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A Dynamic-material-value-based decomposition method for mineral supply chain optimization

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Abstract: A decomposition method is developed to optimize a mineral value chain composed of a one or multiple mines and a material flow circuit. In the proposed decomposition method, the mine production schedule for each mine and the material flow plan for the material circuit are optimized through separated models and a mechanism is developed to coordination all models to maximize the expected NPV of the entire mineral value chain. The proposed approach is tested through a practical-scale case study and the test results show that the proposed method can effectively coordinate the optimizations of the mine production scheduling model and the material flow planning model. The proposed method can also be applied to optimize a mineral value chain with more complex structures and more details.

Keywords: Mine production scheduling, mineral value chain, market uncertainty, decomposition method

1 Introduction

A typical mineral value chain (MVC) consists of one or multiple mines (open-pit and underground) and a material circuit as shown in Figure 1 (Goodfellow 2014). The material flow circuit includes waste dumps, material stockpiles and a complex value-added material processing system that includes a series of transformation processes generating commodities from raw materials. Figure 1 shows the material flow chart of a typical MVC.

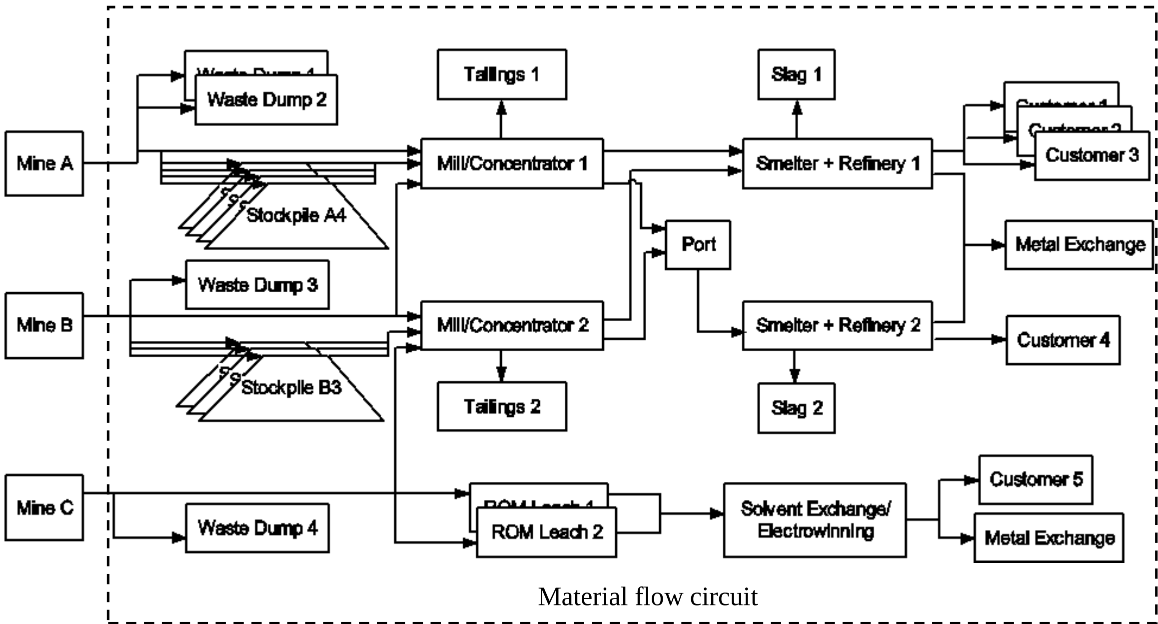


Figure 1. The material flow chart of a typical MVC.

The optimization problems in mining industry usually include the mine production scheduling problem (MPSP) and the material flow planning problem (MFPP). For each planning period, a MPSP for a mine optimizes which areas to mine, while a MFPP optimizes the volume of material flow between each pair of connected nodes in the material flow chart of the entire MVC. A multi-mine MVC (as shown in Figure 1) include multiple MPSPs and a single MFPP.

In an MPSP, the objective is usually maximizing the expected net present value (NPV) of the valuable material mined during the planning horizon subject to the mining precedence constraints and the mining capacity constraints. A feature of an MPSP is the large number of binary variables. In mining engineering, a mine deposit is usually modeled by a large number of “blocks”, and the extraction of a block is a “job” in a mine production scheduling problem. The number of binary variables used to model a real-world mine production scheduling problem can reach hundreds of thousands or even millions. The research of MPSPs focuses on designing algorithms to search the near optimal solution for the problem with a large number of blocks. A review of research on MPSPs can be found in Hochbaum and Chen (2000) and Newman et al. (2010). When the uncertainty of the characters of each block is considered, an MPSP becomes even more complex. A review of research on MPSPs with uncertainty can be found in Lamghari and Dimitrakopoulos (2012), Lamghari et al. (2014) and Ramazan and Dimitrakopoulos (2013).

In a MFPP, the production schedules of mines are often treated as fixed. The objective of a MFPP is usually maximizing the expected NPV given the fixed mine production schedules of all mines subject to the transport capacity of each arc and the processing capacity of each node in the material flow chart of the MVC. The complexity of a MFPP depends on the structure of the MVC and the detailed operations considered. The structure of an MVC is determined the material flow circuit, and can be expanded when other features, such as outsourcing, closed-loop processing, etc., are considered. The details include the nonlinear relationship between input, output and energy consumed at each node, the resource (e.g., labors, equipment, facilities, etc.) allocations, the setup costs, the operational risks, and so on. When market uncertainty is considered, a MFPP becomes even more complex.

The complexities described above make it difficult to integrate MPSP and MFPP in a single big constrained model and search the first-best solution for the entire MVC. Hence, a dynamic-material-value-based decomposition (DMVBD) method is developed herein to allow the MPSPs and the MFPP in an MVC to be solved separately and synchronized at the end so that the final solution for each individual problem is close to its first-best solution for the entire MVC.

In the literature on the MVC optimization, Souza et al. (2010) study a MPSP in consideration of the truck allocation downstream. Hoerger et al. (1999) formulate a model of optimizing an MVC for a gold-mining case with multiple mines, stockpiles and processing facilities. They consider 50 mine sources of which the mine production schedules are assumed to be fixed. Thus, the MVC problem they study is equivalent to the MFPP defined in the present work. Caccetta, & Hill (2003) optimize a MVC with the constraints of mill throughputs, mining capacities, blending requirements, stockpiles and transport system using branch and cut algorithm. Epstein et al. (2012) developed an algorithm that iteratively adds cuts to tighten the linear relaxation of a unified MVC optimization model so that the binary variables in the original model have binary solutions after solving the relaxed model. In order to solve the relaxed model efficiently, the blocks are aggregated to benches or columns, and the downstream MFPP has to be linear. Asad and Dimitrakopoulos (2013) optimize the cut-off grade cutoff grade for MVCs with multiple processing streams in consideration of the uncertainty in material grades. Kizilkale and Dimitrakopoulos (2014) optimize the mining rate of multiple mines inconsideration of the uncertainty of commodity prices. Montiel and Dimitrakopoulos (2015) studied the optimization of a MVC with multiple processing and transportation options.

As cited above, the existing work on MVC optimization in the literature focusing on formulating an MVC with a unified big model so that the upstream scheduling and the downstream planning are optimized at the same time. The drawback of this approach is that the solution method is difficult to be extended for more complex MVC structures, e.g., multiple mines, multiple processing plants, etc., or more detailed operations, e.g., non-linear material transformations, etc. The advantages of the DMVBD method developed herein can be summarized as follows:

First, it is easy to employ the DMVBD method with the existing optimizers designed for solving individual MPSPs and MFPPs. In order to implement the DMVBD method, only the input of MPSP model required to be changed, and so the users do not need to upgrade their MPSP optimizer for the adoption of the DMVBD method. The MFPP model requires the minimum-level modification, but it does not increase the complexity essentially.

Second, it is easy to upgrade the MPSP models and the MFPP model for an MVC after adopting the DMVBD method. Because the MPSPs and the MFPP for an MVC stay independent after the DMVBD method is employed, each model can be upgraded without considering the others. For example, if a user wants to integrate the operational risks for some devices in the MFPP model, there no need to change the MPSP models, but as the function of DMVBD method, the final solutions of the MPSP models will change accordingly.

Finally, with the DMVBD method, it is easy to insert new MPSP models when new mines are added to the MVC. When new MPSP models are added, the DMVBD method will automatically change the optimal solution of other models to account for the change of the MVC structure.

Our sections are organized as follows. In Section 2, a MPSP model and a MFPP model for an example MVC, through which we demonstrate the DMVBD method, are formulated. In Section 3, the DMVBD method is presented. To test the efficiency of the DMVBD method, an MPSP algorithm is developed Section 3.1. A numerical test designed based on a practical-scale case is conducted in Section 4. Finally, a summary of conclusion is presented in Section 5.

2 Models

The method proposed in this work optimizes an MVC that includes one or multiple mines and a material flow circuit as show in Figure 1. Because in our MVC optimization, the mine production schedule (MPS) for each mine and the material flow plan (MFP) for the material flow circuit are optimized using separated models, one or multiple MPS models and a MFP model will be formulated to optimize the MVC.

2.1 The MPS model for each mine

The MPS model for each mine varies because of different mine deposits and production constraints vary from mine to mine. In this section, we focus on a general form of a MPS model, and ad-hoc extensions can be easily made to account for particular settings.

The MPS model optimizes the extraction schedule of a stochastic orebody model. The uncertainty of a mine is represented by a number of orebody scenarios, each of which contains a set of simulated random parameters representing the random characteristics of the deposit. It is a common practice in the literature of mine production scheduling to use simulated scenarios to describe the uncertainties in material tonnage, metal content and so on. In the present work, an orebody model is composed of a number of blocks, and each block represents an undividable unit for production scheduling. The objective of the MPS model is to find out when to extract each block so that the contained valuable elements can generate the maximum expected NPV subject to precedence constraints and capacity constraints. Our MPS model is formulated using the notation listed below.

General	
$T \in \boldsymbol{Z}^+$	The planning horizon.;
$S \in \boldsymbol{Z}^+$	The number of scenarios representing the uncertainty of the mine.
$I \in \boldsymbol{Z}^+$	The number of blocks considered for scheduling.
$J \in \boldsymbol{Z}^+$	The number of material types.
$\boldsymbol{P}_i \subset \{1, \ldots, I\}$	The set of direct predecessors of any block $i \in \{1, \ldots I\}$.
Indices	
$t \in \{1, \ldots, T\}$	The index of a planning period.
$s \in \{1, \ldots, S\}$	The index of a scenario representing the uncertainty of the orebody model. Note that a variable or a parameter with subscript s indicates that it is scenario dependent. In the rest of this article, we omit the description for subscript s when describing a symbol for simplicity.
$i \in \{1, \ldots, I\}$	The index of a block.
Parameters	
$\beta_{ijs} \in \{0,1\}$	The parameter indicating block i belongs to type j.
$p_{jts} \in \boldsymbol{R}^+$	The value of a unit tonnage of type j material extracted in period t.
$c^M \in \boldsymbol{R}^+$	The cost of mining a unit tonnage of material.
$w_{is} \in \boldsymbol{R}^+$	The tonnage of block i.
$\overline{w} \in \boldsymbol{R}^+$	Mining capacity, i.e., the maximum number of blocks that can be extracted in a period.
$\xi \in \boldsymbol{R}^+$	The cost of temporary increase a unit mining capacity, or the penalty for a unit mining capacity shortage.
Variables	
$b_{it} \in \{0,1\}$	Binary variable which equals to 1 if and only if block i is mined in period t.
$w_{ts}^- \in \boldsymbol{R}^+$	The shortage of mining capacity in period t scenario s.

The mine production schedule is optimized through a mixed integer program (MIP) as

$$Maximize\ \frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\frac{1}{[1+\gamma]^t}\left[\sum_{j=1}^{J}\sum_{i=1}^{I}[p_{jts}-c^M]\beta_{ijs}w_{is}b_{it}-\xi w_{ts}^-\right], \tag{1}$$

$$\sum_{t}^{T} b_{it} \le 1\,, \quad i = 1, \ldots, I; \tag{2}$$

$$b_{it} - \sum_{\hat{t}=1}^{t} b_{\varepsilon\hat{t}} \le 0, \quad i = 1, \ldots, I, \;\; t = 1, \ldots, T, \;\; \varepsilon \in P_i; \tag{3}$$

$$\sum_{i=1}^{I} w_{is} b_{it} \leq \overline{w} + w_{ts}^{-}, \quad t = 1, \dots, T, \quad s = 1, \dots, S; \tag{4}$$

$$b_{it} \in \{0,1\}, w_{ts}^{-} \in \mathbb{R}^{+}, \quad \forall i \in \{1, \dots, I\}, \forall t \in \{1, \dots, t\}, \forall s \in \{1, \dots, S\}. \tag{5}$$

In MIP (1)–(5), the objective function (1) is to maximize the expected NPV of the materials extracted at all planning periods. (2) constrains each block to be mined within a single period. (3) constrains any block's predecessors to be mined before that block. (4) constrains the total weight of blocks mined in a period to be within the mining capacity. If the mining constraint is violated, i.e., the excessive part, w_{ts}^{-}, is positive, a penalty will be incurred.

2.2 The MFP model for the material flow circuit

In order to demonstrate our MVC optimization method, we focus on material flow circuit with a simple structure. However, similar to the MPS model, our MFP model can be easily extended to account for more complex material flow circuit. In addition to the symbols defined for the MPS model above, the following symbols are defined for the MFP model.

General	
$S' \in \boldsymbol{Z}^{+}$	The number of scenarios representing the uncertainty in the commodity market.
$K \in \boldsymbol{Z}^{+}$	The number of mines.
Indices	
$s' \in \{1, \dots, T\}$	The index of a scenario representing the uncertainty in the commodity market. Similar to s, a variable or parameter with subscript s indicates that it is scenario dependent, and in the rest of this article, we omit the description for subscript s' when describing a symbol for simplicity.
$k \in \{1, \dots, K\}$	The index of a mine.
Parameters	
$r_{ts'} \in \boldsymbol{R}^{+}$	The spot price of a unit tonnage of commodity in period t.
$c^{H} \in \boldsymbol{R}^{+}$	The unit rehandling cost for sending material to stockpiles.
$e_j \in \boldsymbol{R}^{+}$	The value of type j material stockpiled at the end of the planning horizon.
$\tau \in \boldsymbol{R}^{+}$	The cost of expanding a unit transportation capacity.
$o_{jts}^{k} \in \boldsymbol{R}^{+}$	The cumulated tonnage of type j material extracted in mine k in period t.
$g_j \in (0,1)$	The expected grade of type j material.
$\underline{g}, \overline{g} \in (0,1)$	The lower bound and the upper bound of the head grade of the feeding material accepted by the processing plant.
$\bar{v}^{P} \in \boldsymbol{R}^{+}$	The processing capacity of the processing plant in each planning period.
$\bar{x}^{P} \in \boldsymbol{R}^{+}$	The transportation capacity for moving materials.
Variables	
$x_{jtss'}^{MP} \in \boldsymbol{R}^{+}$	The amount of type j material sent from the mine to the processing plant in period t.
$x_{jtss'}^{MH} \in \boldsymbol{R}^{+}$	The amount of type j material sent from the mine to the stockpile in period t.
$x_{jtss'}^{HP} \in \boldsymbol{R}^{+}$	The amount of type j material sent from the stockpile to the processing plant in period t.
$x \in \boldsymbol{R}^{+}$	The expansion of transportation capacity at the beginning of the planning horizon.
$v_{jtss'}^{H} \in \boldsymbol{R}^{+}$	The stock level of the stockpile of type j material in period t.

The mining complex's processing stream plan is optimized through a linear program as

$$Maximize\ \frac{1}{SS'}\sum_{s=1}^{S}\sum_{s'=1}^{S'}\sum_{t=1}^{T}\frac{1}{[1+\gamma]^t}\left[\underbrace{r_{ts'}\sum_{j=1}^{J}g_j\left[x_{jtss'}^{MP}+x_{jtss'}^{HP}\right]}_{(i)}-\underbrace{c^H\sum_{j=1}^{J}x_{jtss'}^{MH}}_{(ii)}\right] + \underbrace{\frac{1}{SS'[1+\gamma]^T}\sum_{s=1}^{S}\sum_{s'=1}^{S'}\sum_{j=1}^{J}e_j v_{jTss'}^{H}}_{(iii)} - \underbrace{\tau x}_{(iv)}; \tag{6}$$

$$x_{jtss'}^{MP}+x_{jtss'}^{MH}\le\sum_{k=1}^{K}o_{jts}^{k},\quad \forall j,t,s,s'; \tag{7}$$

$$v_{j0ss'}^{H}=0,\quad \forall j,s,s'; \tag{8}$$

$$v_{jtss'}^{H}=v_{j,t-1,s,s'}^{H}+x_{jtss'}^{MH}-x_{jtss'}^{HP},\quad \forall j,t,s,s'; \tag{9}$$

$$\sum_{j=1}^{J}\left[x_{jtss'}^{MP}+x_{jtss'}^{HP}\right]\le\bar{v}^P,\quad \forall t,s,s'; \tag{10}$$

$$\underline{g}^P\le\frac{\sum_{j=1}^{J}g_j\left[x_{jtss'}^{MP}+x_{jtss'}^{HP}\right]}{\sum_{j=1}^{J}\left[x_{jtss'}^{MP}+x_{jtss'}^{HP}\right]}\le\overline{g}^P,\quad \forall t,s,s'; \tag{11}$$

$$\sum_{j=1}^{J}\left[x_{jtss'}^{MH}+x_{jtss'}^{MP}+x_{jtss'}^{HP}\right]\le\overline{x}+x,\quad \forall t,s,s'; \tag{12}$$

$$x_{jtss'}^{MH},x_{jtss'}^{MP},x_{jtss'}^{HP},x,v_{jtss'}^{H}\in\mathbb{R}^+. \tag{13}$$

In the LP (6)–(13), the objective function (6) is to maximize the expected NPV of the processing stream. In the objective function, (i) computes the revenue obtained by selling the commodity in period t. (ii) computes the total rehandling cost incurred in period t. (iii) compute the expected value of the stockpiled materials at the end of the planning horizon. (iv) computes the total investment in expanding the transportation capacity. The investment in transportation capacity expansion is made at the beginning of the planning horizon. (7) constrains that for any type j in any period t, the total flows from the mine should not exceed the total material extracted from the mines. o_{jts}^{k} for any mine k is obtained by aggregating the weight of type j material in each period obtained from the solution of the MPS model as

$$o_{jts}^{k}=\sum_{i=1}^{I}w_{is}^{k}b_{it}^{k}\beta_{ijs}^{k}, \tag{14}$$

where w_{is}^{k}, b_{it}^{k} and β_{ijs}^{k} are w_{is}, b_{it} and β_{ijs}, respectively, in the MPS model for mine k.

By (8) we suppose that the initial stock level for the stockpile of each type of material is 0. (9) constrains that in any period t, the stock level of any stockpile equals to the stock level in the previous period plus the difference of the inflow amount and the outflow amount. (10) constrains that the throughput of the processing plant should not exceed the processing capacity. (11) constrains that the head grade of the feeding material should be within the range accepted by the processing plant in any period. (12) constrains that the total amount of material transported in each period should not exceed the transportation capacity.

3 The DMVBD method

In the DMVBD method proposed herein, the MPS model(s) and the MFP model are solved separately, and then through an iterative procedure, the solutions of different models are synchronized. It should be noticed that because the synchronization mechanism proposed in this work is not limited to the forms of the MPS models and the MFP model in Section 2, various solution methods can be employed to solve each model. In this work, we focus on introducing the synchronization mechanism and the solution methods for the MPS models and the MFP model are introduced in Appendix A.

In our MPS model, the material value, p_{jts}, determines the incentive of mining type j material in period t. That is, when p_{jts} is increased or decreased, the mining complex has an incentive to mine more or less type j material in period t. Based on this relationship between material value and the mining incentive, we propose a synchronization mechanism summarized as in Figure 2.

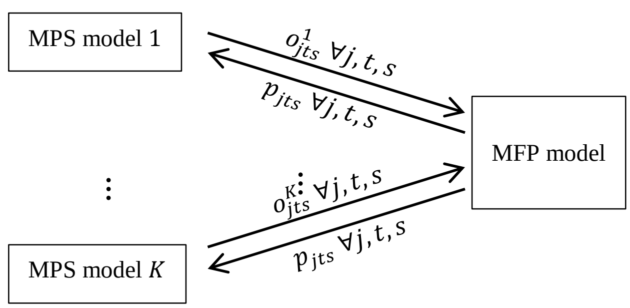


Figure 2. Summary of the proposed synchronization mechanism.

In the proposed mechanism, any MPS model k sends to the MFP model a message which includes the values of o^k_{jts} for all j, t and s obtained from the optimal solution of MPS model. Then, with the received message, the MFP model is solved and then sends back to the each MPS model a message including the updated p_{jts} for all j, t and s. The procedure repeats iteratively until the system converges. The values of o^k_{jts} for all j, t and s can be computed from (14) as stated in Section 2.2. The values of the updated p_{jts} for all j, t and s are obtained by solving a modified MFP model. The modified MFP model preserves the form of the original MFP model except that the objective function (6) and the constraint (7) are replaced with

$$Maximize\ \frac{1}{SS'}\sum_{s=1}^{S}\sum_{s'=1}^{S'}\sum_{t=1}^{T}\frac{1}{[1+\gamma]^t}\left[r_{ts'}\sum_{j=1}^{J}g_j\left[x^{MP}_{jtss'}+x^{HP}_{jtss'}\right]-c^H\sum_{j=1}^{J}x^{MH}_{jtss'}\right.$$
$$\left.+\,o^-_{jts}\left[p_{jts}-\Delta p_{jts}\right]-o^+_{jts}\left[p_{jts}+\Delta p_{jts}\right]\right]-\tau x\,; \tag{15}$$

$$x^{MP}_{jtss'} + x^{MH}_{jtss'} \leq \sum_{k=1}^{K} o^k_{jts} + o^+_{jts} - o^-_{jts}, \quad \forall j, t, s, s'; \tag{16}$$

$$0 \leq o^+_{jts}, o^-_{jts} \leq \overline{o}, \quad \forall j, t, s. \tag{17}$$

In the modified part (15)–(17), Δp_{jts} and $\overline{o}$ are constants. For all j, t and s, we add two new variables, o^+_{jts} and o^-_{jts}, which represent the amounts of purchasing and selling material j in period t scenario s, respectively. The prices for purchasing and selling material j in period t and scenario s are set to $p_{jts} + \Delta p_{jts}$ and $p_{jts} - \Delta \mathrm{p}_{jts}$, respectively. We constrain o^+_{jts} and o^-_{jts} for all j, t and s to be within a small positive upper bound, $\overline{o}$, so that the modification does not make a significant impact on the other variables in the optimal solution. With the proposed modification as described above, the detailed steps of the proposed synchronization algorithm can be described as follows:

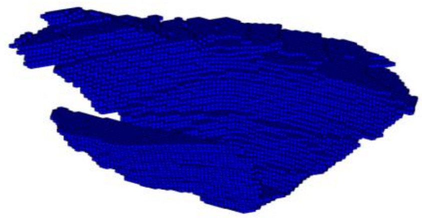
Algorithm 1: Synchronization algorithm.

Step 1: Initialize p_{jts} for all j, t and s based on the available knowledge.
Step 2: Solve the MPS model with the incumbent p_{jts} for all j, t and s, and generate o^k_{jts} for all k, j, t and s from (14) based on the obtained optimal solution.
Step 3: Solve the modified MFP model with the incumbent o_{jts} for all j, t and s, and update p_{jts} for all j, t and s as
IF there is $o^-_{jts} > 0$ for any j, t and s THEN
Set $p_{jts} \leftarrow p_{jts} - \Delta p_{jts}$ for all j, t and s with $o^-_{jts} > 0$; GOTO Step 2;
ELSEIF there is $o^+_{jts} > 0$ for any j, t and s THEN
Set $p_{jts} \leftarrow p_{jts} + \Delta p_{jts}$ for all j, t and s with $o^+_{jts} > 0$; GOTO Step 2;
ELSE
STOP;
END IF

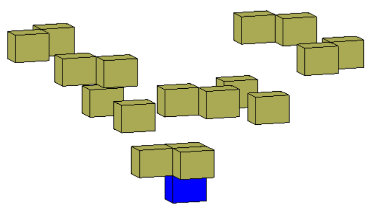
From the synchronization algorithm above, it can be observed that the iteration stops when o^-_{jts} and o^+_{jts} are 0 for all j, t and s.

4 Numerical Test

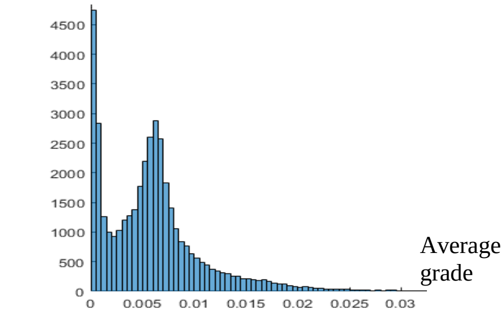
We test the optimization approach for an MVC that includes a copper mine, a number of material stockpiles and a processing plant. The copper deposit in our test case contains 40,044 blocks and is a phase of a larger mine. 20 simulated grade scenarios are available. Figure 3 shows a summary of the copper deposit in our test case. We consider 17 predecessors for each block to be extracted as shown in Figure 3b. From Figure 3c, it can be observed that almost all blocks have average grades below 3%. Hence, we set $J = 15$ types of materials, and for all $j \in \{1, \ldots, J\}$, $g_j = 0.002 \times [j-1]$. A block belongs to type j if its grade falls in $\left[g_j, g_j + 0.002\right)$. If a block has a grade more than 3%, it belongs to type J. The cost for mining a block is ranged as $c^M_i \in [2{,}2.5]$ \$ per ton depending the depth its location. The mining capacity is set to $\overline{w} = 1.45 \times 10^8$ tons per year.



(a) Shape of the deposit



(b) Slope constraints



(c) Grade distribution

Figure 3. The copper deposit in the test case.

The processing capacity of the processing plant is set to $\overline{v}^P = 4 \times 10^7$ tons per year, and the processing cost is set to $c^P = 10\$$ per ton. The lower bound and the upper bound of the material requirements are set to $\underline{g} = 0.002$ and $\overline{g} = 0.03$, respectively. For the stockpiles, the rehandling cost is set to $c^H = 0.1\$$ per ton. The existing transportation capacity is set to $\overline{x} = 1 \times 10^8$ tons per year, and the cost of increasing a unit ton per year capacity is set to $\tau = 1\$$. The discount rate for computing NPV is set to $\gamma = 0.1$. For the commodity market, there are 10 simulated scenarios where the spot commodity price at each period fluctuates in the range of $[4000{,}5500]$ \$ per ton. The simulated price fluctuations are shown in Figure 4. Note that we simulate a trend of increasing commodity prices for the purpose of testing if the proposed method can properly adjust the MPS to account for market prediction.

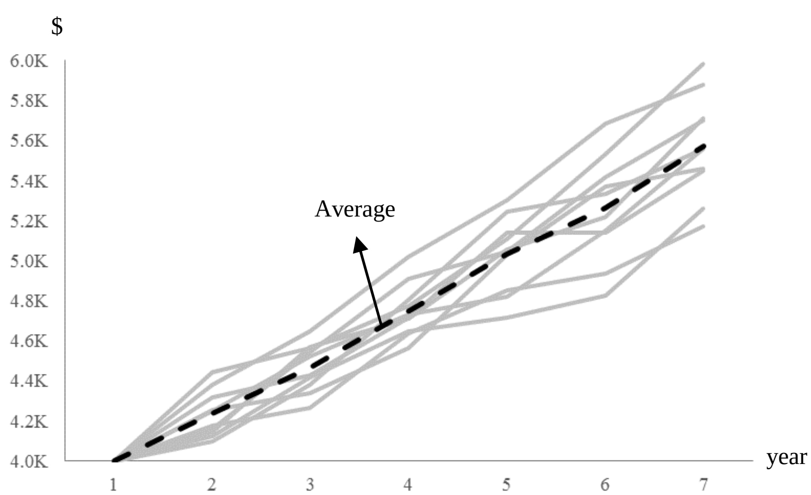


Figure 4. Simulated price fluctuations over the planning horizon.

To implement the proposed synchronization method, the control parameters are set to $\overline{o} = 10$ and $\Delta p_{jts} = \max\{0.1, p_{jts} \times 0.1\}$. The initial value for any type j material is set to

$$p_{jts} = max\left\{\frac{1}{S'}\sum_{s'\in\{1,\ldots,S'\}} r_{ts'}\, g_j - c^P, 0\right\}.$$

The algorithms are programmed using Matlab R2015b and the MIPs and LPs are solved using CPLEX 12.51. The algorithms are tested on a platform of Intel Xeon X5650 with two 2.67GHz processors and 24.0GB RAM.

The MVC plan, including the MPSs of all mines and the MFP of the material flow circuit, obtained by the DMVBD method is compared with the one obtained by the local optimization (LO) method by which the MPS models and the MFP model are optimized separately without synchronization.

Figure 5 shows the MPS generated by the algorithm described in Appendix A. The blue blocks are scheduled to be extracted in the corresponding period. In the test shown in Figure 5, the employed algorithm uses 89.6 seconds to generate the initial MPS and the revising algorithm uses 206.8 seconds to revise the MPS. The revised pit increases the expected NPV by 1.24% from 8.401 billion dollars to 8.505 billion dollars.

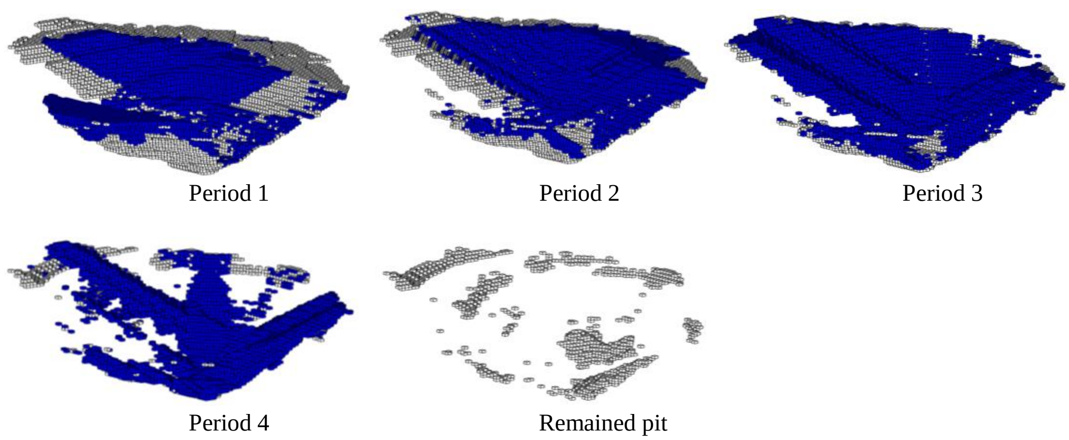


Figure 5. The initial MPS obtained from the LO method.

In Figure 5, the MPS is computed based on the initial material values, is equivalent to the MPS generated by the LO method where the congestions and the market fluctuations in the material flow circuit are not accounted. Using the proposed MVC optimization method, the final mine production schedule is displayed in Figure 6. It can be observed that the proposed optimization method increases the life of mine from 4 years to 7 years. The results indicate that the optimizations of the MPS model and the MFP model are synchronized because at each period, the upstream mining rate generated by the DMVBD method is reduced to account for the congestion at the processing plant while the LO method maximizes the mining rate to maximize the mine's NPV rather than the MVC's NPV.

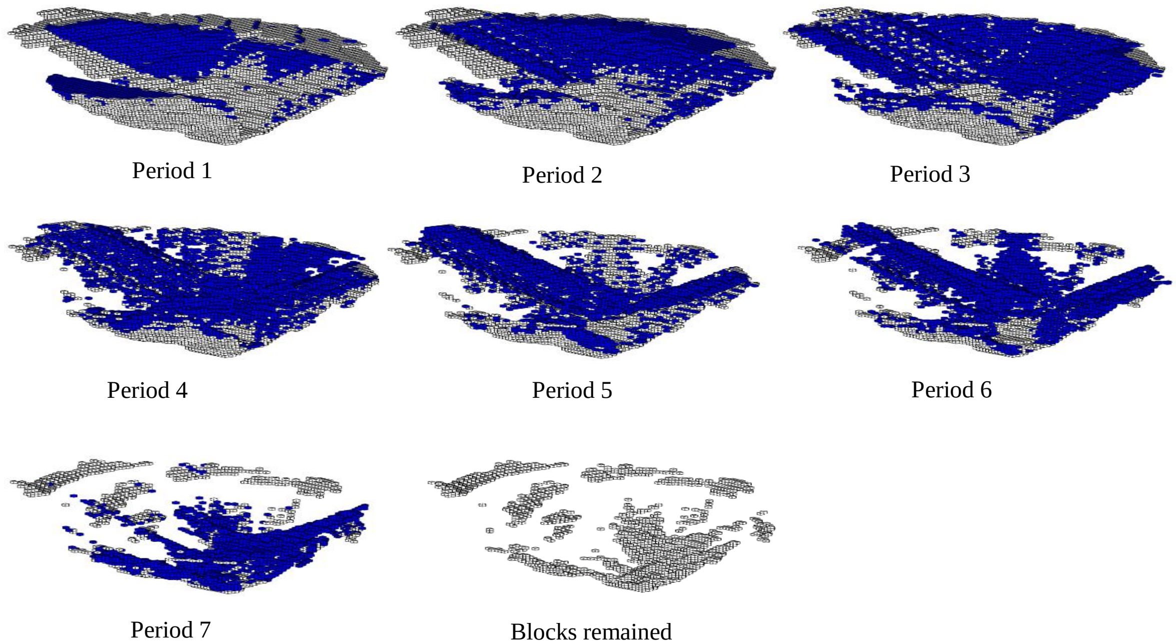


Figure 6. The MPS obtained from the DMVBD method.

To clearly present what has happened downstream under the MPS generated by the two methods, we use Figures 7–9 to compare the difference of the stockpile levels, the mining rates and the processing rates under the MPSs obtained from the two methods. Figure 7 shows that using the DMVBD method, the stockpile level is significantly reduced. The reason for the reduced stockpile can be observed from Figures 8 and 9. In Figure 8, it can be observed that with the MPS obtained from the LO method, the mining rate is maximized to maximize the NPV of the mine. Under the MPS obtained by the DMVBD method, the production rate n is reduced. Figure 9 shows that using either method, the obtained MPS leads to the full utilization of processing capacity. Because with the MPS obtained by the proposed method, the stockpile level in each period is close to 0, we can conclude that the mining rate in each period is reduced to a proper level such that the processing capacity is fully utilized without incurring unnecessary stockpiles, which means that the mineral value chain is leaner after employing the DMVBD method.

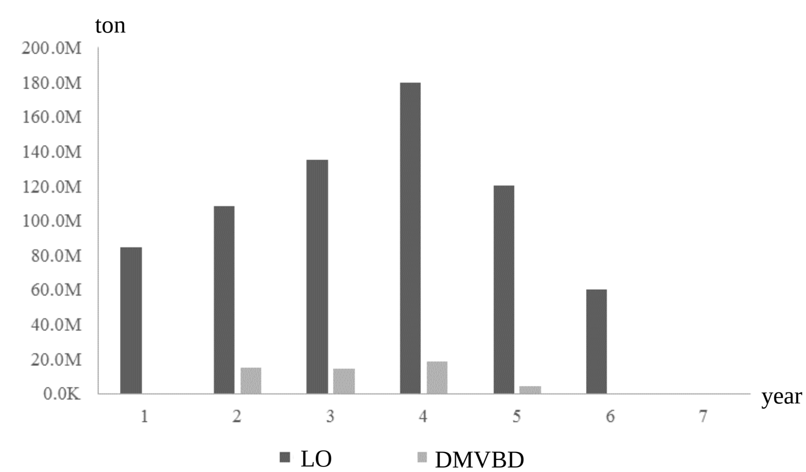


Figure 7. The amounts of materials stocked at stockpiles in each period under the MPSs obtained from the LP method and the DMVBD method.

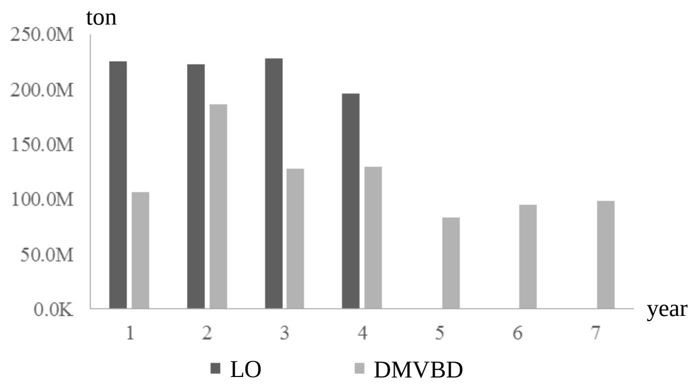


Figure 8. The amounts of materials extracted at the mine in each period under the MPSs obtained from the LP method and the DMVBD method.

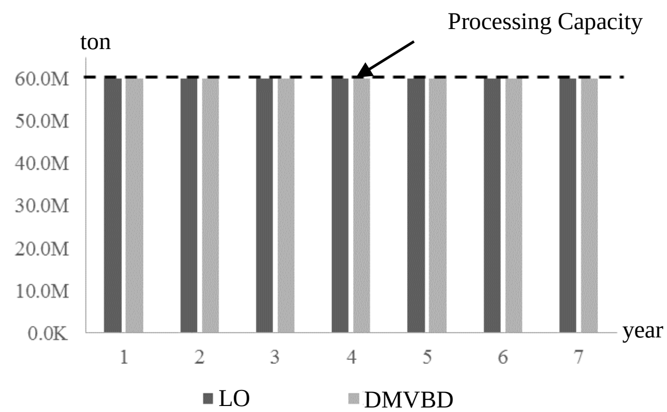


Figure 9. The amounts of materials processed at the plant in each period under the MPSs obtained from the LP method and the DMVBD method.

Figure 10 shows the amounts of commodity produced in each period. It can be observed that the production in each period is following a decreasing trend for both methods because of the 10% discount rate of each year. However, under the MPS obtained from the DMVBD method, the production of the commodity is reduced in the early periods and increased in later periods, which accounts for the increasing commodity price. Using the DMVBD method, some high-grade material is extracted in later periods in consideration of the increasing commodity price for the optimum of long-term planning, which evidences that the proposed method can generate an MPS that accounts for the fluctuation of the commodity market.

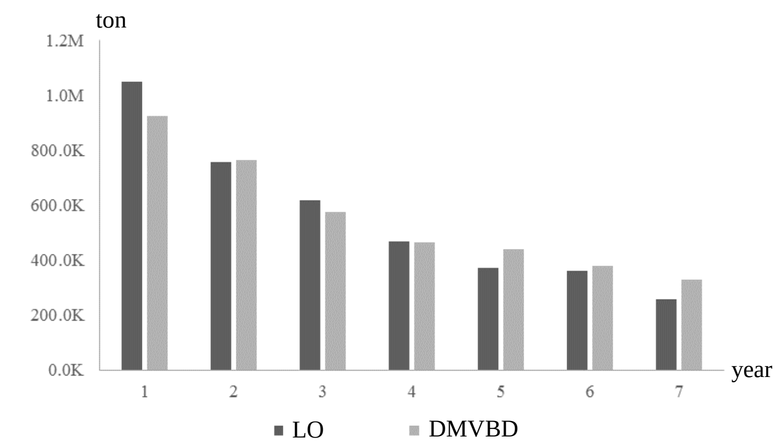


Figure 10. The amounts of commodity produced in each period under the MPSs obtained from the LP method and the DMVBD method.

Figure 11 compares the accumulated expected NPV of the entire value chain at the end of each period with the MPSs obtained from the conventional and the proposed optimization methods. It can be observed that in early periods, the conventional method can generate higher NPVs because of the higher commodity production, but in later periods, the proposed method outperforms the conventional method because of the reduced material stockpiles and the increased commodity production. Over the planning horizon, the total expected NPV of the entire MVC is increased by 2.22% with the MPS obtained from the proposed method.

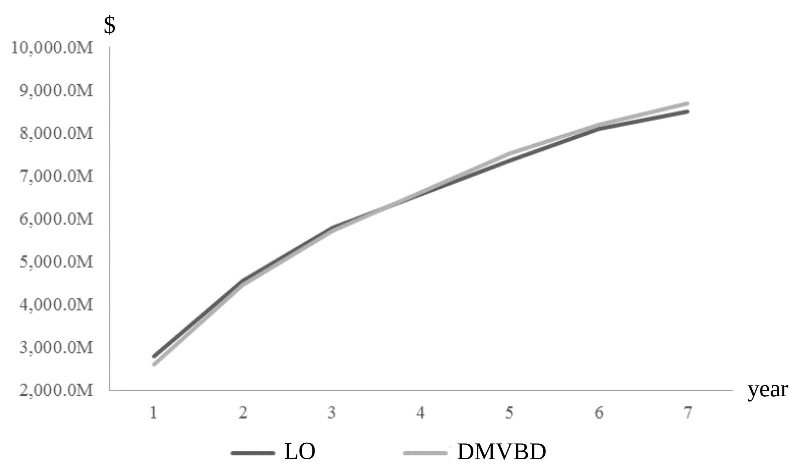


Figure 11. The accumulated expected NPV of the entire MVC at the end of each period under the MPSs obtained from the LP method and the DMVBD method.

The numerical results in our case study show the capability of the DMVBD method in coordinating the optimizations of the MPS model and the MFP model. It shows that, although the MFP model is a black box to the MPS optimizer, with properly changing the material value for each period and each scenario, the proposed method can automatically adjust the MPS to account for the downstream congestions and the market fluctuation. The proposed method is tested by repeating the case study with different settings such as different processing capacities, different commodity price fluctuations, etc. The test results are similar and the conclusions holds, which evidences the generality of the proposed MVC optimization method.

5 Conclusions

In this work, an MVC optimization method is developed. In the proposed DMVBD method, the optimizations of the MPS model and the MFP model are coordinated to optimize the entire MVC instead of each indivisual model. We propose a scheme in which the values of different types of materials are iteratively changed. Based on the updated material values, the MPS model dynamically updates the mine production schedule accordingly. Through a case study, we show that the proposed method is capable of generating a mine production schedule that accounts for the downstream congestion and the commodity market fluctuation.

The proposed method makes it possible to optimize an MVC with complex structures. By extending the MFP model, we can integrated more delicate factors such as the operational and market uncertainties, the nonlinear transformation of materials, resource allocations, etc., and then develop ad hoc solution method to solve the MFP model without worrying about the complexity in solving the MPS model

6 Appendix A. Solving the MPS and the MFP models

In the MPS model formed in Section 2.1, it can be observed that there are $I \times T$ binary variables to be optimized subject to the slope constraints and the mining capacity constraint. Because in practice, the number of blocks, I, is usually too large for an integer programming solver to handle, an MPS model is usually solved by heuristics. In the literature, the heuristic for solving an MPS model has been studied extensively. A detailed review of the different solution approaches for the MPS optimization can be found in Newman et al. (2010). In this work, we develop a solution method which is modified from a bench-phase scheduling method.

The bench-phase method is developed for surface mining. It generates a series of nested pits by gradually reducing the value of each block. The pushbacks are formed based on the generated nested pits. Each pushback is then divided into bench-phases (Chicoisne et al. 2012) based on the vertical levels (or benches). Finally, a new mathematical model is formulated to optimize the extraction schedule of all the bench-phases. The precedents of a bench-phase include the bench-phases with an equal or higher vertical level and within the incumbent or smaller nested pits. Figure A.1 shows the definition of nested pits, pushbacks, bench-phases and an example precedence constraint. The area between each pit shell is a pushback, and the area with the same letter forms a bench-phase. According to the definition of the precedents of a bench-phase, for example, the precedents of bench-phase ‘g’ are bench-phases ‘a’, ‘b’, ‘c’, ‘d’ and ‘f’. Common methods to solve to generate nested pits include Lerch and Grossman method (Lerch and Grossman 1965) and maximum flow method (Picard 1976).

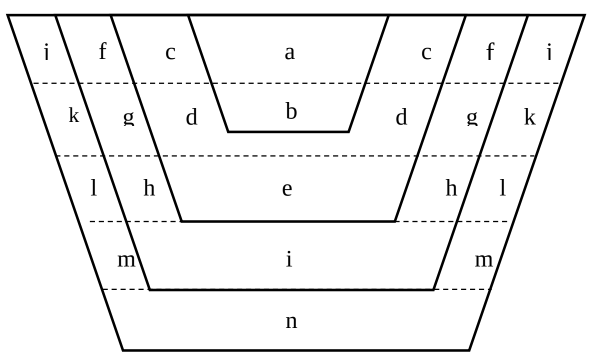


Figure A.1. Bench-phases of an open-pit mine.

In order to increase the accuracy of the MPS from the bench-phase level to the block level, a revising algorithm is developed based on the bubble sorting algorithm. Let B_t be the set of blocks that should be mined in period t for any $t \in \{1, \ldots, T\}$ and, for convenience, let B_{T+1} be the set of wasted blocks that are not mined within the planning horizon. Then, the target of MPS optimization is finding the optimal $\vec{B}: \vec{B} = \{B_1, B_2, \ldots, B_{T+1},\}$ that maximizes the NPV all blocks. The main steps of our MPS optimization algorithm can be summarized as follows:

Algorithm 2: Main steps of MPS.

Step 1: Initialize $\vec{B}$.
Step 2: For t'=1 to T step 1
 For t= T to t' step -1
  Update B_t and B_{t+1}.
 Next t
Next t'

In the pseudo algorithm above, $\vec{B}$ is first initialized using the bench-phase method. Then through a process similar to bubble sorting, we attempt to improve the MPs by iteratively solving a smaller sub-problem.

The detailed algorithm for initializing $\vec{B}$ is described as follows:

Algorithm 3: Initializing $\vec{B}$.

Step 1: : Index the blocks in such a way that a block located at a higher altitude has a lower index, i. Initialize the parameters by setting $t \leftarrow 1$, $B_t \leftarrow \emptyset$ for all $t \in \{1, \dots, T\}$ and $B_{T+1} \leftarrow \{1, \dots, I\}$. Set a control parameter λ.
Step 2: Find the ultimate pit limit (UPL) by solving the UPL model as

$$Maximize \frac{1}{S} \sum_{s \in \{1,\dots,S\}} \sum_{i \in B_{T+1}} [v_{its} - c_i^M - \lambda] w_{is} b_i\,; \tag{18}$$

$$Subject\ to\colon\ \ b_i \le b_\varepsilon, \quad i = 1, \dots, I, \ \ \varepsilon \in \boldsymbol{P}_i; \tag{19}$$

$$b_i \in \{0,1\}, \quad i = 1, \dots, I. \tag{20}$$

Step 3: Run the following loop to move blocks from B_{T+1} to B_t:
FOR EACH $i \in B_{T+1}$ AND $b_i = 1$ in the optimal solution of the UPL model
 IF $\frac{1}{s} \sum_{s \in \{1,\dots,S\}} \sum_{\varepsilon \in B_t \cup \{i\}} w_{is} < \overline{w}$ THEN
  $B_t \leftarrow B_t \cup \{i\}$; $B_{T+1} \leftarrow B_{T+1} \backslash \{i\}$;
 ELSE
  $t \leftarrow t + 1$; GOTO Step 2;
 END IF
NEXT i
Step 4: Reduce λ. If $\lambda \ge 0$, GOTO Step 2; otherwise STOP.

In the algorithm for initializing $\vec{B}$, the UPL model (18)–(20) in Step 2 computes the ultimate pit that maximizes the expected NPV subject to the slope constraints with the incumbent λ. For all blocks $i \in \{1, \dots, I\}$, $b_i = 1$ if block i is included in the ultimate pit. The UPL is computed based on the material value at the incumbent period. In our initial MPS, the blocks in the UPL computed with greater λ is mined first until the mining capacity of the incumbent period is reached.

After initialization, $\vec{B}$ is improved through a procedure that is similar to a bubble sorting algorithm. In the main steps of MPS, the algorithm of updating B_t and B_{t+1} is described as follows:

Algorithm 4: Updating B_t and B_{t+1}.

Step 1: If $t = T$, set $\delta \leftarrow 1$; otherwise, set $\delta \leftarrow 0$. Solve the MIP

$$Maximize \sum_{s \in \{1,\dots,S\}} \sum_{i \in B_t \cup B_{t+1}} \left[\left[v_{its} - c_i^M - \frac{[v_{i,t+1,s} - [1-\delta] c_i^M]}{1+\gamma} \right] w_{is} b_i - \xi w_s^- - \frac{\xi w'^-_s}{1+\gamma} \right]; \tag{21}$$

$$b_i \le b_\varepsilon, \quad \forall i \in B_t \cup B_{t+1}, \ \ \forall \varepsilon \in P_i; \tag{22}$$

$$\sum_{i \in B_t \cup B_{t+1}} w_{is} b_i \le \overline{w} + w_s^-, \quad \forall s \in \{1, \dots, S\}\,; \tag{23}$$

$$\sum_{i \in B_t \cup B_{t+1}} w_{is}[1 - b_i] \leq \overline{w} + w'^{-}_{s} + \delta M, \quad \forall s \in \{1, \dots, S\}, M = +\infty; \tag{24}$$

$$b_i \in \{0,1\}, \forall i \in B_t \cup B_{t+1};\ w_s^-, w'^{-}_{s} \in \mathbb{R}^+, \forall s \in \{1, \dots, S\}. \tag{25}$$

Step 2: Update B_t and B_{t+1} based on the optimal solution of the MIP in Step 1 as

$$B_t = \{i | b_i = 1\}\ and\ B_{t+1} = \{i | b_i = 0\}.$$

The MIP (21)–(25) solves a two-period sub-problem with much fewer binary variables which is usually solvable using existing MIP solvers.

The MFPP model formed herein can be solved efficiently using many linear programming solvers such as CPLEX.

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