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Spatial vs. non-spatial transboundary pollution control in a class of cooperative and non-cooperative dynamic games

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Abstract: We analyze a transboundary pollution differential game where, in addition to the standard temporal dimension, a spatial dimension is introduced to capture the different geographical relationships among regions. Each region behaves strategically and maximizes its welfare net of environmental damage caused by the pollutant stock. The emission-output ratio in each region is reduced by investment in clean technology which is region specific and evolves over time. The spatio-temporal dynamics of the pollutant stock is described by a parabolic partial differential equation. Using aggregate variables for the environmental variables we study the feedback Nash equilibrium of a discrete-space model which could be seen as a space discretization of the continuous-space model. The discrete-space model still presents the three main features of the original formulation: the model is truly dynamic; the decision agents behave strategically; and the model incorporates spatial aspects. For special functional forms previously used in the literature of transboundary pollution dynamic games we analytically characterize the feedback Nash equilibrium and evaluate the impact of the introduction of the spatial dimension in the economic-environmental model. We show that our spatial model is a generalization of the model that disregards the spatial aspects in the sense that the behavior of the environmental variables at the equilibrium in the non-spatial setting can be reproduced as the parameter describing how pollution diffuses among regions tends to infinity and the stocks of pollution in the regions are instantaneously mixed, which is the main hypothesis made in the non-spatial differential game.

Keywords: OR in environment and climate change, transboundary pollution, spatial dynamics, spatially distributed controls, differential games, parabolic differential equations

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1 Introduction

A review of the literature on dynamic models proposed for studying economic and environmental problems clearly shows that these models focus on the temporal aspect and ignore the spatial aspect. The addition of the spatial aspect obviously enriches the model and its possible predictions, but in turn leads to greater technical difficulties in its analysis. However, recently some authors have added the spatial dimension in the analysis of different economic problems such as allocation of economic activity or technological diffusion (Brito (2004), Boucekkine et al. (2009, 2013a, 2013b), Camacho et al. (2008), Brock and Xepapadeas (2008a), Desmet and Rossi-Hansberg (2010), Brock et al. (2014a) and Fabbri (2016)) or environmental and climate problems (Brock and Xepapadeas (2008b, 2010), Brock et al. (2014b), Camacho and Pérez-Barahona (2015), Xepapadeas (2010), Anita et al. (2013), Desmet and Rossi-Hansberg (2015), La Torre et al. (2015) and De Frutos and Martín-Herrán (2017)). All these papers (except De Frutos and Martín-Herrán (2017)) analyze finite or infinite time optimal control problems extended to infinite dimensional state space and focus on the problem of a social planner. The social planner allocates resources to maximize the present value of an objective over the entire spatial domain taking into account the spatio-temporal evolution of the state variable. To the best of our knowledge there is only one recent study (De Frutos and Martín-Herrán (2017)) that considers agents who behave both dynamically and strategically. The present paper tries to contribute to this very limited literature and analyzes an intertemporal transboundary pollution dynamic game where the pollution stock diffuses over a continuum of spatial sites and there are strategic interactions among the decision-makers.

Other contributions which explore the spatial dimension in environmental economics can be found in Anita et al. (2013, 2015), Brock et al. (2014b), Camacho and Pérez-Barahona (2015), Desmet and Rossi-Hansberg (2015), La Torre et al. (2015) and Augeraud-Véron et al. (2017).

Brock et al. (2014b) review the applications of optimal control of diffusive transport processes to environmental and climate problems in economics. Anita et al. (2013) analyze the large-time behavior of a spatially structured economic growth model coupling physical capital accumulation and pollution diffusion. Anita et al. (2015) add to the previous model a possible taxation based on the amount of produced pollution. The taxation rate depends upon the level of pollution at each spatial location and time. La Torre et al. (2015) extend the analysis in Anita et al. (2013) by introducing abatement activities. They introduce a spatial component in the Solow model and in the Ramsey model and analyze the spatio-temporal dynamics through numerical simulations. Desmet and Rossi-Hansberg (2015) analyze the geographic impact of climate change through a model featuring two externalities: technology diffusion and emission from energy used in production. Camacho and Pérez-Barahona (2015) analyze optimal land use from a social planner's point of view who decides the land use activities taking into account that local actions affect the whole space because pollution flows across locations resulting on both local and global damages.

All these contributions focus on the problem of a social planner who allocates resources and hence, disregard the strategic interactions among different decisions-makers. These strategic interactions are taken into account in the dynamic game with spatial effects analyzed in De Frutos and Martín-Herrán (2017). This last paper studies dynamic optimization for the pollution control in a spatial setting with strategic agents and focuses on the equilibrium emission strategies in a multiregional setting. Each economic agent responsible for controlling the emissions at each region takes into account the spatial transport phenomena across space when making the emission decisions at this region in order to maximize his profits.

The present paper shares the main general objective with De Frutos and Martín-Herrán (2017): to investigate the impact of the strategic and spatial dynamic behaviour of the economic agents responsible for controlling the emissions of pollutant on the design of equilibrium environmental policies. However, the functional specifications in the present work allow us to analytically treat the conditions that characterize the Markov-perfect Nash equilibria of the space-discretized differential game. Conversely, in De Frutos and Martín-Herrán (2017) the space-discretized differential game is solved using a numerical algorithm adapted from De Frutos and Martín-Herrán (2015) and the results are illustrated by means of numerical experiments even for the simplest case of two regions. Furthermore, our present functional specification allows us, first, to introduce the possibility of investment in clean technology in order to reduce the emission-output ratio and

hence, to analyze how the availability of new technology could affect the optimal emission strategies and the stock of pollution. Second, the present specification allows us to answer our main research question. We show that our spatial model is a generalization of the standard dynamic model that does not take into account the spatial dimension, in the sense that the behavior of the environmental variables at the equilibrium in the non-spatial setting can be reproduced as a limit case of the spatial setting; in particular, when the parameter describing how pollution diffuses among regions tends to infinity and the stocks of pollution in both regions are instantaneously mixed, which is the main hypothesis made in the non-spatial differential game.

Our paper contributes, on the one hand, to the literature on spatial economics, and more specifically, to the pollution control in a spatial setting previously described by adding the strategic behavior of economic agents. On the other hand, to the literature on transboundary pollution dynamic games (see, for example, Jørgensen et al. (2010) for a survey of this literature) by adding the spatial aspect.

The main objective of the present paper is to evaluate the effect of the strategic and spatially dynamic behaviour of the agents responsible for controlling the emissions of pollutant on the design of equilibrium strategies. Specifically, we aim at comparing the equilibrium strategies, long-run pollution stocks and long-run discounted net welfare of a transboundary pollution dynamic game when the spatial transport phenomena is either taken into account or is ignored. This analysis is carried out both for a non-cooperative and a cooperative formulation of the dynamic game.

The model is originally stated in continuous space and continuous time with two spatial dimensions and one time dimension. There are J players and each player decides the emission level and the investment in clean technology in order to maximize the present value of benefits net of environmental damages due to the concentration of pollutants over his spatial domain. The emission-output ratio in each region rather than assumed to be constant as in most of the papers of the literature of environmental dynamic games (Jørgensen et al. (2010)), is assumed to be a decreasing and strictly convex function of the stock of clean technology of this region. The maximization problem of each region is subject to the temporal evolution of the stock of clean technology which is assumed to be region specific (Jørgensen and Zaccour (2001)) and to the spatio-temporal evolution of the stock of a pollutant. The spatio-temporal evolution of the stock of a pollutant is described by a Diffusion partial differential equation (PDE) and general boundary conditions are assumed. This PDE is a generalization of the PDE describing this evolution in one of the examples presented in Brock et al. (2014b). While their specification is one-dimensional, ours is two-dimensional allowing to better describe the geographical or spatial aspect of the problem.

It is worth noting that in order to maintain the model simple and to focus on the spatial dimension of pollution diffusion we do not allow the spatial diffusion of new technology. With this hypothesis we are assuming that the capital in each region affects instantaneously all the region. This is the standard assumption in the literature when spatial effects are disregarded. Therefore, in order to emphasize the spatial aspect of pollution diffusion, we consider a spatial model where agents behave strategically, the abatement capital evolves over time and the pollution stock evolves across space and over time.

Our original specification is a J -player differential game. Each player aims at maximizing his profits net of environmental damages by choosing his level of emission and investment in new technology at each spatial point in his region and at each time. When making his decisions, each player takes into account the temporal evolution of his stock of clean technology described by a ordinary differential equation (ODE) and the spatio-temporal evolution of the stock of a pollutant described by a partial differential equation (PDE).

Along the same lines as in De Frutos and Martín-Herrán (2017) we apply a spatial discretization approach to simplify the model and characterize the equilibrium outcomes of the transboundary pollution dynamic game with spatial effects. In the space-discretized model each player decides the average total emission in his region and its investment in new technology taking into account the time evolution of the average pollution in each region and the stock of clean technology. The new formulation has $2J$ state variables described by a system of $2J$ ordinary differential equations (ODEs). A similar spatial discretization approach has been recently proposed in Graß and Uecker (2017) in an optimal control framework and applied to the analysis of a shallow lake model with diffusion. The structure of our space-discretized formulation is similar to that proposed in Mäler and Zeeuw (1998) to analyze an acid rain differential game.

The space-discretized model is exactly solved, unlike the model in De Frutos and Martín-Herrán (2017) that has to be numerically solved using a numerical algorithm. From our results we can conclude that the space-discretized formulation is a good first approach to characterize the equilibrium outcomes of the transboundary pollution dynamic game with spatial effects. Our analytical results show that the space-discretized model is a clear generalization of the model which ignores the spatial transport phenomena. Specifically, we analytically prove that for a two-player setting and both for non-cooperative and cooperative frameworks the traditional equilibrium policies derived ignoring the spatial dimension can be reproduced as a limit case of the space-discretized formulation. The limit case is described by the diffusion pollution parameter tending to infinity, when the stocks of pollution in both regions are instantaneously mixed, the assumption implicitly considered in the non-spatial dynamic game.

The paper is organized as follows. In the next section we present the multiregional spatially distributed control of pollution formulated initially as a continuous-space model, and in a second step, as a discrete-space model. Section 3 analyzes a particular specification of the model and presents the characterization of the Markov-perfect Nash equilibrium of the model. Section 4 analytically shows for the case of two players the main differences between the environmental policies in the formulation with and without spatial effects. Section 5 revisits this comparison but for a cooperative setting, where there is a unique decision-maker and the strategic interactions among the players disappear. Section 6 is devoted to present some concluding remarks.

2 The model

Let us denote by Ω a planar domain which is partitioned in J regions Ω_j , $j = 1, \dots, J$, such that

$$\bar{\Omega} = \bigcup_{j=1}^J \bar{\Omega}_j, \quad \Omega_i \cap \Omega_j = \emptyset, \quad i \neq j,$$

where $\bar{\Omega}$ is the closure of Ω . The common boundary between regions Ω_i and Ω_j is given by $\partial_{ij} := \partial\Omega_i \cap \partial\Omega_j = \bar{\Omega}_i \cap \bar{\Omega}_j$, $i \neq j$.

In each region there is one decision-maker. Player i wishes to choose the rate of pollutant emissions in region Ω_i as well as the investment in clean technology to maximize his own payoff. Therefore, the differential game considers J players (regions) and each player has pollution emissions and investments in abatement technology as control variables. The J -player differential game is played non-cooperatively.

Each region i produces a single consumption good. We denote by $Y_i(\mathbf{x}, t)$ the production of the good at time $t \geq 0$ at the particular point $\mathbf{x} \in \Omega$. The instantaneous net social benefits of production of region i are given by $B_i(Y_i(\mathbf{x}, t))$, with function B_i being increasing and strictly concave. The production of $Y_i(\mathbf{x}, t)$ generates pollution emissions. Hence, the industrial activities of the regions create pollution as an undesirable by-product. Let us denote by $E_i(\mathbf{x}, t)$, $i = 1, \dots, J$, the emission rate of region i at time $t \geq 0$ and $\mathbf{x} \in \Omega$. It is convenient to think of $E_i(\mathbf{x}, t)$ and $Y_i(\mathbf{x}, t)$ as densities of emission rate and production which are distributed along the domain Ω . Also it is convenient to assume that although $E_i(\mathbf{x}, t)$ and $Y_i(\mathbf{x}, t)$ are defined for all $\mathbf{x} \in \Omega$, $E_i(\mathbf{x}, t) = 0$, $Y_i(\mathbf{x}, t) = 0$ for $\mathbf{x} \notin \Omega_i$. The emission rate $E_i(\mathbf{x}, t)$ resulting from production of region i is proportional to current output and given by:

$$E_i(\mathbf{x}, t) = \alpha_i(K_i(t))Y_i(\mathbf{x}, t), \tag{1}$$

where $K_i(t)$ denotes the stock of clean technology of region i at time t . Following Ploeg and Zeeuw (1992) and Jørgensen and Zaccour (2001) the emission-output ratio $\alpha_i(K_i(t))$ rather than assumed to be constant as in most of the papers of the literature of environmental dynamic games (see Jørgensen et al. (2010) for a survey of dynamic games models used to analyze transboundary pollution problems), is assumed to be dependent on the stock of clean technology of region i at time t . The idea is that the emission-output ratio can be reduced by investment in new technology. Hence, each function α_i is a decreasing and strictly convex function of the stock of clean technology of region i to account for decreasing returns in the investment activities in new technology. Ploeg and Zeeuw (1992) assume that the stock of clean technology is public

knowledge, while Jørgensen and Zaccour (2001) consider the case where the stock of clean technology is region specific. We follow this last hypothesis and as Jørgensen and Zaccour (2001) from now on we shall refer to $K_i(t)$ interchangeably as the abatement capital or the clean technology of region i at time t . As a first step in the analysis and in order to maintain the model simple and to focus on the spatial dimension of pollution diffusion, we do not allow the spatial diffusion of new technology (abatement capital). With this hypothesis we are assuming that the capital in each region affects instantaneously all the region. This is the standard assumption in the literature when spatial effects are disregarded. Therefore, in order to emphasize the spatial aspect of pollution diffusion, we consider a spatial model where agents behave strategically, the abatement capital evolves over time and the pollution stock evolves across space and over time.

The dynamics of the stocks of abatement capital over time is described by the following differential equations:

$$\dot{K}_i(t) = f(I_i(t), K_i(t)), \quad K_i(0) = K_{i0}, \quad (2)$$

where a dot over a variable denotes its derivative with respect to time, $I_i(t)$ is the investment in abatement capital in region i and K_{i0} is the initial stock of abatement in this region. The cost associated with investment in abatement capital is denoted by $C_i(I_i(t))$, where function C_i is assumed to be increasing and strictly convex.

The emissions accumulate in a stock of pollution denoted by $P(\mathbf{x}, t)$ and defined for all $\mathbf{x} \in \Omega$. In what follows we denote by ∇u the spatial gradient of a scalar function $u : \Omega \rightarrow \mathbb{R}$, and by $\nabla \cdot \mathbf{u} = \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}$ the divergence of a vectorial function $\mathbf{u} = [u_1, u_2] : \Omega \rightarrow \mathbb{R}^2$. The following parabolic partial differential equation describes the spatio-temporal dynamics of the stock of pollution:

$$\begin{aligned} \frac{\partial P}{\partial t} &= \nabla \cdot (b \nabla P) - dP + \sum_{j=1}^J F_j(E_j(\mathbf{x}, t)) \mathbf{1}_{\Omega_j}, \quad \mathbf{x} \in \Omega, \\ P(\mathbf{x}, 0) &= P^0(\mathbf{x}), \quad \mathbf{x} \in \Omega, \\ a(\mathbf{x})P(\mathbf{x}, t) + b(\mathbf{x})\nabla P^T(\mathbf{x}, t)\mathbf{n} &= a(\mathbf{x})P_b(\mathbf{x}, t), \quad \mathbf{x} \in \partial\Omega. \end{aligned} \quad (3)$$

The velocity at which the stock of pollution is diffused away in a particular location \mathbf{x} is measured by $b = b(\mathbf{x})$, a local diffusion coefficient which is assumed to be a smooth function satisfying $b_m \leq b(\mathbf{x}) \leq b_M$, for all $\mathbf{x} \in \Omega$, where $0 < b_m \leq b_M$ are two given constants. The natural decay of the pollutant is represented by the term dP , with $d = d(\mathbf{x}, t) \geq 0$. In the source term $\sum_{j=1}^J F_j(E_j(\mathbf{x}, t)) \mathbf{1}_{\Omega_j}$, F_j , $j = 1, \dots, J$ is a family of smooth functions, with $\mathbf{1}_{\Omega_j}$ denoting the characteristic function of set Ω_j , that is, the function defined to be identically one on Ω_j , and zero elsewhere. With this specification the emission rates of region i contribute to enlarge the stock of pollution only in region i . However, the diffusion process modeled by the state equation (3) transfers part of the pollution to the whole domain Ω . The diffusive character of the state equation implies that the emissions in region Ω_i instantaneously affect each one of the regions Ω_j , $i \neq j$. How much region j is affected by the emissions of region i depends on the time elapsed from the instant when the emissions take place and the distance between regions i and j .

The initial distribution of the stock of pollution along region Ω is described in the second equation in (3) and the boundary condition is stated in the third equation of (3). The boundary condition states that the flux of pollution throughout $\partial\Omega$ is proportional to the difference $P_b(\mathbf{x}) - P(\mathbf{x})$, where $P_b(\mathbf{x})$ is a given function representing the concentration of pollution in the exterior of Ω and \mathbf{n} denotes the normal vector exterior to Ω . Function $a(\mathbf{x})$ is a non-negative smooth function that appears after applying Newton's law of diffusion on the boundary of Ω .

Player i , $i = 1, \dots, J$ aims at maximizing his payoff

$$J_i(E_1, \dots, E_J, I_i, P_0, K_{i0}) = \int_0^{+\infty} \int_{\Omega_i} e^{-\rho t} G_i(E_i, I_i, P, K_i) d\mathbf{x} dt, \quad (4)$$

taking into account the dynamics of the stocks of abatement capital and the pollution stock given in Equations (2) and (3), respectively. In the expression above $\rho > 0$ denotes the time-discount rate. This payoff can be understood as an average over Ω_i of a density of revenue represented by $G_i(E_i, I_i, P, K_i)$. We remark that the emissions in region j , $j \neq i$, enter into the objective functional of player i via the state equation (3).

The standard assumption in dynamic pollution games (see, for example, Jørgensen et al. (2010) for a survey of this literature) establishes that the instantaneous welfare of each region is given by a benefit from consumption ($B_i(Y_i(\mathbf{x}, t))$) minus the cost of the investment in abatement capital ($C_i(I_i(t))$) and the damage caused by the stock of pollution $D_i(P(\mathbf{x}, t))$. The smooth function D_i is commonly assumed in the literature to be a convex of its argument. Therefore, the net benefits from consumption have the form

$$(B_i(Y_i) - C_i(I_i) - D_i(P)) \mathbf{1}_{\Omega_i}. \quad (5)$$

Taking into account the emission-production trade-off function in (1), the instantaneous benefits of region i in (5) can be rewritten in terms of the emission rates and its own stock of abatement capital:

$$G_i(E_i, I_i, P, K_i) = (B_i(\alpha(K_i)^{-1} E_i) - C_i(I_i) - D_i(P)) \mathbf{1}_{\Omega_i}. \quad (6)$$

We restrict the analysis to stationary Markov-perfect Nash equilibria because the strategies supporting this equilibria do not require precommitment to a course of action over time and have been assumed to be a good description of realistic behaviour (see, for example, Haurie et al. (2012) and Jørgensen et al. (2010)). From now on we assume that the dynamic game defined by (2)–(6) has at least one stationary Markov-perfect Nash equilibrium (Başar and Olsder (1999)). From (2) and (3) it can be easily deduced that the optimal strategies of each region will be independent of the stock of abatement capital of the other regions. Therefore, the emission and investment decisions of an agent at any point in time and space only depend on the state of the pollution stock and its own stock of abatement capital at that moment and point in space.

In order to answer our main research questions and focus on the comparison of the equilibrium strategies when the spatial aspects are either taking into account or disregarded, we introduce aggregated variables for the environmental variables and reformulate the model. The discrete-space model derived using these aggregate variables can be obtained along the same lines as in De Frutos and Martín-Herrán (2017). The discrete-space model can also be seen as a space discretization of the continuous-space model. It is worth noting that the discrete-space model is truly dynamic, incorporates spatial aspects and the decision makers behave strategically.

The aggregated stock of pollution and the averaged emissions in each region i are defined by

$$p_i(t) = \frac{1}{m_i} \int_{\Omega_i} P(\mathbf{x}, t) d\mathbf{x}, \quad e_i(t) = \frac{1}{m_i} \int_{\Omega_i} E_i(\mathbf{x}, t) d\mathbf{x}, \quad i = 1, \dots, J, \quad (7)$$

where m_i represents the area of region Ω_i .

Let us define

$$\widehat{G}_i(e_i, I_i, p_i, K_i) = m_i G_i(e_i, I_i, p_i, K_i),$$

which is an approximation to $\int_{\Omega_i} G_i(E_i, I_i, P, K_i) d\mathbf{x}$.

The objective of Player i in the discrete-space model is to maximize the space averaged payoff

$$\widehat{J}_i(e_1, \dots, e_J, I_i, \mathbf{p}^0, K_{i0}) = \int_0^\infty e^{-\rho t} \widehat{G}_i(e_i, I_i, p_i, K_i) dt, \quad (8)$$

taking into account the dynamics of its own stock of abatement given by (2) and the dynamics of the aggregated stock of pollution in each region described by the following system of ordinary differential equations:

$$m_i \dot{p}_i = \sum_{\substack{j=1 \\ j \neq i}}^J b_{ij} (p_j - p_i) + b_{i0} (p_{i0} - p_i) - m_i \delta_i p_i + m_i F_i^i(e_i), \quad i = 1, \dots, J. \quad (9)$$

System (9) is supplemented with initial conditions given by

$$p_i(0) = \frac{1}{m_i} \int_{\Omega_i} P_0(\mathbf{x}) d\mathbf{x} := p_i^0, \quad i = 1, \dots, J,$$

where $P_0(\mathbf{x})$ is the initial data in (3). From now on we use the notation $\mathbf{p}^0 = [p_1^0, \dots, p_J^0]^T$.

Coefficients b_{ij} measure how fast the pollution spreads across boundary ∂_{ij} between regions Ω_i and Ω_j in absence of external transport phenomena. Of course, it is understood that $b_{ij} = 0$ if regions Ω_i and Ω_j have no common boundary and $b_{ij} \neq 0$ in the opposite case when $\partial_{ij} \neq \emptyset$, that is, regions Ω_i and Ω_j share a common boundary. It is implicitly assumed that $b_{ij} = b_{ji}$ for all i, j . Region corresponding to index $i = 0$ represents the exterior of Ω . The stock of pollution p_{i0} can be obtained by aggregation on $\partial\Omega_i \cap \partial\Omega$ of the boundary data P_b in formula (3), so that it is a known function of time in (9). The first two terms in the right hand side of the differential equation in (9) collect the diffusion effect that tends to equilibrate the pollution between regions: the pollution entering Ω_i is proportional to the difference between the stock of pollution in the adjacent regions, the pollution moves from regions with high levels of concentration to regions with low levels of concentration (Flick's law of diffusion). The third term is pollution degradability or natural degradation of the pollution stock. Finally, the fourth term is the flow of emissions.

The discrete-space model described by (2), (8) and (9) is a J -player infinite horizon differential game with two decision variables for each player (the averaged emission rates in his region and the investment in his own abatement capital) and $2J$ state variables (the stock of abatement capital and the averaged stock of pollution in each region) with time evolution described by the system of ODEs in (2) and (9).

3 A particular specification of the discrete-space model

We adopt the simplest version of the economics and environment model that still captures the main ingredients of the more general model described in the previous section. First, the strategic behavior of the players, emissions by one player affects the environment of all; and second, the spatial aspect that allows us to analyze how our results compare to those obtained using standard dynamic game models which disregard the spatial dimension of the problem.

From now on we use the special functional forms proposed in Jørgensen and Zaccour (2001). The functional forms for instantaneous benefits, costs of investment in clean technology (or abatement capital), emission-output ratio and the damage environmental cost are assumed as follows:

$$B_i(Y_i) = \log(Y_i), \quad C_i(I_i) = \frac{1}{2}c_i I_i^2, \quad \alpha_i(K_i) = \sigma_i e^{-\beta_i K_i}, \quad D_i(P) = \varphi_i P, \quad i = 1, \dots, J,$$

where c_i, σ_i, β_i and φ_i are positive constants.

The abatement capital stocks evolve in time according to the standard dynamics:

$$\dot{K}_i(t) = I_i(t) - \mu_i K_i(t), \quad K_i(0) = K_{i0}, \quad i = 1, \dots, J, \quad (10)$$

where μ_i is the rate of depreciation of capital which as usual is assumed to be constant and K_{i0} is a given initial stock of abatement capital for region i .

The source function in (9) is given by:

$$F_i(e_i) = \eta_i e_i, \quad i = 1, \dots, J.$$

Under these hypotheses the J -player differential game played over an infinite-time horizon defined by (2), (8) and (9) particularizes as follows: Each region i chooses its control variables, the average emission rate e_i and the investment in abatement capital I_i in order to maximize

$$\tilde{J}_i(e_i, I_i, \mathbf{p}^0, K_{i0}) = \int_0^\infty e^{-\rho t} (\log(e_i) + \beta_i K_i - \frac{1}{2}c_i I_i^2 - \varphi_i p_i) dt, \quad (11)$$

subject to the dynamics of its own stock of abatement given by (10) and of the aggregated stock of pollution in each region defined by

$$\dot{p}_i = \sum_{\substack{j=1 \\ j \neq i}}^J b_{ij}(p_j - p_i) - d_i(p_i - q_i) + \eta_i e_i, \quad p_i(0) = p_i^0, \quad i = 1, \dots, J, \quad (12)$$

where $d_i = \delta_i + b_{i0}$ and $q_i = b_{i0}p_{i0}/d_i$. From now on we assume that q_i , $i = 1, \dots, J$, are independent of time and, for the ease of notation, we will denote by b_{ij} the diffusion coefficient once it has been normalized by m_i , the total area of region Ω_i .

Let us note that a constant term $-\log(\sigma_i)$ should appear in the objective (11). Because this term does not affect the optimal policies it has been omitted for simplicity.

Next we will use vectorial notation. Then, we introduce vectors $\mathbf{p} = [p_1, \dots, p_J]^T$, $\mathbf{K} = [K_1, \dots, K_J]^T$, $\mathbf{e} = [e_1, \dots, e_J]^T$, $\mathbf{q} = [q_1, \dots, q_J]^T$ and diagonal matrices $\Pi = \text{diag}(\eta_1, \dots, \eta_J)$, $\Gamma = \text{diag}(\mu_1, \dots, \mu_J)$, $\mathbf{D} = \text{diag}(d_1, \dots, d_J)$, and B is the symmetric matrix $B = [b_{ij}]_{i,j=1}^J$, with $b_{ii} = -\sum_{j \neq i} b_{ij}$, so that the dynamics (10)–(12) can be written in the condensed form

$$\dot{\mathbf{K}}(t) = \mathbf{I}(t) - \Gamma \mathbf{K}(t), \quad \mathbf{K}(0) = \mathbf{K}_0, \quad (13)$$

$$\dot{\mathbf{p}}(t) = \mathbf{B}\mathbf{p}(t) - \mathbf{D}(\mathbf{p}(t) - \mathbf{q}) + \Pi \mathbf{e}(t), \quad \mathbf{p}(0) = \mathbf{p}_0. \quad (14)$$

The value function for player i , $V_i = V_i(\mathbf{p}, \mathbf{K})$, $i = 1, \dots, J$, satisfies the stationary Hamilton-Jacobi-Bellman equations

$$\rho V_i = \max_{I_i, e_i} \left\{ \log(e_i) + \beta_i K_i - \frac{1}{2} c_i I_i^2 - \varphi_i p_i + \nabla_{\mathbf{K}} V_i (\mathbf{I} - \Gamma \mathbf{K}) + \nabla_{\mathbf{p}} V_i (\mathbf{B}\mathbf{p} - \mathbf{D}(\mathbf{p} - \mathbf{q}) + \Pi \mathbf{e}) \right\}, \quad (15)$$

where $\nabla_{\mathbf{K}} V_i$ and $\nabla_{\mathbf{p}} V_i$ denote the gradients of V_i with respect to the variables K_1, \dots, K_J and p_1, \dots, p_J , respectively.

With the particular functional forms considered here, the dynamic game belongs to the class of state separable or linear-state differential games. For this class of games it is well-known that the Hamilton-Jacobi-Bellman equations that characterize the non-cooperative feedback Nash equilibrium are satisfied for value functions linear in the state variables, in our case K_i, p_i, p_j for player i (Dockner et al. (2000)). Therefore, we postulate

$$V_i(\mathbf{p}, \mathbf{K}) = \mathbf{M}_i^T \mathbf{K} + \mathbf{R}_i^T \mathbf{p} + X_i, \quad (16)$$

with X_i constant and $\mathbf{M}_i = [M_{i,1}, \dots, M_{i,J}]^T$ and $\mathbf{R}_i = [R_{i,1}, \dots, R_{i,J}]^T$ constant vectors to be determined.

The first-order conditions in (15) proportionate the values

$$I_i = \frac{M_{i,i}}{c_i}, \quad e_i = -\frac{1}{\eta_i R_{i,i}}, \quad (17)$$

for the equilibrium investment and emissions rates. Substituting in (15) and using the linearity of the value function we get that $M_{i,j} = 0$ for $j \neq i$ and $M_{i,i} = \beta_i / (\rho + \mu_i)$. Coefficients \mathbf{R}_i , $i = 1, \dots, J$, satisfy the following system of linear equations

$$(\mathbf{B} - \rho \text{Id} - \mathbf{D}) \mathbf{R}_i = \varphi_i \mathbf{u}_i, \quad i = 1, \dots, J, \quad (18)$$

where Id denotes the $J \times J$ identity matrix and \mathbf{u}_i is the i -th vector of the usual base of \mathbb{R}^J .

System (18) possesses a unique solution because matrix $\mathbf{B} - \rho \text{Id} - \mathbf{D}$ is strictly diagonally dominant. Furthermore, as the diagonal terms of $\mathbf{B} - \rho \text{Id} - \mathbf{D}$ are strictly negative, matrix $\mathbf{B} - \rho \text{Id} - \mathbf{D}$ is negative definite which proves that

$$R_{i,i} = \frac{1}{\varphi_i} \mathbf{R}_i^T (\mathbf{B} - \rho \text{Id} - \mathbf{D}) \mathbf{R}_i < 0, \quad i = 1, \dots, J,$$

and, in consequence $e_i > 0$, $i = 1, \dots, J$.

The constant term X_i in (16) has the expression

$$X_i = \frac{1}{\rho} \left(\log(e_i) - \frac{1}{2} c_i I_i^2 + M_{i,i} I_i + \mathbf{R}_i^T (\Pi \mathbf{e} + \mathbf{D} \mathbf{q}) \right),$$

where e_i and I_i , $i = 1, \dots, J$ are the values given in (17).

The stationary equilibrium of the dynamics (13), (14) subject to (17) is globally asymptotically stable because the matrix system $\mathbf{B} - \mathbf{D}$ is negative definite if at least one of the d_i , $i = 1, \dots, J$ is different from zero.

4 Spatial vs. non-spatial model in a non-cooperative framework

This section compares the equilibrium environmental policies of the spatial transboundary pollution problem stated in the previous sections which takes into account the spatial context with those equilibrium emission rates obtained as the optimal solution of a dynamic game that ignores the spatial transport phenomena. In order to simplify as much as possible this comparison we restrict to the case of two players. Although this assumption could be seen to be restricted it allows us to easily characterize the equilibrium emission and investment rates of both differential games and present clear-cut results from the comparison.

Next proposition completely characterizes the feedback Nash equilibrium and the discounted net welfare of each player in the case of a 2-player differential game.

Proposition 1 *The equilibrium emission and investment rates of the 2-player differential game defined by (10), (11) and (12) are constant and given by:*

$$I_i = \frac{\beta_i}{c_i(\mu_i + \rho)}, \quad e_i = \frac{(d_j + \rho)b + (b + d_j + \rho)(d_i + \rho)}{\eta_i(b + d_j + \rho)\varphi_i}, \quad i = 1, 2, i \neq j,$$

where $b = b_{ij} = b_{ji}$ for $i \neq j$.

The discounted net welfare of player i is given by:

$$V_i(K_i, p_i, p_j) = M_{i,i}K_i + R_{i,i}p_i + R_{i,j}p_j + X_i, \quad (19)$$

where

$$\begin{aligned} M_{i,i} &= \frac{\beta_i}{\mu_i + \rho}, \quad R_{i,i} = -\frac{\varphi_i(b + d_j + \rho)}{(d_i + \rho)(b + d_j + \rho) + b(d_j + \rho)}, \\ R_{i,j} &= -\frac{b\varphi_i}{(d_i + \rho)(b + d_j + \rho) + b(d_j + \rho)}, \quad X_i = -\frac{1}{2c_i\rho(b + d_i + \rho)\varphi_j} (X_{i1} + X_{i2}), \\ X_{i1} &= \frac{2c_i}{(d_i + \rho)(b + d_j + \rho) + b(d_j + \rho)} [(b + d_i + \rho)(b + d_j + \rho)(d_i + \rho + q_i d_i \varphi_i)\varphi_j \\ &\quad + b[b(d_i + \rho)\varphi_i + (b + d_i + \rho)((\rho + d_j)(\varphi_i + \varphi_j) + d_j q_i \varphi_i \varphi_j)], \\ X_{i2} &= -(b + d_i + \rho)\varphi_j \left(\frac{\beta_i^2}{(\mu_i + \rho)^2} + 2c_i \log \left(\frac{(d_i + \rho)(b + d_j + \rho) + b(d_j + \rho)}{\eta_i(b + d_j + \rho)\varphi_i} \right) \right), \end{aligned}$$

for $i, j = 1, 2, i \neq j$.

The steady-state levels of abatement capital and pollution stocks are given by:

$$\begin{aligned} K_i^{SS} &= \frac{\beta_i}{c_i \mu_i (\mu_i + \rho)}, \\ p_i^{SS} &= \frac{Nump_i^{SS}}{(d_i d_j + b(d_i + d_j))(b + d_i + \rho)(b + d_j + \rho)\varphi_i \varphi_j}, \\ Nump_i^{SS} &= (b + d_j)(b + d_i + \rho)(b(d_j + \rho) + (b + d_j + \rho)(d_i + \rho + bq_i d_i \varphi_i))\varphi_j \\ &\quad + b(b + d_j + \rho)\varphi_i(b(d_i + \rho) + (b + d_i + \rho)(d_j + \rho + bq_j d_j \varphi_j)), \end{aligned}$$

for $i, j = 1, 2, i \neq j$, where the superscript *SS* stands for steady-state levels and q_k is the aggregation of the boundary data defined in (12). The steady-state equilibrium is globally asymptotically stable.

As expected the discounted net welfare of each region depends positively on its own stock of abatement capital and negatively on both stocks of pollution. As a result of the linearity of the value function, the equilibrium emission and investment rates are constant with respect to the state variables and hence they are constant over time. It is worth noting that b and q_i , two of the most significant parameters of the spatial diffusion model, affect the equilibrium emission rate and the steady-state levels of the stock of pollution. Next corollary collects the results of the sensitivity analysis with respect to these two main parameters.

Corollary 1

1. The equilibrium emission rate, e_i , increases as the diffusion parameter b increases.
2. The steady-state levels of the stocks of pollution, p_i^{SS} $i = 1, 2$, increase as parameters q_i and q_j increase.
3. In a completely symmetric setting, the steady-state levels of the stocks of pollution p_i^{SS} $i = 1, 2$, increase as the diffusion parameter b increases.

It can be easily shown that the higher b , the faster the stock of pollution diffuses from one region to another, and as a result, the region takes advantage of the opportunity to emit at a higher rate. A greater q_k indicates a greater stock of pollution in the neighbourhood of the region under consideration, and by the Fick's law this region would be the recipient of pollution coming from outside until the stock of pollution inside and outside the region stabilizes. Unfortunately, the effect of the diffusion parameter b on the steady-state of the stock of pollution is unclear and depends on all the model parameters. However, for the symmetric scenario this effect is clearly positive.

In order to evaluate the impact of the introduction of the spatial aspect in the economic-environmental model we briefly describe the differential game proposed by Jørgensen and Zaccour (2001) where the spatial dimension is ignored. For brevity we refer to this model as the non-spatial model and we add a tilde on the variables and parameters linked to the environmental part of the model to distinguish them from those used in the spatial model. We have used identical notation in both models for the variables and parameters related to the investment in clean technology and to the stock of abatement capital because we have not included any spatial aspect in this part of the model.

Region i in the non-spatial model chooses its control variables, the emission and investment rates, \tilde{e}_i and I_i in order to maximize

$$\tilde{J}_i(\tilde{e}_i, I_i, \tilde{P}_0, K_{i0}) = \int_0^\infty e^{-\rho t} \left(\log(\tilde{e}_i) + \beta_i K_i - \frac{1}{2} c_i I_i^2 - \tilde{\varphi}_i \tilde{P} \right) dt, \quad (20)$$

subject to the dynamics of its own stock of abatement given by (10) and of the stock of pollution which evolves over time according to:

$$\dot{\tilde{P}} = \tilde{\eta}_1 \tilde{e}_1 + \tilde{\eta}_2 \tilde{e}_2 - \tilde{d} \tilde{P}, \quad \tilde{P}(0) = \tilde{P}_0. \quad (21)$$

For comparison purposes next proposition characterizes the feedback Nash equilibrium and the discounted net welfare of each player of the non-spatial 2-player differential game.

Proposition 2 *The equilibrium emission and investment rates of the 2-player differential game defined by (10), (20) and (21) are constant and given by:*

$$I_i = \frac{\beta_i}{c_i(\mu_i + \rho)}, \quad \tilde{e}_i = \frac{\rho + \tilde{d}}{\tilde{\eta}_i \tilde{\varphi}_i}, \quad i = 1, 2, i \neq j.$$

The discounted net welfare of player i is given by:

$$\tilde{V}_i(K_i, \tilde{P}) = M_i K_i + \tilde{R}_i \tilde{P} + \tilde{X}_i, \quad (22)$$

where

$$M_i = \frac{\beta_i}{\mu_i + \rho}, \quad \tilde{R}_i = -\frac{\tilde{\varphi}_i}{\rho + \tilde{d}},$$

$$\tilde{X}_i = \frac{1}{2\rho} \left(\frac{\beta_i^2}{c_i(\mu_i + \rho)^2} - \frac{2(\tilde{\varphi}_i + \tilde{\varphi}_j)}{\tilde{\varphi}_j} + 2 \log \left(\frac{\rho + \tilde{d}}{\tilde{\eta}_i \tilde{\varphi}_i} \right) \right),$$

for $i = 1, 2, i \neq j$.

The steady-state levels of abatement capital and pollution stocks are given by:

$$K_i^{SS} = \frac{\beta_i}{c_i \mu_i (\mu_i + \rho)}, \quad \tilde{P}^{SS} = \frac{(\rho + \tilde{d})(\tilde{\varphi}_i + \tilde{\varphi}_j)}{\tilde{d} \tilde{\varphi}_i \tilde{\varphi}_j},$$

for $i = 1, 2, i \neq j$, where the superscript *SS* stands for steady-state levels. The steady-state equilibrium is globally asymptotically stable.

From the comparison of both models it is clear that the equilibrium investment rates and the steady-state levels of the stock of abatement capital are identical. This result is completely expected because the spatial differential game has only incorporated the spatial aspect in the pollution dimension and has disregarded this aspect in the stock of abatement capital. Hence, both models are identical as far as the equilibrium investment rates and the steady-state levels of the stock of abatement capital are concerned. Therefore, from now on we focus on the comparison of the environmental variables, equilibrium emissions rates and steady-state levels of the stocks of pollution, as well as the discounted net welfare.

Next proposition shows one of the main results of our study because it establishes that our spatial model is a generalization of the non-spatial model in the sense that the behaviour of the environmental variables at the equilibrium in the non-spatial setting can be reproduced as a limit case of the spatial setting. In particular, this link is obtained when the parameter describing how pollution diffuses among regions increases unboundedly. Specifically, when the diffusion parameter tends to infinity, the mixing of the stocks of pollution in both regions is instantaneous, which is the main hypothesis in the non-spatial differential game.

In order to have comparable models from now on we assume that the following hypotheses are satisfied:

Hypotheses [H] Parameters ρ , μ_i , β_i , c_i , $i = 1, 2$ are identical in the differential games described by (10), (11) and (12), and (10), (20) and (21), respectively, and

$$\eta_1 = \eta_2 = 2\tilde{\eta}_1 = 2\tilde{\eta}_2; \quad d_1 = d_2 = \tilde{d}; \quad b_{10} = b_{20} = 0; \quad \varphi_i = \tilde{\varphi}_i, \quad i = 1, 2.$$

We remark that hypotheses **H** allow the two models to be comparable. More explicitly the hypothesis $d_1 = d_2 = \tilde{d}$ simply means that the rate of natural decay of the pollution stock is identical in both regions Ω_1 and Ω_2 and corresponds to the rate of decay of the only region in the model defined by (10), (20) and (21). Furthermore, in the non-spatial model the region under consideration is supposed by definition to be isolated from the exterior. This is exactly the meaning of hypothesis $b_{10} = b_{20} = 0$ (see (9)). It is more interesting the meaning of $\eta_1 = \eta_2 = 2\tilde{\eta}_1 = 2\tilde{\eta}_2$. First, we are in both cases in a symmetric scenario ($\eta_1 = \eta_2$ and $\tilde{\eta}_1 = \tilde{\eta}_2$) and, second in the non-spatial dynamics (21) the emissions of both players instantaneously affect the whole domain Ω under consideration. Conversely, in the spatial dynamics (12) the emissions of player i only affect region Ω_i . The size of Ω_i is supposed, if the rest of parameters are identical for both players, to be half the total area of domain Ω . So, the effect of emissions on the rate of change of the pollution stock in domain Ω are comparable for $\eta_1 = \eta_2 = 2\tilde{\eta}_1 = 2\tilde{\eta}_2$.

Proposition 3 Under hypotheses **H**, as the diffusion parameter b tends to infinity, then

- i) the average of the steady-state levels of the pollution stocks $\frac{p_i^{SS} + p_j^{SS}}{2}$ converges to \tilde{P}^{SS} ;
- ii) the equilibrium emission rate e_i converges to \tilde{e}_i .

Furthermore, if both regions are completely symmetric, then $p_i^{SS} = p_j^{SS} = \tilde{P}^{SS}$.

The results in Proposition 3 follow straightforwardly from Propositions 1 and 2 and show that even for this simple model where equilibrium strategies are constant, our formulation of the spatial model is a generalization of the non-spatial version of the model capturing the main ingredients of the spatial dynamics.

In view of these results and in order to evaluate the effect of the diffusion parameter on the equilibrium emissions rates and steady-state levels of the stocks of pollution, next proposition compares the environmental variables under hypotheses **H**.

Proposition 4 *Under hypotheses **H**, for any finite value of the diffusion parameter b :*

i) *the average of the steady-state levels of the pollution stock satisfies*

$$\frac{\tilde{P}^{SS}}{2} \leq \frac{p_i^{SS} + p_j^{SS}}{2} \leq \tilde{P}^{SS};$$

ii) *the equilibrium emission rate e_i satisfies:*

$$\frac{\tilde{e}_i}{2} \leq e_i \leq \tilde{e}_i.$$

The results can be derived taking into account the expressions of \tilde{P}^{SS} and \tilde{e}_i as well as the following differences

$$\begin{aligned} \tilde{P}^{SS} - \frac{p_i^{SS} + p_j^{SS}}{2} &= \frac{(d + \rho)^2(\varphi_i + \varphi_j)}{2d(b + d + \rho)\varphi_i\varphi_j}, \\ e_i - \tilde{e}_i &= -\frac{(d + \rho)^2}{2\tilde{\eta}_i(b + d + \rho)\varphi_i}. \end{aligned}$$

From Proposition 3, the gaps above tend to zero as the diffusion parameter b goes to infinity, which represents an instantaneous mix of the pollution stocks. However, the largest gaps arise when the diffusion parameter is zero, which can be viewed as the extreme case where there is not diffusion of pollution from one region to another.

Finally, we assess the impact of the diffusion parameter on the discounted net welfare. Next proposition compares the discounted net welfare for both spatial and non-spatial dynamic games.

Proposition 5 *Under hypotheses **H**, then,*

- i) *As the diffusion parameter b tends to infinity, the long-run discounted net welfare of player i in the spatial model, $V_i(K_i^{SS}, p_i^{SS}, p_j^{SS})$, converges to the long-run discounted net welfare of player i in the non-spatial model, $\tilde{V}_i(K_i^{SS}, \tilde{P}^{SS})$.*
- ii) *$V_i(K_i^{SS}, p_i^{SS}, p_j^{SS})$ could be greater or lower than $\tilde{V}_i(K_i^{SS}, \tilde{P}^{SS})$.*
- iii) *For the completely symmetric scenario $\varphi_i = \varphi_j = \varphi$, and for any finite value of the diffusion parameter b , $V_i(K_i^{SS}, p_i^{SS}, p_j^{SS})$, is always lower than $\tilde{V}_i(K_i^{SS}, \tilde{P}^{SS})$.*

The difference of the long-run discounted net welfares reads:

$$V_i(K_i^{SS}, p_i^{SS}, p_j^{SS}) - \tilde{V}_i(K_i^{SS}, \tilde{P}^{SS}) = \frac{1}{\rho} \log \left(\frac{2(b + d + \rho)}{2b + d + \rho} \right) - \frac{(d + \rho) [d(d + \rho)\varphi_i + b(2d + \rho(\varphi_i + \varphi_j))]}{d\rho\varphi_j(2b + d)(b + d + \rho)},$$

and items i) and ii) immediately follow.

For the completely symmetric scenario ($\varphi_i = \varphi_j = \varphi$), the difference above simplifies

$$V_i(K_i^{SS}, p_i^{SS}, p_j^{SS}) - \tilde{V}_i(K_i^{SS}, \tilde{P}^{SS}) = \frac{1}{\rho} \log \left(\frac{2(b + d + \rho)}{2b + d + \rho} \right) - \frac{(d + \rho)^2}{d\rho(b + d + \rho)}.$$

It can be easily proved that the RHS of the equation above increases with b , takes a negative value for $b = 0$ and tends to zero as b converges to infinity, and as a result, the difference is always negative.

5 Spatial vs. non-spatial model in a cooperative framework

In order to assess the impact of the spatial aspects on the definition of the emissions and investment policies in a cooperative setting, in this section we focus on the comparison of the cooperative strategies of the transboundary pollution dynamic games with and without spatial effects. For the ease of presentation and

as in the previous section we restrict ourselves to the particular case of two players. However, the main conclusions derived along this section can be easily extended to the more general case where the number of players is greater than 2. Next proposition characterizes the cooperative solution obtained solving an optimal control problem where the unique decision-maker chooses both regions' emission policies as well as the investment in the stock of abatement capital in order to maximize the joint welfare of both regions

$$J^c(e_1, e_2, I_1, I_2, \mathbf{p}^0, K_{10}, K_{20}) = \int_0^\infty \sum_{i=1}^2 e^{-\rho t} \left(\log(e_i) + \beta_i K_i - \frac{1}{2} c_i I_i^2 - \varphi_i p_i \right) dt, \quad (23)$$

subject to the dynamics of both stocks of abatement capital given by (10) and of the aggregated stock of pollution in each region defined by (12).

Proposition 6 *The cooperative emission and investment rates of the optimal control problem defined by (10), (12) and (23) are constant and given by:*

$$I_i^c = \frac{\beta_i}{c_i(\mu_i + \rho)}, \quad e_i^c = \frac{(d_j + \rho)b + (b + d_j + \rho)(d_i + \rho)}{\eta_i((b + d_j + \rho)\varphi_i + b\varphi_j)}, \quad i = 1, 2, i \neq j,$$

where $b = b_{ij} = b_{ji}$ for $i \neq j$ and the superscript c stands for cooperative.

The optimal cooperative discounted net welfare is given by:

$$V^c(K_i, K_j, p_i, p_j) = M^c K_i + N^c K_j + R^c p_i + T^c p_j + X^c, \quad (24)$$

where

$$\begin{aligned} M^c &= M_{i,i} = \frac{\beta_i}{\mu_i + \rho}, \quad N^c = M_{j,j} = \frac{\beta_j}{\mu_j + \rho}, \\ R^c &= -\frac{\varphi_i(b + d_j + \rho) + b\varphi_j}{(d_i + \rho)(b + d_j + \rho) + b(d_j + \rho)}, \quad T^c = -\frac{\varphi_j(b + d_i + \rho) + b\varphi_i}{(d_i + \rho)(b + d_j + \rho) + b(d_j + \rho)}, \\ X^c &= \frac{1}{2\rho} (X_1^c + X_2^c), \\ X_1^c &= -2 - \frac{d_i q_i((d_j + \rho)\varphi_i + b(\varphi_i + \varphi_j)) + d_j q_j((d_i + \rho)\varphi_j + b(\varphi_i + \varphi_j))}{(d_i + \rho)(d_j + \rho) + b(d_i + d_j + 2\rho)}, \\ X_2^c &= \frac{\beta_i^2}{c_i(\mu_i + \rho)^2} + \frac{\beta_j^2}{c_j(\mu_j + \rho)^2} + 2 \log \left(\frac{(b(d_i + \rho) + (b + d_i + \rho)(d_j + \rho))^2}{\eta_i \eta_j ((b + d_j + \rho)\varphi_i + b\varphi_j)((b + d_i + \rho)\varphi_j + b\varphi_i)} \right), \end{aligned}$$

for $i, j = 1, 2, i \neq j$.

The cooperative steady-state levels of abatement capital and pollution stocks are given by:

$$\begin{aligned} K_i^{SSc} &= \frac{\beta_i}{c_i \mu_i (\mu_i + \rho)}, \quad p_i^{SSc} = \frac{Num p_i^{SSc}}{d_i d_j + b(d_i + d_j)}, \\ Num p_i^{SSc} &= b q_j d_j + q_i d_i (b + d_j) \\ &\quad + ((d_i + \rho)(d_j + \rho) + b(d_i + d_j + 2\rho)) \left(\frac{b}{(b + d_i + \rho)\varphi_j + b\varphi_i} + \frac{b + d_j}{b\varphi_j + (b + d_j + \rho)\varphi_i} \right), \end{aligned}$$

for $i, j = 1, 2, i \neq j$, where the superscript SSc stands for cooperative steady-state levels and q_k is the aggregation of the boundary data defined in (12). The steady-state equilibrium is globally asymptotically stable.

Similarly, in the cooperative formulation of the non-spatial model the unique decision-maker chooses both regions' emission policies as well as the investment in the stock of abatement capital in order to maximize the joint welfare of both regions

$$\tilde{J}^c(\tilde{e}_1, \tilde{e}_2, I_1, I_2, \tilde{P}_0, K_{10}, K_{20}) = \int_0^\infty \sum_{i=1}^2 e^{-\rho t} \left(\log(\tilde{e}_i) + \beta_i K_i - \frac{1}{2} c_i I_i^2 - \tilde{\varphi}_i \tilde{P} \right) dt, \quad (25)$$

subject to the dynamics of both stocks of abatement capital given by (10) and of the stock of pollution defined by (21). Next proposition characterizes the cooperative solution of this optimal control problem.

Proposition 7 *The cooperative emission and investment rates of the optimal control problem defined by (10), (21) and (25) are constant and given by:*

$$I_i^c = \frac{\beta_i}{c_i(\mu_i + \rho)}, \quad \tilde{e}_i^c = \frac{\rho + \tilde{d}}{\tilde{\eta}_i(\tilde{\varphi}_i + \tilde{\varphi}_j)}, \quad i = 1, 2, i \neq j,$$

where the superscript c stands for cooperative.

The optimal cooperative discounted net welfare is given by:

$$\tilde{V}^c(K_i, K_j, \tilde{P}) = M^c K_i + N^c K_j + \tilde{R}^c \tilde{P} + \tilde{X}^c, \quad (26)$$

where

$$M^c = M_{i,i} = \frac{\beta_i}{\mu_i + \rho}, \quad N^c = M_{j,j} = \frac{\beta_j}{\mu_j + \rho}, \quad \tilde{R}^c = -\frac{\tilde{\varphi}_i + \tilde{\varphi}_j}{\rho + \tilde{d}},$$

$$\tilde{X}^c = \frac{1}{2\rho} \left(\frac{\beta_i^2}{c_i(\mu_i + \rho)^2} + \frac{\beta_j^2}{c_j(\mu_j + \rho)^2} - 4 + 2 \log \left(\frac{(\tilde{d} + \rho)^2}{\tilde{\eta}_i \tilde{\eta}_j (\tilde{\varphi}_i + \tilde{\varphi}_j)} \right) \right),$$

for $i = 1, 2, i \neq j$.

The cooperative steady-state levels of abatement capital and pollution stocks are given by:

$$K_i^{SSc} = \frac{\beta_i}{c_i \mu_i (\mu_i + \rho)}, \quad \tilde{P}^{SSc} = \frac{2(\rho + \tilde{d})}{\tilde{d}(\tilde{\varphi}_i + \tilde{\varphi}_j)},$$

for $i = 1, 2, i \neq j$, where the superscript SSc stands for cooperative steady-state levels. The steady-state equilibrium is globally asymptotically stable.

As one can note, the cooperative investment rates for both optimization problems are the same as the equilibrium investment strategies obtained in the non-cooperative frameworks. This result implies that the Nash equilibrium investment strategies are Pareto optimal. This comes as a result of the structure of the model where there is no interaction between the investment decisions of the players. The interaction between the emission decisions of the players through their effect on the accumulation of the pollution stock(s) implies that the cooperative emission rates are different from the equilibrium emission strategies obtained in the non-cooperative settings. It can be easily shown that, as expected, the emission levels are lower under cooperation (both for the spatial and non-spatial differential game). The difference between cooperative and non-cooperative emission rates lies in the fact that in the non-cooperative setting each region (player) takes into account only his marginal damage cost (φ_i or $\tilde{\varphi}_i$ for the spatial and non-spatial models, respectively), while in the cooperative setting the decision-maker takes into account both players marginal damage costs (φ_i, φ_j or $\tilde{\varphi}_i, \tilde{\varphi}_j$). Because the emission levels are lower under cooperation, the steady-state level of each stock of pollution is lower under cooperation too.

Next three propositions follow the same patterns as Propositions 3, 4 and 5 but now for the cooperative setting. Similarly to Proposition 3 and under the same assumptions on the model parameters next we show that also for the cooperative framework our spatial model generalizes the non-spatial model in the sense that as the diffusion parameter tends to infinity, the environmental variables at the equilibrium in the spatial model converge to those variables in the non-spatial setting.

Proposition 8 *Under hypotheses H, as the diffusion parameter b tends to infinity,*

- i) *the average of the cooperative steady-state levels of the pollution stocks $\frac{p_i^{SSc} + p_j^{SSc}}{2}$ converges to \tilde{P}^{SSc} ;*
- ii) *the cooperative emission rate e_i^c converges to \tilde{e}_i^c .*

Furthermore, if both regions are completely symmetric, then $p_i^{SSc} = p_j^{SSc} = \tilde{P}^{SSc}$.

The proof is straightforward from Propositions 6 and 7.

Next proposition compares the cooperative emissions rates and cooperative steady-state levels of the stocks of pollution for the spatial and non-spatial models under hypotheses **H**, in order to evaluate the effect of the diffusion parameter on the environmental variables.

Proposition 9 *Under hypotheses **H**, for any finite value of the diffusion parameter b :*

i) the average of the cooperative steady-state levels of the pollution stocks satisfies

$$\tilde{P}^{SSc} \leq \frac{p_i^{SSc} + p_j^{SSc}}{2} \leq \frac{(\varphi_i + \varphi_j)^2}{4\varphi_i\varphi_j} \tilde{P}^{SSc};$$

ii) the cooperative emission rate satisfies

$$\min\left\{\tilde{e}_i^c, \frac{\varphi_i + \varphi_j}{2\varphi_i}\tilde{e}_i^c\right\} \leq e_i^c \leq \max\left\{\tilde{e}_i^c, \frac{\varphi_i + \varphi_j}{2\varphi_i}\tilde{e}_i^c\right\};$$

iii) If the marginal environmental damage costs are identical for the two regions ($\varphi_i = \varphi_j$), then $e_i^c = \tilde{e}_i^c$, and consequently, $\frac{p_i^{SSc} + p_j^{SSc}}{2} = \tilde{P}^{SSc}$.

The results can be derived taking into account the expressions of \tilde{P}^{SSc} and \tilde{e}_i^c as well as the following differences

$$\begin{aligned} \tilde{P}^{SSc} - \frac{p_i^{SSc} + p_j^{SSc}}{2} &= -\frac{(d+\rho)^3(\varphi_i - \varphi_j)^2}{2d(\varphi_i + \varphi_j)((b+d+\rho)\varphi_i + b\varphi_j)((b+d+\rho)\varphi_j + b\varphi_i)}, \\ e_i^c - \tilde{e}_i^c &= -\frac{(d+\rho)^2(\varphi_i - \varphi_j)}{2\tilde{\eta}_i(\varphi_i + \varphi_j)((b+d+\rho)\varphi_i + b\varphi_j)}. \end{aligned}$$

As in the non-cooperative setting, the gaps above tend to zero as the diffusion parameter b goes to infinity (instantaneous mix of the stocks of pollution) and the gaps increase as the diffusion parameter tends to zero (the stock of pollution does not spread from one region to another). It is worth noting that the lower and upper bounds of the equilibrium emission rate e_i^c in item ii) depend on how the marginal environmental damage costs of the regions compare. If $\varphi_i > \varphi_j$, then e_i^c runs between $\frac{\varphi_i + \varphi_j}{2\varphi_i}\tilde{e}_i^c$ and \tilde{e}_i^c , and the cooperative emission rate of region i in the spatial model is always lower than the corresponding rate in the non-spatial model. However, if $\varphi_i < \varphi_j$, then e_i^c runs between \tilde{e}_i^c and $\frac{\varphi_i + \varphi_j}{2\varphi_i}\tilde{e}_i^c$, and the cooperative emission rate of region i in the spatial model is always greater than the corresponding rate in the non-spatial model.

Next proposition compares the long-run discounted net welfare for both spatial and non-spatial cooperative dynamic games.

Proposition 10 *Under hypotheses **H**, then,*

i) As the diffusion parameter tends to infinity, the long-run cooperative discounted net welfare in the spatial model, $V^c(K_i^{SSc}, K_j^{SSc}, p_i^{SSc}, p_j^{SSc})$, converges to the corresponding value in the non-spatial model, $\tilde{V}^c(K_i^{SSc}, K_j^{SSc}, \tilde{P}^{SSc})$.

ii) $TV^c(K_i^{SSc}, K_j^{SSc}, p_i^{SSc}, p_j^{SSc})$ could be greater or lower than $\tilde{V}^c(K_i^{SSc}, K_j^{SSc}, \tilde{P}^{SSc})$.

iii) For the completely symmetric scenario ($\varphi_i = \varphi_j = \varphi$), and for any value of the diffusion parameter $V^c(K_i^{SSc}, K_j^{SSc}, p_i^{SSc}, p_j^{SSc})$ and $\tilde{V}^c(K_i^{SSc}, K_j^{SSc}, \tilde{P}^{SSc})$ are identical.

The difference of the long-run cooperative discounted net welfares reads:

$$V^c(K_i^{SSc}, K_j^{SSc}, p_i^{SSc}, p_j^{SSc}) - \tilde{V}^c(K_i^{SSc}, K_j^{SSc}, \tilde{P}^{SSc}) = \\ \frac{1}{\rho} \log \left(\frac{(2b + d + \rho)^2 (\varphi_i + \varphi_j)^2}{4((d + \rho)\varphi_i + b(\varphi_i + \varphi_j))((d + \rho)\varphi_j + b(\varphi_i + \varphi_j))} \right) \\ - b \frac{(d + \rho)^2 (\varphi_i - \varphi_j)^2}{d(2b + d)((d + \rho)\varphi_i + b(\varphi_i + \varphi_j))((d + \rho)\varphi_j + b(\varphi_i + \varphi_j))}.$$

From the expression above the proof easily follows.

6 Concluding remarks

This paper analyzes a transboundary pollution differential game where, in addition to the standard temporal dimension, a spatial dimension is introduced to capture the different geographical relationships among regions. There is a fairly recent literature devoted to the analysis of different economic and environmental problems by means of dynamic models that include the spatial dimension. However, most of this literature either neglects the strategic interactions among decision makers or neglects the dynamic aspect of the model. In the first case, the papers focus on the problem of a social planner; while in the second case, the decision-makers behave myopically in both the temporal and the spatial dimensions, and hence, agents solve static problems. As far as we know De Frutos and Martín-Herrán (2017) is the only recent study that considers that decision makers behave both dynamically and strategically. In the present paper we follow the same approach and characterize the equilibrium outcomes of an intertemporal pollution problem where there is a continuum of spatial sites and the pollution stock diffuses over these sites. The functional specifications of the present work allow us to analytically treat the conditions that characterize the Markov-perfect Nash equilibrium of the space-discretized differential game. In De Frutos and Martín-Herrán (2017) the results are illustrated by means of numerical experiments even for the simplest case of two regions. Furthermore, our present functional specification allows the regions to invest in clean technology or abatement capital in order to reduce the emission-output ratio. Hence, the present specification allows us to analyze the effect of this new technology on the optimal emission strategies and the stock of pollution.

Our analytical results show, on the one hand, how the equilibrium emission policies in a spatial context differ from those characterized ignoring the spatial dimension. On the other hand, the comparison of the equilibrium emission policies we have obtained in our spatial differential game version and those obtained for the same model when the spatial aspects are disregarded allows us to show that our spatial model can be viewed as a generalization of the non-spatial model. The equilibrium of the non-spatial formulation can be reproduced as a limit case of the spatial differential game. Specifically, the equilibrium environmental policy of the spatial model coincides with the equilibrium policy of the non-spatial model when the diffusion parameter, that describes how pollution diffuses among regions, tends to infinity. In this case, the stocks of pollution in the regions are instantaneously mixed, as is implicitly assumed in the standard hypothesis in a non-spatial setting.

One first further step in the analysis could be to evaluate the impact of the adoption of cleaner technology on the equilibrium emission rates and the long-run value of the pollution stock when the spatial dimension is taken into account. Recently, Benckekroun and Ray Chaudhuri (2014, 2015) have shown that the adoption of a cleaner technology may imply that the countries respond by increasing their emissions resulting in an increase of pollution that may be detrimental to welfare. The strategic behavior of the players may lead at first glance to counterintuitive results when the free-riding effect is exacerbated (Benckekroun and Martín-Herrán (2016) study this effect in a transboundary pollution game with myopic and farsighted players).

In the present formulation pollution has a local dimension as a direct consequence of the production of the consumption good in a particular region. Another possible extension could be to add a second dimension for the pollution and consider that pollution produced in other regions may also harm welfare. In this case, the environmental damage function would depend on the pollution over the entire spatial domain. In a different

framework Camacho and Pérez-Barahona (2015) introduced the local and global dimension of pollution in their study of optimal land use and environmental degradation. This analysis is one of the subjects of our future research.

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