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Time-Frequency Multi-Betas Model
-An Application with Gold and Oil -

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Abstract :

The OLS Estimation of the CAPM suffers a lot of statistical issues. We develop a Time-Frequency Multi-Betas Model with Gold and Oil as supplementary source of risk and with ARMA-EGARCH errors to take into consideration some of these statistical weaknesses. We use 30 french stocks listed on the CAC40 for the daily period from 2005 to 2015. The conjugaison of the wavelets decompositions and the parameters estimates constitutes an significant asset for managers choices according to their view (shor-medium-long term). The results represent a decision support by multiplying the interpretive possibilities, In short-run (''Noise Trader'' and ''High Frequency trader'') 1/3 of the equities are not affected by the Oil and Gold fluctuations, and the estimated Betas parameters related to the market movements are few different compare to the Model without wavelets. At the opposite, in long-run (fundamentalists investors), Oil and Gold affect all the stocks but their impact varies according to the Beta (sensitivity to the market). As example, we highlight that Oil prices negatively affect the stocks in long-run especially the equities with a high market beta (greater than 1) as Banking Stocks. We also observe significant differences between parameters estimated with and without wavelets.

Keywords :

CAPM, Multi-Betas Model, Time-Frequency Analysis, MODWT, Oil, Gold, CAC40

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Modern Portfolio Theory of Markowitz (1952) leds to the Capital Asset Pricing Model (CAPM) development in the 60s by Sharpe, Lintner and Mossin. Its mathematical equation, the Securities Market Line (SML), is similar to a simple regression model between the asset risk premium and the those of the Market. According to the CAPM hypothesis, the market is the only source of risk and the agents have homogeneous investing behavior. The systematic risk is measured by the estimations of the traditional Market Line. We summarize in the followings points the statistical and theoretical weaknesses of the CAPM which have been highlighted by several studies :

- Parameters Estimation Problem

Many authors such as Black, Scholes and Jensen (1972) and then Fama and MacBeth (1973) highlighted several statistical anomalies in the model, more particularly, the non-robustness of the method used resulting from the autocorrelation-heteroscedasticity in the residuals of the estimations, and the potential absence of exogeneous variables in the model.

- The presence of autocorrelation and heteroscedasticity effects in the CAPM has been observed par many studies such as Diebold et al (1988) and Giaccoto et Ali (1982). One of the consequences of this observation lies in the loss of Beta's BLUE statistical properties and more precisely minimal variance and convergence characteristics. The (G)ARCH familly processes of Engel (1982) and Bollerslerv (1986) is currently used to estimate a Beta parameter more consistent than the OLS estimator (Bera et al [1988], Schwert and Seguin [1990] , Corhay and Rad [1996]). The consideration of heteroscedasticity seems, however, to affect only periods of high volatility of the model, as shows Morelli in 2003 by comparing two versions of CAPM (with and without GARCH) for UK stocks. More recently, Bendod *et al* (2017) compare the CAPM and the GARCH-CAPM for the Oil sectors stocks of Arab and Golf countries and conclude that the EGARCH model are better adapted to estimate the Beta. Mestre et Terraza (2018) for french stocks come to similar conclusions. They also specify that the beta differences (between CAPM and EGARCH-CAPM) are not very significant when Betas are less than one whereas a correction is necessary for larger betas (greater than one). For all of these studies, there is a clear improvement of the Market Line residuals characteristics.

- The addition of explanatory variables in the CAPM is known as Multi-Factors or Multi-Betas Model. It was originally initiated and theoretically constructed by Merton (1973) and Ross (1976) (with the Arbitrage Pricing Theory or APT). The number and the choice of selected variables vary according to the authors and analysis. Bantz (1981) and Basu (1983) highlight the importance of considering the effects of accounting variables (specific to each stock) on equity returns, as their capitalization or size (in annual or quarterly frequencies). In an extension of these works, Fama and French (1992-1996) establish a three factors CAPM (generally referred as Fama-French Model) considering the Price-to-Book and the Company Size as supplementary variables (or more precisely, the relative performance of small companies versus big, and of companies with high Price-to-Book versus low

Price-to-Book). Otherwise, Chen *et al* (1986) incorporate macro-economics variables as output or interest rate in the Market-Line equation.

- The behavioral hypothesis of agents homogeneity

This hypothesis represents an important theoretical criticism of CAPM. In practice, investors have heterogeneous behavior resulting in different investing frequency. We can compare the positions of High-Frequency Trader (HFT) having a short-run vision with those of Mutual Funds investing in long-run. These two agents don't valorize the same Market informations but they still use the same models/methods adapting to their appetences to create their own time series. The Wavelets related to the Time-Frequency Analysis represent a response at this type of problematic. The discreet decompositions or MODWT (see. Mallet and Meyer works) appear as the easiest and most suitable tool in this case. Gençay *et al* (2005) with US data and then Mestre et Terraza (2018) with french data showed that wavelets can take in consideration the heterogeneous behavioral hypothesis leading to a Beta differentiation according to various investment horizons.

In this paper, we estimate a Time-Frequency Multi-Betas Model with AR-EGARCH errors to overcome the previous CAPM's limits. We use 30 french equity listed on CAC 40 for the daily period from 2005 to 2015.

In a first part, we estimate parameters of standard Multi-Betas-EGARCH (without wavelets). In a second part, we decompose by the wavelets the variables and we build time-frequency model considering heteregeneous behoviors. We discuss the results and the financial perspectives for portfolio managers in conclusion.

I. Standard Estimation of Multi-Betas Model

The Multi-Betas Model, theorized by Merton (1973), incorporate supplementary variables to improve the original market line estimation and residuals characteristics.

Studies in this direction are numerous but since the oil shocks of the 1980s they especially highlighted the links between financial markets and oil. Huang *et al* (1996), Jones *et al* (2004), Basher and Sadorsky (2006), Boyer and Filion (2007) show the effects of oil prices variations on stocks returns. According to these authors, this variables positively affects Oil and Energy Companies in Oil producing countries. Lee et Zeng (2011), using Quantile Regressions, have similar results for G7 countries.

Gold is also a variable generally considered as « Safe Heaven » with contracyclical variations to the Market as indicate Baur and Lucey (2010) and then Baur and McDermott (2010-2016). There are many studies about Gold-Market relationships confirming this fact. Sumner *et al* (2010) show that the Market affects Gold Prices in Crisis period but the links are weaker in expansion times. Miyazaki *et al* (2012) confirm Gold's interest in portfolio management as a counter-cyclical asset with low correlation to short-term markets. Mirsha *et al* (2010) highlight a bi-causal relationships between Gold Prices and the Indian Stocks Market. More recently, Arfaoui and Ben Rejeb (2017) with US data and Hussain Shahzad *et al* (2017) with a panel of european countries (Greece, Irlande, Portugal, Spain and Italy) confirm this result : Gold Prices influence Financial Market.

This work mainly focuses on the analysis of the Gold-Markets relationships, but few studies directly introduce Gold into the CAPM. Chua *et al* (1990) include Gold in the CAPM as dependant variable. They also consider Gold as an asset similar to an equity but it have a weak Beta. These authors don't study the reverse relation and stocks sensitivities to Gold Prices fluctuations. But Tufano (1998) analyse the CAPM with Gold in explanatory variable for noth-american mining stocks. He concludes that these stocks are greater sensitive to Gold Price than Market variations because the Beta related to Gold is higher. He also highligh the effect of data frequencies on Beta value. Jonhson andLamdin (2016) and then He *et al* (2018) find similar results with more recent UK-US data (2005-2015).

The combination of these different works leads us to retain, in the rest of this article, Oil and Gold Prices as additional factors to the Market.

We take into account the statistical limits observed in model's residuals and also the agents' heterogeneity by estimating a Multi-Betas Model with AR-EGARCH errors. We estimate it for 30 french stocks listed on the CAC40 (used as Market reference) for the daily period 2005-2015. We consider the WTI Oil Price/Barrel listed on New-York Mercantile Exchange and the Ounce Gold Price listed on London Bullion Market. The characteristics of series in log-form and the results of Unit-Root Tests (See. Appendixes A1.1 and A1.2) reject the stationary hypothesis (DS structure). As indicated in the CAPM, the Risk Premium is computed by subtracting the risk-free rate (OAT 10 years rate) from returns (the stationarized variables by the first differences filter). The Appendix A1.3 summarizes the characteristics of risk premia series. These news variables are stationary and centered (See. A1.2).

The Multi-Betas Model is written as follows :

$$r_{i,t} = \beta_{m,i}\, r_{m,t} + \beta_{o,i}\, r_{o,t} + \beta_{g,i}\, r_{g,t} + \varepsilon_{i,t} \qquad (1)$$

Where $r_{i,t}$ is the risk premium of asset i , $r_{m,t}$ the Market Premium, $r_{o,t}$ *and* $r_{g,t}$ are Oil and Gold Premia.

Under the OLS Hypothesis, $\varepsilon_{i,t}$ is a $i.i.d$ (0, σ_ε) process so in this case the $\beta_{m,i}$ $\beta_{o,i}$ and $\beta_{g,i}$ ara consistent estimators. The above studies reject this hypothesis concerning $\varepsilon_{i,t}$. As subsitute, we use an AR(1)-EGARCH(1,1) of Nelson (1991) to characterize it (See. Bibliography 44). The parameters of equation (1) and those of this process ara simultaneously estimated by the Maximum Likelihood methods associated with a non-linear optimization algorithm (See. Ye [1992] et Ghalanos and Theussl [2011]).

The Table 1 summarizes the Model estimations for the 30 equities ranked according to the decreasing value of the β_m.

Table 1 : Multi-Betas-AR-EGARCH Model Estimates

	MB-EGARCH	β_m	TSTAT	β_o	TSTAT	β_g	TSTAT	R2	JB	LB	ARCH
	Essilor	0.532	33.25	-0.0169	-2.18	0.0376	2.2	0.31	16533	2.35	3.27
	Sodexo	0.571	37.31	-0.0303	-3.77	0.0112	0.85	0.35	9494	0.81	0.7
	Ricard	0.634	40.76	-0.0239	-3.13	0.0398	3.12	0.35	9779	5.62	0.47
	Publicis	0.699	44.35	0.0037	0.29	0.0262	2.85	0.43	2014	5.82	0.77
Equities	Danone	0.701	49.39	-0.0109	-1.33	0.0237	1.52	0.41	4726	3.94	2.24
with	Orange	0.727	49.78	-0.0167	-2.21	-0.0427	-3.05	0.43	4609	2.51	0.42
β_m< 1	L'Oréal	0.768	52.25	-0.0254	-3.02	-0.0008	-0.05	0.49	4672	1.47	2.35
	Vivendi	0.791	55.85	-0.0091	-1.04	-0.0091	-0.56	0.52	7738	1.57	2.3
	Veolia	0.833	40.91	-0.0312	-4.95	-0.0195	-0.95	0.39	149850	0.19	1.21
	Air Liquide	0.853	66.28	-0.0052	-1.06	0.0407	3.69	0.65	7500	3.28	0.64
	Total	0.859	72.35	0.0757	10.15	0.0781	5.8	0.69	2419	3	1.06
	Carrefour	0.874	47.28	-0.0155	-1.68	-0.0026	-0.14	0.48	3817	1.39	1.11
	Technip	0.937	38.04	0.1423	11.01	0.1062	4.25	0.41	7486	5.32	4.55
	Airbus	0.946	39.37	0.0071	0.6	0.0217	0.86	0.36	104573	0.17	0.08
	GDF	0.949	28.9	-0.0153	-4.25	-0.0395	-2.52	0.5	153218	6.21	0.18
	Accor	0.959	43.34	-0.0163	-1.35	0.036	1.58	0.48	5618	1.37	1.07
Equities	Bouygues	0.987	47.83	-0.0148	-1.38	0.0417	2.31	0.5	17527	0.59	0.48
with	Gemini	1.008	46.98	-0.0281	-2.23	0.021	0.89	0.48	2815	2.43	0.34
β_m= 1	Michelin	1.023	41.23	-0.0162	-1.2	0.0376	1.81	0.49	3588	4.26	1.44
	LVMH	1.031	76.11	-0.0122	-1.54	0.0192	1.17	0.62	10478	1.69	0.68
	Vinci	1.068	77	-0.017	-2.5	0.0132	0.93	0.67	5157	1.34	0.96
	Alcatel	1.133	34.36	-0.0209	-3.27	-0.0883	-5.23	0.32	14120	0.31	1.49
	PSA	1.14	44.18	-0.0176	-1.19	-0.0535	-1.75	0.39	1557	4.19	1.24
	Schneider	1.195	69	-0.0085	-0.82	0.0193	1.06	0.68	1086	9.99	2.54
Equities	St-Gobain	1.252	67.31	-0.0021	-0.29	0.0057	1.21	0.67	15798	1.99	0.23
with	Renault	1.276	52.46	-0.0175	-1.52	0.0186	0.82	0.55	2303	0.94	1.94
β_m> 1	AXA	1.296	61.17	-0.0386	-3.74	-0.0591	-3.11	0.67	43408	4.34	0.04
	BNP	1.303	76.83	-0.0315	-3.58	-0.0811	-5.55	0.61	41010	2.48	1.09
	SG	1.315	61.39	-0.0015	-0.13	-0.0878	-4.55	0.56	11224	1.5	4.65
	CA	1.356	61.91	-0.0252	-1.97	-0.0712	-2.92	0.56	7966	2.69	1.6

At 5% risk level, Column LB (Ljung-Box test): $\chi^2(5)$=11.1; Column ARCH (ARCH-LM test): $\chi^2(5)$=11,1 Column J-B (Jarque-Bera Line): $\chi^2(2)$=5,99. We use Weighted Tests of Ljung-Box and ARCH-M of Fisher-Gallagher (2012).
Moreover, in this model the non-collinearity between exogeneous variables are tested (See. Appendix A2).

All the Betas parameters are significant and, thanks the determination coefficients (R^2), we note that the three variables explain 30% - 70% of assets total risk. This ranking reveals a significant relationship (R^2=0.33) between the 30 Betas β_m and their corresponding determination coefficients. Equities with strong (high) β_m have globally an high R^2 but this relationship is disrupted by the presence of outliers linked to few stocks, as example Airbus and PSA.

Residuals of the Multi-Betas Model are non-autocorrelated and homoscedastic, but the normality hypothesis is not respected. However, we consider this Model is statistically acceptable.

Significance tests of Oil and Gold Betas are used to appreciate if the Multi-Betas Model is selected for all stocks. In this context, if $\beta_o = \beta_g = 0$ the CAPM is selected for the stock. The overall results leads to the followings comments :

- We observe for 37% of the sample i.e. 11 equities (Danone, Vivendi, Carrefour, Airbus, Accor, Michelin, LVMH, PSA, Schneider, St-Gobain, Renault) the addition of Oil and Gold variables is not relevant because we accept $\beta_o = \beta_g = 0$. In this case, the CAPM results remain valid for these stocks (mainly in agrifood and automotive sectors).

- For the others 19 equities (63% of the sample), there is al lesat one significant additional variable. On notice that $\beta_o = 0$ for 4 equities (Publicis, Air Liquide, Bouygues and SG) and $\beta_g = 0$ for 5 stocks (Sodexo, L'Oréal, Gemini, Veolia et Vinci). Finally, the 2 additional variables are significant for 1/3 of the sample (i.e. 12 equities).

We built an adjustment of OLS Betas in order to quickly appreciate a more consistent β_m without reestimating the model with AR(1)-EGARCH[1] processes, This adjustment remain valid in the Multi-Betas Model case, because we observe no significant differences between the β_m (and residuals) of Multi-Betas Model of the Table 2 and the β_m of a CAPM with AR(1)-EGARCH(1,1) errors (See. Appendixes A3 et A4). The addition of these additional variables has a limited impact for the vast majority of equities because the β_m are not affected. However, the Multi-Betas Model results have an interest to portfolio managers to analyse and interpret the $\beta_o \text{et des } \beta_g$ (their sign and their value) illustrating the sensitivities to Oil and Gold fluctuations.

For a large part of equities having a significant β_o, we notice that estimators are almost negative, however, the sensitivities to Oil movements are relativly low varying between -0.015% and -0.030%. By considering the stocks classification by the β_m, as in Table 1, we observe that the Oil affects the differents equities profiles in the same way. Technip and Total are a notable exceptions to this case because their β_o are positive and higher values compare to the others stocks. Thereby, an Oil Price rise by 1% entails an stock price rise by 0.08% for Total and by 0.14% for Technip. We conclude that stocks of Oil and Gas sectors are the most sensitive to Oil price fluctuations, which is relevant with their activities.

We note significant and negative β_g for 7 stocks (Alcatel, SG, BNP, CA, AXA, Orange and GDF) and positive for 7 others equities (Publicis, Essilor, Ricard, Air Liquide, Bouygues, Total, Technip). The Banking sector equities, classified as ''risky'' because their β_m are greater than one, are negatively affected by the Gold, justifying in part its ''Safe Heaven'' characteristic. We can extend this result to stocks with strong β_m (Banking Sector + Alcatel) having a negative and relatively higher sensitivity to Gold fluctuations. On the contrary, stocks with $\beta_m < 1$ have positive and lower β_g. Once again, Total and Technip are exceptions because it are more sensitive to Gold than others stocks.

By comparing the three sensitivity estimators, the Market represents the major source of risk because the β_m are greater than $\beta_g$and β_o in absolute values. We also note that Gold sensitivity are higher (in absolute values) than Oil sensitivity, particularly in Banking sectors and for stocks with $\beta_m > 1$.

[1] See. Bibliography 42

II. Time-Frequency Multi-Betas Model Estimates

In the financial markets, the assumption of homogeneity of agents' behavior is difficult to maintain. The investment frequency o of a HFT and a Mutual Funds in portfolio, for example, depends on their buying or selling intentions based on various calculation/financial models. These latter don't differentiate the agents and consider only an aggregation of behaviors (i.e an ''average behavior'') from financial time series used. The use of wavelets time-frequency decomposition of these time series is justifed in the Multi-Betas Model framework because the High-Frequencies are related to HFT and the Low-Frequencies to fundamentalist investors. Wavelets represent a relevant solution to analyse the behovior of agents wich use this kind of financial model.

The first paragrap is a brief reminder of wavelets methodology before comparing results of the overall Multi-Betas Model with its time-frequency version in the second paragraph. In a third paragraph,we analyse the frequency sensitivities of stock prices to exogenous additional variables (Oil and Gold).

- **<u>Wavelets Methodology Reminder : the Maximal Overlap Discret Wavelets Transform (or MODWT) :</u>**

A Wavelets-mother $\Psi(\mathrm{t})$ with zero-mean and normalised is written as follow[2] :

$$\int_{-\infty}^{+\infty} \psi(t)\, dt = 0 \quad \text{et} \quad \int_{-\infty}^{+\infty} |\psi(t)|^2\, dt = 1 \quad (2)$$

These properties ensure the Variance/Energy preservation during the decomposition of a series and also guarantee the respect of admissibility condition (Grossman et Morlet [1984]).

This wavelet-mother are shifted by the τ parameter and dilated by scale parameter s to create ''wavelets-daughters'' regrouping in the wavelets familly used as filtering basis :

$$\Psi_{\tau,s}(\mathrm{t}) = \frac{1}{\sqrt{\mathrm{s}}}\Psi\left(\frac{\mathrm{t}-\tau}{\mathrm{s}}\right) \qquad (3)$$

The decomposition of time function x(t) creates/lead to the wavelets coefficients $W(s,\tau)$ as follow :

$$W(s,\tau) = \int_{-\infty}^{+\infty} x(t)\ \frac{1}{\sqrt{s}}\psi^*\left(\frac{t-\tau}{s}\right) dt = \langle x(t), \psi_{\tau,s}(t)\rangle \quad (4)$$

$$\psi^* \; is \;\; the \; complex \; conjuguate \; of \; \psi$$

τ and s parameters indicate the time and frequency localization of the coefficient. Thanks Wavelets, we can represent the temporal localization of the frequency components hence the name of time-frequency analysis. These previous equations are a theoretical presentation of wavelets decomposition based on continuous wavelets. A time discreet version is used to

[2] We use the notation of Mallat (2001)

decompose time series x_t but the principle remains similar because of frequencies are still continuous. The practical use of this kind of decomposition implies important computational time and efforts, consequently, a frequency discretization is realized to fastly decompose time series, it is the MODWT. In this framework, wavelets are defined by a succession repeated J times of High-Pass and Low-Pass filters combination (Mallat Algorithm [1989-2009]). J is decompositio ordre representing the optimal number of repetitions necessary to reconstruct a time series x_t of length N such as $J = \frac{\mathrm{Ln}(N)}{\mathrm{Ln}(2)}$.

Despite this simplified process, the MODWT is still variance/energy preserving. It ensures the perfect reconstruction of the decomposed series, without loses, by adding the High and Low-Frequencies components :

$$x_t = S_{J,t} + \sum_{j=1}^{j=J} D_{j,t} \qquad (5)$$

$S_{J,t}$ is a basic approximation of the series and $D_{j,t}$ are the Details, called also Frequency Bands, of scale j regrouping the frequencies in the interval $[\frac{1}{2}^{j+1} ; \frac{1}{2}^{j}]$.

In Finance, the frequencies interpreation is simplified by translating them in Periods wich have the same time unit as original data (as example, days, weeks etc…). In this case, frequencies represent the differents time investment horizons (Short-Medium-Long run). The Table in Appendix A5 records the frequency bands and the corresponding time horizons in days.

Considering the series lenght, by wavelets decomposition we have 11 frequency bands and one approximation. The High-Frequencies Bands (D1-D2) are related to short run investments whereas the Low-Frequencies illustrate long-run horizon. In order to simplify the analysis, we focus on the fist six frequency bands : D1 bands is related to 2 – 4 days investment horizon (High- Frequencies) whereas D6 band represents a 3-6 months investment (Low-Frequencies).

In the Multi-Betas Model framework, we decompose the dependent and the 3 independent variables by the MODWT. In th Multi-Betas Model, each Frequency Bands of the stock are associated with the corresponding bands of Market, Oil and Gold. By construction, the frequency bands means are equal to zero.

For an asset i, the Time-Frequency Multi-Betas Model is written as follow :

$$\begin{gathered} D_{j,t}^{asset} = \beta_j^m D_{j,t}^{Market} + \beta_j^o D_{j,t}^{Oil} + \beta_j^g D_{j,t}^{Gold} + \varepsilon_{j,t} \\ \forall\ j = 1, \dots\dots, 6\ the\ frequency\ band \\ \varepsilon_{j,t} \sim AR(1) - EGARCH(1,1) \end{gathered} \qquad (6)$$

Betas parameters are estimated in the time-frequency space and represent the asset sensitivities to the three factors considering agents investment frequency thus increases the stock profiling. The differents time-frequency regression models are estimated as previous by conserving the simplied hypothesis of AR(1)-EGARCH(1,1) errors.

The Appendix A6 summarizes all results[3] of Time-Frequency Multi-Betas AR-EGARCH Model estimates.

- **Time-Frequency Multi-Betas Model Estimates :**

β_m coefficients are highly signficant for all equities and frequencies. The coefficients of determination computed on the high-frequencies (D1-D2) are relatively closed to the overall Model of Table 1. But they become more important on medium-low-frequnencies (D4-D5) and they are almost equal to 100% on D6 band. The order of magnitude of the D5-D6 wavelets coefficients are small so range of residuals estimates are also small and then values of R^2 are high on low-frequencies. However, for all frequency bands regression, we notice a deterioration of residuals characteristics. Particularly, the AR-EGARCH process no longer properly captures the heteroscedasticity. By increasing the order of the process we reduce the autocorrelation and heteroscedasticity without significantly modified the values of Betas parameters. Despite these reservations, the Time-Frequency Multi-Betas Model has sufficient statistical properties to analyse its economics results.

Estimates of these three parameters play an important role in investors strategies who question the choice of model according to the parameters significativity. Globally, we remark that stocks with a strong β_m (>1) have negative β_o and β_g for all frequency bands more particularly at long-run. The stock with low β_m (<1) have positive and relatively high β_g whereas β_o still mainly remain negative. Portfolio managers can thus appreciate the different sources of risk affecting their portfolio when making their choices.

The Table2 summarizes the differences between parameters of standard and time-frequency models and represent an additional help to interpret results.

Tableau 2 : Percentages of significantly different Betas between Global Multi-Betas Model (GMB) and Time-Frequency Multi-Betas Model (TFMB)

TFMB - GMB	D1	D2	D3	D4	D5	D6
β_m	20%	16.66%	36.66%	50	73.33	83.33
β_o	20%	20%	30%	46.66%	76.66%	90%
β_g	3.33%	6.66%	23.33%	43.33%	73.33%	73.33%

We test if the difference between the two estimators are significant with Student Test. We count the number of significant differences and we expressed them as a percentage of the total number of actions (i.e. 30)

The Betas estimated without wavelets (GMB) are globally similar to short-run Betas (D1-D2) for the majority of equities and the three factors whereas the differences are more significant in long-run. As example, in long-run (D6), we note that the differences between β_m of GMB and TFMB are significant for 83% of equities.

[3] We use the ''rugarch'' R - package developed by Ghalanos and Theussl (2011).

Wavelets provide a differenciated betas estimates according the investment frequencies usefull to identify and analyse the effects of investment horizon on systematic risk measures/indicators and on sensitivities to different factors. The intensity of β_o and $\beta_g$is greater in long-run than in short-run. For all equities, the three selected variables more strongly affect assets for long-run investments. So, we confirm the results of Gençay *et al (2005)* et par Mestre et Terraza (2018) concerning the interest of wavelets in market model for long-run investments. The Time-Frequency Multi-Betas Model is therefore of strategic interest for long-term investments by estimating its low-frequency parameters.

- **Stocks sensitivities to Oil and Gold movments:**

By testing the significance of frequency parameters β_o and $\beta_g$for all stocks, we establish the following statements :

- If $\beta_o = \beta_g = 0$, the addition of the two variables is not appropriate for this asset. In this case, the time-frequency CAPM[4] is selected.
- If $\beta_o \neq 0$ et $\beta_g \neq 0$, the two additional variables are relevant and the Multi-Betas (MB) Model with Oil and Gold is retained.
- If a one of the two Beta is significant, we always retain the Multi-Betas Model and we indicate by "MB-Oil" or "MB-Gold" the choice of the variable to keep.

We use this framework to summarize in the Tables 3 the results. We count for each frequency band the number of shares for which the CAPM or Multi-Betas with one or two additional variables is retained.

Tables 3 : Analysis of Time-Frequency Multi-Betas Model results

3.1 Number of stocks for which the CAPM or the Multi-Betas is valid (in values and percentage)

In values	D1	D2	D3	D4	D5	D6
CAPM	11	5	6	5	1	1
Multi-Betas	19 including	25 including	24 including	25 including	29 including	29 including
MB Gold	6	3	6	4	1	1
MB Oil	5	15	7	7	3	5
MB Oil-Gold	8	7	11	14	25	23

In % of the sample	D1	D2	D3	D4	D5	D6
CAPM	36.67	16.67	20.00	16.67	3.33	3.33
Multi-Betas	63.33	83.33	80.00	83.33	96.67	96.67
MB Gold	20.00	10.00	20.00	13.33	3.33	3.33
MB Oil	16.67	50.00	23.33	23.33	10.00	16.67
MB Oil-Gold	26.67	23.33	36.67	46.67	83.33	76.67

[4] The time-frequency CAPM was already estimated in the study referenced 44.

3.2 Results per equities

	D1	D2	D3	D4	D5	D6
Accor	CAPM	MB Oil	MB Oil	CAPM	MB	MB
Airbus	CAPM	CAPM	MB	MB Oil	MB	MB
Alcatel	MB Gold	CAPM	MB Gold	MB	MB	MB
Air Liquide	MB Gold	MB Gold	MB Gold	MB Oil	CAPM	MB Oil
AXA	MB	MB	MB	MB	MB	MB
BNP	MB Gold	MB Gold	MB Oil	MB	MB	MB
Bouygues	MB Oil	MB Oil	MB Gold	MB	MB	MB
CA	MB Gold	MB Gold	MB Gold	MB	MB	MB Oil
Carrefour	CAPM	MB Oil	MB Oil	MB	MB	MB
Danone	CAPM	CAPM	MB	CAPM	MB Gold	MB Oil
Essilor	CAPM	MB	MB	CAPM	MB	MB
GDF	MB Oil	MB Oil	MB Oil	MB Oil	MB	MB
Gemini	CAPM	MB Oil	MB Gold	MB	MB Oil	MB
St-Gobain	CAPM	MB Oil	MB Oil	MB Gold	MB	MB
L'Oréal	MB	MB Oil	MB Oil	CAPM	MB Oil	MB
LVMH	CAPM	MB Oil	CAPM	MB Gold	MB	MB
Michelin	CAPM	MB Oil	CAPM	MB	MB	MB
Orange	MB	MB Oil	MB	MB Oil	MB	CAPM
PSA	MB	MB Oil	MB Oil	MB Oil	MB	MB
Publicis	MB	CAPM	CAPM	MB Oil	MB Oil	MB
Renault	MB Oil	MB Oil	MB Gold	CAPM	MB	MB Gold
Ricard	MB	MB	MB	MB Gold	MB	MB
Schneider	CAPM	CAPM	CAPM	MB Gold	MB	MB
SG	MB Gold	MB	MB	MB	MB	MB
Sodexo	MB Oil	MB Oil	CAPM	MB	MB	MB
Technip	MB	MB	MB	MB	MB	MB
Total	MB	MB	MB	MB	MB	MB
Veolia	MB Gold	MB Oil	MB	MB Oil	MB	MB Oil
Vinci	CAPM	MB	CAPM	MB	MB	MB
Vivendi	MB Oil	MB Oil	MB	MB	MB	MB Oil

By reading these tables, it is possible to establish the following comments:

- There is no stocks having the CAPM on all frequency bands, contrary to the full Multi-Betas Model retained for AXA, Technip and Total. There is no stocks having an intermediary Multi-Betas Model (i.e MB-Oil or MB-Gold) on all frequency bands.

- In short-run (D1), the CAPM is valid for 1/3 of the sample so results are similar to the previous estimate of Global Model. This percentage decreases as the time horizon increases, so more stocks retain the Multi-Betas Model in long-run more precisely the Multi-Betas with Oil. Stocks are therefore more impacted/affected by Oil and/or Gold in the long term than in the short term.

The TimeFrequency Multi-Betas Model is therefore of statistical interest for a majority of esquities whatever the investment horizons. This model is complementary to the CAPM results by introducing the decompositions of Oil and Gold variables. It can be a decision support for investors by synthesizing the results of the stocks sensitivities to Oil and Gold.

The Table 4 synthesize, for each frequency bands, the percentage of β_o and $\beta_g$significantly greater, lower or equal to zero and also their means. To improve the analysis, we also indicate the mean of the corresponding β_m.

Table 4 : Synthesis of $\beta_o$and β_g signs and means

	D1	D2	D3	D4	D5	D6
% of β_o > 0	23.08	18.18	16.67	28.57	17.86	42.86
Mean of β_o	0.06	0.07	0.12	0.06	0.09	0.1
Mean of β_m	0.88	0.94	0.94	0.92	0.99	0.85
% of β_g > 0	42.86	60.00	58.82	55.56	50.00	50.00
Mean of β_g	0.05	0.04	0.05	0.08	0.09	0.12
Mean ofβ_m	0.79	0.8	0.86	0.97	1	0.95
% of β_o < 0	76.92	81.82	83.33	71.43	82.14	57.14
Mean of β_o	-0.02	-0.03	-0.03	-0.05	-0.06	-0.11
Mean of β_m	0.89	0.97	0.97	1.1	1.07	1.1
% of β_g < 0	57.14	40.00	41.18	44.44	50.00	50.00
Mean of β_g	-0.07	-0.15	-0.08	-0.1	-0.09	-0.16
Mean of β_m	1.13	1.33	1.16	1.22	1.11	1.11

The parameters estimates of Oil and Gold of Time-Frequency Multi-Betas Model lead to the following general comments :

- The number of Multi-Betas Model having a positive $\beta_g$is relatively stable on all frequecy bands (around 50%) whereas it increases for Model with postive β_o. Furthermore, we note that their means increase from D1 (short-run) to D6 (long-run). Stocks with positive β_g and/or β_0 have in average a β_m less or equal to one. The Oil-sector stocks (Total, Technip and Air Liquide) are strongly and positively sensitive to Oil and Gold for all frequency bands. The mean value of positive β_o are higher than mean value of negative β_o. To a lesser extent, Automotive, Luxury sectors (LVMH, L'Oréal and Ricard), Large Distribution and Catering Services (Carrefour, Danone and Sodexo) are positively affected by Gold and Oil in long-run (only D6).

- The number of stock negatively sensitive to Oil is higher in short-term (D1-D2) than long-term (D6), whereas it is stable for Gold. The intensity of β_g and β_0 increases as the investment horizon increases (in average). We note similar results for stock with $\beta_o < 0$, the mean of β_m increases accross frequencies, it is lesser than one until the D3-Bands but greater beyond. Banking Equities (SG, BNP, CA and AXA) and stokcs related to Electricity and energy management (Veolia, GDF, Saint-Gobain, Schneider) are not sensitive to Oil short-run variations (except AXA). However, these stocks have

strongly negative β_o when investment horizon increases (D5-D6). We observe an opposit result for automotive sector stocks. The equities negatively sensitive to Gold have in average a β_m greater than one. However, we notice that the intensity of negative β_g is greater (in average) than the positive β_g mean. Gold negatively affects the stocks with an important systematic risk as Banking sector and Alcatel.
During an expansion period, a rise in the market leads to a stronger increase in the stocks price and a decrease in the gold price confirms the upward dynamic. At the opposite, during crises period, the decreasing trend of the market pushes down the stocks prices In this context, investors close their positions and buy Gold. So, gold demand becoming more and more important, its price naturally increases confirming the investors choices so stocks prices decreases So the « Safe Heaven » characteristic of Gold are partially justified even if the half of stocks have positive β_g.

III. Conclusion

The Time-Frequency Multi-Betas Model effectively complements the different instruments used by stock investors to build their portfolios. In the first hand, it can substitute the CAPM by considering the residual anomalies by using ARMA-EGARCH processes to model the errors of the regression. In the second hand, it improves the CAPM by adding exogeneous variables and it considers the heterogeneity of agents behaviors by the wavelets decompositions. Despite of some statistical shortcomings, particularly those concerning the characteristics of its frequency residuals, this model brings a significant gain of information to model the risk premiums.

- In short-run, the β_m parameter of the Time-Frequency Multi-Betas Model, measuring the sensitivity to market fluctuation, is not significantly differents to the Global Multi-Betas Model and the CAPM. For a short-run investors, the use of the CAPM can be sufficient to make investment choices based on the β_m. However, he can consider the Multi-Betas Model and the sensitivities to Gold and Oil in order to modulate its choices.

- The Global Multi-Betas Model (without wavelets) is retained for the 2/3 of the stocks in its full version (Gold and Oil) or one of the intermediary versions (Gold or Oil). The stocks sensitivities to Oil and Gold are lower than sensitivity to market, but we can appreciate potential positive effects on some sectors as Petroleum/Gaz-stocks and Banking. As example, Oil negatively affects the majority of stoks, however, its impact is stronger for high β_m equities than low β_m equities. Gold negatively and more strongly affects equities with a high systematic risk (like the banking sector) but the effect is reversed for equities with a β_m lower than one (like luxury and petroleum sectors). The Time-frequency Multi-Betas Model multiplies the analysis by crossing the betas and the sectors with the investment horizons.

- Ultimately, the Time-Frequency Multi-Betas Model is more useful for fondamentalists investors (long-run). At low-frequencies (D6), the CAPM is retained for only one stock whereas for the others, Oil and Gold variables have significant effects on equities. Their effects increases as the time horizons increases.

- Wavelets represent a powerfull tool to differentiate the stocks sensitivities to various factors according the agents investment horizons. The combination of the time-frequency estimates of the Multi-Betas Model improves investment choices possibilities and risk analysis.

APPENDIXES

Tables A1 : Equities characteristics

A1.1 : Means and standard deviations of Ln(Prices)

Stocks Prices	Means	Std-Dev	Skewness	Kurtosis
Accor	3.15	0.34	0.15	2.76
Airbus	3.15	0.52	0.28	2.18
Alcatel	1.20	0.74	0.19	2.05
Air Liquide	4.05	0.40	-0.15	2.16
AXA	2.48	0.31	0.08	2.71
BNP	3.72	0.25	-0.98	3.68
Bouygues	3.13	0.26	0.10	2.17
CA	2.29	0.48	-0.46	2.87
CARREFOUR	3.23	0.28	-0.74	3.31
DANONE	3.70	0.23	-0.39	2.78
ESSILOR	3.84	0.47	0.27	2.00
GDF	2.76	0.19	-0.11	2.75
Gemini	3.51	0.37	0.78	3.22
St Gobain	3.48	0.26	-0.08	3.22
LVMH	4.42	0.41	-0.10	1.79
Michelin	3.90	0.35	0.04	2.28
L'Oréal	4.36	0.36	0.50	2.30
Orange	2.29	0.19	-0.10	3.09
PSA	2.72	0.56	-0.47	2.36
Publicis	3.46	0.43	0.29	1.98
Renault	3.81	0.47	-0.62	2.87
Ricard	4.12	0.31	-0.03	2.21
Schneider	3.51	0.43	-0.16	1.88
SG	3.70	0.50	0.05	2.51
Sodexo	3.77	0.41	-0.06	2.28
Technip	3.85	0.36	-0.65	3.21
Total	3.43	0.18	0.29	2.88
Veolia	2.77	0.49	0.03	2.24
Vinci	3.45	0.30	-0.03	2.84
Vivendi	2.57	0.18	0.68	3.10
Gold	6.64	0.40	-0.44	2.00
Oil	4.05	0.25	-0.58	2.81
CAC	8.33	0.19	-0.02	2.31

A1.2 : Phillips-Perron stationarity test on Ln(Prices) and on risk premia

Ln(Prices)	M3	M2	M1	Premia	M3
CAC	-1.89	-1.85	0.23	CAC	-56.11
Oil	-2.57	-2.13	-0.07	Oil	-57.32
Gold	-1.22	-2.23	1.77	Gold	-56.18
Accor	-3.1	-2.06	-1.11	Accor	-52.93
Airbus	-1.53	-0.52	0.93	Airbus	-53.39
Alcatel	-1.15	-1.74	-1.47	Alcatel	-51.26
Air Liquide	-3.27	-1.51	2.08	Air Liquide	-60.49
AXA	-2.41	-2.05	0.72	AXA	-51.22
BNP	-2.48	-2.48	0.24	BNP	-53.9
Bouygues	-1.88	-1.88	0.44	Bouygues	-55.34
CA	-1.5	-1.49	-0.51	CA	-51.42
CARREFOUR	-1.87	-1.67	-0.03	CARREFOUR	-54.48
DANONE	-3.06	-2.19	1.34	DANONE	-56.78
ESSILOR	-2.32	-0.45	2.38	ESSILOR	-57.77
GDF	-3.09	-3.02	0.33	GDF	-54.64
Gemini	-1.58	-0.81	1.4	Gemini	-53.5
St Gobain	-2.36	-2.36	0.03	St Gobain	-54.64
LVMH	-1.96	-0.65	1.73	LVMH	-59.99
Michelin	-2.54	-1.41	1.1	Michelin	-55.7
L'Oréal	-2.46	-1.57	0.69	L'Oréal	-52.28
Orange	-1.42	-1.38	0.32	Orange	-54.42
PSA	-1.36	-1.46	-0.59	PSA	-49.27
Publicis	-1.97	-0.7	1.26	Publicis	-54.03
Renault	-1.27	-1.25	0.3	Renault	-49.21
Ricard	-2.64	-1.48	1.43	Ricard	-55.28
Schneider	-2.53	-1.91	1.19	Schneider	-57.69
SG	-1.72	-1.49	-0.33	SG	-48.76
Sodexo	-3.19	-1.6	2.04	Sodexo	-54.67
Technip	-1.89	-2.04	0.24	Technip	-53.73
Total	-3.31	-2.77	0.78	Total	-55.66
Veolia	-1.1	-1.14	0.04	Veolia	-510257
Vinci	-2.59	-1.99	1.48	Vinci	-56.9
Vivendi	-2.17	-1.86	0.56	Vivendi	-56.31

Critical	Values
1%	-3.96
5%	-3.41
10%	-3.13

For tests on premia, the test statistics for model 1 and 2 are similar to model 3 values.

A1.3 : Means and standard deviations of risk premia

Premia	Means	Nullity tests	Std-Dev	Skewness	Kurtosis
CAC	-0.0000494	0.19	0.0143	0.02	9.55
Oil	0.0000309	0.07	0.0232	-0.01	8.44
Gold	0.000392	1.78	0.0118	-0.47	8.27
Accor	0.000355	0.93	0.0205	0.17	7
Airbus	0.000316	0.74	0.0228	-0.95	16.65
Alcatel	-0.000506	0.88	0.0307	-0.26	9.46
Air Liquide	0.000368	1.34	0.0147	0.04	7.34
AXA	0.000351	0.72	0.026	0.45	12.15
BNP	0.0000518	0.11	0.0254	0.27	11.53
Bouygues	0.000105	0.26	0.0212	0.31	10.41
CA	-0.000212	0.41	0.0275	0.21	9.03
DANONE	0.000245	0.9	0.0145	-0.05	7.18
CARREFOUR	-0.0000784	0.23	0.0185	-0.06	6.34
ESSILOR	0.000466	1.84	0.0136	0.36	9.08
GDF	0.0000471	0.13	0.019	1.12	23.11
Gemini	0.000425	1.07	0.0212	0.02	6.65
St Gobain	-0.0000298	0.07	0.0234	0.04	9.52
L'Oréal	0.00031	1.12	0.0148	0.23	8.75
LVMH	0.000298	0.89	0.018	0.09	8.53
Orange	-0.0000008	0	0.0158	0.28	6.68
Michelin	0.000228	0.56	0.0219	-0.1	6.59
PSA	-0.000269	0.54	0.0267	-0.02	5.42
Publicis	0.000271	0.92	0.0157	0.01	6.39
Renault	0.000123	0.25	0.0261	-0.16	7.44
Ricard	0.000293	0.95	0.0165	-0.33	12.43
Schneider	0.000351	0.88	0.0214	0.09	7.92
SG	-0.000179	0.34	0.0281	-0.07	9.04
Sodexo	0.00047	1.65	0.0152	-0.11	8.88
Technip	0.0000608	0.14	0.024	-0.34	8.17
Total	0.000137	0.45	0.0162	0.17	10.01
Veolia	-0.0000174	0.04	0.0209	-0.79	17.07
Vinci	0.000379	1.05	0.0194	0.27	10.59
Vivendi	0.0000598	0.21	0.0155	0.07	7.5

Tables A2 : Multicolineartity analysis

A2.1 Matrix of Correlation

	CAC	Oil	Gold
CAC	1	0.15	0.28
Oil	0.15	1	-0.07
Gold	0.28	-0.07	1

A2.2 Variance Inflation Factors (ViF)

ViF		
CAC	Oil	Gold
1.099	1.12	1.038

Table A3 : MEDAF-EGARCH Estimates

	MEDAF-EGARCH	β_m	T-stat	R2	LB	ARCH	JB
$\beta_m < 1$	Essilor	0.525	33.527	0.31	2.02	3.59	16301
	Sodexo	0.559	38.830	0.35	0.41	0.72	9359
	Ricard	0.627	35.094	0.355	5.79	0.386	9427
	Danone	0.697	49.456	0.41	4.11	2.26	4732
	Publicis	0.7	43.134	0.43	6	0.76	2019
	Orange	0.722	51.499	0.43	2.27	0.46	4566
	L'Oréal	0.755	55.404	0.486	1.34	2.04	4675
	Vivendi	0.788	57.146	0.52	1.51	0.16	7736
	Veolia	0.838	41.000	0.39	0.24	1.35	152130
	Air Liquide	0.851	65.115	0.65	2.9	0.21	7518
	Carrefour	0.866	49.282	0.48	1.32	1.14	3813
	Total	0.887	73.458	0.67	2.47	0.68	2273
	GDF	0.942	62.613	0.49	11.6	0.33	148170
	Airbus	0.945	42.384	0.35	3.94	0.17	105930
	Accor	0.954	41.580	0.48	1.4	1.07	5617
$\beta_m = 1$	Technip	0.991	35.798	0.39	3.22	1.22	6331
	Bouygues	0.994	51.512	0.5	0.24	0.84	17744
	Gemini	0.996	46.108	0.484	2.27	0.35	2782
	Michelin	1.017	44.993	0.49	4.46	2.41	3560
$\beta_m > 1$	LVMH	1.028	72.783	0.62	1.79	0.31	10806
	Vinci	1.062	72.543	0.67	1.12	0.875	5150
	Alcatel	1.131	37.546	0.32	0.846	1.56	14264
	PSA	1.135	46.521	0.39	3.63	0.07	1539
	Schneider	1.192	70.704	0.68	9.93	2.533	1063
	St-Gobain	1.25	64.465	0.67	2.04	0.23	15825
	Renault	1.268	54.896	0.55	1.16	2.05	2262
	AXA	1.288	70.891	0.67	4.38	0.013	43151
	BNP	1.289	77.316	0.61	2.94	0.96	40217
	SG	1.305	62.943	0.56	5.07	4.53	10525
	CA	1.351	37.302	0.56	2.62	1.58	7842

At 5% risk level, Column LB (Ljung-Box test): $\chi^2(5)=11.1$; Column ARCH (ARCH-LM test): $\chi^2(5)=11{,}1$ Column J-B (Jarque-Bera Line): $\chi^2(2)=5{,}99$.

We use Weighted Tests of Ljung-Box and ARCH-M of Fisher-Gallagher (2012).

Table A4 : Comparison of β_m between MEDAF-EGARCH and Multi-Betas-EGARCH

	Stocks	Beta (CAPM-EGARCH)	Beta (MB-EGARCH)	Differences	
$\beta_m < 1$	Essilor	0.53	0.53	0	NS
	Sodexo	0.58	0.57	0.01	NS
	Danone	0.7	0.7	0	NS
	Ricard	0.63	0.63	0	NS
	Publicis	0.7	0.7	0	NS
	L'Oréal	0.75	0.77	-0.02	NS
	Orange	0.72	0.73	-0.01	NS
	Vivendi	0.79	0.79	0	NS
	Air Liquide	0.85	0.85	0	NS
	Carrefour	0.87	0.87	0	NS
	Veolia	0.83	0.83	0	NS
	Total	0.89	0.86	0.03	NS
	GDF	0.95	0.95	0	NS
	Airbus	0.94	0.95	-0.01	NS
	Accor	0.95	0.96	-0.01	NS
$\beta_m = 1$	LVMH	1.02	1.03	-0.01	NS
	Gemini	0.99	1.01	-0.02	NS
	Technip	0.99	0.94	0.05	NS
	Bouy	0.99	0.99	0	NS
	Michelin	1.03	1.02	0.01	NS
$\beta_m > 1$	Vinci	1.06	1.07	-0.01	NS
	PSA	1.13	1.14	-0.01	NS
	Alcatel	1.13	1.13	0	NS
	Schneider	1.19	1.2	-0.01	NS
	St-Gobain	1.25	1.25	0	NS
	Renault	1.27	1.28	-0.01	NS
	BNP	1.29	1.3	-0.01	NS
	CA	1.35	1.36	-0.01	NS
	SG	1.31	1.32	-0.01	NS
	AXA	1.29	1.3	-0.01	NS

NS= Non-Significant Differences

<u>*Table A5– Frequency Bands corresponding days*</u>

	Horizons en Jours	
Level J	Inferior border	Superior border
D1	2	4
D2	4	8
D3	8	16
D4	16	32
D5	32	64
D6	64	128
D7	128	256
D8	256	512
D9	512	1024
D10	1024	2048
D11	2048	4096
S11	4096	-

Table A6: Time-Frequency Multi-Beta-AR-EGARCH Estimates

$\beta_m = Market\ Beta\ \ \beta_o = Oil\ Beta\ \ \beta_g = Gold\ Beta$

Stocks	Bands	β_m	T-STAT	β_o	T-STAT	β_g	T-STAT	R2	JB	LB	ARCH
	D1	0,922	53,568	-0,0131	-1,3	0,0019	0,099	0,64	2204	720,423	5,574
	D2	0,984	50,781	-0,0235	-2,493	0,0017	0,521	0,565	1296	1316,592	12,735
	D3	1,033	65,3	-0,027	-2,944	0,0217	1,151	0,828	312	1813,83	291,959
ACCOR	D4	1,021	58,074	0,0051	0,554	-0,0123	-0,791	0,943	403	1991,38	0,644
	D5	1,06	102,876	-0,0786	-15,065	0,0876	10,301	0,985	54	3550,87	9,751
	D6	0,922	179,691	0,077	29,068	0,1309	21,159	0,997	133	5414,858	10,955
	D1	0,921	34,132	0,0096	0,533	0,0053	0,25	0,592	15604	685,551	3,232
	D2	0,988	317,141	0,0209	0,309	0,0246	0,29	0,423	10082	1344,368	25,083
AIRBUS	D3	1,052	48,233	0,0339	2,911	0,0718	5,568	0,766	3670	1950,566	361,547
	D4	0,938	59,123	0,0477	4,836	0,0081	0,358	0,93	1814	1940,658	0,376
	D5	1,127	74,908	-0,0123	-2,129	0,0405	4,801	0,983	98	3427,716	17,265
	D6	1,057	100,269	-0,0559	-12,332	-0,2061	-23,259	0,995	1244	5139,662	1,388
	D1	1,13	15,55	-0,006	-0,429	-0,0745	-2,665	0,55	4055	676,418	9,678
	D2	1,162	131,953	-0,0182	-0,827	-0,0307	-0,455	0,407	2053	1239,125	14,845
	D3	1,212	40,039	0,0015	0,097	-0,0645	-2,15	0,766	823	1916,485	227,714
ALCATEL	D4	1,195	72,691	-0,045	-6,653	-0,1012	-6,578	0,928	229	1983,44	6,144
	D5	1,608	87,96	0,0603	9,718	-0,1421	-7,578	0,983	77	3739,435	9,791
	D6	1,056	77,267	0,0952	11,826	-0,4725	-43,9	0,996	110	4906,294	12,773
	D1	0,879	77,935	-0,0047	-0,597	0,0467	3,601	0,785	3658	754,676	2,569
Air	D2	0,838	75,114	-0,007	-1,911	0,0214	2,494	0,685	1151	1322,802	18,965
	D3	0,866	67,361	-0,0124	-1,657	0,0542	3,986	0,859	774	1827,23	356,415
Liquide	D4	0,873	85,264	0,0163	4,512	-0,0154	-1,839	0,955	5918	1962,898	15,448
	D5	0,823	63,518	0,0013	0,195	0,0123	0,928	0,989	1156	3476,632	19,158
	D6	0,735	151,265	0,0731	22,043	-0,0002	-0,056	0,996	90	5375,606	46,855
	D1	1,302	92,734	-0,0154	-2,122	-0,0712	-5,291	0,774	18964	687,71	6,967
	D2	1,359	73,298	-0,0217	-2,53	-0,1259	-7,542	0,722	14559	1229,758	19,739
AXA	D3	1,401	81,679	-0,0196	-3,315	-0,0839	-5,976	0,867	5239	1949,991	306,684
	D4	1,415	88,816	-0,0271	-5,592	-0,1252	-7,886	0,967	2520	2104,741	10,316
	D5	1,184	97,609	-0,0405	-15,744	-0,1378	-12,881	0,987	10038	3495,235	6,593
	D6	1,35	203,93	-0,2821	-92,108	-0,3023	-53,248	0,998	3249	4671,677	10,535
	D1	1,287	71,344	-0,0074	-0,978	-0,0786	-5,334	0,767	10672	682,734	15,477
	D2	1,316	84,514	-0,0026	-1,061	-0,1227	-7,886	0,653	12603	1304,48	18,303
	D3	1,331	71,332	-0,0655	-8,661	-0,0424	-1,315	0,851	12069	1916,993	235,842
BNP	D4	1,332	83,583	-0,0739	-11,28	-0,1264	-9,242	0,957	11237	2007,968	0,508
	D5	1,18	121,582	-0,0536	-11,056	-0,0681	-10,048	0,982	7760	3282,641	2,759
	D6	1,235	143,006	-0,1418	-28,493	-0,1477	-41,589	0,996	25827	4609,296	0,004
	D1	0,971	53,72	0,0221	2,435	0,0262	1,424	0,689	6075	714,38	3,226
	D2	0,95	41,002	0,0328	3,353	0,032	1,814	0,564	3648	1265,106	16,355
Bouygues	D3	1,003	59,313	-0,0016	-0,153	0,0436	2,398	0,814	20125	1821,523	359,875
	D4	0,998	91,394	-0,0362	-6,964	0,0257	2,704	0,944	2357	1900,468	2,986
	D5	0,912	67,347	0,0249	3,418	-0,0555	-4,247	0,982	112	4102,425	8,078
	D6	0,932	140,057	-0,1614	-46,496	0,0633	7,727	0,996	31	5110,38	22,158

Stocks	Bands	β_m	T-STAT	β_o	T-STAT	β_g	T-STAT	R2	JB	LB	ARCH
	D1	1,274	44,119	0,0041	0,363	-0,1051	-5,129	0,698	3074	712,218	8,442
	D2	1,271	52,67	0,0145	1,336	-0,194	-8,933	0,607	7921	1340,081	26,383
	D3	1,378	70,074	0,0244	1,693	-0,1689	-8,915	0,842	6246	1886,868	232,375
CA	D4	1,467	117,298	-0,0932	-10,943	-0,058	-2,638	0,954	1299	1967,495	1,588
	D5	1,28	101,315	-0,1038	-15,193	-0,1333	-14,336	0,983	1990	3102,17	0,686
	D6	1,434	151,138	-0,1313	-17,748	0,0082	0,871	0,997	5665	4782,805	0,063
	D1	0,897	63,492	0,0002	0,024	0,0141	0,912	0,673	1139	713,39	8,053
	D2	0,896	59,47	-0,0096	-2,177	-0,0112	-1,02	0,557	1207	1229,213	16,74
Carrefour	D3	0,858	45,131	-0,0277	-5,541	-0,0178	-1,572	0,788	505	1920,435	307,689
	D4	0,943	51,338	0,0163	2,733	-0,0513	-4,142	0,944	951	2084,816	5,033
	D5	0,966	97,56	-0,099	-18,71	0,0533	3,854	0,984	119	3567,823	8,862
	D6	0,917	228,951	-0,0716	-20,656	0,1206	26,796	0,995	6	5442,434	46,885
	D1	0,737	61,417	-0,0082	-1,665	-0,0018	-0,577	0,637	2964	765,416	23,292
	D2	0,709	58,165	-0,0014	-0,471	0,0039	0,54	0,492	2952	1280,469	22,841
	D3	0,674	61,165	-0,0479	-6,423	0,0313	2,189	0,742	3969	1832,224	294,52
Danone	D4	0,561	52,063	0,0007	0,074	0,0195	1,289	0,918	186	2076,038	16,175
	D5	0,696	97,018	0,0014	0,671	-0,0241	-4,306	0,978	440	3740,374	8,759
	D6	0,725	169,421	0,0921	24,367	-0,0014	-0,917	0,997	2	4852,443	25,67
	D1	0,546	36,593	-0,0134	-1,794	-0,025	-1,762	0,575	3009	833,8	1,718
	D2	0,553	44,366	-0,0224	-3,815	0,0347	5,343	0,378	2017	1204,631	15,767
Essilor	D3	0,533	37,298	-0,0226	-3,211	0,0742	6,076	0,709	16012	1824,622	366,232
	D4	0,561	10,105	-0,0256	-0,689	-0,0029	-0,032	0,93	3151	2072,106	5,2
	D5	0,643	49,295	-0,0542	-13,963	0,0715	9,365	0,982	11167	3644,675	0,1
	D6	0,791	148,989	-0,0935	-30,069	0,1178	17,876	0,994	399	5445,626	2,35
	D1	0,909	65,558	-0,0324	-4,463	-0,0049	-0,329	0,65	71465	735,042	4,969
	D2	0,913	58,982	-0,0319	-3,89	-0,0191	-1,141	0,549	25261	1251,034	17,128
GDF	D3	0,983	72,713	-0,0313	-3,484	0,0022	0,149	0,791	2683	1831,72	301,399
	D4	1,017	48,169	-0,0369	-5,717	0,0449	1,863	0,944	3114	2007,483	4,381
	D5	0,898	99,075	-0,0614	-14,877	-0,0985	-8,745	0,983	763	3506,152	11,128
	D6	0,878	223,197	-0,1281	-15,478	-0,1865	-46,133	0,996	93	4680,596	21,709
	D1	1,002	33,979	-0,0136	-0,999	0,0138	0,615	0,652	745	739,851	7,738
	D2	1,025	58,859	-0,0387	-6,889	-0,0169	-1,17	0,559	778	1278,93	23,591
Gemini	D3	1,037	58,898	-0,0072	-0,855	-0,0437	-3,356	0,818	112	1840,925	319,023
	D4	1,037	52,898	-0,0812	-9,945	0,0731	5,776	0,944	213	1990,637	3,3
	D5	1,202	91,467	-0,0961	-9,763	0,0356	1,703	0,985	516	3346,097	7,629
	D6	1,247	180,589	-0,0754	-20,896	-0,1584	-35,876	0,997	274	5026,435	1,805
	D1	1,248	81,412	-0,0056	-1,54	-0,0072	-1,087	0,788	3975	717,355	6,754
	D2	1,256	75,235	-0,025	-3,345	-0,0013	-0,075	0,728	2835	1192,911	19,451
Saint	D3	1,299	96,76	-0,032	-3,99	-0,012	-1,286	0,87	10593	1933,088	336,118
Gobain	D4	1,247	43,44	-0,008	-1,414	-0,05	-6,234	0,965	20313	1823,361	2,7
	D5	1,42	46,91	-0,0242	-2,973	-0,0714	-5,877	0,99	1591	3779,978	17,416
	D6	1,321	139,715	-0,0847	-19,028	-0,0434	-15,043	0,998	1704	4978,199	10,059

Stocks	Bands	β_m	T-STAT	β_o	T-STAT	β_g	T-STAT	R2	JB	LB	ARCH
	D1	0,806	49,172	-0,0242	-5,081	0,0172	1,998	0,68	2875	721,902	4,962
	D2	0,796	80,283	-0,0395	-3,776	0,0033	0,183	0,544	937	1276,811	11,204
L'Oréal	D3	0,704	46,502	-0,0353	-4,784	0,0113	0,704	0,79	177	1875,929	368,543
	D4	0,704	67,297	-0,0048	-1,343	-0,0038	-0,302	0,934	139	2062,251	8,404
	D5	0,698	5,612	-0,0596	-15,37	0,029	0,393	0,981	174	3779,932	17,939
	D6	0,623	138,419	0,0328	12,872	-0,0103	-2,478	0,997	74	4897,552	27,416
	D1	0,99	70,254	-0,0195	-1,85	0,0139	0,989	0,752	3855	738,863	5,546
	D2	1,058	75,921	-0,0259	-3,519	0,0236	1,723	0,681	1817	1277,3	11,425
LVMH	D3	1,017	79,718	0,0106	1,889	0,0004	0,24	0,875	432	1818,208	239,076
	D4	0,994	102,042	-0,0095	-1,347	0,0788	6,933	0,956	712	1946,948	2,814
	D5	1,17	111,387	-0,0659	-18,094	-0,0309	-6,721	0,988	1194	3673,455	28,345
	D6	0,852	151,765	0,1614	53,827	0,1628	41,282	0,997	63	4948,791	15,012
	D1	0,99	24,292	-0,0326	-1,821	0,0364	1,913	0,643	1553	805,246	9,807
	D2	1,061	61,831	-0,0275	-2,153	-0,0092	-0,439	0,558	1522	1254,669	20,26
Michelin	D3	1,108	63,356	-0,0162	-1,465	-0,0235	-1,19	0,831	654	1902,326	268,755
	D4	1,137	88,262	-0,0653	-12,329	0,0353	2,835	0,938	2541	1861,805	2,606
	D5	1,049	113,278	-0,0436	-13,227	0,0897	9,103	0,983	30	3545,693	14,065
	D6	0,78	46,648	0,1171	16,409	0,1776	12,621	0,996	1463	4783,524	2,389
	D1	0,803	66,52	-0,0171	-3,975	-0,0442	-3,074	0,657	984	642,411	3,546
	D2	0,772	96,492	-0,0205	-2,228	-0,0177	-0,548	0,498	522	1303,186	17,907
Orange	D3	0,75	48,777	-0,0255	-4,569	-0,022	-2,686	0,758	543	2005,264	299,768
	D4	0,689	58,285	-0,0642	-8,598	-0,0096	-0,431	0,926	353	1997,156	1,108
	D5	0,746	91,543	-0,0232	-7,063	0,0573	10,542	0,978	417	3398,805	2,83
	D6	0,734	4,885	-0,0513	-0,892	0,0961	0,624	0,994	49685	4644,284	0,027
	D1	1,11	49,5	-0,0321	-3,072	-0,0756	-3,311	0,577	699	749,887	11,407
	D2	1,212	59,276	-0,0516	-4,214	-0,016	-0,634	0,487	1061	1233,383	11,813
	D3	1,243	59,351	-0,0338	-3,108	0,0151	0,721	0,796	860	1868,307	262,055
PSA	D4	1,219	64,159	-0,0434	-4,992	-0,0142	-1,195	0,935	362	2079,779	0,835
	D5	1,344	97,359	-0,0618	-17,868	0,1681	4,455	0,982	305	3747,039	3,86
	D6	1,191	160,654	0,0318	14,274	-0,224	-38,913	0,995	1043	4734,492	0,6
	D1	0,694	42,273	-0,0194	-2,112	0,0327	2,76	0,596	1224	757,493	6,301
	D2	0,707	45,385	-0,0137	-1,504	0,027	1,614	0,493	473	1287,14	7,925
Publicis	D3	0,741	46,288	0,0172	1,716	0,0019	0,113	0,808	109	1795,001	384,774
	D4	0,817	59,203	0,0171	2,533	0,0149	0,84	0,943	2051	1949,901	1,736
	D5	0,76	88,772	-0,0495	-9,643	0,0015	0,126	0,983	7950	3447,421	0,471
	D6	0,824	86,228	0,026	6,827	-0,0232	-4,281	0,996	157	5151,618	7,091
	D1	1,238	56,505	-0,0477	-5,216	-0,0317	-1,382	0,681	897	755,725	20,346
	D2	1,302	53,378	-0,0409	-4,056	0,0003	0,012	0,61	1495	1341,472	18,856
Renault	D3	1,335	65,015	0,004	0,33	0,0424	2,124	0,855	552	1802,368	346,576
	D4	1,363	4,817	-0,01	-0,148	0,0369	0,544	0,953	528	2035,71	4,813
	D5	1,425	85,697	-0,0958	-18	0,073	7,306	0,986	127	3772,753	7,318
	D6	1,796	46,037	0,0135	1,516	0,0904	15,57	0,998	1839	5054,13	8,789
	D1	0,61	39,674	-0,0277	-4,981	0,0338	2,379	0,578	3633	720,146	6,28
	D2	0,627	42,105	-0,0236	-5,819	0,0275	3,95	0,441	2238	1295,382	21,609
	D3	0,611	48,038	-0,0163	-2,576	0,113	9,528	0,762	7591	1798,997	306,814
Ricard	D4	0,692	65,392	0,0058	1,141	0,0364	2,499	0,925	2444	1867,326	0,227
	D5	0,668	73,462	0,0274	4,784	-0,0382	-2,71	0,981	499	3521,147	13,891
	D6	0,861	163,407	-0,0445	-18,583	0,0947	27,602	0,997	5425	4551,731	0,741

Stocks	Bands	β_m	T-STAT	β_o	T-STAT	β_g	T-STAT	R2	JB	LB	ARCH
Schneider	D1	1,25	78,748	-0,0119	-1,261	-0,0141	-0,451	0,804	298	751,308	6,225
	D2	1,214	62,874	-0,002	-0,408	-0,015	-1,054	0,717	350	1299,377	14,378
	D3	1,183	112,767	-0,0252	-1,861	0,0227	0,733	0,863	485	1939,112	381,906
	D4	1,216	68,8	-0,0015	-0,159	0,0745	8,035	0,966	96	2017,251	12,497
	D5	1,241	142,445	-0,0518	-7,797	0,0532	5,872	0,989	24	3584,71	5,187
	D6	1,121	143,908	-0,1078	-21,842	0,0962	20,502	0,997	12	5337,106	19,574
Société Générale	D1	1,299	65,073	-0,001	-0,085	-0,0728	-4,716	0,691	12655	713,99	6,397
	D2	1,372	65,863	-0,0311	-5,715	-0,1159	-7,495	0,585	15743	1254,541	14,104
	D3	1,316	69,691	-0,0261	-3,157	-0,1266	-6,782	0,833	8311	1965,056	264,734
	D4	1,427	81,925	-0,0399	-4,734	-0,2014	-14,498	0,951	4963	2004,685	6,148
	D5	1,522	122,87	-0,1777	-28,857	-0,1438	-10,264	0,986	1904	3037,177	5,423
	D6	1,603	178,765	-0,1447	-31,842	-0,1056	-7,988	0,998	4839	4808,233	0,031
Sodexo	D1	0,591	38,955	-0,026	-3,343	-0,0041	-0,59	0,564	2380	717,009	7,007
	D2	0,624	46,632	-0,0261	-3,033	0,016	0,782	0,461	1592	1237,021	9,203
	D3	0,518	29,946	0,0015	0,856	0,0028	0,822	0,756	637	1844,456	298,796
	D4	0,602	46,281	-0,0225	-8,268	0,1157	11,177	0,93	257	2136,183	4,415
	D5	0,622	71,037	-0,0089	-2,043	0,1273	15,91	0,98	221	3514,833	18,342
	D6	0,689	74,584	0,0158	8,174	0,1747	25,146	0,995	16	4915,132	16,46
Technip	D1	0,893	36,736	0,1112	8,041	0,1011	4,313	0,614	2414	756,339	4,167
	D2	0,917	40,842	0,1545	11,967	0,0848	4,49	0,494	1022	1275,475	20,702
	D3	0,982	40,088	0,1983	15,557	0,0419	2,407	0,795	2157	1903,7	394,174
	D4	1,032	57,138	0,1747	14,672	0,2132	6,624	0,944	54	2094,344	6,915
	D5	0,825	41,628	0,2057	26,016	0,2193	12,162	0,982	114	3463,723	3,564
	D6	0,795	93,768	0,3087	39,955	0,0275	5,944	0,996	3240	4875,62	0,832
Total	D1	0,873	62,711	0,0614	9,36	0,0649	4,275	0,802	2401	767,264	13,434
	D2	0,852	73,628	0,0851	13,567	0,0649	5,584	0,729	1314	1275,056	14,515
	D3	0,794	75,237	0,1246	21,436	0,0334	2,551	0,88	348	1908,366	334,542
	D4	0,89	65,314	0,1159	14,129	0,1264	9,641	0,964	2681	1932,746	1,687
	D5	0,926	110,936	0,1224	26,215	0,0312	7,279	0,991	118	3410,849	8,209
	D6	0,95	110,29	0,1425	45,248	0,1957	40,782	0,998	75745	4211,004	0,003
Veolia	D1	0,8	51,954	-0,0034	-0,381	-0,0169	-2,083	0,601	19209	712,128	3,719
	D2	0,863	46,134	-0,0316	-3,047	0,0247	1,487	0,448	8043	1230,49	16,82
	D3	1,01	72,509	-0,0391	-6,752	-0,0676	-5,585	0,786	10245	1806,172	279,243
	D4	1,013	66,798	-0,05	-5,947	-0,0053	-0,325	0,942	2550	2044,105	3,863
	D5	1,039	68,723	-0,0509	-8,522	0,0534	4,281	0,982	1380	3352,049	9,374
	D6	1,072	143,174	-0,1706	-46,97	0,0075	1,052	0,997	476	4518,798	1,925
Vinci	D1	1,053	76,824	0,0031	0,997	0,0068	1,386	0,795	2187	745,545	9,871
	D2	1,041	91,259	0,0224	4,651	0,0152	2,504	0,724	3144	1227,291	10,726
	D3	1,108	85,355	0,0031	0,395	0,0158	1,103	0,871	2623	1899,902	459,31
	D4	1,096	91,975	-0,0234	-5,028	0,0439	3,879	0,958	359	2109,606	1,025
	D5	1,045	121,723	-0,0222	-5,911	-0,0666	-8,804	0,985	182	3692,986	16,208
	D6	0,987	167,461	-0,0105	-2,136	-0,0166	-2,887	0,998	129	5239,722	25,223
Vivendi	D1	0,835	65,841	-0,0107	-2,41	0,0053	1,124	0,705	1484	744,222	8,912
	D2	0,804	51,11	-0,0283	-3,634	0,0082	0,561	0,584	751	1296,013	21,059
	D3	0,768	52,12	-0,0347	-5,687	0,0325	2,108	0,805	661	1984,072	404,616
	D4	0,777	85,901	-0,0183	-4,74	-0,0528	-4,837	0,945	219	1934,851	7,703
	D5	0,84	118,075	-0,0243	-7,507	-0,0987	-17,753	0,983	249	3433,533	7,929
	D6	0,814	126,653	-0,1442	-61,332	-0,004	-0,65	0,996	180	5393,483	0,917

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