Distributive Online Channel Assignment for Hexagonal Cellular Networks with Constraints

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Abstract

In cellular networks, channels must be assigned to call requests so that interference constraints are respected and bandwidth is minimized. The number of call requests per cell is continually changing, making channel assignment naturally an online problem. We describe two new online channel assignment algorithms for networks based on a regular hexagonal layout of cells, where interference levels depend only on the distance between cells. Such networks can be modelled by so-called *hexagon graphs*. Our model incorporates different *separation constraints*, prescribed minimal differences between channels assigned to cells within a certain distance of each other. The algorithms presented are the first to take into account separation constraints between non-adjacent cells in this type of layout. The algorithms are distributed in nature: each cell server will need only a limited exchange of information with cells in its proximity to make decisions on its channel assignment.

Résumé

Dans les réseaux cellulaires, il faut affecter des canaux de communication aux appels de façon à ce que les contraintes d'interférence soient respectées, et que la largeur de bande soit minimisée. Le problème d'affectation de canaux est de nature dynamique, comme le nombre d'appels par cellule varie constamment.

Nous décrivons deux nouveaux algorithmes dynamiques d'affectation de canaux pour des réseaux basés sur une disposition régulière de cellules hexagonales. Nous supposons que l'interférence dépend seulement de la distance entre les cellules. Notre modèle considère trois *contraintes de séparation* différentes, qui sont des différences prescrites entre les canaux affectés aux cellules dont l'une est à une certaine distance de l'autre.

Les algorithmes proposés sont les premiers à considérer des contraintes de séparation entre cellules non-adjacentes dans ce type de disposition. Il s'agit d'algorithmes distribués: chaque serveur peut prendre ses décisions sur l'affectation de canaux dans sa cellule seulement d'après un échange limité d'information entre la cellule et des cellules dans sa proximité.

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1 Introduction

The *Channel Assignment Problem* in a cellular network is the problem of assigning frequency channels to communication links. Requests for links vary over time and throughout the network. Available bandwidth is limited and costly, motivating operators to optimize *channel reuse*. Channel reuse is the simultaneous usage of the same channel at different locations in the network. However, reuse is limited by interference constraints, which restrict the channels that can be used simultaneously in particular cells.

Good channel assignment is essential for all voice, data and other communication cellular networks which are based on FDMA (Frequency Division Multiple Access). In FDMA systems, the available radio spectrum is divided into small frequency bands of a prescribed bandwidth. Each frequency band corresponds to a radio channel, and each communication link is assigned one such channel. (Advanced systems, such as GSM, assign more than one link to each channel by using time sharing. However, this does not change the basic premise of the system.) Channels located close to each other in the spectrum are more likely to interfere, and therefore cannot be used in cells that are geographically close together. FDMA is used by most of the earlier cellular telephone networks and forms the basis of a large part of the new generation of PCS (Personal Communication Service) systems.

The Channel Assignment Problem was first studied in the late 1970s. Most work has concentrated on the static problem of finding a global channel assignment for a certain network and pre-specified, static parameters. Methods from graph theory and combinatorial optimization have been used to develop heuristics, and lately to develop exact methods, to solve specific instances. More recently, an algorithmic approach to channel assignment has been developed. Here, a cellular network is regarded as a dynamic system where channel assignment decisions are made locally, not planned globally, and assignments must continually be updated to accommodate changing parameters.

A cellular network is treated here as a distributed network. Every transmitter or base station is a server, and our channel assignment algorithms are treated as if they were running simultaneously on these servers. Moreover, the algorithms are shown to be also suited for the online channel assignment problem. Here, the demand for channels is subject to change, and servers must adapt to these changes by making minimal changes to the existing assignment. General online channel assignment algorithms are described in [JKM99, Ray91, ESG82, CR82], among others. In [JK00], theoretical limits on the competitive ratio of general online channel assignment algorithms are derived. In [JDNS98], a framework is described for the evaluation of channel assignment algorithms in the context of distributed and online algorithms. We will follow this framework in our paper.

In this paper, online distributed channel assignment algorithms are given for cellular networks with a regular hexagonal layout. The interference constraints are modelled by a set of *separation constraints*. These constraints dictate the separation, in the radio spectrum, that must exist between channels assigned to cells that within a particular distance of each other. In [JDNS98], online distributed channel assignments are given for hexagonal cellular networks, with the only constraint that channels assigned to the same or adjacent cells must be distinct. In [SUZ97] and [JN99], global and static channel assignment algorithms are given for hexagonal networks with different separation constraints for channels assigned to the same cell and adjacent cells, respectively. In [Ger99], the same is done for networks that can be modelled by bipartite graphs. In [FS98], a distributed online algorithm is given for hexagonal networks without separation constraints, but with the requirement that all channels assigned to cells at distance two or less from each other must be distinct (adjacent cells are assumed to be at distance one). This paper describes two new distributed online algorithms for hexagonal networks with different separation constraints for channels assigned to the same or adjacent cells, and for channels assigned to cells at distance two from each other. These are the first algorithms given for this type of network and constraints.

1.1 Channel Assignment Definitions

A cellular network can be conveniently represented by a graph and a set of constraint parameters. In this paper we represent a cellular network by a graph G with node set V and edge set E, where the nodes correspond to the cells of the network, and edges correspond to pairs of neighbouring cells. For the basic definitions of graph theory we refer to [BM76]. In terminology consistent with the network which the graph represents, a node adjacent to node v is called a **neighbour** of v, and the set of all neighbours of v is the **neighbourhood** of v.

A class of graphs with particular importance in the context of channel assignment are the hexagon graphs. A *hexagon graph* is an induced subgraph of the triangular lattice. Here, the triangular lattice is considered to be a graph whose nodes are the points of the lattice, and edges exist precisely between neighbouring points. Hexagon graphs are named thus because they can be used to represent a cellular network with regular, hexagonal cells. Since decay of a radio signal is proportional to the distance from a transmitter, the coverage area of a transmitter will naturally have a circular form. Hexagonal cells have the advantage that they approximate this shape and yet are easily stacked. In practice, limitations imposed by the terrain may force the cellular network to deviate from the ideal hexagonal layout. However, for the new generation of cellular networks based on LEO (Low Earth Orbit) satellites, the hexagonal layout is especially relevant. Figure 1 shows an example of a hexagon graph and the underlying cellular network.

We will assume that the lattice is generated by the vectors $\mathbf{x} = (1, 0)$ and $\mathbf{y} = (1/2, \sqrt{3}/2)$, and we will use the lattice coordinates to identify the nodes of our graphs. More precisely, node (i, j) will denote the node corresponding to lattice point $i\mathbf{x} + j\mathbf{y}$.

The amount of interference possible between channels used in different cells generally depends on the distance between the cells. Since in our model adjacency is based on neighbouring cells, graph distance can be used to approximate the distance, and hence the level of possible interference, between cells. The *graph distance* $d(u, v)$ between two nodes is the length of the shortest path between these nodes. Thus the graph distance between adjacent nodes equals 1, and $d(u, u) = 0$ for every node u.

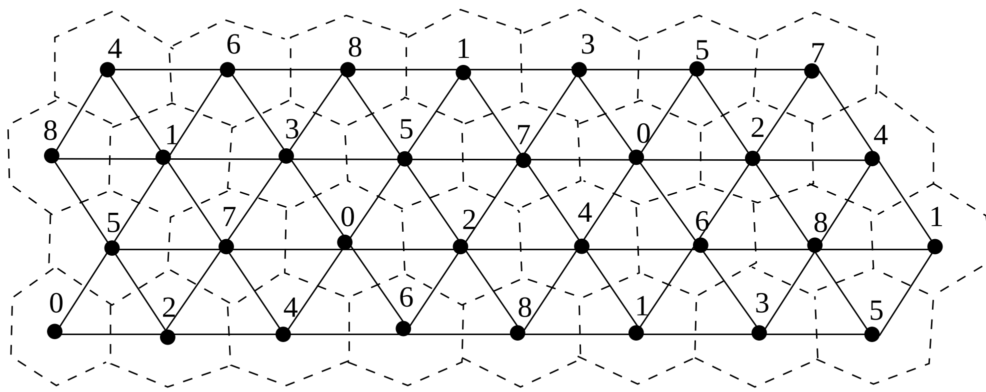

Figure 1: An example of a hexagon graph with an arithmetic assignment generated by 2 and 5, with a modulus of 9.

If signals are carried by radio frequencies that are close together, they generally will have higher interference levels than otherwise. Hence, the common requirement in cellular networks that channels assigned to cells that are geographically close must be spaced further apart in the radio spectrum. This can be included in our graph model by adding *separation constraints*. Firstly, channels may be represented by integers. Then, for each pair of cells, the separation constraint prescribes the minimum difference between any pair of channels assigned to these cells. For a detailed description of the model, see for example [Lee97].

A **constrained graph** $G = (V, E, c_0, \ldots, c_k)$ is a graph $G = (V, E)$ and positive integer parameters $c_0, \ldots, c_k$. Here c_0 represents the minimum separation between pairs of channels assigned to the same node, c_1 gives the separation constraint between adjacent nodes, and, in general, c_i gives the separation constraint between pairs of nodes at graph distance i of each other.

The last parameter of the cellular network that must be incorporated in the graph model is the demand for channels, i.e. the number of calls to be serviced in each cell. In the static case, this demand will usually represent a prognosis about future demands. In the online model, the weights are considered to be in constant change, and can thus be used to model actual call requests.

A **constrained, weighted graph** is a pair (G, w) where G is a constrained graph and w is a positive integral weight vector indexed by the nodes of G. The component of w corresponding to node u is denoted by $w(u)$ and called the **weight** of node u. The weight of node u represents the number of calls to be serviced at node u. We use w_{max} to denote $max\{w(v) \mid v \in V\}$ and w_{min} to denote the corresponding minimum weight of any node in the graph.

In the context of this graph model, we now give a formal definition of a channel assignment. A **channel assignment** for a constrained, weighted graph (G, w) where

$G = (V, E, c_0, \ldots, c_k)$ is an assignment f of sets of non-negative integers (which represent the channels) to the nodes of G which satisfies the conditions:

$$\begin{array}{ll} |f(u)| = w(u) & (u \in V), \\ i \in f(u) \text{ and } j \in f(v) \Rightarrow |i-j| \geq c_\ell & \text{if } d(u,v) = \ell. \end{array}$$

The goal of good channel assignment is to minimize bandwidth, represented by the span of the assignment. The **span** of a channel assignment f of a constrained weighted graph is the difference between the lowest and the highest channel assigned by f. The span of a constrained, weighted graph G and a positive integer vector w indexed by the nodes of G is the minimum span of any channel assignment for (G, w).

1.2 Distributed online algorithms and their evaluation

The algorithms given in this paper are *distributed* and *deterministic*. Each node is considered to be an independent server, and the algorithm is running simultaneously on these servers. Each server computes its own local assignment of channels deterministically, based only on a small amount of precomputed information (not dependent on the weights), and local information of the state of the network. More precisely, a server at a node knows the weight of that node, and can obtain knowledge about the weight of the nodes that are within a certain distance of it. Following the definitions from [JDNS98], the maximum graph distance over which a node server can acquire information is a parameter of the algorithm.

Formally, a distributed channel assignment algorithm is *k-local* if each node server computes its assignment knowing only the pre-computed information and the weight at all nodes at graph distance k or less from it.

The concept of locality takes on a special meaning in the context of online channel assignment algorithms. Here, the input of the algorithm is a constrained graph and a sequence w_t of weight vectors. At each time instance t, each node v is presented independently with its new weight $w_t(v)$. If $w_t(v) < w_{t-1}(v)$, then certain calls in the corresponding cell have been dropped. If $w_t(v) > w_{t-1}(v)$, then new call requests have come in, and channels must be assigned to them in such a way that interference constraints between new and old assigned channels are satisfied.

In the simplest model, no reassignment of channels is allowed, and the only freedom the node server has is to decide which channels will be used for new call requests. However, most modern network have the possibility of *channel reassignment*, so a call in progress can be transferred to another channel without noticeable interruption of service.

A natural requirement for online, distributed algorithms with possibility of channel reassignment is that channels are only reassigned in reaction to local changes of weight. Hence the definition of k-locality extends to online algorithms in the following way. An online distributed channel assignment algorithm is *k-local* if, at each time instance t, each node server can obtain knowledge about changes in weight at all nodes at distance k or less from it, and can respond to these changes by reassigning its calls to other channels.

For the evaluation of our algorithms, we use the well-known criteria of performance ratio (for static algorithms) and competitive ratio (for online algorithms). An approximation algorithm for channel assignment has **performance ratio** p when the span of the assignment produced by the algorithm on (G, w) is at most $pS(G, w) + \Theta(1)$, where $S(G, W)$ is the minimum span of any assignment for (G, w). Here we consider the span to be a function of the weights and the size of the graph, so the $\Theta(1)$ term can include terms that depend on the constraints c_i. An online channel assignment algorithm is *c-competitive* when for any weight sequence w_t, the span used by the algorithm never exceeds $c \cdot max_t S(G, w_t) + \Theta(1)$.

To evaluate the performance ratio and competitive ratio we use a folklore bound derived from the maximum weight on a clique in a graph. Precisely, if U is a collection of nodes of a constrained graph $G = (V, E, c_0, c_1, \ldots, c_\ell)$ which are all at graph distance d or less from each other, then the span of any assignment of (G, w) is at least $c_d(\sum_{u \in U} w(u) - 1)$.

In this paper, we will only describe static algorithms. This also solves the online problem, because of the following lemma.

Lemma 1.1 *(***from [JDNS98]***) Let A be a k-local static approximation algorithm for channel assignment with performance ratio p. Then A can be converted to a k-local p-competitive online channel assignment algorithm.*

In fact, the online algorithms derived from static algorithms with performance ratio p will have the stronger property that for any weight sequence w_t, at any time instance t the span of the assignment produced by the algorithm does not exceed $pS(G, w_t) + \Theta(1)$.

2 Algorithms

In this section, we describe two distributed online or static channel assignment algorithms for a constrained hexagon graph $G = (V, E, a, a, b)$, where $a \geq b > 0$. The first algorithm is based on an initial assignment which would be optimal if all the weights were equal, combined with a borrowing phase to account for differences in weight. The second algorithm uses an auxiliary algorithm designed for constraints $a = 1$, $b = 0$ to find an assignment for general parameters a and b. Its performance and locality will depend on the auxiliary algorithm that is used.

2.1 Arithmetic Borrowing (AB) Algorithm

We first describe an algorithm which is valid when $a \geq 2b$. This algorithm is based on an initial labelling f of the nodes derived from the coordinates of the nodes in the triangular lattice. Precisely,

$$f(i, j) = ia + j(3a + b) \pmod{N},$$

where $N = 5a + 3b$. The labelling is shown in Figure 2. Labellings of this type were introduced in [vdHLS98], where they were used as channel assignments for unweighted graphs.

Note that f has the property that $N \geq a$, $a \leq f(i, j) \leq N - a$ for all neighbours of $(0, 0)$, and $b \leq f(i, j) \leq N - b$ for all nodes at graph distance 2 from $(0, 0)$. (Since $f(0, 2) = a - b$

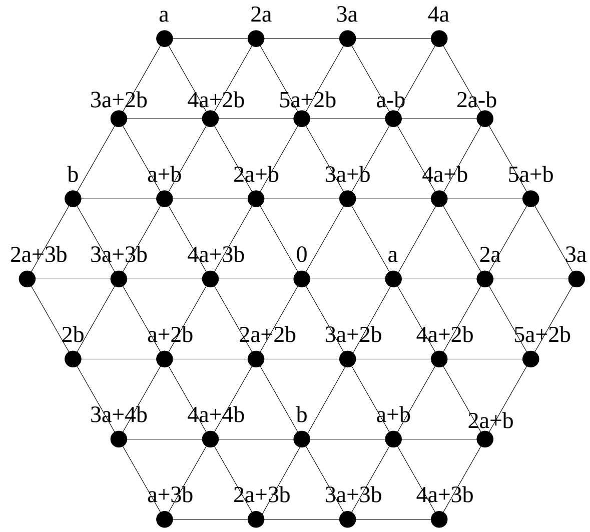

Figure 2: The arithmetic assignment used in the AB algorithm.

this only holds if $a \geq b$.) Hence f naturally generates a channel assignment for G. Namely, assign to each node (i, j) only channels from the set $\{f(i, j) + kN \mid k \in \mathbb{N}\}$. Now for any pair of channels γ_1, γ_2 assigned to nodes (i, j) and (i', j') at graph distance 2 we have that $|\gamma_1 - \gamma_2| \equiv |f(i', j') - f(i, j)| \equiv |f(i' - i, j' - j)| \bmod N$. Since $(i' - i, j' - j)$ has graph distance 2 to $(0, 0)$, it follows that $|\gamma_1 - \gamma_2| \geq b$. A similar argument holds for nodes at graph distance 1.

It is also the case that $b \leq f(i, j) \leq N - b$ even for nodes (i, j) at graph distance 3 of $(0, 0)$. So, any channel assignment derived from f has the property that the nodes at graph distance 3 also have separation at least b.

2.1.1 Local Information Every node $v = (i, j)$ knows its value under f, $f(i, j)$, and is able to identify its neighbours and their position with respect to itself, and receive information about their weight. Specifically, v is able to identify the neighbours $(i + 1, j)$ and $(i + 1, j - 1)$, and to calculate the maximum weight on a clique among its neighbours. More precisely, v can calculate $T(v)$, where

$$T(v) = \max\{\sum_{u \in C} w(u) \mid C \text{ a clique}, d(u, v) \leq 1 \text{ for all } u \in C\}$$

2.1.2 Description The AB algorithm begins by assigning channels according to f, followed by two 'borrowing phases' where nodes with high demand borrow unused channels from their neighbours. The assignment of channels to each node $v = (i, j)$ is done in three phases.

Phase 1. Node v receives channel $f(i, j) + kN$, $0 \leq k \leq min\{w(v), T(v)/3\}$.

Phase 2. If v has weight higher than $T(v)/3$, say $w(v) = T(v)/3 + \alpha$, $\alpha > 0$, then v will try to borrow channels from its neighbour $x = (i+1, j)$. Precisely, if $w(x) < T(v)/3$, then v receives channels $f(i+1, j) + kN$, $w(x) \leq k < min\{w(x) + \alpha, T(v)/3\}$. Let $\beta = max\{0, T(v)/3 - w(x)\}$ be the maximum number of channels that v receives in this phase.

Phase 3. If v still has unfulfilled demand after the last phase, in other words, if $\alpha > \beta$, then v borrows the remaining channels from its neighbour $y = (i+1, j-1)$. Precisely, in this phase v receives channels $f(i+1, j-1) + kN$, $T(v)/3 - \alpha + \beta \leq k < T(v)/3$.

2.1.3 Correctness The channels assigned to a node in Phase 1 will be called the *base channels*, and the channels assigned in Phases 2 and 3 the *borrowed channels*. By the argument given at the introduction of f, there is no possible conflict (violation of the separation constraints) between base channels.

We saw earlier that f has the additional property that for any nodes (i, j) and (i', j') at graph distance 3, $b \leq f(i, j) - f(i', j') \leq N - b$, so two base channels assigned to nodes which are at graph distance 3 from each other also have a separation of at least b between them. Since any node borrows from a neighbour, this implies that there can be no violation of a b-separation constraint between a borrowed channel and a base channel. Moreover, since the two nodes from which a node can borrow are also neighbours, there can be no violation of a b-constraint between borrowed channels, except possibly when two nodes at graph distance 2 actually borrow the same channel. An examination of Figure 2 shows that the latter is impossible.

Let us then consider possible violations of the a-separation constraint. Since a node only borrows from its neighbours, and the cosite constraint, a, is the same as the separation constraint between neighbours, no conflict can occur between channels assigned to the same node.

Since our algorithm is based on an arithmetic assignment, and the assignment at each node is identical up to translation along the lattice, it suffices to consider possible conflicts that involve node $v = (0, 0)$. Each neighbour (i, j) of v has the property that $2a \leq f(i, j) \leq N - a$, so there is no conflict between the base channels of the neighbours of v and the channels (of the form $a + kN$) borrowed by v from its neighbour $x = (1, 0)$ in Phase 2.

At first sight there might be a possible conflict between the channels borrowed by v from its neighbour $y = (1, -1)$ in Phase 3 (of the form $3a + 2b + kN$) and the base channels on node $z = (0, 1)$ (of the form $3a+b+kN$). Suppose then that $w(v) = T(v)/3+\alpha$ where $\alpha > 0$, and $\beta < \alpha$, where $\beta = max\{0, T(v)/3 - w(x)\}$ is the maximum number of channels v could have borrowed from x in Phase 2. Then, in Phase 3 v borrows the channels $f(1, -1) + kN$, for $T(v)/3 + \alpha - \beta \leq k < T(v)/3$, while the base channels assigned to z are the channels $f(0, 1) + kN$, for $0 \leq k < min\{w(z), T(z)/3\}$. If the highest value of k for z is less than the lowest value of k for v, then there is no possibility for conflict. Let $T = w(v) + w(x) + w(z)$. Since $\{v, x, z\}$ is a clique with all its members within graph distance 1 of v, $T(v) \geq T$. Now, $w(z) = T - w(x) - w(v) \leq T(v) - w(v) - w(x) = 2T(v)/3 + \alpha - w(x) \leq T(v)/3 + \alpha - \beta$, so there is no conflict.

Lastly, we consider possible conflict between borrowed channels. Since the assignment in invariant under translation, and since the base channels form a valid assignment, there can be no conflict between channels borrowed both in Phase 2 or in Phase 3. However, there might be a possible conflict between a channel assigned to v in Phase 3 with channels assigned in Phase 2 to $z = (0, 1)$ or to $u = (-1, 1)$. Suppose that v borrows in Phase 3. Then $w(v) + w(x) > 2T(v)/3$. Let $T = w(v) + w(x) + w(z)$. Then $w(z) \leq T - 2T(v)/3 \leq T/3 \leq T(z)/3$, so z does not borrow any channels.

Let α and β be as before, and suppose that $w(u) = T(u)/3 + \alpha'$, where $\alpha' > 0$. Then u borrows channels $3a + b + kN$, $w(z) \leq k < min\{w(z) + \alpha', T(u)/3\}$ from its neighbour z, while v borrows channels $3a+2b+kN$, $T(v)/3-\alpha+\beta \leq k < T(v)/3$ from $y = (1, -1)$. Let $T = w(v) + w(z) + w(u)$. Then $w(z) + \alpha' = w(z) + w(u) - T(u)/3 \leq w(z) + w(u) - T/3 = 2T/3 - w(v) \leq 2T(v)/3 - w(v) = T(v)/3 - \alpha$. So the highest channel borrowed by u lies below the lowest channel borrowed by v, and thus there is no conflict.

2.1.4 Performance ratio

No channel higher than $(T(v)/3)N$ is assigned to any vertex v, while for each v, a channel greater than $(T(v)/3 - 1)N$ is assigned to a node in the neighbourhood of v. So the span of this assignment is $\omega(5a + 3b)/3 - \Theta(1)$, where ω is the maximum weight on any clique of G. By the lower bound mentioned in Section 1.2, $S(G, w) \geq a\omega - \Theta(1)$. So the performance ratio equals $\frac{\omega(5a+3b)/3}{\omega a} = 5/3 + b/a$.

We have thus obtained the following result.

Theorem 2.1 *The AB algorithm is a 1-local distributive channel assignment algorithm for parameters a, b where $a \geq 2b$, with performance ratio $5/3 + b/a$. Moreover, it can be converted into a 1-local online channel assignment algorithm with competitive ratio $5/3 + b/a$.*

2.2 The Extended Multicolouring (EM) Algorithm

When the constraint between nodes at graph distance 2 is dwarfed by the constraint between neighbouring nodes, then channel assignments derived from *multicolourings* of the hexagon graphs are expected to give good performance. A multicolouring is an assignment of sets of integers (‘colours’) to the nodes of a weighted graph (G, w), so that each node v receives a set of $w(v)$ colours, and colour sets assigned to adjacent nodes are disjoint. Hence, a multicolouring of a graph (G, w), where $G = (V, E)$, corresponds to a channel assignment of of the weighted constrained graph (G', w), where $G' = (V, E, 1, 1)$. By multiplying all colours of a multicolouring by a, we obtain a channel assignment for (G'', w), where $G'' = (V, E, a, a)$.

With some adjustments, we can also obtain a channel assignment for a constrained graph (V, E, a, a, b) from a multicolouring. The EM algorithm uses as a basis any multicolouring algorithm. It also uses a (unweighted) colouring of $G^2 - G$. (The graph $G^2 - G$ is the graph which has the same node set as G, and two nodes are adjacent in $G^2 - G$ precisely when they have graph distance 2 in G.) A colouring is an assignment of one colour to each node, so that colours on adjacent nodes are distinct. A minimal colouring for $G^2 - G$ is shown in Figure 3. Let $h : V \to \{0, 1, 2, 3\}$ denote this colouring.

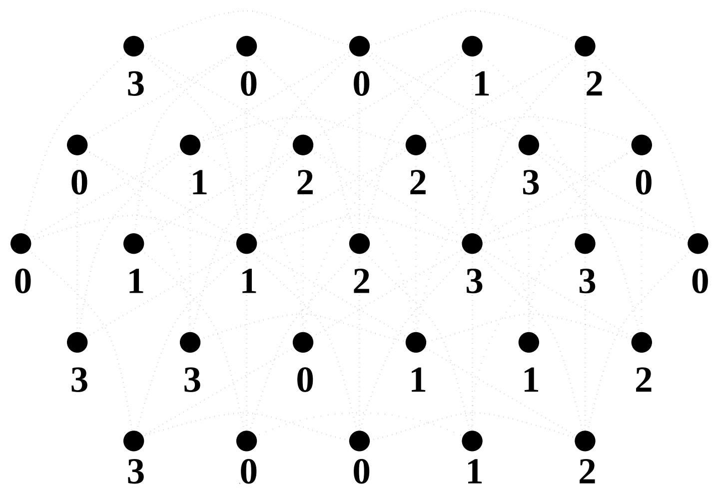

Figure 3: A colouring of G^2 using four colours.

2.2.1 Local Information Every node v knows its colour $h(v)$ in the colouring of $G^2 - G$. Moreover, any local information for required by the multicolouring algorithms is available to v.

2.2.2 Description Every node v finds the set $f(v)$ of colours that would be assigned to it by the multicolouring algorithm (operating on the unconstrained graph, with the same weight vector). The final assignment g is then found by combining h and f in the following way:

$$g(v) = \{(a + 3b)i + bh(v) \mid i \in f(v)\},$$

for all $v \in V$.

2.2.3 Correctness It follows from the definition that any two channels assigned to the same node v differ by at least $a+3b$. Let $x_1 \in g(v)$ and $x_2 \in g(u)$ be two channels assigned to nodes u and v, and assume that $x_1 \geq x_2$. Then there exist distinct channels $i \in f(u)$ and $j \in f(v)$ such that $x_1 = (a + 3b)i + bh(u)$ and $x_2 = (a + 3b)j + bh(v)$.

If $d(u, v) = 1$, then $f(u)$ and $f(v)$ have no elements in common. So $x_2 - x_1 = (a + 3b)(i - j) + b(h(v) - h(u)) \geq (a + 3b) - 3b = a$. If $d(u, v) = 2$, then $h(u) \neq h(v)$. If $i \neq j$, then as in the previous case, $x_2 - x_1 \geq a \geq b$. If $i = j$, then $x_2 - x_1 = b(h(u) - h(v)) \geq b$.

2.2.4 Performance ratio The performance ratio of the EM algorithm will depend on the performance ratio of the underlying multicolouring algorithm. Most multicolouring algorithms use the weighted clique number as a lower bound to evaluate the performance. For a weighted graph (G, w) the **weighted clique number**, $\omega(G, w)$, is the maximum sum of weights of any clique, so

$$\omega(G, w) = max\{\sum_{v \in C} w(v) \mid C \text{ is a clique in } G\}.$$

Suppose the multicolouring algorithm never uses more than $k\omega(G,w)+\Theta(1)$ colours. By the bound mentioned in Section 1.2, $a\omega(G,w)-a$ is a lower bound on the span of any channel assignment of (G,w), where $G=(V,E,a,a,b)$. The span of the channel assignment found by the EM algorithm is $(a+3b)s-\Theta(1)$, where s is the number of colours that would have been used by the multicolouring algorithm on the unconstrained graph with the same weight vector. Hence, the EM algorithm has performance ratio

$$\frac{(a+3b)\omega(G,w)+\Theta(1)}{a\omega(G,w)-a}=k(1+\frac{3b}{a})+\Theta(1).$$

We see that the locality and the performance ratio of the EM algorithm depend on those of the multicolouring algorithm used. The best known results for multicolouring of hexagon graphs are stated in below.

Theorem 2.2 (From [JDNS98]) *For hexagon graphs there exists a 4-local multicolouring algorithm that uses at most* $4/3\omega$ *colours, a 2-local algorithm that uses at most* $17/12\omega$ *colours, and a 1-local algorithm that uses at most* $3/2\omega$ *colours. Here* $\omega=\omega(G,w)$*, the weighted clique number of the graph and weight vector.*

We summarize these results in the following theorem.

Theorem 2.3 *The EM algorithm, used with a* $k-local$ *multicolouring algorithm which uses at most* $p\omega(G,w)+\Theta(1)$ *colours on any weighted hexagon graph* (G,w)*, is a* k*-local distributed channel assignment algorithm for constrained hexagon graphs, with performance ratio* $p(1+\frac{3b}{a})$*. Using the multicolouring algorithms from Theorem 2.2, we can obtain the following values of* k *and* p*.*

k	p
4	$\frac{4}{3}+4\frac{b}{a}$
2	$\frac{17}{12}+\frac{17b}{4a}$
1	$\frac{3}{2}+\frac{9b}{2a}$

Moreover, the EM algorithm, used with a k*-local multicolouring algorithm, can be converted into a* k*-local* p*-competitive online channel assignment algorithm, where* k *and* p *take values from the table above.*

3 Conclusions

Two distributed online algorithms for hexagon graphs with separation constraints are given. The AB algorithm is 1-local, and (13/6) competitive if $a/b=2$, (the smallest a/b ratio for which the algorithm is valid), and close to being 5/3-competitive when a/b becomes large. The EM algorithm is derived from a multicolouring algorithm, and its competitive ratio depends on the locality required. The best competitive ratio that can be achieved with the known multicolouring algorithms is $4/3 + 4(b/a)$. For $a/b > 9$, this is the best competitive ratio available.

For the case where $1 \leq (a/b) \leq 2$, the AB algorithm is not valid, and the best EM algorithm gives a competitive ratio of at least 10/3 (when $(a/b) = 2$), and at worst 16/3. In [FS98], Feder and Shende give a distributed online algorithm for the case where $a = 1, b = 1$ with competitive ratio 7/3. This algorithm can be easily modified to give a $(7a)/(3b)$-competitive algorithm for the general case, since any assignment for a constrained graph (V, E, a, a, a) also is an assignment for (V, E, a, a, b). Hence, for this range of (a/b) their algorithm is best known. (We note here that the Feder-Shende algorithm, though distributed, does not fit the definition of k-locality, since node servers need to know the actual assignment at the nodes within graph distance 2, not just the weights of those nodes.) However, better competitive ratios may be achieved by a more specialized algorithm, and we recommend that further work concentrate on the case where $b \leq a \leq 2b$.

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