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An Output Orientated Non Parametric Measure of Economies of Scope

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Abstract

Economies of scope are present when producing jointly a diversified basket of goods and services is less costly than their separate production. Data Envelopment Analysis (DEA) can estimate economies of scope by the difference between input requirement sets of diversified and specialized firms and are applicable only when both specialized and diversified firms are present in the sample. We develop a measure of economies of scope to an output orientated DEA model even when the sample comprises only diversified firms, to obtain radial estimation of economies of scope. Our method puts in evidence that economies of scope are influenced by scale inefficiencies, and if these inefficiencies are left aside, diseconomies of scope are impossible.

Keywords : Economies of scope, DEA.
JEL : C14, D24.

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INTRODUCTION

Economies of scope exist when producing jointly a variety of goods and services costs less than having them produced separately. This concept has become increasingly important namely in banking, particularly to understand if regulation changes allowing financial institutions to offer a more diversified output have led to more efficient output choices. Indeed, scope economies are crucial in evaluating the performance of multi-products firms, and for deciding if lines of products should be maintained or abandoned. Empirical evaluation of scope economies is often based on parametric estimation of a cost function, the translogarithmic forms being frequently a best choice because of its ability to approximate most types of function.[1] One caveat of this method is that, in order to keep tractable the number of parameters to be estimated, the translog can deal only with a small number of outputs and inputs, thus limiting its ability to cope with the highly diversified output of some industries, like banking for instance.[2] Moreover, since parametric estimation do not deal with cost differences arising because of inefficiencies, there is a need to develop an alternative based on a frontier approach to adjust firms' performance to take account this effect.

Because of its ability to accommodate a wide range of goals and activities with few technological restrictions, Data Envelopment Analysis (DEA) has become the most popular method for estimating efficiency. This non parametric method developed by Charnes, Cooper and Rhodes (1978) (thereafter CCR) identifies by linear programming the best performing peers to determine a firm's highest achievable results on the frontier, the firm's "target". The distance between the actual performance and the target provides a basis to estimate the degree of inefficiency. Besides its easiness of use, DEA is popular also because it allows to make diagnostics as to the causes to inefficiencies. For example, given that data on input and output prices are available it is possible to distinguish between technical and allocative inefficiencies. Also, since Banker, Charnes, and Cooper (1984), the variable returns to scale DEA model (known as the BCC model) makes possible to determine how the scale of operation hinders the

[1] The translogarithmic is sometimes used in its pure form or some after some transformation has been made. See for example Clark and Speaker (1994) or Mitchel and Onvural (1996)
[2] Is k is the number of variables in the cost model, the number of parameters is $k(k\text{-}1)/2 + 2k + 1$.

firm's performance. This allows to decompose technical inefficiencies into scale inefficiencies and pure technical inefficiencies.

Two methods have been proposed to adapt DEA to evaluate how a larger spectrum of output impacts on cost. Ferrier and al. (1993) introduce the concept of economies of diversification, which differ from economies of scope in the fact that at least one output is produced by all firms. They estimate the cost advantage made possible by output diversification by comparing the input requirement set of diversified firms with a calculated additive cost frontier of firms that do not produce some outputs. Morita (2003) deals directly with economies of scope by estimating separately the frontier first on a sample comprised only of diversified firms then to a second sample constructed by adding up the inputs required by specialized firms. Comparison of both input requirement sets allows to identify if diversified firms require less or more inputs. However, in order to apply these procedures, one needs data coming from both specialized and diversified firms. They are then not applicable when the sample contains only multi-outputs firms. Our goal is to develop a non parametric method that can complement the efficiency scores obtained from DEA to estimate economies of scope in any type of samples.

To achieve this goal, we will adopt an approach based on output expansion allowed by output diversification. In the next section we show that with constant returns to scale and without economies of scope, the production possibility curve would be a linear convex combination of specialized firms' output. A direct consequence is that economies of scope is then proportional to the relative distance between the actual frontier and this linear convex combination. In the third section, we develop a measure of economies of scope based on a proportional expansion of outputs that we call a radial measure of scope economies. The fourth section shows that the method still applies with variable returns to scale if firms' output is adjusted for the scale efficiency of the production. It will also pinpoint the fact that as usually defined, scope economies are potentially distorted by scale efficiency and that if all firms were scale efficient, diversification of output would never be cost increasing. In the fifth section we illustrate the methodology with a sample collected with Canadian Credit unions.

ECONOMIES OF SCOPE, THE PRODUCTION POSSIBILITY CURVE AND ECONOMIES OF SCALE

Suppose a multi-output and multi-input production function $F:\mathbb{R}_+^n \to \mathbb{R}_+^m$ written $y = F(x)$, where $y \in \mathbb{R}_+^m$ is the efficient output and $x \in \mathbb{R}_+^n$ the input. Under the definition of Panzar and Willig (1981), economies of scope are present if the joint production of these *m* outputs is less costly than producing them separately. Let us suppose that the input price vector *w* is given so that we can neglect input price as argument of the conditional demand for factors of a profit maximizing firm. We define this function as $x = x(y)$ so that the cost of y is $C(y_1, y_2, \cdots, y_m) = w'x$. If the associated cost function is $C(y_1, y_2, \cdots, y_m)$, there are overall economies of scope if :

$$C(y_1, y_2, \cdots, y_m) < C(y_1, 0, \cdots, 0) + C(0, y_2, 0, \cdots, 0) + \cdots + C(0, 0, \cdots, y_m) \qquad (1)$$

Conversely, there are overall diseconomies of scope when the inequality is reversed, that is, when the joint production costs more than the same production made by specialized firms. Following Baumol, Panzar and Willig (1982), economies of scope *s* is the proportional cost difference between a diversified output mix and the same production made by separate firms, that is :

$$s = \frac{\left[C(y_1, 0, \cdots, 0) + C(0, y_2, 0, \cdots, 0) + \cdots + C(0, 0, \cdots, y_m)\right] - C(y_1, y_2, \cdots, y_m)}{C(y_1, 0, \cdots, 0) + C(0, y_2, 0, \cdots, 0) + \cdots + C(0, 0, \cdots, y_m)} \qquad (2)$$

Reciprocally we could also say that economies of scope can be measured by the proportional output gain resulting from producing simultaneously many products rather than separately, given a constant factor cost. This is the approach we will take and in order to make that possible we shall characterize economies of scope in terms of production rather than cost.

For simplicity, let assume for now that there are only two goods y_1 and y_2 and that the production function is homogeneous of degree 1, that is, there are constant returns to scale (CRS), assumptions that we will later relax to deal with a more general case. We suppose that each firm (or DMU for Decision Making Unit) has the option of specializing its production in any output or to produce a diversified output mix while keeping cost constant. If we define $z_j, j = 1, 2$ as the technically efficient specialized production of good 1 and 2, that implies :

$$C(z_1, 0) = C(0, z_2) = w'x \,. \tag{3}$$

Since the DMU can also diversify its production, let consider that the firm seeks to produce an interior linear convex combination $(\lambda_1 z_1, \lambda_2 z_2)$ with $0 < \lambda_1 < 1$ and $\lambda_1 + \lambda_2 = 1$. Note that with the usual hypothesis that the production set is convex, this combination is feasible. Applying (1) implies that there are economies of scope if $C(\lambda_1 z_1, \lambda_2 z_2) < C(\lambda_1 z_1, 0) + C(0, \lambda_2 z_2)$. With CRS, cost is proportional to output so that we can write, $C(\lambda_1 z_1, 0) = \lambda_1 C(z_1, 0)$ and $C(0, \lambda_2 z_2) = \lambda_2 C(0, z_2)$. From (3) these equalities become $C(\lambda_1 z_1, 0) = \lambda_1 w'x$ and $C(0, \lambda_2 z_2) = \lambda_2 w'x$. Finally, because $\lambda_1 + \lambda_2 = 1$ economies of scope imply that $C(\lambda_1 z_1, \lambda_2 z_2) < w'x$. It other words, the interior linear convex combination $(\lambda_1 z_1, \lambda_2 z_2)$ requires less inputs than any specialized production. Therefore, given a constant input cost, any such diversified mix is inefficient and an output expansion is feasible.

This simple result requires some discussion. First, it must be clear that with CRS, the usual hypothesis of a convex feasible set makes it impossible to have diseconomies of scope. Moreover, economies of scope are necessarily present if the feasible set is strictly convex. In order words, the usual hypothesis made in analyzing production leads to the conclusion that output diversification reduces cost. This is in fact the basic result in micro textbook that since specialization becomes, at the margin, too costly, a diversified basket is preferable.

Our comparison between diversified and specialized firms differs from that of Panzar and Willig on the size at which firms operate when specializing of diversifying their output. Economies of scope compare costs of two diversified firms with those of a merge of these two firms into a single one producing simultaneously both outputs. Necessarily, the diversified firm must operate at a larger input scale than each specialized firm. In contrast, we make our comparison with firms operating at the same input cost and we convert the technology adopted by specialized firms to a smaller scale to apply the definition of Panzar and Willig. Under CRS assumption, this reduction in scale causes a proportional reduction of inputs, and the conclusion that a strictly convex feasible set implies scope economies.

Defining scale is somewhat ambiguous here. Although per-unit cost reduction is a by-product of diversification, some of cost economies realized by the diversified firm may occur because of different input scale. Indeed, the arguments developed by Panzar and Willig suggest that: *"It is intuitively appealing to link economies of scope to the existence of sharable inputs; that is, inputs which, once procured for the production of one output, would also be available (either wholly or in part) to aid in the production of other outputs."*[3] Examples they give is power generation or building that can be made available for more than one product. Intuitively, power generation or building have a minimum efficient scale so that the procurement for only one line of product may exceed the needs so that firms have excess capacity available to use in another product. For instance, we argue here that diseconomies of scale are necessary to explain the existence of specialized firms with a strictly convex feasible set. Indeed, we showed that under CRS, strict convexity implies that output diversification is less costly. So let suppose that, instead of CRS there are increasing returns to scale. Then, a larger diversified firm can benefit at the same time of both scale and scope economies, to that its cost advantage is amplified over specialized firms. Thus, in order for specialized firms to be competitive over large diversified ones which benefits from scope economies, it requires that operating at a smaller scale generate benefits from specialization. Consequently, diseconomies of scale must be present to have specialized firms.

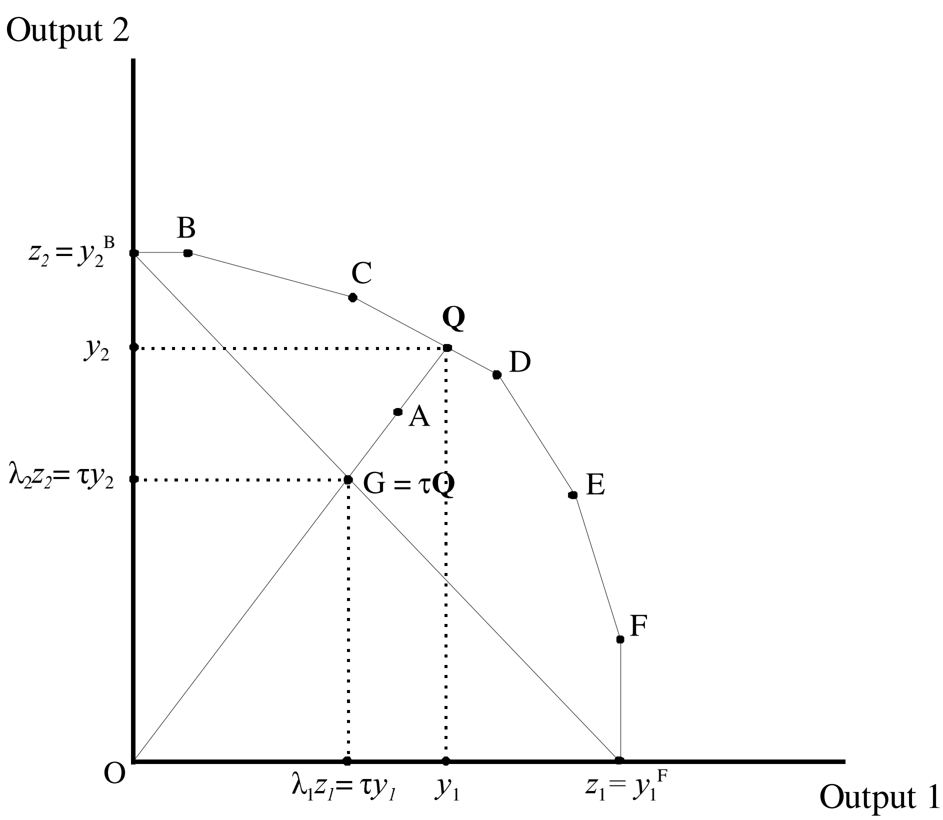
It is clear that comparing the costs of specialized firms with those of diversified firms will depend crucially on the treatment given to economies of scale. We will further discuss the issue in a subsequent section. Before doing that however, we will left aside considerations regarding economies of scale to illustrate, in the case of constant returns to scale, how we can measure economies of scope by output expansion.

[3] Panzar, J. C. and R. D. Willig (1981), Economies of Scope, *The American Economic Review*, 71(2), p. 269.

ESTIMATING ECONOMIES OF SCOPE WITH CONSTANT RETURNS TO SCALE

Let consider an efficient output mix (y_1, y_2) proportional to $(\lambda_1 z_1, \lambda_2 z_2)$. If τ is the proportion between the linear convex combination and the efficient level of production, we can write $(\lambda_1 z_1, \lambda_2 z_2) = \tau(y_1, y_2)$. As stated above, economies of scope imply that the efficient output exceeds the linear convex combination, that is, τ is lower than one. Its value shows how the average output of specialized firms compares with that of a diversified one. It can be called a <u>radial measure of scope economies</u>. It is equivalent to measuring how the average cost of production of a diversified firm compares with that of specialized firms. Obviously, there is neither economies nor diseconomies of scope if $C(\lambda_1 z_1, \lambda_2 z_2) = w'x$. It remains to show how to measure τ.[4]

Figure 1: Measure of economies of scope



[4] We fall short in describing a case for diseconomies of scope with CRS since that would not be consistent with a convex set of possibility of production.

Consider in figure 1 the hypothetical case of 6 DMUs designed by A, B, C, D, E and F all producing y_1 and y_2 in different proportions with the same amount of a single factor x and let assume that we analyse their performance with an output-orientated DEA model.[5] Given the usual hypothesis of a convex set, the DEA frontier is composed of linear segments passing between the 5 efficient firms B, C, D, E and F. The frontier is completed by applying the hypothesis of free disposition to those closest to the axis.[6] The highest proportion of good 1 is made by firm F, which indicates a maximum production of the first good equals to y_1^F while firm B has the highest proportion of the second good at a level y_2^B. Thus, the frontier intersects the axis of goods 1 and 2 at points $z_1 = y_1^F$ and $z_2 = y_2^B$ which represent the maximum production of goods 1 and 2 if all resources are devoted to only one good. As to observation A, it is below the frontier and is therefore inefficient. Since a proportional expansion of all outputs is possible for firm A, it can reach a level of production (y_1, y_2) at point Q on the frontier, so that its score of radial efficiency is the ratio OA/OQ.

Now we infer how economies of scope benefit to diversified firms by comparing a proportional expansion of outputs with respect to a linear convex combination of specialized firms' production. Geometrically the linear convex combination, the point G on figure 1, is located at the intersection of the segment between points $(z_1, 0)$ and $(0, z_2)$ with the ray passing trough point Q. We found the coordinates of Q by solving the set of equations given by $(\lambda_1 z_1, \lambda_2 z_2) = \tau(y_1, y_2)$, that is, $\tau y_1 = \lambda_1 z_1$ and $\tau y_2 = \lambda_2 z_2$ subject to $\lambda_1 + \lambda_2 = 1$. Note that our goal being to measure economies of scope, we are interested in solving this set of equations only for the value taken by τ. If we isolate λ_1 and λ_2 in each equation, so that $\lambda_1 = \tau\, y_1/z_1$ and $\lambda_2 = \tau\, y_2/z_2$, and we sum these two equalities we obtain $\lambda_1 + \lambda_2 = \tau\, y_1/z_1 + \tau\, y_2/z_2$. But since $\lambda_1 + \lambda_2 = 1$, this equality becomes $1 = \tau\, y_1/z_1 + \tau\, y_2/z_2$. Finally, isolating τ gives :

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[5] Since we assume CRS, we can generalize to a situation in which DMUs do not use the same level of input. Indeed, a similar figure could be traced with the output divided by input cost so that the production would be measured as the number of units of output obtained for spending one dollar in inputs. This is what we do in the empirical part.

[6] As is well known, there is output slack at points $(z_1, 0)$ and $(0, z_2)$. Thus, considering these positions as efficient specialized peers underestimate the maximum specialized production and overestimate economies of scope. However, without information regarding the rate of transformation for firms more specialized than B or F, it is not possible to infer the real position of completely specialized firms.

$$\tau = \frac{1}{y_1/z_1 + y_2/z_2} \tag{4}$$

Geometrically, τ is the ratio OG/OQ in figure 1, thus the name radial measure of economies of scope. This measure is easy to calculate with the CCR model. As an illustration, consider that in figure 1, the efficient level of production at point Q is $y_1 = 6$ and $y_2 = 8$ while the intercepts are $z_1 = 9$ and $z_2 = 9$. Then $\tau = 1/(6/9 + 8/9) = 9/14$, that is, the cost of producing simultaneously in the same proportions as A is only 64.3% of the cost of the separate production. Or otherwise, we could say that because of output diversification A's production can expand in a proportion 14/9 = 1.556, that is, an increase of 55.6% over the production by specialized firms.

The generalization to *m* products is straightforward. Let define *z* and *y* as *m*-component vectors representing respectively the maximum specialized production of each output and the efficient production of a diversified output mix. The linear convex combination of specialized output is an hyperplane in $\mathbb{R}_+^m$ whose equation is given by $\lambda' z$, where λ is a *m*-component non-negative vector of weights such that $\lambda' e = 1$, with e a vector of one. The intersection between the hyperplane and a bundle with an output mix identical to *y* is found by solving the system of equations $\tau y = \lambda' y$, with τ the proportionality constant. Summing all *m* equations and using the condition $\lambda' e = 1$ give the following solution for τ:

$$\tau = \frac{1}{\sum_{j=1}^{m} \left(y_j / z_j \right)} \tag{5}$$

ADAPTING THE PROCEDURE TO VARIABLE RETURNS TO SCALE

Our comparison points of specialized firms operate at a larger scale than those in Panzar and Willig's definition of scope economies. With CRS, this difference in input use does not modify their performance but this is no longer true with variable returns to scale (VRS) since size becomes a cause to cost discrepancies. For each firm having increasing or decreasing return to scale, relaxing the hypothesis of CRS makes necessary to adjust its output or input in order to cancel the impact of scale, that is, we must consider if they are scale inefficient. Our strategy here is to find a productivity transformation which cancels the scale inefficiency so that the transformed relationship between input and output is scale invariant. In doing so, we will make possible to apply the procedure of the previous section not to the initial output and input but rather to a transformed space which satisfy the CRS hypothesis.

Let express the production of firm *i* as $y_i = F(x_i)$ and suppose that, in order to minimize long run cost, a firm must operate at the level of input x_0 so that the optimal long run output level is $y_0 = F(x_0)$. Locally, the scale-efficient firms operate with constant returns to scale while smaller (larger) firms face increasing (decreasing) returns to scale and are thus scale inefficient. Now, we define an output scaling factor β_i and an input scaling factor θ_i, indicating how output and input of firm *i* differ from their scale-efficient levels, such that $y_i = \beta_i y_0$ and $x_i = \theta_i x_0$.[7] This implies that production of firm *i* can be written:

$$\beta_i y_0 = F(\theta_i x_0) \tag{6}$$

By definition of the efficient scale, any move away from the optimal size will cause an output change that cannot be proportionally more than input's, that is the inequality $\beta_i \leq \theta_i$ must prevail, while $\beta_i = \theta_i$ when returns to scale are constant. The scale efficiency of firm *i*, noted SE_i, is the ratio between the maximum productivity at its actual scale and the maximum productivity if it adopted an efficient scale, that is :

$$\frac{(y_i/x_i)}{(y_0/x_0)} = \frac{\beta_i}{\theta_i} = SE_i \tag{7}$$

[7] For a thorough discussion of the link between scale efficiency and returns to scale, see Førsund (1996). It is important here to keep in mind that *y* is not the actual output but its projection on the frontier at the actual scale, that is, the pure technically efficient level of production, although not necessarily scale-efficient.

Now let us define a "virtual" output y_i^* adjusted for the impact of size on firm i's performance[8], such that $y_i^* = y_i / SE_i = (\theta_i/\beta_i) y_i$, and a production function $G(\bullet)$ indicating the relation between y_i^* and input, that is, $y_i^* = G(x_i)$, or otherwise $(\theta_i/\beta_i) y_i = G(x_i)$. Substituting y_i by $\beta_i y_0$ and x_i by $\theta_i x_0$ gives $\theta_i y_0 = G(\theta_i x_0)$, which shows that $G(\bullet)$ has constant returns to scale. Therefore, the procedure described in the previous section to estimate economies of scope can be applied if we transform the output for scale efficiency.

Measuring scale efficiency is now a standard procedure with DEA. First, an output orientated VRS model is estimated which assign to each firm an efficiency score e_i^{VRS}. If actual output is $y_i^{'}$ then $e_i^{VRS} = y_i^{'}/y_i$ (the ratio OA/OQ in figure 1), that is, the proportion between firm i's actual productivity $y_i^{'}/x_i$ and its maximum productivity y_i/x_i at its actual input scale. Then an output orientated CRS model is estimated to get a different efficiency score e_i^{CRS}. This new score indicates the ratio between the actual productivity $y_i^{'} = y_i/x_i$ and the maximum productivity y_0/x_0 allowing for optimal scale adjustment, which is equal to $e_i^{CRS} = (y_i^{'}/x_i)/(y_0/x_0)$. Multiplying and dividing by y_i and substituting equation (7) implies that $e_i^{CRS} = (y_i^{'}/y_i)\bullet SE_i$. If $y_i^{'}/y_i$ is replaced by e_i^{VRS}, it is straightforward to show that the scale efficiency is equal to the ratio of both efficiency scores, that is, $SE_i = e_i^{CRS}/e_i^{VRS}$.

Now let consider that $y_i = y_i^{'}/e_i^{VRS}$ so that $y_i^* = (y_i^{'}/e_i^{VRS})/SE_i$. If we substitute SE_i by e_i^{CRS}/e_i^{VRS} it is possible to cancel e_i^{VRS} so that the transformed production function $y_i^* = G(x_i)$ becomes:

$$y_i^{'}/e_i^{CRS} = G(x_i) \tag{8}$$

Thus, a better solution is simpler still. An output oriented CRS model applied on non transformed data allows to find the efficient level of output y_i^* on which we can base the

[8] Indifferently we could choose not to transform output but to create instead a virtual input vector $x_i^* = (\beta_i/\theta_i) x_i$.

calculation of economies of scope as described in the previous sections. If SE_{z_j} is the scale efficiency of the technically efficient specialized production of good j and we define $z_j^* = z_j \big/ SE_{z_j}$ then economies of scope with VRS can be calculated with this generalization of equation (5):

$$\tau_i = \frac{1}{\sum_{j=1}^{m} \left(y_{ij}^* \big/ z_j^* \right)} = \frac{1}{\sum_{j=1}^{m} \left(\frac{y_{ij} \big/ SE_i}{z_j \big/ SE_{z_j}} \right)} \tag{9}$$

where y_{ij}^* indicates the scale and technically efficient production of good j by firm i.

We must point out that despite the adjustment for scale efficiency, the value of τ_i cannot once again be greater than one. Indeed, since the projections in the CRS space forms a convex set, diseconomies of scope are still impossible. Note also that, contrary to the CRS case, the measure provided by equation (9) is not directly compatible with Panzar and Willig's definition. This is so because specialized firms operate at a different input level in our measure than in their definition, and that may change their scale efficiency. Since there is no observation on specialized firms producing each product at the level produced by the diversified firm i, we cannot measure their scale efficiency so that the appropriate adjustment cannot be done.[9] It must be stated again, may be with more precision now that we have described the role of scale efficiency, that the kind of output diversification envisioned by Panzar and Willig, and more generally all measure of cost subadditivity, involve a change in the level of input, and therefore a potential change in the scale efficiency when the DEA model has an output orientation. In other words, any measure of scope economies compatible with the definition presented in equation (1)

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[9] If the production of firm i is the m-tuple $(y_{i,1}, y_{i,2}, \cdots, y_{i,m})$ we would need to calculate the scale efficiency of m firms producing $y_{i,j}$ for $j = 1, 2, \cdots, m$ and to repeat that for each firm to adapt the calculation to the definition of Panzar and Willig. Some might argue that because our measure of output gain from diversification is not directly compatible with this definition, we make an improper use of the term economies of scope. However, we were not able to find another term that seem appropriate given that the term economies of diversification as another meaning,. We must point out that the methods of Ferrier & al and Morita are also non directly compatible with the formal definition of economies of scope, since specialized firms used to calculate the input requirement set do not operate at the appropriate scale. No discussion is made as to the possible bias that can be involved.

is affected by the possible difference between the scale efficiency of the diversified larger firm and those of specialized firm. We suggest here that since scale efficiency is easily measured with DEA, it may be more appropriate to disentangle cost savings from adjusting the number of outputs from cost savings caused by an expansion of input. Our procedure makes this separation.

Although the presentation of the procedure and the accompanying discussion are a bit tedious, the procedure we propose is very easy to implement. In order to illustrate this, we applied it in the next section on data for a network of small Canadian credit unions.

AN ILLUSTRATION WITH BANKING DATA

Our data were provided by two Canadian credit unions networks: The *Fédération des caisses populaires Desjardins* operating in the province of Quebec and the *Fédération des caisses populaires acadiennes*, in the province of New Brunswick. These two federations were operating together in 2005 a network of over 500 local credit unions. The use of this sample will present the additional advantage of making it possible to compare the result of the method we propose with a parametric estimation of scope economies. Indeed, using a similar database, Fortin, Leclerc and Thivierge (2000) proceeded to the estimation of a translogarithmic cost function and found significant overall economies of scope, the separate production being 88% more expensive than the joint production.[10]

Since the main purpose of this text is the development of a methodology to measure economies of scope in a non-parametric setting, we select only in this section a small number of inputs and outputs to keep the model simple. The credit unions are placed in a setup where they produce 4 outputs using only 2 inputs as presented in table 1. Our database was at the beginning made up of all the caisses populaires operating in 2005. However, a small number has been eliminated because of exceptional circumstances that cannot be reproduced (for example, some had access to free commercial space). Our final database is thus composed of 500 credit unions whose assets

[10] Fortin, M., A, Leclerc and C. Thivierge (2000), «Économies d'échelle et de gamme dans les Caisses Desjardins», *L'actualité économique* 76(3), 393-421.

vary from $3.9 millions (Canadian $) to $863.3 millions. Data for the measurement of these variables come mainly from Desjardins' information system.[11]

Table 1 : List of variables

Inputs	**Unit of Measurement**
1- Labour	Number in full-time equivalent
2- Buildings and equipments	Insured value
Outputs	**Unit of Measurement**
1- Personal Loans	Value
2- Business Loans	Value
3- Investments	Value
4- Transactions	Number

On the inputs side, labour is measured by total wages and other employees' cost (social security, insurances and other advantages) while capital is the expenditures on the exploited buildings and on equipments. Our decision to measure input not in physical quantity but rather by their expenditure comes from the necessity to identify the most specialized units as those that produce the most for each input used. In a multiple inputs setting, the aggregation of many inputs into a single number would have as many solutions as the possible weight given to each input. Since the most natural solution for profit maximization consists in weighting by the unit cost, this is what we impose in solving for input expenditures.

Outputs are grouped in 4 different financial services. Personal loans (1st output) combine three types of loans, that is mortgage loans, fixed term personal loans and variable credit (credit margins). Business loans (output #2) include commercial loans to business and institutional such as churches, municipalities and so on. Investments (output #3) represent local credit unions deposit to the Federation when the demand for credit is too low to absorb all funds available for loans. Loans and investments are measured as the average book value using a twelve months average. A yearly average is the best way to connect the quantity of the variable input, measured

[11] Since the two federations have historically developed major business relations, they use the same information system.

on a trimester basis, and loans negotiated during the year with the co-operative members. Finally, all types of transactions (manual or automated for businesses or households) are grouped in one product (output 4).[12] Appendix 1 presents descriptive statistics on these variables.

To apply the method we estimate an output oriented DEA models with CRS and use the projection of the CRS model to generate the efficient production of each firm, adjusted for its scale efficiency. Although not strictly necessary, we estimate also a VRS model in order to have a value of each unit's scale efficiency.[13] This value will provide an indication of the impact of *SE* on the calculated economies of scope.

After the model has been estimated, these projected values were divided by the sum of expenditures on labour and capital, to get the efficient production of each output for one dollar spent on input. Table 2 presented in the first rows these projections for four randomly selected credit unions, named A, B, C and D (the full table has 500 rows). The identification of the highest output j for one dollar spent is the next step. These reference units are presented in the last four rows. The highest production of output 1, indicated in the row identified by z_1 is \$59.62 per dollar spent in inputs. This is produced simultaneously with \$0.02 of output 2, \$4.98 of output 3 and 0.669 transactions, all valued that are indicted is smaller character between brackets. These real values are projected on the axis by assuming no production of outputs 2, 3 and 4, thus the value 0 indicated in the same cells were the real values are indicated. By selecting the highest production of output 2 (\$40.04), 3 (\$39.55) and 4 (0.95), we complete the identification of the four specialized z_i.

The last column indicates the scale efficiency. In these cases, units A to D have scale efficiency scores that vary between 0.946 and 0.999 while all four specialized firms are scale efficient. We must point out here that the fact that these references units are scale efficient is an accident, since they could as easily be scale inefficient. What happens here is that most units have almost scale

[12] Automated transactions comprise: check payments, direct withdrawals and deposits, banking machine withdrawals and transfers, wage deposits, debit card payments, AccèsD (trademark for online services) invoice payments and transfers, and a number of fixed-cost transactions. Manual transactions include: withdrawals and deposits at the teller, invoice payments and deposits at the automated teller machine, invoice payments with AccèsD with personnel intervention and processing files (opening an account, mortgage renegotiation...).

[13] For our estimations, we used the CCR output oriented and BCC output orientated routines of DEA-Solver-PRO version 4.1made by Saitech.

efficient, since the average is 98.7% while the median is 99.8% and 43 (8.6%) have a score of 1. Only 10 (2%) have a scale efficiency score smaller than 0.90, the minimum score being 0.724. As we indicated in the preceding section, it is only with highly scale inefficient specialized units that our calculation of scope economies may differ from the measure suggested by Baumol, Panzar and Willig. Given the scale efficiency scores are so close to one, this difference cannot be important.

Table 2 : Projection of the CRS model per dollar of input

DMU	Output 1	Output 2	Output 3	Output 4	SE_i
A	27.8263	20.2336	11.2982	0.5737	0.9904
B	20.9992	27.6098	4.5200	0.5785	0.9993
C	19.3242	22.2310	5.4256	0.6245	0.9643
D	26.7190	4.0854	3.0220	0.8721	0.9460
z_1	**59.6231**	0 (0.0229)	0 (4.9755)	0 (0.6690)	1.0000
z_2	0 (13.6331)	**40.0418**	0 (4.0864)	0 (0.3890)	1.0000
z_3	0 (52.7448)	0 (2.4606)	**39.5525**	0 (0.3062)	1.0000
z_4	0 (24.8604)	0 (3.1478)	0 (2.5196)	**0.9506**	1.0000

Table 3 shows the calculated value of economies of scope. In each column we calculated the ratio between the actual production of each good and the maximum specialized production, that is, the ratio y_{ij}^* / z_j^* and, in the last column, the inverse of the sum of these ratios. For the 4 units used to illustrate the results, τ_i varies between 0.5372 and 0.6477. In the total sample, the scores of scope economies range from a minimum of 0.441 to a maximum of 0.766 with an average of 0.586. This average indicates that complete specialization would reduce production by 41.4%. Therefore, complete specialization would increase per-unit cost by 70.6%.[14]

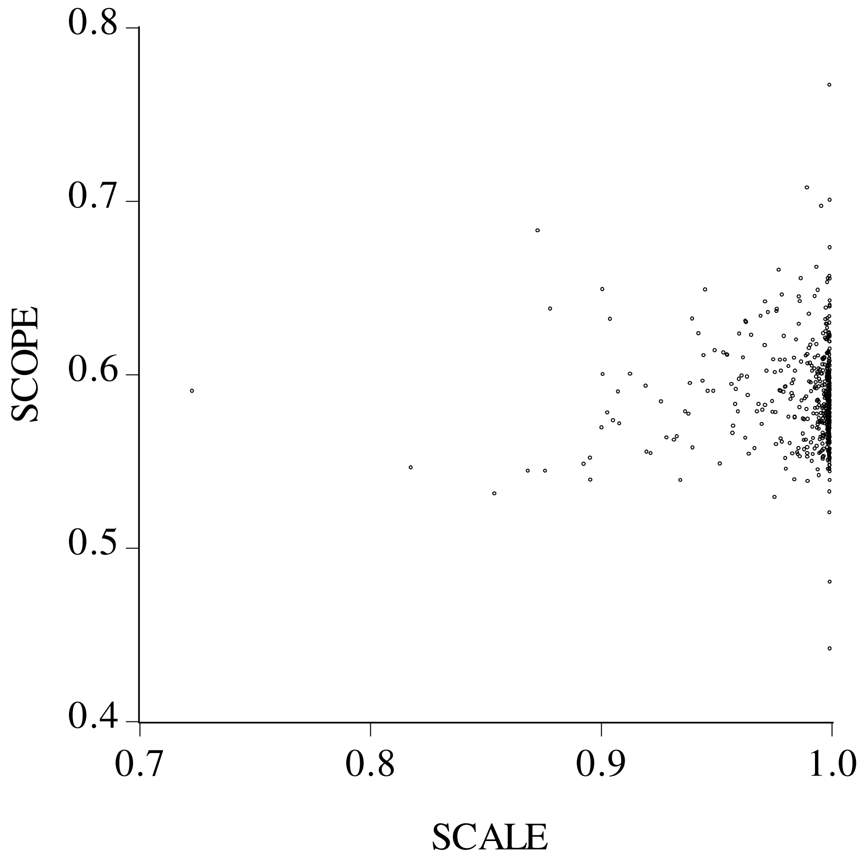
[14] This value is not too different from the 88% cost increase found in Fortin, Leclerc and Thivierge (2000). Note however that the results cannot be directly compared because even if the data are from the same organization, the sample has changed considerably between the year 1997 used in Fortin, Leclerc and Thivierge and the actual sample of 2005. This changes occur mainly because a wave of mergers has cut in half the number of local credit unions. Also, we do not have exactly the same definition of products. Finally, around 84 of local credit unions, usually the smallest, have delegated part of commercial financial services to subsidiaries called Financial Center for Businesses. Partner credit unions in those subsidiaries share cost in proportion to loans financed with each credit union's liquidities and the cost and activities of the Financial Centers are integrated in those credit union's data.

Table 3 : Calculation of economies of scope

	y_{i1}^{*}/z_{1}^{*}	y_{i2}^{*}/z_{2}^{*}	y_{i3}^{*}/z_{3}^{*}	y_{i4}^{*}/z_{4}^{*}	$\tau_i = 1/\sum_{j=1}^{m}(y_{ij}^{*}/z_{j}^{*})$
A	0.4667	0.5053	0.2857	0.6035	0.5372
B	0.3522	0.6895	0.1143	0.6086	0.5667
C	0.3241	0.5552	0.1372	0.6570	0.5976
D	0.4481	0.1020	0.0764	0.9174	0.6477

As a last information, we present in figure 2 the relation between scale efficiency and scope economies scores. No correlation can be found between both measures. The more scale inefficient units have scope economies close to the average value while the highest and lowest scores of scope economies are for (almost) scale efficient units. This indicates that extreme values of scope economies are not explained by unusually scale inefficient units.

Figure 2 : Scale efficiency and scope economies



CONCLUSION

We have developed a measure of economies of scope based on the efficient production as projected by a constant return to scale output orientated DEA model. The method is straightforward to use and can be applied by simply using a spreadsheet to calculate the output gain made possible by output diversification over a linear convex combination of specialized production. An important part of our paper is the role of returns to scale in measuring scope economies. Indeed, we have showed that diseconomies of scope are impossible with a CRS technology. We have also demonstrated that in a context of variable return to scale, it is not possible to use the usual definition of economies of scope without having to deal with the scale efficiency of the DMU that are compared. Since parametric measures do not address the question of efficiency, they cannot help much in solving the issue, while others non parametric methods, based on a comparison of input requirement sets, do not discuss the question of scale efficiency. On the contrary, our approach indicate how scale efficiency theoretically modifies the results. Given that the information on the cost of specialized production is usually not available, we suggest that it is preferable to use a measure of scope economies that is independent of scale efficiency.

In developing the method, we use the hypothesis of free disposal to project the production of the most specialized units to complete specialization, resulting in an overestimation of the scope economies proportional to the output slack of most specialized DMU. Although this bias is an inconvenient, it must be balanced with the limits encountered by other methods since there is no elegant solution to this problem. Without truly specialized units, the method based on the comparison of input requirements of specialized and diversified DMU cannot be applied. In the same context, parametric estimation of a cost function can be used but the projection of the properties of the functional form outside the range of data is susceptible to create an approximation bias of unknown importance. Illustration with banking data found that complete specialization would allow to produce on average only 58.6% of the output obtained with output diversification, a result of the same order of magnitude than one obtained with a similar sample when estimating a translog cost function. This illustration should not be used as a position we take on the way banking production should be measured since our paper's purpose is mainly methodological.

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APPENDIX 1: DESCRIPTIVE STATISTICS

	Mean	Standard deviation	Minimum	Maximum
Total expenses on labour	2,169,607	1,871,014	75,018	11,774,056
Total expenses on buildings and equipments	1,018,607	929,713	32,497	5,707,900
Personal Loans (consumer and mortgage)	83,712,889	79,645,522	2,577,941	451,833,008
Business Loans	38,016,275	42,257,344	16,537	277,235,469
Investments	16,387,077	16,066,891	160,544	113,354,562
Number of transactions	2,003,188	1,881,068	56,507	12,283,532