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The Tube Challenge

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Abstract: The Tube Challenge consists of visiting all stations of the London Underground in the least possible time. The competition started in 1959 and the current record was established in August 2013. This paper shows that under some simplifications, this problem and some of its variants can be cast as a Generalized Traveling Salesman Problem, as a Traveling Salesman Problem, as a Rural Postman Problem or as a Chinese Postman Problem defined on a directed graph.

Key Words: Tube Challenge, London Underground, directed Generalized Traveling Salesman Problem, directed Traveling Salesman Problem, directed Rural Postman Problem, directed Chinese Postman Problem.

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1 Introduction

The Tube Challenge consists of traveling through all stations of the London Underground within the least possible time (Wikipedia, 2014). This excludes the stations of the London Overground, of the Docklands Light Railway and of the Emirates Airline gondola lift, for a total of 270 stations at the time of writing. There are three pairs of stations with the same name (Edgware Road, Hammersmith and Paddington) which are in fact distinct from each other and must each be visited. A station can only be counted if the train actually stops there, as opposed to just passing through. Contestants may use the Underground, or they can travel by foot or by public bus between stations. The rules of the contest can be found in http://www.tubeforum.co.uk/rules.html (2014). The current record belongs to Geoff Marshall and Anthony Smith, from the United Kingdom, who accomplished their feat in 16 h, 20 m and 27 s on 16 August 2013 (Geofftech.co.uk, 2014). It is officially entered in the Guinness Book of Records (2013). The competition has been going on since 1959, but was only officially recognised in 1960. It is difficult to accurately compare the relative performance of the teams because the network evolves over time.

Various websites and blogs vaguely mention the connection between the Challenge and the Shortest Path Problem (SPP) and the Traveling Salesman Problem (TSP), two topics which are closely related to my research and teaching. The SPP consists of determining a shortest path from a source node to a sink node on a graph; the TSP consists of computing a least cost Hamiltonian circuit or cycle on a graph. The problem is clearly not an SPP, nor can it be defined as a TSP over all stations of the Underground. It is, however, closely related to the TSP, as I will show, and some variants of the problem can be formulated as a TSP or as other known routing problems. The purpose of this paper is to establish these connections.

My interest in this exercise is to motivate the modeling of these problems through a concrete example, namely in a vehicle routing course I give each year. This being said, it is extremely difficult to define the problem through a neat mathematical model for a number of reasons. First, the Challenge is defined in terms of duration minimization, as opposed to distance minimization. This creates several methodological difficulties since travel times are highly stochastic, even if the London Underground operates in principle according to a predefined schedule. During the course of a typical day, multiple train service delays and interruptions occur, and it is also impossible to accurately measure the transfer times between platforms or between stations when foot travel or buses are used. In other words, the problem is highly stochastic and the probability distributions of random events are unknown. Although the performance of contestants can be measured, the combinatorial problem they are indirectly solving cannot be modeled with accurate data. For this reason, I will make a number of simplifications in my exposition, including that of working with deterministic parameters.

The remainder of this problem is organized as follows. Section 2 is devoted to the problem of visiting all stations of the London Underground, whereas the problem of traversing all links of the system is treated in Section 3. Conclusions follow in Section 4.

2 Visiting all stations

Figure 1 depicts the map of the London Underground, including some overground lines that are not considered in the challenge. This stylised representation of the network does not constitute an accurate representation of reality since it distorts the actual geographic distances. For example Stanmore (Jubilee line) and Edgware (Northern line) are much closer to each other than the map indicates. Similarly, Wimbledon (District line) and Morden (Northern line) are not very far apart. This suggests that it may sometimes pay to walk or use buses between stations. In their website (Geofftech.co.uk, 2014), the current record holders never reveal their winning solution. They write that they solve a TSP as a first approximation and then check the feasibility of the solution in terms of scheduling with the TFL Journey Planner (2014). They do not specify the graph on which the TSP is solved, but one can assume that they work with an undirected graph defined over all stations and use estimates of travel times as travel costs. Since such a model ignores transfer times between platforms of a station (which are not negligeable), its costs underestimate the true solution cost. If transfer costs are added a posteriori to the TSP solution cost, one then obtains an upper bound on the optimal cost of a solution that would have incorporated transfer times in the model.

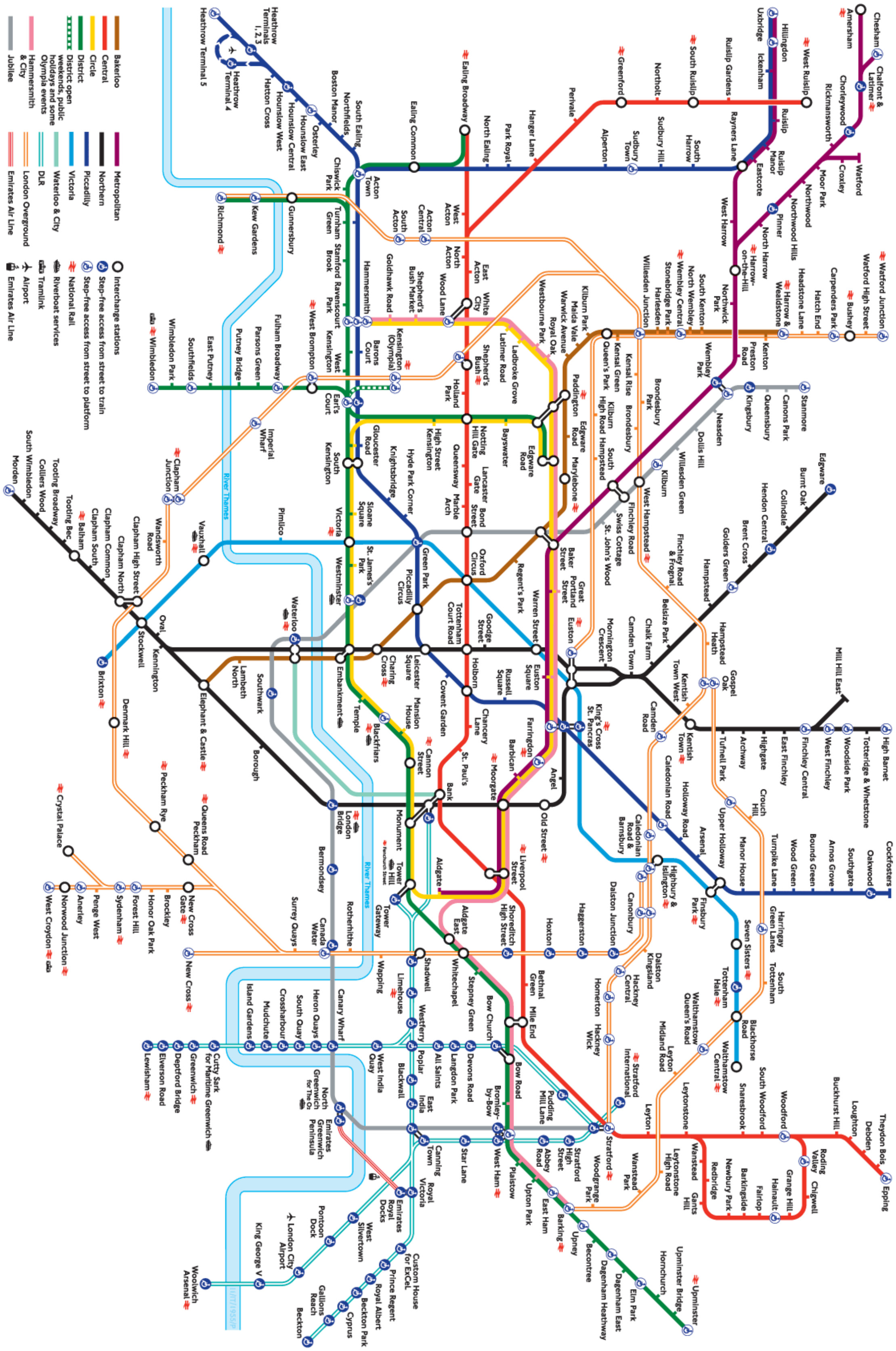

Figure 1: The London Underground

In order to properly account for transfer times, it is necessary to use a finer graph representation in which the nodes are *platforms* as opposed to *stations*. For example, up to 2 l platform nodes are obtained whenever l lines meet at a station (Figure 2), and all arcs between them must be generated. The problem can then be defined on a directed graph $G = (N, A)$, where N is the set of all platform nodes, and A is the set of arcs linking the nodes in either direction. The cost c_{ij} of arc (i, j) is an approximation of the shortest travel time from i to j using the underground or overground travel. Let C_k be the *cluster* consisting of the set of all platform nodes of station k. Then the problem can be cast as a directed *Generalized Traveling Salesman Problem* (GTSP) (in its "at least" version) which consists of determining a least cost circuit containing at least one node per cluster. Exact algorithms for the directed GTSP are provided in Laporte et al. (1987), for example.

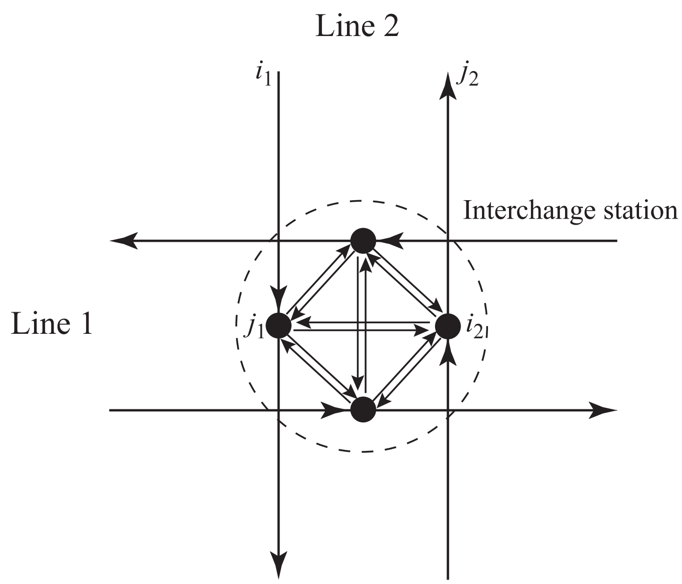

Figure 2: Four platform nodes ● are generated when two lines meet at a station

A meaningful variant of the Tube Challenge would be to require that each platform be visited. The problem would then be an asymmetric TSP on the graph induced by the set of all platforms. This problem, which involves a few hundred nodes, is well within the reach of asymmetric TSP solvers (Lodi and Punnen, 2002).

Another interesting variant, applicable to both the "all stations" or "all platforms" cases would be to prohibit ground transportation. This means that all travel would have to take place exclusively on the Underground. In this case, an important graph simplification can be made. Indeed, it is never optimal to change directions between two interchange stations. This means that all stations at which a transfer is not possible can be eliminated from the graph, and the travel costs can therefore be recomputed between consecutive interchange stations. This yields a reduced node set $N' \subseteq N$. In this context, an interchange station is not only a station that allows a change of lines, but also a station that allows branching on the same line, for example Camden Town on the Northern line. The stretch between Acton Town and Heathrow Terminal 5, both on the Piccadilly line, can also be condensed since one can easily figure out the best order in which to visit Heathrow Terminal 4.

Finally observe that if the timetables of the TFL journey planners were fully respected, all the above problems could be cast in their time-dependent versions, since travel times depend on the time of day.

3 Traversing all links

A further variant of the Tube Challenge is obtained by requiring that each *link* of the graph be traversed in either direction. Here, a link is defined as an undirected edge between two stations. One way to solve this problem is again to transform it into a directed GTSP. As in Section 2, first define node clusters representing the platforms of the same station. Then replace each edge with two opposite arcs and associate a node with each arc (Figure 3). A pair of opposite arcs (i_1, i_2) and (j_1, j_2) then defines a cluster $\{i_1i_2, j_1j_2\}$, where i_1i_2 and j_1j_2 are called *arc nodes*. The travel cost from node i_1i_2 to node j_1j_2 is that of a least cost path from i_2 to j_1. The problem is then a directed GTSP consisting of visiting at least one node in each cluster.

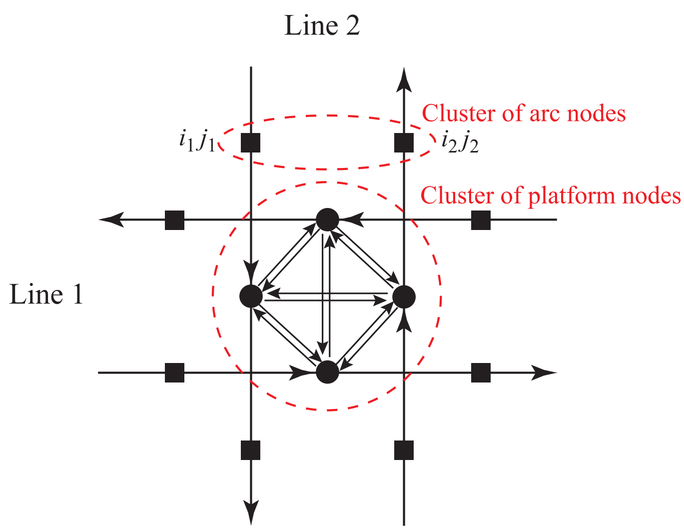

Figure 3: Directed graph representation showing platform nodes ● and arc nodes ■

One may also define yet another version of the problem by requiring that all tracks of the system be traversed. This can be achieved by using a directed graph transformation, where each track is represented by an arc and all platform nodes within the same station are again mutually connected by two opposite arcs. When defining this directed graph, care must be taken to account for parallel tracks linking two stations in the same direction. One such case is the Hammersmith to Acton Town stretch where the Piccadilly and the District lines run in parallel. Let A' be the set of arcs used in this representation, which can be partitioned into $\{A'_R, A'\backslash A'_R\}$, where A'_R is the set of inter-station arcs and $A'\backslash A'_R$ is the set of arcs linking two platforms of the same station. The problem can then be cast as a directed Rural Postman Problem (RPP) on $G = (N', A')$, where the set of required arcs is A'_R (Figure 4) and the arcs of $A'\backslash A'_R$ are non-required. The directed RPP consists of determining a least cost closed traversal of all required arcs of a graph. Several exact algorithms and heuristics also exist for the directed RPP (Corberán et al., 2014).

Finally, the problem of visiting all tracks and all inter-platform arcs of the system is simply a directed Chinese Postman Problem on the graph $G = (N', A')$. This problem consists of determining a least cost closed traversal of all arcs of G. It can be solved in polynomial time by first making the graph Eulerian through the solution of a transportation problem, and then traversing the augmented graph by means of the end-pairing algorithm (Edmonds and Johnson, 1973).

4 Conclusions

The Tube Challenge is a rather difficult competition partly because of its size but also because travel times on the London Underground and transfer times between stations are not deterministic. By making some simplifications, it is possible to cast this problem as a directed Generalized Traveling Salesman Problem. Several variants of the problem can also be modeled in a similar way or through the use of classical routing problems like the Traveling Salesman Problem, the Rural Postman Problem or the Chinese Postman Problem.

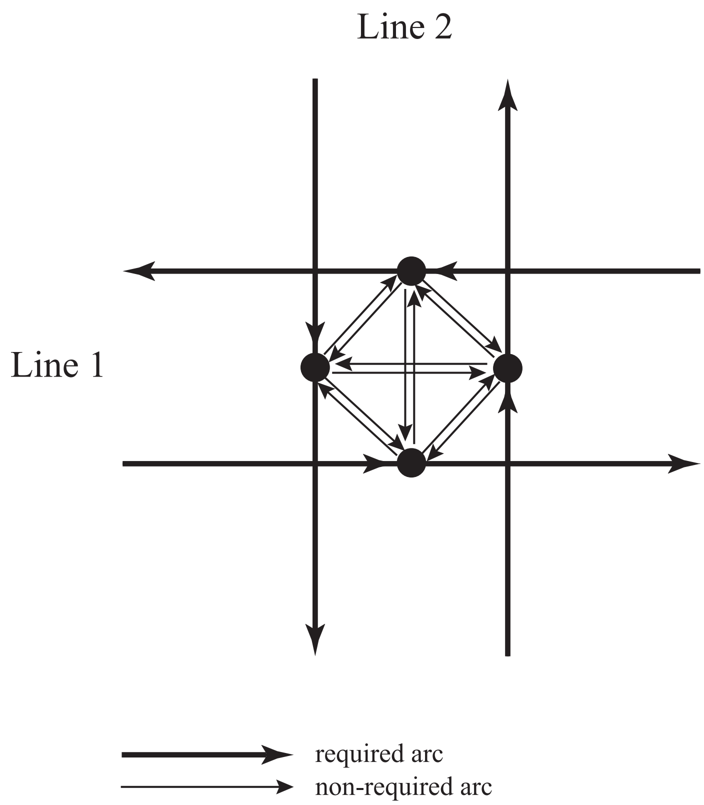

Figure 4: Directed Rural Postman Problem

I believe this problem provides an interesting motivation for the study of such difficult combinatorial optimization problems in a vehicle routing course and confirms the power of the GTSP as a modeling tool.

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