deviation incentive is high enough, implementing the reward-penalty mechanism either outperforms or delivers the same recovery performance and environmental impact as the penalizing mechanism. Otherwise, none of these two regulatory approaches dominates the other one.

In this research, we considered one OEM who has the financial capability and technical infrastructure to remanufacture its own products. However, it may not be the case for mid-size and small companies. If this is the case, the OEM is under the threat of independent remanufacturer(s) who profits from the used products and reduces the sale of the brand new products. This scenario is worth studying not only because of the comparing the best mechanisms and policies in a competitive market, but also to see if the competition can partially replace regulations, as some research studies claimed. Moreover, the introduction of more than one player to the market raises the question of if coordination outperforms competition in an environmental sense.

## Appendix

### A Demand functions

In the first period, there is only one product in the market. The normalized demand function for the first period is

$$q_{N1} = 1 - p_{N1}.$$

The utility functions for each potential customer in the second period from new and remanufactured products are  $U_{N2} = 1 - (1-x)t - p_{N2}$  and  $U_R = \rho - xt - p_R$ , respectively, and  $x$  is the position of a consumer on the line. The indifference point is the position of the consumer whose utility from buying new and remanufactured products is equal.

$$U_{N2} = U_R \rightarrow 1 - (1-x)t - p_{N2} = \rho - xt - p_R \rightarrow x^* = \frac{\rho - 1 + p_{N2} - p_R + t}{2t}.$$

Assuming that the market coverage is full, both utility values at the indifference point must be positive, that is,

$$\begin{aligned} U_{N2} = 1 - (1-x^*)t - p_{N2} &= 1 - \left(1 - \frac{\rho - 1 + p_{N2} - p_R + t}{2t}\right)t - p_{N2} \geq 0 \Rightarrow \\ 1 - \frac{t - \rho + 1 - p_{N2} + p_R}{2} - p_{N2} &= 1 - t + \rho - p_{N2} - p_R \geq 0 \rightarrow \rho - p_R \geq p_{N2} + t - 1. \end{aligned}$$

Therefore,  $p_{N2} + p_R \leq \rho - t - 1$  is the condition for full market coverage. Under the full market coverage condition, the quantity demanded for the new product ( $q_{N2}$ ) is  $1 - x^*$  and the quantity demanded for the remanufactured product ( $q_R$ ) is  $x^*$ .

The objective function of the manufacturer has the form

$$\Pi = p_{N2}q_{N2} + p_Rq_R + h,$$

where  $h$  represents all other costs, which are either linear in  $p_{N2}$  and  $p_R$  or independent from them. Assume that the market is fully covered. The Hessian of the objective function in terms of  $p_{N2}$  and  $p_R$  is given by

$$\begin{pmatrix} \frac{\partial^2 \Pi}{\partial p_{N2}^2} & \frac{\partial^2 \Pi}{\partial p_{N2} \partial p_R} \\ \frac{\partial^2 \Pi}{\partial p_{N2} \partial p_R} & \frac{\partial^2 \Pi}{\partial p_R^2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{t} & \frac{1}{t} \\ \frac{1}{t} & -\frac{1}{t} \end{pmatrix}.$$

On the one hand, observe that the first principle component is negative while the second principle component is zero. Consequently,  $p_{N2}$  and  $p_R$  are dependent while the objective function is concave. Moreover, the off-diagonal elements are positive, concluding that the objective function is jointly increasing in  $p_{N2}$  and  $p_R$ . Therefore, in order to maximize the concave objective function, we need to jointly increase  $p_{N2}$  and  $p_R$ . On the other hand, however, we have assumed that the market is fully covered by stipulating an upper bound on the total value of prices:  $p_{N2} + p_R \leq \rho + 1 - t$ . Thus, any value for  $p_{N2}$  and  $p_R$  which result in  $p_{N2} + p_R < 1 + \rho - t$  is not optimal as we can increase one of them to achieve a higher profit. Hence, assuming full market coverage, the profit is maximized if and only if  $p_{N2} + p_R = 1 + \rho - t$ . Substituting  $p_N = 1 + \rho - t - p_R$  in  $x^*$ , which is equal to the demand for the remanufactured products, results in:

$$q_R = x^* = \frac{\rho - 1 + p_{N2} - p_R + t}{2t} = \frac{\rho - 1 + (1 + \rho - t - p_R) - p_R + t}{2t} = \frac{\rho - p_R}{t}.$$

Therefore, the demand for the new products would be:

$$q_{N2} = 1 - x^* = 1 - \frac{\rho - p_R}{t} = \frac{t + p_R - \rho}{t}.$$

From the full market coverage condition we know that  $t + p_R - \rho = 1 - p_{N2}$ . Therefore, the demand function for the new products is:

$$q_{N2} = \frac{1 - p_{N2}}{t}.$$

Therefore, full market coverage results in a situation where the demand function of each product could be represented only by its own price. However, cannibalization is embedded in the inter-dependency of price values.

## B Optimal solutions and conditions in Tables 4 and 5

To find the optimal solutions, the first step is to assume that the *primary* conditions hold (Those addressed in Table 3). After obtaining the optimal solutions under the assumed conditions, the second step is to verify the primary conditions. The optimal solutions hold if and only if the verified conditions hold.

For region BY, first-order conditions (FOCs) are applied on the unconstrained objective function. Note that assuming full market coverage, objective function (8) is concave in terms of  $p_{N1}$ ,  $p_A$  and  $p_R$ . The proof of concavity is straightforward and is not mentioned here. The optimal values are

$$p_{N1} = \frac{1 + c_N + \pi_A \theta_A}{2}, \quad (24)$$

$$p_R = \frac{3\rho - 2t + c_R - c_N + 1 - \beta\theta_R}{4}, \quad (25)$$

$$p_A = \frac{\theta_A - \pi_R \theta_R}{2}. \quad (26)$$

The primary conditions of BY obligates  $q_R < q_A$  and  $q_A < \delta q_{N1}$ . Replacing the optimal solutions leads to

$$\frac{\rho - 1 + 2t - c_R + c_N + \beta\theta_R}{2\alpha t} + \pi_R \theta_R \leq \theta_A \leq \frac{\alpha \pi_R \theta_R + \delta(1 - c_N)}{\alpha + \delta \pi_A}.$$

The reason to separate  $\theta_A$  from other parameters is to simplify recognizing the impact of one of the regulatory parameters on the feasibility regions.

Similar approach is taken to find the optimal solutions and feasibility conditions for other regions. In region BX, the OEM remanufactures all acquired products. This could be the case if the optimal solution of the unconstrained problem,  $q_A$ , falls below the lower bound  $q_R$ . In other words,

$$\frac{\rho - 1 + 2t - c_R + c_N + \beta\theta_R}{2\alpha t} + \pi_R \theta_R \leq \theta_A,$$

does not hold. In this case, since the objective function is concave, it is optimal for the OEM to activate this constraint and remanufacture all acquired products. Knowing that  $q_R = q_A \Rightarrow (\rho - p_R)/t = \alpha p_A \Rightarrow p_A =$

$(\rho - p_R)/\alpha t$ , the optimal values of  $p_R$  and  $p_{N1}$  could be obtained by applying FOCs. Therefore, the optimal values are

$$p_{N1} = \frac{1 + c_N + \pi_A \theta_A}{2}, \quad (27)$$

$$p_R = \frac{\alpha t(3\rho - 2t + 1 - c_N + c_R - \theta_A - \beta\theta_R + \pi_R\theta_R) + 2\rho}{2(2\alpha t + 1)}. \quad (28)$$

The second condition is obtained by verifying  $q_A < \delta q_{N1}$ , that is,

$$\theta_A \leq \frac{\delta(2\alpha t + 1)(1 - c_N) - \alpha(\rho + 2t - 1 + c_N - c_R + \beta\theta_R - \pi_R\theta_R)}{\alpha + \delta\pi_A(2\alpha t + 1)}.$$

Combining the two addressed conditions for region BX yields

$$\theta_A \leq \min \left\{ \frac{\delta(2\alpha t + 1)(1 - c_N) - \alpha(\rho + 2t - 1 + c_N - c_R + \beta\theta_R - \pi_R\theta_R)}{\alpha + \delta\pi_A(2\alpha t + 1)}, \right. \quad (29)$$

$$\left. \frac{\rho - 1 + 2t - c_R + c_N + \beta\theta_R}{2\alpha t} + \pi_R\theta_R \right\}. \quad (30)$$

In region AY, the OEM acquires all acquirable products but does not remanufacture all of them. This is the case if the optimal collected quantity in the unconstrained problem (BY) is greater than its upper bound  $\delta q_{N1}$ . In other words, it happens if

$$\theta_A \leq \frac{\alpha\pi_R\theta_R + \delta(1 - c_N)}{\alpha + \delta\pi_A}, \quad (31)$$

does not hold. In this scenario, as the objective function is concave, the OEM activates the upper bound constraint by putting  $p_A = \delta(1 - p_{N1})/\alpha$ . Applying the first-order conditions leads to

$$p_{N1} = 1 + \frac{c_N + \delta\pi_R\theta_R + \theta_A(\pi_A - \delta) - 1}{2(1 + \delta^2/\alpha)}, \quad (32)$$

$$p_R = \frac{3\rho - 2t + c_R - c_N + 1 - \beta\theta_R}{4}. \quad (33)$$

The only conditions remained to verify is  $q_R < q_A$ , which yields

$$\theta_A \geq \frac{(1 + \delta^2/\alpha)(\rho + \beta\theta_R + c_N - c_R + 2t - 1)}{2\delta t(\delta - \pi_A)} + \frac{c_N - 1 + \delta\pi_R\theta_R}{\delta - \pi_A}. \quad (34)$$

Together with the previous conditions, we conclude that the necessary and sufficient conditions to acquire all acquirable products but remanufacturing part of them are

$$\theta_A \geq \max \left\{ \frac{\alpha\pi_R\theta_R + \delta(1 - c_N)}{\alpha + \delta\pi_A}, \frac{(1 + \delta^2/\alpha)(\rho + \beta\theta_R + c_N - c_R + 2t - 1)}{2\delta t(\delta - \pi_A)} + \frac{c_N - 1 + \delta\pi_R\theta_R}{\delta - \pi_A} \right\}$$

To find the solutions of the AX region, the FOCs should be found in regards to a single variable. It could be the case if (i) both conditions of the unconstrained problem are not satisfied, (ii) the OEM activates the upper bound by setting  $q_A = \delta q_{N1}$  but the lower bound condition is not verified, and (iii) the OEM activates the lower bound by setting  $q_R = q_A$  but the upper bound condition is not verified. Therefore, whatever conditions that could not be placed into the three addressed sets of conditions for BY, BX and AY would be automatically associated with the AX region.

## C Proofs of Propositions 1 to 5

**Proof of Propositions 1 to 4.** For the BY, BX and AY regions, and for the collection-based and the remanufacturing-based cases, we define  $\Delta EI$  and  $\Delta PRP$  values as the difference between the  $EI$  and  $PRP$  values of regulated and unregulated markets.

$$\Delta EI = EI_{Regulated} - EI_{Unregulated}.$$

$$\Delta PRP = PRP_{Regulated} - PRP_{Unregulated}.$$

Therefore, for example, if  $\Delta EI$  is decreasing in  $\theta_A$ , that means  $EI_{Regulated}$  is decreasing in  $\theta_A$ . In a similar manner, increasing and decreasing of  $\Delta PRP$  is equivalent to increasing and decreasing of  $PRP_{Regulated}$ , respectively. The reason to consider the differences rather than the original values is two-fold. Firstly, if the sign of the difference value is determined, it is also bearing a meaning; and the meaning is that implementing the performance-based mechanism either outperforms or underperforms an unregulated market. If the sign cannot be determined exclusively, the unregulated market is ignored and the conclusions are made assuming that the regulation is already in place. Secondly, it illustrates the impact of the regulatory parameters on the outcome. Finally, note that the deviation incentive values are assumed to be positive in this analysis. Denote  $\psi_R = \pi_R \theta_R$ ,  $\phi_R = \beta \theta_R$  and  $\psi_A = \pi_A \theta_A$ . The required parametric conditions for each case are presented in the table below:

**Table 14: Required parametric conditions for target-based analysis**

Cases	Conditions
Collection-based	$\psi_R = 0, \phi_R = 0$
Remanufacturing-based	$\theta_A = 0$

Note that for the sake of brevity, we shortened some of the equations and replaced the cut-off parts with constant values  $C$ . These values do not include the regulator parameters  $\theta$ ,  $\beta$  and  $\pi$ . For each equation, we put  $C_1$  and  $C_2$  to denote different values. However, the values of  $C_1$  and  $C_2$  in one equation are not equal to the values of  $C_1$  and  $C_2$  in another equation. Furthermore,  $C_1$  is a value which can be positive or negative. However, it could be easily shown that the performance of the market in the partial-remanufacturing regions is better than the full-remanufacturing cases. First assume that the only difference comes from the value of  $\theta_A$  (See Table 3). This assumption is put as we need to be able to compare the *same markets* under minimal different regulations. The firm uses the opportunity to remanufacture a certain quantity of products. If the addressed quantity is greater than what she has already acquired, that means she has to stick with full-remanufacturing strategy. In other words, the collected and remanufactured quantities are less than those of the partial remanufacturing case. Therefore, the quantities of the collected and remanufactured products are higher, and the quantity of the first period production is lower for the partial-remanufacturing cases. Thus, the environmental impact and product recovery performance in BY and AY are better than those of BX and AX, respectively. In conclusion, and for the sake of emphasis, from positivity of  $\Delta PRP$  and negativity of  $\Delta EI$  in BX and AX, it could be concluded that a regulated market always outperforms an unregulated market.

**Calculations for environmental performance:** We start by presenting the results for **partial-remanufacturing partial-collection (BY)** case:

*Collection-based:*

$$\Delta EI = C_1 - \frac{\theta_A(\alpha(1 - \omega) + \pi_A t)}{2t},$$

where  $C_1$  is a constant. Since  $\alpha((1 - \omega) + \pi_A t)/2t$  is always positive,  $\Delta EI$  is increasing in  $\pi_A$  and  $\theta_A$ .

*Remanufacturing-based:* Since  $\rho - 1 + c_N - c_R + 2t > 0$ , region BY is not feasible (See Table 5).

For **full-remanufacturing partial-collection (BX)** case, we have the following solutions:

*Collection-based:*

$$\Delta EI = -\frac{\psi_A + 2\alpha t(\theta_A + \psi_A)}{2\alpha t + 1}.$$

Therefore,  $\Delta EI$  is decreasing in deviation incentive and target level of collection. Negativity of  $\Delta EI$  means the regulated market has better environmental impact of the unregulated market.

*Remanufacturing-based:*

$$\Delta EI = -\frac{\alpha \theta_R(\beta - \pi_R)}{2\alpha t + 1}.$$

Since  $\beta = 1$  is required to assess target level for remanufacturing,  $\Delta EI$  is decreasing in  $\theta_R$  but increasing in  $\pi_R$ . Negativity of  $\Delta EI$  means the regulated market has better environmental impact of the unregulated market.

Finally, we calculate  $\Delta EI$  for **AY** region wherein all of the acquirable products are being acquired but only a proportion of them are remanufactured. To see if a regulated market outperforms an unregulated market for partial-remanufacturing full-collection strategy, the performance indicators for regulated market under AX are also considered (but not mentioned here for the sake of brevity).

*Collection-based:*

$$\Delta EI = \frac{2\alpha^2 t^2 \delta \theta_A (\delta - \pi_A) (\delta(1 - \delta^2)(2 + t) + 1 - \delta(1 - \omega)) + C_1}{C_2},$$

where  $C_1$  and  $C_2$  are constants and  $C_2$  is positive. Since  $\delta > \pi_A$  and  $\delta(1 - \delta^2)(2 + t) + 1 - \delta(1 - \omega) > 0$ , the production-caused pollution increases by the deviation incentive but decreases by the target level. For the AX region and assuming  $\delta \geq 1/2$ ,  $\Delta EI$  is always negative; it increases in the deviation incentive but decreases in the target level. Therefore, in this case, the regulated market outperforms an unregulated market.

*Remanufacturing-based:* According to Table 5, AY is not feasible for the remanufacturing-based case. Since collection is not addressed, the firm collects only to remanufacture.

Thus, for all cases, the regulated market outperforms the unregulated market in terms of environmental performance.

**Calculations for product recovery performance:** Now, the same calculations need to be done for the product recovery performance  $\Delta PRP$ . We start by **partial-remanufacturing partial-collection (BY)** case.

*Collection-based:*

$$\Delta PRP = \frac{2\gamma\alpha t\theta_A + C_1}{C_2},$$

where  $C_1$  and  $C_2$  are constants and  $C_2$  is positive. Therefore, increasing the collection deviation incentive increases product recovery while the collection target plays no role.

*Remanufacturing-based:* As stated before, BY does not contain the optimum value for the remanufacturing-based case.

For the **full-remanufacturing partial-collection (BX)** we have:

*Collection-based:*

$$\Delta PRP = \frac{\alpha\theta_A}{2(\alpha t + 1)}.$$

Apparently, the deviation incentive for collection increases product recovery but the collection target has no impact on it.

*Remanufacturing-based:*

$$\Delta PRP = \frac{\alpha\theta_R(\beta - \pi_R)}{2(2\alpha t + 1)}.$$

Since  $\beta > \pi_R$ , product recovery performance improves by increasing  $\theta_R$ , but deteriorates by higher value of  $\pi_R$ .

Since in all of the cases  $\Delta PRP$  is positive for BX, it could be concluded that a regulated market outperforms an unregulated market in terms of total surplus.

Finally, for the **partial-remanufacturing full-collection (AY)** we have:

*Collection-based:*

$$\Delta PRP = \frac{2\alpha t \delta \gamma \theta_A (\delta - \pi_A) + C_1}{C_2},$$

where  $C_1$  and  $C_2$  are constants and  $C_2$  is positive. Therefore, product recovery increases by  $\theta_A$  but decreases by  $\pi_A$ .  $\Delta PRP_{AX}$  is always positive, concluding that if this is the case, the regulated market outperforms an unregulated market.

*Remanufacturing-based:* As stated before, if only the remanufacturing is addressed, the firm collects only to remanufacture. Therefore, partial remanufacturing is not the optimum strategy.  $\Delta PRP_{AX}$  is always positive, concluding that if this is the case, the regulated market outperforms an unregulated market.

Thus, implementing the regulation improves the product recovery performance.  $\square$

**Proof of Proposition 5.** The optimal solutions in the regions S5, S7 and S8 are required to be compared to the optimal solutions in the regions BX, BY and AY, respectively.

**Comparing BX and S5:** Define  $\Delta EI = EI_{BX} - EI_{S5}$  and  $\Delta PRP = PRP_{BX} - PRP_{S5}$ . Therefore, if  $\Delta EI$  is positive, it means the penalty-based mechanism performs better environmentally; and if  $\Delta PRP$  is positive, that means the award-penalty mechanism outperforms the penalty-based mechanism in terms of product recovery. Substituting the optimal values, we obtain  $\Delta PRP = 0$  and  $\Delta EI = -\psi_A/2$ , concluding that the incentive mechanism performs better environmentally and yield the same outcome of product recovery.

**Comparing BY and S7:** Substituting the optimal values, we obtain  $\Delta PRP = 0$  and  $\Delta EI = 0$ . Thus, for the partial collection and remanufacturing case, both mechanisms have the same performance.

**Comparing AY and S8:** Substituting the optimal values, we obtain:

$$\Delta PRP = C_1 (\delta(\alpha + \pi_A^2)(1 - c_N + \theta_A(\delta - \pi_A) + \pi_A(\alpha + \delta^2)(\pi_A + c_N - 1)),$$

where  $C_1$  is a positive constant. The value of  $\Delta PRP$  could be positive or negative. However, sufficient condition to have positive is that the collection rate is high enough to ensure  $\pi_A \geq 1 - c_N$ . For the environmental performance, we calculate:

$$\Delta EI = C_1 ((\pi_A^2 - \delta\gamma(\alpha + \pi_A^2))(1 - c_N + \theta_A(\delta - \pi_A)) + (1 - \gamma\pi_A)(\alpha\pi_A + \delta^2(c_N + \pi_A - 1)) + \alpha(\pi_A + \theta_A(\delta - \pi_A) - \gamma\pi_A c_N)),$$

where  $C_1$  is a positive constant. The value of  $\Delta EI$  could be positive or negative. However, decreasing  $\pi_A$  and increasing  $\theta_A$  increase the chance that  $\Delta EI$  is negative, concluding that under some conditions, the incentive mechanism could perform better than the penalty-based mechanism.  $\square$

## D Optimal solutions and conditions for the penalizing mechanism (Table 13)

This appendix is devoted to find the optimal solutions and the necessary and sufficient conditions for scenarios 1 to 8 in the extended model. For each scenario, first we solve the unconstrained problem under the set of conditions associated with that scenario (sub-problem). Second, those conditions are verified with the optimum values found in the first step. The third step is to figure out the conditions whom violation results in moving from one sub-problem to another. Next, the necessary and sufficient conditions for each sub-problem are obtained. Finally, inclusion and exclusion of the sets of conditions are checked to verify the results and the associated region for each sub-problem. Before proceeding to the proofs, note that the monopoly price for the new products is  $\frac{1+c_N}{2}$  and for the remanufactured products is  $\frac{\rho+c_R}{2}$ . Since at the monopoly price exactly the whole market should be covered ( $q_N, q_R \leq 1$ ),  $c_N + 2t - 1 \geq 0$  and  $c_R + 2t - \rho \geq 0$  must hold true. Otherwise, the OEM obtains nothing by decreasing the price to the monopoly price level.

Assume that the market coverage is partial, supply of used products is not restricting and the regulation is no enforcing (S1). The OEM maximizes

$$\Pi = (1 - p_{N1})(p_{N1} - c_N) + \frac{1}{t}(1 - p_{N2})(p_{N2} - c_N) + \frac{1}{t}(\rho - p_R)(p_R - c_R - p_A).$$

As the OEM is not enforced by the regulations, she remanufactures all of the acquired products, therefore  $p_A = \frac{\rho - p_R}{\alpha t}$ . Using the first-order conditions we have:

$$p_{N2}^* = p_{N1}^* = \frac{1 + c_N}{2}, \quad (D1)$$

$$p_R^* = \frac{\rho}{2} + \frac{\alpha t c_R + 1\rho}{2(\alpha t + 1)}. \quad (D2)$$

Now the three addressed conditions should be checked. First, the market coverage needs to be partial  $p_{N2} + t - 1 \geq \rho - p_R$ , that is,

$$\frac{\alpha t(\rho - c_R)}{2(\alpha t + 1)} \leq \frac{1 + c_N}{2} + t - 1 \iff (\alpha t + 1)(c_N + 2t - 1) \geq \alpha t(\rho - c_R). \quad (D3)$$

Next, the supply limitation constraint redundancy should be checked  $q_R \leq \delta q_{N1}$ , that is,

$$\frac{\alpha(\rho - c_R)}{2(\alpha t + 1)} \leq \frac{\delta(1 - c_N)}{2} \iff \delta(1 - c_N)(\alpha t + 1) \geq \alpha(\rho - c_R). \quad (D4)$$

The final condition is that the regulation should not be enforcing  $q_R \leq \pi_A q_{N1}$ , that is,

$$\frac{\alpha(\rho - c_R)}{2(\alpha t + 1)} \geq \frac{\pi_A(1 - c_N)}{2} \iff \pi_A(1 - c_N)(\alpha t + 1) \leq \alpha(\rho - c_R). \quad (D5)$$

Now assume that constraint (D4) does not hold while (D3) holds (S2). Since the model is concave in terms of  $p_R$ , it is optimal for the OEM to remanufacture as many as possible  $\delta q_{N1} = q_R$ , therefore  $p_{N1} = 1 - \frac{\rho - p_R}{\delta t}$ .  $p_A = \frac{\rho - p_R}{\alpha t}$  is still holding because the regulation is assumed to be non-enforcing. Using the first-order conditions, we obtain:

$$p_{N2}^* = \frac{1 + c_N}{2}, \quad (D6)$$

$$p_R^* = \rho - \frac{\alpha \delta^2 t(\rho - c_R) + \alpha \delta t(1 - c_N)}{2\alpha \delta^2 t + 2\alpha + 2\delta^2}. \quad (D7)$$

Since  $\delta \geq \pi_A$  by definition, the only condition to check is partial market coverage condition  $\rho - p_R \leq p_{N2} + t - 1$ :

$$\frac{\alpha \delta^2 t(\rho - c_R) + \alpha \delta t(1 - c_N)}{2\alpha \delta^2 t + 2\alpha + 2\delta^2} \leq \frac{1 + c_N}{2} + t - 1 \iff (\delta^2 \alpha t + \alpha + \delta^2)(c_N + 2t - 1) \geq \delta \alpha t(1 - c_N + \delta(\rho - c_R)). \quad (D8)$$

Therefore, if (D3) and (D8) holds but (D4) does not hold, the market coverage is partial, supply constraint is restricting and the regulation is not enforcing.

Suppose that (D3) holds but (D5) does not (S3). Therefore if the OEM want to set the prices as of S1, she has to pay penalty. First we introduce the penalty term to the objective function. Thus, the OEM maximizes:

$$\Pi = (1 - p_{N1})(p_{N1} - c_N) + \frac{1}{t}(1 - p_{N2})(p_{N2} - c_N) + \frac{1}{t}(\rho - p_R)(p_R - c_R) - q_A p_A - \theta(\pi_A q_{N1} - q_A).$$

Since the OEM is forced to acquire more but remanufacturing more may not be desirable,  $q_A \geq q_R$ . The new objective function is concave in terms of  $p_R$ ,  $p_{N2}$ ,  $p_{N1}$  and  $p_A$ . Applying the first-order conditions yields:

$$p_{N2}^* = \frac{1 + c_N}{2}, \quad (D9)$$

$$p_R^* = \frac{\rho + c_R}{2}, \quad (D10)$$

$$p_{N1}^* = \frac{1 + c_N + \pi_A \theta}{2}, \quad (D11)$$

$$p_A^* = \frac{\theta}{2}. \quad (\text{D12})$$

By adding the unconditional penalty term to the objective function we have also assumed that the OEM pays penalty, therefore  $\pi_A q_{N1} > q_A$ , that is,

$$\pi_A(1 - c_N - \pi_A \theta) > \alpha \theta. \quad (\text{D13})$$

Supply limitation constraint is obviously redundant. Applying the partial market coverage results in

$$\rho - p_R \leq c_N + 2t - 1. \quad (\text{D14})$$

Note that if (D14) holds, holding (D3) is guaranteed. Therefore (D3)  $\subset$  (D14). Thus, if (D3), (D14) and (D13) hold (which could be reduced to (D14)  $\cap$  (D13)) but (D5) does not hold, market coverage is partial, supply constraint is redundant and the regulation enforces the OEM to pay penalty. Note that, if  $\theta \geq \sqrt{\frac{1}{\alpha t}(\rho - c_R + 2t + c_N - 1)}$  does not hold, then  $q_A < q_R$ . So we have assumed that if the social planner enforces the regulation, he sets the per-unit penalty high enough.

In calculating the optimum values and obtaining conditions in S3, we assumed that the OEM pays penalty  $\pi_A q_{N1} > q_A$ . If introducing the penalty term forces the OEM to avoid paying the penalty, the solutions of S3 must violate (D13), concluding that the optimum point for the acquisition quantity is  $\pi_A q_{N1}$  (S4). Therefore, if all of the S3 conditions hold but (D13) is violated, it could be concluded that  $q_A = \pi_A q_{N1}$ :

$$p_{N2}^* = \frac{1 + c_N}{2}, \quad (\text{D15})$$

$$p_A^* = \frac{\pi_A(1 - p_{N1})}{\alpha}, \quad (\text{D16})$$

$$p_{N1}^* = \frac{2\pi_A^2 + \pi_A \alpha + \alpha + \alpha c_N}{2(\pi_A^2 + \alpha)}, \quad (\text{D17})$$

$$p_R^* = \frac{\rho + c_R}{2}. \quad (\text{D18})$$

Thus, if (D14) holds but (D5) and (D13) do not hold, then market coverage is partial, supply constraint is redundant, the regulation is enforcing but the OEM avoids paying penalty by acquiring as many as required by law.

Suppose that none of the partial coverage conditions hold. Thus, the only remaining scenario is full market coverage  $p_{N2} + t - 1 = \rho - p_R$ . In order to find the general conditions of full market coverage, we need to find the whole region which is not covered by S1, S2, S3 and S4.

- In the set of conditions for S1, if (D3) does not hold, the market coverage cannot be partial.
- In the set of conditions for S2, if neither (D4) nor (D8) holds, the market coverage cannot be partial.
- In the set of conditions for S3 and S4, if neither (D5) nor (D14) holds but (D3) does, the market coverage cannot be partial.

Therefore, the set of conditions that take us from partial to full market coverage is

$$(\text{D3})' \cup ((\text{D3}) \cap (\text{D4})' \cap (\text{D8})') \cup ((\text{D3}) \cap (\text{D5})' \cap (\text{D14})').$$

Now that the market coverage is full  $\rho - p_R = p_{N2} + 2t - 1$ , supply constraint and penalty terms are to be investigated. Assume that the supply constraint is redundant and the firm does not pay penalty, which means  $q_A = q_R$ . Therefore, the OEM maximizes

$$\Pi = (1 - p_{N1})(p_{N1} - c_N) + \frac{1}{t}(2t + p_R - \rho)(1 + \rho - 2t - p_R - c_N) + \frac{1}{t}(\rho - p_R)(p_R - c_R) - q_A p_A.$$

Applying the first-order conditions on this concave function yields

$$p_{N1}^* = \frac{1 + c_N}{2}, \quad (\text{D19})$$

$$p_R^* = \rho + \frac{\alpha t(c_R + 1 - \rho - 2t - c_N)}{2(2\alpha t + 1)}. \quad (\text{D20})$$

Next, the assumed conditions should be verified. Note that the following optimum value for  $p_A$  is obtained using the non-enforcing legislation condition:

$$p_A^* = \frac{(\rho - p_R)}{\alpha t}. \quad (\text{D21})$$

Redundant supply constraint means

$$q_R \leq \delta q_{N1} \Rightarrow \frac{\alpha(c_N + 2t - 1 + \rho - c_R)}{2(2\alpha t + 1)} \leq \delta \frac{1 - c_N}{2} \iff \alpha(c_N + 2t - 1 + \rho - c_R) \leq \delta(1 - c_N)(2\alpha t + 1). \quad (\text{D22})$$

In the same way, the OEM is assumed to avoid penalty  $q_R \geq \pi_A q_{N1}$ :

$$\alpha(c_N + 2t - 1 + \rho - c_R) \geq \pi_A(1 - c_N)(2\alpha t + 1). \quad (\text{D23})$$

Thus, if the market coverage is full and (D22) and (D23) hold, the supply constraint is redundant and the OEM is not enforced by minimum acquisition law.

In S6, it is assumed that the market coverage is full, the supply constraint is restricting and therefore the OEM is not enforced by the minimum acquisition legislation as  $\pi_A \leq \delta$  by definition. The OEM maximizes:

$$\Pi = (1 - p_{N1})(p_{N1} - c_N) + \frac{1}{t}(2t + p_R - \rho)(1 + \rho - 2t - p_R - c_N) + \frac{1}{t}(\rho - p_R)(p_R - c_R) - q_A p_A,$$

where  $q_R = q_A$ . Since the objective function is concave in  $q_R$ , the optimum point is at the supply constraint boundary  $q_R = \delta q_{N1}$ . Thus, the objective function only becomes a concave function of  $p_R$ . Applying the first-order conditions yields:

$$p_R^* = \rho - \frac{\alpha \delta^2 t(\rho - c_R + c_N + 2t - 1) + \alpha \delta t(1 - c_N)}{4\alpha \delta^2 t + 2\alpha + 2\delta^2}.$$

The optimum values for  $p_{N2}$ ,  $p_{N1}$  and  $p_A$  could be obtained by full market coverage, restricting supply constraint and non-enforcing legislation conditions, respectively.

Suppose that the conditions for full market coverage hold but the non-enforcing legislation condition (D23) does not hold (S7). Therefore, two possible scenarios could be taken into account: the OEM pays the penalty or she acquires as many as required by law. Assume that the former is the case. Similar to S5, the objective function needs to be revised to:

$$\Pi = (1 - p_{N1})(p_{N1} - c_N) + \frac{1}{t}(1 - p_{N2})(p_{N2} - c_N) + \frac{1}{t}(\rho - p_R)(p_R - c_R) - q_A p_A - \theta(\pi_A q_{N1} - q_A),$$

where  $p_{N2} = 1 + \rho - 2t - p_R$ . The objective function above is a concave and differentiable function of  $p_{N1}$ ,  $p_A$  and  $p_R$  whose optimum values could be found by the FOCs:

$$p_{N1}^* = \frac{1 + c_N + \pi_A \theta}{2}, \quad (\text{D24})$$

$$p_A^* = \frac{\theta}{2}, \quad (\text{D25})$$

$$p_R^* = \frac{c_R + 3\rho + 1 - 2t - c_N}{4}. \quad (\text{D26})$$

The next step is to see if the enforcing legislation condition  $\pi_A q_{N1} \geq q_A$  holds.

$$(1 - c_N - \pi_A \theta) \geq \alpha \theta,$$

which is called constraint (D13). Thus, in a market with full market coverage where (D23) does not hold but (D13) does, the OEM prefers to pay penalty.

The only possible remaining scenario is where the market coverage is full and the OEM acquires just as many as required by law (S8). It obviously is the case where neither (D13) nor (D23) holds.

$$\pi_A q_{N1} = q_A \Rightarrow p_A = \frac{\pi_A(1 - p_{N1})}{\alpha}.$$

The optimum values of  $p_{N2}$  and  $p_R$  are the same as of S7. Substituting  $p_A = \frac{\pi_A(1-p_{N1})}{\alpha}$  in the objective function, it becomes a concave function of  $p_{N1}$  whose optimum value is obtained using FOCs:

$$p_{N1}^* = \frac{2\pi_A^2 + \pi_A\alpha + \alpha + \alpha c_N}{2(\alpha + \pi_A^2)}.$$

The condition for full market coverage is  $(D3)' \cup ((D3) \cap (D4)' \cap (D8)') \cup ((D3) \cap (D5)' \cap (D14)')$  which does not have overlap with any of the partial market coverage regions. S1 and S2 are exclusive in (D4). S3 and S4 are exclusive in (D13). S1 is completely separated from S3 and S4 by (D5). Finally, since  $(D4)' \subset (D5)$ ,  $(D5)' \subset (D4)$  and consequently  $(D5)' \cap (D4)' = \emptyset$ , S2 does not have overlap with S3 and S4. For the sub-problems with full market coverage, S5 is differentiated from S6 by ((D22)). S7 and S8 are exclusive in (D13). S5 does not have overlap with S7 and S8 because of (D23). Finally, since  $(D22)' \subset (D23)$ ,  $(D23)' \subset (D22)$  and consequently  $(D22)' \cap (D23)' = \emptyset$ , S6 does not have overlap with S7 and S8. Thus, each combination of the model parameters implies only one sub-problem.

To show inclusion (i.e. each combination of the model parameters at least refers to one sub-problem), we need to show that the union of all regions will be the universal set. Remember that  $(D14) \subset (D3)$ ,  $(D22)' \subset (D23)$ ,  $(D23)' \subset (D22)$ ,  $(D4)' \subset (D5)$  and  $(D5)' \subset (D4)$ . The union of the partial market coverage regions is:

$$\begin{aligned} & ((D3) \cap (D4) \cap (D5)) \cup ((D3) \cap (D4)' \cap (D8)) \cup ((D5)' \cap (D14) \cap (D13)) \cup ((D5)' \cap (D14) \cap (D13)') = \\ & ((D3) \cap (D4) \cap (D5)) \cup ((D3) \cap (D4)' \cap (D8)) \cup ((D5)' \cap (D14)) = ((D3) \cap (D4)' \cap (D8)) \cup \\ & (((D3) \cup (D5)') \cap ((D3) \cup (D14)) \cap ((D4) \cup (D5)') \cap ((D4) \cup (D14)) \cap ((D5) \cup (D5)') \cap ((D5) \cup (D14))) = \\ & ((D3) \cap (D4)' \cap (D8)) \cup (((D3) \cup (D5)') \cap (D3) \cap (D4) \cap ((D4) \cup (D14)) \cap ((D5) \cup (D14))) = \\ & ((D3) \cap (D4)' \cap (D8)) \cup ((D3) \cap (D4) \cap ((D5) \cup (D14))). \end{aligned}$$

Since  $(D22)' \subset (D23)$  and  $(D23)' \subset (D22)$  and  $(D22) \cup (D23) = S$ , the union of the full market coverage regions is reduced to set A:

$$(D3)' \cup ((D3) \cap (D4)' \cap (D8)') \cup ((D3) \cap (D5)' \cap (D14)').$$

The union of full and market coverage regions is simplified to:

$$\begin{aligned} & (((D3)' \cup (D4)) \cap ((D3)' \cup (D5) \cup (D14))) \cup (((D3) \cap (D4)') \cup ((D3) \cap (D5)' \cap (D14)')) = \\ & ((D3) \cap (D4)') \cup ((D3) \cap (D5)' \cup (D14)') \cup ((D3)' \cup (D5) \cup (D14)) = S. \end{aligned}$$

Thus, every combination of model parameters will be included in at least one region. This result (inclusion) together with the exclusion feature proves that each combination of the model parameters (each feasible point) refers to one and only one sub-problem. Thus, the conditions (regions) presented in Table 15 are necessary and sufficient.

**Table 15: Sub-problems and conditions for the penalizing mechanism model**

Subproblems	Conditions	Market Coverage	Supply Limitation	Regulation Enforcing	Paying Penalty
S1	$(D3) \cap (D4) \cap (D5)$	Partial	No	No	No
S2	$(D3) \cap (D4)' \cap (D8)$	Partial	Yes	No	No
S3	$(D5)' \cap (D14) \cap (D13)$	Partial	No	Yes	Yes
S4	$(D5)' \cap (D14) \cap (D13)'$	Partial	No	Yes	No
S5	$A \cap (D22) \cap (D23)$	Full	No	No	No
S6	$A \cap (D22)'$	Full	Yes	No	No
S7	$A \cap (D23)' \cap (D13)$	Full	No	Yes	Yes
S8	$A \cap (D23)' \cap (D13)'$	Full	No	Yes	No

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