Les Cahiers du GERAD ISSN: 0711–2440

On the design of a wind farm collection network when several cable types are available

A. Hertz, O. Marcotte, A. Mdimagh,
M. Carreau, F. Welt

G–2015–23

March 2015

GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

On the design of a wind farm collection network when several cable types are available

Alain Hertz[a]
Odile Marcotte[b]
Asma Mdimagh[a]
Michel Carreau[c]
François Welt[d]

[a] *GERAD & Département de mathématiques et génie industriel, Polytechnique Montréal, Montréal (Québec) Canada, H3C 3A7*

[b] *GERAD & Département d'informatique, UQAM, Montréal (Québec) Canada, H3C 3P8*

[c] *Hatch, Montréal (Québec) Canada, H3B 2G2*

[d] *Hatch, Niagara Falls (Ontario) Canada, L2E 7J7*

alain.hertz@gerad.ca
odile.marcotte@gerad.ca

March 2015

Les Cahiers du GERAD
G–2015–23

Abstract: We consider the problem of designing a collection network for a wind farm, assuming that the locations of the turbines and the potential cables are known and several cable types are available. We propose a mixed integer quadratic program to model the network design problem and then linearize the quadratic program because the latter is too difficult to solve using a standard mathematical programming software. We describe several classes of valid inequalities for the resulting mixed integer linear program. Finally we present the results of computational experiments with several versions of our model.

Key Words: Network design, wind farm, mixed integer programming, valid inequalities.

Résumé : Dans cet article nous étudions le problème de concevoir un réseau de collecte pour un parc éolien, dans le cas où la localisation des turbines et des câbles potentiels est connue et plusieurs types de câbles sont disponibles. Nous montrons que ce problème peut être modélisé comme un programme quadratique mixte en nombres entiers et nous donnons un modèle linéaire équivalent (puisque le programme quadratique peut difficilement être résolu par un logiciel standard de programmation mathématique). Nous décrivons ensuite des familles d'inégalités valides pour le programme mixte en nombres entiers résultant de cette linéarisation. Finalement nous présentons les résultats obtenus pour différentes versions de notre modèle.

Mots clés : Conception de réseau, parc éolien, programmation en nombres entiers mixte, inégalités valides.

Acknowledgments: This work was supported by the NSERC discovery grant of Alain Hertz and through MITACS projects partially financed by Hatch.

1 Introduction

Because of the increase in the demand for energy and the depletion of traditional energy sources, the design of renewable energy systems (in particular the design of wind farms) has become an urgent question for energy producers and indeed society as a whole. The design of such systems is a daunting task and requires solving different types of optimization problems (see Baños et al. [1] for a survey of optimization methods used in the field of renewable energy). In the specific case of wind energy, one must consider criteria for choosing a wind farm location (see for instance Yeh et al. [22]), evaluate the wind resource by analyzing the wind data (see for instance Coughlin et al. [4]), and find models for the wake effects of upstream turbines (see Son et al. [18]).

Usually one wishes to choose the turbines locations so as to minimize wake effects or, equivalently, maximize the power production of the wind farm. This is the so-called wind farm layout optimization problem or WFLOP for short: we refer the reader to Samorani [17] for a survey on this problem and Tesauro et al. [19] for a survey of optimization methods used to solve it. Of particular interest are the approaches using mathematical programming to model the WFLOP, such as those found in Donovan [5], Kiranoudis et al. [11], Ozturk and Norman [15], Pérez et al. [16], and Turner et al. [21].

Once the location and energy output of each turbine within the wind farm have been ascertained, one must find a way of routing the energy produced by all the turbines so that it reaches a public transmission network. This is sometimes referred to as the collector system design problem (CSDP) or the wind farm cable layout problem. Dutta and Overbye [6] present an algorithm based on clustering for solving the CSDP; they also present an algorithm for locating the trenches to be dug for installing the cables (see Dutta and Overbye [7]). Berzan et al. [3] decompose the CSDP into several subproblems and use graph-theoretical methods for solving them. Mathematical programming techniques can be used as well for modelling and solving the CSDP: for instance Lumbreras and Ramos [12] model the problem of designing the electric system of an offhsore wind farm and propose a Benders' decomposition approach for solving it. Bauer and Lysgaard [2] consider a slightly different problem, in the sense that the network to be designed must be planar. They formulate it as a vehicle routing problem with planarity constraints.

Sometimes one does not design a network from scratch but rather tries to expand an existing network. Trötscher and Korpås [20] present a model that extends the standard mixed integer linear programming approach to the transmission expansion problem. Finally we mention that in his thesis, Fagerfjäll [8] considers both the wind farm layout problem and the collector system design problem. In this article we address a version of the CSDP, i.e., we assume that the locations of the turbines and the potential locations of the cables are known. The problem we consider was proposed to us by a company (Hatch) and is related to many network design problems studied in the operations research literature. Note that in Hertz et al. [10], we were considering a more restricted version of the current problem, since we were not taking into account the energy losses (see Section 2) and were assuming that all the cables between two given nodes were of the same type. The first of the two models presented in Section 2 has a quadratic objective function and is therefore much more difficult to solve than the model in Hertz et al. [10].

The choice of a design for a transportation, telecommunications, or energy network can often be formulated as a mixed integer program. There is a vast literature on these formulations and the methods used for solving them. In particular we refer the reader to the surveys by Gendron et al. [9], Magnanti and Wong [13], and Minoux [14]. Our article is organized as follows: Section 2 contains a precise statement of our network design problem and two models, Section 3 some inequalities for strenghtening the models, Section 4 the results of our computing experiments, and Section 5 our conclusion.

2 Statement and modelling of the design problem

In this article we assume that the location of wind turbines and the potential network links have already been selected. Some of these links constitute an *underground network*, i.e., a network where some of the links will be used to install underground cables. The other links are part of the *above-ground network* and correspond to road segments (each of them between two geographical points) where transmission lines can be installed. We will use the word "cable" to mean either a cable or a transmission line. One of the nodes of the above-ground network is the *sub-station*, where the energy produced by the turbines will be made available to the public power grid. Both subnetworks (i.e., the underground and above-ground networks) can be represented by a graph whose nodes include the turbines, the sub-station, and intermediate nodes and whose set of arcs includes the potential (underground or above-ground) cables. A toy example is pictured in Figure 1.

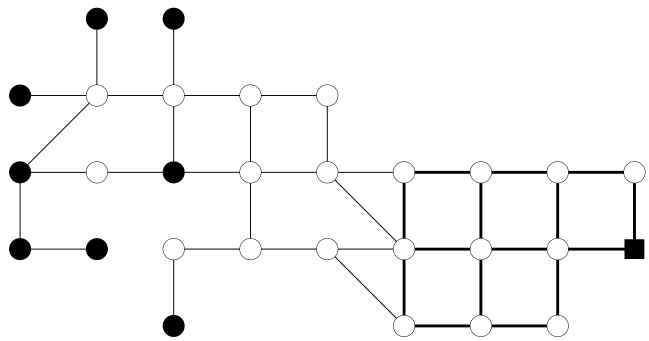

Figure 1: A picture showing the potential cables of the underground and above-ground subnetworks. The black circles represent the turbines, the black square the sub-station, and the white circles the intermediate nodes. The light (resp. bold) edges represent the possible locations of undergroud (resp. above-ground) cables.

Note that one can install several cables on a given link, in either subnetwork, and that the energy may flow in either direction on a given cable. Thus it will be convenient to consider a directed multigraph (defined below) rather than an undirected graph such as the one in Figure 1. Note also that there are several types of cables and that the energy loss incurred when energy flows through a cable depends upon the cable characteristics: length, capacity, whether the cable is under the ground or above ground, etc. Those characteristics are summarized in a coefficient denoted by B and the energy loss is estimated by the formula $B \cdot X^2$, where X denotes the flow through the cable. When several cables of the same type are installed on a link, the cost of the second cable is at most the cost of the first one, the cost of the third one is at most the cost of the second one, and so on.

In general, not every cable type may be used on a given link. Assuming that $\{1, 2, \dots, q'\}$ is a set of indices representing all the cable types, we let $H_{uv} \subseteq \{1, 2, \dots, q'\}$ denote the set of cable types that can be used between the nodes u and v. In many cases there will be a set of cable types for the underground network (denoted by H_{ug}) and another set of cable types for the above-ground network (denoted by H_{ag}). In those cases H_{uv} will equal H_{ug} or H_{ag} for every pair $\{u, v\}$. We will not assume this, however, unless stated otherwise. The cost of the kth cable installed between u and v, assuming that it is of type $h \in H_{uv}$, will be denoted by c_{uv}^{kh}. The capacity of a cable of type $h \in H_{uv}$ installed between u and v will be denoted by C_{uv}^{h} and the energy loss coefficient for such a cable by b_{uv}^{h}.

Assuming that we know how much energy is produced by each turbine, the problem is to choose a set of cables through which the energy produced by the wind farm will be transported to the sub-station. Naturally the flow must be conserved at any node that is not a turbine or the sub-station. Actually the design of electrical circuits requires that a further condition be satisfied: the flow must be *unsplittable*, that is, if a quantity Q of energy flows from node u to node v (where v is not the sub-station), then this same quantity must be routed through an arc of the form (v, w) for some node w. We refer the reader to Hertz et al. [10] for more details on unsplittability and its relation to graph-theoretic concepts. Of course there are many feasible solutions of the design problem, in terms of cable types and cable locations, and the company wishes to minimize an objective function that includes both the cost of installing the cables and the total energy loss (calculated over a 5-year, 10-year, or 20-year horizon).

We now describe a mathematical programming model for the collection network design problem. Let m denote the maximum number of cables that can be installed between two given nodes. We introduce a directed multigraph $G = (V, A)$ defined as follows.

1. The set of nodes of G (i.e., V) is the union of three disjoint sets: R_1, the set of nodes of the underground network (which includes the set T of turbines); R_2, the set of nodes of the above-ground network (excluding the sub-station); and $\{0, s\}$ (where s denotes the sub-station and 0 a fictitious node considered as the "source of the network").
2. An arc of G is denoted by a triple (u, v, k), where u is the tail of the arc, v its head, and k a number comprised between 1 and m and representing an "instance" of the couple (u, v). The set of arcs of G (i.e., A) is the union of the following sets of arcs (which are pairwise disjoint):
   - the arcs of the form $(0, u, 1)$ for u in T;
   - the arcs (u, v, k) and (v, u, k) for all k in $\{1, 2, \ldots, m\}$, where u and v are the endpoints of a potential underground cable (thus u and v belong to R_1);
   - the arcs (u, v, k) for all k in $\{1, 2, \ldots, m\}$, where u belongs to R_1, v belongs to R_2, and u and v are the endpoints of a potential (underground) cable between the underground and above-ground subnetworks;
   - the arcs (u, v, k) and (v, u, k) for all k in $\{1, 2, \ldots, m\}$, where u and v are the endpoints of a potential above-ground cable (thus u and v belong to R_2); and
   - the arcs (u, s, k) for all k in $\{1, 2, \ldots, m\}$, where u belongs to R_2 and u and s are the endpoints of a potential (above-ground) cable.

In the sequel we will need to distinguish the "unidirectional" arcs from the other ones. Thus we introduce the following notation:

$$A_1 = \{(u, v, k) \in A \mid (v, u, k) \notin A\}.$$

For each arc of G one must decide whether to install a cable on it or not, and if the answer is "yes", what type of cable to install. Hence the most important variables in our model will be the binary variables t_{uv}^{kh}, where t_{uv}^{kh} equals 1 if and only if a cable of type h is installed on the arc (u, v, k). The model will include a variable t_{uv}^{kh} for each (u, v, k) in A and h in H_{uv}. It will also include the binary variables t_{uv}^{k}, where t_{uv}^{k} equals 1 if and only if there is a cable (of any type) on the arc (u, v, k).

In order to formulate the constraint that the selected arcs enable one to collect all the energy produced by the turbines, we also need so-called flow variables: x_{uv}^{kh} denotes the flow carried through the arc (u, v, k) by a cable of type h. To express the constraint that the flow is unsplittable, we need to introduce variables that will indicate how to route the flow on a given arc (u, v, k). Indeed the unsplittability constraint means that every flow unit on (u, v, k) must leave node v through the same arc, denoted by (v, w, k'). Let $\mathcal{P}_2$ denote the set of all paths of length 2 in G, i.e., the set of all couples $((u, v, k), (v, w, k'))$ such that u and w are distinct nodes. We introduce the binary variable $y_{uvw}^{kk'}$ for every $((u, v, k), (v, w, k'))$ in $\mathcal{P}_2$, where $y_{uvw}^{kk'}$ equals 1 if and only if the entire flow on (u, v, k) is routed on (v, w, k'). Similarly $z_{uvw}^{kk'}$ denotes the amount of flow routed on arc (u, v, k) and then arc (v, w, k'). Here is the mixed integer quadratic program that we propose in order to model the collection network problem. It will be denoted by $(MIQP)$ in the sequel.

$$\min \sum_{(u,v)\in A} \sum_{k=1}^{m} \sum_{h\in H_{uv}} c_{uv}^{kh} t_{uv}^{kh} + \sum_{(u,v)\in A} \sum_{k=1}^{m} \sum_{h\in H_{uv}} b_{uv}^{h} \left(x_{uv}^{kh}\right)^2 \tag{1}$$

such that

$$\sum_{\{vv,u)\in A\}} \sum_{k=1}^{m} \sum_{h\in H_{uv}} x_{vu}^{kh} - \sum_{\{vu,v)\in A\}} \sum_{k=1}^{m} \sum_{h\in H_{uv}} x_{uv}^{kh} = \begin{cases} |T| & \text{if } u = s \\ -|T| & \text{if } u = 0 \\ 0 & \text{if } u \neq 0, s \end{cases} \tag{2}$$

$$\sum_{h\in H_{uv}} x_{0v}^{1h} = 1 \qquad \forall v \in T \tag{3}$$

$$x_{uv}^{kh} \leq C_{uv}^{h} t_{uv}^{kh} \qquad \forall (u,v,k) \in A,\ \forall h \in H_{uv} \qquad (4)$$

$$\sum_{h \in H_{uv}} t_{uv}^{kh} = t_{uv}^{k} \qquad \forall (u,v,k) \in A \qquad (5)$$

$$t_{uv}^{k} + t_{vu}^{k} \leq 1 \qquad \forall (u,v,k) \in A \backslash A_1 \qquad (6)$$

$$t_{uv}^{k+1} \leq t_{uv}^{k} \qquad \forall (u,v,k) \in A_1 \qquad (7)$$

$$t_{uv}^{k+1} + t_{vu}^{k+1} \leq t_{uv}^{k} + t_{vu}^{k} \qquad \forall (u,v,k) \in A \backslash A_1 \qquad (8)$$

$$\sum_{u \in P(v)} \sum_{k=1}^{m} z_{uvw}^{kk'} = \sum_{h \in H_{vw}} x_{vw}^{k'h} \qquad \forall (v,w,k') \in A,\ v \neq 0 \qquad (9)$$

$$\sum_{w \in S(v)} \sum_{k'=1}^{m} z_{uvw}^{kk'} = \sum_{h \in H_{uv}} x_{uv}^{kh} \qquad \forall (u,v,k) \in A,\ v \neq s \qquad (10)$$

$$z_{uvw}^{kk'} \leq \min\left(\max_{h \in H_{uv}} C_{uv}^{h}, \max_{h \in H_{vw}} C_{vw}^{h} \right) y_{uvw}^{kk'} \qquad \forall ((u,v,k),(v,w,k')) \in \mathcal{P}_2 \qquad (11)$$

$$\sum_{w \in S(v)} \sum_{k'=1}^{m} y_{uvw}^{kk'} = t_{uv}^{k} \qquad \forall (u,v,k) \in A,\ v \neq s \qquad (12)$$

$$x_{uv}^{kh} \geq 0 \qquad \forall (u,v,k) \in A,\ \ \forall h \in H_{uv} \qquad (13)$$

$$z_{uvw}^{kk'} \geq 0 \qquad \forall ((u,v,w),(v,w,k')) \in P_2 \qquad (14)$$

$$t_{uv}^{k} \in \{0,1\} \qquad \forall (u,v,k) \in A \qquad (15)$$

$$t_{uv}^{kh} \in \{0,1\} \qquad \forall (u,v,k) \in A,\ \ \forall h \in H_{uv} \qquad (16)$$

$$y_{uvw}^{kk'} \in \{0,1\} \qquad \forall ((u,v,w),(v,w,k')) \in P_2 \qquad (17)$$

The objective function (1) is the sum of two terms: a term representing the total cost of the cables installed and a term representing the total energy loss over a certain period. Note that using a sum to combine the two objectives (minimizing the total cost and minimizing the energy loss) allows some flexibility, since one may scale the b_{uv}^{h} to obtain a different combination. Constraints (2) are flow conservation constraints. Constraints (3) express the fact the each turbine produces exactly one energy unit. (The right-hand sides of some of these constraints could be replaced by other integers if the corresponding turbines produced more than one unit.) Constraints (4) enforce the requirement that the flow on a given cable be at most the capacity of that cable. Constraints (5) enforce the definition of t_{uv}^{k}.

Constraints (6) enforce the requirement that there be at most one cable on a "bidirectional" arc. Constraints (7) enforce the requirement that a cable be installed on (u,v,k) before one is installed on $(u,v,k+1)$ (for a unidirectional arc (u,v,k)). Constraints (8) enforce the same requirement for bidirectional arcs. Constraints (9) and (10) relate the variables $z_{uvw}^{kk'}$ to the variables $x_{vw}^{k'h}$ and x_{uv}^{kh}, respectively. Note that $P(v)$ (resp. $S(v)$) denotes the set of predecessors (resp. successors) of v, i.e., the set of nodes u such that (u,v,k) (resp. (v,u,k)) belongs to A. Constraints (11) and (12) relate the variables $y_{uvw}^{kk'}$ to the variables $z_{uvw}^{kk'}$ and t_{uv}^{k}, respectively. The last five groups of constraints enforce the requirement that certain variables be non-negative (or binary). Note that Constraints (13) (resp. (14)) could be replaced by the constraints $x_{uv}^{kh} \in \mathbb{Z}$ (resp. $z_{uvw}^{kk'} \in \mathbb{Z}$): indeed it can be shown that the values of the x_{uv}^{kh} and $z_{uvw}^{kk'}$ are integral in any optimal solution of $(MQIP)$. We give a formal proof of this statement since it will be used to linearize the model.

Proposition 2.1 *In any optimal solution of $(MIQP)$ the variables x_{kh}^{uv} and $z_{uvw}^{kk'}$ have integral values.*

Proof. In what follows x_{uv}^{k} denotes $\sum_{h \in H_{uv}} x_{uv}^{kh}$ (for any triple (u,v,k)). Let us define a directed graph $H = (V', A')$ as follows: V' is the set $\{(u,v,k) \in A \mid t_{uv}^{k} = 1\}$ and A' the set

$$\{((u,v,k),(v,w,k')) \mid (u,v,k) \in A, (v,w,k') \in A, \text{ and } y_{uvw}^{kk'} = 1\}.$$

We first prove that H is an acyclic graph. Constraints (12) imply that every node in H has at most one successor, i.e., for every (u,v,k) in V', there is at most one node (v,w,k') such that $((u,v,k),(v,w,k'))$ is in A'. This fact and Constraints (4), (5), and (11) for a given (u, v, k) enable us to conclude that only one term in the left-hand side of Constraint (10) is different from 0. Finally Constraints (9) and (10) imply that if (v, w, k') is the successor of (u, v, k) in H, then the value of x^k_{uv} in the optimal solution is at most equal to the value of $x^{k'}_{vw}$. Assume that C is a cycle in H. It follows that the values of the x^k_{uv} for (u, v, k) in C are all equal. There are two cases to consider:

1. C is a connected component of H: then no path from a turbine to the sub-station goes through any (u, v, k) in C and all the cables corresponding to nodes in C can be removed from the current solution, contradicting the hypothesis that this solution is optimal;
2. C is not a connected component of H: then there is a node (u, v, k) in C such that
   - either there is some (v, w, k') not in C such that $((u, v, k), (v, w, k'))$ is an arc in H, contradicting the fact that each node in H has a single successor,
   - or there is some (w, u, k') not in C such that $((w, u, k'), (u, v, k))$ is an arc in H. But then (u, v, k) has also a predecessor (w', u, ℓ) in C and $x^k_{uv} = x^\ell_{w'u}$ holds. Constraints (2) imply that $x^{k'}_{wu}$ equals 0, contradicting the assumption that the solution is optimal (because $t^{k'}_{wu}$ can be assigned the value 0).

Therefore H is an acyclic subgraph and we may assign numbers to its nodes in such a way that if $((u, v, k), (v, w, k'))$ is an arc in H and s_1 (resp. s_2) is the number assigned to (u, v, k) (resp. (v, w, k')), then s_1 is smaller than s_2. (Such a numbering is a topological order of the nodes.) Also we observe that if x^{kh}_{uv} is integral and the node (u, v, k) has a successor in H, then all variables of the form $z^{kk'}_{uvw}$ are integral. Indeed, if these assumptions hold, only one term in the left-hand side of Constraint (10) is different from 0. This term, for the node (u, v, k) that we are considering, is of the form $z^{kk'}_{uvw}$ and actually equal to the integer $\sum_{h\in H_{uv}} x^{kh}_{uv}$. We conclude that every variable of the form $z^{kk'}_{uvw}$ is integral if x^{kh}_{uv} is integral and the node (u, v, k) has a successor in H.

We shall now prove, by induction on the number assigned to a node, that the variables x^{kh}_{uv} and $z^{kk'}_{uvw}$ take integer values in the optimal solution considered. If x^{kh}_{uv} equals 0, there is nothing to prove. If x^{kh}_{uv} is greater than 0, then (u, v, k) is a node in H and one of the following cases must hold:

1. the node (u, v, k) does not have any predecessor. Then it must be of the form $(0, v, 1)$ and x^{1h}_{0v} equals 0 or 1 by Constraints (3), (4), and (5);
2. the node (u, v, k) has at least one predecessor. By the induction hypothesis, every predecessor (w', u, ℓ) of (u, v, k) is such that $x^{\ell h}_{w'u}$ and $z^{\ell k}_{w'uv}$ have integer values. Then Constraints (4), (5), and (9) imply that x^{kh}_{uv} equals either 0 or $\sum_{w'\in P(u)} \sum_{\ell=1}^{m} z^{\ell k}_{w'uv}$, proving that x^{kh}_{uv} has indeed an integer value (for any h).

As noted above, the assumption that x^{kh}_{uv} is integral implies that every variable of the form $z^{kk'}_{uvw}$ (if it exists, i.e., the node (u, v, k) has a successor in H) has an integral value. This completes our inductive proof. □

Using Proposition 2.1 we replace $(MIQP)$ by an equivalent integer linear program. Indeed the fact that the x^{kh}_{uv} are integral allows us to express each x^{kh}_{uv} as $\sum_{r=1}^{C^h_{uv}} r\alpha^{kh}_{uvr}$, where α^{kh}_{uvr} is a binary variable equal to 1 if and only if x^{kh}_{uv} equals r in a feasible solution of $(MQIP)$. Recall that C^h_{uv} is the capacity of a cable of type h installed between u and v and thus the possible values of x^{kh}_{uv} are $0, 1, \ldots, C^h_{uv}$. If x^{kh}_{uv} equals 0, all the α^{kh}_{uvr} equal 0 and we may also assume that t^{kh}_{uv} equals 0 (otherwise the solution is not optimal). If x^{kh}_{uv} is greater than 0, exactly one of the α^{kh}_{uvr} equals 1 and t^{kh}_{uv} also equals 1. Therefore we remove the variables x^{kh}_{uv} from the model and introduce into it the variables α^{kh}_{uvr} and the constraint $\sum_{r=1}^{C^h_{uv}} \alpha^{kh}_{uvr} = t^{kh}_{uv}$ for each t^{kh}_{uv}. Here is the integer programming model (denoted by (IP)) that is equivalent to $(MQIP)$. Note that because of Proposition 2.1, the constraints $z^{kk'}_{uvw} \in \mathbb{Z}$ can be replaced by $z^{kk'}_{uvw} \geq 0$ when solving the model. Note also that R^h_{uv} denotes the set $\{1, 2, \ldots, C^h_{uv}\}$.

$$\min \sum_{(u,v)\in A} \sum_{k=1}^{m} \sum_{h\in H_{uv}} c^{kh}_{uv} t^{kh}_{uv} + \sum_{(u,v)\in A} \sum_{k=1}^{m} \sum_{h\in H_{uv}} b^h_{uv} \sum_{r=1}^{C^h_{uv}} r^2 \alpha^{kh}_{uvr} \tag{18}$$

such that

$$\sum_{r=1}^{C_{uv}^h} \alpha_{uvr}^{kh} = t_{uv}^{kh} \qquad \forall (u,v,k) \in A,\ \forall h \in H_{uv} \tag{19}$$

$$\sum_{\{vv,u) \in A\}} \sum_{k=1}^{m} \sum_{h \in H_{uv}} \sum_{r=1}^{C_{uv}^h} r\alpha_{vur}^{kh} - \sum_{\{vu,v) \in A\}} \sum_{k=1}^{m} \sum_{h \in H_{uv}} \sum_{r=1}^{C_{uv}^h} r\alpha_{uvr}^{kh} = \begin{cases} |T| & \text{if } u = s \\ -|T| & \text{if } u = 0 \\ 0 & \text{if } u \neq 0, s \end{cases} \tag{20}$$

$$\sum_{h \in H_{uv}} \sum_{r=1}^{C_{uv}^h} r\alpha_{0vr}^{1h} = 1 \qquad \forall v \in T \tag{21}$$

$$\sum_{r=1}^{C_{uv}^h} r\alpha_{uvr}^{kh} \leq C_{uv}^h t_{uv}^{kh} \qquad \forall (u,v,k) \in A,\ \forall h \in H_{uv} \tag{22}$$

$$\sum_{h \in H_{uv}} t_{uv}^{kh} = t_{uv}^k \qquad \forall (u,v,k) \in A \tag{23}$$

$$t_{uv}^k + t_{vu}^k \leq 1 \qquad \forall (u,v,k) \in A \backslash A_1 \tag{24}$$

$$t_{uv}^{k+1} \leq t_{uv}^k \qquad \forall (u,v,k) \in A_1 \tag{25}$$

$$t_{uv}^{k+1} + t_{vu}^{k+1} \leq t_{uv}^k + t_{vu}^k \qquad \forall (u,v,k) \in A \backslash A_1 \tag{26}$$

$$\sum_{u \in P(v)} \sum_{k=1}^{m} z_{uvw}^{kk'} = \sum_{h \in H_{vw}} \sum_{r=1}^{C_{vw}^h} r\alpha_{vwr}^{k'h} \qquad \forall (v,w,k') \in A,\ v \neq 0 \tag{27}$$

$$\sum_{w \in S(v)} \sum_{k'=1}^{m} z_{uvw}^{kk'} = \sum_{h \in H_{uv}} \sum_{r=1}^{C_{uv}^h} r\alpha_{uvr}^{kh} \qquad \forall (u,v,k) \in A,\ v \neq s \tag{28}$$

$$z_{uvw}^{kk'} \leq \min\left(\max_{h \in H_{uv}} C_{uv}^h, \max_{h \in H_{vw}} C_{vw}^h \right) y_{uvw}^{kk'} \qquad \forall ((u,v,k),(v,w,k')) \in \mathcal{P}_2 \tag{29}$$

$$\sum_{w \in S(v)} \sum_{k'=1}^{m} y_{uvw}^{kk'} = t_{uv}^k \qquad \forall (u,v,k) \in A,\ v \neq s \tag{30}$$

$$\alpha_{uvr}^{kh} \in \{0,1\} \qquad \forall (u,v,k) \in A, \forall h \in H_{uv}, \forall r \in R_{uv}^h \tag{31}$$

$$z_{uvw}^{kk'} \in \mathbb{Z} \qquad \forall ((u,v,k),(v,w,k')) \in P_2 \tag{32}$$

$$t_{uv}^k \in \{0,1\} \qquad \forall (u,v,k) \in A \tag{33}$$

$$t_{uv}^{kh} \in \{0,1\} \qquad \forall (u,v,k) \in A,\ \ \forall h \in H_{uv} \tag{34}$$

$$y_{uvw}^{kk'} \in \{0,1\} \qquad \forall ((u,v,w),(v,w,k')) \in P_2 \tag{35}$$

Observe that (IP) contains many more variables than $(MQIP)$ but that an integer linear program can in general be solved more easily than an integer quadratic program.

3 Strengthening the model

Both $(MQIP)$ and (IP) are "difficult" models, in the sense that each of them contains a large number of binary variables. Thus it is desirable to strengthen them by including in each model inequalities that are satisfied by at least one optimal solution (although some of them render infeasible some solutions that were previously feasible). In particular there is a lot of symmetry in each model and we have looked for inequalities that mitigate the problem of symmetry. Propositions 3.1 and 3.2 state that their respective classes of inequalities do not eliminate all optimal solutions (note that Proposition 3.2 is identical to a proposition in Hertz et al. [10] but was not formally proved in our first article).

Proposition 3.1 *For any cable type h, any node pair $\{u, v\}$, and any indices k and k' such that $k \leq k'$ holds, the following constraint is satisfied by at least one optimal solution of $(MQIP)$ or (IP).*

$$t_{uv}^{k'h} + t_{vu}^{kh} \leq 1 \tag{36}$$

Proof. Let the phrase "uv-cable" denote a cable in which the energy flows from u to v. Also let us use the phrase "the slot k is occupied by a uv-cable" to mean that t_{uv}^{kh} equals 1 for some h. Finally consider the pair $\{u, v\}$ and a fixed cable type h. Inequalities (36) amount to saying that among the cables of type h between u and v, every uv-cable occupies a slot k that is smaller than any slot occupied by a vu-cable. It is clear that if an optimal solution of $(MQIP)$ or (IP) does not satisfy Inequalities (36) for some pair $\{u, v\}$ and some cable type h, then the cables of type h between u and v can be permuted so that every uv-cable has a smaller slot that any vu-cable. This permutation does not affect any other node pair or make the solution infeasible; it also preserves the objective function value. Therefore $(MQIP)$ or (IP) has at least one optimal solution satisfying Inequalities (36). □

Proposition 3.2 *Assume that only one type of cable is allowed between u and v (resp. between v and w). For any $((u, v, k), (v, w, k'))$ in $\mathcal{P}_2$ and $((u, v, \ell), (v, w, \ell'))$ in $\mathcal{P}_2$ such that $k < \ell$ and $k' > \ell'$ hold, the following constraint is satisfied by at least one optimal solution of $(MQIP)$ or (IP).*

$$y_{uvw}^{kk'} + y_{uvw}^{\ell\ell'} \leq 1 \tag{37}$$

Proof. We consider a feasible solution of $(MQIP)$ or (IP) and assume that there is only one type of cable allowed between u and v (the proof is similar in the other case, i.e., when only one cable type is allowed between v and w). Assume that $y_{uvw}^{kk'} + y_{uvw}^{\ell\ell'}$ is greater than 1 for some indices k, k', ℓ, and ℓ' with $k < \ell$ and $k' > \ell'$ (i.e., (k, k', ℓ, ℓ') is a *skewed quadruple*). Among all the skewed quadruples there is one that maximizes the index k' (say, the quadruple $(k_1, k_1', \ell_1, \ell_1')$) and one that maximizes the index ℓ (say, the quadruple $(k_2, k_2', \ell_2, \ell_2')$). Then the quadruple $(k_1, k_1', \ell_2, \ell_2')$ is also a skewed quadruple since $k_1 < \ell_1 \leq \ell_2$ and $k_1' \geq k_2' > \ell_2'$ hold. To simplify the notation we will denote the latter quadruple by (k, k', ℓ, ℓ').

In that case we may swap cables (u, v, k) and (u, v, ℓ) in the current solution in order to obtain a feasible solution with the same objective function value as the current solution. Swapping those two cables amounts to letting the flow of cable (u, v, ℓ) go through cable (v, w, k') and the flow of cable (u, v, k) go through cable (v, w, ℓ'). In particular we swap the values of $y_{uvw}^{kk'}$ and $y_{uvw}^{\ell k'}$, that is, $y_{uvw}^{kk'}$ takes the value 0 and $y_{uvw}^{\ell k'}$ the value 1. Similarly we swap the values of $y_{uvw}^{\ell\ell'}$ and $y_{uvw}^{k\ell'}$ (resp. $z_{uvw}^{kk'}$ and $z_{uvw}^{\ell k'}$, $z_{uvw}^{\ell\ell'}$ and $z_{uvw}^{k\ell'}$). We also have to swap the values of variables of the form $y_{w'uv}^{jk}$ and $y_{w'uv}^{j\ell}$ (resp. $z_{w'uv}^{jk}$ and $z_{w'uv}^{j\ell}$). Finally we swap the values of x_{uv}^{kh} and $x_{uv}^{\ell h}$ in $(MQIP)$ and the values of α_{uvr}^{kh} and $\alpha_{uvr}^{\ell h}$ (for every r) in (IP).

In the new feasible solution, there are no skewed quadruples involving the arcs (u, v, ℓ_1) or (v, w, k_1') for any $\ell_1 \geq \ell$ or $k_1' \geq k'$. Therefore iterating the transformation we have just described will eliminate all skewed quadruples for the triple (u, v, w). This procedure can be repeated for all the triples, thus eliminating every skewed quadruple from the solution. The resulting feasible solution has the same objective value as the initial feasible solution: hence Constraints (37) do not remove every optimal solution from the model. □

Proposition 3.3 *Let (u, v) be an arc such that $S(v)$ is not empty and assume that all the cables incident upon node v are of the same type, i.e., have the same capacity. Then the following constraint is satisfied by every optimal solution of $(MQIP)$ or (IP).*

$$\sum_{i=1}^{m} t_{uv}^{i} \leq \sum_{\substack{w \in S(v) \\ w \neq u}} \sum_{i=1}^{m} t_{vw}^{i} \tag{38}$$

Proof. Consider a feasible solution of $(MQIP)$ or (IP) that does not satisfy Inequality (38). Then there must exist indices i, j, and i' and a node w such that i is different from j and $y^{ii'}_{uvw}$ and $y^{ji'}_{uvw}$ are both equal to 1. This means that the sum of the flows on arcs (u,v,i) and (u,v,j) is at most the capacity of the arc (v,w,i'), and since the capacity of either (u,v,i) or (u,v,j) equals that of (v,w,i'), we can merge the flows on arcs (u,v,i) and (u,v,j) and only use the arc (u,v,i), thereby reducing the cost of the current solution. We conclude that in an optimal solution, the function that assigns to an arc (u,v,i) with $t^i_{uv}=1$ the only arc (v,w,i') with $y^{ii'}_{uvw}=1$ is a one-to-one function, i.e., Inequality (38) holds. □

Let us now introduce two subsets of arcs of G, denoted respectively by D_s and D_b: D_s consists of all the arcs of the form (u,s,k) (where s is the sub-station) and D_b of all the border arcs, i.e., the arcs (u,v,k) with u in the underground subnetwork and v in the above-ground subnetwork. Observe that each of D_s and D_b is a directed cut in the graph G. For a solution of $(MQIP)$ or (IP) to be feasible, the total capacity of the cables installed on the arcs in D_s (resp. D_b) must be at least T, the number of turbines. Recall that the set of cable types is the same for all above-ground links (in particular, H_{us} is the set H_{ag} for all u). Then one can express this requirement as follows.

$$\sum_{(u,s,k)\in D_s}\sum_{h\in H_{\text{ag}}} C^h_{us} t^{kh}_{us} \geq |T| \tag{39}$$

Assume, without loss of generality, that H_{ag} is the set $\{1,2,\ldots,q\}$, that the set of cable capacities is $\{C^h_{\text{ag}}\}^q_{h=1}$, and that $C^h_{\text{ag}} < C^{h+1}_{\text{ag}}$ holds for $h=1,2,\ldots,q-1$. Propositions 3.4 and 3.5 are easy consequences of Inequality (39).

Proposition 3.4 *For any j in H_{ag}, the inequality*

$$\sum_{(u,s,k)\in D_s}\sum_{h\in H_{\text{ag}}} \left\lceil \frac{C^h_{\text{ag}}}{C^j_{\text{ag}}} \right\rceil t^{kh}_{us} \geq \left\lceil \frac{|T|}{C^j_{\text{ag}}} \right\rceil \tag{40}$$

is satisfied by every feasible solution of $(MQIP)$ or (IP).

Proposition 3.5 *For any j in H_{ag}, the inequality*

$$\sum_{(u,s,k)\in D_s}\sum_{h=1}^{j} t^{kh}_{us} \geq \left\lceil \left(|T| - \sum_{(u,s,k)\in D_s}\sum_{h=j+1}^{q} C^h_{\text{ag}} t^{kh}_{us} \right) / C^j_{\text{ag}} \right\rceil \tag{41}$$

is satisfied by every feasible solution of $(MQIP)$ or (IP).

We now consider the directed cut D_b. Recall that the set of cable types is the same for all underground links (that is, H_{uv} equals H_{ug} for all pairs $\{u,v\}$ featured in D_b). Then one can express the directed cut requirement as follows.

$$\sum_{(u,v,k)\in D_b}\sum_{h\in H_{\text{ug}}} C^h_{uv} t^{kh}_{uv} \geq |T| \tag{42}$$

Assume, without loss of generality, that H_{ug} is the set $\{q+1,q+2,\ldots,q'\}$, that the set of cable capacities is $\{C^h_{\text{ug}}\}^{q'}_{h=q+1}$, and that $C^h_{\text{ug}} < C^{h+1}_{\text{ug}}$ holds for $h=q+1,q+2,\ldots,q'-1$. The following propositions can be derived from Inequality (42) in the same way as Propositions (3.4) and (3.5) were derived from Inequality (39).

Proposition 3.6 *For any j in H_{ug}, the inequality*

$$\sum_{(u,v,k)\in D_b}\sum_{h\in H_{\text{ug}}} \left\lceil \frac{C^h_{\text{ug}}}{C^j_{\text{ug}}} \right\rceil t^{kh}_{uv} \geq \left\lceil \frac{|T|}{C^j_{\text{ug}}} \right\rceil \tag{43}$$

is satisfied by every feasible solution of $(MQIP)$ or (IP).

Proposition 3.7 *For any j in H_{ug}, the inequality*

$$\sum_{(u,v,k)\in D_b} \sum_{h=q+1}^{j} t_{uv}^{kh} \geq \left\lceil \left(|T| - \sum_{(u,v,k)\in D_b} \sum_{h=j+1}^{q'} C_{\text{ug}}^{h} t_{uv}^{kh} \right) / C_{\text{ug}}^{j} \right\rceil \tag{44}$$

is satisfied by every feasible solution of $(MQIP)$ *or* (IP).

4 Experimental results

In order to test our model, we used instances of the network collection problem that were provided by Hatch. There are three groups of instances: in each group the underlying directed graph (including the nodes representing the turbines) is the same. For instance the graph underlying the instances in the first group contains 40 turbines, 143 nodes, and 384 arcs (see Table 1). In each of the first two instances there are only two cable types for underground cables and two cable types for above-ground cables. Underground cables may have a capacity of 6 or 13 and above-ground cables may have a capacity of 8 or 19. In the third instance (labelled 1-H20), the capacity of an underground cable must belong to the set $\{4, 6, 8, 13, 20\}$ while the capacity of an above-ground cable must belong to the set $\{5, 8, 13, 19, 24\}$. Another difference between the instances 1-H05, 1-H10, and 1-H20 is their *horizon*: their respective horizons are 5 years, 10 years, and 20 years. This difference manifests itself through the coefficients b_{uv}^h: when the horizon is multiplied by 2, so is b_{uv}^h for every h and every couple (u, v). In Table 2 we give the number of variables and the number of constraints for each instance and each of the two models.

To solve the models presented in this article, we used the 12.5 version of CPLEX (with the parameters being set to their implicit values). All our tests were carried out on an Intel Core i7 Computer with 2.7 GHz and a memory of 9G. Although CPLEX allows one to solve certain types of quadratic programs, our experiments with $(MQIP)$ yielded results that were inferior to those obtained with (IP). It turned out also that Inequalities (40), (41), (43), and (44) were not useful in solving (IP). Therefore we only report here experiments carried out with (IP) and Inequalities (36) and (37). Note that in the case where we added

Table 1: Characteristics of the instances.

Instance	Nr. turbines	Nr. nodes	Nr. arcs	UG Capacities	AG Capacities
1-H05	40	143	384	(6,13)	(8,19)
1-H10	40	143	384	(6,13)	(8,19)
1-H20	40	143	384	(4,6,8,13,20)	(5,8,13,19,24)
2-H05	33	64	160	(6,20)	(8,24)
2-H10	33	64	160	(6,20)	(8,24)
2-H20	33	64	160	(4,6,8,13,20)	(5,8,13,19,24)
3-H05	42	91	232	(6,20)	(8,24)
3-H10	42	91	232	(6,20)	(8,24)
3-H20	42	91	232	(4,6,8,13,20)	(5,8,13,19,24)

Table 2: Numbers of variables and constraints.

Instance	Variables (MQIP)	Constraints (MQIP)	Variables (IP)	Constraints (IP)
1-H05	30400	24720	65664	27632
1-H10	30400	24720	65664	27632
1-H20	38656	32976	129152	40016
2-H05	10680	8295	27068	9443
2-H10	10680	8295	27068	9443
2-H20	13728	11343	48912	14015
3-H05	15888	12816	38312	14504
3-H10	15888	12816	38312	14504
3-H20	20448	17376	66864	21344

Inequalities (37) to the model, we included all of these inequalities, even if they did not satisfy the hypothesis of Proposition 3.2. This means that in order to accelerate the resolution of the model, we were willing to include into it some inequalities that are not valid (i.e., they may remove all of the optimal solutions). On the other hand, the inclusion of Inequalities (36) always leaves at least one optimal solution. In each case we let our program run until it had run out of memory or proved that the current solution was optimal.

The results are displayed in Table 3. We compared three versions of our model: the linearized model (IP), the same model with Inequalities (36), and finally (IP) with Inequalities (36) and (37). The first column displays the instance label and the second column the best solution found. Note that for each of the first six instances, every version of our program found the same "best solution." This solution is not necessarily optimal and the third (resp. fifth, seventh) column contains an upper bound on the gap between that solution and the optimal solution. Note that solving (IP) enables us to find the optimal solution of Instances 1-H05, 1-H10, 1-H20, and 2-H10, and it is very likely that the solutions found by (IP) for 2-H05, 2-H20, and 3-H20 are also optimal. In the case of instances 3-H05, 3-H10, and 3-H20, our program ran out of memory before it found a feasible solution, except for the solution of 3-H20 by (IP) and the solution of 3-H10 by the model (IP) & (36). A "?" indicates that the program ran out of memory. The inclusion of Inequalities (36) either reduces the solution time to a large extent or increases it by a small percentage (with the exception of instance 2-H05); the inclusion of these inequalities also enables us to find an optimal solution of 3-H10. If we add both Inequalities (36) and (37) to the model, the solution time is reduced even further for the first two instances.

Table 3: Results of the experiments.

		Linear. Model (IP)		**Model (IP) & (36)**		**(IP) & (36), (37)**	
Instance	Best Sol.	Gap	CPU (s)	Gap	CPU (s)	Gap	CPU (s)
1-H05	2169201.8	0.0	41222	0.0	13427	0.0	9188
1-H10	2977032.8	0.0	23397	0.0	19120	0.0	4817
1-H20	3966841.8	0.0	215477	0.28	231724	0.55	29343
2-H05	983985.0	0.95	9706	0.77	25998	0.82	22914
2-H10	1298614.8	0.0	29	0.0	52	0.0	61
2-H20	1775457.6	0.22	83221	0.25	31180	0 12	266026
3-H05	?	?	2847	?	4762	?	5850
3-H10	2464129.0	?	4353	0.0	129275	?	3136
3-H20	3452628.6	0.38	87760	?	5279	?	2636

Because our model has many symmetries (i.e., many feasible solutions that are essentially equivalent), we expect that the program will take some time before finding a feasible solution, some more time to find a good solution, and a lot of time to prove that the best solution found so far is actually optimal. In Table 4 we present some information on the solutions found for the first six instances. The third column of this table contains the gap (in percentage) between the first feasible solution of (IP) found by the program and the best solution of (IP) found by the program. For instance there is a 3.9% gap between the first feasible solution for 1-H05 and the best solution found by (IP) for 1-H05. The fourth (resp. fifth) column gives the time spent by the program in order to compute the first feasible solution (resp. best solution) found by (IP). These times are denoted by T1 and T2, respectively. The other columns contain similar information for the two other versions of our program. Overall the information displayed in Table 4 demonstrates that our models find good solutions fairly quickly but that the program spends a lot of time trying to prove that the best solution found so far is indeed optimal.

5 Conclusion

In this article we have modelled a version of the collector system design problem as a quadratic mixed integer linear program, which we then transformed into an integer linear program. We have shown that solving the latter model with a commercial solver enabled one to find very good solutions, even when it is not possible to prove that the current solution is optimal. It seems that any mathematical programming model of this

Table 4: Times required to find good solutions.

		Linear. Model (IP)			**Model (IP) & (36)**			**(IP) & (36), (37)**		
Instance	Best Sol.	Gap	T1	T2	Gap	T1	T2	Gap	T1	T2
1-H05	2169201.8	3.9	1375	1527	21.2	76	1145	0.3	2550	2840
1-H10	2977032.8	12.4	1314	11812	12.8	313	760	8.3	242	3692
1-H20	3966841.8	2.1	963	1540	2.1	5023	19925	13.6	3210	6467
2-H05	983985.0	7.2	206	263	12.2	82	8224	4.7	63	489
2-H10	1298614.8	0.2	6	26	4.6	7	24	2.9	26	37
2-H20	1775457.6	1.1	84	97	1.9	89	402	10.2	61	276

problem will have the drawback of "symmetry," i.e., many of its feasible solutions will be equivalent. Further research is needed to mitigate this drawback, by eliminating solutions that are equivalent to those retained in the model. In particular introducing new constraints into the model or exploring different branching rules might enhance the performance of our approach.

References

[1] Baños, R., Manzano-Agugliaro, F., Montoya, F.G., Gil, C., Alcayde, A., Gómez, J. 2011. Optimization methods applied to renewable and sustainable energy: A review. Renewable and Sustainable Energy Reviews, 15(4), 1753–1766.

[2] Bauer, J., Lysgaard, J. 2015. The offshore wind farm array cable layout problem: A planar open vehicle routing problem. Journal of the Operational Research Society, 66, 360–368.

[3] Berzan, C., Veeramachaneni, K., McDermott, J., O'Reilly, U.-M. 2011. Algorithms for cable network design on large-scale wind farms. MSRP Technical Report, Massachusetts Institute of Technology, Cambridge, MA, USA.

[4] Coughlin, K., Murthi, A., Eto, J. 2014. Multi-scale analysis of wind power and load time series data. Renewable Energy, 68, 494–504.

[5] Donovan, S. 2006. An improved mixed integer programming model for wind farm layout optimisation. ORSNZ'06: Annual Conference of the Operational Research Society of New Zealand, 143–152.

[6] Dutta, S., Overbye, T.J. 2011. A clustering based wind farm collector system cable layout design. Proc. 2011 IEEE Power and Energy Conf. at Illinois, Feb. 2011.

[7] Dutta, S., Overbye, T.J. 2013. A graph-theoretic approach for addressing trenching constraints in wind farm collector system design. Proc. 2013 IEEE Power and Energy Conf. at Illinois, Feb. 2013.

[8] Fagerfjäll, P. 2010. Optimizing Wind Farm Layout – More Bang for the Buck Using Mixed Integer Linear Programming. Master's Thesis, Department of Mathematical Sciences, Chalmers University of Technology and Gothenburg University, Göteborg, Sweden.

[9] Gendron, B., Crainic, T., Frangioni, A. 1999. Multicommodity capacitated network design. Telecommunications Network Planning 4 (P. Soriano and B. Sansò, eds.), Kluwer Academic Publishers, 1–19.

[10] Hertz, A., Marcotte, O., Mdimagh, A., Carreau, M., Welt, F. 2012. Optimizing the Design of a Wind Farm Collection Network. INFOR, 50, 95–104.

[11] Kiranoudis, C.T., Voros, N.G., Maroulis, Z.B. 2001. Short-cut design of wind farms. Energy Policy, 29(7), 567–578.

[12] Lumbreras, S., Ramos, A. 2011. A Benders' decomposition approach for optimizing the electric system of offshore wind farms. PowerTech, 2011 IEEE Trondheim, 1–8.

[13] Magnanti, T.L., Wong, R.T. 1984. Network design and transportation planning: models and algorithms. Transportation Science, 18, 1–55.

[14] Minoux, M. 1989. Network synthesis and optimum network design problems: models, solution methods and applications. Networks, 19, 313–360.

[15] Ozturk, U.A., Norman, B.A. 2004. Heuristic methods for wind energy conversion system positioning. Electric Power System Research, 70(3), 179–185.

[16] Pérez, B., Mínguez, R., Guanche, R. 2013. Offshore wind farm layout optimization using mathematical programming techniques. Renewable Energy, 53, 389–399.

[17] Samorani, M. 2014. The Wind Farm Layout Optimization Problem. Handbook of Wind Power Systems – Energy Systems 2013, 21–38.

[18] Son, E., Lee, S., Hwang, B., Lee, S. 2014. Characteristics of turbine spacing in a wind farm using an optimal design process. Renewable Energy, 65, 245–249.

[19] Tesauro, A., Réthoré, P.-E., Larsen, G.C. 2012. State of the Art of Wind Farm Optimization. Proceedings of EWEA 2012 – European Wind Energy Conference & Exhibition. European Wind Energy Association (EWEA).

[20] Trötscher, T., Korpås, M. 2011. A framework to determine optimal offshore grid structures for wind power integration and power exchange. Wind Energy, 14(8), 977–992.

[21] Turner, S.-D.O., Romero, D.A., Zhang, P.Y., Amon, C.H., Chan, T.C.Y. 2014. A new mathematical programming approach to optimize wind farm layouts. Renewable Energy, 63, 674–680.

[22] Yeh, T.-M., Huang, Y.-L., 2014. Factors in determining wind farm location: Integrating GQM, fuzzy DEMATEL, and ANP. Renewable Energy, 66, 159–169.