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# Underground long-term mine production scheduling with integrated geological risk management

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**Abstract:** A stochastic integer-programming (SIP) model is presented to optimize long-term scheduling of underground mine operations while considering geological uncertainty. To integrate this uncertainty, a set of stochastic simulations is generated, corresponding to representations of the deposit, and is used as primary inputs to optimization. The two-stage SIP model developed considers a variable cut-off grade and accounts for maximum development, material handling flow conservation, mill and mine capacity, and activity precedencies for an underground nickel mine. The results show that the schedule generated has a higher expected value when considering and managing grade risk. They also demonstrate the benefits of risk control, which this approach allows.

**Key Words:** Underground mine, stochastic optimisation, production scheduling, long-term-planning, geological uncertainty.

**Résumé :** Un modèle de programmation linéaire stochastique est présenté pour optimiser la planification à long terme dans une mine souterraine en tenant compte de l'incertitude géologique. Pour intégrer cette incertitude, un ensemble de simulations stochastiques est généré, correspondant à la représentation du gisement, et utilisé comme intrant au modèle d'optimisation. Le modèle stochastique développé considère une teneur de coupure variable, la capacité des équipements et la précédence entre les activités d'une mine de nickel. Les résultats montrent que les horaires de production générés offrent une valeur présente nette plus élevée lorsqu'on tient compte de l'incertitude géologique.

**Acknowledgments:** The authors acknowledge the industrial partner for sharing the data used for the case study. Without their generosity, the application of this approach would not have been validated. They also express their gratitude to Jean Collard, who contributed to this paper by developping and analysing the deterministic model used for comparison purposes in this project. Finally, the work in this paper was funded by Natural Sciences and Engineering Research Council of Canada (NSERC) and the COSMO Stochastic Mine Planning Laboratory.

# 1 Introduction

Over the last decades, stochastic optimization methods have been developed to deal with geological uncertainty in open pit mine design and life-of-mine production scheduling (e.g. Menabde et al., 2005; Boland et al., 2008; Dimitrakopoulos, 2011; Goodfellow and Dimitrakopoulos, 2013; others). These efforts stem from earlier documentations of the adverse effects of geological uncertainty of mining projects and production performance (e.g. Dowd 1994, 1997; Morley et al., 1999; Valee 2000; Dimitrakopoulos et al., 2002) and aim to provide effective geological risk management, leading to more robust mine production forecasts and improved net present value assessments. Similar efforts in underground mine planning and production scheduling are very limited, as the required optimization developments are dependent, unlike in open pit mines, on the mining method employed, making generalizations of solutions challenging.

Optimization methods in underground mining were introduced in the 1980's (Lisotte and Elbrond, 1985; Chatterjee and Sridhar, 1986) and to date remains largely conventional. Mixed integer linear programming (MILP) approaches were further developed in the 1990's to consider mine production scheduling, including haulage capacity and backfill (Trout, 1997; Topal 1998), optimization of stope geometry (Ovanic, 1998), large scale production scheduling in sublevel caving iron mines (Topal, 2003) and block caving mining (Rahal et al., 2003), but were, in general, unable to solve realistic industrial mining scenarios. Additional information can be found in a review by Alford et al. (2007). More recently, Nehring et al. (2010) present a new MIP model for underground mine planning aiming to accelerate the computational time needed for practical applications. Accordingly, while binary variables are defined to represent all four typical production phases (development, drilling, extraction and backfill), the authors simplify the model by assigning a single variable for the entire production, thereby significantly reducing the model complexity and computational time. Little et al. (2013) consider the impact of the interaction between underground stope design and long term scheduling, and O'Sullivan and Newman (2014) propose a model for underground mine scheduling, which aim to maximize metal production and respect resource constraints, while considering sequencing of backfill and rock extraction operations. A heuristic is used to provide a solution to the formulation. Notable is the approach in Roberts and Bloss (2014), who show an adaptation of BHP Billiton's BLASOR (e.g. Stone et al., 2004), an open pit planning optimiser and life-of-mine production scheduling software based on MILP, to the optimization of underground strategic mine planning. Other deterministic optimization applications in underground mining include decline optimization (Brazil et al., 2003), cost optimization of mining networks (Brazil et al., 2005), and joint optimization of designing access and scheduling (Sirinanda et al., 2014). Geological risk is not considered in any of the past work, with initial efforts limited to quantifying risk in stope designs (Myers et al., 2005) and integer programming formulations incorporating probabilities above given cut-off grades (Grieco and Dimitrakopoulos, 2007).

This paper presents a new optimization formulation integrating geological uncertainty that allows for assessment of its impact on the financial performance, including capital investments and operational costs, while meeting production expectations of a mining project considering a group of underground mines with orebodies in the form of lenses. The new formulation is based on stochastic integer programming (SIP) with recourse (Birge and Louveaux, 1997), similar to those developed for open pit mines (Ramazan and Dimitrakopoulos, 2005, 2013; Benndorf and Dimitrakopoulos, 2013; Goodfellow, 2014; other). The deterministic equivalent of the formulation herein is described in Collard (2013), where a project extension in a group of underground nickel mines is considered to assess economically mineable sectors of the individual mines and the optimal sequence of extraction. The formulation is developed to maximize Net Present Value (NPV), while including constraints for activity precedencies, material handling and mineral resources' limits. Figure 1 shows a schematic section of an underground mine, as considered herein, and includes mineralized lenses, ramps that need to be excavated and require capital expenditure (CAPEX) and development sections needed to access and extract the materials of the mineralized lenses, requiring corresponding operational expenditure (OPEX). The mining method considered is a hybrid, combining cut and fill (CF) and long-hole's (LH), and is detailed in the case study section.

The SIP model presented here generates a long-term schedule which improves the value of the project and also assesses which mines to open, when considering geological uncertainty. The latter uncertainty is described in terms of grades (nickel) and it is based on the well-established stochastic or geostatistical simulations described in textbooks (David, 1988; Goovaerts, 1997; Remy et al., 2009). Stochastic simulation methods generate equally probable scenarios of the mineral deposit in the ground, based on and reproducing all the available data, information, as well as the local spatial variability of grades. A group of these equally probable scenarios or realizations represent the quantified geological uncertainty in a mineral deposit and is the input to stochastic optimizers. Note that conventional representations of deposits using estimation methods provide smooth representations of the actual deposit considered and mislead deterministic optimization approaches (Ravenscroft 1992; Dowd 1994; Dimitrakopoulos et al., 2002; Godoy and Dimitrakopoulos, 2004), a topic also discussed herein.

The SIP model considers the case of several potential mines, each involving a set of several mineralized lenses and determines the economically minable lenses, the optimal cut-off grade to be used, the beginning of activities (CAPEX and OPEX; development, mine opening lenses' extraction) and production rate, while managing and minimizing deviations from production expectations in terms of ore and metal production.

Figure 1 is a schematic representation of a typical overall underground mine, regardless of the mining method. This elemental sketch shows the main activities along with the precedencies between each of them. In this case, the term CAPEX (Capital Expenditure) stands for the ramp excavation only and OPEX (Operational Expenditure) represents, for an extractable lens (mineralized zone), the development section corresponding. If a lens is not profitable, the OPEX associated to it is not extracted either. Also, this particular case comprises two mining methods, where Cut and Fill (CF) segment has to be extracted before Long-hole's (LH). More detailed explanations will be found in the case study section.

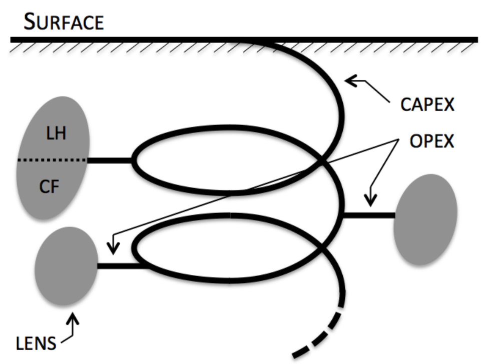


Figure 1: Typical underground mine setup

The stochastic integer programming model (SIP) presented aims to generate the optimal long-term schedule of an underground project extension, while considering geological uncertainty. This project comprises five potential mines, each involving a set of several mineralized lenses. More precisely, the mathematical model determines the economically minable lenses and the cut-off grade associated, the beginning of activities (CAPEX and OPEX development, mine opening, the lenses' extraction) and finally the production rate. All of these values are optimised by using a set of conditional simulations as an input. We assume they all reproduce original data in situ variability and grade distribution, and consequently have an equal chance to be the actual orebody representation. However, we generate the best possible schedule to maximize NPV and reduce the impact of geological uncertainty. In this section, notations will be presented, followed by the objective-function and constraints.

In the following sections, the proposed SIP formulation is detailed and then a case study is presented. The case study includes the results from the proposed method and comparison to its deterministic equivalent, including the results in Collard (2013). Conclusions and recommendations for further work follow.

# 2 Method

## 2.1 Notations

### Index

Subscripts:

- $s$ is the scenario index, with $s = 1, ..., S$;
- $t$ is the period index, with $t = 1, ..., T$ and where $T$ is the planning horizon;
- $m$ is the mine index, with $m = 1, ..., M$;
- $\ell$ is the lens index;
- $r$ is the ramp index;
- $r_m^1$ is the index representing the first ramp of mine $m$;
- $g$ is the cut-off grade index.

Superscripts:

- $R$, $O$, and $P$ are superscript indexes corresponding to development of ramps (CAPEX), development of accesses to lens (OPEX), and production activities, respectively.
- $Beg$ and $End$ are superscript indexes to indicate the opening and the closure of a mine, respectively.

### Sets

- $\mathcal{G}$ is the set of cut-off grades;
- $\mathcal{L}$ is the set of lenses;
- $\mathcal{L}_m$ is the set of lenses in mine $m$;
- $\mathcal{L}_r^{succ}$ is the set of lenses successors to ramp $r$;
- $\mathcal{R}_m$ is the set of ramps in mine $m$;
- $\mathcal{R}_r^{succ}$ is the set of ramps successors to ramp $r$;
- $\mathcal{R}_\ell^{pred}$ is the set of ramps that have to be extracted before mining lens $\ell$;

### Parameters

- $O_{\ell gs}$ corresponds to the tonnage of ore that is available in lens $\ell$ if cut-off grade $g$ is selected in scenario $s$;
- $\overline{O}_t$ is the maximum tonnage of ore that can be extracted during year $t$;
- $N_{\ell gs}$ corresponds to the tonnage of nickel in lens $\ell$ if cut-off grade $g$ is selected in scenario $s$;
- $\overline{N}_t$ is the maximum tonnage of nickel that can be extracted during year $t$;
- $L_r^R$ is the length of ramp $r$;
- $L_{\ell gs}^O$ length of drifts to excavate to access lens $\ell$ if cut-off grade $g$ is selected in scenario $s$;
- $L_{\ell gs}^P$ length of excavation that must be done in lens $\ell$ if cut-off grade $g$ is selected in scenario $s$;
- $\overline{L}_t$ is the maximum length that can be excavated during year $t$;
- $V_r^R$, $V_{\ell gs}^O$, $V_{\ell gs}^P$ correspond to volumes ($m^3$) of waste material (valueless);
- $C_r^R$,$C_{\ell gs}^O$ and $C_{\ell gs}^P$ correspond to costs of mining;
- $C_m^{Beg}$ and $C_m^{End}$ are investment required for starting and closing mine $m$, respectively;
- $C_m^{Fix}$ is the maintenance cost (\$/year) fixed to keep the mine $m$ operating;
- $C^W$ is the excess waste material handling cost (\$/$m^3$);
- $C^{Ore}$, $C^N$, $C^L$, $C^A$ are penalties (costs) associated with excess variables for ore and nickel production, development length and percentage of advancement (capacity constraints being relaxed);
- $f_1^t$ and $f_2^t$ are respectively the economic and geological discount factors;

- $R_t$ is the revenue per ton of Ni ($/t), which considers market price, treatment cost and recuperation rate at the mill;
- $F^S$ is the swelling factor for waste material;
- $F^L$ is the loading factor for extracted lenses;
- $LM$ is the maximal development length ($m$) mined in straight line per period.

## 2.2 Variables

### Binary

- $y^R_{rt}$, $y^O_{\ell t}$, and $y^P_{\ell t}$are equal to 1 when the corresponding activity starts at time $t$, and 0 otherwise;
- $y^{Beg}_{mt}$ and $y^{End}_{mt}$ are equal to 1 if mine $m$ is opened or closed at time $t$
- $y^G_{\ell g}$ equal 1 if cut-off grade $g$ is selected for lens $\ell$.

### Continious

- $x^R_{rt}$, $x^O_{\ell tg}$, and $x^P_{\ell tg}$ correspond to the proportion of the activity completed during period $t$;
- $w^{out}_{mt}$ and $w^{in}_{mt}$ correspond to the waste volume ($m^3$) that has to be handled respectively up to surface (out) and down in the underground mine (in). Note that these two variables cannot be equal to zero simultaneously.;
- $e^{Ore}_{ts}$, $e^N_{ts}$, $e^L_{ts}$, $e^A_{\ell ts}$, $e^{out}_{mts}$ and $e^{in}_{mts}$ correspond to excess variables used to ensure a feasible solution;

## 2.3 Proposed model

### Definition of the objective function

$$
\begin{aligned}
\max Z = \frac{1}{S}\sum_s\sum_t f_1^t\Big( & \overbrace{R_t\sum_\ell\sum_g (N^P_{\ell gs}x^P_{\ell tg})}^{\text{part 1}} \\
& \overbrace{-\sum_r C^R_r x^R_{rt} - \sum_\ell\sum_g C^O_{\ell gs}x^O_{\ell tg}}^{\text{part 2}} \\
& -\overbrace{\sum_\ell\sum_g (C^P_{\ell g}x^P_{\ell tg})}^{\text{part 3}} \\
& -\overbrace{\sum_m (C^{Beg}_m y^{Beg}_{mt} + C^{End}_m y^{End}_{mt})}^{\text{part 4}} \\
& \overbrace{-\sum_m C^{Fix}_m (\sum_{u=1}^{t} y^R_{r^1_m u} - \sum_{u=1}^{t} y^{End}_{mt})}^{\text{part 5}} \\
& -\overbrace{\sum_m C^W (w^{in}_{mt} + w^{out}_{mt})}^{\text{part 6}} \Big) \\
-f_2^t\Big( & \overbrace{C^{Ore}e^{Ore}_{ts} + C^N e^N_{ts} + C^L e^L_{ts} + \sum_\ell C^A e^A_{\ell ts} + \sum_m C^W(e^{in}_{mts} + e^{out}_{mts})}^{\text{part 7}} \Big)
\end{aligned} \tag{1}
$$

$$
\sum_t y^R_{rt} \leq 1 \qquad \forall r;\ t \in T \tag{2}
$$

$$\sum_t y^O_{\ell t} \leq 1 \qquad \forall \ell;\ t \in T \tag{3}$$

$$\sum_t y^P_{\ell t} \leq 1 \qquad \forall \ell;\ t \in T \tag{4}$$

$$\sum_t y^{Beg}_{mt} \leq 1 \qquad \forall m;\ t \in T \tag{5}$$

$$\sum_t y^{End}_{mt} \leq 1 \qquad \forall m;\ t \in T \tag{6}$$

$$\sum_g y^G_{\ell g} = 1 \qquad \forall \ell;\ g \in \mathcal{G} \tag{7}$$

$$\sum_t x^O_{\ell tg} \leq y^G_{\ell g} \qquad \forall \ell;\ \forall g;\ t \in T \tag{8}$$

$$\sum_t x^P_{\ell tg} \leq y^G_{\ell g} \qquad \forall \ell;\ \forall g;\ t \in T \tag{9}$$

$$\sum_{u=1}^{t} y^{End}_{mu} \leq \sum_{u=1}^{t} y^{Beg}_{mu} \qquad \forall m;\ \forall t \tag{10}$$

$$\sum_{u=1}^{t} x^R_{ru} \leq \sum_{u=1}^{t} y^R_{ru} \qquad \forall r;\ \forall t \tag{11}$$

$$\sum_g \sum_{u=1}^{t} x^O_{\ell ug} \leq \sum_{u=1}^{t} y^O_{\ell u} \qquad \forall \ell;\ \forall t;\ g \in \mathcal{G} \tag{12}$$

$$\sum_g \sum_{u=1}^{t} x^P_{\ell ug} \leq \sum_{u=1}^{t} y^P_{\ell u} \qquad \forall \ell;\ \forall t;\ g \in \mathcal{G} \tag{13}$$

$$x^R_{rt} \leq 1 - \sum_{u=1}^{t} y^{End}_{mu} \qquad \forall m;\ \forall r \in \mathcal{R}_m;\ \forall t \tag{14}$$

$$\sum_g x^O_{\ell tg} \leq 1 - \sum_{u=1}^{t} y^{End}_{mu} \qquad \forall m;\ \forall \ell \in \mathcal{L}_m;\ \forall t;\ g \in \mathcal{G} \tag{15}$$

$$\sum_g x^P_{\ell tg} \leq 1 - \sum_{u=1}^{t} y^{End}_{mu} \qquad \forall m;\ \forall \ell \in \mathcal{L}_m;\ \forall t;\ g \in \mathcal{G} \tag{16}$$

$$\sum_{u=1}^{t} y^R_{iu} \leq \sum_{u=1}^{t} x^R_{ru} \qquad \forall r;\ \forall t;\ \forall i \in \mathcal{R}^{succ}_r \tag{17}$$

$$\sum_{u=1}^{t} y^O_{iu} \leq \sum_{u=1}^{t} x^R_{ru} \qquad \forall r;\ \forall t;\ \forall i \in \mathcal{L}^{succ}_\ell \tag{18}$$

$$\sum_{u=1}^{t} y^P_{\ell u} \leq \sum_g \sum_{u=1}^{t} x^O_{\ell ug} \qquad \forall \ell;\ \forall t;\ g \in \mathcal{G} \tag{19}$$

$$\sum_\ell \sum_g O_{\ell gs} x^P_{\ell tg} - e^{Ore}_{ts} \leq \overline{O}_t \qquad \forall t;\ \forall s;\ \ell \in \mathcal{L};\ g \in \mathcal{G} \tag{20}$$

$$\sum_\ell \sum_g N_{\ell gs} x^P_{\ell tg} - e^{N}_{ts} \leq \overline{N}_t \qquad \forall t;\ \forall s;\ \ell \in \mathcal{L};\ g \in \mathcal{G} \tag{21}$$

$$\sum_r L^R_r x^R_{rt} + \sum_\ell \sum_g (L^O_{\ell gs} x^O_{\ell tg} + L^P_{\ell gs} x^P_{\ell tg}) - e^L_{ts} \leq \overline{L}_t \qquad \forall m;\ \forall t;\ \forall s;\ \forall r \in \mathcal{R}_m;\ \ell \in \mathcal{L};\ g \in \mathcal{G} \quad (22)$$

$$\sum_r (L^R_r/LM) x^R_{rt} + \sum_g (L^O_{\ell gs}/LM) x^O_{\ell tg} + \sum_g (L^P_{\ell g}/LM) x^P_{\ell tg} - e^A_{\ell ts} \leq 1 \qquad \forall \ell;\ \forall t;\ \forall s;\ r \in \mathcal{R}^{pred}_\ell;\ g \in \mathcal{G} \quad (23)$$

$$F^S \Big\{ \sum_r V^R_r x^R_{rt} + \sum_\ell \sum_{\forall g} V^O_{\ell gs} x^O_{\ell tg} \Big\} + w^{in}_{mt} + e^{in}_{mts} - F^L \Big\{ \sum_\ell \sum_g V^P_{\ell gs} x^P_{\ell tg} \Big\} - w^{out}_{mt} - e^{out}_{mts} = 0 \qquad \forall m;\ \forall t;\ \forall s;\ r \in \mathcal{R}_m;\ \ell \in \mathcal{L}_m;\ g \in \mathcal{G} \quad (24)$$

The objective function equation (1), upon considering all periods and equally probable scenarios, aims to maximize profit while minimizing cost of deviation from selected targets. More precisely, the first part computes revenue (Part 1) and the next five parts indicate the costs of development (Part 2), production (Part 3), opening and closing of mines (Part 4), keeping the mine in operation (Part 5) and material handling (Part 6). The last section (Part 7) is used for geological risk management purposes, as we penalize the objective-function by applying costs for deviation. In this case, the indicators chosen to be controlled are: the ore and metal tonnage, the length of development, the percentage of advancement and the extra material handling.

In addition, the first block of constraints ensures the uniqueness of the beginning of activities. Indeed, (2), (3) and (4) correspond to the fact that extraction of a CAPEX, OPEX, or production segment starts its extraction no more than once, and (5) and (6) take into account mine opening and mine closure. We assume, for the sake of simplicity, that a mine cannot be temporarily closed and reopened during the time horizon of the project. If unprofitable, either a lens may never be extracted, or a mine may never be in operation. As for (7), it ensures that only one cut-off grade value (COG) is assigned to a lens. Indeed, the mine scheduler selects several COG to be tested and potentially chosen depending on the lens.

Links between OPEX development and production extraction rate to the selected cut-off grade regarding a lens $i$ are ensured using (8) and (9). Indeed, the OPEX development and the production required for a lens depend on its COG. This last value directly influences the size of the economically minable lens and the corresponding operational development.

The dependency between the mine opening and its closure is ensured by (10): if a mine has to open, it will also have to be closed eventually. Also, (11), (12) and (13) link continuous and binary variables. These constraints mean that in period $t$, for lens $i$, the extraction rate of both CAPEX and OPEX development, and production cannot be positive if the activity has not begun. From the time $t$, where the decision to begin a task is taken, the related continuous variable will be allowed to be positive. Their sum will not exceed 1, which corresponds to 100% extracted. In the case where an activity never starts, we are certain that the corresponding mining rate will be 0.

Also, activities associated to mine $m$ have to be accomplished before the mine closure; this is constrained by (14), (15) and (16).

The activities are sequenced with respect to the precedence, which is ensured by (17), (18) and (19).

According to the simulation, the volume and the average tonnage will change from one scenario to another. This makes it difficult, even impossible, to satisfy the following three blocks of constraints for ten different scenarios. Slack and surplus variables are introduced into the model to ensure feasibility. These variables represent deviation from the production targets.

The next block of constraints is called "Global constraints". For each period $t$, they aim to limit the quantity of ore extracted (20), the metal produced (21) and the annual development (22). For each of these limitations, deviation variables are added to assess the gap between the resulting value and the corresponding target. They allow relaxation of constraints and capture the excess amounts, which are then penalized in the objective-function.

Constraint (23) limits advancement in straight lines, regarding the logical sequence of extraction. It ensures that the maximum annual production is not exceed along any given path that goes from the first ramp segment to a lens. A surplus variable is added so that it is possible to apply cost when in a scenario $s$, the percentage of advancement is over the real operating capacity.

Finally, Equation (24) is called "Flow conservation", as it aims to control and ensure the balance of material handling. The quantities of waste brought from the surface plus what is produced underground have to balance the rock handled to the surface or used for backfill. Since the model must satisfy this constraint for several scenarios, each of them having a different tonnage of waste, surplus and shortage variables are added for the same reasons mentioned above.

# 3 Case study

## 3.1 Deposit

The case study concerns a potential extension of an existing nickel mine site, which corresponds to forty-seven mineralized lenses, divided into five underground mines. Since the actual mining operations are still active and mill capacity and resources have to be shared, the maximum production regarding the new project is 0 for the first two years and increases progressively to finally attain its maximum limit during period nine. The two selected methods used are long-hole drilling (LH) and cut and fill (CF). We consider that every lens is mined using a combination of these techniques. The proportion of each method is mainly determined by considering the overall dip. A very strong dip value means that this vertical lens will have the minimum possible proportion of cut and fill technique, let us assume 10 %. This is a simplification of the model. In fact, this corresponds to the case where the lens is totally mined as long hole and the cut and fill aims for the bottom haulage drift.

A set of representations of the orebody of interest is used as primary input in the stochastic mathematical model. It has been shown (Albor and Dimitrakopoulos, 2009) that 10 to 15 simulations of a deposit generate stable solutions. Indeed, fewer simulations would not allow the mine planner to properly assess the geological risk, while more would provide no significant increase of the value.

Figure 2 represents an example of a grade tonnage curve used, which corresponds to an arbitrary lens. The dark line named "Base" shows data regarding the conventional representation of the orebody (interpolation technique) and the lighter lines "Sim", represent the simulations. One of the interesting points here is the first set of simulation curves, which correspond to the ore tonnage. They are located under the grade-tonnage curve of the conventional estimated model and go over as the cut-off grade increases. The second set

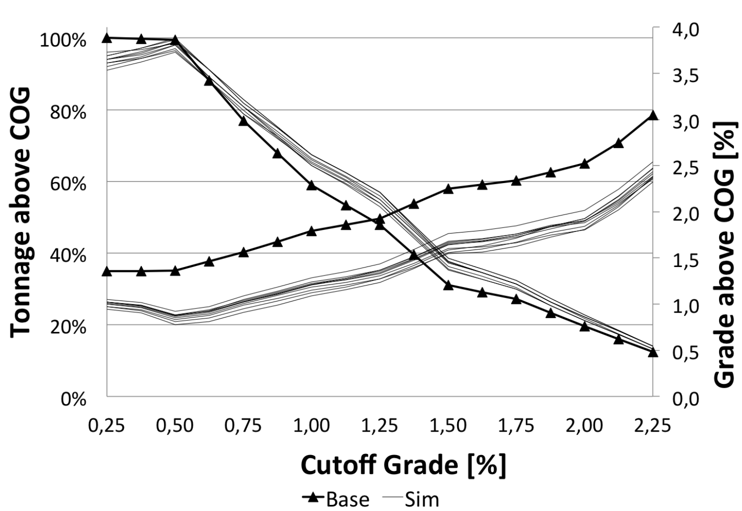


Figure 2: Typical grade-tonnage curves corresponding to a lens

represents grade above cut-off grade and shows lower values for the simulated scenarios. Consequently, nickel tonnages resulting from the scenarios are always under the corresponding values for the estimated model. This characteristic will be noted in the results that follow.

## 3.2 Production scheduling

The SIP formulation presented in Section 2 is applied in this case study, where five potential mines are evaluated, each of those being composed of several mineralized lenses. These large-scale problems cannot be solved to optimality in a reasonable time. For example, deterministic problem resolution takes over a week. Since the stochastic model is substantially larger than the deterministic model, an efficient solution strategy must be used. Collard (2013) has proposed several strategies to reduce the computational time. The most efficient one was a decomposition approach where the mines are introduced into the solution approach progressively. This strategy has been used here. A pre-treatment is completed to assess the individual value of each mine separately; their insertion into the model corresponds to the decreasing order of their NPV. The computational time went from over one week to only two hours using the deterministic approach. The stochastic problem being substantially more complexe, we expect an even more important improvement of the resolution. Deterministic optimisation concludes that four mines should be opened, and the stochastic approach made the same conclusion for only three of the five mines. Indeed, in this case study, all simulations represent the grade-tonnage curve of lenses and have lower economical potential. Since revenues are lower, but cost is comparable, it is logical that mines have lower probability to open through stochastic optimisation process.

To ensure data confidentiality, the results have been scaled to fit a range with an arbitrary maximum value of 100 million dollars. Figure 3 shows the cumulative expected NPV over the full horizon project while using the stochastic optimisation process. During the first year, no development is made since production is allowed to start only in year 3. Indeed, the discount aspect being part of the optimization process, costs are delayed as much as possible since it is not a critical task. Mines opening arise in year 2, 4 and 7, which explains curve behaviour at these periods. The first four years are mainly marked by the CAPEX development, while the production is limited due to resource sharing with existing operations. The graph shows a six-year expected payback period. From there, this profitable project should be increasing in value until year 12. Production continues for the following three years but mines are closing and thus, this lead to unavoidable costs and consequently, a slight decrease of the total NPV.

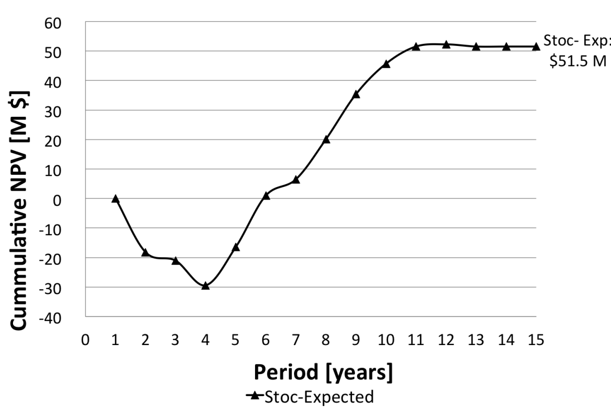


Figure 3: Cumulative NPV from the stochastic schedule

## 3.3 Risk management and geological discount rate

A risk control process allows the mine planner to manage the production indicators selected: ore production and nickel production. The mathematical model is also set up to allow the control of the length of development, percentage of advancement and extra material handling. However, in this case study, these indicators are not limiting elements. This will be discussed later in this section. Recall that in the SIP model, deviations from the deterministic schedule are essential, otherwise the lack of flexibility will lead to an unfeasible problem. By applying cost, the deviation is controlled to not surpass a certain limit selected by the planner. In this case, we accept 10% over the production targets. This percentage is not part of the mathematical model, but is used instead during the risk control process through trial and error. While testing the geological discount factor and the deviation cost combination, the mine planner verifies resulting quantities and ensures that the deviations do not exceed the limit chosen.

In the objective function of the stochastic mathematical model, two parameters help the optimiser directly controlling risk profiles: deviation costs ($C^{Ore}$, $C^N$, $C^L$, $C^A$) penalize the model for variation from target values and geological risk discount factor ($f_2^t$) assesses the importance of meeting the target in early periods. A good starting point to select deviation costs are the real prices to be paid. As an example, the deviation cost applied to extra material handling may correspond to the cost of moving material plus additional charges due to overtime. In addition, the discount rate can be used for the geological rate if we want to prioritize early period performance in proportion to the time value of money. However, it is necessary to select the best combination of these parameters in order to generate a schedule that fits the production targets. Results are compiled for observation purposes to ensure that certain limits are not exceeded and that the sequence is feasible. A final schedule is then obtained when the mine planner judges the risk profiles to be satisfactory and the project as realizable and profitable.

Referring to the constraints of the stochastic model, with the exception of the production indicators chosen, other deviation variables are added to soften the robust constraints that may compromise resolution feasibility. In fact, from constraint 22, the resources needed for horizontal development are never a limiting element, and this applies to any scenario of grade-tonnage curves and production periods. This means that regardless of the simulated grade-tonnage curve utilized, the maximum length allowed is never exceeded; in other words, the deviation variable is never positive. This means that the corresponding constraint is not active and consequently, there is no geological risk involved.

Constraint 23, which assesses task succession, also shows the same behaviour. Due to the influence of the scenario on the segment length of OPEX development, we modify the constraint to a soft type for relaxation purposes. Indeed, slack variables are added to assess any excess extraction progress regarding mine capacities. However, regarding all scenarios and periods, the limit is respected regardless of the deviation cost applied.

The figures presented in this section demonstrate the benefit of risk management through the stochastic optimization process used herein. In each of them, the dark lines represent the production target, the rhombus accounts for the production planned from its schedule and the short lines show the expected quantities regarding the set of simulations. Although graphs generated from a conventional approach are displayed here, the project value comparison to determinist schedule will be covered in the next section.

Target lines in Figure 4(a) correspond to an upper bound for the ore production capacity increase for the project extension considered in this study. Sharing resources with existing operations, ore production only starts in Period 3 and reaches its full capacity in year 9, at 500,000 tonnes. In the stochastic model, Figure 4(b), the ore tonnage variability is not significant. The schedule being fixed, lenses extracted in a conventional base case scenario are also mined regarding the ten other scenarios. The tendency of ore tonnage production follows the target line because, naturally, ore extraction is proportional to profitability. However, in Periods 4 and 5, the capacity will not be fully utilized. Of course, a part of this is explained by the set of constraints that globally impedes more ore extraction.

As opposed to the stochastic approach, the deterministic model does not contain any parameter that forces production to be as close as possible to the target. However, using the maximum of the resources available is clearly an advantage in a mining project. This is particularly true in the early periods because higher profit in the early period increases the cumulative NPV and also because it is better to fully utilize resources and then decrease its availability if not required. For example, in 4(a) the model shows that one can plan a decrease of resources used in year 12, but this can be done in Period 10 using the stochastic schedule, as seen in 4(b).

In addition, in this last figure, we obtain a smoother ore production curve that better fits the target line. Also, production stops two years earlier, as compared to the deterministic schedule, in year 12. Indeed, the

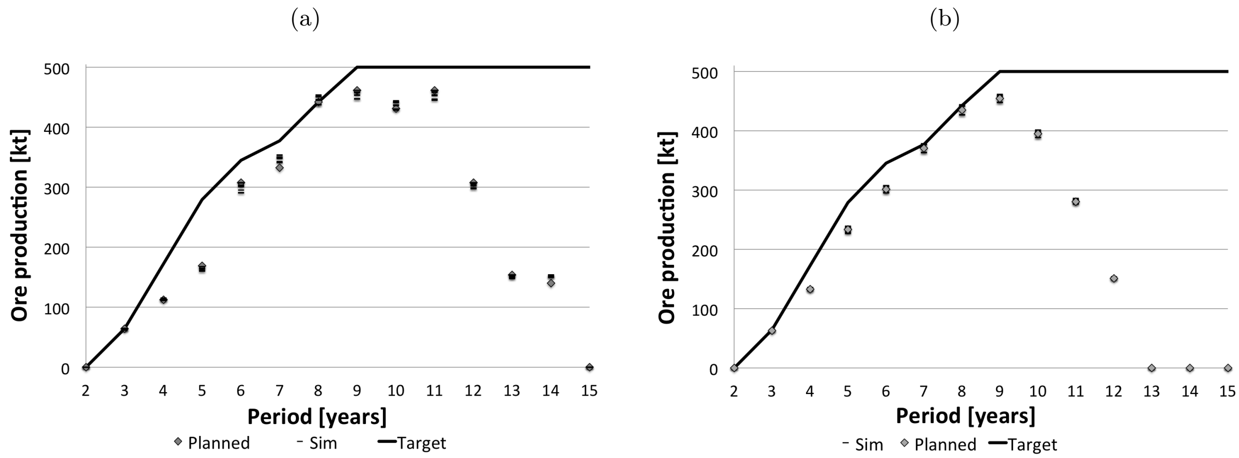


Figure 4: Production target (solid line) and risk profiles of forecasted ore production from (a) the deterministic schedule and (b) the stochastic schedule

SIP model concludes that there are only three mines worth opening instead of four, but it is explained as well by the fact that the resources available are better used. Thus, another conclusion is that a stochastic approach leads to a reduction in the life of the mine.

Moreover, the issue with not including stochastic aspects during mine scheduling is clearly manifested in Figure 5(a). Overestimation of deposit richness is apparent on the graph, as planned values are persistently over the expected nickel production. Once again, this misevaluation gives an overly optimistic view of the project, but it could also be the other extreme. Indeed, it will depend on how the grade distribution has been distorted following the estimated average based on orebody reproduction. That is how the ore-waste proportion is influenced by smoothing the grade distribution. The target line, corresponding to the nickel processing capacity, is ascending due to an increase of treatment resources available for mine extension needs until it reaches a constant at full capacity, specifically 15,000 tonnes.

A common tendency of both deterministic 5(a) and stochastic 5(b) planned curves is the two distinct parts: constant ascending link to the increase of resources at hand and next, a decreasing slope related to

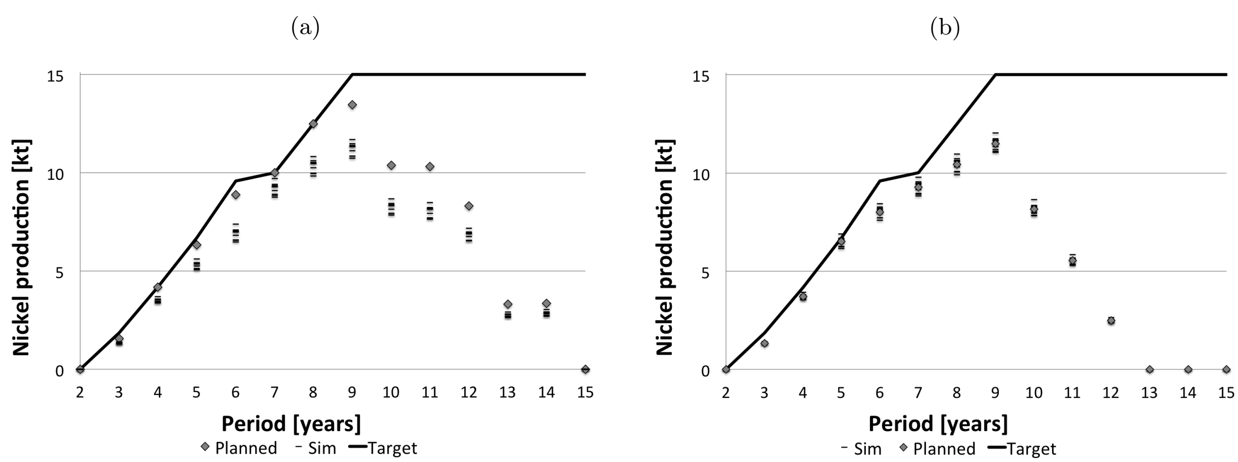


Figure 5: Production target (solid line) and risk profiles of forecasted nickel production from (a) the deterministic schedule and (b) the stochastic schedule

a decline of ore availability. This shows that both models properly assess the fact that a mining schedule is usually optimised if the mill capacity is fully used when possible.

## 3.4 Further comparisons to the deterministic life of mine (LOM) plan

A deterministic production schedule is generated by using the estimated orebody model. It is tested through each scenario by fixing the resulting binary variables: the beginning of activities, the COG and the mine opening and closure. Continuous variables stay free to ensure resolution feasibility. The results are then compiled regarding the profit and project indicators selected: ore and nickel production. To reiterate, the purpose of this study is to prove that the expected project's value is higher when considering the stochastic simulations through the optimization process, instead of a single estimated orebody representation that does not assess the geological uncertainty.

In general, conventional deterministic approaches do not take into account geological data variability and grade distribution. Thus, they can either over or under evaluate the economical potential of a mining project. In our particular project, regardless of the cut-off grade, nickel tonnage corresponding to simulations are overwhelmingly below the base case scenario corresponding value, as explained above. This can be seen in Figure 6, where the set of lines corresponding to simulations are below the line named "Det-Planned" (Figure 6), which are incomes planned with a conventional approach. In this case study, deterministic scheduling optimisation overestimates cumulative NPV. The average of the cumulative NPV from tested simulations with the deterministic schedule is named "Det-Expected" (Figure 6). By not considering the uncertainty of geological data using the model, the planner may end up with half the project's original value, which in this case is 46.9%. The last curve, named "Stoc-Expected" (Figure 6), shows profits to be gained with the employment of the stochastic schedule. It is observed that the mine extension promises a 22.6% increase of the NPV, as compared to what is expected conventionally. In addition, on this same figure, there are negative values in the first periods. This is partly due to capital expenditures, such as ramp development, but also because of the limited production of the first years, as mentioned earlier. The stochastic schedule is marked by a negative cash flow in Period 4, which account for the lesser ore tonnage that characterized the simulations.

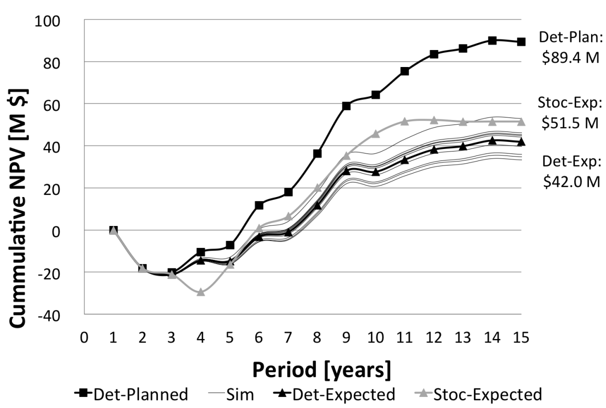


Figure 6: Cumulative NPV from the deterministic and stochastic schedules

## 3.5 Additional tests

Finally, some other tests are performed to assess the impact of the project size and complexity on the NPV, while integrating geological uncertainty. To do so, several cases are considered with one, three and five mines at the same time and two different type of cut-off grade: variable and fixed. For the first one, we used an extra discrete variable for cut-off grade variability with a range from 0.5% to 2.0%. The scheduling optimization process determines which COG is optimal for every lens. On the other hand, a fixed COG is pre-set to 1.5% and aims to represent the average value of the variable COG range. Also, the mining horizon for one and three mines is 15 years, as opposed to 20 for five mines.

Table 1 shows project value comparison regarding deterministic and stochastic optimisation process for six cases. It is demonstrated that improvement is gained from stochastic optimisation and grows with size of the model complexity. In other words, the impact of including geological uncertainty information into the model is proportional to its size and detail. The bigger the mine project, the more it becomes likely that negative financial consequences will result from not taking into account geological risk. Indeed, as the size of the problem grows, so too does the risk of not meeting the target, which has major corresponding economic consequences.

Table 1: Cumulative expected NPV comparison between deterministic and stochastic approaches for the six cases evaluated

| | **1 mine** | | **3 mines** | | **5 mines** | |
|---|---|---|---|---|---|---|
| **Type of COG** | **Fixed** | **Variable** | **Fixed** | **Variable** | **Fixed** | **Variable** |
| Deterministic approach (M$) | 26.8 | 27.4 | 40.1 | 39.3 | 44.4 | 42.0 |
| Stochastic approach (M$) | 28.6 | 30.1 | 46.6 | 48.1 | 50.1 | 51.5 |
| Project comparison (%) | 6.6 | 10.0 | 16.0 | 22.5 | 12.9 | 22.6 |

In addition, one notices that the stochastic approach leads to a greater increase of the NPV, while using a variable COG. For deterministic schedules tested with the scenarios, it can be the opposite. This is mainly due to the fact that optimal COG varies not only from one lens to another, but also depends on the scenario. The corresponding binary variables being fixed using a single representation of the orebody, the probability of compromising schedule optimisation increases since it is not the optimal for all scenarios. As an example, a particular lens can have a low COG assigned, but for a given scenario, it would have been optimal to use a high COG. The size of the lens to be mined should have been smaller and this leads to extraction of uneconomic zones. The reverse situation, where a smaller COG would have been optimal, is also problematic. It would mean mining insufficient ore material and leaving behind economical mining zones. Finally, another interesting point is the stochastic optimisation impact that significantly increases if the cut-off grade is fixed or variable. In fact, the precision and results are improved when this important parameter can be selected through the optimisation process.

## Conclusions

A stochastic underground mine scheduling problem has been presented in this paper. The adaptation of a deterministic mathematical model to assess geological uncertainty provides an interesting tool for the mine planner considering the capability to manage risk related to predefined production targets. The proposed stochastic model uses as primary inputs the data corresponding to several simulations of the deposit, unlike the conventional approaches that consider a single estimated geological model. Information related to local variability, which contributes to the assessment of uncertainty, is unfortunately not employed when using the estimated average based model. In fact, by using several scenarios, a higher expected project value, as well as an improvement in controlling production risk, is guaranteed. The performance of the process proposed relies on the availability of stochastic equally probable simulations, which all represent the orebody.

It is a parameter too often underestimated in mine scheduling. Regardless, the simplicity of the technology explains why it is often used. Indeed, conventional deterministic approaches utilize these types of deposit representation as a main input.

The case study covered in this paper demonstrates the efficiency of the stochastic approach through the NPV increase and a better use of the nickel treatment and ore production resources. In other words, the capacities were almost fully used and the risk of not achieving the target decreased. Following this application, we concluded that stochastic scheduling:

- Increases expected net present value of a project;
- Allows production risk control by managing model parameters to respect the chosen limits and to prioritize, as wished, target reaches in early periods;

- Leads to shortest mine life, by using the full resource capacity and by focussing on meeting the target as early as possible.

Also, assessing the variability of the different extraction zones can be used as a scheduling indicator. The more variable the data, the more uncertainty is present and the chances of not meeting the target are high. Having mentioned the importance of meeting the target during the early periods, this can lead the planner to prioritize zones with less variability. In the case considered, assigning a priority factor to every lens can do this. Additional work can be done by adding details to the schedule, like stope size and extraction sequence. Also, it can be interesting to integrate other uncertain parameters (market price, performance of the plant, etc.) to assess their impact.

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