Optimization Based Decision Support for Order Promising in Supply Chain Networks

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Abstract

We are interested in order promising models for supply chain management in the context of firms involved in eCommerce. In this context, buyers and suppliers constantly exchange information. All parties update production, warehousing, and distribution plans based on negotiations involving product price, features, and due dates. One interesting question, from the supplier's point of view, is how a supplier firm should take part in these negotiations. Of particular interest to us at this point in time is to build an optimization based decision support system (DSS) that will help an agent of the firm promise orders, i.e., quote due dates and prices. We present a model and illustrate it with a small example to show how the DSS addresses a useful need.

Key Words

Supply Chain Management, Order Promising, Demand Planning, ATP, Decision Support Systems, Multi Commodity Network Flows, Network Programming

Introduction

This paper deals with order promising in supply chain networks. Order promising involves the quoting of due-dates to customers and when quotes are accepted, entering them as firm orders or commitments. This function is also sometimes referred to as ATP (available-to-promise) or AATP (Advanced ATP). Pibernik (2005) presents a classification of order promising methods. The order promising system has a link with manufacturing and distribution planning because order intake through such a system forms the basis of manufacturing and distribution planning. However, in order to effectively promise orders, their impact on manufacturing and distribution plans must be considered. The contribution of this paper is to that effect.

We consider supply chain management in the eCommerce context. For an understanding of this context and the role of quantitative models for planning and operations, the reader is referred to Keskinocak and Tayur (2000). All firms in such a context are involved in inputs transformation processes, and outputs. It is clear that in such a system, one firm only controls a small part of the global supply chain, with certain resources (production and warehousing facilities, transportation infrastructure, manpower, etc.) directly under its influence, and most resources out of its control (they are under the control of other firms) .

According to Shapiro (1999), production planning is one of the challenges in the modeling of such systems and associated with that is demand forecasting and demand and order management. Within the order management function of a supplying firm, the firm and its buyers have constant (often automated) information exchange. A buyer inquires about products, features, and prices. The firm gives out this information. The buyer asks for an RFQ (request for quotation) on product price, quantities, and due dates. The firm submits quotes. Several rounds of negotiations take place, and a purchasing contract is signed. The framework for such systems has been discussed in Montreuil et al. (2000) and Frayret et al. (2001).

One interesting question, from the supplier's point of view, is how a supplier firm should take part in these negotiations; namely, how the supplying firm can quote and negotiate on prices, products and product features, and due dates. In this paper, we show how an optimization based decision support system can be implemented to support these features. We cannot overemphasize the importance of demand management for the following reasons: a) it is the driver for all production and logistical planning in a firm's supply chain and b) it is very important for the survival and growth of the firm; if a customer does not receive good quality products on time, future orders may be in jeopardy.

Literature Review and Research Motivation

An argument can be made for using quantitative models in demand planning and due date quoting, the focus of this paper. Motivation for such models is found in McClelland (1998) who makes the case for tying order promising and the master production schedule. Master production scheduling or master planning involves mid-term decision making for effective utilization of production and transportation capacities and the balancing of supply and demand (Rhode and Wagner, 2000). Related to master planning is the function of order promising which includes available-to-promise (ATP) (Kilger and Schneeweiss, 2000). The model in the current paper could be viewed as bridging the gap between master planning and order promising.

There has been some interest in models for order processing of late. Kawtummachai and Hop (2005) look at the problem of allocating products to suppliers at an operation level and try to minimize purchasing cost based on service satisfaction history. They also mention that very few researchers have tried the optimization approach to order allocation. Pibernik (2005) states that quantitative methods for quantity and due date promising are not addressed extensively in the literature. He then goes on to build a theoretical framework for models and algorithms to support due date quoting. The model in our paper fits nicely into the framework.

There are models in the literature dealing with due date or order promising though most do not have the same focus as this paper. Abid et al. (2002) look at the order management problem at a factory with capacity limits facing customer orders. They try to assign these orders to time periods so that customer satisfaction is maximized. Their method is optimization based and includes side constraints like product dependency (for example, product A may not follow product B in the delivery schedule). Another relevant paper on due date assignment in manufacturing is by Luss and Rosenwein (1993). They generate candidate schedules for each order at a manufacturing plant and model due date assignment as an integer programming problem. Computational results are shown and a case is made for batch order promising. Chen et al. (2001) model the available to promise (ATP) problem as a mixed integer programming problem and look at the effects of batching interval size and material reserve policy on performance of their system. Stochastic approaches to the due date setting problem include the papers by Hegedus and Hopp (2001) and Harris and Pinder (1995). D'Amours et al. (1999) study the impact of information sharing in networked manufacturing. They consider a scenario where firms bid for production operations, storage, and transportation. Bids are structured and scheduled by a virtual enterprise 'master', which solves a linear program (LP) model with underlying network flow structure. The authors show how performance is improved when there is a high degree of collaboration and their model is of value to virtual enterprise receiving quotes from suppliers, subcontractors, transporters, or warehouses. Our interest is complementary and the DSS we develop is relevant to firms making such bids.

Since buying and selling is not a new phenomenon, there are numerous papers and treatises on the theory. A primer on the topic of negotiation has been written by Raiffa (1982). Focussing on the issue of two firms negotiating for the sale/purchase of a product, there is the generally accepted belief that the price of the product is a function of lead time (Rosenfeld et al., 1985). In this paper, the author develops cost-lead time curves (see Figure 1) for both buyer and seller based on the earlier work by Raiffa (1982). The underlying hypothesis is that as the buyer gives room to the supplier (lead time increases), the price of the product goes down. The rationale for such a belief is that the supplier has the room to optimize the production and logistics processes involved (i.e., the supply chain under the supplier's control), and can pass off some of these savings to the buyer. Similarly, the buyer's willingness to pay is a decreasing function of time. The buyer may be indifferent to early delivery but is usually very sensitive to not receiving a product on time, especially if the buyer is a firm with its own delivery schedule where receiving materials late could be catastrophic. This sets up a potential transaction between buyer and seller depending on their cost functions and the ability of the two sides to split the cost differential shown in Figure 1. When we say that a buyer may be indifferent to early delivery, we mean that buyers are not responsible for holding costs. In the Just-In-Time context, early deliveries are not desirable (Askin and Goldberg, 2002) and the curve should be modified to show higher costs when the lead time is shorter than the ideal lead time.

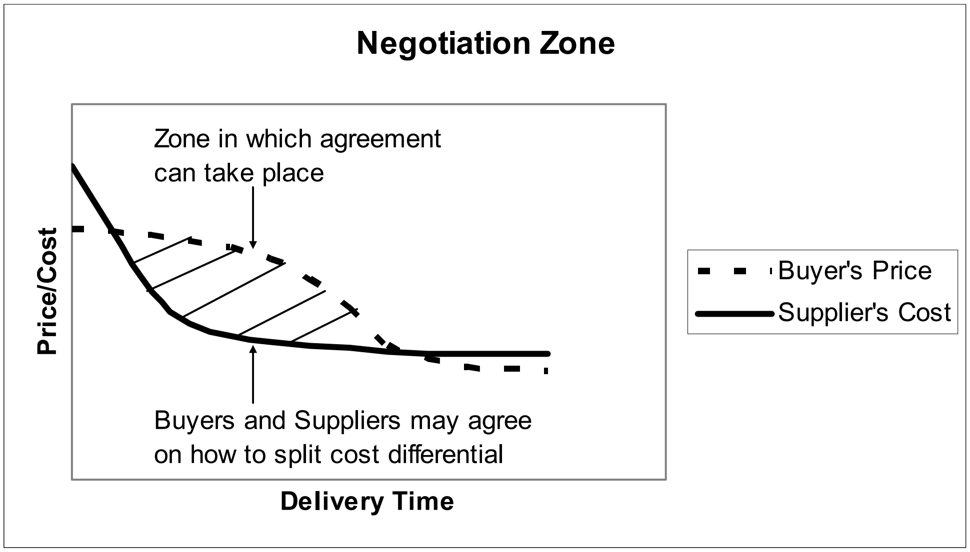

Figure 1: Supplier's production cost and customer's willingness to pay (as in Rosenfeld et al., 1985)

While we will later see why the curves, such as in Figure 1 are simplistic, the main problem for a supplying or buying firm is that such curves are impossible to build without a DSS model describing firm operations. For example, it is not clear how much of a price break a supplier should give if the buyer is willing to wait longer. Similarly, it is difficult to estimate the value of a shipment to a buyer as time progresses and the buyer's assembly schedules get impacted.

Moodie and Bobrowski (1999) model a job shop responding to bids from buyers. They assume that there is no relationship between buyer and the job shop and that the buyer and the job shop participate in setting due dates. They use a simulation model to describe the behaviour of the system under various policies. The paper assumes simplified step down function (calculated with simple parameters) to approximate Raiffa's curve and there is no indication of how the parameters may be estimated. Also, the supplier in this case is a job shop, not a supply chain firm. This paper tries to address this gap.

Model for Demand Planning in a Distribution Network

We now present a simple model for demand planning in a distribution network. The model is a development of work in Srinivasan et al (1991). Let us consider a simple supply chain firm producing multiple products. Each product enters the system, usually at one or more plants and undergoes process transformation through various stages. Finished products come out of plants and they are transported to national and regional distribution centers before being delivered to the customer. Our model can take into account direct shipping where products may skip one or more distribution stages of the supply chain and get directly to the customer.

Demand Scenarios

Customers ask for products regularly and are quoted due dates, depending on quantity. We also assume that the product is shipped to the customer site by the firm. At any given point, the firm has committed demand that it must try and satisfy as much as possible, but depending on contractual agreements the model can allow lateness in committed demand so long as a penalty is incurred. When a new customer request comes in, the firm quotes prices and due-dates. After some negotiation, the order may be placed. Negotiation can be represented by a series of dialogues. A dialogue is a series of questions and answers. Hence, several questions may be asked by the buyer or an internal planner at the supplying firm. In our first implementation, our decision support system can address the following questions (the first question may be asked by the buyer, the second is a question that the planner may ask his or her own firm; the third questions can be asked by either party, and the fourth is of concern to the planner):

1. Will the supplier be willing to commit to an order for a given combination of product, time, and quantity?
2. What is the maximum quantity (for given product and given time) that the firm can commit to a potential order without delaying the current shipment schedule?
3. What is the marginal cost of each product unit (again, for a given time) if the firm were to commit to a given combination of product, time, and quantity?
4. How does cost vary by lead time?

We believe that answers to questions like this should be available to any negotiating agent, human or otherwise and that is the goal of the DSS. For the time being, we set aside the issue of contractual obligations and long-term relationships between buyer and supplier.

Supply Chain Representation

Our demand planning model is a classic network model with nodes, arcs, and commodities. Plants, distribution centers, warehouses, etc. are the nodes in the network. The nodes themselves could be as aggregate or detailed as needed. For example, a node could be the whole plant, an assembly line, or an assembly or processing station. The arcs are transportation possibilities (direct allowable possibilities only) between nodes. Further, we assume that the system can be broken up into planning horizons of fixed length depending on the level of temporal aggregation needed in the planning.

Let the resources in the multiple source multiple destination precedence network be N= {1, 2,…, j, …,n}. Let the planning periods be T= {1, 2,.., t,...T}. Let the products in the system be represented by the set P= {1, 2, …, m,…M). The nodes in the model represent a time period for a given resource, i.e., (tj). Let the arcs represent precedence constraints, that is, allowable direct routings for product m from the output of one resource j to the input of resource k. There are two types of arcs. The first type of arc represents temporal flow between two successive resources for a product, i.e., (t,j,t+l(j,k,m),k,m), where l(j,k,m)is the lead time between nodes j and k for product m. Multiple modes of transport are also possible. In such a case, these lead times can be further indexed by mode of transport. The second arc type represents carryover of inventory from resource j in period t to resource j in period t+1, i.e., (t, j, t+1, j).

Let $A^{m}{}_{j}$ and $B^{m}{}_{j}$ represent direct successors and predecessors respectively of j for m in period t. The set of nodes with no successors (by product) is the demand set D^{m}; the set of nodes with no predecessors is the supply set S^{m}. Each node in the network has a specified initial inventory (when t=0). It is assumed generally to be positive, representing an initial supply or work in process.

Basic Demand Planning Model

We will now write an LP model to answer the four questions in the previous section. Obviously, when facing new demand, the supplying firm will try to satisfy it while trying to incur the least amount of cost possible. All previously committed demand will be part of set D^{m} and the current inquiry made by the customer is entered in the set $D_1{}^{m}$.

Further, let us assume that all costs are linear (dependent on the number of units), as is the case with most operational large scale ERP (enterprise resource planning) systems. ERP implementations usually have cost information, so it should be safe to assume that these costs can be extracted. Nodes have product dependent costs like the cost of holding and processing. Some processing costs may be product independent. For example, all engine blocks in an engine making plant may need cleaning sandblasting and the size or weight of the engine may not matter. Similarly, costs on arcs represent transportation costs and holding cost of products in transit. Costs on inventory carryover arcs represent WIP or finished inventory holding costs.

Since the network is capacitated, only so many products can be produced, transported, or stored during a period. We will impose capacities on nodes in this model and ignore arc capacities.

Sets

M = {1, 2,…,m,…}	Products or Commodities
T = {1, 2,…t, …}	Time Periods
N = {1, 2,…, j,...k,…}	Nodes
IP	Production nodes, a subset of N
IT	Transshipment nodes, a subset of N
R = {1,2,…,r,…}	A set of recipes or processes with defined input and output products
IP_r	All input products needed for recipe r
OP_r	All output products produced from recipe r
$\Gamma_{t,j,t+l(j,k,m),k}$	Arc between node (t,j) and node (t+l(j,k,m),k) for product m.
S^m = {1, 2,…, j,…}	Suppliers for product m
D^m = {1, 2, …, j,…}	Customers for product m
A^m_j	Direct successors of node j for item m
B^m_j	Direct predecessors of node j for item m

Parameters

l(j,k,m)	Lead time between node j and node k for product m
d^m_{tj}	Committed demand for product m at node (t,j)
$d_1{}^m_{tj}$	Demand of product m at node (t,j) being negotiated.
$o^m_{t,j,,t+l(j,k,m),k}$	Purchasing cost of one unit of product m (usually in raw state) so that it reaches node k at time t+l(j,k,m) from node j at time t.
a_j	Product independent unit cost incurred at node j (per period). Represents handling or overhead costs.
c_j^m	Product dependent unit cost at node j for product m. Represents production or overhead costs.
h^m_j	Unit holding cost of product m at node j (per period)
b_{jk}^m	the unit arc cost (sum of transportation and holding costs) between nodes j and k for product m (per period)
L^m_{tj}	Unit lateness cost applicable to lateness in committed demand at node j in period t
u^m_j	Capacity utilized by one unit of product flowing through node j
LI^m_{tj}	Lower bound on the inventory of product m in node j at time t
UI^m_{tj}	Upper bound on the inventory of product m in node j at time t
u^m_{tjr}	Units of commodity m required for one run of recipe r in node j ($m \in IP_r$)
v^m_{tjr}	Units of commodity m produced from one run of recipe r in node j ($m \in OP_r$)

w_{jr}	Units of capacity used by one run of recipe r at node j

Additional comments on two of the parameters in the model are in order. Modeling the ordering cost $o^{m}_{t,j,,t+l(j,k,m),k}$ this way allows us to let suppliers charge more for express orders, i.e., there may be several alternatives to obtain product m at node k from supplier j as a function of lead time.

Also, we have defined lateness cost L^{m}_{tj} as depending only on product, node, and time period because at this point in our research, we are actually interested in 'demand' management, not 'order management'. An order is defined as a combination of products ordered in one contract by a customer. It may span deliveries over several time periods. The difference between demand and order management is that in demand management, we do not keep track of individual orders and penalties resulting from delaying orders. In reality, the lateness penalty may depend on orders. If so, orders have to be modeled specifically and that is beyond the scope of this paper.

Instead, we match supply to demand by using an average lateness penalty factor L^{m}_{tj} for all demand faced by customer j in period t for product m. If the lateness penalty factor does not depend on the customer and time period, we can substitute L^{m}_{tj} by L^{m}.

Finally, it is also possible to allow orders to get delayed by no more than n periods, as seen in the adaptation of the Wagner and Whitin (1958) single item dynamic lot-size mode (Shapiro, 1993). By default, we allow for demand to be delivered early at the customer site.

Decision Variables

$f^{m}_{t,j,t+l(j,k,m),k}$	Flow on arc $\Gamma_{t,j,t+l(j,k,m),k}$
δ_{tj}^{m}	The amount of committed demand backlogged at node j in time t for

	product m
I^m_{tj}	Inventory of product m in node j ($j \notin S$) at the start of period t
N_{tjr}	Runs of recipe r made at node j in time period t, $j \in IP$

<u>Model DPP</u>

The objective is to minimize:

$$\sum_{t\in T}\sum_{j\in S^m}\sum_{k\in A_j^m}\sum_{m\in P} o^m_{t,j,t+l(j,k,m),k} f^m_{t,j,t+l(j,k,m),k} + \sum_{t\in T}\sum_{j\in N} a_j \sum_{m\in P}\sum_{k\in A_j^m} f^m_{t,j,t+l(j,k,m),k} +$$
$$\sum_{t\in T}\sum_{j\in N}\sum_{m\in P} c^m_j \sum_{k\in A_j^m} f^m_{t,j,t+l(j,k,m),k} + \sum_{t\in T}\sum_{j\in N}\sum_{m\in P}\left(h^m_j \frac{I^m_{t+1,j} + I^m_{t,j}}{2}\right) + \tag{1}$$
$$\sum_{t\in T}\sum_{j\in N}\sum_{m\in P}\sum_{k\in A_j^m} b^m_{jk} f^m_{t,j,t+l(j,k,m),k} + \sum_{t\in T}\sum_{j\in D^m} \delta^m_{tj} L^m_{tj}$$

The holding cost expression in the above function assumes that all material arrivals or departures at a node happen exactly mid way through the period. The node holding cost is therefore based on average inventory.

A feasible solution to the model is one which satisfies the flow conservation equations (2, 3, 4 and 5), limits on production or shipment capacities at each node (6, 7, and 8), boundary conditions on initial and final inventories (9), and non-negativity constraints (10) shown below:

$$I^m_{t+1,j} = I^m_{tj} + \sum_{k\in B_j^m} f^m_{t-l(k,j,m),k,t,j} - \sum_{k\in A_j^m} f^m_{t,j,t+l(j,k,m),k} \qquad \forall j \in \{IT\},\quad \forall t, \forall m \tag{2}$$

$$I^m_{t+1,j} = I^m_{tj} + \sum_{r} v^m_r N_{tjr} - \sum_{k\in A_j^m} f^m_{t,j,t+l(j,k,m),k} \qquad \forall j \in \{IP\},\quad \forall t, \forall m \in \{OP_r\} \tag{3}$$

$$I^m_{t+1,j} = I^m_{tj} + \sum_{k\in B_j^m} f^m_{t-l(k,j,m),k,t,j} - \sum_{r} u^m_r N_{tjr} \qquad \forall j \in \{IP\},\quad \forall t, \forall m \in \{IP_r\} \tag{4}$$

$$I^{m}_{t+1,j} = I^{m}_{tj} + \sum_{k \in B^{m}_{j}} f^{m}_{t-l(k,j,m),k,t,j} - d^{m}_{tj} - d^{\mathrm{m}}_{1\,\mathrm{tj}} + \delta^{\mathrm{m}}_{\mathrm{tj}} - \delta^{\mathrm{m}}_{\mathrm{t\text{-}1,\,j}}$$

$$\forall j \in \{D^{m}\}, \forall t, \forall m \qquad (5)$$

$$\sum_{k \in A^{m}_{j}} f^{m}_{t,j,t+l(j,k,m),k} \le I^{m}_{tj} \qquad \forall j \in \{IT \cup IP\}, \forall t, \forall m \qquad (6)$$

$$\sum_{m \in M} \sum_{k \in A^{m}_{j}} u^{m}_{j} f^{m}_{t,j,t+l(j,k,m),k} \le \mathrm{C}_{\mathrm{tj}} \qquad \forall j \in \{S^{m} \cup IT\}, \forall t \qquad (7)$$

$$\sum_{j} \sum_{r} w_{jr} N_{tjr} \le \mathrm{C}_{\mathrm{tj}} \qquad \forall j \in \{IP\}, \forall t \qquad (8)$$

$$LI^{m}_{tj} \le \mathrm{I}^{m}_{tj} \le UI^{m}_{tj} \qquad \forall j \notin \{D^{m}\}, \forall t, \forall m \qquad (9)$$

$$All \left\{ I^{m}_{tj}, \delta^{m}_{tj}, f^{m}_{t,j,t^{'}>t,k} \right\} \ge 0 \qquad (10)$$

Constraint sets (2), (3), and (4) are flow balance constraints for transshipment and production nodes. For transshipment nodes, the ending inventory is equal to the beginning inventory plus inflow less outflow. For production nodes, constraint set (3) specifies that the ending inventory of output products is a result of beginning inventory, outflow, and production. Similarly, constraint set (4) also applies to production nodes and it specifies that ending inventory of input products is a result of beginning inventory, inflow, and consumption in production.

Constraint set (5) is also a flow balance constraint; it models demand at demand point or customer (tj). We could think in terms of a bin at the customer's site that needs to be filled. The bin may be initially empty (if nothing is outstanding from the past), negative (if some items fall in the MRP past-due category), or positive (if for some reason early deliveries are made). The bin is emptied by the customer in each time period depending on what was committed to in that time period. I^{m}_{tj} is the level of inventory in the bin at the start of period t. The ending inventory is the beginning inventory plus the inflows (from predecessor nodes) minus the outflows (demands). The expression for demand is $d_{tj}^{m} + d1_{tj}^{m} - \delta_{tj}^{m} + \delta_{t-1,j}^{m}$, which can be justified as follows: $d_{tj}^{m} + d1_{tj}^{m}$ is the real demand. We can subtract δ_{tj}^{m} from it because this is backlogged demand in period t. However, we need to make good backlogged demand coming from period t-1 which is $\delta_{t-1,j}^{m}$.

Constraint set (6) is a limit on outflow at transshipment nodes. It is imposed because of the assumption we make that all in and out flows at (tj) occur in the middle of period t. Therefore, we must make sure that the outflow from (tj) should be less than initial inventory I_{tj}^{m}, i.e., flow that comes in at the middle of period t must wait at least one period before being allowed to go on.

Constraint set (7) allows us to model throughput capacities at resources like warehouses and plants. Also, suppliers cannot be expected to produce beyond their limits. For example, each unit of product at a warehouse will require shipping and there might be a limit on how much shipping resources are available at the warehouse. The same is true of plants.

Constraint set (8) allows us to model production capacities in plants at the aggregate level. The units of capacity used when one run of recipe r is made at node j is w_{jr}. We can model manpower and other resource requirements at an aggregate level with this parameter. While it is not a very detailed view of the manufacturing system, it does impose an aggregate limit on production.

LI^{m}_{tj} and UI^{m}_{tj} in constraint set (9) are upper and lower bounds on the level of stock of product m at (tj). Lower levels are dictated by safety stock concerns (one should be permitted to dip into this under exceptional circumstances implying that this restriction can be removed depending on the situation and the state of negotiation with the buyer). Upper levels are dictated by space and safety restrictions. When t=1, these bounds are set to initial inventory. Constraint set (10) enforces that all variables in the model be non negative.

Problem Solution

The above problem (we call it DPP for Demand Planning Problem) is a linear network flow problem with side constraints that can be solved using standard LP solvers. The solution tends to be degenerate in some cases because of the underlying network structure in the problem. Through our experimentation, we found that solvers that handle degeneracy solve the problem more easily than others which tend to get stuck for large problem instances. Let us now consider how the four questions it was designed to help answer can be answered.

<u>Will the supplier be willing to commit to an order for a given combination of product, time, and quantity?</u>

The answer to this question depends on the solution to the problem. There are two cases:

1. Delaying committed customer orders is never desirable:

   In this case, the δ^m_{tj} variables are dropped from DPP in the objective function (1) and constraint set (5). If there is a feasible solution to the problem, the answer is a possible yes. It would depend on economic feasibility for which some more investigation (proper allocation of direct and indirect costs) would be necessary.

2. Delaying committed customer orders is permissible with a penalty (L^m_{tj}), per unit per period, for product m, customer j, and time period t):

Here, DPP is solved and the δ^m_{tj} variables inspected. If all the δ^m_{tj} variables are zero, the current shipment schedule can be maintained and the current inquiry can be entertained. The question is again one of economic feasibility.

If any of the δ^m_{tj} variables are non zero, the current shipment schedule may have to be delayed. The solution cannot directly tell us what and how much to delay of current orders and the current request; simple rules can be written to make this determination post priori.

What is the maximum quantity (for given product and time) that the firm can commit to a potential order without delaying the current shipment schedule?

This is the simpler case of a more general problem. The buyer and supplier are negotiating possibilities regarding one given product at one given time. To answer this question, solve the problem MSD (maximum serviceable demand) below.

Model MSD

Maximize d^m_{1tj}

Subject To (2), (3), (4), (5), (6), (7), (8), (9), and (10) with δ^m_{tj} variables dropped from constraints (5) and (10)

What is the marginal cost of each product unit (again, for a given set of time periods) if the firm were to commit to a given combination of product, time, and quantity?

This is an interesting question for a supplying firm that is not only negotiating delivery dates with a buyer, but also the price. Price setting is a complex issue with several factors: potential revenue, potential for acceptance by buyer, specifications in current contract (if any), loyalty rewards like quantity discounts, cost allocation methods to take into account overhead, safety stock costs, etc. But the first step is estimating the direct marginal costs. The procedure outlined in the next paragraph can estimate marginal costs.

The simpler case is the one dealing with one product required in a given quantity at different time periods. To simplify further, let us drop the δ^m_{tj} variables from DPP and constraint set (9) on beginning and ending inventories. The resulting problem can be reformulated as an arc-path problem, i.e., the flow variables take the form f^m_k, where the k's are complete paths from source to sink. An artificial source variable is created and connected to each of the source nodes for every product as well as any node (j,t=1) which has positive initial inventory. Similarly, the all demand points are connected to the sink node. Both the source and sink nodes have infinite capacity and all arcs out of or into them (respectively) have zero costs.

The arc-path formulation has capacity constraints, non negativity constraints, and demand constraints. This can be solved using the modified Ford-Fulkerson column generation technique developed by Tomlin (1966). In this method, paths are entered into the system progressively and flow augmentations take place up to the level of bottleneck capacity. The only paths that are admissible are those which spend at least one time unit at a node (see constraint 4). Once the flow is known, that gives the marginal value of product costs from one break point to the next. Our assumption states that the d^m_{jt} demand type takes precedence over the d^m_{1jt} demand type. With this assumption, a simple tracking subroutine can make sure that the d^m_{1jt}'s are correctly accounted for. It basically has to check the flow augmentation taking place at any step in the modified Ford-Fulkerson algorithm. Four cases are possible:

1. The augmentation does not pertain to a point with only a $d^m{}_{jt}$: there is nothing to do.
2. The augmentation pertains to a point with only a $d^m{}_{1jt}$: the marginal cost is recorded along with the range.
3. The augmentation pertains to a point with both $d^m{}_{jt}$ and $d^m{}_{1jt}$ and $d^m{}_{jt}$ has not yet been filled: don't do anything.
4. The augmentation pertains to a point with both $d^m{}_{jt}$ and $d^m{}_{1jt}$ and $d^m{}_{jt}$ has already been filled or will be filled in this iteration: fill the $d^m{}_{jt}$ demand. If the iteration involves augmentation beyond this, the marginal cost of the $d^m{}_{1jt}$ is recorded along with the range.

Note: there will be a complete set of break points for each $d^m{}_{1jt}$.

<u>How does cost vary by lead time?</u>

Model DPP can be solved for any customer j repeatedly by entering a demand to be negotiated, $d_1{}^m{}_{jt}$ for $t = t_1$ to t_2. From the solution, the cost lead time trade-off curve can be built. This curve could serve as the basis for quoting lead times to customers.

Model Application

Figure 2 shows how the model can be used in practice. Customers (represented by customer nodes) send RFQs. A centralized planner having control over the entire supply chain uses RFQs and committed demands to build model DPP. DPP is solved and quotations are sent to the customers. If accepted, a quotation becomes committed demand. The solution to model DPP also indicates the supply plan, i.e., how much raw material should be purchased from each supplier.

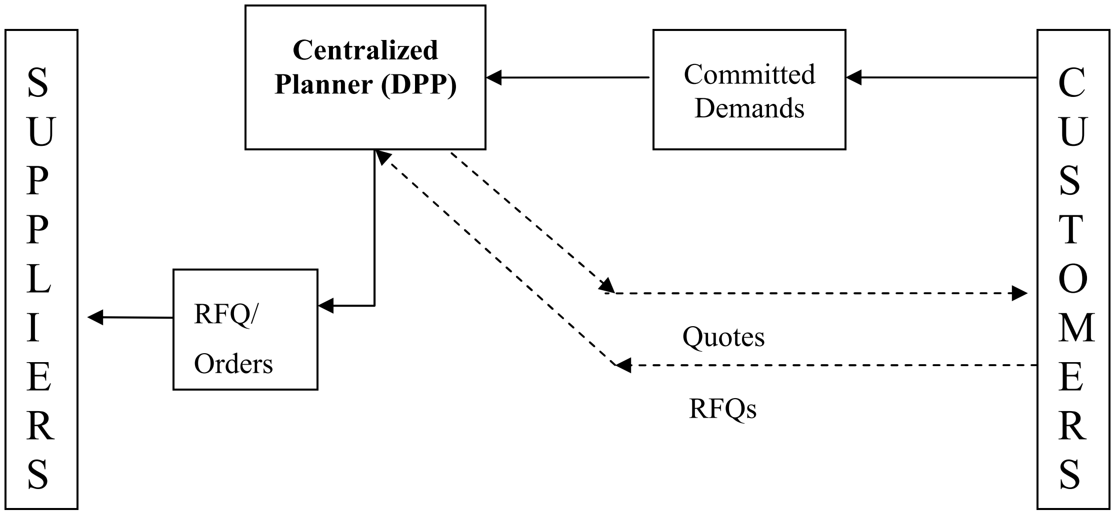

Figure 2: Order Promising in Proposed System

The model was implemented using ILOG OPL Studio 3.7 and ILOG Cplex 9.0.

Illustrative Example

Consider the case of a supply chain network comprising one supplier, an assembly plant, two warehouses, and two customers. Figure 3 shows the overall static supply chain network. This network is replicated over the planning horizon. We consider an aggregate planning schedule broken up into days.

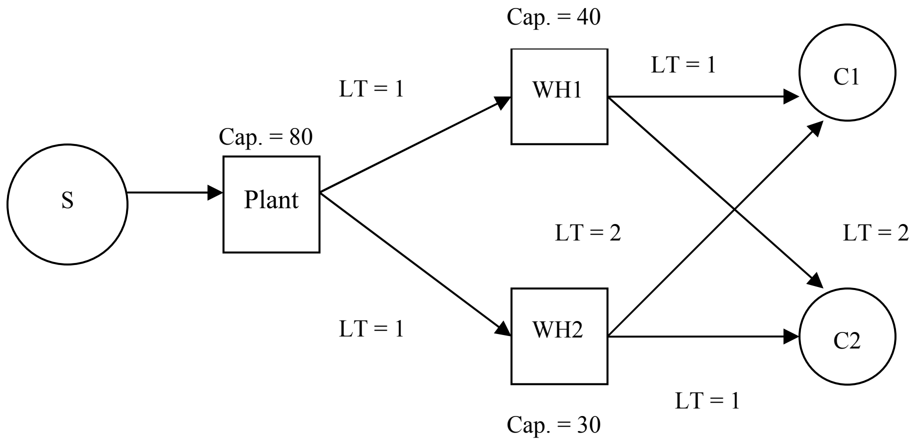

Figure 3: Illustrative supply chain network

The assembly plant is where assemblies are made from kits at fixed workstations by a group of workers, each specializing in a certain subsystem of the overall assembly. Obviously the time to assemble varies from product to product, but we will assume that the plant has a well developed and reliable rotation schedule. We model the entire plant as a node and assume that the production rate of outgoing assemblies is fairly constant. The assemblies come in about 25 different SKUs which can for aggregate planning purposes be grouped into two families (of value $1,500 and $1,400 respectively). We will assume that production capacity at the plant is unconstrained, but the transportation capacity from the plant to the warehouses is 80 units per day. Each product consumes different units of this transportation capacity due to the difference in size.

Further, we suppose that incoming raw materials (kits) to the assembly plant can be ordered from a supplier. We assume that the firm's supplier is a Rosenfeld supplier, i.e., receiving costs decrease as requested lead time increases.

After production and inspection, assemblies are sent to one of two warehouses and from there on to the two customers. The transportation lead time from the plant to both warehouses is a day. At the warehouses, capacity is dictated by space and again we assume that there are slightly different space requirements for the families.

We assume that transportation, holding, production, and warehouse handling costs are linear and known. Finally, we assume initial inventories for both products. While this supply chain network is fairly simple, solving the model for larger networks involves much the same process.

Due-Date Quotation

In the first scenario, we are interested in quoting due-dates to customers. Table 1 shows committed demands for the two customers, and an RFQ from customer 1 of 20 units of product 1 in period 5, 10 units of product 1 in period 6 and 20 units of product 2 in period 6.

Customer	Product	Period	Committed Demand	RFQ
1	1	3	5	0
1	1	4	10	0
1	1	5	5	20
1	1	6	5	10
1	2	3	5	0
1	2	4	0	0
1	2	5	5	0
1	2	6	10	20
2	1	3	5	0
2	1	4	5	0
2	1	5	10	0
2	1	6	5	0
2	2	3	10	0
2	2	4	0	0
2	2	5	5	0
2	2	6	0	0

Table 1: Committed Demands and RFQ

The first question is what the response to the RFQ should be. We assumed a lateness cost of 550 per unit of backlogged demand. DPP is solved for the problem and the optimal solution has no backlogged demand. To obtain the cost of the RFQ, DPP is run twice, once with all the RFQ and once without. When DPP is solved with all the RFQ, the optimal solution has a value of $ 38,407.79. However when DPP is solved without any of the RFQ, the optimal solution goes down to $15,987.50. Therefore, the marginal cost of the RFQ is the difference, i.e., $ 22,420.29. The average cost per unit for the 50 units of demand in the RFQ is therefore $22,420.29/50 = $448.4057. This figure can be used as the basis to quote prices, since it is common practice to mark-up the total cost to obtain price.

In the second scenario based on Table 1, customer 1 has an RFQ of 60 units of product 1 in time period 6 instead of 10 units previously requested. Running DPP yields an infeasible solution if customer orders cannot be delayed. There are two options at this point:

1. DPP can be run for a smaller RFQ and no committed demands can be delayed. In this option, MSD is run to find the maximum RFQ that can be tolerated. This value turns out to be 34.99 units at customer 1 for product 1 in time period 6. Thus, the firm can quote 34 units in response to the RFQ of 60 units. If 34 units are quoted, the total cost of the solution is found to be $50,028.98, an increase of $34,041.48 over the case without any RFQ. The average cost for all 74 units in the RFQ will be $34,041.48/74=$460.02.
2. In this option customer commitments are allowed to be delayed. DPP yields a solution involving a delay of 13 and 10 units to the total demand (committed

demand and RFQ) of customer 1 in period 6 for product 2 and product 1 respectively. Looking at the RFQ and demand tables, it is seen that there is already an RFQ of 20 and 60 units at these nodes respectively. Assuming that committed demand takes precedence over RFQ, we can quote 20 units of product 1 in period 5, 50 units of product 1 in period 6 and 7 units of product 2 in period 6. The value of the optimal solution is $63,794.50, an increase of $47,807 over the case without any RFQ.

Once the firm responds to either of these quotes, they become committed demand and the process is repeated all over again when the next RFQ arrives.

Maximum Serviceable Demand

We first solved MSD for each product and each time period to find out how much demand is serviceable at each point combination of single customer (customer 1) and product (products 1 and 2). Figure 4 shows availability as a function of lead time. As expected, as lead time goes up, more assemblies are available to the customer.

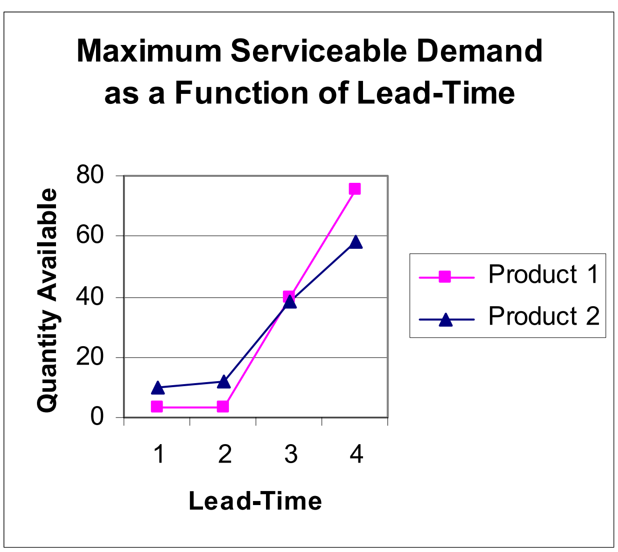

Figure 4: Availability as a function of lead time

Determining Marginal Costs

Finally, we solved DPP for product family 1 and customer 1 with different RFQ values for time period 6. We were able to break out the marginal costs as a function of RFQ and as expected, marginal costs either stay the same or increase (see Figure 5). When there is a low cost arc chain not fully assigned to demands or RFQ, the marginal cost of an RFQ remains the same. Once the arc chain gets capacitated, additional RFQ can only be satisfied by a higher cost arc chain resulting in an increase in the marginal cost. The granularity in Figure 5 is 5 units of RFQ at a time. Any granularity may be used, but using a finer granularity implies more computational effort.

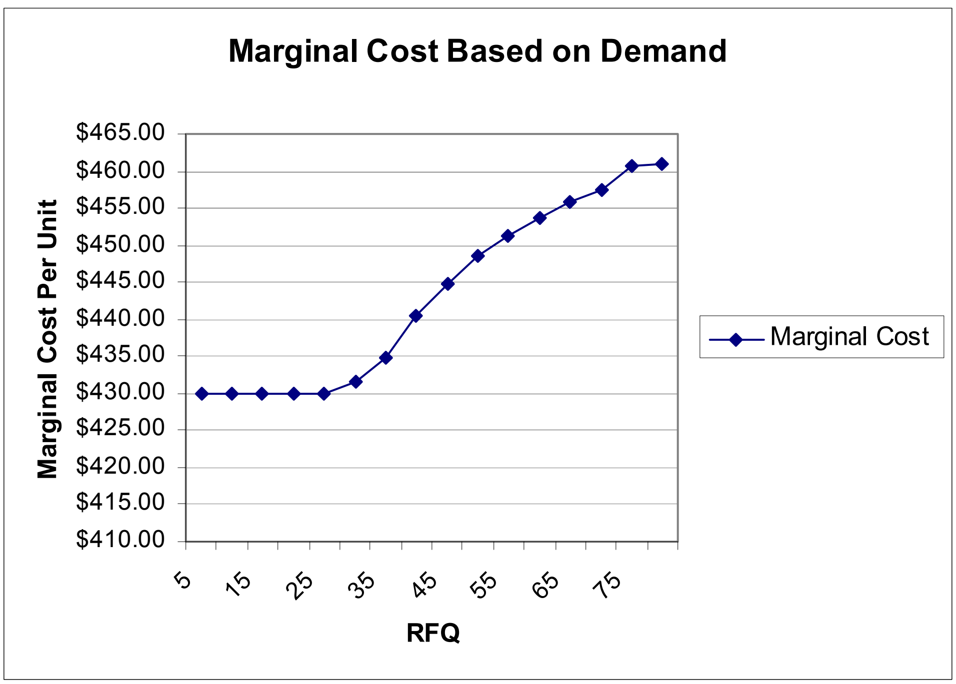

Figure 5: Marginal Cost as a function of RFQ

Cost Lead-Time Trade-Off for a given RFQ

We took product family 2 as an example and solved DPP repeatedly for an RFQ of 12 units for customer 1 in periods 4, 5, and 6. Figure 6 shows the cost lead-time trade-off when holding costs are not considered in the model at all. As seen, total cost decreases with lead time. It should be mentioned that the 12 unit RFQ is below maximum serviceable demand at each of these time periods. As can be seen, Rosenfeld's hypothesis holds for this case. This is because there is some opportunity for the supplying firm to optimize its supply chain because product can be produced and distributed through the lower cost arc-chains in the network. Even when holding costs were considered, the same pattern was observed (Figure 6).

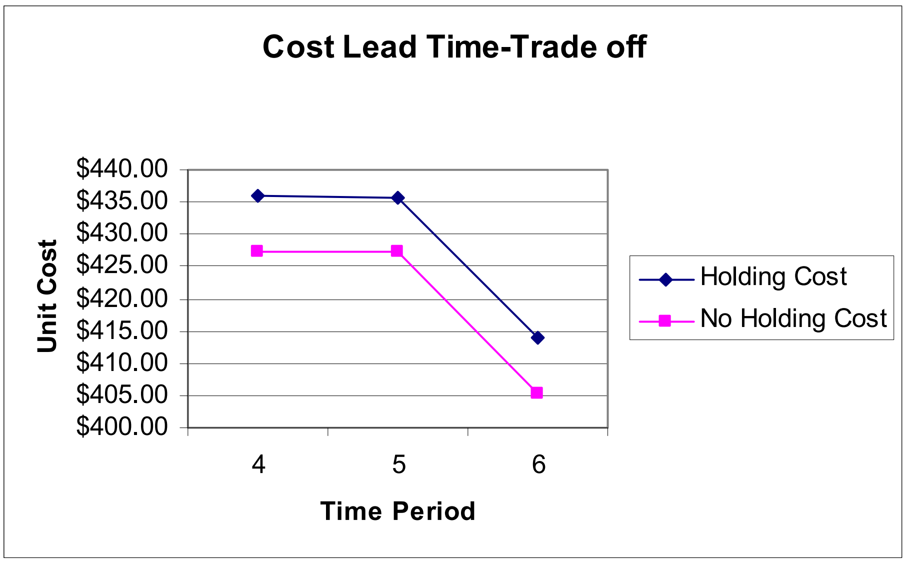

Figure 6: Cost lead-time tradeoff with and without holding costs

Effect of Supplier Lead Time Policy on Cost Lead-Time Curve

We discovered that initial inventory has an effect on the cost lead-time curve. We took the base case with committed demand and set the RFQ equal to 15 units for periods 4, 5, and 6, one at a time. For the case of customer 2 and product family 2 DPP was solved for each time periods and one of the three types of raw material suppliers:

1. Rosenfeld raw material supplier: one who reduces costs at a reducing rate as lead time increases. A Rosenfeld supplier charges price $P = pe^{-kt}$, where p and k are constants and t is the lead time.

2. Constant price raw material supplier: one who always charges the same amount for a product. This supplier charges $P = p$.

3. Increasing price raw material supplier: one who increases price as lead time increases; such a supplier was considered for hypothetical purposes only. Such a supplier charges price $P = pe^{kt}$, where p and k are constants and t is the lead time.

For all cases, given the data used, we were able to observe Rosenfeld's hypothesis (see Figure 7). However the trade-off curve was found to be the steepest when the firm's raw material supplier is a Rosenfeld supplier. A decrease of \$230.22 was observed on shifting the demand from time period 4 to time period 6 in case of Rosenfeld supplier while decreases of \$154.80 and \$ 133.80 were observed with the constant price and increasing price suppliers respectively.

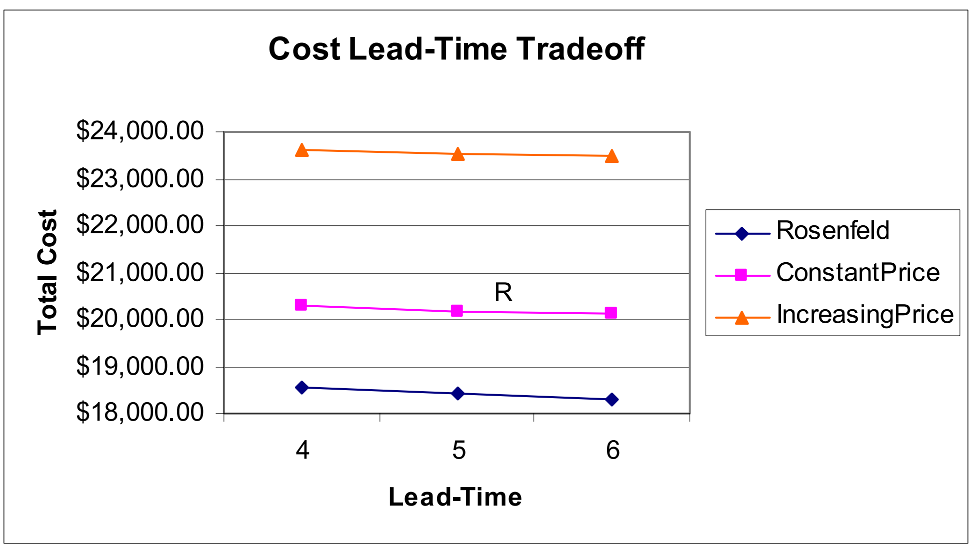

Figure 7: Cost lead-time assuming zero RFQ for three types of raw material suppliers

For the Rosenfeld supplier, the price decreases with increase in lead time. This means that orders should be placed as early as possible, i.e, with lead time as high as possible. In Table 2, it can be seen that 10 units of product 1 and 3 units of product 2 are ordered with a lead time of 2 in response to an RFQ of 15 in period 4. For the same RFQ for a constant price supplier, fewer units of product 1 (5 units) and the same number of units of product 2 (3 units) are ordered with a lead time of 2. Because the price is the same regardless of lead-time, products are ordered later from the supplier in order to save on holding costs (see Table 3). This effect is even more pronounced for the increasing price supplier. Not only are holding costs occurred when lead time is high, the purchase price also goes up. Therefore, with such a supplier, it is advantageous to order as much as possible with a lower lead time. In Table 4, it can be seen that no orders are placed with a lead time of 2 units in response to the RFQ in period 4. Tables 2, 3, and 4 also show the same pattern in response to RFQ in time periods 5 and 6.

RFQ=15	**Period = 4**		**Period =5**		**Period = 6**	
Products	**Lead Time=1**	**Lead Time=2**	**Lead Time=1**	**Lead Time=2**	**Lead Time=1**	**Lead Time=2**
Product 1	10	10	10	10	10	10
Product 2	14	3	12	5	5	12

Table 2: Ordering pattern for Rosenfeld Supplier

RFQ=15	**Period = 4**		**Period =5**		**Period = 6**	
Products	**Lead Time=1**	**Lead Time=2**	**Lead Time=1**	**Lead Time=2**	**Lead Time=1**	**Lead Time=2**
Product 1	15	5	15	5	15	5
Product 2	14	3	12	5	5	12

Table 3: Ordering Pattern for Constant Price Supplier

RFQ=15	Period = 4		Period =5		Period = 6	
Products	**Lead Time=1**	**Lead Time=2**	**Lead Time=1**	**Lead Time=2**	**Lead Time=1**	**Lead Time=2**
Product 1	20	0	20	0	20	0
Product 2	17	0	17	0	17	0

Table 4: Ordering Pattern for Increasing Price Supplier

Conclusions and Future Work

We believe that in the eCommerce era, agent based negotiation will be common and agents will need decision support based on live enterprise data. While the cost-lead time trade-off curve has been talked about in the literature for a long time, our proposed method will generalize the curve and help build real instances.

Throughout the paper, we have made a number of simplifying assumptions: costs are linear and continuous; there are no fixed costs or setups, components for assembly are not considered; and so on. All these are possibilities for future expansion.

Also, we have not modeled customer orders explicitly. An order promising (advanced ATP) model for customer orders would be a natural extension.

An interesting extension of the model is to allow for customers to 'reserve' capacity. Issues of interest are what price to charge for reserving capacity, how much capacity should one allocate for such reservation, what procedure should be followed for reservation, and how would this reservation affect other customers.

Finally, the model proposed in this paper focuses on the internal supply chain of a suppler firm. Using the model as a building block, this approach can be extended to modeling a supply chain with several supplier firms. This would help us better understand cooperation and competition between supplier firms in multi-firm supply chains.

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