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Delay analysis of multichannel opportunistic spectrum access MAC protocols

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Abstract: We provide a comprehensive delay and queueing analysis for two baseline medium access control protocols for multi-user cognitive radio networks with homogeneous users and channels and investigate the impact of different network parameters on the system performance. In addition to an accurate Markov chain, which follows the queue status of all users, several lower complexity queueing theory approximations are provided. Accuracy and performance of the proposed analytical approximations are verified with extensive simulations. It is observed that using an Aloha-type access to the control channel, a buffering MAC protocol, where in case of interruption the CR user waits for the primary user to vacate the channel before resuming the transmission, outperforms a switching MAC protocol, where the CR user vacates the channel in case of appearance of primary users and then compete again to gain access to a new channel. The reason is that the delay bottleneck for both protocols is the time required to successfully access the control channel, which occurs more frequently for the switching MAC protocol. It is thus shown that a clustering approach, where users are divided into clusters with a separate control channel per cluster, can significantly improve the performance by reducing the competitions over control channel.

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1 Introduction

Opportunistic spectrum access (OSA) communication models [1], implemented by cognitive radios (CR), offer both the capacity to decrease the communication infrastructure expenses by using the vacant portions of the spectrum and to improve the quality of wireless communications by permitting wireless CR nodes¹ to switch to a better channel when the channel quality is not satisfactory. These notable features have made cognitive-radio based wireless networks a technology of choice for incoming wireless technologies. Frequent spectrum handovers along with new requirements such as spectrum sensing and spectrum decision distinguish a cognitive radio network from its predecessors. New medium access control (MAC) protocols which take into account the inherent nature of cognitive radios are thus indispensable. For this aim, several medium access protocols have been proposed in the literature [2–4]. Different protocols may differ in details such as the existence of a common control channel, contention management, number of radios required for the nodes and sensing algorithm (please see [5, 6] and references therein for a detailed survey of CR MAC protocols). However, what is common in most of the proposed protocols is the spectrum management and channel handover, which can be broadly classified as either staying on the same channel (buffering) or switching to a new channel (switching) in case of PU appearance. Those two extreme possibilities are representative because any other policy will also be a combination of those two baseline policies. We thus provide a comprehensive delay analysis and comparison for two baseline medium access control protocols discussed in [6, 7] which in turn provide performance bounds for other MAC protocols.

Related work In the literature, most of the attention has been focused on the throughput analysis of different medium access protocols in the presence of saturated traffic (e.g., [2–4, 6–8]), while little work has been done on delay and jitter analysis. Throughput analysis provides an upper bound for the performance of the network, which can be used for evaluation and comparison purposes of different protocols. However, delay analysis and packet level performance evaluation are required to investigate how a protocol, along with its parameter settings, behaves for delay-sensitive applications, such as multimedia communications [9, 10]. Further, for cognitive radio networks, which consists of alternating operating and interruption periods, throughput analysis in the presence of saturated traffic is often insensitive to the period lengths while packet delay for unsaturated traffic varies significantly [11]. It is therefore crucial to possess delay analysis tools to assess the performance of cognitive networks, which is the motivation for this paper.

In the few papers available on delay analysis (e.g., [12–17]), detailed system parameters are not considered or the CR network under consideration is simple, such that full insights into the impact of different parameters on the network performance have not been provided. The target has also been specific networks; e.g., sensor networks, with specific requirements [18, 19], such that the results can not be easily extended to general models. In [19], two different switching models for sensor networks are discussed and the buffering model is not investigated. Instead of a common control channel, in [20], a hopping model has been addressed and the focus is to evaluate the proposed proactive spectrum switching model. Instead of delay and throughput, [21] proposed effective bandwidth as the performance metric, but focus is on spectrum sensing and adaptive transmission and the discussion is provided for a single cognitive radio user.

In [11, 22], we proposed a general M/G/1 queueing model [23] for a cognitive radio link where the server is subject to interruptions [24]. The channel (i.e., the server of the queue) is subject to interruptions because the CR node has to suspend its communication session for the period of spectrum handover (switching) or primary users' activity (buffering). The proposed models in those papers were for a single node with continuous-time operating and interruption periods. However, those models did not address the complex problem of multi-node operation in a multi-channel environment.

Approach and objectives The focus of this work is the spectrum handover and interruptions which occur in opportunistic spectrum access. Our objective in this paper is thus to fill the delay analysis gap. From the spectrum management and channel handover point of view, staying on the same channel after an interruption or switching to a new channel can be considered as the two extreme possibilities and are thus representative. We extend the throughput analysis in the presence of saturated traffic for buffering and switching policies presented in [6, 7], and provide a comprehensive delay analysis in the presence of unsaturated traffic with an

¹Node and user are used interchangeably.

arrival process and assuming a queue (buffer) for the users. Note that the models described in those papers are not detailed MAC models for practical purposes. The intent is rather to have models with enough details, yet general enough, to use as baseline reference models for families of MAC protocols and investigate the effect of different parameters, specially the parameters related to channel handover, on the throughput of a cognitive radio network. In this paper, we have the same objective for the delay analysis and thus only relax the saturated traffic assumption and keep the same other assumptions and modeling details to analyze the average delay. Some simplifying assumptions are therefore made to keep the analysis complexity tractable. In the first part, a buffering MAC model [6, 7], in which a node stays on the channel in case of interruptions until the transmission of the packet is finished, is investigated. In the second part, a switching MAC model [7], where the node leaves the interrupted channel and participates in a new reservation competition every time that an interruption occurs, is investigated.

Following the models presented in [6, 7], and since most of the proposed MAC schemes assume a dedicated control channel and multiple radios which increase the hardware and complexity cost, we investigate in this paper a realistic cognitive radio network with a common control channel, distributed sensing appropriate for ad-hoc networks, and a single radio per node. Similar to throughput analysis works for multi-user multi-channel networks, users and channels are assumed to be homogeneous. For both MAC models, we derive an exact Markov chain model which can be solved to obtain the system time distribution and any moment (delay, jitter, etc). Since the number of states in the Markov chains grows exponentially with the number of nodes, we also propose lower complexity approximations based on discrete-time M/G/1 queueing theory results. In both MAC models, the service time of a packet is composed of two parts: the time spent to compete with other users to reserve a channel (for the switching MAC model, the reservation period may occur multiple times during the service of a single packet), and the transmission time. We use Markov chain models, combined with Renewal theory results, to derive approximate distributions of both parts of the packet service time. The service time moments are then used to find the approximate average system time of packets in the OSA network for both MAC protocols. We also provide numerical results to validate the analysis and provide insight on the delay performance of the MAC protocols for different network parameters.

Novelty and contributions The main contribution of this work is thus providing a comprehensive analytical modeling and delay analysis for a multichannel cognitive radio network with both buffering and switching recovery policies where the impact of different network parameters (number of users, number of channels, control channel access probability, channel availability, and arrival rate) on the performance is considered. The results thus provide insights and highlight the trend of performance variation as a function of different network parameters. The results can also be used in network optimization and design problems to select optimal parameters. However, due to high dimension and complexity of the problem, we are not able to provide closed-form or accurate results in all scenarios, and the contribution is thus limited to approximations.

While there are very few papers on delay analysis in multi-user multi-channel cognitive radio networks, no work focused on the baseline channel handover policies presented in [6, 7]. Since the system model is almost the same, naturally some of the general insights are similar to those obtained in [6, 7]. However, in addition to new accurate and approximate results for delay analysis which can be used in optimization models, network design and more in-depth network performance analysis, some novel observations are obtained, such as the significance of clustering in the context of cognitive radio networks, comparison of buffering and switching and impact of traffic load (which can not naturally be discussed in the presence of saturated traffic).

Paper organization The paper proceeds as follows. In Section 2, the system model is presented and related parameters are introduced. The next two sections discuss the buffering model. In Section 3, an accurate Markov chain for queue occupancy is proposed from which different parameters of interest can be derived. A service cycle analysis is provided in Section 4. Section 5 discusses the switching model where the same approach is followed: first an accurate Markov chain for queue occupancy is proposed and then a delay cycle analysis and related approximations are provided. Numerical and simulation results for both buffering and switching models are discussed in Section 6. Discussions on the limitation of this work and how it can be extended to other scenarios as future research direction is provided in Section 7. Finally Section 8 concludes the paper.

2 System model

In this section, we provide a summary of the two generic multichannel OSA MAC protocols for which we derive the delay analysis under unsaturated traffic in later sections. More details on those protocols, as well as their throughput performance under saturated traffic, can be found in [6, 7]. It is assumed that there are N homogeneous nodes in an ad-hoc cognitive radio network and M OSA channels where one channel is the dedicated control channel and the remaining $M_C = M - 1$ similar channels are used for data transmission. Without loss of generality and for notation simplicity, we assume that the N nodes have intended receivers that are not part of those nodes. A maximum of $s_{max} = \min(N, M_C)$ links might thus simultaneously exist.

Operations in the OSA network are time-slotted, and primary and secondary users are fully synchronized (common assumption to make the analysis tractable [20, 25]). In the beginning of each timeslot, there is a short quiet period during which the whole cognitive network stops operation to sense the channels. Primary users (PU) activity in any channel, including the control channel, is modeled by a Bernoulli random variable: in each timeslot and independent of the other timeslots and channels, a channel is unavailable for CR users (occupied by primary users or due to false alarms) with a probability p_c . For an ON/OFF model, the probability of channel being PU empty is equal to θ and we have $p_c = 1 - \theta(1 - p_m) + \theta p_f$, where p_m and p_f are, respectively, the probability of miss detection and false-alarm in the sensing process.

We also use a packet capture model whereby a transmission during an available timeslot is successful, with probability η for the data channels and probability η_C for the control channel. η is a function of the fading model (outage probability) and sensing errors (miss detections resulting in collisions with PU). Channel availability (PU appearance) and packet captures are assumed to be independent.

To model the unsaturated traffic, a geometric (Bernoulli) arrival process is assumed for all nodes with a probability of packet arrival in a timeslot λ . The packet length L is given in the number of timeslots required to transmit the packet and has a geometric distribution with parameter q .

A node is in the busy state when it has a reserved data channel for transmission, otherwise the node is in the idle state (with an empty or non-empty packet queue). After the sensing period, if the control channel is available, the g idle nodes with a non-empty packet queue attempt to reserve a channel using an Aloha-type competition over the control channel. That is, each of the g nodes will transmit in the timeslot a reservation request with a probability p over the control channel. The competition is successful if only one reservation request is transmitted and it is correctly received. The probability of success in competition is given by [6]:

$$P_s(g) = gp(1-p)^{g-1}\chi, \quad (1)$$

where $\chi = (1-p_c)\eta_C$ is the probability of a successful transmission on the control channel without considering collisions. The probability of success for a marked node among g nodes is $\frac{P_s(g)}{g}$. It is assumed that the list of channels not being used by CR nodes is given. Without this assumption, a collision may occur when a CR node selects a channel already in use by another CR user. This results in a variable rate queue (capacity is shared) and is out of the scope of this paper. So, a success in competition in this timeslot is followed by reserving the channel which is on the top of the list by the competition winner, to be used from the beginning of the next timeslot. An exception is when all channels are occupied in this timeslot. If a channel is released at the end of this timeslot, it will be assigned to the successful node in competition, otherwise the successful node in competition is unsuccessful in reserving a channel and returns for a new competition in the next timeslot. It is therefore important to distinguish the event of *success in competition*, which depends solely on the number of competing users, from the event of *success in reservation*, which depends also on the status of channels. In case that a node is successful in reserving a channel, it starts transmitting one packet in the reserved channel from the beginning of the next timeslot. Other nodes can not make any interference on the reserved channel, but fading, sensing errors and PU activities may interrupt the transmission.

Two baseline MAC protocols are considered in this paper. In the buffering MAC protocol [6], a node stays on its reserved channel, even when it becomes occupied by PUs, until the packet is entirely transmitted. In each timeslot, a successful transmission therefore occurs with probability $\psi = (1-p_c)\eta$. For the buffering MAC protocol, the (enlarged) packet service time X therefore consists of a reservation period of length X_R followed by a transmission time X_T which consists of successful transmission, unsuccessful transmission and unavailable timeslots, until the entire packet is transmitted.

In the switching MAC protocol [7], a node that senses its channel occupied by PUs leaves the channel and returns to the idle state to participate in the competition to reserve a new channel in this timeslot. For the switching MAC protocol, the (enlarged) packet service time X therefore consists of several reservation periods and the successful transmission and unsuccessful transmission timeslots required to transmit the entire packet.

For both MAC protocols, once the packet transmission is terminated, the node releases the channel and returns to the idle state. It will enter the competition at the next timeslot if it has another packet in its queue. A service-resume transmission model is assumed for both MAC protocols where the remaining part of the packet is transmitted after each interruption. What we mean here by packet is a general concept including a block of data to be transmitted itself composed of smaller identifiable parts. As an example, we can mention a MAC frame composed of several higher OSI layers' data blocks or jumbo frames composed of smaller frames, such as in the MAC Service Data Unit (MSDU) aggregation and MAC Protocol Data Unit (MPDU) aggregation in IEEE 802.11n networks where multiple wireless or Ethernet frames are merged to form a larger super frame. Naturally, if an interruption occurs, there will be no need to retransmit the wireless/Ethernet frames which have already been delivered. Note that due to the packet length geometric distribution model, the delay analysis is similar for a service-repeat model where the entire packet must be retransmitted after each interruption.

3 Buffering MAC protocol: Queue occupancy markov chain analysis

The discrete and memoryless nature of the system model and MAC protocols described in Section 2 enables us to propose a general discrete-time Markov chain (MC) which tracks all arrivals, departures and interruption events for all nodes. The different queue performance metrics can then be directly computed using the steady-state MC solution. The state variable is given in the form $(\vec{n}, \vec{b}) = (n_1, n_2, \dots, n_N, b_1, b_2, \dots, b_N)$ where n_i ($n_i = 0, 1, \dots$) is the current number of packets in the system (waiting in the queue or being transmitted) of node i and b_i ($b_i = 0, 1$) is a status flag indicating whether node i has a reserved channel ($b_i = 1$) or is idle ($b_i = 0$). For each state A , we also define the variables $k_A = \sum I_{(b_i=1, n_i>0)}$ and $g_A = \sum I_{(b_i=0, n_i>0)}$, where I is an indication function with $I_{true} = 1$, $I_{false} = 0$, which are, respectively, the total number of busy nodes and the total number of idle nodes with a non-empty queue. An MC example with $N = 1$ is illustrated in Figure 1.

Because of the geometric arrival process and since at maximum one packet can be served in one timeslot over each channel, the queue status changes at most by one packet in two consecutive slots. That is $n_i(t) - 1 \leq n_i(t+1) \leq n_i(t) + 1$, where $n_i(t)$ is the node i queue occupancy at time t . However, as a function of the status flag, the following refinements can be made:

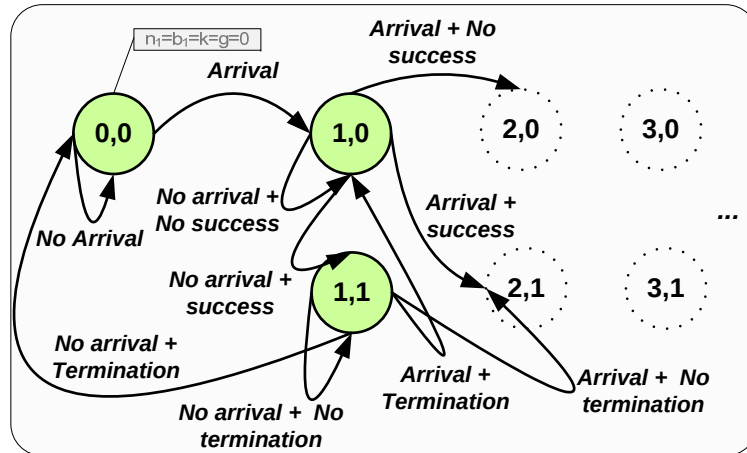


Figure 1: An example of the proposed queue occupancy Markov chain for $N = 1$ and $M_C > 0$.

1. If $n_i = 0$ then $b_i = 0$, and only an arrival event can occur, in which case $n_i(t+1) = 1$, otherwise $n_i(t+1) = 0$. Furthermore, the next status state is also $b_i = 0$, irrespectively of an arrival event occurrence.
2. If $n_i > 0$ and $b_i = 0$, two events of reservation success and an arrival can occur at the same timeslot and the events are independent. We have $n_i(t) \leq n_i(t+1) \leq n_i(t) + 1$ depending on arrival or no arrival. Reservation success only changes the value of b_i to 1.
3. If $n_i > 0$ and $b_i = 1$, two events of packet transmission termination and an arrival can occur at the same timeslot and the events are independent. We have $n_i(t) - 1 \leq n_i(t+1) \leq n_i(t) + 1$. The case $n_i(t+1) = n_i(t) + 1$ occurs with an arrival and no transmission termination, and the case $n_i(t+1) = n_i(t) - 1$ occurs with no arrival and transmission termination. Meanwhile, a node will have the same number of packets in the queue in the next timeslot for two events: transmission termination and an arrival, or no transmission termination and no arrival. A transmission termination results in changing the value of b_i to 0, irrespectively of an arrival event occurrence.

Furthermore, all busy nodes and idle nodes with an empty queue operate independently, so the joint probability of different events can easily be found by multiplication.

For two states $A = (\vec{n}, \vec{b}) = (n_1, \dots, n_N, b_1, \dots, b_N)$ and $B = (\vec{m}, \vec{c}) = (m_1, \dots, m_N, c_1, \dots, c_N)$, $\mathbb{IS}$, the set of impossible transitions, can then be defined as follows:

$$\mathbb{IS} = \left\{ (A, B) = ((\vec{n}, \vec{b}), (\vec{m}, \vec{c})) \mid \exists n_i, m_i (n_i = 0 \ \& \ b_i = 1) \parallel (n_i = 0 \ \& \ c_i = 1) \parallel (m_i = 0 \ \& \ c_i = 1) \parallel (Num_{s,AB} > 1) \parallel (m_i \notin (n_i - 1):(n_i + 1)) \parallel \left(\{k_A = s_{max}\} \ \& \ \{Num_{(s,AB)} > 0\} \ \& \ \{Num_{(t,AB)} = 0\} \right) \right\}, \quad (2)$$

where $Num_{(s,AB)} = |find(\vec{c} - \vec{b} == 1)|$ ($|find(\vec{x} - \vec{y} == 1)|$ is the number of positions for which the difference between the elements of $\vec{x}$ and $\vec{y}$ is 1) is the number of nodes which started transmitting in the new timeslot from state A to B and $Num_{(t,AB)} = |find(\vec{b} - \vec{c} == 1)|$ is the number of busy nodes in state A whose transmission is finished in state B .

The transition probabilities of going from state A to a new state B can then be computed using Algorithm 1. For $(A, B) \in \mathbb{IS}$, the transition probability is zero. In Algorithm 1, $\mathbb{S}_g$ is the set of nodes participating in competition in state A whose size is given by $|\mathbb{S}_g|$. If $\mathbb{S}_g$ is empty, the last part of the algorithm is not run. $AllBusyNoTer = 1$ represents the case where all channels are busy and no termination occurs.

From this exact Markov chain, any performance parameter of interest such as the first (related to delay) and second moment (related to jitter) of the number of packets in the buffer can be found. This model is general and can also address several system model extensions, such as non-homogeneous nodes with different arrival rates or different packet length. However, as there is no buffer limit, the number of states is infinite and grows exponentially with the number of nodes. To be able to solve the Markov chain numerically a buffer limit $Q_{s_{max}}$ should be set and the truncated Markov chain is analyzed instead. If the transition probabilities are written for a truncated version of the Markov chain with the buffer size $Q_{s_{max}}$, the states with $\exists i$ s.t. $n_i = Q_{s_{max}}$ should be treated separately because for cases $b_i = 0$, or $b_i = 1$ and no termination, the queue occupancy in the next state with arrival or without any arrival (with probability equal to one) will still be $m_i = Q_{s_{max}}$.

Even for the truncated MC, the queue occupancy MC model suffers from exponential growth of the number states and is thus not scalable. For example, for a buffer limit of $Q_{s_{max}}$, the number of states will be $[2(1 + Q_{s_{max}})]^N$ (e.g., 484 states for $N = 2$ and $Q_{s_{max}} = 10$). In the next section, the homogeneity of the nodes is taken into account to propose an approximate but scalable combined Markov chain which will be used for a service cycle analysis.

Algorithm 1 The algorithm to find the transition probabilities $P_{A=(\vec{n},\vec{b}) \rightarrow B=(\vec{m},\vec{c})}$.

```

if ( $k_A == s_{max}$  &  $Num_{(t,AB)} == 0$ ) then
     $AllBusyNoTer = 1$ 
for  $i = 1 : N$  do
    if ( $n_i == 0$ ) then
         $P_i = \lambda^{m_i - n_i} (1 - \lambda)^{1 - (m_i - n_i)}$ 
    else
        if  $b_i == 1$  then
            if ( $c_i == 1$ ) then
                 $P_i = (1 - q\psi) \lambda^{m_i - n_i} (1 - \lambda)^{1 - (m_i - n_i)}$ 
            else
                 $P_i = (q\psi) \lambda^{1 - (n_i - m_i)} (1 - \lambda)^{n_i - m_i}$ 
        else
            Put  $i$  in  $\mathbb{S}_g$  ;  $P_i = \lambda^{m_i - n_i} (1 - \lambda)^{1 - (m_i - n_i)}$ 
    ***** Handling Competitions ***** ( $\mathbb{S}_g \neq \emptyset$ )
    Find  $j$  s.t. ( $c_{\mathbb{S}_g[j]} == 1$ )
    if  $j > 0$  then
         $P_{\mathbb{S}_g[j]} = P_{\mathbb{S}_g[j]} * \frac{P_s(|\mathbb{S}_g|)}{|\mathbb{S}_g|}$ 
    else
        if ( $AllBusyNoTer == 0$ ) then
             $P_{\mathbb{S}_g[1]} = P_{\mathbb{S}_g[1]} * [1 - P_s(|\mathbb{S}_g|)]$ 
    FindProb( $A, B$ ) =  $\prod_i^N P_i$ 

```

4 Buffering MAC protocol: Service cycle analysis

Since a geometric arrival process is assumed, we have for any node a discrete-time M/G/1 queue with geometric arrivals called Geom/G/1. Continuous-time M/G/1 and Geom/G/1 become equivalent when the timeslot is short with respect to the arrival rate. Furthermore, since the arrival process and packet length distribution are identical for all nodes, the queues are homogeneous. To solve a tagged node queue, we should therefore only find the service time $X = X_R + X_T$ for the buffering MAC protocol. X_R and X_T are independent, so any moment of the service time X can easily be found from the moments of X_R and X_T . Discrete version of the Pollaczek?-Khinchine formula [26] can then be used to find the queue performance metrics of interest such as expected delay and delay variance (jitter). The exact distribution of X_T and approximations for the distribution of X_R are derived in Sections 4.1 and 4.2, respectively.

4.1 Transmission time

For the buffering MAC protocol, the transmission time is started after the timeslot in which a channel has been reserved with success and lasts until the end of the packet transmission. Since the node is using a reserved channel, there is no interference from other CR nodes and in each timeslot the probability of a successful transmission is given by $\psi = (1 - p_c)\eta$ (see Section 2). Therefore, conditioned on the packet length $L = n$, n successful timeslots are required, and the transmission time of the packet X_T has a negative binomial distribution given by:

$$Pr(X_T = k | L = n) = \binom{k-1}{n-1} \psi^n (1 - \psi)^{k-n}, \quad k = n, n+1, \dots \quad (3)$$

The packet length L has a geometric distribution such that the unconditional distribution of X_T is given by:

$$Pr(X_T = k) = \sum_n Pr(X_T = k | L = n) f_L(n) = \sum_{n=1}^{\infty} \binom{k-1}{n-1} \psi^n (1 - \psi)^{k-n} (1 - q)^{n-1} q. \quad (4)$$

The first and second moment of X_T can then be found as:

$$E[X_T] = \frac{1}{q\psi}, \quad E[X_T^2] = \frac{2 - q\psi}{(q\psi)^2}. \quad (5)$$

4.2 Reservation periods

At the beginning of every timeslot, nodes can be divided into three groups: busy nodes with a reserved channel, idle nodes with an empty queue and idle nodes with a non-empty queue. The probability of reservation success at the end of the timeslot depends on the number of nodes in the last group, except for the case where all channels are occupied. Due to packet arrival and packet transmission termination events, the number of idle nodes with a non-empty queue changes from one timeslot to the next one such that the probability of reservation success in a timeslot is time-varying. We will proceed as follows to find the reservation period distribution. First, we will propose a combined Markov chain to find the distribution and transition probabilities of the number of nodes competing in the reservation procedure. We will then find the reservation period distribution by first conditioning it on the number of competing nodes in the first reservation timeslot for the tagged node, and then using the steady-state probabilities to obtain the unconditioned distribution.

4.2.1 Combined Markov chain

Since the sum of nodes at every timeslot is fixed to N , the state variable of the proposed combined Markov chain is given by a 2-tuple (k, g) where k is the number of busy nodes and g is the number of idle nodes with a non-empty queue. The number of idle nodes with an empty queue is then given by $N - k - g$. We have the following facts about the system:

- Every idle node with an empty queue may independently become a node with a non-empty queue in the next timeslot with the probability $P_a = \lambda$, which is the probability of one arrival during one timeslot.
- Among the idle nodes with a non-empty queue, at most one node may become busy with the probability of reservation success $P_s(k, g)$ and all others stay in the same group. $P_s(k, g)$ is the probability of getting a channel when g nodes participate in competition and k channels are already reserved. It is given by:

$$P_s(k, g) = \begin{cases} P_s(g) & k < s_{max} \\ P_s(g)(1 - T_k^{(0)}) & k = s_{max} \end{cases} \quad (6)$$

where $T_k^{(0)}$ is the probability of no transmission termination among the k ongoing communication sessions.

- Every busy node may independently finish in a timeslot its packet transmission with probability $q\psi$ and then, with probability P_0 , the node joins the idle nodes with an empty queue. $1 - P_0$ converges to the steady-state queue occupancy of a node queue and is the same for all busy nodes because nodes are homogeneous and have a geometric memoryless packet length.

The number of participants in the competition in the next slot compared to the previous slot may thus decrease at most by one, but may increase to any larger value up to N depending on the number of idle nodes with an empty queue with an arrival and the number of busy nodes that finish their transmission and have a non-empty queue.

Using these facts and the results in [6, Eq. (16)], the transition probabilities of the combined Markov chain can be given by:

$$P_{(k,g) \rightarrow (z,h)} = \begin{cases} 0 & \text{if } z > k+1, \forall g, h, \\ 0 & \text{if } \forall k, z, h < g-1, \\ T_k^{(0)} S_g^{(1)} Y_{(h-g+1)}^{(k-z+1), (N-k-g)} & \text{if } z = k+1, h \geq g-1, \\ T_k^{(k-z)} S_g^{(0)} Y_{(h-g)}^{(k-z), (N-k-g)} \\ \quad + T_k^{(k-z+1)} S_g^{(1)} Y_{(h-g+1)}^{(k-z+1), (N-k-g)} & 0 < z \leq k, k+z \neq 2s_{max}, h \geq g-1, \\ T_k^{(k-z+1)} S_g^{(1)} Y_{(h-g+1)}^{(k-z+1), (N-k-g)} \\ \quad + T_k^{(k-z)} Y_{(h-g)}^{(k-z), (N-k-g)} & \text{if } z = k = s_{max}, h \geq g-1, \\ T_k^{(k)} S_g^{(0)} Y_{(h-g)}^{(k-z), (N-k-g)} & \text{if } z = 0, h \geq g-1. \end{cases} \quad (7)$$

$T_k^{(j)}$ and $S_g^{(j)}$ are respectively defined in [6, Eqs. (14),(15)] as the probability of j transmission terminations among k ongoing communication sessions and the probability of j success in competition among g competitors (note that $S_g^{(1)} = P_s(g)$).

$Y_{(a)}^{b,c}$ is the probability that a new nodes join the group of idle nodes with a non-empty queue given that there are b busy nodes that terminate their transmission and c idle nodes with an empty queue. This happens if i nodes out of the b busy nodes that terminate their transmission have a non-empty queue at the end of the timeslot and j nodes out of the c idle nodes with an empty queue have a packet arrival in the timeslot, where $i+j=a$. $Y_{(a)}^{b,c}$ is thus given by:

$$Y_a^{b,c} = \sum_{(i,j)|i+j=a, 0 \leq i \leq b \text{ \& } 0 \leq j \leq c} \binom{b}{i} (1-P_0)^i (P_0)^{b-i} \binom{c}{j} (P_a)^j (1-P_a)^{c-j}. \quad (8)$$

Assuming that P_0 is known, the steady-state probabilities $\pi_{k,g} \forall (k, g)$ of the combined Markov chain can be found. We explain in Section 4.2.3 how P_0 can be found.

4.2.2 Reservation period distribution

We now derive the distribution of X_R for a tagged node belonging to the group of idle nodes with a non-empty queue. Let $X_R^{k,g}$ denote the reservation length given that the system is in the state (k, g) , $g > 0$ at the beginning of the reservation period. The probability of a successful reservation for the marked node in the first timeslot, i.e., $\Pr(X_R^{k,g} = 1)$ is equal to $P_s^m(k, g) = \frac{P_s(k, g)}{g}$. However, if the reservation is unsuccessful, the system transits to a new state (z, h) with the probability $P_{(k,g) \rightarrow (z,h)} | (\text{No-Success})$. In this state, the probability of successful reservation and thus have $X_R^{k,g}$ equal to 2 timeslots is now $P_s^m(z, h)$. Otherwise, the system transits to a new state and the process repeats. In general, $\Pr(X_R^{k,g} = i)$ can be found in a recursive manner from the probability of transition to other states (z, h) and $\Pr(X_R^{z,h} = i-1)$ from other states, given that the node was not successful in reservation.

It is important to note that for this analysis, the transition probabilities of the combined MC discussed in the previous section can not be directly used because it is implicitly assumed that the marked node was not successful. We therefore need a different MC with transition probabilities denoted by Q conditioned on the fact that at each transition, the marked node was not successful, and that if the reservation competition was a success, it was one of the other $g-1$ competitors who was successful. The transition probabilities Q are given by (7) where $P_s(g)$ given in (1) is replaced by $(g-1)p(1-p)^{g-1}\psi$.

The distribution of $X_R^{k,g}$ is then given by:

$$\Pr(X_R^{k,g} = i) = \begin{cases} P_s^m(k, g) & i = 1 \\ (1 - P_s^m(k, g)) \sum_{(z,h)} Q_{(k,g) \rightarrow (z,h)} P_s^m(z, h) & i = 2 \\ (1 - P_s^m(k, g)) \sum_{(z,h)} Q_{(k,g) \rightarrow (z,h)} (1 - P_s^m(z, h)) \\ \quad \sum_{(a,b)} Q_{(k,g) \rightarrow (a,b)} P_s^m(a, b) & i = 3 \\ \dots & \end{cases} \quad (9)$$

$X_R^{k,g}$ can then be unconditioned to obtain X_R as follows:

$$X_R = \sum_{(k,g), g>0} X_R^{k,g} \pi_{(k,g)}^N. \quad (10)$$

Since when we mark a node to find X_R , we implicitly assume that the node is idle with a non-empty queue, the states in which the number of competing users is zero ($g = 0$) should therefore be eliminated to obtain the steady-states probabilities for the states $(k, g), g > 0$ as $\pi_{(k,g)}^N = \frac{\pi_{(k,g)}}{1 - \sum_{(k,g)|g=0} \pi_{(k,g)}}$.

4.2.3 Solution procedure

To find the combined MC transition and steady-state probabilities, P_0 , the steady-state probability of a node's queue being empty, is required. P_0 can be computed from the Geom/G/1 queue relation $(1 - P_0) = \lambda E[X] = \lambda E[X_R + X_T]$. However, $E[X_R]$ depends on the combined MC solution. We therefore proposed an iterative solution where an initial value is set for P_0 . The combined MC chain can then be solved and $E[X_R]$ computed. A new value of P_0 is then obtained and used for the next iteration. The iteration continues until the value of P_0 converges. To compute the initial value of P_0 , we propose to use the lower bound case for X_R where only the tagged node participates in the competition. The distribution of X_R is then geometric and $E[X_R] = (p\chi)^{-1}$.

Note that the number of competing nodes in one state is not independent of the other states. However by using the steady-state probabilities in (10) combined with the iterative solution procedure, this dependency is ignored. Furthermore, the different functions involve in this system are not necessarily convex. The combined-MC approach is therefore inherently an approximate solution. For example, we have observed that in some cases the combined-MC solution converges to a non-zero value of P_0 while simulations indicated that the system is unstable. However, all studies that we have performed showed that in the region where the system is stable, the combined-MC approximation is very accurate (see results presented in Section 6).

4.2.4 Other approximations

The proposed approach based on the combined Markov chain is scalable. However, the number of slots until reservation success is infinite and the distribution of X_R should be truncated which results in an approximation. Furthermore, even when the distribution of X_R is truncated, the computations are intensive. We therefore propose in this section three different approaches to reduce the computational complexity required to calculate the moments of X_R .

Reduced Markov chain The number of states in the Markov chain can be reduced by assuming that idle nodes will independently have a non-empty queue at the beginning of a timeslot, and therefore participate in the competition, with probability $1 - P_0$. This assumption therefore ignores the dependency between timeslots of the status of an idle node and we don't need to track in the MC state variable the number of idle nodes with a non-empty queue. That is, the combined MC chain presented in Section 4.2.1 can be replaced with the single dimension Markov chain proposed in [6] which uses the single state variable k , the number of busy nodes. In each timeslot, $N - k$ nodes then participate in the competition with an Aloha access probability given by $p_m = p(1 - P_0)$. The distribution of X_R can then be computed as explained in Section 4.2.2 where $P_s(g)$ given in (1) is replaced by $(N - k - 1)p_m(1 - p_m)^{N-k-1}\psi$, the summations in (9) and (10) are over a single variable, and the modified transition probabilities in (9) and steady state probabilities in (10) are obtained from the MC in [6]. The solution procedure is then the same as the one described in Section 4.2.3. This approximate model, denoted by 'Pawelczak-MC' in the numerical results, reduces the computational complexity since the state space is smaller.

Approximate X_R distribution We now propose an approximate approach which significantly reduces the complexity associated with finding the distribution of X_R so that it can be used in scenarios with limited computation capacity. The complexity in (9) arises from the fact that the reservation success probability is time-varying and has memory. As an approximation, we propose to simply assume that the number of participating nodes during a reservation period is always n . Even though the assumption is not realistic,

it helps finding an approximation for X_R . With this assumption, X_{nR} , the reservation time given that the number of participating nodes is n at each timeslot, has a geometric distribution with probability $(1 - H_{s_{max}})P_s^m(k, n|k < s_{max}) + H_{s_{max}}P_s^m(k = s_{max}, n)$, where $H_{s_{max}}$ is the probability of k being equal to s_{max} and is defined as follows:

$$H_{s_{max}} = \sum_{(k,g)|k=s_{max}} \pi_{(k,g)}^N. \quad (11)$$

From the combined MC chain presented in Section 4.2.1, we can compute $Pr(g = n|g \neq 0)$ ($n = 1, \dots, N$), the distribution of the number of nodes participating in the competition excluding the cases where there is no competitors. We can then approximate the distribution of X_R as follows:

$$X_R \approx \sum_n Pr(g = n|g \neq 0) X_{nR}. \quad (12)$$

Given that X_{nR} is geometric, it is then easy to compute the moments of X_R . Since P_0 is not known, the same solution procedure as the one described in Section 4.2.3 should be used. This approximate model is denoted by Combined-MC-Dist in the numerical results.

Average X_R distribution This third approach further simplifies the computations by calculating the reservation time with the average number of idle nodes in competition with a non-empty queue. Let define $\bar{G} = \sum_{(k,g)|g \neq 0} g \pi_{k,g}^N$, where the steady-state probabilities $\pi_{k,g}^N$ are obtained from the combined MC chain presented in Section 4.2.1. Then X_R is approximated with a geometric distribution with probability $(1 - H_{s_{max}})P_s^m(k, \bar{G}|k < s_{max}) + H_{s_{max}}P_s^m(s_{max}, \bar{G})$. Since P_0 is not known, the same solution procedure as the one described in Section 4.2.3 should be used. This approximate model is denoted by Combined-MC-Avg in the numerical results.

At the end of this section, it is worth noting that even though a simplified system model has been addressed, the problem is still too complex to be solved fully analytically. That is the reason that we require approximations. In other words, we modeled the problem and related all engaged parameters, but providing an accurate closed-form solution was intractable (except for small-size models where the accurate occupancy Markov chain approach can be used). The relation between network parameters and performance (delay, jitter, etc) can not be represented with closed-form relation due to the complexity and dimension of the problem. For optimization purposes, the approaches proposed in this paper can be implemented to find the optimal performance by a mix of approximate numerical and analytical results.

5 Switching MAC protocol delay analysis

In this section, we provide the delay analysis for the multichannel OSA switching MAC protocol. The same approach as for the buffering MAC protocol is followed: we first provide a system occupancy Markov chain in Section 5.1 and then we derive an Geom/G/1 queue analysis in Section 5.2. The only difference between the buffering and switching MAC protocols is that in the switching protocol, if a SU senses that its reserved channel is occupied by a PU, it immediately returns to the group of idle nodes with non-empty queue and participates in the competition. For sake of brevity, we omit details in the analysis that are similar to the buffering MAC protocol analysis.

5.1 Occupancy Markov chain

The states of the queue occupancy Markov chain are similar to the buffering case and are given by $(\vec{n}, \vec{b})$. However, to facilitate the analysis, the observation time where the Markov chain has been imbedded is immediately after the initial sensing period when the status of the channels has already been determined. We therefore have that when $b_i = 1$ the reserved channel for node i is available in this timeslot for transmission (the transmission may however be unsuccessful due to the packet capture model).

Similar to the buffering case, for a state A , we can define k_A as the total number of busy nodes who have a channel and whose channels are available in this timeslot and g_A as the total number of idle nodes with non-empty queue. The switching model facts are very similar to the buffering model with the difference

that a node who transmitted on its assigned channel in this timeslot may, if it does not finish its packet transmission, come back to the competition with probability p_c if a PU is detected on its assigned channel.

Transition probabilities for this Markov chain can be found using Algorithm 2 with the same set of impossible states (IS). Similar to the buffering model, when a node i is busy ($b_i = 1$), its transitions are independent of other nodes. For $b_i = 0$ and $n_i > 0$, the node participates in competition. Compared to the buffering model, the main change is for the case where there is a transition between $b_i = 1$ and $c_i = 0$ because this transition may occur in the switching model due to both a service termination or a PU positive sensing. Moreover, we can see that the probability of success is updated by $1 - p_c$ because a success in competition should also be followed by the availability of the reserved channel to be a reservation success.

For the case where we have $k = s_{max}$, the success depends on the nodes who had a transition from $b_i = 1$ to $c_i = 0$. Any of them may have a transition due to missing the channel and PU arrival (kept in vector P_m) or a termination (kept in vector P_t). P_{mt} keeps the total probability of both events. Then, P_{Allm} is defined to be the probability that all transitions from $b_i = 1$ to $c_i = 0$ have been due to PU arrival. In this special case, no reservation success is possible. If at least one node among them has finished its transmission, with the probability P_{10T} , then a competition success may result into a reservation success with the probability $1 - p_c$.

5.2 Service cycle analysis

The nodes queue in the multichannel OSA network with the switching MAC protocol can also be modeled with a Geom/G/1 queue. As illustrated in Figure 2, the Geom/G/1 service time in this queue is composed of alternating renewal processes: uncompleted packet transmissions (a part of the packet however can be transmitted) and competitions for channel reservation alternate until the packet is entirely transmitted. Since the packet length is geometric and the occurrence of an interruption is a Bernoulli event, and both are independent, the uncompleted packet transmission time is thus memoryless and identical. The unsuccessful transmission attempts therefore form a renewal process. The reservation periods X_R during the transmission of a packet are also identical and form the second renewal process. Let X be the service time of the Geom/G/1 queueing model, which can be given by

$$X = X_R + L_e + nX_R = L_e + mX_R, \quad (13)$$

where L_e is the enlarged packet length (in number of slots), which is geometrically distributed with the probability $q\eta$, and $n = m - 1$ is the number of switching during the transmission of the packet. To find the service time, we must find the distribution of X_R and the number of renewals n , which are addressed in Section 5.2.1 and Section 5.2.2, respectively.

5.2.1 Reservation period

The approach to find the distribution of X_R for the switching MAC protocol is similar as the one followed for the buffering model in Section 4.2. The state variable of the combined Markov chain is still given by a 2-tuple (k, g) , however, the observation time where the combined Markov chain is embedded for the switching MAC protocol is after the initial sensing period at the beginning of each timeslot such that the status of the channels for the current timeslot is known. k is therefore the number of reserved channels on which there

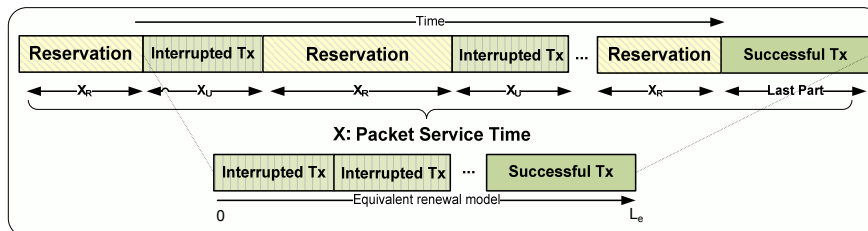


Figure 2: Unsuccessful transmission and reservation periods create alternating renewal processes.

Algorithm 2 The algorithm to find the transition probabilities ($\text{FindProb}(A,B)$) in the switching model. Note that $f_{ap}^0(n_i, m_i) = \lambda^{m_i - n_i} (1 - \lambda)^{1 - (m_i - n_i)}$ ($m_i \geq n_i$) and $f_{ap}^1(n_i, m_i) = \lambda^{1 - (n_i - m_i)} (1 - \lambda)^{n_i - m_i}$ ($m_i \leq n_i$).

```

if ( $k_A == s_{max}$  &  $Num_{(t,AB)} == 0$ ) then
     $AllBusyNoTer = 1$ 
for  $i = 1 : N$  do
    if ( $n_i == 0$ ) then
         $P_i = f_{ap}^0(n_i, m_i)$ 
    else
        if  $b_i == 1$  then
            if ( $c_i == 1$ ) then
                 $P_i = (1 - q\eta)(1 - p_c)f_{ap}^0(n_i, m_i)$ 
            else
                if ( $k_A! = s_{max}$ ) then
                     $P_i = (1 - q\eta)(p_c)f_{ap}^0(n_i, m_i) + (q\eta)f_{ap}^1(n_i, m_i)$ 
                else
                    Put  $i$  in  $\mathbb{S}_{10}$  ;  $h = h + 1$ 
                     $P_i = 1$  ;  $P_{t,h} = (q\eta)f_{ap}^1(n_i, m_i)$ 
                     $P_{m,h} = (1 - q\eta)(p_c)f_{ap}^0(n_i, m_i)$ 
                     $P_{mt,h} = (1 - q\eta)(p_c)f_{ap}^0(n_i, m_i) + (q\eta)f_{ap}^1(n_i, m_i)$ 
            else
                Put  $i$  in  $\mathbb{S}_g$  ;  $P_i = f_{ap}^0(n_i, m_i)$ 
for  $i = 1 : |\mathbb{S}_{10}|$  do
     $P_{10T} = P_{10T} + \prod_{h=1}^{h-1} P_{m,h} P_{t,h} \prod_{h+1}^{|\mathbb{S}_{10}|} P_{mt,h}$ 
 $P_{Allm} = \prod_{h=1}^{|\mathbb{S}_{10}|} P_{m,h}$ 
 $\text{FindProb}(A,B) = \prod_i P_i \prod P_{mt,h}$ 
    ***** Handling Competitions ***** ( $IF\mathbb{S}_g \neq \emptyset$ )
if ( $k_A! = s_{max}$ ) then
    Find  $j$  s.t. ( $c_{\mathbb{S}_g[j]} == 1$ )
    if  $j > 0$  then
         $P_{\mathbb{S}_g[j]} = P_{\mathbb{S}_g[j]} * \frac{P_s(|\mathbb{S}_g|)}{|\mathbb{S}_g|} (1 - p_c)$ 
    else
        if ( $AllBusyNoTer == 0$ ) then
             $P_{\mathbb{S}_g[1]} = P_{\mathbb{S}_g[1]} * [1 - P_s(|\mathbb{S}_g|)(1 - p_c)]$ 
else
    Find  $j$  s.t. ( $c_{\mathbb{S}_g[j]} == 1$ )
    if  $j > 0$  then
         $P_{\mathbb{S}_g[j]} = P_{\mathbb{S}_g[j]} * \frac{P_s(|\mathbb{S}_g|)}{|\mathbb{S}_g|} (1 - p_c)$  ;  $P_{\mathbb{S}_m[1]} = P_{10T}$ 
    else
        if ( $AllBusyNoTer == 0$ ) then
             $P_{\mathbb{S}_g[1]} = P_{\mathbb{S}_g[1]} * [P_{Allm} + P_{10T}(1 - P_s(|\mathbb{S}_g|)) + P_{10T}P_s(|\mathbb{S}_g|)(1 - p_c)]$ 
     $\text{FindProb}(A,B) = \prod_i P_i$ 

```

will be a transmission attempt in this timeslot and g is the number of idle nodes with a nonempty queue which participate in the competition on the control channel for reservation. The number of idle nodes with an empty queue is thus given by $N - g - k$.

To find transition probabilities, we use four auxiliary variables i, j, n and m . Note that in the following, k and g refer to $k(t)$ and $g(t)$ for the current timeslot t . Among k busy nodes, i nodes independently finish the transmission of their packets, each with the probability $q\eta$. We use the notation $T_k^i = \binom{k}{i} q\eta^i (1 - q\eta)^{k-i}$ for the probability of this event. Any of those i nodes may join in the next timeslot, with probability $1 - P_0$, the idle nodes with a nonempty queue. Otherwise, they join the idle nodes with an empty queue. The probability

that n nodes out of the i finishing nodes will be among the $g(t+1)$ nodes in the next timeslot is thus given by $\binom{i}{n} P_0^{i-n} (1 - P_0)^n$. For the remaining $k - i$ nodes which do not finish their transmission, j nodes may leave their channels due to PU presence sensing with probability $F_{k-i}^{k-i-j} = \binom{k-i}{j} (1 - p_c)^{k-i-j} (p_c)^j$. Those nodes also join the $g(t+1)$ idle nodes with a nonempty queue after the sensing period. The remaining $k - i - j$ nodes stay on their channels, so they are among the $k(t+1)$ nodes.

Considering g participants in competition, we have the same results for competition success as for the buffering case. One node may then be successful and join the $k(t+1)$ nodes with probability $P_s(k, g)(1 - p_c)$. The $1 - p_c$ term is included because the node may be successful in competition and reservation during this timeslot, but the reserved channel might be unavailable in the next timeslot. Therefore, without starting the transmission, the node has to come back to competition. All other $g - 1$ nodes remain in the group of idle nodes with a nonempty queue, i.e., they are among $g(t+1)$ nodes. Finally, any of $N - k - g$ idle nodes with an empty buffer may receive a packet and join the participants in competition in the next timeslot. The probability to have m such nodes is given by $\binom{N-k-g}{m} \lambda^m (1 - \lambda)^{N-k-g-m}$.

We can thus, for the switching MAC protocol, express the transition probabilities of the combined Markov chain in the form $P_{(k,g) \rightarrow (z,h)}$, as in (14), where $Y_a^{b,c}$ is given in (8).

$$P_{(k,g) \rightarrow (z,h)} = \begin{cases} 0 & \text{if } z > k + 1, \forall g, h, \\ 0 & \text{if } \forall k, z, h < g - 1, \\ T_k^{(0)} F_k^k S_g^{(1)} (1 - p_c) Y_{(h-g+1)}^{(k-z+1), (N-k-g)} & \text{if } z = k + 1, \\ \sum_{i=0}^{k-z} T_k^i F_{k-i}^z (1 - S_g^{(1)} (1 - p_c)) Y_{(h-g-(k-i-z))}^{i, (N-k-g)} \\ + \sum_{i=0}^{k-z+1} T_k^i F_{k-i}^{z-1} (S_g^{(1)} (1 - p_c)) Y_{(h-g-(k-i-z))}^{i, (N-k-g)} & \text{if } 0 < z \leq k, k + z \neq 2s_{max}, \\ \sum_{i=0}^{k-z} T_k^i F_{k-i}^z Y_{(h-g-(k-i-z))}^{i, (N-k-g)} \\ + \sum_{i=0}^{k-z+1} T_k^i F_{k-i}^{z-1} (S_g^{(1)} (1 - p_c)) Y_{(h-g-(k-i-z))}^{i, (N-k-g)} & \text{if } z = k = s_{max}, \\ \sum_{i=0}^{k-z} T_k^i F_{k-i}^z (1 - S_g^{(1)} (1 - p_c)) Y_{(h-g-(k-i-z))}^{i, (N-k-g)} & \text{if } z = 0. \end{cases} \quad (14)$$

Given the transition probabilities for the combined Markov, the distribution of X_R can be obtained as explained in Section 4.2.2. The second and third approximations given in Section 4.2.4 to find the distribution of X_R can also be used (the approximation based on the model presented in [6] is not applicable for the switching MAC protocol).

5.2.2 Renewal process

As illustrated in Figure 2, X_U , any part of the whole service time where the node has a reserved channel, is the inter-arrival time of the channel unavailability events. Let us use the random variable A_u to denote the inter-arrival time of those events. A_u has a geometric distribution with probability p_c . From the Renewal Theory point of view [27], $n = m - 1$ is the number of renewals from 0 to L_e where the process which is renewed is A_u . The number of renewals can be found from Renewal Theory results [27].

However, as we have geometric distributions for both L_e and A_u , we can use the following simpler approach. Due to the memoryless packet length, the remaining part of the packet in any operating period is geometrically distributed with parameter q . The total number of renewals is thus the number of trials until success, which is a geometric distribution of type-II, with the probability of success in a period being $Pr(L_e \leq A_u)$, the probability of sending the whole packet in the given period. However, the number of reservation periods which appear in the calculation of service time is $m = n + 1$ because the first reservation period is not counted as a renewal (see Figure 2). We thus have:

$$Pr(n == k) = (1 - Pr(L_e \leq A_u))^k Pr(L_e \leq A_u). \quad (15)$$

For two type-I geometrically distributed random variables, we have:

$$Pr(L_e \leq A_u) = \frac{q\eta}{1 - (1 - q\eta)(1 - p_c)}, \quad (16)$$

and thus

$$E[n] = \frac{1 - \frac{q\eta}{1-(1-q\eta)(1-p_c)}}{\frac{q\eta}{1-(1-q\eta)(1-p_c)}} = \frac{p_c(1-q\eta)}{q\eta}. \quad (17)$$

$E[n^2]$ can be found similarly. For the second moment of X , we have a random sum and the two parameters n and L_e are not independent. Using the facts that $\sum_{k=1}^{\infty} k(1-q)^{k-1} = \frac{1}{q^2}$ and $\sum_{k=1}^{\infty} k(k-1)(1-q)^{k-2} = \frac{2}{q^3}$, the term $E[L_e n]$ can be given by:

$$\begin{aligned} E[L_e n] &= \sum_{k=1}^{\infty} E[L_e n | (L_e = k)] Pr(L_e = k) = \sum_{k=1}^{\infty} k E[n | (L_e = k)] Pr(L_e = k) \\ &= \sum_{k=1}^{\infty} k(k-1) p_c q \eta (1-q\eta)^{k-1} = \frac{2p_c(1-q\eta)}{(q\eta)^2}. \end{aligned} \quad (18)$$

We have used the fact that the number of renewals in the interval $[0, k]$ of a type-I geometric with the parameter p_c is equal to $(k-1)p_c$.

From the moments of n and X_R , it is then straightforward to find the moments of X and solve the Geom/G/1 queue. Note that since P_0 is initially unknown and is required to find the distribution of X_R , the solution procedure outlined in Section 4.2.3 can be followed. The initial value of P_0 is computed with the lower bound of X_R obtained for the case where only a single node is competing and for which $E[X_R] = (p\chi(1-p_c))^{-1}$.

6 Simulation and numerical results

In this section, we verify the accuracy of the proposed models and approximations by comparing simulation results with numerical evaluations of the theoretical results presented in this paper. We also study the impact of several parameters on the multi-channel OSA MAC protocols performance. Simulations are done with Matlab where each simulation scenario is run for 350000 timeslots and the same scenario is repeated 10 times. Unless indicated otherwise, we have $M_C = N$ data channels. In Sections 6.1 to 6.4, we investigate the buffering MAC protocol while in Section 6.5 we present results for the switching MAC protocol.

6.1 Impact of arrival rate

Figure 3 shows the average system time as a function of the arrival rate for the buffering MAC protocol and for different system parameter values. It can be first observed that, as expected, the queue occupancy Markov chain provides accurate results compared with simulation results, called 'truncated simulation', obtained with the same limited buffer size assumed to numerically solve the occupancy Markov chain. All approximations also provide acceptable results at low arrival rate while for larger values, when the system gets close to instability, their accuracy decreases. For example, for $N = 10$ users, the combined-MC approximation is accurate within 3% for a normal operation region below 80% of maximum loading. It was observed through several scenarios that, as expected, the combined Markov chain provides the best approximation while the average based and distribution based approximations both diverge sooner than the two other schemes when the queue approaches the instability region. In the following, to help the figures readability, we will only present some of the results obtained with the different approximations.

6.2 Impact of medium access probability

The probability of medium access on the control channel has a major influence on the performance of the network. A very small p or a very large p makes the system unstable because, in the former case, a packet has to wait several slots before the node tries to reserve a channel, and, in the latter case, there will be a lot of collisions such that the reservation success probability will be very low. In Figure 4, we can see that for any given scenario of the buffering MAC protocol, there is an optimal value for p which depends on the arrival rate. This optimal value decreases when the total traffic load on the control channel increases due to either an increase in the arrival rate, an increase in the number of nodes or smaller packets. Interestingly, it can be

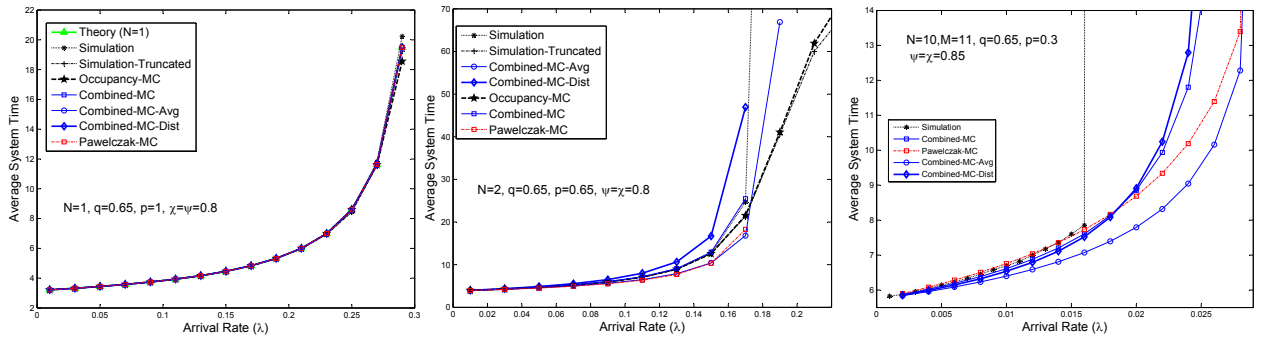


Figure 3: Performance comparison of the proposed schemes versus the variation of the arrival rate for the buffering MAC protocol.

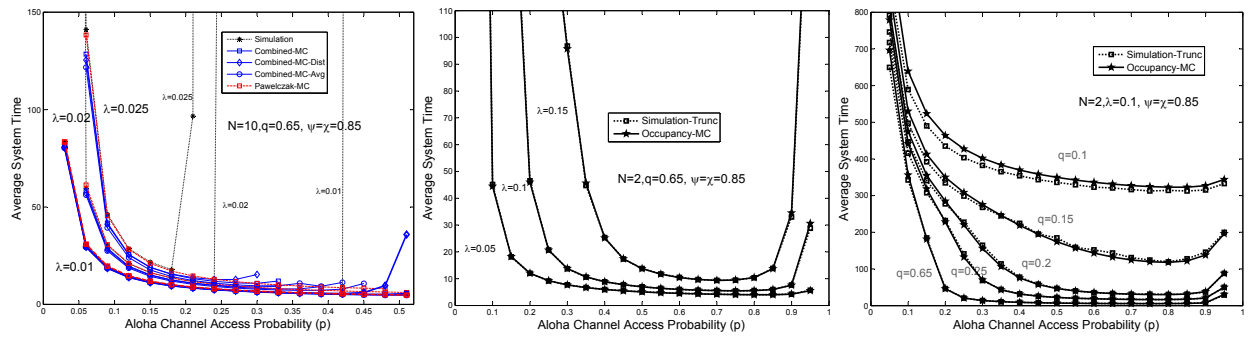


Figure 4: Average system time for the buffering MAC protocol versus the Aloha-type probability of control channel access, for different arrival rate and packet length values.

observed that by increasing the packet length (smaller q), even though the total system load and the delay increase because the packets are now longer, the optimal probability of channel access p also increases. This is due to the fact that with longer packets, the nodes return less frequently to the control channel. There is thus less competition for channel reservation and a higher probability of control channel access is optimal.

It is also observed that the optimal control channel access probability is significantly larger than for the saturated traffic model where the optimal value of p is very small [6]. There is also a large range of values of p where the performance is similar. However, this range decreases with an increase in traffic load of the system.

6.3 Impact of packet length

As observed in Figure 4, the system time increases when the system load increases either due to an increase of arrival rate or packet length. In Figure 5, we investigate the impact of both parameters on the buffering MAC protocol when the traffic load λ/q is fixed. It can be observed that as the packet length increases (and the arrival rate decreases), the system time decreases. This is due to the fact that in the network model with an Aloha-type medium access algorithm, the main bottleneck is the competition on the control channel to reserve a data channel. For a fixed traffic load, when the packet length increases, fewer nodes will therefore participate in the competition and the nodes with a reserved channel spend more time on the reserved channel and less time waiting on the control channel to get a reservation success.

6.4 Impact of transmission capacity

We investigate in this section the impact of transmission capacity which depends on the number of available channels and the channel availability ψ . In Figure 6, the number of nodes is fixed to $N = 10$ and the number of data channels is increasing from $M_C = 1$ to $M_C = 10$, for two different values of packet length. As

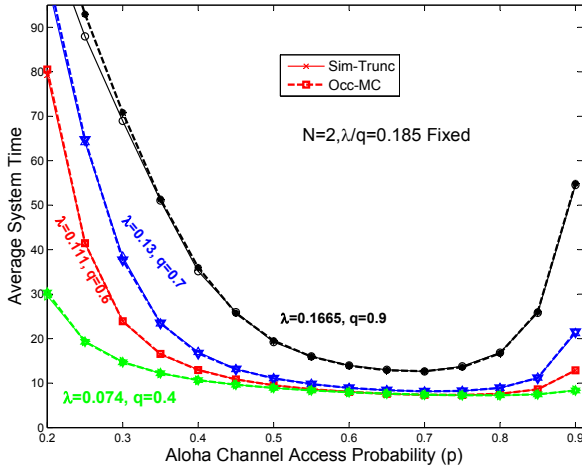


Figure 5: Performance comparison for the buffering MAC protocol for a fixed traffic load of $\lambda/q = 0.85$ and $\chi = \psi = 0.85$.

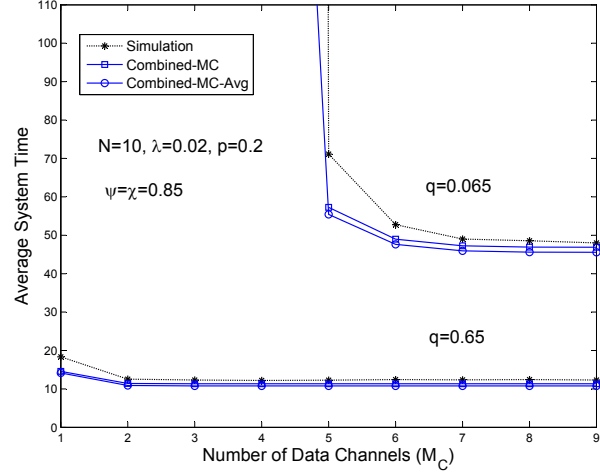


Figure 6: System time as a function of the number of data channels M_C for the buffering MAC protocol.

discussed previously, when the number of nodes is high the probability of success in competition decreases and all nodes spend a longer time for reserving a channel. Even though there are available channels, no node can make a reservation; therefore, the increase of the number of channels provides no gain for the network. When the packet length is small ($q = 0.65$), the packet transmission time is short, so a winner again joins quickly the competing users and the probability to have even two channels busy at the same time is low. With a longer packet length ($q = 0.065$), fewer users compete and data channels are more utilized. The system time therefore decreases until 6 data channels are available and marginal gains are achieved for larger values of M_C .

The observation that the main bottleneck of an Aloha-based cognitive radio MAC protocol is the channel reservation competition on the control channel leads us to propose the concept of channel and node clustering. That is, instead of having a single control channel the CR nodes and channels can be divided into clusters where each cluster has its own control channel. The results presented in Figure 7 show the performance of such a clustered system. We assumed a network with 10 nodes and 10 channels and compared the performance when there is a single control channel and all nodes operate under a single entity of the buffering MAC protocol (i.e., $N = 10$ and $M_C = 9$) with the case where the nodes and channels are divided into two clusters and each cluster has its own control channel and operate under an independent entity of the buffering MAC protocol (i.e., $N = 5$ and $M_C = 4$). The probability of medium access, p , has been adjusted to the value which minimizes the delay for each case. It can be observed that even though one additional channel is now used as a control channel in the two-cluster case, and we thus have fewer data channels, the system time has significantly decreased for all values of arrival rate. Furthermore, a larger arrival rate can be served when there are two clusters. Those results confirm our hypothesis that the delay bottleneck is due to the reservation competition on the control channel and the fact that, depending on the system parameters, it might be more beneficial to add control channels than to add data transmission capacity.

Another important issue is the trade-off between the number of channels and their availability, when, for instance, a cognitive radio network has the option of utilizing a spectrum with a large number of channels with a lower availability per timeslot and another spectrum with a smaller number of channels with a higher availability. In Figure 8, we investigate this tradeoff where the channel availability ψ decreases when M_C increases. Note that we kept the control channel availability constant. The interesting point to observe is that there is an optimal point where the average delay is minimized. This is due to the fact that because of the control channel access bottleneck, an increase in the number of channels does not provide any significant gain for the network (see results presented in Figure 6) while the decrease in availability of the assigned channel increases the transmission time of the packet. Such an optimal point exists for any network as a function of

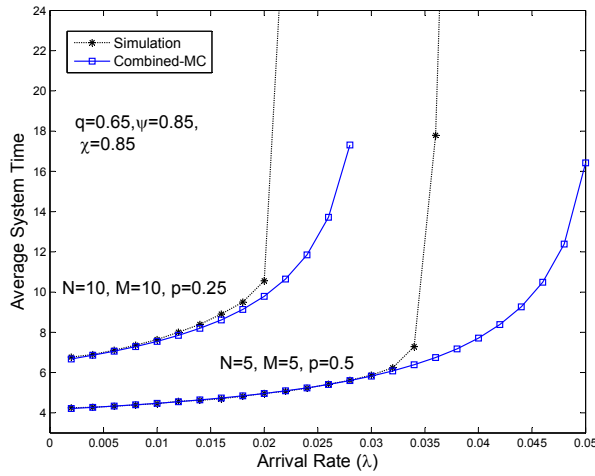


Figure 7: Performance comparison for the buffering MAC protocol of a network with $M = N = 10$ and the clustered network with two clusters of $N = M = 5$.

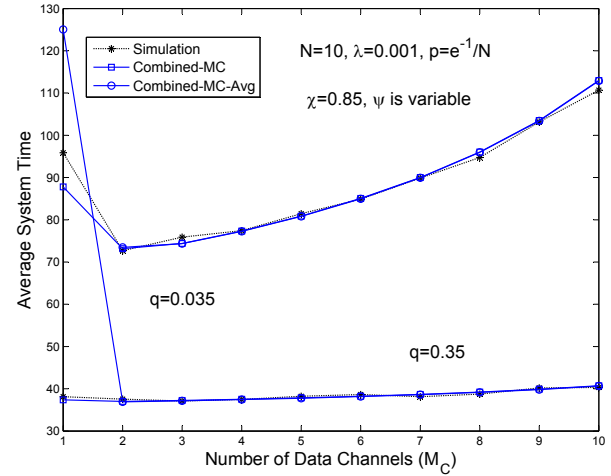


Figure 8: Impact of the variation of the number of data channels M_C when the channel availability, ψ , decreases for the buffering MAC protocol. For $M_C = [1 \dots 10]$, we have $\psi = [0.85, 0.8, 0.75, 0.7, 0.65, 0.6, 0.55, 0.5, 0.45, 0.4]$.

the system parameters, so the analytical queueing results presented in this paper enable the network designer to select the best operating channel set.

6.5 Switching MAC protocol

In this section we investigate the performance of the switching MAC protocol. Figure 9 shows the system time as a function of the arrival rate for $N = 10$ nodes. It can first be observed that the analytical approximations provide a good prediction of the system performance. Those results also show that there is a significant performance degradation, approximately 50% higher delay for similar parameters, versus the results presented previously for the buffering MAC protocol. This is due to the fact that for a packet transmission a node has to incur several reservation periods, which are the delay bottleneck, and furthermore this creates more competition on the control channel, which further increases the reservation delay.

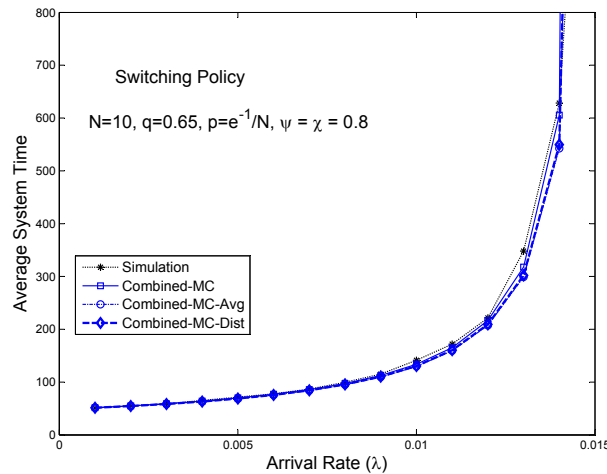


Figure 9: System time for the switching MAC protocol as a function of the arrival rate.

In Figure 10, the performance versus the variation of the number of data channels and channel availability ψ is illustrated. It can easily be seen that the performance of the switching policy significantly degrades especially when the packet length is long because the longer the packet are and the more frequent the transmission occurs, the more often the nodes return to the control channel. However, we can still observe for large packets ($q = 0.035$) an initial improvement when the number of channels increases from one to two.

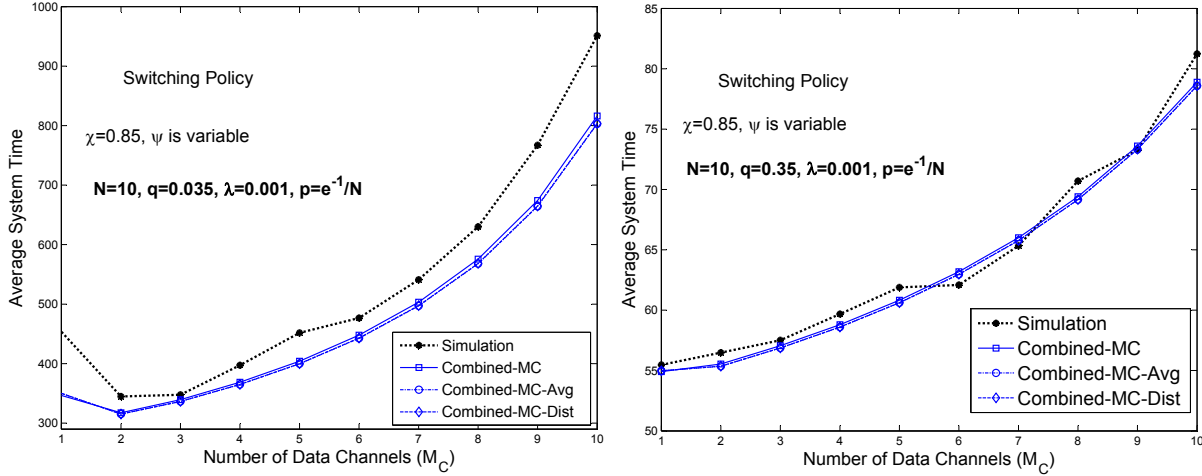


Figure 10: Performance comparison versus the variation of the number of data channels M_C when the channel availability, ψ , decreases for the switching MAC protocol. For $M_C = [1 \dots 10]$, we have $\psi = [0.85, 0.8, 0.75, 0.7, 0.65, 0.6, 0.55, 0.5, 0.45, 0.4]$.

Figure 11 shows the impact of Aloha access probability on the packet system time for the switching MAC protocol. Those results first validate the occupancy Markov chain model. It is also interesting to observe that even though the switching policy is more sensitive to an increase in packet length due to the frequent returns to the control channel, there are some scenarios, as illustrated in the figure, in which having a lower arrival rate with longer packets provides a better performance than having a higher arrival rate of shorter packets.

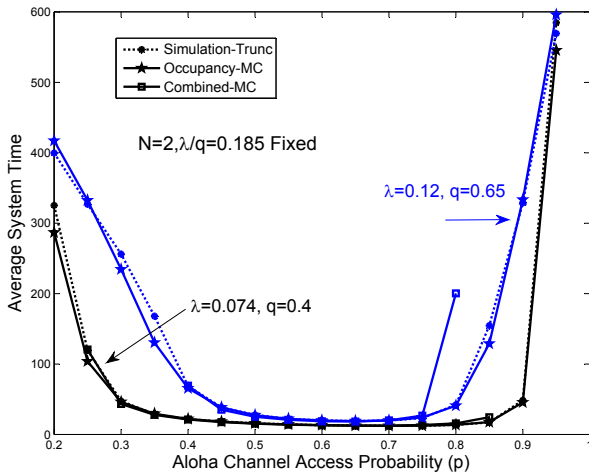


Figure 11: Average system time for the switching MAC protocol versus the Aloha-type probability of control channel access. ($M_C = N, \chi = \psi = 0.8$).

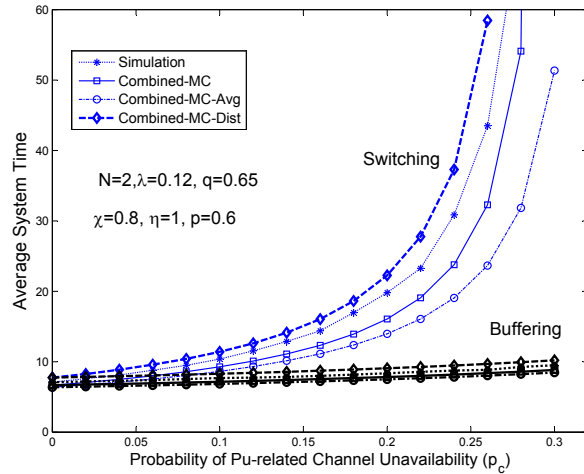


Figure 12: Average system time for the switching and buffering MAC protocols as a function of PU probability of activity (p_c).

Since the switching occurs due to PU arrivals and not fading, we investigate the impact of variation of p_c on the system time. The results for the switching and buffering MAC protocols are illustrated in Figure 12. When $p_c = 0$, the two policies are naturally the same. When p_c increases, the performance of the buffering MAC policy only slightly deteriorates due to a small increase in packet transmission time. However, for the switching MAC policy, each PU interruption incurs a large penalty of channel reservation. Therefore, the performance quickly degrades as a function of p_c .

Comparing the results of the buffering and switching policies reveals that in all scenarios, the buffering MAC protocol outperforms the switching. This is due to the homogeneity of the channels and memoryless PU presence. That is, with the switching policy if the channel is not available, the node tries to reserve a channel. In the best case, the reservation process will last one timeslot. However, the reserved channel has the same probability of being available in the next timeslot as if the node stayed on the same channel with the buffering policy. So there is no way for a node to decrease its transmission delay by switching.

7 Discussion

The assumptions in this paper are made for simplicity of presentation and for consistency with the assumptions in [6, 7] to be able to compare and verify some of the results. In this section, we discuss how approximate and simplified results can be employed in practice, and how other scenarios and assumptions can be addressed as an extension to this work.

Approximations in practice As discussed, the relation between network parameters and performance can not be represented with nice closed-form relations due to the complexity and dimension of the problem. For practical purposes, the solution procedure will similarly use a combination of approximate and accurate pure analytical and numerical analysis. An entity may observe and log network activities for a given period to find different network parameters can statistically. The next step will be fitting simplified distributions on the statistical distributions to be able to represent the complex observed parameters with simplified but approximate simpler distributions. The central entity will then be able to use the results of this paper in order to derive the approximate performance metrics and performance trends, and exploit them for optimization and network design purposes. This cycle of observation, approximation and act revision is repeated, and the period of re-computation will depend on network dynamism, observation overhead and processing limitations. As the cumbersome step is to find the distribution of reservation periods, X_R , it is worth reminding that we provided a framework and Markov chain model, which provides an opportunity for future work to come up with other solutions/approximations for finding the distribution of reservation periods.

Memory-aware distributions Memoryless distributions decrease the state complexity and simplify the analysis. On the node side, a memory-aware arrival process results in a $G/G/1$ queue, which is known to be complex and intractable [23]. However, a bulk Poisson arrival can be addressed and will induce additional transitions in the queue occupancy Markov chain with no significant change in the delay analysis approach. If the packet length is assumed to be generally distributed, an additional state variable is required to keep the length of the remaining part of the packet for all users in service, which increases the state complexity. For the channel, a packet capture model with memory (e.g., a Markov chain channel fading model [28]) can be addressed as follows. When a node starts using a channel, the user has no knowledge of the state of the channel in the previous timeslot. Therefore, X_T (similarly for X_u in the switching model) is found conditioned on the channel state in the first timeslot. Then using the steady state probabilities, X_T can be found. To solve the queue, the state of the channel of all transmitting (busy) users should be kept. The delay cycle analysis approach will not be scalable anymore even for a two-state channel model. A multi-state channel model needs more investigation because from the queueing point of view it results in a queueing model with variable service rate. Addressing a Markov chain PU availability model with two states (geometrically distributed ON/OFF periods) will be similar and results in the same approach and complexities. The approach will be again to conditionally define the current state based on the unknown previous state, and then using the steady-state probabilities to drop the condition. Generally distributed ON/OFF periods will be more complicated since the state space will be larger and not scalable.

Collision among secondary users We made the assumption that the list of channels not being used by the secondary network is available, so a competition winner takes the first channel in the list and no collision may happen among secondary users. This is the only limiting assumption on the spectrum knowledge, which is however a common assumption in the literature on multi-channel cognitive radio networks. On the other hand, if two or more secondary users operate concurrently on a channel, two cases may be considered: 1) Channel capacity is shared among users and they may have a successful transmission with a lower rate, 2) All users will have an unsuccessful transmission. The first case results in a variable rate, which, as discussed, is complicated and remains as future work. The second case implies that the probability of successful transmission, η , will also be a function of the probability of reservation success and the probability that a channel occupied by a secondary user is selected by the competition winner. Therefore, by changing the definition of η , the obtained results will be valid.

Heterogeneous users Throughout this work, we assumed that all users are homogeneous (same arrival rate and packet length distribution). The queue occupancy Markov chain approach can easily address the case with heterogeneous users by assuming a different λ and q for each user. For the delay cycle analysis, the combined Markov chain should be enlarged to consider the heterogeneity of the users. Based on the number of user classes, we need to have more state variables which increases the complexity.

CSMA MAC model In our system model, an Aloha contention management model was assumed. Considering a more realistic CSMA model remains as future work. The approach will be to combine the CSMA Markov chain proposed by Bianchi [29] and the Markov chains proposed in this work together to have a bigger multi-dimensional Markov chain.

Distributed algorithms The proposed medium access models do not rely on any central entity, so they are appropriate for distributed control. The only information which is assumed to be known in the whole network is the list of channels not being used by the cognitive radio network, which can be obtained from broadcast control messages. Otherwise, collisions between secondary users need to be considered.

8 Conclusion

In this paper, a delay and queueing analysis for a multi-node network with homogeneous users and an Aloha-type medium access model was provided. Two different recovery models were considered: a waiting and buffering recovery policy where the CR node waits for the primary user to vacate the channel and continues the transmission on the same channel, and a switching policy where after the appearance of primary user, a channel handover occurs. It was observed that access to the control channel to reserve a data channel is the major bottleneck in Aloha, so any approach which decreases the number of competitors, such as having fewer but longer packets and node clustering, improves the performance. The probability of medium access in Aloha should also be adjusted carefully to have the minimum delay. With the assumption of having homogeneous and memoryless channels, a buffering policy always outperforms the switching policy.

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