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Designing Logistics Networks in Divergent Process Industries: A Methodology and its Application to the Lumber Industry

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Abstract. This paper presents a generic methodology to design the production-distribution network of divergent process industry companies in a multinational context. The methodology uses a mathematical programming model to map the industry manufacturing process onto potential production-distribution facility locations and capacity options. The industrial process is defined by a directed multigraph of production and storage activities. The divergent nature of the process is modeled by associating one-to-many recipes to each of its production activities. Each facility may use different layouts and the plants capacity is specified by selecting appropriate technological options. Seasonal shutdowns of these capacities are possible and finished product substitutions are taken into account. The objective is to maximize global after tax profit in a predetermined currency. The methodology is illustrated by applying it to the case of the softwood lumber industry. Guidelines for the use of the methodology are provided. The resolution of the mathematical model with commercial optimization software is also discussed.

Key words. Supply Chain Engineering, Mathematical Programming, Production-distribution Network, Divergent Process Modeling, Product Substitution.

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1 Introduction

Supply chains are networks of logistic and manufacturing activities starting with raw material sourcing and ending with the distribution of finished goods to markets. The performance of a supply chain for a given product-market critically depends on the structure of its production-distribution network, i.e. the number, location, mission, technology and capacity of the facilities of the firms involved. The exact nature of the logistics network design problems encountered in practice depends very much on the industrial context in which they occur. The design problem to solve for a high volume make-to-stock manufacturer is very different from the problem found in a highly customized make-to-order products industry or in a slow moving repair parts distribution context. When manufacturing resource acquisition, deployment and/or allocation decisions are considered, the nature of the manufacturing process must also be taken into account. In some industries, manufacturing processes are divergent: several products being made from a common raw material (e.g. lumber industry, meat industry, etc.). In other sectors the manufacturing processes are convergent: several raw-materials and components are assembled into finished products. Networks covering several countries lead to much more complex design problems than single-country networks. Factors such as exchange rates, duties and income taxes must then be taken into account. This paper presents a generic methodology to design international production-distribution networks for make-to-stock products with divergent manufacturing processes.

In industries such as the lumber or the meat industry, the raw material used (stems or carcasses) is obtained from nature and its exact properties are not known before the trees are cut or the animals are slaughtered. These natural raw materials can then be cut or separated in various ways to get several finished products and by-products. The present paper studies the design of the production-distribution network of this type of divergent process industries. This critical strategic planning decision may have a significant impact on company competitiveness. Since, from one industrial context to another, the nature of manufacturing processes can be very different, it was necessary to develop a generic methodology which could be applied in any context. In order to do this, a formalism is proposed and it is illustrated with an example from the lumber industry. This formalism associates production and storage activities to the nodes of a directed multigraph.

Natural resource industries such as those considered here are often affected by economic fluctuations and by international trade disputes, and the supply of the raw material they transform is often heavily regulated. For these reasons, drastic network capacity expansions, other than by the acquisition of a competitor, are rare and companies tend rather to adapt to market fluctuations either by closing facilities temporarily, by reorganizing the layout of their production facilities, by modernizing their production technology or by relocating their distribution centers. Also, due to the nature of the products involved, it is often possible in these industries to upgrade the products demanded by customers. All these aspects of the problem are explicitly taken into account by the proposed mathematical programming model.

An abundant literature exists on location, capacity acquisition and technology selection problems. A review of the early work done in these fields is found in Verter and Dincer (1992). The first location-allocation model proposed (Geoffrion and Graves, 1974) was a single echelon single period model to determine the distribution centers to use, as well as the assignment of products and clients to these centers, in order to minimise the total cost of the system in a domestic context. Several extensions to this model were then made to take into account multiple echelons (Cohen and Lee, 1989; Pirkul and Jayaraman, 1996; Martel and Vankatadri, 1999; Vidal and Goetschalckx, 2001; Martel, 2005), multiple production seasons (Cohen et al., 1989; Arntzen et al., 1995; Dogan and Goetschalckx, 1999; Martel, 2005), capacity acquisition and technology selection (Eppen et al., 1989; Verter and Dincer, 1995; Mazzola and Neebe, 1999; Paquet et al., 2004; Martel, 2005), economies of scale (Cohen and Moon, 1990, 1991; Mazzola and Schantz, 1997; Martel and Vankatadri, 1999; Martel, 2005), after tax net revenue maximization in an international context (Cohen et al., 1989; Arntzen et al., 1995; Vidal and Goetschalckx, 2001; Martel, 2005) and product development and recycling (Fandel and Stammen, 2004). Geoffrion and Powers (1995) and Shapiro et al. (1993) discuss the evolution of strategic supply chain design models and Vidal and Goetschalckx (1997) present many of these models. Shapiro (2001) provides an excellent coverage of several supply chain modeling issues. The models proposed by Arntzen et al. (1995), Fandel and Stammen, (2004) and Martel (2005) are among the most complete presented to date. Commercial software products based on some of these models are also available on the market.

Some authors proposed models for specific assembly process industries (Brown et al., 1987; Dogan and Goetschalckx, 1999; Philpott and Everett, 2001) and others used activity graphs to represent supply chains (Lakhal et al., 1999, 2001) but, to our knowledge, the approach presented here is the first generic methodology proposed to design production-distribution networks for divergent process industries. The proposed modeling approach is an adaptation and an extension of the production-distribution network design optimization framework proposed by Martel (2005), for international make-to-stock assembly industries, to the case of international make-to-stock divergent manufacturing process industries. The paper also presents a realistic lumber industry case (Virtu@l-Lumber), conceived in partnership with three large lumber companies of Canada (Domtar, Kruger and Tembec), two Canadian forest industry research centers (FOR@C and Forintek) and Quebec Ministry of Natural Resources, to demonstrate the feasibility and the usefulness of the approach.

The paper is organised as follows. Section 2 presents the proposed production-distribution network design approach. Section 3 develops the mathematical programming model which is the corner stone of the approach. Section 4 discusses the solution of the model and section 5 provides guidelines for the use of the methodology in various process industry contexts.

2 Production-distribution Network Design Approach

In order to address the type of production-distribution design problem considered in this paper, it is necessary to obtain detailed information on the products, markets, manufacturing processes and logistic resources of the company or companies involved and to use powerful decision support tools. The proposed approach involves five steps:

1. The definition of the product-markets, sourcing context and planning horizon;
2. The definition of product families and the elaboration of the manufacturing-storage activities process graph;
3. The definition of potential network resources (facilities location, layouts, technologies and capacity options) and of technology dependent recipes for production activities;
4. The definition of the revenues and costs associated to the network design and activity decisions;
5. The optimal mapping of the process graph onto the potential network resources.

In the following sections, the facets of the supply chain design problem associated to each of these steps are discussed and illustrated with the case of the lumber industry in the province of Quebec in Canada.

2.1 Products-markets, sourcing and planning horizon

The appropriate characterization of the product-markets of the company considered is an important design task. This characterization depends on the type of products sold to different market segments and on the geographical dispersion of customer ship-to-locations. It is assumed that the company operates national divisions in several countries $o \in O$, and that each of these divisions is constituted of several demand zones $d \in D_o$. A given demand zone is characterized by a geographical region and a market segment, the latter being defined by a product category, and particular price and service policies. Each product category includes several finished products which can be classified into a set FP of product families to keep the size of the problem manageable. It is assumed that the largest demand the company can expect for product family $p \in FP$ in demand zone $d \in D_o$ can be forecasted, and that the company has minimum market penetration objectives for each of its product-markets.

In the lumber industry, three main market segments are usually distinguished: the spot market, large retailers and industrial customers. The products sold to the industrial customers (Machine Stressed

Rated - MSR lumber) are of higher quality and value than those sold to retailers (Premium lumber) and these are also of higher value than those sold to the spot market (Dimension Lumber). For this reason, the manufacturer can use higher quality products to satisfy the demand for lower quality products when a sale is made. For example, a manufacturer could sell Premium lumber on the spot market simply by declaring it as Dimension Lumber. The substitution possibilities for the Quebec producer's case are illustrated in Table 1. As can be seen in this table, each segment includes several finished product families based on the lumber dimensions: sections of 2x6, 2x4 or 2x3 inches and 8 foot length or random length (RL), which means longer than 8 foot and up to 16 feet. Note also that there are by-product markets for chips, short lumber and planks (one inch thick lumber).

Markets / Products	Markets									
	Spot markets			Contracts						Pulp & Paper mills
				Retailers			Industrial customers			
Dimension	*2x6*	*2x4*	*2x3*	*2x6*	*2x4*	*2x3*	*2x6*	*2x4*	*2x3*	
Dimension Lumber & Stud	8 RL	8 RL	8 RL							
Premium	8,RL	8,RL	8,RL	8,RL	8,RL	8,RL				
MSR	8,RL	8,RL	8,RL				8,RL	8,RL	8,RL	
Plank							Planks			
Short							Shorts			
Chip										Chips

8: eight feet long lumber; RL: Random length lumber.

Table 1: Product-Markets with Possible Product Substitutions

As indicated in the introduction, dramatic network capacity expansions are rare in natural resource based industries because the availability of the natural resource is usually heavily regulated. In the province of Quebec, for example, the government manages 90 % of the forest area and allocates it to lumber companies every 5 years. Sawmills are tied by Forest Management and Supply Contracts defining annual allowable cut. In fact, these contracts do not only specify upper bounds on the supply of raw material from a given source, but they also force companies to use a large proportion of the trees available. The problem is further complicated by the fact that the properties of the trees available are not known exactly before they are cut so that sawmills, at best, know only the proportions of stems or logs of various types they can expect to get from a given forest area. Producers therefore have little control over their supply of raw material.

For the Quebec lumber industry, since most of the available forest area is already allocated, major expansion plans can be considered only if a competitor abandons its CAAF, which is uncommon. As indicated, our approach is not intended for such decisions but rather to permit companies to adapt to market fluctuations either by closing facilities temporarily, by reorganizing the layout of their production facilities, by modernizing their production technology or by modifying the location of their distribution centers. In such a context, using a planning horizon of a year or two is appropriate. To take seasonal demand into account properly, however, the planning horizon is divided into seasons and decisions on how much of a product needs to be made and stocked at the different sites must be seasonal. Seasonal inventories can also be kept to smooth production.

The following notation is used to define the business environment of the company:

FP	=	Set of product families sold on the market ($p \in FP$).
SP^p	=	Set of substitutes for product family $p \in FP$ ($SP^p \subset FP$).
SP_p	=	Set of product which can be substituted by product family $p \in FP$ ($SP_p \subset FP$).
D	=	Set of demand zones serviced by the company ($d \in D$).
D_p	=	Set of demand zones requiring product family $p \in FP$ ($D_p \subset D$).
$x_{pdt}^{\max}$	=	Largest expected demand for product $p \in FP$ in zone $d \in D_p$ during season $t \in T$.
$x_{pdt}^{\min}$	=	Minimum market penetration objective for product $p \in FP$ in zone $d \in D_p$ for season $t \in T$.
V	=	Set of raw material supply sources ($v \in V$).
O	=	Set of countries covered by the logistic network ($o \in O$).
$o(n)$	=	Country of geographical location n.
T	=	Set of seasons in the planning horizon ($t \in T$).

2.2 Product families and manufacturing process multigraph

Divergent manufacturing processes can be represented by an acyclic directed multigraph Γ defined by a set of nodes $A = \{a\}$ corresponding to activities, and a set of directed arcs $\Psi = \{(p, a, a')\}$ where $a, a' \in A$ is a pair of adjacent activities and $p \in P$ is the product family associated to the arc. The set of nodes A can be partitioned into four mutually exclusive subsets:

- The root node $a = 1$ corresponding to the raw materials supply market;
- The set of production activities A^p ;
- The set of storage activities A^s ;
- The sink node $a = \bar{a} = |A|$ corresponding to the products sale market.

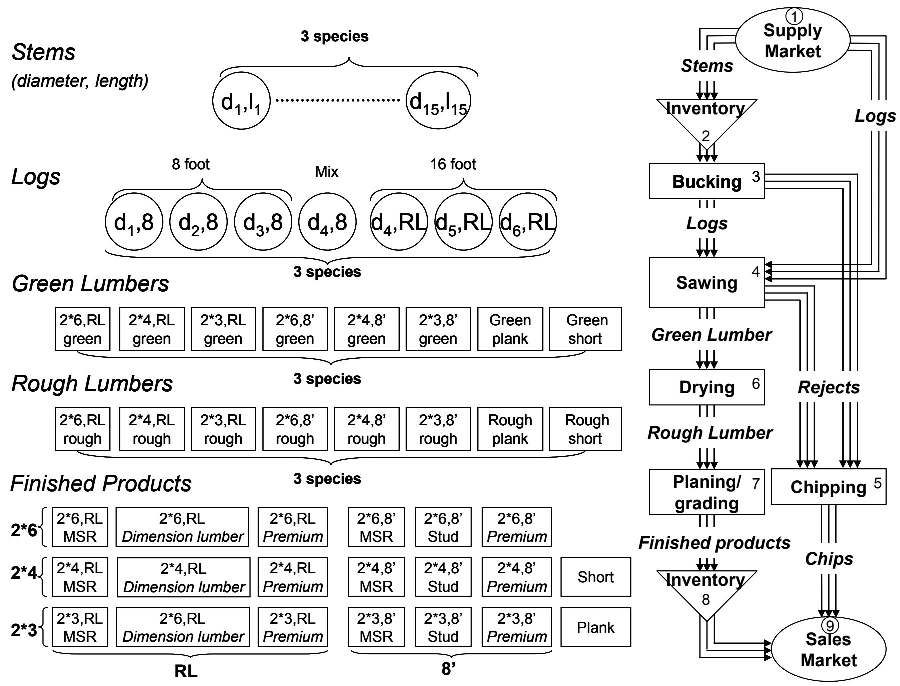


Figure 1: Quebec Lumber Industry Manufacturing Process Multigraph.

Figure 1 shows the manufacturing process multigraph of the Quebec lumber industry. In the graphical formalism used, rectangles represent production activities, triangles storage activities and ellipses the source and sink activities. This graph is a conceptual representation of the manufacturing process and it is independent of the current physical implementation of the company. The product families associated to the arcs are defined on the left-hand side. The finished product families ($FP \subset P$) correspond to those defined in Table 1. Semi-finished products and raw material families are defined to capture the essence of the manufacturing process while respecting market segment characteristics. In our case, wood species are distinguished and families are defined based on the physical characteristics of the products (diameter, length).

Note that a process multigraph including the supply market, a series of storage activities and the sales market describes a multi-echelon distribution network. Hence, our approach could also be used to design pure distribution networks. The following notation is required to model the manufacturing process multigraph $\Gamma = (A, \Psi)$:

P = Set of product families ($p \in P$).
A = Set of activities ($a \in A$).
Ψ = Set of directed arcs $\{(p, a, a')\}$ in the multigraph.
A^p = Set of production activities ($A^p \subset A$).
A^s = Set of storage activities ($A^s \subset A$).
A_a^{in} = Set of immediate predecessors of activity a ($A_a^{in} \subset A$).
A_a^{out} = Set of immediate successors of activity a ($A_a^{out} \subset A$).
P_a^{in} = Input product families of activity a ($P_a^{in} \subset P$).
P_a^{out} = Output product families of activity a ($P_a^{out} \subset P$).

2.3 Potential network resources and production recipes

The production and storage activities defined in the process multigraph must be performed in manufacturing and/or distribution facilities. Some facilities may already be in use by the company, but potential sites may also be considered for the construction, purchase or rent of other facilities. It may also be possible to transform existing facilities. As illustrated in Figure 2, it is the assignment of the activities of the process multigraph to the potential facility sites that defines the company logistics network. In the resulting directed network, the nodes correspond to supply sources (*V*), potential production-distribution centers (S^{pd}), potential distribution centers (S^d) or demand zones (*D*). The arcs represent the flow of products between nodes. In practice, the inbound flow arcs in ($V \times S$), the internal flow arcs in ($S \times S$) and the outbound flow arcs in ($S \times D$) are generally not all feasible. In particular, the size of the outbound arc set ($S \times D$) depends very much on the delivery policy of the company, since this set contains only the arcs which are short enough to comply with a given delivery time. For this reason, sets of potential node predecessors and successors must also be defined. The following notation is required to define potential facilities and potential moves in the logistic network:

S = Set of potential network sites ($s \in S$).
S_o = Set of sites located in country $o \in O$ ($S_o \subset S$).

S^{pd} = Set of potential production-distribution center sites ($s \in S^{pd} \subset S$).
S^{d} = Set of potential distribution center sites ($s \in S^{d} \subset S$).
S^{o}_{ps} = Set of potential sites (output destinations) which can receive product p from site s.
S^{i}_{pn} = Set of potential sites (input sources) which can ship product p to location $n \in S \cup D_p$
V_s = Set of vendors which can supply site $s \in S$ ($V_s \subset V$).
V_{ps} = Set of vendors which can supply product p to site $s \in S$ ($V_{ps} \subset V_s$).
D_{ps} = Set of demand zones which can receive product p from site $s \in S$ ($D_{ps} \subset D_p$).

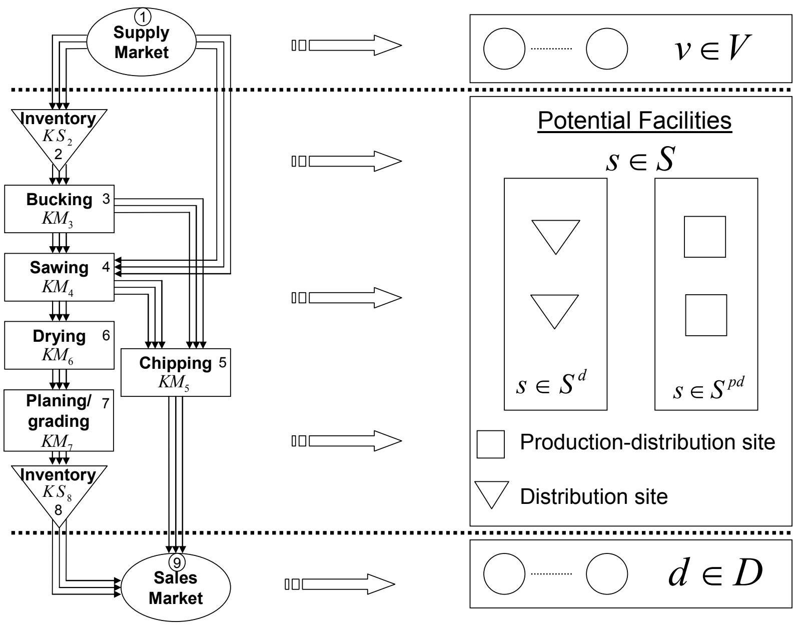


Figure 2: Mapping the Manufacturing Process onto the Potential Network Nodes.

The production and storage activities defined in the process multigraph can be performed with different *technologies*. A *technology* is considered as a class of equipment which can be used to produce/store a given set of products. It is assumed that the amount of resources consumed when a production activity is performed depends on the technology used. It is also assumed that the output quantities obtained with a given input product when a production activity is performed is technology dependent. The input-output quantities associated to the use of a given technology to perform an activity are defined by *recipes*. The recipe i used when activity a is performed with technology k can be selected from a set of potential recipes R_{ak}. It is in fact through the choice of appropriate recipes, that management is able to match supply and demand in the type of industries considered. As illustrated in Figure 3, each recipe $i \in R_{ak}$ is characterized by one input product p_i, a set of output products P_i^{out}, yield factors $g^{i}_{p_i p}, p \in P^{out}_{i}$, and a resource consumption factor q^i. In the lumber industry, recipes take different forms for different activities. For bucking ($a=3$) and sawing ($a=4$) activities, recipes correspond to the different cutting patterns which can be selected. Typical stem and log cutting patterns are illustrated in Figure 4. For planing/grading ($a=7$), recipes are associated to lumber sorting options, and for chipping ($a=5$) and drying ($a=6$), one-to-one recipes define process yield.

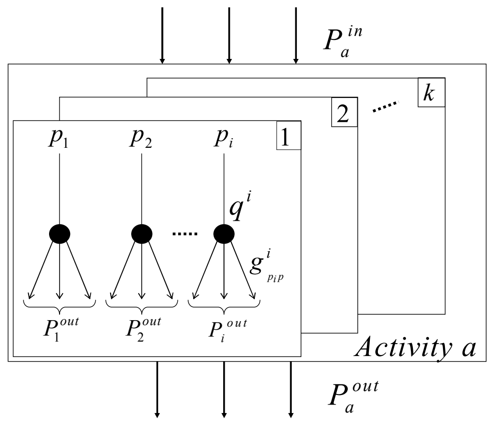


Figure 3: Technology Dependent One-to-Many Recipes for a Production Activity

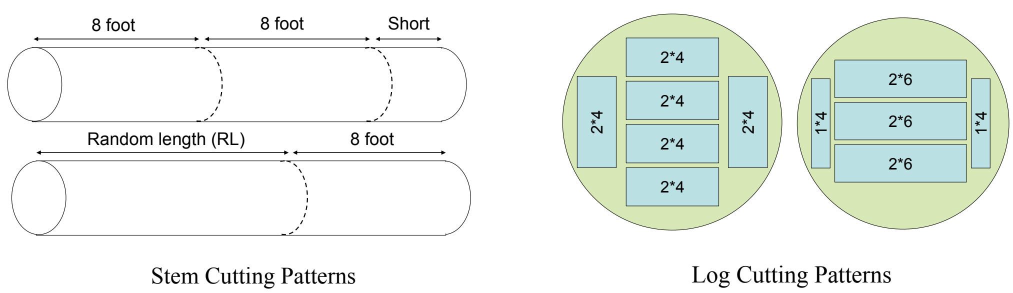


Figure 4: Cutting Patterns Corresponding to Bucking and Sawing Recipes

No one-to-many recipe needs to be defined for storage activities since input and output products are identical. Also, the storage technologies used for a given activity $a \in A^s$ are assumed to be flexible: they can be used to store any of its input products $p \in P^{in}_{a}$, and their resource consumption rates are measured in the same units. For a product p associated to a storage activity a, it is therefore sufficient to specify a single resource consumption rate q_{pa}. The following notation is required to define technologies and recipes:

KM_{sa} = Production technologies which can be used to perform activity $a \in A^p$ on site s ($k \in KM_{sa}$).

KS_{sa} = Storage technologies which can be used on site s to perform activity $a \in A^s$ ($k \in KS_{sa}$).

R_{ak} = Set of recipes available to perform production activity $a \in A^p$ with technology k. These sets uniquely define the activity *a* and technology *k* of recipes $i \in R_{ak}$.

p_i = Input product for recipe $i \in R_{ak}$.

P_i^{out} = Set of output products obtained with recipe $i \in R_{ak}$.

$g^i_{p_i p}$ = Quantity of product p obtained from one unity of product p_i with recipe $i \in R_{ak}$.

q^i = Production capacity required to process one unit of product p_i with recipe $i \in R_{ak}$.

q_{pa} = Capacity consumption rate per unit of product $p \in P_a^{in}$ for storage activity $a \in A^s$.

Note that the sets KM_{sa} and KS_{sa} can be used to restrict the mission of a given site. If the set KM_{sa} is empty, for example, it implies that activity $a \in A^p$ cannot be performed on site $s \in S^{pd}$. Note also that, by definition, $KM_{sa} = \varnothing, \forall s \in S^d, a \in A^p$. In order to ensure that the specification of the previously defined sets is coherent, for each activity $a \in A^p$, the following must hold true:

$$\bigcup_{s \in S^{pd}} \bigcup_{k \in KM_{sa}} \bigcup_{i \in R_{ak}} \{p_i\} = P_a^{in} \text{ and } \bigcup_{s \in S^{pd}} \bigcup_{k \in KM_{sa}} \bigcup_{i \in R_{ak}} P_i^{out} = P_a^{out} .$$

The capacity of the potential network facilities depends on the technologies implemented in the space available on their site. For the production-distribution sites ($s \in S^{pd}$), various *facility layouts* can be considered and various *capacity options* can be selected. A layout $l \in L_s$ is characterized by an area available E_{ls} for the installation a set J_{ls} of predetermined potential *capacity options*. The layouts considered for a given production-distribution site can correspond to the status-quo layout, if there is already a facility on the site, or to alternative layouts for new construction or reconfiguration opportunities. By convention, index $l = 1$ is used for the status-quo layout. A set of alternative capacity options can be considered to implement a given technology. An option $j \in J_s$ can correspond to capacity already in place, to a reconfiguration of an installed equipment to increase its capacity or to the addition of new resources. In this last case, different options can be associated to equipment of different size to reflect economies of scale. Moreover, the simultaneous inclusion of dedicated capacity options and flexible capacity options allow for the modeling of economies of scope. When dealing with a potential equipment replacement/reconfiguration, the options associated to the new potential equipment cannot be selected at the same time as the status-quo option, which leads to the definition of mutually exclusive sub-sets of options $JR_{ls}^{n}, n = 1, \ldots, N_{ls}$, for some facility layouts. Each option $j \in J$ is characterized by a seasonal capacity, b_{jt}, stated in the units of its technology, by the floor space e_j required to install it and by a fixed cost and a variable cost per product. In order to be able to adapt production capacity to demand fluctuations, an important aspect of the problem in our context is that the capacity options selected do not have to be used in every season: seasonal shutdowns are possible.

Distribution sites ($s \in S^d$) are assumed to be pre-configured, which means that the technology $k \in KS_{sa}$ they use and the capacity available for these technologies in a given season b_{skt}, are known *a priori*. This simplifying assumption is made because it often applies in practice, mainly when public warehouses are used. However, the generalisation to the case of alternative layouts and capacity options presents no difficulty. The notation required to define facility layouts and capacity options is the following:

L_s = Potential facility layouts for site $s \in S^{pd}$ ($l \in L_s$).

J_s = Potential capacity options which can be installed on site $s \in S^{pd}$ ($j \in J = \bigcup_{s \in S^{pd}} J_s$).

J_{ks} = Potential technology k capacity options which can be installed on site $s \in S^{pd}$ ($J_{ks} \subseteq J_s$).

J_{ls} = Potential capacity options which can be installed on site $s \in S^{pd}$ when layout $l \in L_s$ is used ($J_{ls} \subseteq J_s$).

JR_{ls}^{n} = Mutually exclusive options sub-set in J_{ls} ($n = 1, \ldots, N_{ls}$).

N_{ls} = Number of mutually exclusive option subsets (equipment replacement/reconfiguration) in J_{ls}.

E_{ls} = Total area of the layout l for site s.

e_j = Area required to install capacity option j.

b_j^t = Capacity of the technology associated to option j available for season t.

b_{skt} = Technology k capacity available for season t for distribution site $s \in S^d$.

2.4 Relevant revenues and expenses

A large volume of cost and price information is required to calculate the total revenues and expenses associated with logistic network design. This is particularly true in the international business context. In order to properly evaluate potential solutions, the following assumptions are made:

- The prices and cost associated to the nodes of the network are given in local currency. The costs associated to the arcs of the network are given in source currency. Exchange rates are known and constant during the planning horizon considered.

<table>
<tr><td colspan="2"></td><td colspan="2">Do not use the site</td><td colspan="2">Use the current layout ($l = 1$)</td><td colspan="2">Use a new layout ($l > 1$)</td></tr>
<tr><td colspan="2">Initial state</td><td>Decision</td><td>Fixed cost (A_{0s})</td><td>Decision</td><td>Fixed cost (A_{1s})</td><td>Decision</td><td>Fixed cost (A_{ls})</td></tr>
<tr><td rowspan="3">Current facility</td><td>Owned</td><td>Close</td><td>• Closing cost</td><td>Status-quo</td><td>• Capital recovery
• Opportunity cost
• Operating cost</td><td>Change layout</td><td>• Set-up cost
• Capital recovery
• Opportunity cost
• Operating cost</td></tr>
<tr><td>Rented</td><td>Close</td><td>• Closing cost
• Lease penalty</td><td>Status-quo</td><td>• Rent
• Operating cost</td><td>Change layout</td><td>• Set-up cost
• Rent
• Operating cost</td></tr>
<tr><td>Public</td><td>Stop</td><td>• Stopping cost</td><td>Status-quo</td><td>• Operating cost</td><td>Change layout</td><td>• Operating cost</td></tr>
<tr><td rowspan="3">Potential site</td><td>New facility or purchase & renovated</td><td>Do not use</td><td>• Zero</td><td rowspan="3" colspan="2"></td><td>Build/ Buy</td><td>• Set-up cost
• Capital recovery
• Opportunity cost
• Operating cost</td></tr>
<tr><td>Rented facility</td><td>Do not use</td><td>• Zero</td><td>Rent</td><td>• Set-up cost
• Rent
• Operating cost</td></tr>
<tr><td>Public</td><td>Do not use</td><td>• Zero</td><td>Use</td><td>• Starting cost
• Operating cost</td></tr>
</table>

Table 2: Facility Layout Fixed Costs in Different Contexts

- The fixed costs A_{ls} associated to facility layouts reflect potential changes of state (closing an existing facility, building or buying a new facility, changing the layout of a facility…) and fixed operating expenditures, and they depend on the practical context of each potential node. Relevant fixed costs for different contexts are listed in Table 2. These costs are based on the engineering economy principles of capital recovery plus return over the planning horizon (Frabrychy and Torgersen, 1966). The fixed costs a_j^1 associated to the installation of potential capacity options also cover capital recovery and opportunity costs expenditures, but they do not include fixed operating costs. Fixed capacity option operating costs $\hat{a}_{jt}$ are charged on a seasonal basis when the option is in use. When existing equipment is disposed off, a fixed removal cost a_j^0 may also be charged. The approach proposed in Table 2 to compute layout fixed costs can also be used, with minor modifications, to obtain capacity options fixed costs.

- Each time products cross a border, tariffs and duties are charged on the flow of merchandise and these are paid by the importer. In other words, tariffs are calculated on the inflow to a given site from a foreign country of origin.
- The transportation costs on the network arcs are paid by the origin. It is assumed that they are linear with respect to seasonal product flows.
- Transfer prices for products sent in the internal network are fixed by the accounting department of the company.
- The income taxes paid in a country are calculated on the sum of the *net revenues* (Total revenue - Total logistic network costs) made by all facilities in this country. If a facility reports a loss, this loss is deducted from the total profit of the subsidiary before taxes. It is also assumed that the corporate taxes paid by the parent company are deferred until it pays dividends and that the decision to pay out dividends is independent of the design of the network.
- The company wishes to maximize its global after tax net revenues in a predetermined currency.

The notation for the costs and revenues is as follows:

A_{ls} = Fixed cost of using layout l on site $s \in S^{pd}$ for the planning horizon.

A_{0s} = Fixed cost of disposing of production-distribution site $s \in S^{pd}$ at the beginning of the planning horizon.

A_s = Fixed cost of using distribution site $s \in S^{d}$ for the planning horizon.

a_j^0 = Fixed cost of disposing of capacity option j at the beginning of the planning horizon.

a_j^1 = Fixed cost of installing of keeping capacity option j for the planning horizon.

$\hat{a}_{jt}$ = Fixed cost of using capacity option j during season t.

$c^i_{p_i st}$ = Cost of producing one unit of product p_i with recipe i on site s during season t.

m_{pst} = Unit handling cost for the transfer of product p to or from its stock in production-distribution site s during season t.

f^{o}_{psnt} = Unit cost of the flow of product p between site s and node n paid by origin s during season t (this cost includes the customer-order processing cost, the shipping cost, the variable transportation cost and the inventory-in-transit holding cost).

f^{t}_{psnt} = Unit transportation cost of product p from site s to node n during season t (this cost is included in f^{o}_{psnt}).

f^{d}_{pnst} = Unit cost of the flow of product p between node n and site s paid by destination s during season t (this cost includes the supply-order processing costs and the receiving cost).

$f^{v}_{pv(s,a)t}$ = Unit cost of the flow of product p between vendor v and activity a on site s paid by destination s during season t (this cost includes the product's price and the variable transportation cost).

h_{pst} = Unit inventory holding cost of product p in facility s during season t.

π_{pst} = Transfer price of product p shipped from site s during season t.

$e_{oo'}$ = Exchange rate, i.e. number of units of country o currency by units of country o' currency (the index $o = 0$ is given to the base currency, whether it is part of O or not).

δ_{pns} = Import duty rate applied to the CIF price of product p when transferred from the country of node n to the country of site s.

τ_{o} = Income tax rate of country o.

P_{pdt} = Amount received for the sales of product p to demand zone d in season t.

In order to compute inventory holding costs, the following parameter, which is the inverse of the familiar inventory turnover ratio, is also required:

ρ_{pst} = Number of seasons of inventory (order cycle and safety stocks) of product p kept at site s for season t.

2.5 *Mapping of the process graph onto the potential network resources*

In the previous sections, graph and set based constructs, as well as material and financial resource consumption parameters were defined to represent divergent process industry companies internal and external business environment, the technological opportunities they have at their disposal to improve competitiveness, as well as the financial information required to evaluate these opportunities in an international context. The last step of the proposed approach is to use a mathematical programming model to select the opportunities maximizing the overall after tax net revenues of the company considered. As illustrated in Figure 2, this involves a series of network design decisions to map the company manufacturing process multigraph onto its potential logistic network resources. Specifically, some of the questions to be answered are:

- Which potential production and distribution sites should the company use?
- Which production-storage activities should be assigned to each of the selected sites?
- Which layout and capacity options should be implemented on the production-distribution sites?
- Should some of the installed capacity options be shutdown during certain seasons to adapt to market demand and price fluctuations?
- Which product should be manufactured and stored on each site, taking potential product substitutions into account?
- How much seasonal raw material and finished product inventories should be kept to help absorb supply and demand fluctuations, taking recipe selection possibilities into account?
- Which demand zones should be supplied from the various sites?
- Which raw material sources should supply each production site?

To answer such questions, the following decision variables must be used:

Y_{ls} = Binary variable equal to 1 if layout $l \in L_s$ is used for site $s \in S^{pd}$ and to 0 otherwise.

Y_{0s} = Binary variable equal to 1 if production-distribution site $s \in S^{pd}$ is not used and to 0 otherwise (i.e. layout $l = 0$ implicitly corresponds to a closed facility).

Y_s = Binary variable equal to 1 if potential distribution center $s \in S^d$ is used and to 0 otherwise.

Z_j = Binary variable equal to 1 if capacity option j is installed and to 0 otherwise.

$\hat{Z}_{jt}$ = Binary variable equal to 1 if capacity option j is used during season t and to 0 otherwise.

$F_{p(n,a)(n',a')t}$ = Flow of product $p \in P$ between activity a at location $n \in V \cup S$ and activity a' at location $n' \in S \cup D_p$ during season $t \in T$.

$F_{pp'(s,a)dt}$ = Outbound flow of finished product $p' \in FP$, used to satisfy the demand for product $p \in FP$, between activity a in site s and demand zone $d \in D_p$ during season $t \in T$.

$X^i_{p_i st}$ = Quantity of product p_i processed with recipe $i \in R_{ak}$ in production-distribution site s during season $t \in T$.

I_{pkst} = Seasonal inventory of product $p \in P$ stored on site s with technology $k \in KS_{sa}$ at the end of season $t \in T$.

Although the binary variable Y_{0s} implies that one could decide to discard an existing production-distribution facility or consider the addition of new facilities, as indicated earlier, our approach is not intended to make such decisions. In fact, in most cases, this 0-1 variable would be fixed to 0 *a priori*, and the analysis would concentrate on the choice of appropriate layouts and capacity options. Also, although the production and the flow variables defined above lead to the specification of optimal seasonal production and transportation quantities, as well as to the definition of optimal recipe selection profiles, these would not be implemented *per se* in practice. These decisions would be finalized in the shorter term, taking specific supplier and customer orders into account. They are important however because they indicate the products which should be manufactured on each site, the substitution which should be considered and the customers to serve from each sites. Their optimal value also permits the anticipation of the economic impact of the design decisions made. The next section presents the optimization model conceived to answer the design questions raised previously.

3 Mathematical Programming Model

This section presents the various elements of the generic mathematical programming model proposed to optimize logistics networks in divergent process industries. It covers the modeling of the supply market, of production and storage activities and of the demand market. The section ends with the formulation of the model objective function. The application of this generic model to the Quebec lumber industry case is also discussed.

3.1 Modeling the supply market

The raw material supply market corresponds to the root node ($a = 1$) of the manufacturing process multigraph Γ. Raw materials flow from the vendors in this supply market to the sites performing production-storage activities $a \in A_1^{out}$. Let F^1 be the vector of these inbound raw material flows, i.e.

$$F^1 = \left[F_{p(v,1)(s,a)t} \quad \forall p \in P_1^{out}, \forall a \in A_1^{out}, \forall s \in S, \forall v \in V_{ps}, \forall t \in T \right]$$

and let Ω^1 be the set of all the feasible inbound raw material flows in the context considered. Then, to remain generic, the supply market conditions can be stated simply as:

$$F^1 \in \Omega^1 \tag{1}$$

Since supply conditions tend to be context dependent, the set Ω^1 must be defined specifically for each application. In the simplest cases, Ω^1 can be defined by bounds on seasonal or annual inflows but in some instances it is much more complex. To illustrate, let us consider the case of the Quebec lumber industry described in Figure 1. For this case, $A_1^{out} = \{2,4\}$. Quebec sawmills are tied by Forest Management and Supply Contracts defining annual upper bounds on the supply of raw materials for a given forest, and minimum procurement quantities. Also, sawmills know only the proportions of stems or logs of different type they can expect to get from a given source. To define the set Ω^1 of inbound flows satisfying these constraints, the following specific notation is required:

$\mathrm{Pr}_{pv(s,2)}$ = Proportion of products of family $p \in P_2^{in}$ in the stems supplied by source $v \in V$ to site $s \in S^{pd}$, when bucking in done in the sawmill.

$\mathrm{Pr}_{pv(s,4)}$ = Proportion of products of family $p \in P_4^{in}$ in the logs supplied by source $v \in V$ to site $s \in S^{pd}$, when bucking in done in the forest.

$b_{v(s,a)t}^{\max}$ = Upper bound on the seasonal shipments of raw material between source $v \in V$ and activity $a \in A_1^{out}$ on site $s \in S^{pd}$ for season t.

$b_{vs}^{\min}$ = Annual minimum level of raw material to be shipped between source $v \in V$ and site $s \in S^{pd}$ in order to comply with supply contracts with government.

Using this notation, the set of feasible inbound flows Ω^1 can be defined as follows:

$$\sum_{p \in P_1^{out} \cap P_a^{in}} F_{p(v,1)(s,a)t} \le b_{v(s,a)t}^{\max} \quad a \in A_1^{out}, s \in S^{pd}, v \in V_s, t \in T \tag{2}$$

$$\sum_{t \in T} \sum_{a \in A_1^{out}} \sum_{p \in P_1^{out} \cap P_a^{in}} F_{p(v,1)(s,a)t} \ge b_{vs}^{\min} \quad s \in S^{pd}, v \in V_s \tag{3}$$

$$F_{p(v,1)(s,a)t} = \mathrm{Pr}_{pv(s,a)} \sum_{p' \in P_1^{out} \cap P_a^{in}} F_{p'(v,1)(s,a)t} \quad a \in A_1^{out}, p \in P_1^{out} \cap P_a^{in}, s \in S^{pd}, v \in V_s, t \in T \tag{4}$$

3.2 Modeling production-distribution facility layouts and capacity options

Using the plant layout selection variables Y_{ls}, the following constraints must be included in the model to ensure that at most one layout is selected for each production-distribution site:

$$\sum_{l \in L_s} Y_{ls} + Y_{0s} = 1 \quad s \in S^{pd} \tag{5}$$

Using the capacity option selection variables Z_j, the following constraints must also be included to ensure that, for a given site, the area required by the selected options does not exceed the area available in the selected layout, and that mutually exclusive options are not selected:

$$\sum_{j \in J_{ls}} e_j Z_j \le E_{ls} Y_{ls} \quad s \in S^{pd}, l \in L_s \tag{6}$$

$$\sum_{j \in RL_{ls}^n} Z_j \le 1 \quad s \in S^{pd}, l \in L_s, n = 1, \ldots, N_{ls} \tag{7}$$

Since the capacity options selected can be shutdown during some seasons, constraints are also required to ensure that a capacity option can be used in a season only if it was in use:

$$\hat{Z}_{jt} \le Z_j \quad s \in S^d, j \in J_s, t \in T \tag{8}$$

Note finally that, since distribution centers are assumed to be pre-configured, there is no layout and capacity options decision to make for sites $s \in S^d$.

3.3 Modeling flows and inventories

In addition to deciding the sites, layouts and capacity options to use during the planning horizon, tactical decisions must be made on the quantity of products to manufacture, the seasonal stocks to accumulate and the internal flow of products in the network. This requires the modeling of flows and inventories in the network facilities and the consideration of capacity constraints.

Any valid network optimization model must ensure the equilibrium between the flows of material entering an activity, its transformation or stocking in the activity and the flow of products exiting the activity. For production activities, one must ensure that the material processed does not exceed the material received from preceding activities in the same site or in other sites, i.e. that the following relations are satisfied:

$$\sum_{k\in KM_{sa}} \sum_{i\in R_{ak}|p_i=p} X^{i}_{p_i st} \le \sum_{a'\in A^{in}_a} \sum_{s'\in S^{i}_{ps}\cup V_{ps}} F_{p(s',a')(s,a)t} + \sum_{a'\in A^{in}_a} F_{p(s,a')(s,a)t} \qquad a\in A^p, p\in P^{in}_a, s\in S^{pd}, t\in T \tag{9}$$

One must also ensure that the material flowing out of the production activity does not exceed the amounts produced, i.e. that the following constraints are respected:

$$\sum_{a'\in A^{out}_a} \sum_{s'\in S^{o}_{ps}\cup D_{ps}} F_{p(s,a)(s',a')t} + \sum_{a'\in A^{out}_a} F_{p(s,a)(s,a')t} \le \sum_{k\in KM_{sa}} \sum_{i\in R_{ak}|p\in P^{out}_i} g^{i}_{p_i p} X^{i}_{p_i st} \qquad a\in A^p, p\in P^{out}_a, s\in S^{pd}, t\in T \tag{10}$$

Similarly, for the storage activities, additions and withdrawals from the seasonal inventory must be accounted for. This yields the following inventory accounting equations.

$$\sum_{k\in KS_{sa}} I_{pkst} = \sum_{k\in KS_{sa}} I_{pkst-1} + \sum_{a'\in A^{in}_a} F_{p(s,a')(s,a)t} + \sum_{a'\in A^{in}_a} \sum_{s'\in S^{i}_{ps}\cup V_{ps}} F_{p(s',a')(s,a)t} - \sum_{a'\in A^{out}_a} \sum_{s'\in S^{o}_{ps}\cup D_{ps}} F_{p(s,a)(s',a')t} - \sum_{a'\in A^{out}_a} F_{p(s,a)(s,a')t} \qquad a\in A^s, p\in P^{in}_a, s\in S^{pd}\cup S^d, t\in T \tag{11}$$

Seasonal stocks are used to allow the smoothing of production over the planning horizon. As illustrated in Figure 5, the seasonal stocks at the beginning and at the end of the horizon must therefore be the same, i.e. we must have:

$$I_{ps0} = I_{ps|T|} \quad a\in A^s, p\in P^{in}_a, s\in S^{pd}\cup S^d \text{ where } I_{pst} = \sum_{k\in KS_{sa}} I_{pkst} \tag{12}$$

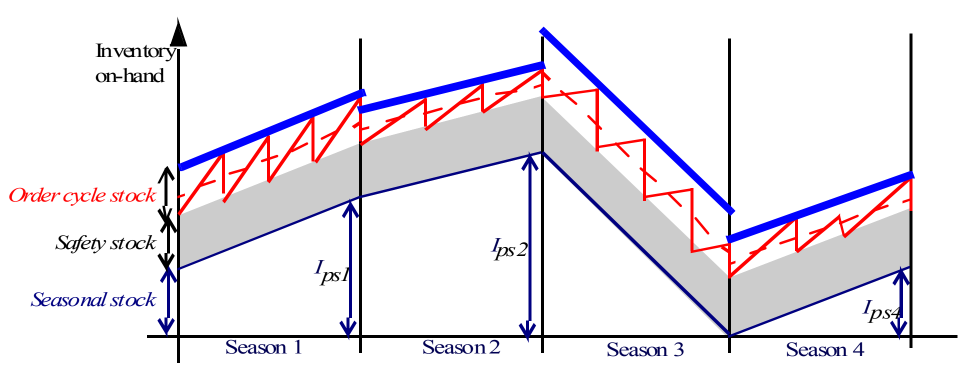


Figure 5: Behaviour of Product *p* Inventory in a Storage Activity on Site *s*

In addition to seasonal inventory, the level of safety stocks and order cycle stocks generated by the network design must be taken into account. These stock levels depend on the inventory management policies and rules used by the company and on the ordering behaviour of customers. It is assumed here that the impact of these policies is reflected by the inventory turnover ratio of the product on a given site. This implies that the average level $\overline{I}_{pst}$ of the order cycle and safety stock of product p during season t at site s can be calculated with the following expression:

$$\overline{I}_{pst} = \rho_{pst}[\sum_{a' \in A_a^{out}} (F_{p(s,a)(s,a')t} + \sum_{s' \in S_{ps}^i \cup D_{ps}} F_{p(s,a)(s',a')t})] \qquad a \in A^s, p \in P_a^{out}, s \in S^{pd} \cup S^d, t \in T \tag{13}$$

The quantity of products which can be processed during a season by an activity in a production-distribution center is limited by the capacity options selected for that center. This imposes the following production and storage capacity constraints:

$$\sum_{i \in R_{ak}} q^i X^i_{p_i st} \le \sum_{j \in J_{ks}} b^t_j \hat{Z}_{jt} \qquad a \in A^p, s \in S^{pd}, k \in KM_{sa}, t \in T \tag{14}$$

$$\sum_{p \in P_a^{in}} q_{pa} (\sum_{a' \in A_a^{out}} \sum_{s' \in S_{ps}^o \cup D_{ps}} F_{p(s,a)(s',a')t} + \sum_{a' \in A_a^{out}} F_{p(s,a)(s,a')t}) \le \sum_{k \in KS_{as}} \sum_{j \in J_{ks}} b^t_j \hat{Z}_{jt} \qquad a \in A^s, s \in S^{pd}, t \in T \tag{15}$$

Note that in (15), the storage capacity is expressed in terms of a maximum throughput and not in terms of the storage space available. This does not present any problem since the inventory turnover ratio can be used to convert the space available into a maximum seasonal flow. Similarly, for distribution centers, the storage capacity available depends on the installed storage technologies. This yields the following capacity constraints:

$$\sum_{p \in P_a^{in}} q_{pa} (\sum_{a' \in A_a^{out}} \sum_{s' \in S_{ps}^o \cup D_{ps}} F_{p(s,a)(s',a')t} + \sum_{a' \in A_a^{out}} F_{p(s,a)(s,a')t}) \le (\sum_{k \in KS_{as}} b_{skt}) Y_s \qquad a \in A^s, s \in S^d, t \in T \tag{16}$$

Finally, finished product flows to the sales market must be modeled. There is a lower and an upper bound on product demand for each of the demand zones in the sales market. Also some finished products can be substituted by others. This leads to the following constraints:

$$F_{p(s,a)(d,\bar{a})t} = \sum_{p' \in SP_p} F_{p'p(s,a)dt} \quad a \in A_{\bar{a}}^{in}, p \in P_{\bar{a}}^{in}, s \in S^{sd} \cup S^d, d \in D_{ps}, t \in T \tag{17}$$

$$x_{pdt}^{\min} \le \sum_{p' \in PS^p} \sum_{s \in S_{p'd}^i} \sum_{a \in A_a^{in}} F_{pp'(s,a)dt} \le x_{pdt}^{\max} \quad p \in P_{\bar{a}}^{in}, d \in D_p, t \in T \tag{18}$$

3.4 Objective function

In an international context, in order to take transfer prices and taxes into account correctly, it is necessary to derive an income statement for each network facility. The revenues and expenses of the production-distribution centers and the distribution centers, in local currency, are presented in Table 3. The expression for the transfer costs of material inflows is obtained by first converting the transfer prices and transportation costs in local currency and then by adding the applicable duties. A similar approach is used to calculate other revenues and expenses. Let:

C_s = Total site s expenses for the planning horizon.

R_s = Total site s revenues for the planning horizon.

Then, using the expenditure and revenue elements in Table 3, it is seen that:

$$C_s = a) + b) + c) + d) + e) + f) + g) + h) + i) + j) \quad s \in S^{pd} \tag{19}$$

$$C_s = a) + b) + c) + e) + f) + g) + i) + j) \quad s \in S^d \tag{20}$$

$$R_s = k) + l) \quad s \in S^{pd} \cup S^d \tag{21}$$

The operating income for each national division $o \in O$ is given by $M_0 = \sum_{s \in S_o} (R_s - C_s)$ and the corporate net revenues before taxes in the reference currency are $\sum_{0 \in O} e_{0o} M_o$. However, to calculate corporate after tax profits, the divisions with positive margins must be distinguished from those with negative margins because there is no income tax to pay on losses. To do this, M_0 must be separated in its negative and positive parts by defining

$$\text{Operating Income} = M_0^+ - M_0^- \quad o \in O$$

where the operating profit $M_0^+ = M_0$ if $M_0 > 0$ and the operating loss $M_0^- = -M_o$, otherwise. Given this, the after tax net revenues of the corporation in its reference currency is given by the expression

$\sum_{0 \in O} e_{0o}[(1-\tau_0)M_o^+ - M_o^-]$.

		Distribution center (S^d)	Production-distribution center (S^{pd})
Expenses	a) Inflow transfer cost	$\sum_{t \in T} \sum_{a,a' \in A-\{\bar{a}\}} \sum_{p \in P_a^{out} \cap P_{a'}^{in}} \sum_{s' \in S_{ps}^i} (1+\delta_{ps's}) e_{o(s)o(s')} (\pi_{ps't} + f_{ps'st}^t) F_{p(s',a')(s,a)t}$	
	b) Raw materials	$\sum_{t \in T} \sum_{a \in A-\{1\}} \sum_{p \in P_a^{in}} \sum_{v \in V_{ps}} (1+\delta_{pvs}) e_{o(s)o(v)} f_{pvst}^v F_{p(v,1)(s,a)t}$	
	c) Receptions from other sites	$\sum_{t \in T} \sum_{a,a' \in A-\{\bar{a}\}} \sum_{p \in P_a^{out} \cap P_{a'}^{in}} \sum_{n \in V_{ps} \cup S_{ps}^i} f_{pnst}^d F_{p(n,a')(s,a)t}$	
	d) Production		$\sum_{t \in T} \sum_{a \in A^p} \sum_{k \in KM_{sa}} \sum_{i \in R_{ak}} c_{p_i st}^i X_{p_i st}^i$
	e) Facilities and options cost	$A_s Y_s$	$\sum_{l \in L_s \cup \{0\}} A_{ls} Y_{ls} + \sum_{j \in J_s} (a_j^1 Z_j + a_j^0 (1 - Z_j)) + \sum_{t \in T} \sum_{j \in J_s} \hat{a}_{jt} \hat{Z}_{jt}$
	f) Order cycle and safety stocks	$\sum_{t \in T} \sum_{a \in A^s} \sum_{p \in P_a^{out}} h_{pst} \rho_{pst} (\sum_{a' \in A_a^{out}} (F_{p(s,a)(s,a')t} + \sum_{n \in S_{ps}^o \cup D_{ps}} F_{p(s,a)(n,a')t}))$	
	g) Seasonal stocks	$\sum_{t \in T} \sum_{a \in A^s} \sum_{p \in P_a^{in}} h_{pst} \sum_{k \in KS_{sa}} I_{pkst}$	
	h) Handling		$\sum_{t \in T} \sum_{a \in A^s} \sum_{p \in P_a^{in}} m_{pst} \sum_{a' \in A_a^{out}} F_{p(s,a)(s,a')t}$
	i) Outflows to other sites	$\sum_{t \in T} \sum_{a,a' \in A-\{\bar{a}\}} \sum_{p \in P_a^{out} \cap P_{a'}^{in}} \sum_{s' \in S_{ps}^o} f_{pss't}^o F_{p(s,a)(s',a')t}$	
	j) Outflows to demand zones	$\sum_{t \in T} \sum_{a \in A_a^{in}} \sum_{p \in P_a^{in}} \sum_{d \in D_{ps}} f_{psdt}^o F_{p(s,a)(d,\bar{a})t}$	
Revenues	k) Outflows to other sites	$\sum_{t \in T} \sum_{a,a' \in A-\{\bar{a}\}} \sum_{p \in P_a^{out} \cap P_{a'}^{in}} \sum_{s' \in S_{ps}^o} (\pi_{pst} + f_{pss't}^t) F_{p(s,a)(s',a')t}$	
	l) Outflows to demand zones	$\sum_{t \in T} \sum_{a \in A_{\bar{a}}^{in}} \sum_{p \in P_{\bar{a}}^{in}} \sum_{p' \in SP^p} \sum_{d \in D_{ps}} e_{o(s)o(d)} P_{pdt} F_{pp'(s,a)dt}$	

Table 3: Facilities Expenses and Revenues in Local Currency

Based on previous statements, the complete mixed-integer programming model proposed to optimize the structure of the logistic network of the company takes the following form:

$$\textit{Maximize} \quad \sum_{0 \in O} e_{0o}[(1-\tau_0)M_o^+ - M_o^-] \qquad \text{(MIP)}$$

subject to

- Supply market constraints (1)
- Facility layout, space and exclusive options constraints (5), (6) and (7)
- Seasonal capacity option usage constraints (8)
- Production activities flow equilibrium constraints (9) and (10)
- Storage activities inventory accounting constraints (11) and (12)
- Production and storage capacity constraints (14), (15) and (16)
- Sales market constraints (17) and (18)
- Facilities total cost and revenue definitions (19), (20) and (21)
- National divisions operating income definition

$$\sum_{s \in S_o} (R_s - C_s) - M_0^+ + M_0^- = 0 \quad o \in O \tag{22}$$

- Non-negativity constraints

$$\begin{aligned}
&Y_{ls} \in \{0;1\} \quad s \in S^{pd}, l \in L_s \qquad Y_{0s} \in \{0;1\} \quad s \in S^{pd} \qquad Y_s \in \{0;1\} \quad s \in S^d \\
&Z_j \in \{0;1\} \quad s \in S^{pd}, j \in J_s \qquad \hat{Z}_{jt} \in \{0;1\} \quad t \in T, s \in S^{pd}, j \in J_s \\
&F_{p(n,a)(n',a')t} \geq 0 \quad p \in P, (n,a) \in (V \cup S) \times A, (n',a') \in (V \cup D_p) \times A, t \in T \\
&F_{pp'(s,a)dt} \geq 0 \quad p \in PF, p' \in SP^p, (s,a) \in S \times A, d \in D_p, t \in T \\
&X_{p_i st}^i \geq 0 \quad s \in S^{pd}, a \in A^p, k \in KM_{sa}, i \in R_{ak}, t \in T \\
&I_{pkst} \geq 0 \quad p \in P, s \in S^{pd}, a \in A^s, k \in KS_{sa}, t \in T
\end{aligned} \tag{23}$$

4 Finding the Model Optimal Solution

In order to test the solvability and the applicability of mathematical program, MIP was used to solve several instances of the Virtu@l-Lumber case developed with our partners of the Quebec forest products industry. The base case involves a moderate size lumber company operating three sawmills in the province of Quebec and selling lumber in Canada and in the United States. The product-markets of the company and the finished products substitution possibilities considered were defined in Table 1. The manufacturing process of the company was illustrated in Figure 1: it involves 138 product families (raw materials, semi-finished and finished products). The log and stem supply sources available and the potential network facility layouts considered are illustrated in Figure 6. The proportion of wood species in supply from each forest and their volume are given. For each site, the figure also distinguishes the current layout from an alternative potential layout. The alternative layout for Chicoutimi would be used to increase the 8' sawing capacity and the alternative layout for Scott-Jonction would permit the implementation of MSR grading technology. The addition of a warehouse in Montreal is also considered. The location of the bucking activity is predetermined, which affects flows and activities for each site. For example, activities 2 and 3 (activity numbers from Figure 2) are not considered in Scott-Jonction because all the bucking is done in the forest. A single technology is considered for each activity except for sawing, which can use *eight foot* (8') and/or *random length* (RL) technologies, and for planning/grading which can use *classic* and/or *MSR* technologies. Note that because of the nature of supply contracts with government, the definitive closing of a sawmill is ruled out. Four three months seasons are considered. The mixed-integer program to solve includes 227 binary variables, 8 234 continuous variables and 4 206 constraints.

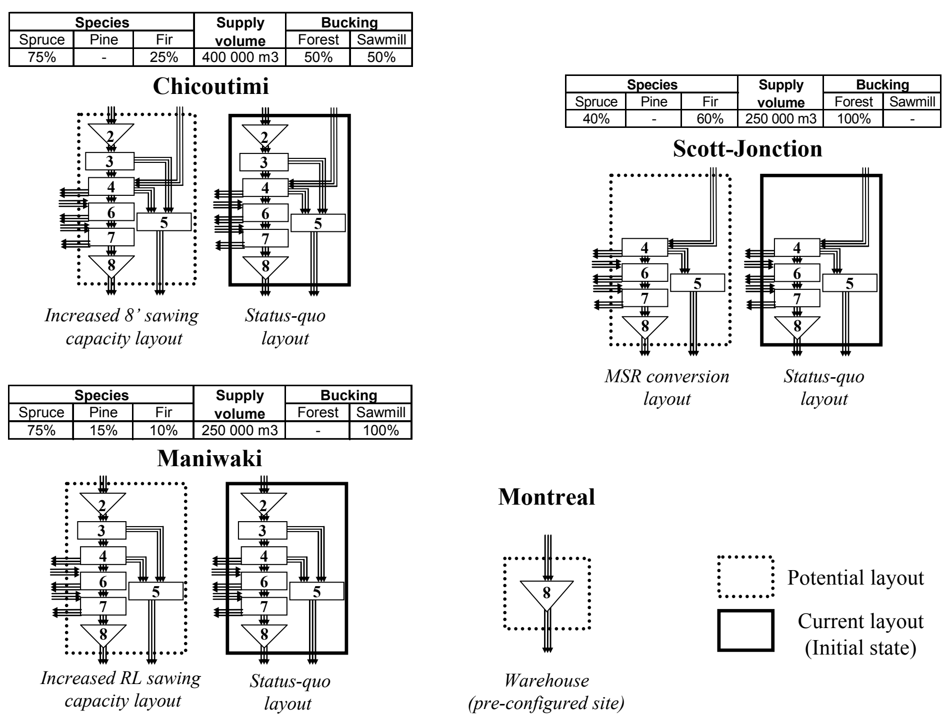


Figure 6: Forest Supply and Potential Facility Layouts for the Virtu@l-Lumber Case

The mathematical programs were solved with CPLEX 9.0 on a 1.9 GHz computer. In order to study the solvability of the model with CPLEX, the product prices, the fixed seasonal operating costs and the (total demand)/(total potential capacity) ratio of the base case were varied to generate extreme test problems. These factors were chosen because they tend to have a significant impact in practice, on the capacity options considered and on the way companies are organized. Eight problem instances were generated (Table 4). It was found that the default CPLEX settings lead to relatively long computation times. However, experimentation with CPLEX settings (see ILOG CPLEX 9.0 User's Manual) lead to the reduction of computation time by a factor of 30 for some problem instances. The best CPLEX solution strategy found for our model was the following:

- Give more importance to feasibility than to analysis and proof of optimality by setting the *MIPemphasis* parameter to 1.
- Set the *Probe* parameter to 3 in order to increase the search of logical implications after the preprocessing and before the solution of the root relaxation.

Table 4 gives the resolution times obtained in seconds. The results show that our model can be solved efficiently for realistic cases with commercial optimization software such as CPLEX. They also show that the solution times are not very sensitive to product prices and to the demand-capacity gap but that they are quite sensitive to the value of the seasonal fixed operating costs.

Overcapacity (0.65*Capacity ≈ Demand)		**Price** ($Can/MBF)	
		Low (320)	*High (460)*
Seasonal fixed cost	*Zero*	66 s	70 s
	2% of annual operating cost	185 s	148 s

Capacity ≈ Demand		**Price** ($Can/MBF)	
		Low (320)	*High (460)*
Seasonal fixed cost	*Zero*	54 s	83 s
	2% of annual operating cost	146 s	240 s

Table 4: Computational Times (in seconds) for Extreme Problem Instances

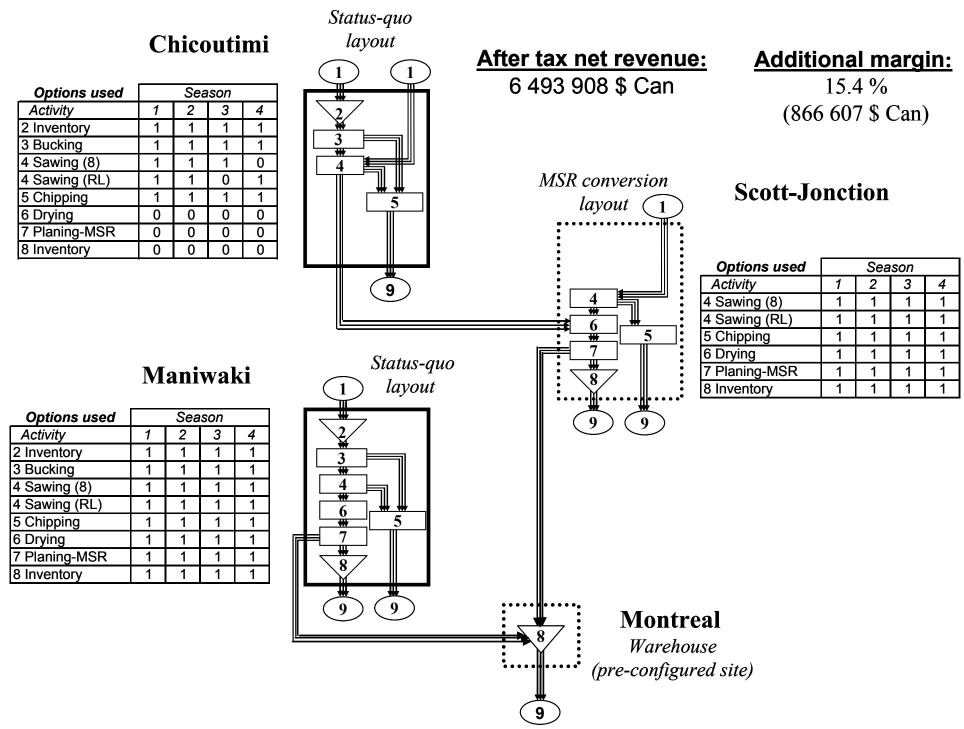
5 Guidelines for the use of the methodology

The design methodology proposed is adequate to make plant and logistic network reconfiguration decisions in a context where:

- the implementation of these decisions requires significant efforts and budgets, so that companies are prepared to make them only occasionally;
- product prices and demand follow a predictable seasonal pattern, and the company is prepared to keep seasonal inventories and to temporarily shut down some activities to adapt to these patterns;

- product substitution and alternative production recipes can be used to allow a better match between supply and demand, which may lead to the implementation of more expensive but more flexible capacity options.

At a more tactical level, the approach is also adequate to guide sourcing decisions and demand management decisions.



Chicoutimi

Options used	Season			
Activity	1	2	3	4
2 Inventory	1	1	1	1
3 Bucking	1	1	1	1
4 Sawing (8)	1	1	1	0
4 Sawing (RL)	1	1	0	1
5 Chipping	1	1	1	1
6 Drying	0	0	0	0
7 Planing-MSR	0	0	0	0
8 Inventory	0	0	0	0

Scott-Jonction

Options used	Season			
Activity	1	2	3	4
4 Sawing (8)	1	1	1	1
4 Sawing (RL)	1	1	1	1
5 Chipping	1	1	1	1
6 Drying	1	1	1	1
7 Planing-MSR	1	1	1	1
8 Inventory	1	1	1	1

Maniwaki

Options used	Season			
Activity	1	2	3	4
2 Inventory	1	1	1	1
3 Bucking	1	1	1	1
4 Sawing (8)	1	1	1	1
4 Sawing (RL)	1	1	1	1
5 Chipping	1	1	1	1
6 Drying	1	1	1	1
7 Planing-MSR	1	1	1	1
8 Inventory	1	1	1	1

Figure 7: Virtu@l-Lumber and results.

These contextual properties are all present in the Virtu@l-Lumber case, which describes the business environment of a realistic Canadian lumber company. Figure 7 summarizes the main design recommendations resulting from the application of model (MIP) to the base Virtu@l-Lumber case, with a one year planning horizon divided into four seasons. The model recommends implementing the MSR conversion layout at the Scott-Jonction sawmill and replacing the classic planing/grading capacity option in place by a MSR capacity option. The *status-quo* layout is kept at Chicoutimi and Maniwaki. However, the model suggest closing the drying, planing and finished inventory storage activities at Chicoutimi, and shipping all the green lumber produced in Chicoutimi to the Scott-Jonction mill for final processing. The model further recommends the implementation of RL and 8' sawing lines in the three mills, but with a shutdown of the RL sawing line in season 3 and of the 8' sawing line in season 4 at Chicoutimi. In addition, the use of the Montreal warehouse is recommended. The model also suggests keeping a seasonal inventory of several raw materials and finished products. Several product substitutions are also recommended and the demand zones to supply from each plant and from the warehouse are specified. These recommendations result in a 15.4% increase of the company after tax profits.

Clearly, before implementing such recommendations, one would have to be very confident that the cost, price, capacity, supply and demand parameter values used for the year considered reflect a durable yearly pattern and, even then, some sensitivity analysis should be done to confirm the robustness of the solution obtained. Note that the model decision variables fall into two categories: design variables Y_{ls}, Y_s and Z_j and the seasonal activity anticipation variables $\hat{Z}_{jt}$, $F_{p(n,a)(n',a')t}$, $F_{pp'(s,a)dt}$, $X^i_{p_ist}$ and I_{pkst}. The later are included in the model mainly to reflect the impact of the design decisions on seasonal activities and they would not be acted upon except maybe for the first season. The model can then be used as a tactical planning tool by fixing the design variables and running it on a rolling horizon basis to adapt seasonal decisions to up-to-date information and forecasts. If the business environment price and demand pattern is not stable, then one would have to use a two or three year planning horizon to properly anticipate the impact of the design on seasonal activities. When a longer horizon is used, prices, exchange rates and demands become much more difficult to forecast and several potential business environment scenarii must be considered. A good example of how to use the type of model presented here in such a context is given by Körksalan and Süral (1999).

The Virtu@l-Lumber case illustrates the use of the design methodology proposed to reorganize the current production-distribution network of a company, but the approach can be used in several other contexts. For example, it could be used to evaluate the value of a potential merger, the acquisition of a competitor's plant or a joint venture. It could be used by a company to investigate the impact of a change of its transfer prices, within the limits permitted by custom authorities. The model proposed could also be used as an econometric tool by governments to investigate the impact of a change of natural resources availability regulations on an industry sector.

6 Concluding Remarks

As was demonstrated in the previous section, the methodology proposed in this paper can effectively support the design of the production-distribution networks of divergent process industries. The model elaborated is a mixed integer programming problem, that can effectively be solved with commercial solvers in a reasonable amount of time, for realistic business cases. Further work may be required to obtain an efficient solution approach for very large business cases, but we believe that the paper provides the basis required to develop a good strategic decision support system.

Several extensions to the model proposed can be considered, some trivial and others more demanding. A simple extension would be to incorporate the possibility of moving some existing equipment between plants. Another one would be the generalization of the approach to the case of many-to-many recipes for the process activities. An important extension would be to model product-markets in more details by considering important sub-markets such as the spot market, long term contracts and VMI agreements explicitly.

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