Mathematics Geometry IV

Circles and Right Triangles

Québec ⬛⬛

Mathematics
Geometry IV

Circles and Right Triangles

1.   INTRODUCTION

This Definition of the Domain for Summative Evaluation describes and classifies the essential and representative elements of the secondary-level adult education Mathematics program and, more specifically, of the course entitled Geometry IV (Circles and Right Triangles). As such, it gives an overview of the program, but should by no means replace the program itself. The purpose of defining the domain is to ensure that all summative evaluation instruments are consistent with the overall program.

The Definition of the Domain for Summative Evaluation for each course in this program is organized in a similar manner; however, the content of this definition of the domain is specific to the course entitled Geometry IV (Circles and Right Triangles).

The goal of the Definition of the Domain for Summative Evaluation is to prepare examinations that are valid from one version to another or from one school board to another, taking into account the responsibilities shared by the Ministère de l'Éducation and the school boards.

2. PROGRAM ORIENTATIONS AND CONSEQUENCES FOR SUMMATIVE EVALUATION

ORIENTATIONS	CONSEQUENCES
The main objective of the secondary-level adult education Mathematics program is to help students fully understand mathematical concepts.	Evaluation should involve verifying whether the student has fully understood the different concepts.
The program is designed to help students master the use of certain mathematical tools used in the field of science and technology or in different trades.	Evaluation items should pertain to situations in the field of science and technology or to situations related to trades.
The program aims to provide students with the skills they need to process information by applying mathematical models and appropriate strategies for solving problems.	Evaluation items should involve performing tasks that require the students to classify information, use mathematical models and solve problems.
The program also aims to improve the students' ability to clearly relate information using mathematical language.	Evaluation items should involve performing tasks that require the use of mathematical language. The appropriateness and clarity of the language used should be taken into account in the marking process.
The program is intended to help students develop a systematic work method.	Evaluation items should require the students to present their work in a clear and structured manner. This should be taken into account in the marking process.
The program will help students master the use of technological tools.	The use of a scientific calculator is permitted for the examinations related to this course.

3. CONTENT OF THE PROGRAM FOR PURPOSES OF SUMMATIVE EVALUATION

Concepts

Relationships governing measurements in circles
- geometric principle related to a relationship in a circle
- geometric principle related to the ratio of the measurements of two circles or to measurements of angles or arcs in a circle
- measurement of various elements and of the existing ratio of one or two circles
- measurement and characteristics of angles in a circle
- relationships between the segments or arcs in a circle
- relationships governing measurements in two circles
- equivalence of two expressions describing relationships between measurements of arcs and angles in a circle
- problem involving the relationships governing measurements within the same circle or between two circles

Relationships governing measurements in right triangles
- theorem or corollary related to a relationship governing measurements in a right triangle
- measurement of angles or of lengths in a right triangle
- relationship between the measurements of segments in a right triangle
- problem involving the relationships governing measurements in one or two right triangles

Skills

Each skill is defined within the context of a mathematics program.

Structuring   Being familiar with the fundamentals of mathematics, understanding some mathematical concepts and establishing simple relations among them.

                   Possible actions: to associate, classify, compare, complete, describe, define, contrast, distinguish, state, enumerate, group, name, rank, organize, recognize, arrange, and so on.

Mathematizing   Interpreting a given situation using a mathematical model (arithmetic, algebraic or graphical).

                   Possible actions: to formalize, illustrate, represent, schematize, symbolize, translate, transpose, and so on.

Operating   Performing a given operation or transformation.

                   Possible actions: to calculate, construct, break down, perform, estimate, evaluate, isolate, measure, reconstruct, solve, draw, transform, verify, and so on.

Analyzing   Demonstrating, in an organized fashion, the complex connections between concepts or definitions and their related actions and illustrations.

                   Possible actions: to conclude, correct, deduce, derive, demonstrate, explain, extrapolate, infer, justify, and so on.

Synthesizing   Effectively integrating a variety of concepts and skills to solve a problem.

                   Possible actions: to solve a problem.

4. TABLE OF DIMENSIONS

CONCEPTS SKILLS	RELATIONSHIPS GOVERNING MEASUREMENTS IN CIRCLES 55%	RELATIONSHIPS GOVERNING MEASUREMENTS IN RIGHT TRIANGLES 45%
STRUCTURING **15%**	Identify the geometric principle related to a relationship in a circle. **1** **5%**	
	Identify the geometric principle related to the ratio of the measurements of two circles or to measurements of angles or arcs in a circle. **2** **5%**	Choose the geometric principle pertaining to a relationship governing measurements in a right triangle. **10** **5%**
MATHEMATIZING **5%**	Determine the measurement of an element of a circle or right triangle. **3** **5%**	
OPERATING **30%**	Determine the measurement or the ratio of various elements in one or two circles. **4** **10%**	Determine the measurement of angles or of lengths in a right triangle. **11** **10%**
	Determine the measurement of various angles or arcs in a circle. **5** **10%**	
ANALYZING **20%**	Verify relationships between the segments or arcs in a circle. **6** **5%**	Verify relationships between the measurements of segments in a right triangle. **12** **5%**
	Verify relationships governing measurements in two circles. **7** **5%**	
	Verify the equivalence of two expressions describing relationships between measurements of arcs or angles in a circle. **8** **5%**	
SYNTHESIZING **30%**	Solve a problem involving the relationships governing measurements within the same circle or between two circles. **9** **10%**	Solve two problems involving the relationships governing measurements in one or two right triangles. **13** **20%**

5.   OBSERVABLE BEHAVIOURS

Examination items should be formulated on the basis of the observable behaviours listed below. The requirements and restrictions specified in the dimensions and the objectives of the program must be observed.

Dimension 1

Identify the geometric principle that makes it possible to justify a relationship governing measurements in a circle. A figure is included along with the principle.
(structuring)   /5

Dimension 2

Identify the geometric principle that makes it possible to justify a ratio of the measurements of two circles, the measurement of an angle or the measurement of an arc in a circle. A figure is included along with the principle.
(structuring)   /5

Dimension 3

Determine the measurement of an element of a circle or a right triangle. The given measurements for the figure are literal.
(mathematizing)   /5

Dimension 4

Determine the measurement of two elements or of the ratio of the elements, given certain measurements or certain ratios. The elements may be a radius, a diameter, a segment, a chord, an area, a circumference or an arc, and they may belong to the same circle or to two circles. The students must identify the geometric principle that justifies their answer.
(operating)   /10

Dimension 5

Determine the measurement of arcs or angles, given an illustration of a circle with measurements of angles or arcs. Three measurements must be determined. The students must identify the geometric principle that justifies their answer.
(operating)   /10

Dimension 6

Given a circle, whose arcs, chords, segments or tangents are illustrated and described, and statements describing the relationship between segments or arcs, determine which statements are true, justify the answer and indicate the numbers of the appropriate theorems or corollaries. The measurements provided for certain elements are literal.
(analyzing)   /5

Dimension 7

Given a relationship governing measurements in two circles and statements describing other relationships governing measurements between these two circles, determine which statements are true.
(analyzing)   /5

Dimension 8

Given a circle, whose angles or arcs are illustrated and described, demonstrate that two expressions describing the relationship between measurements of angles or between measurements of arcs are equivalent. The measurements provided for certain elements are literal. The students must clearly show all their work and identify the geometric principles that justify their answer, if applicable.
(analyzing)   /5

Dimension 9

Solve a problem involving the relationships governing measurements within the same circle or between two circles. A figure must be included along with the statement of the problem. The students must clearly show all their work and identify the geometric principles that justify their answer, if applicable.
(synthesizing)   /10

Dimension 10

Identify the geometric principle that makes it possible to justify a relationship governing measurements in a right triangle. A figure is included along with the principle.
(structuring)   /5

Dimension 11

Determine the measurement of angles or lengths, given an illustration of a right triangle whose measurements are indicated. Three measurements must be determined. The students must identify the geometric principle that justifies their answer.
(operating)   /10

Dimension 12

Given a right triangle, whose elements are identified, and literal statements describing a relationship between the elements of this triangle, determine which statement is true. The students must justify their answer by identifying the appropriate geometric principle.
(analyzing)   /5

Dimension 13

Solve problems involving relationships governing measurements in right triangles. The figure may contain two right triangles and must include the statement of the problem. The students must clearly show all their work and identify the geometric principles that justify their answer, if applicable.
(synthesizing)   /20

6. JUSTIFICATION OF CHOICES

In the examination, 15% of the items test the students' **STRUCTURING** skills by verifying their understanding of certain concepts:
– the geometric principle related to a relationship in a circle
– the geometric principle related to the ratio of the measurements of two circles or to measurements of angles or arcs in a circle
– the geometric principle related to a relationship governing measurements in a right triangle

In the examination, 5% of the items test the students' **MATHEMATIZING** skills by verifying whether they are able to translate a given situation into a mathematical model:
– translating a measurement of a circle or a right triangle into symbolic language

In the examination, 30% of the items test the students' **OPERATING** skills by verifying whether they have mastered certain operations or transformations:
– determining the measurement of various elements in one or two circles
– determining the measurement of various angles or arcs in a circle
– determining the measurement of angles or lengths in a right triangle

In the examination, 20% of the items test the students' skill in **ANALYZING** information; they involve verifying whether the students have the ability to make connections:
– by verifying relationships between the segments or arcs in a circle
– by verifying relationships governing measurements involving two circles
– by verifying relationships between the measurements of segments in a right triangle
– by verifying the equivalence of two expressions describing relationships between measurements of arcs and angles in a circle

In the examination, 30% of the items test the students' **SYNTHESIZING** skills by verifying their ability to:
– solve problems
– use a rigorous work method
– communicate clearly using mathematical language

7. DESCRIPTION OF THE EXAMINATION

A. TYPE OF EXAMINATION

The summative examination will be a written examination consisting of multiple-choice, short-response or extended-response items.

The items should take into account the restrictions and the requirements specified in the dimensions and the objectives of the program. The weighting of marks should be consistent with the percentages set out in the table of dimensions.

B. CHARACTERISTICS OF THE EXAMINATION

The examination will be administered in a single session lasting no more than two and a half hours.

Students are permitted to use a scientific calculator; however, they are not permitted to use a graphing calculator.

A list of geometric principles will be provided (see appendix).

C. PASS MARK

The pass mark is set at 60 out of 100.

APPENDIX

GEOMETRIC PRINCIPLES

RELATIONSHIPS GOVERNING MEASUREMENTS WITHIN A CIRCLE

1. Any perpendicular bisector of a chord is a diameter of a circle.
2. The longest chord of a circle is a diameter.
3. In a circle, any radius perpendicular to a chord divides that chord into two congruent segments.
4. In a circle, any radius perpendicular to a chord divides the subtended arc into two congruent arcs.
5. In a circle, arcs located between two parallel chords are congruent.
6. Two chords are congruent if they are equidistant from the centre of the circle.
7. In a circle, congruent chords subtend congruent arcs and, conversely, congruent arcs are subtended by congruent chords.
8. Any line tangent to a circle is perpendicular to the radius that shares the point of tangency.
9. For any circle, two tangent segments originating from the same exterior point are congruent. (The segments are measured from that exterior point to their respective points of tangency.)
10. Two parallel lines, be they tangents or secants, intercept congruent arcs of a circle.

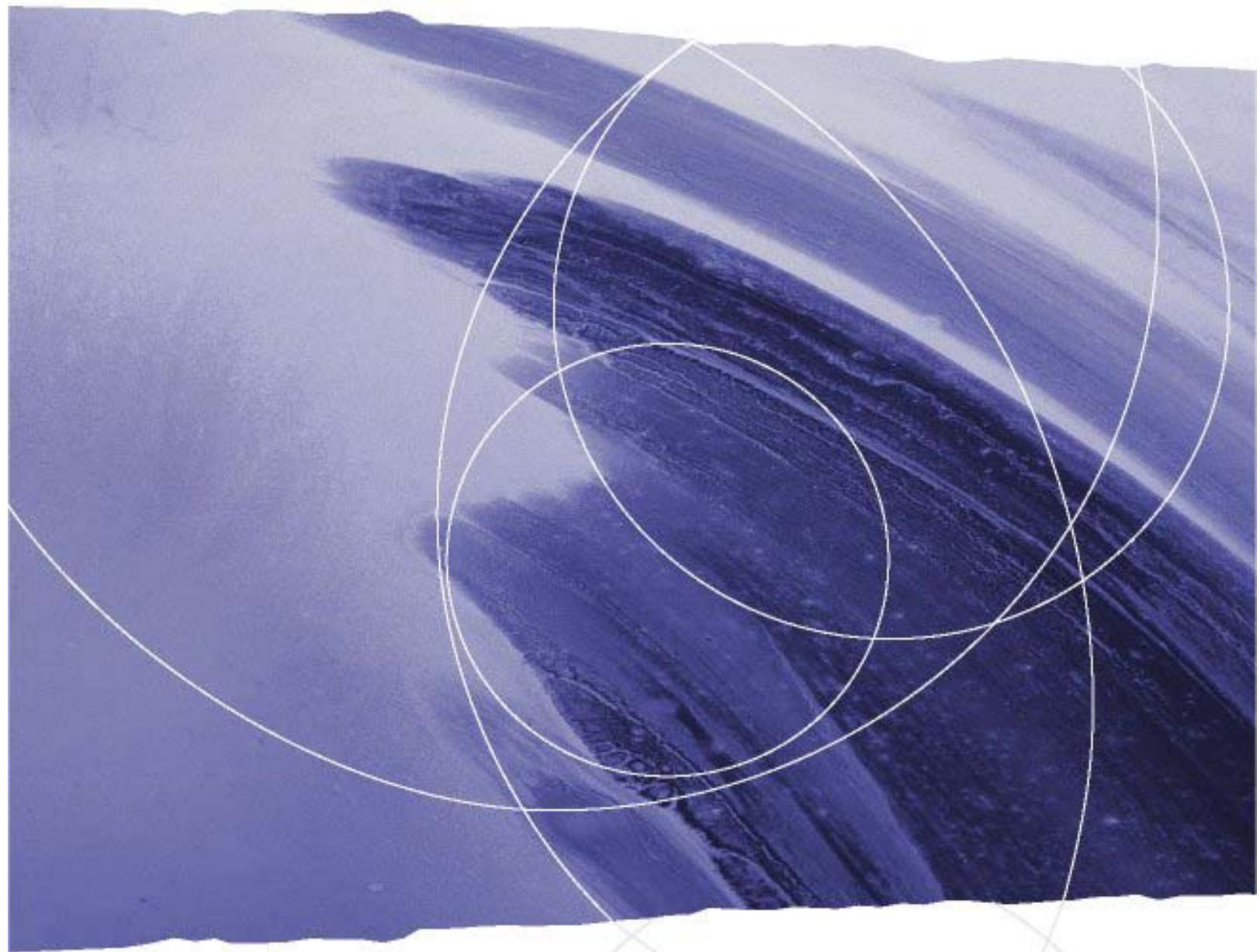### RELATIONSHIPS GOVERNING MEASUREMENTS BETWEEN TWO CIRCLES

11. The circumferences of two circles have the same ratio as their radii.
12. The areas of two circles have the same ratio as the squares of their radii.
13. The measures of similar arcs of two circles have the same ratio as their radii.

RELATIONSHIPS GOVERNING ANGULAR MEASUREMENTS WITHIN A CIRCLE

14. In a circle, the measure of a central angle is equal to the measure of its intercepted arc.
15. In a circle, the measure of an inscribed angle is one-half the measure of its intercepted arc.
16. The measure of an angle formed by two chords intersecting in the interior of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
17. The measure of an angle formed by two tangents, a tangent and a secant, or two secants is one-half the difference of the measures of the intercepted arcs.

RELATIONSHIPS GOVERNING MEASUREMENTS IN RIGHT TRIANGLES

18. The hypotenuse of a right triangle inscribed in a circle is always a diameter of that circle.
19. In a right triangle, the length of the median to the hypotenuse is one-half the length of the hypotenuse.
20. In a right triangle with a 30° angle, the length of the side opposite this angle is one-half the length of the hypotenuse.
21. A right triangle and the altitude to its hypotenuse form two right triangles that are similar to the given triangle and to each other.
22. The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse.
23. The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse.
24. The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse.
25. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides (Pythagorean theorem).