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System-of-systems approach to air transportation design using nested optimization and direct search

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Abstract: Aircraft sizing, route network design, demand estimation and allocation of aircraft to routes are different facets of the air transportation optimization problem that can be viewed as individual "systems," since they can be conducted independently. In fact, there is a large body of literature that investigates each of these as a stand-alone problem. In this regard, the air transportation design optimization problem can be viewed as an optimal system-of-systems (SoS) design problem. The resulting mixed variable programming problem may not be solvable using an all-in-one (AiO) approach because its size and complexity grow rapidly with increasing number of network nodes. In this work, we use a decomposition-based nested formulation and the Mesh Adaptive Direct Search (MADS) optimization algorithm to solve the optimal SoS design problem. The two-stage expansion of a regional Canadian airline's network to enable national operations is considered as a demonstrating example.

Key Words: Systems of systems, aircraft sizing, network design, route allocation, derivative-free optimization, mesh adaptive direct search.

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1 Introduction

Air transportation design involves objectives and constraints related to multiple disciplines such as aircraft sizing, route network configuration, demand estimation and allocation of aircraft to routes. After the deregulation of the U.S. airline industry in the late 1970s, revenue margins have shrunk due partially to the extremely competitive business environment. Airlines have to ensure lean, cost-effective operations to stay in business, as summarized by Bazargan's "survival of the fittest" motto [1]. Operations research and optimal system design are key tools to optimize airline operations.

Traditionally, the majority of research work in this area has focussed on improving performance in individual disciplines separately by treating extradisciplinary quantities as fixed parameters (for example, allocate aircraft to routes assuming fixed aircraft and network designs). Given the interactions among the different disciplines involved in air transportation design, an integrated approach to the associated optimization problem may be beneficial.

1.1 Air transportation as a system of systems

While there is no universally accepted definition for a System of Systems and its properties, the main characteristic of an SoS is that it is "larger than the sum of its parts" in the sense that it can provide functionalities that none of the systems it consists of can provide on their own [2]. In addition, what differentiates the SoS comprising systems from subsystems is the fact that they are independently operational in the sense that they can be considered as stand-alone entities [3]. In this regard, aircraft sizing, route network configuration, demand estimation and allocation of aircraft to routes can be viewed as systems that are integrated to form an air transportation system of systems. Each of these systems have individual constraints that must be satisfied to obtain a feasible solution to the SoS problem. An SoS objective function is formulated to drive the design process while accounting for possible "local" system objectives.

There are only a few reports of integrated approaches to air transportation design in the literature. Taylor and de Weck proposed a methodology for linking the design and allocation problems in an overnight air-cargo delivery system [4]. Mane, Crossley and Nusawardhana presented a decomposition-based SoS approach to aircraft design and allocation [5] . They extended this approach to aircraft design and operations planning under uncertainty for a fleet of single aircraft type for a fractional management company [6]. In his Ph.D. dissertation, Kotegawa analyzed the air transportation SoS using network theory and generated a multilayer network with distinct levels for demand, mobility and capacity [7]. Finally, Davendralingam and Crossley developed a dynamic econometric model to account for changes in demand and ticket prices over time[8].

These works did not consider route network configuration and its impact to the air transportation SoS design. Moreover, Mane et al. considered only hub-spoke networks in their investigations. In this paper, we introduce network design to the SoS problem formulation and address the aspect of unfulfilled demand in allocating aircraft to routes as the network is being re-configured to minimize total direct operating cost. We also treat aircraft passenger capacity as an additional SoS variable. Finally, we enhance a demand estimation model of the literature to account for geographic aspects. We use a nested formulation to decompose our SoS problem formulation since its size and complexity can grow exponentially with increasing number of network nodes. In addition, we utilize the derivative-free Mesh Adaptive Direct Search (MADS) optimization algorithm to solve the overall SoS optimization problem since we can not rely on gradient approximations.

We consider the example of a regional airline expanding its operations to become a national carrier to demonstrate our air transportation SoS design approach. A new aircraft that can fly longer routes and satisfy increased demand must be inducted into the fleet. If the airline were to consider only existing fleet and fly only on fixed routes, an integer programming problem could be formulated and solved using existing allocation algorithms. However, when we consider a new aircraft design corresponding cost coefficients are unknown. Furthermore, if we are free to configure the expanding network routes, passenger demand will be a function of network structure. The SoS design optimization problem is therefore a mixed variable program involving continuous and integer variables.

The paper is organized as follows. We present the SoS problem formulation, the demand estimation model, and the optimization subproblems related to aircraft sizing, network configuration and aircraft to routes allocation. We then consider a two-stage expansion of an eastern regional 5-city network to a national network for the Canadian airspace: we first introduce two major destinations in East Canada, and then more than double the number of cities in the network from seven to fifteen. We present results and offer a comparison to both the hub-spoke networks presented in [5] and actual data for a medium-sized airline. We conclude with a summary and a discussion of future work.

2 Problem formulation

We present the formulation of the air transportation SoS design optimization problem using the example of a Canadian regional airline expanding to national operations. Specifically, the regional airline is assumed to be operating in the western part of Canada, connecting five major cities (Vancouver, Victoria, Kelowna, Calgary and Edmonton). The considered expansion will connect this local network to the main two metropoles on the east of Canada: Toronto and Montreal. Figure 1 shows the seven cities. For a network of n cities, it is possible to have $n(n-1)/2$ routes.[1] For the current problem we have 7 cities and therefore 21 possible routes.

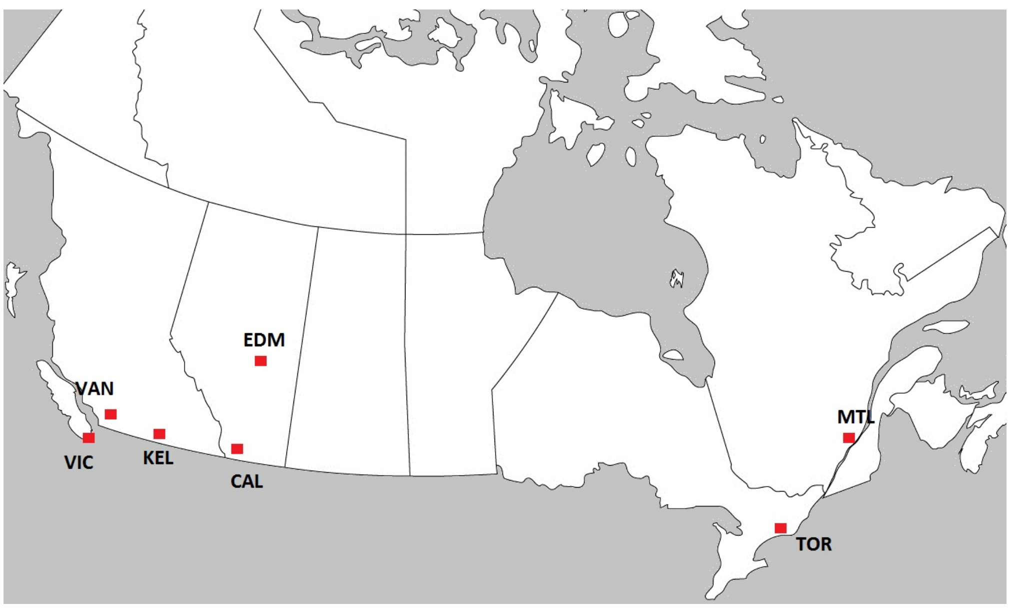

Figure 1: The 7 cities of the considered network

2.1 Demand estimation

The first thing we need to estimate is passenger demand for each possible route. Bhadra developed a semi-log linear model for origin-destination (OD) pairs [9]. His model was calibrated against real air passenger traffic data, but depends on several variables that are specific to the U.S.. As we are interested in OD demand figures for Canada, we need a general model. Jorge-Calderón developed a gravity model for OD passenger estimation [10]. This model is based on variables such as population, income, hub airports and their proximity, distance, and existence of body of water between cities. Grosche, Rothlauf and Heinzl expanded upon this model to consider geographic, demographic and socioeconomic variables and take into account competing airports in a specific airport's demand figures [11]. This model has been shown to be more accurate than the one of Jorge-Calderón. In our study we incorporate a slightly modified version of the Grosche et al. model by introducing two additional variables (language factor and ocean influence) to reflect differences in Franco-

[1] We assume that flights between cities are reciprocal.

and Anglo-phone passenger travel behavior and the denser population near the two coasts of the Canadian landscape, respectively. The model is given by

$$V_{ij} = \frac{1}{c} P_{ij}^{\alpha} C_{ij}^{\beta} B_{ij}^{\gamma} G_{ij}^{\delta} D_{ij}^{\epsilon} T_{ij}^{\zeta} L_{ij}^{\eta} O_{ij}, \tag{1}$$

where i and j denote origin and destination, respectively. Table 1 lists the variables used in Eq. (1). Grosche et al. improved the accuracy of their model by classifying the routes into distance intervals, with each interval calibrated separately. Three distance intervals are considered: short (less than 800 km), moderate (between 800 and 2,000 km) and long (more than 2,000 km). The language and ocean factors were held constant for each interval.

Table 1: The variables of the modified demand model

Variable	Description
$P_{ij} = P_i P_j$	Population
$C_{ij} = C_i C_j$	Catchment
$B_{ij} = B_i + B_j$	Buying power index
$G_{ij} = G_i G_j$	Gross domestic product
D_{ij}	Geographical distance
T_{ij}	Average travel time
L_{ij}	Language factor
O_{ij}	Ocean influence

The most recent Canadian data for air traffic between all domestic OD pairs was released in 1999. However, in its 2011 report, Transport Canada released daily seat numbers for the top 25 city pairs based on domestic demand [12]. We used these data to calibrate the model of Eq. (1), i.e., we used non-linear least-squares regression to determine the coefficients for each variable, including the normalizing coefficient c. We then used the model to generate route demand for the considered network (Table 2).

Table 2: Demand model coefficients for different distance intervals

Distance	α	β	γ	δ	ϵ	ζ	η	c
$<$ 800 km	1.393	9.123E-6	3.528	-0.859	0.259	-0.240	-2.504	1.491E+6
800-2000 km	1.565E-5	0.478	0.240	-0.112	3.578	-6.385	-2.504	1.499E+8
$>$ 2000 km	0.351	4.223E-5	0.0001	0.409	1.732	-1.547	-2.504	1903E+8

2.2 SoS design optimization problem

The objective of the air transportation SoS design optimization problem is to minimize the daily direct operating cost of the entire aircraft fleet (DOC_F) necessary to satisfy demand as close as possible. The existing fleet of the regional airline consists of two aircraft types, A and B, whose range is $2,000$ mi and $2,400$ nmi, respectively. We assume that a third, new, aircraft type X will be necessary for the expansion to a national network. The new aircraft must be sized with respect to passenger capacity P_X and range R_X to minimize its direct operating cost DOC_X for each possible route. In addition, we need to design the route network and allocate the entire aircraft fleet (types A, B and X) to the active routes to minimize DOC_F. Therefore, in our design model,

$$DOC_F = f(DOC_X(\mathbf{d}_X, R_X, P_X), \mathbf{l}, \mathbf{x}, \mathbf{y}), \tag{2}$$

where $\mathbf{d_X}$ are aircraft design variables, $\mathbf{l} = \{l_{ij}\}$ are binary variables defining whether a route is active (i.e., whether there is a direct flight between cities i and j), $\mathbf{x} = \{x_{pij}\}$ are allocation variables of aircraft of type $p \in \{A, B, X\}$ to route connecting cities i and j and $\mathbf{y} = \{y_{ij}\}$ are unfulfilled demand variables for route connecting cities i and j.

In addition to minimize DOC_F with respect to all the variables shown in Eq. (2), we have to satisfy a number of aircraft sizing, network design and allocation constraints. The associated optimization problem is a constrained nonlinear mixed variable program, whose size and complexity grow exponentially with increasing number of network nodes (cities). Therefore, we have decomposed this problem using a nested formulation as shown schematically in Figure 2. We call this a nested formulation because aircraft sizing, network design and aircraft allocation are all optimization problems themselves.

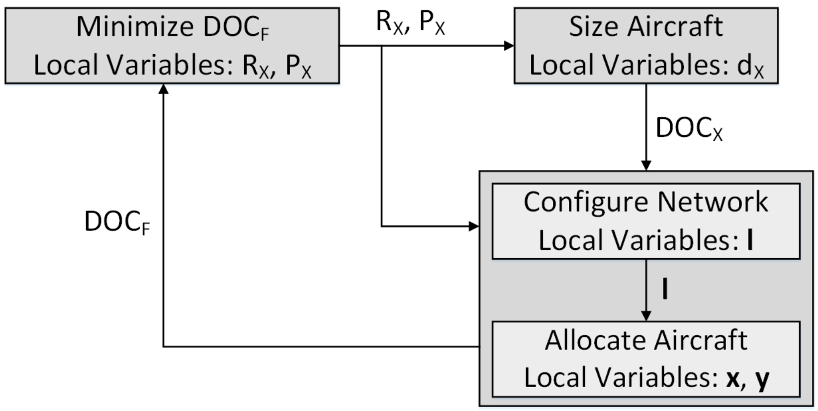

Figure 2: Nested SoS problem formulation

Eq. (2) shows the explicit dependence of DOC_F only with respect to optimization variables; fixed parameters, such as direct operating cost for aircraft types A and B and demand are not shown explicitly. Table 3 lists demand, distance and DOC for existing fleet (types A and B) along all possible routes. Note that aircraft A can not fly the route MTL–VIC because the route distance is greater than the aircraft range; therefore, the corresponding entry is empty. The corresponding 3-tuple $\langle A, 1, 4\rangle$ is included in the set $\xi(\mathbf{R})$ introduced later in Eq. (6).

Table 3: Demand, distance (km) and DOC ($) values

Route No.	OD Pair	Demand	Distance	DOC_A	DOC_B
1	TOR–VAN	3,433	3,358	42,793	38,443
2	TOR–MTL	5,602	504	11,604	10,930
3	TOR–CAL	2,457	2,710	35,709	32,194
4	TOR–EDM	2,101	2,705	35,654	32,145
5	TOR–VIC	720	3,389	43,127	38,737
6	TOR–KEL	491	3,093	39,897	35,888
7	VAN–MTL	893	3690	46,419	41,642
8	VAN–CAL	2,870	674	13,467	12,574
9	VAN–EDM	1,904	819	15,045	13,965
10	VAN–VIC	875	94	7,121	6,975
11	VAN–KEL	630	270	9,046	8,673
12	MTL–CAL	646	3,023	39,136	35,217
13	MTL–EDM	549	2,977	38,629	34,770
14	MTL–VIC	187	3,730	——	42,029
15	MTL–KEL	128	3,421	43,478	39,048
16	CAL–EDM	1,812	279	9,149	8,764
17	CAL–VIC	1,059	729	14,067	13,103
18	CAL–KEL	574	406	10,532	9,984
19	EDM–VIC	1,022	894	15,864	14,687
20	EDM–KEL	560	579	12,429	11,658
21	VIC–KEL	242	325	9,652	9,207

To get an idea of the behavior of the DOC_F, we conducted a modest full-factorial design-of-experiments with respect to range and capacity. The obtained response surface is depicted in Figure 3. The empty (white) regions of the DOC_F response surface represent failure of the nested optimization problems to return a value. Without going into details, since we have not yet presented the nested problem formulations, this can happen for one of the following reasons:

1. The aircraft sizing problem fails to
   (a) satisfy its constraints or
   (b) perform a cost analysis for some route.
2. The allocation problem becomes integer-infeasible.

Moreover, we can see that the DOC_F response surface is monotonically decreasing in capacity and extremely flat in the range dimension for a large part of the considered design space ($> 2,000$ nmi). In fact, if we zoom in the 1-d projection of the response surface on the range design space we can see that while DOC_F is practically constant for fixed capacity along the range interval [2000, 3000] nmi, there are two local minima around 2,150 and 2,900 nmi. These observations and the unavailability of gradients for the "analysis" defined by the nested problems with respect to R_X and P_X necessitate the use of a derivative-free optimization method for the outer-loop SoS problem. We use the Mesh Adaptive Direct Search (MADS) algorithm [13, 14, 15] because of its rigorous convergence properties and its ability to solve moderate-size problems with tens of variables.[2] We used the Matlab interface to the C++ implementation of MADS offered in the publicly available OPTI toolbox [16].

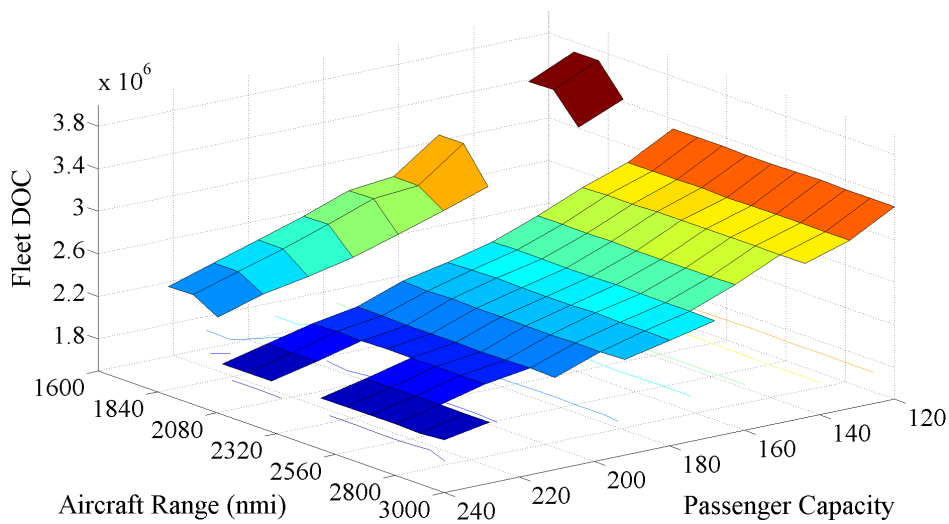

Figure 3: DOC_F response surface

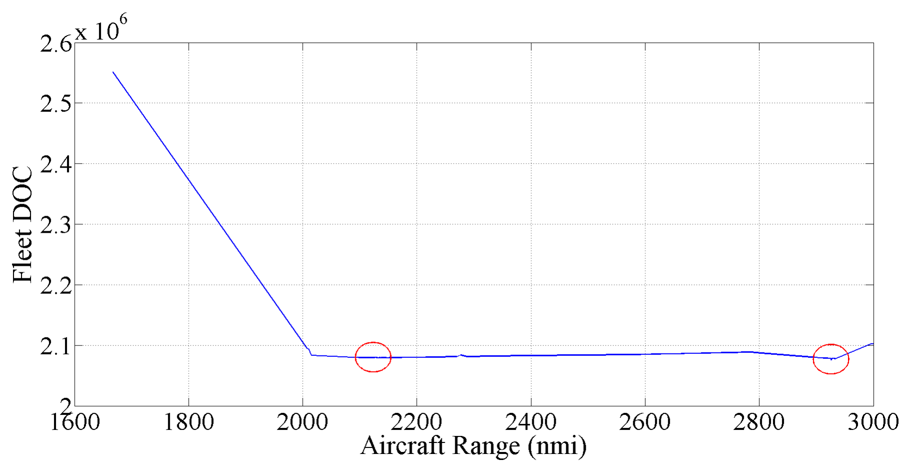

Figure 4: DOC_F vs. range for fixed capacity

[2]We use MADS to solve the network design optimization problem as well.

2.3 Aircraft sizing

The airline needs to expand its aircraft fleet to satisfy demand along new routes. It is desired to design an aircraft that best fulfills the demand requirements of the new route network while minimizing its operating cost. We use NASA's Flight Optimization System (FLOPS) that is routinely used for aircraft sizing and cost estimation in the aeronautics industry [17]. It consists of several optimization routines for complete aircraft design that can be implemented with few input variables; the remaining variables are computed internally through relations described in [18]. For our design problem, we used the Broyden-Fletcher-Goldfarb-Shano (BFGS) option.

Before formulating and solving our aircraft sizing problem, we sized four mid-range domestic aviation aircraft using FLOPS data from [19]. Our goal was to select the best possible set of values for the parameters and variable bounds of the aircraft sizing problem. Figure 5 reports relative error for aircraft empty weight, which are in the same order of magnitude as those reported in [20]. Based on these results, we selected the

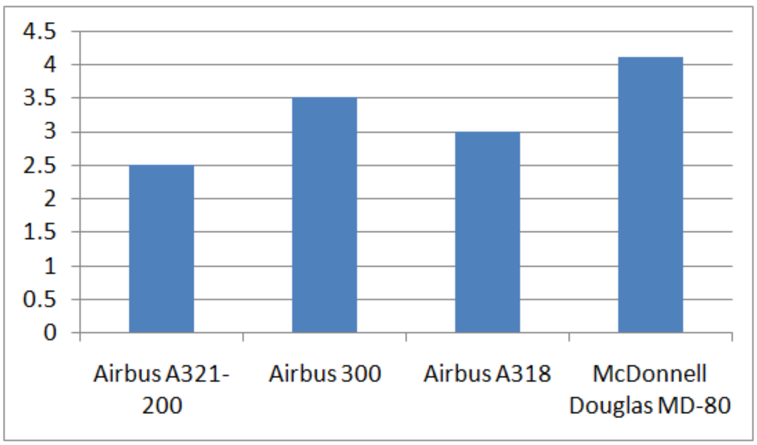

Figure 5: Relative error of aircraft empty weight (FLOPS vs. real data)

Airbus A321-200 aircraft as it exhibited the smallest relative error. Figure 6 presents a comparison between FLOPS output and actual Airbus A321-200 design data from [19].

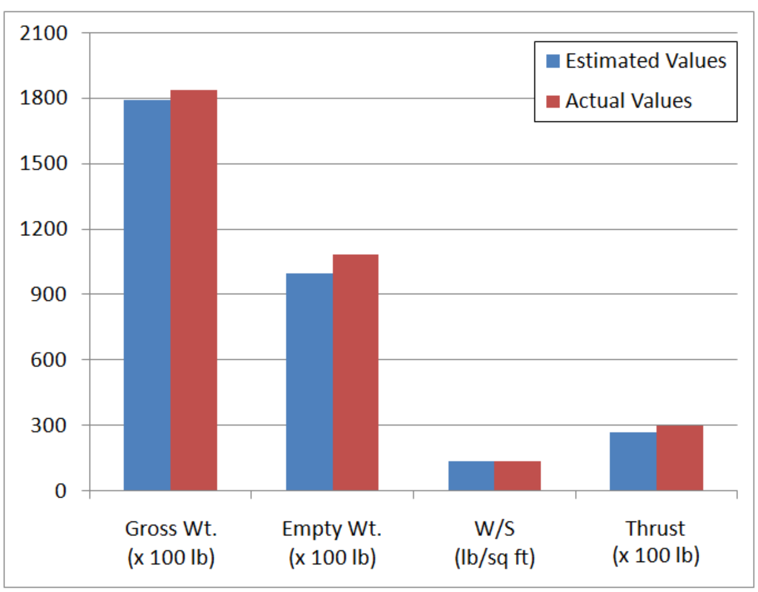

Figure 6: FLOPS vs. real data for Airbus A321-200

The aircraft sizing optimization problem is formulated to minimize aircraft DOC_X as a function a set of aircraft design variables d_X, namely wing aspect ratio $(A/R)_X$, thrust to weight ratio $(T/W)_X$ and wing loading $(W/S)_X$. Note that range R_X and capacity P_X are held fixed in the FLOPS aircraft sizing as they are determined in the outer-loop SoS problem. A constraint on aircraft take-off length is imposed to ensure that the aircraft to be added to the existing fleet can operate in all airports; the right hand side is the smallest airfield length among all airports considered here. The aircraft sizing problems formulation is

$$\begin{aligned} \min_{\mathbf{d}_X} \quad & DOC_X(\mathbf{d}_X; R_X, P_X) \\ \text{subject to} \quad & 6.5 \leq (A/R)_X \leq 10.5 \\ & 98 \leq (W/S)_X \leq 150 \\ & 0.3 \leq (T/W)_X \leq 0.5 \\ & s_{TO}(R_X, P_X, (T/W)_X, AR_X, (W/S)_X) \leq 8{,}990 \ \text{ ft} \end{aligned} \tag{3}$$

Figure 7 summarizes schematically the aircraft sizing problem.

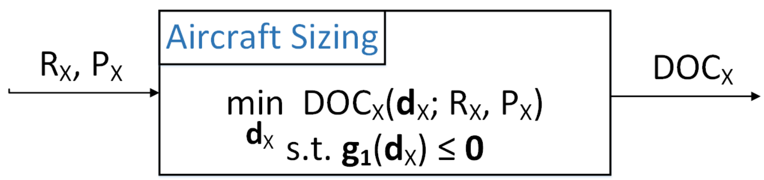

Figure 7: Aircraft sizing problem

To compute DOC_X for all possible network routes, a mission profile must be defined in FLOPS. The basic mission profile used here includes a quick ascent to cruise altitude of 30,000 ft, cruise-climb at constant velocity of Mach 0.82, and a quick descent to the airport for landing. We consider a loiter time of 20 minutes for the scenario where the aircraft does not get immediate clearance for landing.

2.4 Network design

The routes along which an airline operates can be modeled as a network so that the nodes represent cities and the links between different nodes represent direct flights. The network design problem consists of finding the best configuration that connects the different cities to minimize the operating cost of the whole fleet (DOC_F). We make the following assumptions:

1. Not all cities are necessarily connected to each other; for a pair of cities connected by a direct flight, the corresponding link is active and vice-versa.
2. Demand is symmetric between each pair of cities: the number of passengers traveling along a flight path is same in both directions.
3. Each airport in the network has sufficient resources to handle all incoming and outgoing traffic.
4. The network is undirected, simple and connected. This means that each city has at least one link that originates/terminates there and that there can be no more than one link connecting two cities.

We define the binary symmetric linking matrix $\mathbf{l}$ to represent network configuration:

$$l_{ij} = \begin{cases} 1, & \text{if there exists a direct flight between } i \text{ and } j \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

An airline does not operate direct flights between all cities it operates in due to the indirect costs associated with operating a route (IOC). Therefore, airlines operate only along profitable routes. Since profits depend on pricing strategies that vary among airlines, we minimize DOC_F as a surrogate for maximizing profit. By omitting pricing and IOC estimates, we are able to construct a general model that can be adopted by all airlines.

If l_{ij} is 0 for a pair of two cities i and j, then the corresponding demand has to be routed through other cities using connecting flights. Since direct flights are always less expensive than flights routed through other cities, the network configuration problem favors a large number of active links in the network. However, as discussed previously this is practically not feasible due to associated indirect operating cost. The number of active links in the network is therefore bounded. For demand that is routed through other cities, passengers have to change flights. For a practical solution that reflects actual passenger handling and preferences, it is required to limit the number of flight connections.

It is also observed that certain links are critical to the functioning of the network as there are multiple flights routed through them. In order to identify and preserve these links during different iterations we use standard network properties such as Betweenness and Eigencentrality by posing related constraints. The network configuration problem is formulated as

$$\begin{array}{rc} \min\limits_{\mathbf{l}} & DOC_F(\mathbf{l}; R_X, P_X, DOC_X) \\ \text{subject to} & m_l \leq \sum_{i=1}^{n} \sum_{j=i+1}^{n} l_{ij} \leq m_u \\ & \sum_{i=1}^{n} l_{ij} \geq 1 \\ & l_{ij} = 1 \quad \forall \; i,j \in \Pi \\ & LFC(\mathbf{l}) \leq \lambda \\ & b_l \leq \sum_{i=1}^{n} \sum_{j=i+1}^{n} b_{ij}(\mathbf{l}) \leq b_u \\ & c_l \leq \sum_{i=1}^{n} c_i(\mathbf{l}) \leq c_n, \end{array} \tag{5}$$

In the problem of Eq. (5) the first constraint sets the bounds for the number of active links in the network. The second constraint ensures that each city has at least one active link. If a city has to serve as a hub, it is an element of the set Π; the third constraint ensures that it is connected to all other cities. The fourth constraint bounds the allowable least number of flight changes for any pair of cities. Finally, the last two constraints bound betweenness and eigencentrality measures. Table 4 lists the values of the bounds for the 7-city problem. The bound for least flight changes LFC is based on actual passenger movements. Other values in Table 4 are based on the formulation described by Liu et al. [21]. Current literature does not prescribe means for setting upper and lower bounds for the number of active links in the network without considering IOC and pricing estimates. For the problem described here, we select a lower bound that is greater than a minimum spanning network. The upper bound is based on actual observed flight itineraries. It is possible to define a more rigorous procedure for selecting these bounds but not doing so does not have any effect on the usefulness or the validity of the presented methodology.

Table 4: Parameter values of the 7-city network configuration problem

Parameter	m_l	m_u	λ	b_l	b_u	c_l	c_u
Value	9	16	2	0.8	2	2	3

The objective function value for DOC_F is computed at the aircraft allocation problem, which takes as input the network configuration and allocates aircraft to minimize DOC_F while satisfying demand on all active routes as close as possible. The network design and aircraft allocation are thus nested. We use MADS to solve the outer-loop of this nested optimal network configuration problem, whose schematic is depicted in Figure 8.

2.5 Allocation of aircraft to routes

Figure 9 depicts the respective schematic of the aircraft to routes allocation problem. The goal of the allocation problem is to assign aircraft to all active routes in a given network while satisfying passenger

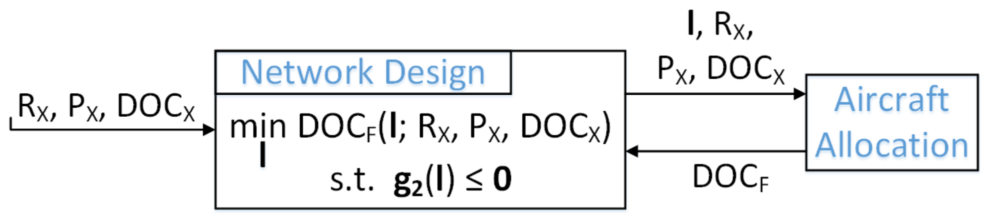

Figure 8: Network configuration problem

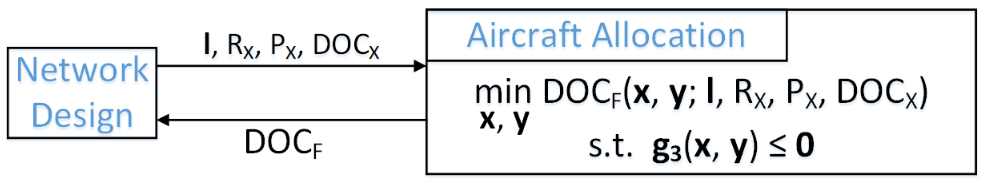

Figure 9: Aircraft to route allocation problem

demand. Since not all links are necessarily active, it is possible that passenger demand between two nodes may need to be routed through other active links. Aircraft allocation first routes this demand along the shortest paths and then determines the best allocation for satisfying this demand. Since the number of aircraft and unfulfilled demand are integers, the resulting problem is an integer program.

We solve this integer program using the model described in [22]. The objective function is to minimize the fleet operating cost, and the optimization variables are i) number of aircraft x_{pij} of a given type $p \in T = \{A, B, X\}$ assigned along each active route and ii) unfulfilled demand y_{ij} along each active route. Since the number of aircraft is limited, an upper bound is imposed for the total number of allocated aircraft for each type. We assign a high penalty weight w_{ij} for unfulfilled route demand, and also impose a bound for the maximum allowable unfulfilled demand along any route. The airline must meet demand along all active routes. An additional constraint is imposed on x_{pij} to limit the number of aircraft of a given type flying along a route; this is necessary since in practice each aircraft requires a different maintenance crew and other operations before take-off and after landing. Not all aircraft can fly each route in the network; thus, a constraint is added to the formulation to ensure that no aircraft is allocated to a route that is longer than the aircraft range. The allocation problem formulation is

$$\begin{aligned} \min_{\mathbf{x},\mathbf{y}} \quad & \textstyle\sum_{p\in T}\sum_{i=1}^{n}\sum_{j=i+1}^{n}[l_{ij}DOC_{pij}x_{pij} + w_{ij}y_{ij}] && (6)\\ \text{subject to} \quad & \textstyle\sum_{p\in T}\sum_{i=1}^{n}\sum_{j=i+1}^{n}[P_p x_{pij} + y_{ij}] \geq V_{ij} \\ & \textstyle\sum_{i=1}^{n}\sum_{j=i+1}^{n} x_{pij} \leq s_p \quad \forall \;\; p \in T \\ & y_{ij} \leq \delta_{ij} \quad \forall \;\; i,j \text{ with } i > j \\ & x_{pij} \leq h_{pij} \quad \forall \;\; p,i,j \text{ with } i > j \\ & x_{pij} = 0 \;\; \forall \; \langle p,i,j\rangle \in \xi(R_A, R_B, R_X), \end{aligned}$$

where

$$w_{ij} = \frac{2.5}{\sum\limits_{p\in T} f_{pij}}\left[\sum_{p\in T} f_{pij}\left(\frac{DOC_{pij}}{P_p}\right)\right] \tag{7}$$

and $\xi(R_A, R_B, R_X) = \{\langle A, 3, 6\rangle, \ldots\}$ is the set of 3-tuples describing infeasible aircraft range of an aircraft type for a given route.

For the 7-city problem we set w_{ij} equal to 2.5 times the DOC-per-passenger averaged over all aircraft as shown in Eq. (7). The variable f_{pij} in Eq. (7) is a binary variable whose default value is 1 but becomes 0 for any $\{pij\}$ in the set $\xi(R_A, R_B, R_X)$.

The right hand side of the first constraint represents the routed demand D_{ij} along that route. For routes that are not active in $\mathbf{l}$ this demand becomes zero. For an active link l_{ij}, routed demand D_{ij} is the sum of demand between cities i and j and the demand for any other pair of cities r and s that are not connected directly and the shortest path between them includes the link l_{ij}.

The second constraint ensures that the number of aircraft allocations does not exceed fleet size. The 3rd and 4th constraints prescribe bounds on y_{ij} and x_{pij}, respectively. The 5th constraint ensures the range feasibility constraints are always met. Allocation x_{pij} where $\{pij\}$ belongs to the set of 3-tuples is always zero. For example, $\xi(R_A, R_B, R_X)$ includes $\{A36\}$ – this is because the corresponding route distance is greater than range R_A of aircraft A.

Table 5 lists the bound values for the problem of Eq. (6). We have a constant value for δ_{ij} for all routes but it is possible to bound y_{ij} along individual routes. Similarly, allocation of aircraft of a given type along all routes has a constant upper bound which may be replaced with individual route bounds.

The allocation problem is solved using the GNU Linear Programming Kit (glpk) adapted for Matlab [23].

Table 5: Bound values for the allocation problem

Bound	s_p	$\delta_{ij}\ \forall i,j$	$h_{pij}\ \forall i,j$
Value	$\{70, 100, 100\}$	20	$\{25, 10, 20\}$

3 Results

We solved the outer-loop SoS optimization problem using 4 different initial guesses for capacity and range. For each of these runs, the initial network configuration of the nested optimal network design problem was the one shown in Figure 10.

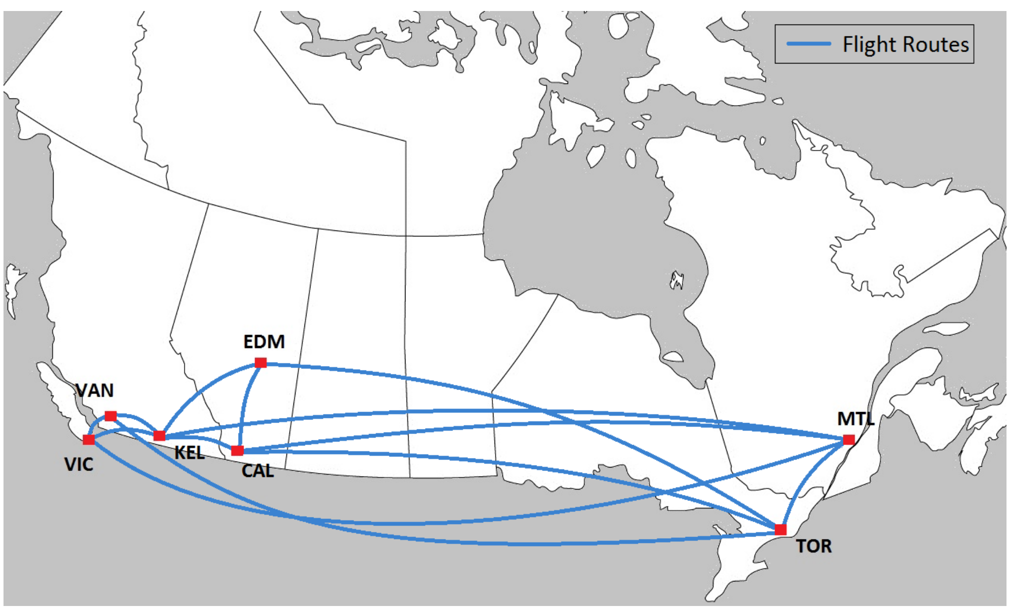

Figure 10: Initial network configuration for the 7-city problem

Table 6 reports the results obtained for each optimization run. We can see that the 4 runs produced two local optima, confirming the information obtained by investigating the DOC_F response surface (Figure 4). This also demonstrates the ability of MADS to find "good" minima when the objective function is extremely "flat" in at least one of the variables.

Figure 11 depicts the optimal network configuration. While this figure highlights the new routes, note that some "baseline" routes have been removed (e.g., KEL-VAN and KEL-CAL). The optimal aircraft design $\mathbf{d_X}$ is reported in Table 7. The corresponding DOC_X is listed in Table 8 for all routes, regardless of whether they are active in the optimal network. The optimal aircraft allocation for all aircraft types and the unfulfilled demand for all routes is listed in Table 9.

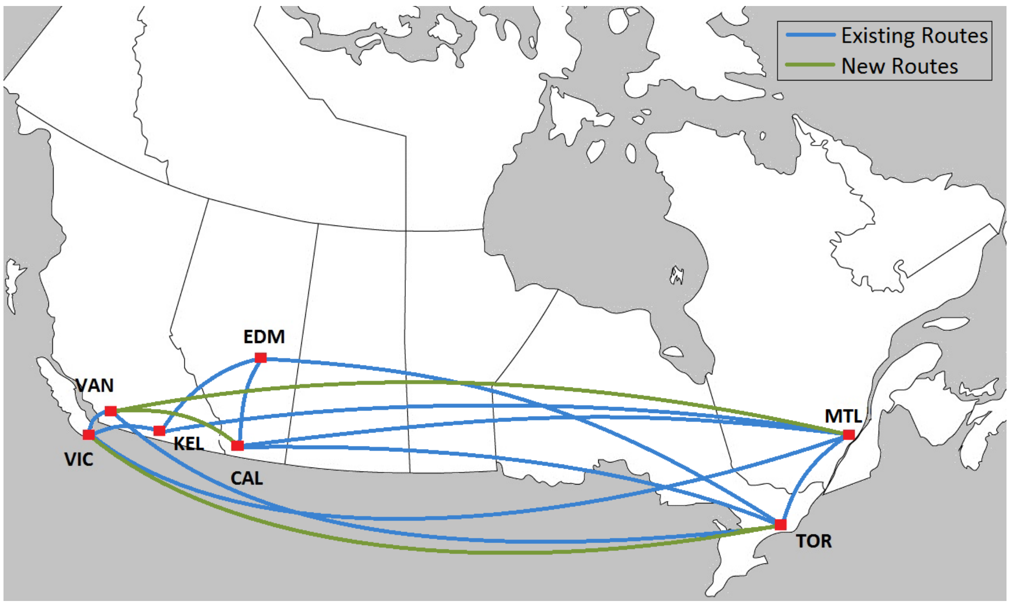

Figure 11: Optimal network configuration for the 7-city problem

Table 6: Results for the 7-city problem

Variable	Run 1	Run 2	Run 3	Run 4
Initial R_X (nmi)	2,015	2,933	2,500	1,800
Initial P_X	240	240	170	170
Initial DOC_F ($)	3,090,187	2,114,174	2,816,993	3,028,026
Optimal R_X (nmi)	2147	2926	2147	2147
Optimal P_X	240	240	240	240
Optimal DOC_F ($)	2,079,355	2,075,566	2,079,355	2,079,355
No. of active routes	14	14	14	14

Table 7: Optimal design of aircraft X for the 7-city problem

Variable	Value
Range (nmi)	2,147
Gross weight (lb)	181,600
Wing aspect ratio (-)	9.69
Thrust per engine (lb)	27,000
Wing area (sq. ft)	1,311
Cruise velocity (Mach)	0.82
Wing loading (lb / sq. ft)	138.50
Thrust to weight ratio (-)	0.31
Take-off distance (ft)	8,990

Table 8: Optimal DOC_X ($) for the 7-city problem

	VAN	MTL	CAL	EDM	VIC	KEL
TOR	25,461	7,979	21,461	21,430	25,650	23,822
VAN		27,518	9,020	9,900	5,471	6,548
MTL			23,393	23,107	27,768	25,850
CAL				6,606	9,355	7,380
EDM					10,357	8,440
VIC						6,887

Table 9: Optimal aircraft allocation and unfulfilled demand for the 7-city problem (inactive routes left out for the sake of brevity)

Route Number	Aircraft A	Aircraft B	Aircraft X	Unfulfilled Demand
1	—	—	15	—
2	9	1	20	—
3	—	—	11	—
4	—	—	11	11
5	—	—	3	—
7	—	—	4	—
8	—	—	20	—
10	14	—	—	—
12	—	—	3	—
14	—	—	1	—
15	—	—	1	—
16	25	—	1	—
20	—	—	7	—
21	4	—	3	—

3.1 Solving the SoS problem all in one

It is possible to combine the aircraft sizing, network design and allocation subproblems to solve the SoS design optimization problem using an all-in-one (AiO) approach. The AiO problem formulation is a mixed integer nonlinear program (MINLP), which can be more difficult to solve and require longer computation time. We solved the AiO 7-city problem using NOMAD with the same initial guesses used to solve the aircraft sizing, network design and allocation subproblems in the nested formulation. The obtained results differed only for the optimal range, i.e., optimal capacity, aircraft sizing, active network routes and allocation of aircraft to routes were the same as for the decomposed problem formulation. As summarized in Table 10, the local optimal range values for the AiO problem are very near the ones for the decomposed problem; the best "AiO" objective value is only 0.1% worse then the best "decomposed" objective value. However, the AiO problem required more than 4 times longer computation time than the one of the decomposed formulation (80 and 17 minutes, respectively, on a 64-bit Intel 7 processor with 4 cores and 8192 Mb of RAM.) We anticipate that this discrepancy will grow rapidly when considering larger-size problems. In fact, as we will explain later, we were not able to obtain a solution to the AiO problem for a network of fifteen cities.

Table 10: Results for the AiO problem formulation (7-city problem)

Variable	Run 1	Run 2
Initial R_X (nmi)	2015	1800
Optimal R_X (nmi)	2124	2920
Optimal DOC_F($)	2,124,600	2,078,618

3.2 Comparison to hub-spoke networks

Mane et al. considered fixed hub-spoke network configurations in a thirty-one route problem [5]. They employed a sequential decomposition approach that has only P_X as the SoS-level variable. The aircraft range R_X is assumed to be the longest route distance in the network. Further, their allocation algorithm does not allow for unfulfilled demand. Mathematically this implies that an aircraft is allocated to a route for even small residual demand. Allowing for unfulfilled demand allows us to model airline transportation more realistically since airlines tend to ensure highest possible capacity utilization for each aircraft.

Following the approach of Mane et al., we solved the 7-city problem using fixed hub-spoke networks, sizing the new aircraft for a fixed range of 2015 nmi. We considered two different hub-spoke network configurations (one with Calgary as the hub-city and one with Toronto (see Figure 12), and compare the results to the "free" network configuration approach described in the previous sections. Since non-hub cities are not connected to each other, the demand between them is routed through the hub-cities.

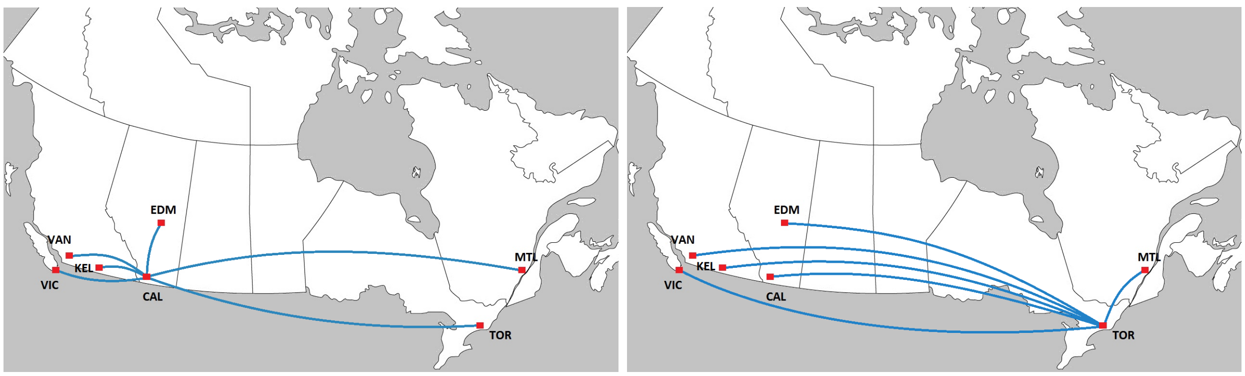

Figure 12: Networks with Calgary or Toronto as hub-cities for the 7-city problem

The allocation and DOC_F results are given in Tables 11 and 12, respectively. While the hub network with Calgary as the hub city is 7% less expensive to operate, it is dramatically (almost three times) more expensive than the optimal "free" network configuration.

Table 11: Optimal aircraft allocation and unfulfilled demand for Calgary and Toronto single-hub networks for the 7-city problem (route numbers kept same but only possible routes shown)

Route Number	Calgary				Toronto			
	A_j	B_j	X_j	y_j	A_j	B_j	X_j	y_j
1	—	—	—	—	40	2	20	4
2	—	—	—	—	36	1	12	—
3	50	30	20	4	33	—	20	—
4	—	—	—	—	22	1	20	—
5	—	—	—	—	—	—	17	25
6	—	—	—	—	—	—	11	—
8	40	2	20	4	—	—	—	—
12	23	—	20	—	—	—	—	—
16	36	—	12	29	—	—	—	—
17	—	—	17	25	—	—	—	—
18	—	—	11	—	—	—	—	—

Table 12: DOC_F for single-hub networks for the 7-city problem

Hub City	DOC_F
Calgary	$\$5,957,316$
Toronto	$\$6,399,722$

4 Increasing the network size: A 15-city network

Medium-sized airline companies fly several hundred routes per day using different aircraft. For example, Westjet Airlines operates nearly 425 flights daily using 114 aircraft of 4 different types. It is therefore interesting to apply the nested formulation to a larger problem.

We consider an extended network consisting of fifteen cities and 105 possible routes as shown on Figure 13. While the number of variables for the SoS-level problem and aircraft sizing remains unchanged, the number of variables for the network design and aircraft allocation problems increases 5-fold from 21 and 84 to 105 and 420, respectively.

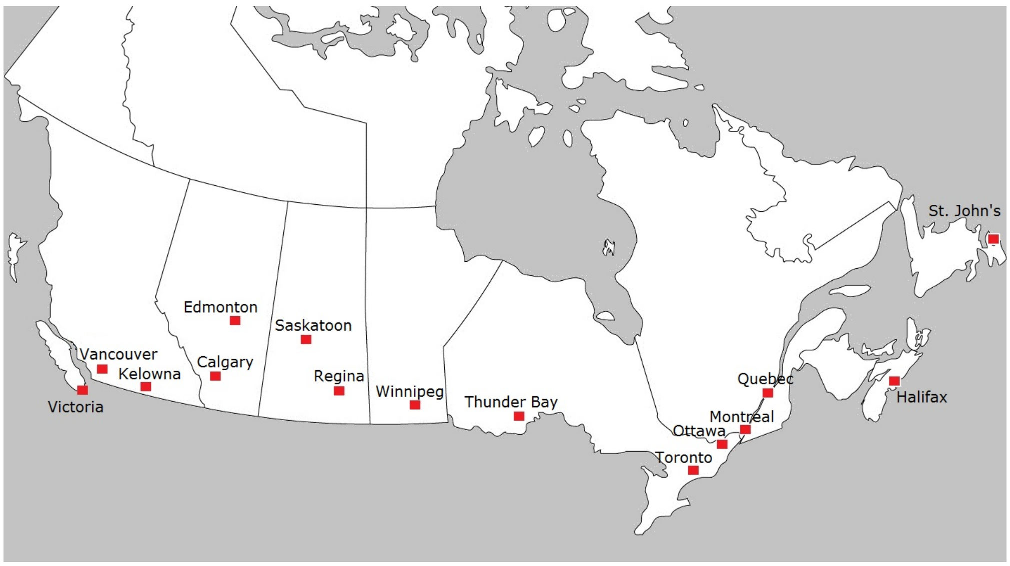

Figure 13: 15-city network

The estimated demand (computed using the model outlined in Section 2.1) for the new possible routes is listed in Table 13.

Table 13: Demand for the additional possible routes of the 15-city problem

	OTT	HAL	WIN	STJ	QBE	SKT	REG	TBY
TOR	3717	1655	1553	435	734	587	476	1086
VAN	1107	489	901	311	206	1041	914	145
MTL	902	659	408	272	380	156	128	303
CAL	796	361	1153	232	149	828	816	398
EDM	678	307	1138	196	127	766	773	395
OTT	–	1253	895	485	359	191	157	655
HAL	–	–	161	595	187	89	75	34
WIN	–	–	–	108	167	553	514	359
VIC	–	–	–	73	43	550	486	31
STJ	–	–	–	–	138	58	49	24
KEL	–	–	–	–	30	581	524	20
QBE	–	–	–	–	–	36	30	124
SKT	–	–	–	–	–	–	260	343
REG	–	–	–	–	–	–	–	360

Additional constraints are included in the nested network design and aircraft allocation problems to account for the following.

1. Aircraft B can not operate from Thunder Bay (Ontario), and aircraft X can not use all airports because of runway length limitations.[3]
2. There exist several routes that are longer than the range of the existing fleet (aircraft A and B) but lie within the bounds of R_X.

These constraints are implement by expanding the set of impermissible 3-tuples as follows

$$\langle p, i, j \rangle \ \in \ \xi(R_p, s_{TO_p}) \ \text{ if } \ s_{TOp} > \{r_i, r_j\} \ \text{ or } \ R_p < D_{ij} \ \ \forall \ p \ \in \{A, B, X\},$$

where the parameters r_i and r_j denote runway lengths for cities i and j, respectively. Based on the aircraft range and runway length constraints, the maximum number of feasible routes for each aircraft is given in Table 14 (upper bound values were used for range and runway length of aircraft X). The set ξ contains at least 83 3-tuples of infeasible aircraft-route combinations. Additional tuples are introduced to this set when R_X is smaller than the distance of an otherwise feasible route. The sharp drop in possible routes for aircraft X is due to i) the runway length constraint and ii) the fact that it can serve a maximum of 11 cities (depending on R_X).

Table 14: Maximum possible routes for each aircraft type for the 15-city problem

Aircraft Type	A	B	X
No. of Routes	91	86	55

The network design problem is also modified to include feasibility constraints that ensure that routes common to all three aircraft in the set ξ are always inactive. Mathematically, this is formulated by

$$l_{ij} = 0 \quad \forall \ \ i, j \quad \text{such that} \ \ \langle p, i, j \rangle \in \xi \ \ \forall \ p \in \{A, B, X\}. \tag{8}$$

Finally, for the larger-sized 15-city problem we used the Mixed Integer Linear Programming (MILP) solver in IBM ILOG CPLEX for increased computational efficiency [24]. The bound values for the allocation problem were modified as listed in Table 15 to ensure fulfilling the increased demand.

Table 15: Revised bound values for the allocation problem (15-city problem)

Bound	s_p	δ_{ij} $\forall i, j$	h_{pij} $\forall i, j$
Value	$\{350, 500, 500\}$	40	$\{50, 20, 40\}$

4.1 Results

Figure 14 depicts the DOC_F response surface for the 15-city problem. The empty regions indicate points where the aircraft sizing problem fails to perform a cost analysis for one or more routes. Figure 15 depicts a contour map of the same response surface. We can see that the response surface is substantially more multi-modal than the one for the 7-city problem, and can pose a large challenge to an optimization algorithm.

We solved the 15-city problem with an initial guess for capacity and range equal to the optimal values of the 7-city problem. The minimum spanning network was used as the initial network configuration for the nested optimal network design problem. Each iteration of the MADS algorithm for the SoS design optimization problem (including aircraft sizing, network design and aircraft allocation) takes nearly 90 minutes to compute on a 64-bit Intel 7 processor with 4 cores and 8192 Mb of RAM. This is in contrast to 17 minutes for finding the optimal solution to the 7-city problem (using the decomposed formulation).[4] The MADS algorithm generated 134 feasible solutions for the decomposed formulation of the 15-city problem (as opposed to 62 for the 5-city problem). The best feasible solution was obtained for $R_X = 2388$ and $P_X = 240$. This solution is

[3] We ignore Halifax otherwise the number of feasible routes for aircraft X falls to 45.

[4] As mentioned earlier, we could not solve the problem using the AiO approach (which took 80 minutes for the 7-city problem) despite several attempts as computations were exceeding 24 hours and Matlab was crashing due to memory allocation problems.

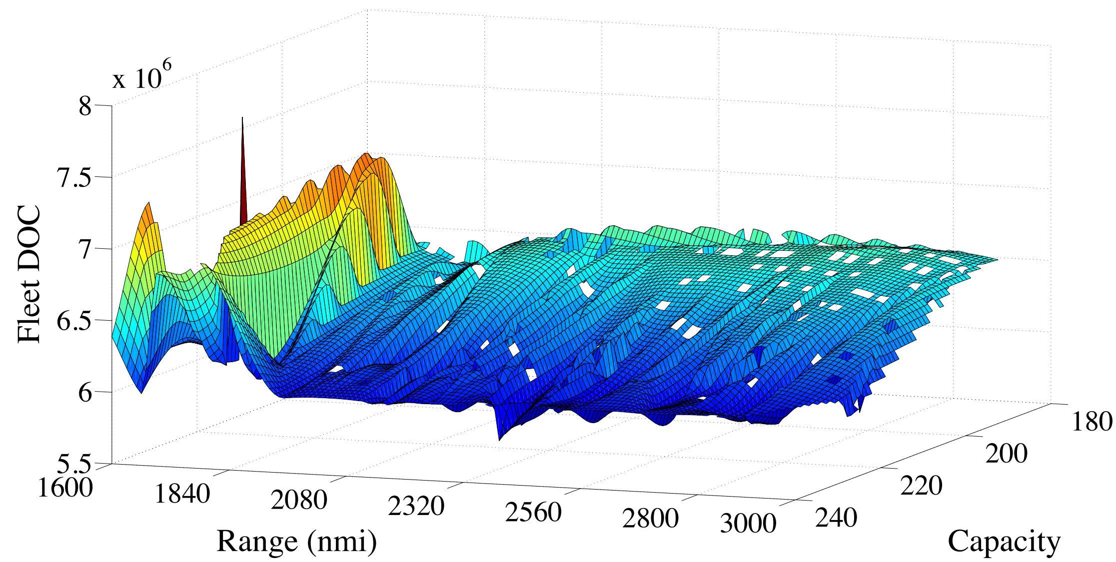

Figure 14: DOC_F response surface for the 15-city problem

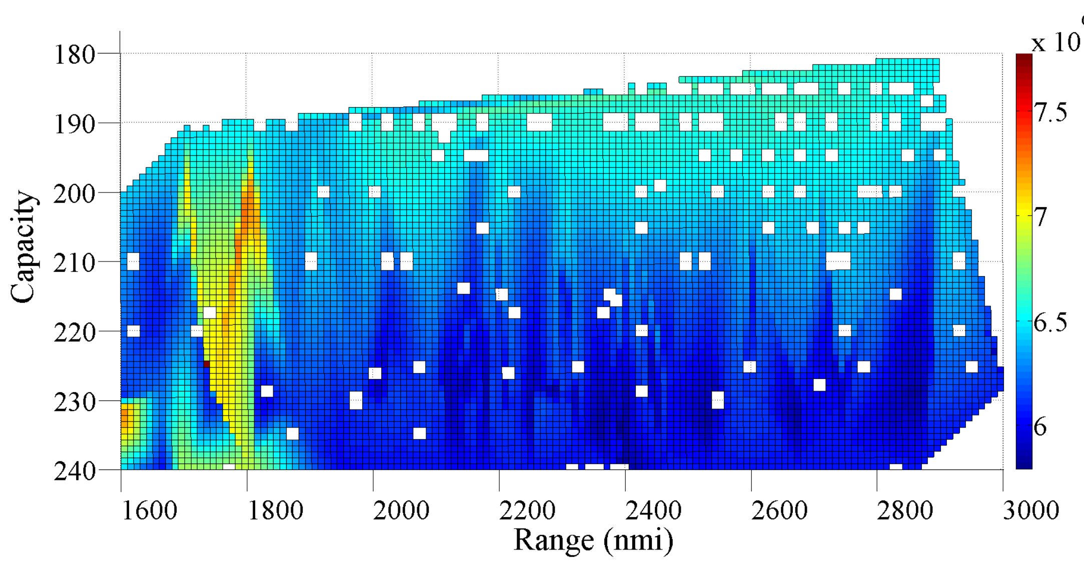

Figure 15: Contour map of the DOC_F response surface for the 15-city problem

Table 16: Best feasible solution for the 15-city problem

R_X	P_X	DOC_F	Active routes	Aircraft allocated	Total unfulfilled demand
2388	240	\$5,642,754	55	418	215

confirmed by inspecting the response surface, and is similar to the 7-city problem where the optimal solution was found at the upper bound for passenger capacity. The optimal aircraft design $\mathbf{d_X}$ is reported in Table 17 and contrasted to the design obtained for the 7-city problem. The corresponding DOC_X is listed in Table 18 for all routes regardless of whether they are active in the optimal network.

Figure 16 depicts the optimal network configuration. Table 19 lists the optimal aircraft allocation and the unfulfilled demand for all routes in the network.

Table 17: Optimal design of aircraft X

Variable	15-city problem	7-city problem
Range (nmi)	2,388	2,147
Gross weight (lb)	193,133	181,600
Wing aspect ratio (-)	9.20	9.69
Thrust per engine (lb)	33,000	27,000
Wing area (sq. ft)	1,289	1,311
Cruise velocity (Mach)	0.82	0.82
Wing loading (lb / sq. ft)	149.83	138.50
Thrust to weight ratio (-)	0.34	0.31
Take-off distance (ft)	8,990	8,990

Table 18: Optimal DOC_X for every possible route of the 15-city problem

	VAN	MTL	CAL	EDM	OTT	HAL	WIN	VIC	STJ	KEL	QBE	SKT	REG	TBY
TOR	25,522	7,996	21,513	21,482	7,061	12,835	14,163	25,712	17,828	23,880	9,371	18,524	17,401	10,548
VAN	–	27,586	9,039	9,922	26670	3,2339	16,327	5,482	31,092	6,562	4,908	12,242	13,050	19,990
MTL	–	–	23,449	23,163	5926	9,906	16,071	27,835	14,752	25,912	6,337	20,296	19,355	12446
CAL	–	–	–	6,620	22,556	28,125	12,272	9,375	31,650	7,395	24,068	8,122	9,007	15,936
EDM	–	–	–	–	22,324	27,653	12,223	10,380	30,881	8,458	23,657	7,875	9,188	15,827
OTT	–	–	–	–	–	10,919	15,169	26,908	15,743	25,001	7,229	19,433	18,454	11,524
HAL	–	–	–	–	–	–	20,809	32,632	10,186	30,645	8,943	24,888	24,074	17,269
WIN	–	–	–	–	–	–	–	16,574	24,694	14,681	16,767	9,273	8,191	8,577
VIC	–	–	–	–	–	–	–	–	36,255	6,902	28,523	12,575	13,307	20,222
STJ	–	–	–	–	–	–	–	–	–	34,203	13,470	28,375	27,806	21,443
KEL	–	–	–	–	–	–	–	–	–	–	26,563	10,602	11,406	18,342
QBE	–	–	–	–	–	–	–	–	–	–	–	20,866	20,023	13,238
SKT	–	–	–	–	–	–	–	–	–	–	–	–	6,356	12,897
REG	–	–	–	–	–	–	–	–	–	–	–	–	–	11,847

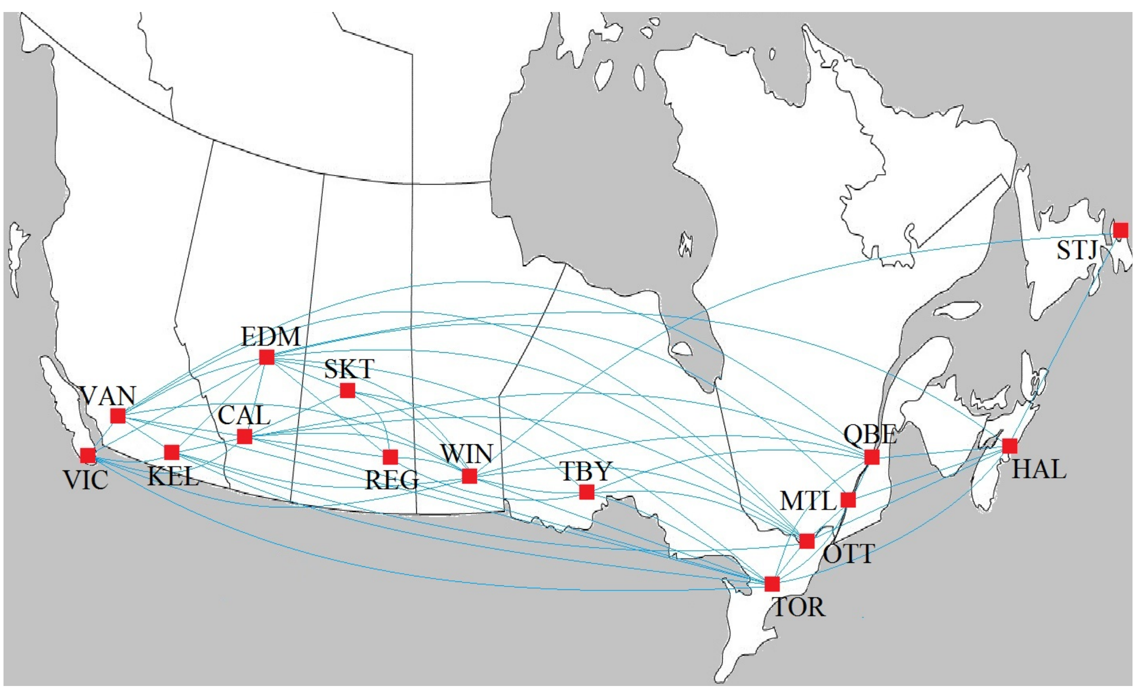

Figure 16: Optimal network configuration for the 15-city problem

Table 19: Optimal aircraft allocation and unfulfilled demand for the 15-city problem (inactive routes left out for the sake of brevity)

Route	A	B	X	U	Route	A	B	X	U
TOR–VAN	–	–	15	–	CAL–SKT	20	2	–	–
TOR–MTL	–	–	24	–	CAL–REG	19	1	–	–
TOR–CAL	–	–	11	–	EDM–OTT	–	–	3	–
TOR–EDM	–	–	9	–	EDM–HAL	–	–	2	24
TOR–OTT	–	–	16	–	EDM–WIN	–	–	7	–
TOR–HAL	–	–	9	–	EDM–VIC	–	–	5	–
TOR–WIN	–	–	11	–	EDM–KEL	–	–	3	–
TOR–VIC	–	–	3	–	EDM–SKT	5	1	–	–
TOR–KEL	–	–	2	11	EDM–REG	5	1	–	–
TOR–QBE	–	–	3	15	OTT –HAL	–	–	8	–
TOR–TBY	8	–	–	–	OTT –WIN	–	–	9	8
VAN–CAL	–	–	20	26	OTT –KEL	–	–	2	–
VAN–EDM	–	–	8	–	OTT –TBY	7	–	–	36
VAN–OTT	–	–	5	–	HAL–STJ	18	–	–	7
VAN–WIN	–	–	5	–	HAL–QBE	–	–	9	–
VAN–VIC	–	–	6	–	WIN –VIC	–	–	3	22
VAN–KEL	–	–	3	–	WIN –STJ	3	1	–	–
VAN–QBE	–	–	10	–	WIN –KEL	–	–	3	–
MTL–EDM	–	–	3	–	WIN –QBE	–	–	3	–
MTL–OTT	–	–	13	–	WIN –SKT	12	–	–	–
MTL–HAL	–	–	4	–	WIN –REG	10	–	–	29
MTL–QBE	–	–	8	–	WIN –TBY	10	–	–	–
CAL–EDM	–	–	8	–	VIC–KEL	–	–	1	3
CAL–OTT	–	–	6	3	QBE–TBY	1	–	–	–
CAL–WIN	–	–	8	–	SKT–REG	2	–	–	–
CAL–VIC	–	–	9	–	SKT–TBY	3	–	–	–
CAL–KEL	–	–	7	–	REG–TBY	3	–	–	–

4.2 Comparison to real data

We compared the obtained results to real data for WestJet Airlines, who expanded from being a regional operator to a national carrier. Since our SoS problem formulation does not take into account aircraft scheduling and intensive revenue models that affect airline operations, we compare the principal aviation hubs and routes of the network using the properties defined in Section 2.4.

Calgary and Edmonton in the western sub-network and Toronto in the eastern sub-network emerge as principal aviation hubs for the network. The corresponding nodes have the highest nodal degree, nodal betweenness and number of passengers enplaned. This is in agreement with WestJet's operations statistics that list Calgary, Toronto and Vancouver as the principal operating bases accounting for nearly 46% of its domestic market share in terms of daily flights [25].

Finally, we use route betweenness to identify principal operational routes in the network. In the absence of route-specific data for WestJet Airlines, we make a comparison with the overall observed passenger movement (for all airline carriers in Canada). Table 20 reports the four busiest routes (domestic non-stop flights) in Canada [12] and those obtained from solving the SoS design optimization problem. The agreement for three out of these four busiest routes is remarkable: the ranking dissimilarities occur because our method assigned a significantly higher passenger number to the CAL-TOR route, which can be attributed to demand that is routed through for other routes of "our" optimal network configuration.

4.3 Comparison to hub-spoke networks

Similar to the comparison presented for the 7-city problem, we considered two different fixed hub-spoke network configurations for the 15-city problem. For each case the new aircraft was sized for a fixed range equal to the longest route distance in the corresponding network. The DOC_F for hub-spoke network is given in Table 21. The Calgary and Toronto hub-spoke networks are now approximately 65% and 135% more expensive, respectively, for the 15-city problem (as opposed to both being approximately 200% more

Table 20: Principal routes in the 15-city network

	Real Data		Air Transportation SoS Design	
Route	Daily Passengers	Rank	Daily Passengers	Rank
MTL–TOR	5,547	1	5,602	1
OTT–TOR	3,743	2	3,717	3
VAN–TOR	3,496	3	3,433	4
CAL–VAN	3,101	4	4,826	2

Table 21: DOC_F for single-hub networks (15-city problem)

Hub City	DOC_F
Calgary	$\$8,925,466$
Toronto	$\$13,102,547$

expensive for the 7-city problem), i.e., the Toronto hub-spoke network is now twice more expensive to operate than the Calgary hub-spoke network; this may be due to a large number of cities being located in the western part of the network.

5 Conclusions

We presented a new, enhanced air transportation SoS design optimization problem formulation that, compared to the existing literature, introduces range as a SoS design variable, conducts optimal network configuration and treats unfulfilled demand as additional optimization variable in the aircraft to routes allocation problem. We demonstrated our methodology using the example of a two-stage expansion of a regional airline network consisting of 5 cities to a small national network consisting of 7 cities that was then more than doubled in size to a network of 15 cities.

Given the rapid grow of problem size and complexity, we solved the SoS design optimization problem using a decomposition approach with a nested formulation. The Mesh Adaptive Direct Search (MADS) algorithm was used to solve the outer-loop SoS optimization problem, NASA's FLOPS code was used for aircraft sizing and a combination of MADS with either GNU's linear programming kit (for the 7-city problem) or IBM ILOG CPLEX (for the 15-city problem) was used for the nested network configuration / aircraft allocation optimization problem.

For the 7-city problem, both the nested formulation and the all-in-one (AiO) approach two local optima that differ in the range value. The respective local optima of the two approaches are almost identical. However, the nested formulation reduced computation time by a factor of more than 4 compared to the AiO approach. A comparison to fixed hub-spoke networks demonstrated that the free network configuration can decrease daily fleet direct operating cost by a factor of almost 3. At the same time, the choice of single-hub city does not seem to alter the fleet direct operating cost for small networks dramatically.

For the 15-city problem, computational challenges prevented the solution of the AiO problem. The results of the nested formulation agree closely with actual observed passenger movement and show that the formulation presented here can be used to obtain practical results. Moreover, comparisons to single-hub networks showed that while the latter are still significantly more expensive to operate, the cost ratio of single-hub to free networks decreases with increasing network size. At the same time, the location of the cities in the network, in conjunction with demand, can have a large impact on which city should be considered as the single hub.

For the allocation problem we assume that each airline can arrive or depart from any airport throughout the day. In practice, there are strict guidelines for aircraft traffic close to an airport for safety reasons. Consequently, each aircraft gets a limited window of time for take-off and landing and associated operations such as taxi-in, taxi-out, deplaning etc. Therefore, for a more accurate model, aircraft scheduling needs to be considered as a part of the allocation problem. This can be be done for example by incorporating block-hours availability as constraints in the allocation step. As a preliminary attempt in this direction, we limit the number of aircraft (of a given type) that can fly along a given route; this is especially important since different aircraft have different crew handling ground operations after landing and before take-off. A possible means of incorporating scheduling into the problem could be modeling the scheduling problem as a network where arcs indicate the block-hours available for aircraft movement.

The symmetric demand assumption in the model described here is quite realistic. However, it has been observed that deviation from the symmetric demand assumption can significantly impact operating costs. This becomes especially important when considering aircraft scheduling since unsymmetrical demand would leave balance aircraft at an airport, leading to reduction fluctuations in daily demand that can be serviced with a given fleet size.

Finally, the cost analysis estimates presented here consider only the direct operating cost. In practice, an airline would also need to factor other indirect costs such cost of acquisition, maintenance costs , crew requirements and costs for flying additional (new) routes. Of these costs, the first and the last are typically of higher order of magnitude, and need to be accounted for in operating costs. The cost of acquisition is adjusted in revenue model through ticket pricing. However to our best knowledge, there does not exist a comparable method for assigning cost of including a route in the network in the literature.

Despite these limitations, the presented model illustrates the usefulness of a SoS approach to air transportation and provides a starting point for development of higher fidelity models. Further, this type of analysis would hold well for air-cargo services where the assumptions described above are more likely to be satisfied - unlike passengers, air-cargo traffic is not sensitive to flight-timings, shipment quantities are usually known beforehand and pricing model follows simpler constraints.

Other topics of possible future work include considering more than one type of new aircraft to be designed and allocated as well as incorporating demand uncertainty in cos analysis for airline operations (variable ticket pricing is a good example of how airlines account for uncertain demand in civil aviation).

References

[1] M. Bazargan. Airline operations and scheduling. Ashgate Publishing, 2010.

[2] A. P. Sage and C. D. Cuppan. On the systems engineering and management of systems of systems and federations of systems. Information, Knowledge, Systems Management, 2(4):325–345, 2001.

[3] M. W. Maier. Architecting principles for systems-of-systems. Systems Engineering, 1(4):267–284, 1998.

[4] C. Taylor and O. L. de Weck. Coupled vehicle design and network flow optimization for air transportation systems. Journal of Aircraft, 44(5):1478–86, 2007.

[5] M. Mane, W. A. Crossley, and Nusawardhana. System-of-systems inspired aircraft sizing and airline resource allocation via decomposition. Journal of Aircraft, 44(4):1222–35, 2007.

[6] M. Mane and W. A. Crossley. Allocation and design of aircraft for on-demand air transportation with uncertain operations. Journal of Aircraft, 49(1):141–50, 2012.

[7] T. Kotegawa. Analyzing the evolutionary mechanisms of the air transportation system-of-systems using Network Theory and machine learning algorithms. Phd thesis, Purdue University, January 2012.

[8] N. Davendralingam, W. A. Crossley, and Nusawadharna. An approximate dynamic programming approach to aircraft allocation incorporating passenger demand models. In 10th AIAA Aviation Technology, Integration and Operations Conference 2010, ATIO 2010, Ft. Worth, Texas, 2010.

[9] D. Bhadra. Demand for air travel in the united states: bottom-up econometric estimation and implications for forecasts by origin and destination pairs. Journal of Air Transportation, 8(2):19–56, 2003.

[10] J. D. Jorge-Calderón. A demand model for scheduled airline services on international european routes. Journal of Air Transportation Management, 3(1):23–35, 1997.

[11] T. Grosche, F. Rothlauf, and A. Heinzl. Gravity models for airline passenger volume estimation. *Journal of Air Transport Management*, 13(4):175–183, 2007.

[12] Transportation in Canada 2011: Comprehensive review. Annual Report Cat. No. T1-23A/2011E-PDF, Transport Canada, Ottawa, 2011.

[13] C. Audet, S. Le Digabel, and C. Tribes. NOMAD user guide. Technical Report, Les Cahiers du GERAD G–2009–37, 2009.

[14] M.A. Abramson, C. Audet, G. Couture, J.E. Dennis, Jr., S. Le Digabel, and C. Tribes. The NOMAD project. Software available at http://www.gerad.ca/nomad.

[15] S. Le Digabel. Algorithm 909: NOMAD: Nonlinear optimization with the MADS algorithm. ACM Transactions on Mathematical Software, 37(4):1–15, 2011.

[16] J. Currie and D. I. Wilson. OPTI: Lowering the Barrier Between Open Source Optimizers and the Industrial MATLAB User. In Nick Sahinidis and Jose Pinto, editors, Foundations of Computer-Aided Process Operations, Savannah, Georgia, 8–11 January 2012.

[17] L. A. McCullers. FLOPS: Flight Optimization System, release 8.23. NASA Langley Research Center, 2011.

[18] D. P. Raymer. Aircraft Design: A Conceptual Approach, volume 3. American Institute of Aeronautics and Astronautics, 1999.

[19] P. A. Jackson, K. Munson, and L. T. Peacock. Jane's All the World's Aircraft 2009-2010. Janes Information Group, 2009.

[20] M. Mane. Allocation of Variable Resources and Aircraft Design using Multidisciplinary Design for systems of Systems. MS Thesis, Purdue University, August 2005.

[21] H. Liu, X. B. Hu, S. Yang, K. Zhang, and E. Di Paolo. Application of complex network theory and genetic algorithm in airline route networks. Transportation Research Record: Journal of the Transportation Research Board, 2214(1):50–58, 2011.

[22] G. B. Dantzig and M. N. Thapa. Linear Programming, volume 1. Springer, 1997.

[23] A. Makhorin. GNU linear programming kit, June 2012. http://www.gnu.org/software/glpk/.

[24] IBM Corporation (US). IBM ILOG CPLEX Optimization Studio, Distribution 12.5 , 2010.

[25] R. Maslen. WestJet Continues to Grow International Services, August 2010. http://www.routesonline.com/news/29/breaking-news/94249/westjet-continues-to-grow-international-services.