Chaire de recherche Industrielle Alliance
sur les enjeux économiques
des changements démographiques

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PENSIONS, ANNUITIES, AND LONG-TERM CARE INSURANCE: ON THE IMPACT OF RISK SCREENING

CAHIER DE RECHERCHE
WORKING PAPER

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Mars / March 2016

UNIVERSITÉ LAVAL
Faculté des sciences sociales

ESG UQÀM

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Dépôt légal : Bibliothèque et Archives nationales du Québec et Bibliothèque et Archives Canada, 2016. ISSN 2368-7207

Pensions, annuities, and long-term care insurance: On the impact of risk screening*

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First draft: February 2015
This draft: March 2016

Abstract

We examine the interaction between the choice of a retirement vehicle and the purchase of long-term care insurance in a world where agents learn about their longevity and long-term care risk over time. In our setting, and absent any long-term care issues, acquiring a retirement product before learning one's risk type would be preferred by risk averse agents. When we introduce the possibility of needing long-term care, some agents will prefer to wait until they know their health status (i.e., their risk type) before purchasing a retirement product (a situation akin to having a defined contribution pension plan), whereas others will opt to purchase their retirement product before learning their health status (a situation akin to having a defined benefit pension plan). The preference of one retirement vehicle over the other depends, *inter alia*, on the level of information asymmetry on the market, on an agent's risk aversion, and on the probability of needing long-term care and its potential cost. When agents purchase their retirement vehicle after (resp. before) knowing their their health status, then agents will choose a contract that provides them with (resp. less than) full long-term care insurance coverage.

*We are indebted to Sharon Tennyson for discussions on a-many related topic. We also gratefully acknowledge the continuing financial support of CIRANO and of the Direction de la recherche at HEC Montréal. This paper was written while the first author was a visiting scholar at the College of Human Ecology at Cornell University.

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JEL Classification: D82, G22, H55.

Keywords: Asymmetric information, Information acquisition, Adverse Selection, Longevity risk, Screening, Pooling Equilibrium.

1 Introduction

1.1 Motivation

Planing for retirement is a complex endeavor; individuals are bound to ask themselves many questions. Should I invest in bonds, stocks, or target retirement date products? What will be my actual consumption needs upon retirement in 20 years (or 30 or 40 years)? Do I have bequest motives or any legacy I want to leave? What if I become sick and need help with activities of daily living? Should I acquire information about my genetic pre-disposition to have some future degenerative disease? Should I work for an employer that offers a defined benefit or a defined contribution plan? Do I care? Should I ask of my employer to offer a defined benefit plan?

Irrespective of whether or not individuals ask all these questions explicitly, retirement planning should not be done haphazardly. Retirement planning is even more important that individuals are exposed to two potentially catastrophic financial risks upon reaching old age: Longevity risk − which is represented by the possibility of outliving one's savings − and long-term care risk − which is represented by possibility of needing help with activities of daily living such as bathing, eating and getting dressed.

Long-term care is an important component of health-related expenditures especially for the elderly population that has to compose with a relatively small fixed income. To put the problem of long-term care in perspective, the total expenditure on long-term care in the United States in 2012 was 220 billion USD. This represents 8.7% of total health care expenditures according to Konetzka *et al.* (2014). Brown and Finkelstein (2009) report that between one-third and one-half of retirees in the United States will need some form of nursing home services, 10% to 20% of them needing it more than five years. Given that the annual cost of a nursing home is approximately four times the average annual gross income of retirees, we see that long-term care represents a potential catastrophic financial event for a household. In the United Kingdom, it is estimated that 10% of individuals aged 65 and over will incur life-time care costs in excess of 100 000 GBP (see Wittenberg 2014). In Canada, De Donder and Leroux (2015) report the results of a 2012 study conducted by the Canadian Life and Health Insurance Association in which it is predicted that one in 10 Canadians aged 55 will need long-term care. The proportion of Canadians needing long-term care increases to one in three when aged 65, and to one in two when aged 75.

The question we address in this paper, and which has not been examined before to our knowledge in the insurance economics and finance litterature is related to the interaction between the desire to have protection against potential catastrophic losses associated with the need to obtain long-term care, and the structure and the individual choices of retirement vehicles. More specifically, and akin to Cocco and Lopes (2011) who show the existence of a link between an individual's earnings characteristics and his choice of a pension plan structure, we seek to model the interaction between an individual's potential future need for long-term care and his choice of a retirement vehicle. Although we do not use data calibration (see Cocco and Lopes 2011 for such an approach) we will nonetheless draw inferences about the impact of long-term care risk on the ex ante choice between enrolling in a defined benefit or in a defined contribution pension plan. In other words, we develop a model of pension plan choice taking into account an individual's potential future need of long-term care services as well as information asymmetry with regards to an individual's health conditions. Individuals in our model will eventually differ with respect to their health conditions insofar as it affects their probability of surviving until old age and their unconditional probability of needing long-term care since such care is needed only once old age is reached. At the time of choosing their retirement vehicle, however, they do not know their future health status or risk of living to an advanced age. Rather, they only learn about their survival probability upon entering retirement. We consider the acquisition of a pension plan before learning one's risk type as being very similar to that of choosing to work for an employer who offers a defined benefit (DB) pension plan. If an agent chooses to wait to know his risk type before acquiring an appropriate retirement vehicle, then it is as if he was choosing to have a defined contribution (DC) pension plan.[1]

Our results contribute to the understanding of the interaction between longevity risk, long-term care risk, and the retirement financing decision.[2] In particular, we provide a theoretical basis for the self-selection of individuals into different types of retirement vehicles based only on the agents' possible risk of needing long-term care. Depending on the distribution of risk types in the economy, our model provides an explanation as to why defined benefit and defined contribution pension plans can coexist in the economy. Our results can be summarized as follows:

1. If agents acquire information about their health status before purchasing a retirement vehicle (DC plan), then all agents will purchase full long-term care insurance, if at all;

2. If agents acquire their retirement vehicle before knowing their health status (DB plan), then all agents will purchase less than full long-term care insurance, if at all;

[1] The model we develop can be modified to address the topic of whether one should get a genetic test before or after purchasing his retirement vehicle. Genetic testing is very much at the forefront of current developments in public policy. For more on the impact of genetic testing on insurance contract, see Doherty and Thistle (1996), Doherty and Posey (1998), Zick et al. (2005), Adams et al. (2013, 2014) and Peter et al. (2013).

[2] See Davidoff (2009) for the relationship between annuities, long-term care insurance and home equity. See Murtaugh et al. (2001) for the case of bundling long-term care insurance with annuities and Webb (2009) for the bundling of retirement vehicles in general.

3. Low-risk individuals signal their health status by having less insurance than the high-risk individuals;

4. Individuals are more likely to prefer a DB plan to a DC plan, *ceteris paribus*, when a) they are more risk averse, b) they face greater potential long-term care expenditures, c) they are more likely to need long-term care, d) they are more likely to be high risks (or the proportion of high risks in the economy is larger), and e) they are poorer.

The remainder of the paper is organized as follow. Before presenting the full setup of the model in Section 2, we offer a survey of the literature related to long-term care and the provision of defined benefit and defined contribution pension plans. In Section 3 we solve for the full information retirement and long-term care contracts. We introduce adverse selection in Section 4. We first examine the case where insurers are constrained to make no profit in expectation on each contract and for each risk-type, and then we examine the case where insurers are only constrained to make no profit in expectation for the whole bundled-contract market (i.e., we allow for the subsidization across risk-types and markets). We provide simulation results in Section 5. Alternate model specifications which yield the same generic results are provided in Section 6. Finally we conclude with a discussion of potential future avenues of research in Section 7.

1.2 The retirement (DB/DC) decision.

Early theoretical research by Yaari (1965) suggests that it would be best for risk averse individuals to annuitize a considerable part of their wealth upon reaching retirement. Despite this clear result, which holds under many other model specifications including bequest motives, the demand for individual annuities is so low that some financial economists are referring to it as the "annuity puzzle" (Mitchell *et al.* 1999; Davidoff *et al.* 2005). One reason to explain why the annuity market is so small is that adverse selection problems are too pronounced. A well-known result of the Rothschild and Stiglitz (1976) model of adverse selection is that a competitive market will collapse provided that the proportion of high risks exceeds a certain threshold. Even though there is ample empirical evidence for adverse selection in the annuity market, there is also evidence of advantageous selection in the sense that more risk-averse individuals take better care of themselves, live longer, and are more likely to buy annuities (see Finkelstein and Poterba 2004), and to buy long-term care insurance (Finkelstein and McGarry 2006).

Many types of retirement vehicles are available to individuals, including defined benefit and defined contribution pension plans.[3] While corporate defined benefit pension plan have been on the decline in the

[3] Even though it is rare that a given employer offers a choice between different retirement vehicles such as a defined benefit and a defined contribution pension plan, in a global compensation framework the type or pension plan is one of the criteria

United States, that is not necessarily the case everywhere[4] - and it is not obvious what are the reason for the decline of corporate defined benefit plans. Does the fault rest with employers who welcome the reduction in their future liability? Are inadequate government rules and regulations to blame for discouraging the use of defined benefit plans? Or are workers themselves responsible for trading a retirement vehicle that offers more in terms of consumption guarantees upon retirement in favor of one that provides more investment flexibility and encourages mobility (see Aaronson and Coronado 2005)? Few studies have investigated the individuals' preference over retirement vehicles. This means that the motivations behind an individual's choice of a defined benefit versus a defined contribution plan are still not very well understood - or, as Brown and Weisbenner (2014) put it,

> "relatively little is known about what types of employees choose a DC over a DB plan when given the option to do so, and even less is known about why individuals make these choices" (p.35).

Brown and Weisbenner (2014) conduct a survey amongst individuals which, at some point, had to choose between a defined benefit and a defined contribution plan in the State Universities Retirement System (SURS) of the state of Illinois. Based on the respondents' personal characteristics as well as their answered reasons for the choice of the retirement vehicle, the authors highlight the role that education and financial literacy plays in a household's behavior toward retirement vehicles. In particular, more educated individuals are more likely to choose a defined contribution pension plan over a defined benefit pension plan. In contrast, Drolet and Morissette (2014) use Canadian pension survey data to not only highlight that the higher is an individual's level of education, the more likely he or she is to be covered by any type of pension plan in general, but more importantly that individuals with higher education are more likely to be covered by a defined benefit pension plan.[5]

Despite the general decline in the use of defined benefit plans in the private sector, the great majority of public employees remain covered by a plethora of different employer-specific defined benefit pension plans. At the end of fiscal year 2013, the 99 largest public retirement systems in the United States - representing 85% of all public pension funds - were administering defined benefit pension plans for over 20 million Americans, including 12.65 million active public employees (see Public Fund Survey[6], and Mohan and Zhang 2014).

when choosing an employer. At the margin, a firm's pension plan might become the deciding factor. It is in this perspective that we say that an individual "chooses" a retirement vehicle.

[4] In the case of Canada, for instance, Drolet and Morissette (2014) find that the drop in DB coverage from 1977 until 2011 is much less pronounced than in the United States, and almost negligible for women as 33% of working women in 1977 were covered by a defined benefit plan compared to 30% in 2011.

[5] For more on the choice between DB and DC see Clark and Pitts (2002) and Kanemasu *et al.* (2014).

[6] http://www.publicfundsurvey.org/publicfundsurvey/scorecard.asp. Last accessed 24 Feb 2015.

Figure 1 illustrates the total assets under management in different OECD countries as a function of whether the occupational pension plan is classified as defined benefit or defined contribution. As we can clearly see, there is quite a lot of variation in the ways that countries have chosen to structure their occupational pension plan industry.

Percentage of total assets in different retirement vehicle for selected OECD countries at the end of 2013								
Only Defined contribution			Mostly Defined contribution			Mostly or only Defined Benefit		
	DB	DC		DB	DC		DB	DC
Chile	0	100	Denmark	7	93	United States	57	43
Czech Rep.	0	100	Italy	7	93	Turkey	61	39
Estonia	0	100	Australia	10	90	Israel	70	30
France	0	100	Mexico	13	87	Korea	73	27
Greece	0	100	New Zealand	20	80	Luxembourg	80	20
Hungary	0	100	Iceland	25	75	Portugal	85	15
Poland	0	100	Spain	28	72	Canada	97	3
Slovak Rep.	0	100				Finland	100	0
Slovenia	0	100				Germany	100	0
						Switzerland	100	0

Figure 1: Percentage of assets in defined benefit and defined contribution pension plans at year end 2013. Occupational pension plans only. Source: OECD Pension Markets in Focus 2014

An individual's choice of a defined contribution pension plan over a defined benefit pension plan has attracted a fair amount of research. For instance, Clark and Pitts (2002) study an individual worker's decision over the type of retirement plan at one given employer. That is also the approached used in Lachance *et al.* (2003). Using an option-like pricing approach they examine the situation of pension plans that allow their participants enrolled in a defined contribution plan to buy their way back into a defined benefit pension plan (see also Milevsky and Promislow 2004). Although they do not examine the characteristics of the individuals who exercised that option to buy back into the defined benefit plan, surely there are personal characteristics that determine one's decision to switch in addition to the pure monetary aspect. More recently, Kanemasu *et al* (2014) examine the response of California public pension plan participants to a potential switch from a defined benefit to a defined contribution plan. Brown and Weisbenner (2014) conclude that, in higher education, over 50% of U.S. states offer some employees the choice between a defined contribution and a defined benefit plan.

Our interest in studying the choice of a particular retirement vehicle over another is based on the fact that wealth at retirement is an important component of the overall level of poverty in a nation. To see why, it is interesting to compare the importance of pension plan income relative to a household's total retirement income as in Purcell (2008). It appears that retirement income that originates from defined benefit pension plans represents between 15% and 20% of the wealthiest retired households' income, but merely 3% of the poorest 20% of households' total retirement income. Defined benefit plans are a very important contributor to the social welfare of the elderly population according to Porell and Almeida (2009) who estimate that the number of poor and near poor households would have increased by 43% were it not for the households' access a defined benefit pension plan. The same analysis for social security and for defined contribution plans shows that the number of poor and of near poor households would have increased by approximately 15% and 1% had those households been denied access to social security and a defined contribution plan respectively. Accordingly defined benefit plans play a very important role in keeping retired households out of poverty.

Cocco and Lopes (2011) highlight the important role that defined benefit pension plans play in the wealth distribution of the nation's retired households. Does that mean that choosing a defined contribution plan over a defined benefit plan is a mistake? Not necessarily for a given individual, but for an entire nation, that may be problematic (see Brown 2014). Gerrans and Clark (2013) report that the welfare debate on the choice of a defined benefit rather than a defined contribution plan is associated to the idea that individuals are irrational so that they either overly discount the future or they misunderstand the differences between the two types of retirement plans (Clark *et al.* 2012). In a study of Australian pension plan participants, Gerrans and Clark (2013) find that individuals that switched from defined benefit to defined contribution were typically younger and wealthier.[7] Brown and Weisbenner (2014) find similar results in the state of Illinois, and add education as a factor increasing the likelihood of picking a defined contribution plan.

1.3 The markets for long-term care insurance

In contrast to the multi-billion bollar industry that surrounds the provision of retirement wealth, the market for long-term care insurance is very small despite the potential catastrophic losses associated with long-term

[7] Recent reforms aimed at introducing fair-value in pension accounting will surely increase the volatility of the financial statements of pension plan sponsors, especially for those that have an important duration mismatch between their assets and liabilities. As the volatility of a firm's implied debt-to-equity ratio increases because of the (potential) underfunding of pension liabilities, it seems natural to expect firms, that face some level of solvency risk, to wish to shift the burden of the retirement package they offer their workers from a defined benefit (which is a potential liability for the firm) to a defined contribution plan (which essentially does not increase the firm's leverage or its risk of financial distress and insolvency). But of course closing DB plans in favour of DC plans does not eliminate the funding risk; it only shifts it from the employer to the employee.

care costs and the considerable impact that access to long-term care insurance (or the lack thereof) can have on an individual's ability to smooth his lifetime income. Survey data suggests that those who outlive their age-gender-cohort also need long-term care for a longer-than-average period of time as compared to their respective cohort. In other words, longer-lived individuals are more likely to use long-term care (Yang *et al.* 2003; Crimmins and Beltrán-Sánchez 2011; Hurd *et al.* 2014). In addition to a more likely need for long term care when an individual gets older, the total cost of long-term care also increases with age as highlighted in Spillman and Lubitz (2000, 2002), Manton *et al.* (2006) and Hurd *et al.* (2014). According to the American Association of Long-Term Care Insurers,[8]

> "Most long term care insurance claims begin after a person reaches age 70... 22.7% of new claims begin between ages 70-79 ... 67.4% of all new long-term care insurance claims begin after age 80".

The same message is provided by the National Association of Insurance Commissioners who claim[9] that "the longer you live, the more likely it is that you will need long–term care". Since individuals do not seem to buy long-term care insurance, the empirical question that needs anwering is therefore associated with their apparent disregard for long-term care risk. According to Zhou-Richter *et al.* (2010), it may be due to a misperception of the risk. Given the potential catastrophic nature of this risk, however, it is hard to imagine that individuals would not want to acquire the relevant information about that risk or do their proper research into the nature of long-term care risk. Another explanation is that individuals are only slightly risk averse, if not even risk neutral, with respect to long-term care risk. This hypothesis is in conflict, however, with the results presented in Sydnor (2010) whereby individuals appear to be extremely risk averse over small gambles, which makes it unlikely that they would be risk neutral over one as large at that of long-term care risk. Boyer (2015) proposes instead that an individual does not see the cost of long-term care (or of nursing home) as being borne by himself, but rather by his younger spouse or heir (this is equivalent to including a bequest motive in a long-term care framework as in Lockwood 2010, and Brunner 2012). Put differently, the cost of long-term care only reduces the size of the bequest that an individual leaves behind so that it should be the heir who should be buying long-term care insurance to protect the bequest that is left behind by his elderly and rapidly aging parent.

The projected need of long-term care insurance as one grows older holds despite the apparent counter-evidence of compressed morbidity whereby the number of years during which an individual is sick is decreas-

[8] http://www.aaltci.org/long-term-care-insurance/learning-center/life-span-calculator.php, 27 January 2015.
[9] http://www.naic.org/documents/consumer_alert_ltc.htm, 27 January 2015.

ing. The debate surrounding the compression of morbidity is best presented in the surveys of Crimmins and Beltrán-Sánchez (2010) and Fries *et al.* (2011). Even if morbidity is compressed, the out-of-pocket cost associated with the last two years of life increases exponentially as the individual gets older. One possible reason is that individuals prefer to use the professional services of nursing homes instead of the natural services of family and informal care-givers (Robine *et al.* 2007, Tennyson and Yang 2014). This shift in the demand for geriatric-specific health services will necessarily have a very important impact on the future demand for long-term care insurance. The projected rise in the demand for formal long-term care is associated with the rapid growth in the elderlier population, changes in their relative independence from their spouse and children (if any), and with a substitution effect across public and private services of geriatric-specific medical care (see the recent De Donder and Leroux 2015 for more on the political economy of long-term care, and Brown and Finkelstein 2011 and Cremer *et al.* 2009 for more extensive surveys).

A longer period spent in need of long-term care is equivalent to a higher present value of future long-term care insurance benefits, hence a higher loss severity in the long-term care segment. Thus, instead of considering different probabilities to eventually need long-term care, it seems straightforward to model different loss severities instead. Doherty and Jung (1993) propose a model of adverse selection in loss probabilities when loss severity is positively correlated with the loss probability. We built on both theirs and on Webb's (2009) model to examine the equilibrium properties of stand-alone annuities and long-term care insurance market equilibria where agents have private information with respect to their survival probability (i.e., their risk type in the annuity market) and their severity of loss on the long-term care market.[10] Based on this model we then examine the situation when the two markets are integrated at the individual risk level as well as at the entire risk portfolio level.

2 The Model Setup

2.1 Timing of the model

Our model starts from the premise that individuals enrolled early in a defined benefit pension plan do not yet have any information about their longevity risk type, so that they have nothing to signal to the providers of retirement income. In other words, an individual enrolled in a defined benefit pension plan has an income path upon retirement that is independent of his future health status since it is contracted at the time he starts his career. This means that at the time he enters a defined benefit pension plan, an individual is so far away

[10] In contrast to Doherty and Jung (1993) our model does not require a positive correlation between the longevity and the extent to which long-term care is needed. Assuming that a healthy individual will live longer and will also live longer in long-term care simply seems to better reflect empirical evidence.

from retirement that he surely has no information whatsoever about his future health condition. In contrast, when the time comes to annuitize their accumulated wealth individuals enrolled in defined contribution pension plans have acquired information on their health. As a result, they have more and better information about their health and longevity risk. Put differently, an individual who accumulates vast sums of money will eventually want to transfer it into a income-generating vehicle upon retirement (say an annuity)[11]. Surely by the point when individuals head into retirement they will have lived long enough to accumulate some private information regarding their health status.

In such a world, and absent any other retirement market characteristics (see Bodie *et al.* 1988), a defined benefit pension plan would always dominate a defined contribution pension plan. The reason is that the distribution of wealth in a defined contribution plan can be seen as a mean-preserving spread of the distribution of wealth in a defined benefit pension plan. Opting for a defined contribution pension plan becomes the equivalent of acquiring a lottery with a mean payoff of zero. To see why, note that agents enrolled in a defined benefit plan often become part of a pool of individuals so early in their career that they have not acquired any private information about their health status upon reaching retirement. And the little information they may have is pooled with that of all the other members of the pension plan. Moreover, any information they learn about true health status over time does not really affect the benefits they receive when they retire. With a DC plan, however, agents accumulate wealth over time, and acquire information about their health status, such that at the time of retirement, when they are considering purchasing an annuity contract, they have valuable private information, which affects their choice of an annuity. Given this delayed albeit forecasted adverse selection problem, it can be easily shown that initially uninformed agents are better off purchasing the contract before they acquire any information. In the case of the defined benefit versus defined contribution plan, our assumption means that abstracting from pension portability, autonomy, vesting, investment risk, interest rate risk, inflation risk, incentives, and every other factor (see Poterba *et al.* 2007 for more on the different risk components associated with the different retirement vehicles), a defined benefit plan is always preferred to a defined contribution plan for risk averse agents.

As it has become obvious, we are starting with a model of pension plan dynamics that is tilted toward the dominance of defined benefit over defined contribution pension plans. Our agents will be fully rational and will only care about their expected utility over final wealth. There is no investment risk or wage risk; inflation and interest rates are both zero.

[11] Rothschild (2015) provides an adverse selection model of annuity purchase in a market where all retirement savings must be annuitized upon retirement. See also Villeneuve (2003) and Finkelstein *et al.* (2009).

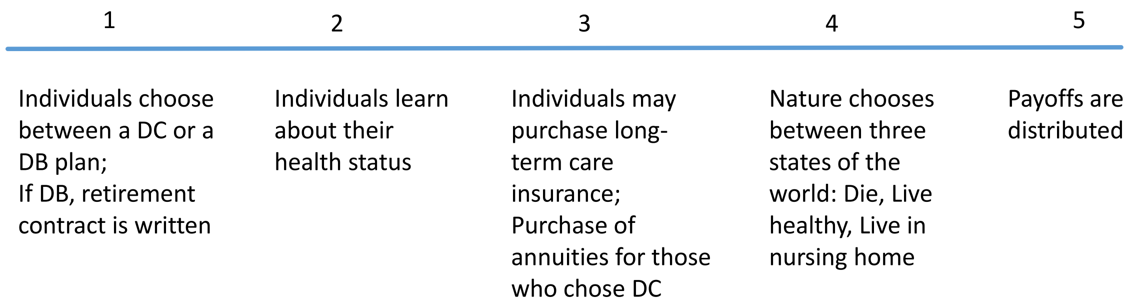

Figure 2: Time line of decision taken by the individuals and the actions of nature in distributing information about types and in choosing the end state of the world.

In addition to facing a longevity risk that can be managed through either a defined benefit or a defined contribution pension plan, we will also assume that individuals face a health risk that requires the potential disbursement of a large amount of money during the retirement period. In other words, with a certain probability individuals will be in need of costly long-term care. They will have the possibility of acquiring long-term care insurance upon reaching retirement, when they privately know their longevity-risk type.

Figure 2 presents the time line more succinctly. Adverse selection becomes relevant starting in step 2, that is after individuals have privately learned their health status. We therefore choose to simplify the economic model to its simplest expression by concentrating exclusively on the combination of longevity and long-term care risk. This means that we do not model the exact investment choice of bonds versus stock versus housing (see Davidoff 2009 and Cocco 2005), nor do we include bequest or legacy motives (see Pauly 1990). We concentrate only on the choice between retirement saving vehicles (DB versus DC) and on the purchase of long-term care insurance in a world where individuals acquire information about their health status over time.

Our results will show that one advantage of waiting to have information about one's health status before acquiring a retirement-income contract (i.e., the DC way) is that full long-term care insurance will be obtainable for all the agents in the economy. If on the other hand individuals acquire their retirement-income contract before their health status is known, then full long-term care insurance will not be chosen by any individual upon retirement.

Our model differs from the multidimensional screening model of Crocker and Snow (2011) in the sense that our agents' private information is does not span the entire dimensionality of the type-space problem. In other words, Crocker and Snow (2011) use an n-dimensional vector screening devices for n-possible types of losses whereas we have only one screening device for a two-dimensional risk (see also Fluet and Pannequin 1997).

2.2 Individual decision and variable definition

The initial setup of our model finds its inspiration in Webb (2009) whereby individuals care about obtaining a decent income upon retirement and protection against the possibility of needing long-term care. Without any retirement vehicle or long-term care insurance, an agent's vonNeumann-Morgenstern expected utility over final wealth in autarchy is given by

$$\Omega_i = U(W_0) + p_i \left[(1-\pi) U(W) + \pi \int_0^\Lambda U(W-\lambda) g(\lambda) d\lambda \right]$$

where W_0 and W are the agent's wealth endowments in period 0 and 1 respectively, with $W_0 >> W \geq 0$. λ is the cost of long-term care; it is distributed according to some function $g(\lambda)$ over the range $\lambda \in [0, \Lambda]$. π is the probability of needing long-term care and p_i, which is agent specific, is the probability of living past retirement. If the agent dies, we let his utility be zero.[12] Of course we suppose that agents are risk averse and prefer to consume more than less so that $U'(C_t) > 0$ and $U''(C_t) < 0$. We assume no discounting of future periods.

We will assume that agents can be of two types, $i \in \{L, H\}$, with a proportion x of agents of type $i = H$. The unconditional probability that any individual will live past retirement is therefore given by $xp_H + (1-x) p_L$, whereas the unconditional probability that any individual will need long term care is $[xp_H + (1-x) p_L] \pi$. Note that we assume that individuals do not have any private information regarding their conditional probability of needing long-term care (π).

Individuals who choose to acquire a retirement vehicle BEFORE knowing their type will receive a payment of B in period 1. Individuals who choose to acquire a retirement vehicle AFTER knowing their type will receive a type-specific payment of A_i in period 1. Irrespective of their retirement contract individuals also get to acquire some level of long-term care insurance protection d_i, which is defined as the proportion of the long-term care cost that is reimbursed by the insurer (or equivalently, they choose an amount $D_i = d_i \lambda$ of protection) AFTER learning about their risk type. Consequently d_i is conditional on each agent's type i.

[12] Alternatively, we could also give the agents in this model a bequest motive from which they extract utility $V(\Gamma)$ for a given amount of wealth bequested Γ. Their expected utility function would then be given by

$$\Omega_i = U(W_0 - \Gamma) + (1-p_i) V(\Gamma) + p_i \left[(1-\pi) U(W) + \pi \int_0^\Lambda U(W-\lambda) g(\lambda) d\lambda \right]$$

The main results of our model are not affected by the inclusion of bequest motives since it will appear only as a reduction in the total wealth. See Pauly (1990) for more on bequest motives.

We let β represent the premium of the pre-information (DB) retirement contract, whereas α_i represents the premium of the post-information (DC) retirement contract. δ_i is then the long-term care insurance contact premium. The expected utility of agents are then equal to

$$\Omega_i^{before} = U\left(W_0 - \beta - \delta_i\right) + p_i \left[(1-\pi) U\left(W + B\right) + \pi \int_0^{\Lambda} U\left(W + B - (1-d_i)\lambda\right) g(\lambda)\, d\lambda \right]$$

if the agent chooses the retirement option before knowing his type (DB) and

$$\Omega_i^{after} = U\left(W_0 - \alpha_i - \delta_i\right) + p_i \left[(1-\pi) U\left(W + A_i\right) + \pi \int_0^{\Lambda} U\left(W + A_i - (1-d_i)\lambda\right) g(\lambda)\, d\lambda \right]$$

if the agent chooses his retirement package after knowing his type (DC).

We assume that both the retirement-provision and the long-term care insurance markets are perfectly competitive in the sense that neither the suppliers of retirement income nor the long-term care insurers make any profit in expectation. Even though markets are assumed to be competitive, insurers face transaction and other types of costs. We shall account for theses costs through three parameters that behave as fixed loading: μ denotes the fixed loading on the long-term care market, whereas m_A (resp. m_B) denotes the fixed loading on the retirement contracts after (resp. before) the agent has learned about his risk type. Loadings represent marketing, management, underwriting, and claims-handling expenses which need to be recouped somehow.

Given the competitive nature of the markets, the premium that individuals must pay for the pre-information (DB) retirement contract is given by $\beta = \left[x p_H + (1-x) p_L\right] B + m_B$. That is, with average probability $\left[x p_H + (1-x) p_L\right]$ the agent lives to the next period in which case he is entitled to receive a payment equal to B. With average probability $1 - \left[x p_H + (1-x) p_L\right]$ he dies. With respect to the premium paid for the retirement contract purchased after the information is learned (DC), it must be set to $\alpha_i = p_i A_i + m_A$, with $i \in \{L, H\}$. The premium is of course agent-type specific since agents who face different probability of living to the next period may not choose the same payment, and surely will not pay the same premium. Finally, in the case of the long-term care insurance contract, the premium is given by $\delta_i = p_i \pi \int_0^{\Lambda} d_i \lambda g(\lambda)\, d\lambda + \mu$. Again the premium is agent-type specific so that with probability $p_i \pi$ the agent needs long-term care. If this happens, the insurer pays a proportion d_i of the agent's loss λ, which is distributed according to some function $g(\lambda)$. In the case of the long-term care contract, the fixed loading is given by μ.

3 Full Information Contracts

We first establish a benchmark by examining the situation where all information is common knowledge. In such a situation, there is no difference between obtaining a retirement contract before or after learning private information since there is no private information. Without loss of generality, let us use the notation of the post-information (DC) retirement vehicle. With full information, the problem that agent $i \in \{L, H\}$ faces is given by

$$\max_{\substack{A_i,\alpha_i,\\ d_i,\delta_i}} \Omega_i = U\left(W_0 - \alpha_i - \delta_i\right) + p_i \left[(1-\pi) U\left(W + A_i\right) + \pi \int_0^\Lambda U\left(W + A_i - (1-d_i)\lambda\right) g\left(\lambda\right) d\lambda \right]$$

subject to the following two zero-profit constraints for the insurers

$$\alpha_i = p_i A_i + m_A; \qquad \delta_i = p_i \pi \left(\int_0^\Lambda d_i \lambda g\left(\lambda\right) d\lambda \right) + \mu,$$

which we substitute in the maximization problem to write the entire problem as

$$\begin{aligned} \max_{A_i,d_i} \Omega_i &= U\left(W_0 - (p_i A_i + m_A) - \left(p_i \pi \left(\int_0^\Lambda d_i \lambda g\left(\lambda\right) d\lambda \right) + \mu \right) \right) \\ &\quad + p_i \left[(1-\pi) U\left(W + A_i\right) + \pi \int_0^\Lambda U\left(W + A_i - (1-d_i)\lambda\right) g\left(\lambda\right) d\lambda \right]. \end{aligned}$$

The first order conditions are

$$\begin{aligned} \frac{\partial}{\partial A_i} \quad &: \quad 0 = -U'\left(W_0 - (p_i A_i + m_A) - \left(p_i \pi \left(\int_0^\Lambda d_i \lambda g\left(\lambda\right) d\lambda \right) + \mu \right) \right) p_i \\ &\quad + p_i \left[(1-\pi)' U\left(W + A_i\right) + \pi \int_0^\Lambda U'\left(W + A_i - (1-d_i)\lambda\right) g\left(\lambda\right) d\lambda \right] \end{aligned} \tag{1}$$

and

$$\begin{aligned} \frac{\partial}{\partial d_i} \quad &: \quad 0 = -U'\left(W_0 - (p_i A_i + m_A) - \left(p_i \pi \left(\int_0^\Lambda d_i \lambda g\left(\lambda\right) d\lambda \right) + \mu \right) \right) p_i \pi \left(\int_0^\Lambda \lambda g\left(\lambda\right) d\lambda \right) \\ &\quad + p_i \pi \int_0^\Lambda \lambda U'\left(W + A_i - (1-d_i)\lambda\right) g\left(\lambda\right) d\lambda. \end{aligned} \tag{2}$$

From these conditions, it can be easily shown that $d_i = 1$ is a solution so that full insurance on the LTC market is best. This takes us to the first proposition of the paper:

Proposition 1 *Under full information and a fixed loading factor, the first best allocation is obtained such that*

$$d_i = 1 \text{ and } A_i = \frac{W_1 - p_i W - m_A - \mu - p_i \pi \left(\int_0^\Lambda \lambda g\left(\lambda\right) d\lambda \right)}{1 + p_i}.$$

Proof. From Equations 1 and 2, we isolate $U'\left(W_0-(p_iA_i+m_A)-\left(p_i\pi\left(\int_0^\Lambda d_i\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)$ *to find that*

$$\frac{\int_0^\Lambda \lambda U'(W+A_i-(1-d_i)\lambda)\,g(\lambda)\,d\lambda}{\int_0^\Lambda \lambda g(\lambda)\,d\lambda}=(1-\pi)\,U'(W+A_i)+\pi\int_0^\Lambda U'(W+A_i-(1-d_i)\lambda)\,g(\lambda)\,d\lambda.$$

This equality holds only when $d_i=1$.

From Equation 1, we obtain

$$U'\left(W_0-(p_iA_i+m_A)-\left(p_i\pi\left(\int_0^\Lambda d_i\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)=U'(W+A_i),$$

which means that $A_i=\frac{W_0-W-m_A-\mu-p_i\pi\left(\int_0^\Lambda \lambda g(\lambda)d\lambda\right)}{1+p_i}$.•

The first best allocation gives the agent perfect income smoothing so that, whatever is the state of the world, his consumption is equal to $C=W+A_i=\frac{W_0+p_iW-m_A-\mu-p_i\pi\left(\int_0^\Lambda \lambda g(\lambda)d\lambda\right)}{1+p_i}$. Given that there is no discounting of future periods, we see that W_0+p_iW is the total expected wealth of the agent (an agent who is dead, with probability $1-p_i$, does not receive any wealth). The agent who purchases annuities and long-term care insurance has to incur the fixed loading factors for both contracts, $\mu+m_A$, and must pay the premium of the long-term care insurance contract $p_i\pi\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right)$ given his choice of $d_i=1$. The agent's total lifetime expected consumption is then $(1+p_i)\,C$.

4 Information asymmetry

We will assume from now on that there are two types of individuals that differ only with respect to their probability of reaching retirement age. We assume that high risk individuals (denoted H) are more likely to reach period 1 than low risk individuals (denoted L) so that $p_H>p_L$. Conditional upon reaching period 1, all agents face the same risk of needing long-term care. In other words the probability of needing long-term care and the potential cost of long-term care to the agents is the same for the two types of agents.[13]

In Subsection 4.1, we consider the case of DC plans whereby individuals have accumulated some wealth W_0 needed to finance consumption at retirement. During the wealth-accumulation phase of their lives, individuals also acquire private information about their health status, which translates into a private information about their longevity risk. Providers of retirement schemes, such as annuity providers in a Yaari-type world, know that individuals have acquired such information but ignore the exact nature of such information. They

[13] We make this assumption regarding the distribution of long-term care losses even though we are aware theat there is empirical evidence to suggest that agents that are more at risk to live a longer life are also more likely to face high cost of long-term care (see Spillman and Lubitz 2000, Yang et al. 2003, and Hurd et al. 2014).

only know that a proportion x of agents learned that they are high risk (i.e., have a probability p_H of reaching retirement age), the remaining proportion $1-x$ of agents learning that they have a probability p_L of reaching retirement age. In addition to purchasing the annuity contract that maximizes their expected utility, individuals must also decide whether they want to purchase long-term care insurance.

In Subsection 4.2, we consider the case associated with defined benefit retirement plans. In that situation no information about risk types is known at the time the retirement contract is purchased, except for the average probability of living past retirement, $xp_H + (1-x)\,p_L$. Although individuals do not know their type when choosing their retirement vehicle, they do know it when the time comes to purchasing long-term care insurance. As in the defined contribution situation, long-term care insurers know that individuals know their type, and they know the distribution of types in the economy.

4.1 Adverse Selection with Post-information retirement choices

In this section, we examine the situation where agents purchase retirement vehicles and long-term care insurance after they have learned about their risk type. We will start with the situation where each contract (retirement or long-term care insurance) must make zero-profit in expectation. We will then allow for cross-subsidization across agent types and products.

4.1.1 Zero-profit for each type, for each product

Given our assumption of zero-profit in expectation for each product and each type $i \in \{L, H\}$, and a fixed loading of m_A in the annuity market and of μ in the long-term care market, we have premium functions given by $\alpha_i = p_i A_i + m_A$ and $\delta_i = p_i \pi \left(\int_0^\Lambda d_i \lambda g(\lambda)\, d\lambda \right) + \mu$. The maximization problem then writes as

$$\max_{\substack{A_L, d_L, \alpha_L, \delta_L \\ A_H, d_H, \alpha_H, \delta_H}} \Omega_L^{after} = U(W_0 - \alpha_L - \delta_L) + p_L \left[(1-\pi) U(W + A_L) + \pi \int_0^\Lambda U(W + A_L - (1-d_L)\lambda)\, g(\lambda)\, d\lambda \right]$$

subject to four zero-profit conditions (agent-type specific and insurance-product specific)

$$\alpha_L = p_L A_L + m_A$$

$$\alpha_H = p_H A_H + m_A$$

$$\delta_L = p_L \pi \left(\int_0^\Lambda d_L \lambda g(\lambda)\, d\lambda \right) + \mu$$

$$\delta_H = p_H \pi \left(\int_0^\Lambda d_H \lambda g(\lambda)\, d\lambda \right) + \mu.$$

We further add to the problem an incentive compatibility constraint (see below) that guarantees that high-risk agents are better off purchasing the contracts designed for them than purchasing the contracts designed for low-risk agents

$$\begin{aligned} 0 &< U\left(W_0-\alpha_H-\delta_H\right)+p_H\left[\left(1-\pi\right)U\left(W+A_H\right)+\pi\int_0^\Lambda U\left(W+A_H-\left(1-d_H\right)\lambda\right)g\left(\lambda\right)d\lambda\right] \\ &\quad -U\left(W_0-\alpha_L-\delta_L\right)-p_H\left[\left(1-\pi\right)U\left(W+A_L\right)+\pi\int_0^\Lambda U\left(W+A_L-\left(1-d_L\right)\lambda\right)g\left(\lambda\right)d\lambda\right]. \end{aligned}$$

Solving this problem yields the following proposition.

Proposition 2 *In a market where providers of services are restricted to offer menus of contracts, each of which makes no profit in expectation, when there are two types of agents that differ with respect to their probability of living long, and when agents privately know that information when making their contract choice, then all agents will be fully indemnified for losses on the long-term care market. On the retirement market, the high risk agent receives his first best allocation (perfect consumption smoothing) whereas the low risk agent must signal his type by accepting an allocation where he still faces some retirement consumption risk.*

Proof. See Appendix 9.1 •

This proposition tells us that full insurance on the long-term care market is possible even if agents have private information about their probability of needing long-term care. Agents are able to obtain full long-term care insurance but not perfect consumption smoothing because, once agents have signalled their type on the longevity market, there is no information asymmetry left to prevent first best long-term care insurance purchase. Consequently, conditional on learning the agents' longevity risk type (p_L or p_H), π and $g(\lambda)$ are the same for all the agents in the economy. In other words, the only difference across agents comes from the probability of reaching retirement age and not from the conditional probability of needing long-term care.

4.1.2 Zero-profit in the industry

There are two ways to look at the problem when cross-subsidization of types is allowed. Either providers on each market make no profit in expectation (so that within a market, cross-subsidization is permitted) or they make no profit in expectation on the joint markets (so that cross-subsidization is permitted across agent types and across insurance products). Although the insurance providers' ability to cross-subsidize across agent types has been well documented, it has been more natural to examine insurance products separately. Given our setup it should be clear that in terms of Pareto efficiency the industry structure that is preferable is the one where subsidization across types and products is permitted. We will therefore only present and discuss the nature of the contract and the equilibrium in the economy in which providers are allowed to cross-subsidize types and products provided they make zero profit on their entire book of business in expectation. The other case of cross-subsidization across types but not across products is available from the authors Results are similar in their nature.

Letting ξ_i represent the total premium that agent of type $i \in \{L, H\}$ must pay to purchase the retirement vehicle and long-term care insurance, the problem can be written as

$$\max_{\substack{A_H, A_L, d_L, d_H, \\ \xi_H, \xi_L}} \Omega_L^{after} = U(W_0 - \xi_L) + p_L \left[(1-\pi) U(W + A_L) + \pi \int_0^\Lambda U(W + A_L - (1-d_L)\lambda) g(\lambda) d\lambda \right]$$

subject to the unique zero-profit condition

$$\begin{aligned} x\xi_H + (1-x)\xi_L &= xp_H \left[A_H + \pi d_H \int_0^\Lambda \lambda g(\lambda) d\lambda \right] + \mu + m_A \\ &\quad + (1-x) p_L \left[A_H + \pi d_H \int_0^\Lambda \lambda g(\lambda) d\lambda \right] \end{aligned}$$

and the incentive-compatibility constraint

$$\begin{aligned} 0 &< U(W_0 - \xi_H) + p_H \left[(1-\pi) U(W + A_H) + \pi \int_0^\Lambda U(W + A_H - (1-d_H)\lambda) g(\lambda) d\lambda \right] \\ &\quad - U(W_0 - \xi_L) - p_H \left[(1-\pi) U(W + A_L) + \pi \int_0^\Lambda U(W + A_L - (1-d_L)\lambda) g(\lambda) d\lambda \right]. \end{aligned}$$

The Lagrangian is

$$\begin{aligned} \max_{\substack{A_H, A_L, \alpha_L, \alpha_H, \\ \mathcal{B}_H, \mathcal{B}_L}} \Omega_L^{after} &= U(W_0 - \xi_L) + p_L \left[(1-\pi) U(W + A_L) + \pi \int_0^\Lambda U(W + A_L - (1-d_L)\lambda) g(\lambda) d\lambda \right] \\ &\quad + \eta \left[\begin{array}{c} x\xi_H + (1-x)\xi_L - xp_H \left[A_H + \pi d_H \int_0^\Lambda \lambda g(\lambda) d\lambda \right] - \mu - m_A \\ - (1-x) p_L \left[A_L + \pi d_L \int_0^\Lambda \lambda g(\lambda) d\lambda \right] \end{array} \right] \\ &\quad + \gamma \left[\begin{array}{c} U(W_0 - \xi_H) + p_H \left[\begin{array}{c} (1-\pi) U(W + A_H) \\ + \pi \int_0^\Lambda U(W + A_H - (1-d_H)\lambda) g(\lambda) d\lambda \end{array} \right] \\ - U(W_0 - \xi_L) - p_H \left[\begin{array}{c} (1-\pi) U(W + A_L) \\ + \pi \int_0^\Lambda U(W + A_L - (1-d_L)\lambda) g(\lambda) d\lambda \end{array} \right] \end{array} \right]. \end{aligned}$$

The following proposition[14] highlights the characteristics of the optimal contract using this modelling approach.

Proposition 3 *When providers of retirement and long-term care insurance services are allowed to cross-subsidize across agent types and insurance products, then the optimal allocation is such that agents are fully*

[14] It must of course be that the allocation under this contract is preferred to, or at least as good as, the one in the standard Rothschild-Stiglitz case if only because the Rothschild-Stiglitz case is a more constrained problem with zero expected profit on each contract type.

insured on the long-term care market (i.e., $d_L = d_H = 1$). Furthermore, the high risk individual is receiving a retirement package that perfectly smooths his consumption in all states of the world whereas the low risk must signal his type by accepting a contact whereby his consumption is not smoothed.

Proof: See Appendix 9.2. •

Similarly to the first Rothschild-Stiglitz equilibrium contract we presented (that is, zero profit in expectation for each product line and for each agent type), this proposition tell us that irrespective of the agents' risk type full insurance against the potential catastrophic expenses associated with long-term care should be purchased. As before, the reason is that there is no conditional asymmetric information on the long-term care market since agents only differ with respect to the probability of reaching retirement age ($p \in \{p_L, p_H\}$) and not with respect to their probability of needing long-term care (π).

Given that there is no conditional asymmetric information on the long-term care market so that all agents are fully insured on that market, it must be that the signalling and all the cross-subsidization across risk types occur on the retirement product market. We may wonder what happens if we take this cross-subsidization to the extreme by forcing high-risk and low-risk type to acquire a contract that completely pools their risk of reaching retirement age altogether. As we will see in the next section, the complete cross-subsidization through a pooling contract will not yield the same full insurance results on the long-term care market.

4.2 Adverse Selection with Pre-information retirement choices

In this section, we examine the situation where agents purchase retirement vehicles before knowing their type, but learn about their type before deciding to purchase long-term care insurance. Again, we will start with the situation where each contract (retirement or long-term care insurance) must make zero-profit in expectation. We will then allow for cross-subsidization across agent types and products.

4.2.1 Zero-profit for each type on the long-term care insurance market

Given our assumption of zero-profit in expectation for each risk type in the long-term care insurance market, it must be that for $i \in \{L, H\}$, $\delta_i = p_i \pi \left(\int_0^\Lambda d_i \lambda g(\lambda) \, d\lambda \right) + \mu$. On the retirement vehicle market, the zero profit condition for all is $\beta = [x p_H + (1-x) p_L] B + m_B$. At the time they are choosing the structure of their retirement vehicle product, agents are able to forecast what will be their long-term care insurance decision once they learn information about their type. Given that agents have no information about their type, they then face the following retirement vehicle maximization problem:

$$\max_{\beta,B}\Omega_L^{before} = (1-x)\left(U\left(W_0-\beta-\widehat{\delta}_L\right)+p_L\left[\begin{array}{c}(1-\pi)U(W+B)\\+\pi\int_0^\Lambda U\left(W+B-\left(1-\widehat{d}_L\right)\lambda\right)g(\lambda)\,d\lambda\end{array}\right]\right)$$
$$+x\left(U\left(W_0-\beta-\widehat{\delta}_H\right)+p_H\left[\begin{array}{c}(1-\pi)U(W+B)\\+\pi\int_0^\Lambda U\left(W+B-\left(1-\widehat{d}_H\right)\lambda\right)g(\lambda)\,d\lambda\end{array}\right]\right)$$

subject to the zero profit conditions

$$\beta=[xp_H+(1-x)p_L]B+m_B$$

and the projected choices of $\widehat{\delta}_L,\widehat{\delta}_H,\widehat{d}_L,\widehat{d}_H$ that are the solution to the maximization problem exposed in Appendix 9.3.

The "values" of $\widehat{\delta}_L,\widehat{\delta}_H,\widehat{d}_L,\widehat{d}_H$ will therefore be anticipated by the agent when choosing his retirement vehicle. The rational expectation problem he faces at that point in time (after substituting in all the relevant zero-profit constraints on the retirement and the long-term care markets) is written as

$$\max_{B,d_L,d_H,\gamma}\Omega_L^{before} = (1-x)\left(\begin{array}{c}U\left(W_0-([xp_H+(1-x)p_L]B+m_B)-\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)\\+p_L\left[(1-\pi)U(W+B)+\pi\int_0^\Lambda U(W+B-(1-d_L)\lambda)g(\lambda)\,d\lambda\right]\end{array}\right)$$
$$+x\left(\begin{array}{c}U\left(W_0-([xp_H+(1-x)p_L]B+m_B)-\left(p_H\pi\left(\int_0^\Lambda d_H\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)\\+p_H\left[(1-\pi)U(W+B)+\pi\int_0^\Lambda U(W+B-(1-d_H)\lambda)g(\lambda)\,d\lambda\right]\end{array}\right)$$

and the projected first order conditions that solve the long-term care insurance problem:

$$\begin{aligned}0 = &-U'\left(W_0-([xp_H+(1-x)p_L]B+m_B)-\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)p_L\pi\left(\int_0^\Lambda\lambda g(\lambda)\,d\lambda\right)\\&+p_L\pi\int_0^\Lambda\lambda U'(W+B-(1-d_L)\lambda)g(\lambda)\,d\lambda\\&+\gamma\left(\begin{array}{c}U'\left(W_0-([xp_H+(1-x)p_L]B+m_B)-\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)p_L\pi\left(\int_0^\Lambda\lambda g(\lambda)\,d\lambda\right)\\-p_H\pi\int_0^\Lambda\lambda U'(W+B-(1-d_L)\lambda)g(\lambda)\,d\lambda\end{array}\right)\end{aligned}$$

$$\begin{aligned}0 = &-\left(\int_0^\Lambda\lambda g(\lambda)\,d\lambda\right)U'\left(W_0-([xp_H+(1-x)p_L]B+m_B)-\left(p_H\pi\left(\int_0^\Lambda d_H\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)\\&+\int_0^\Lambda\lambda U'(W+B-(1-d_H)\lambda)g(\lambda)\,d\lambda\end{aligned}$$

$$\begin{aligned}0 = &\;U\left(W_0-([xp_H+(1-x)p_L]B+m_B)-\left(p_H\pi\left(\int_0^\Lambda d_H\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)\\&+p_H\pi\left(\int_0^\Lambda U(W+B-(1-d_H)\lambda)g(\lambda)\,d\lambda-\int_0^\Lambda U(W+B-(1-d_L)\lambda)g(\lambda)\,d\lambda\right)\\&-U\left(W_0-([xp_H+(1-x)p_L]B+m_B)-\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right).\end{aligned}$$

Of course this problem is much more complicated than the previous one because of the timing of information. From the last equation, suppose that $d_L = d_H = 1$, just as in the case where individuals wait to learn their risk type before choosing their retirement and their long-term care insurance contracts. We then have that the following must hold

$$\begin{aligned} 0 &= U\left(W_0 - \left([xp_H + (1-x)\, p_L]\, B + m_B\right) - \left(p_H \pi \left(\int_0^\Lambda \lambda g(\lambda)\, d\lambda\right) + \mu\right)\right) \\ &\quad - U\left(W_0 - \left([xp_H + (1-x)\, p_L]\, B + m_B\right) - \left(p_L \pi \left(\int_0^\Lambda \lambda g(\lambda)\, d\lambda\right) + \mu\right)\right). \end{aligned}$$

This is not possible. Consequently full insurance on the long-term care market is not an equilibrium. We state this as our next proposition:

Proposition 4 *When the retirement contract is signed before agents know their risk type, then the optimal allocation on the long-term care market is such that full insurance is no longer optimal for neither agent, and the low-risk must signal his type by accepting a less than full insurance contract. In other words, the equilibrium on the long-term care market is such that $d_L < d_H < 1$.*

Proof: The fact that the low-risk agent must now signal his type on the long-term care market by accepting a contract that is less generous (i.e., $d_L < d_H$) is easily shown. What is harder is to show that $d_H < 1$. To do so consider the following constraint

$$\begin{aligned} 0 &= -\left(\int_0^\Lambda \lambda g(\lambda)\, d\lambda\right) U'\left(W_0 - \left([xp_H + (1-x)\, p_L]\, \widehat{B} + m_B\right) - \left(p_H \pi \left(\int_0^\Lambda d_H \lambda g(\lambda)\, d\lambda\right) + \mu\right)\right) \\ &\quad + \int_0^\Lambda \lambda U'\left(W + \widehat{B} - (1-d_H)\, \lambda\right) g(\lambda)\, d\lambda \end{aligned}$$

for which we let $d_H = 1$. We then find that

$$U'\left(W_0 - \left([xp_H + (1-x)\, p_L]\, \widehat{B} + m_B\right) - \left(p_H \pi \left(\int_0^\Lambda \lambda g(\lambda)\, d\lambda\right) + \mu\right)\right) = U'\left(W + \widehat{B}\right),$$

which would be a solution provided that

$$\widehat{B} = \frac{W_0 - W - m_B - \mu - p_H \pi \left(\int_0^\Lambda \lambda g(\lambda)\, d\lambda\right)}{1 + [xp_H + (1-x)\, p_L]}$$

But this retirement amount $\widehat{B}$ assumes that agents already know their type since the premium paid by the high risk agents is included in the calculation of $\widehat{B}$. That cannot be the case, however, since agents do not know their type at that time. This means that the choice of a retirement contract before the agents learn about their risk type must be such that $B^* > \widehat{B}$. It follows that $d_H < 1$ since there is a negative relationship between *the choice of annuities and the choice of the long-term care indemnity. As the pre-information choice of the annuity is greater than the annuity choice if $d_H = 1$, it must then be that that full insurance is no longer a possibility on the long-term care contract, even for the high risk.*•

Assuming that this problem can be solved (by using an explicit utility function for instance), all that is left to do is compare the expected utility of the agents across the two problems. In other words, do agents prefer to learn about their type before deciding on their retirement vehicle and their long-term care purchase, or do they prefer to hedge that risk completely by acquiring a retirement vehicle before knowing their type? The preference for one or the other will depend on many factors, including the difference between the high risk's and the low risk's probabilities of reaching old age, the probability of needing long-term care and and the agents' risk aversion.

4.2.2 Zero-profit overall on the long-term care insurance market

When insurers are able to cross-subsidize insurance purchases on the long-term care market, that changes a bit the structure of the contract because it allows the low risk individual to sacrifice some wealth (which would be given to the high risk individual) in exchange for more protection. The problem on the retirement market does not change in itself, except for the anticipated contract that will be offered to the different agents when it comes time to purchase long-term care insurance. The problem for the agent before information is learned is still given by

$$\begin{aligned}\max_{\substack{B,d_L,d_H,\\ \delta_L,\delta_H}} \Omega_L^{before} &= (1-x)\begin{pmatrix} U\left(W_0 - \left(\left[xp_H + (1-x)\,p_L\right]B + m_B\right) - \delta_L\right) \\ +p_L\left[(1-\pi)\,U\left(W+B\right) + \pi\int_0^\Lambda U\left(W+B-(1-d_L)\,\lambda\right)g\left(\lambda\right)d\lambda\right]\end{pmatrix} \\ &+x\begin{pmatrix} U\left(W_0 - \left(\left[xp_H + (1-x)\,p_L\right]B + m_B\right) - \delta_H\right) \\ +p_H\left[(1-\pi)\,U\left(W+B\right) + \pi\int_0^\Lambda U\left(W+B-(1-d_H)\,\lambda\right)g\left(\lambda\right)d\lambda\right]\end{pmatrix}\end{aligned}$$

subject to

$$\begin{aligned}\{d_L, d_H, \delta_L, \delta_H\} \in\; & \arg\max U\left(W_0 - \left(\left[xp_H + (1-x)\,p_L\right]B + m_B\right) - \delta_L\right) \\ & +p_L\left[(1-\pi)\,U\left(W+B\right) + \pi\int_0^\Lambda U\left(W+B-(1-d_L)\,\lambda\right)g\left(\lambda\right)d\lambda\right] \\ & +\eta\begin{pmatrix} x\delta_H + (1-x)\,\delta_L - xp_H\left[A_H + \pi d_H\int_0^\Lambda \lambda g\left(\lambda\right)d\lambda\right] \\ -(1-x)\,p_L\left[A_H + \pi d_H\int_0^\Lambda \lambda g\left(\lambda\right)d\lambda\right] - \mu\end{pmatrix} \\ & +\gamma\begin{pmatrix} U\left(W_0 - \left(\left[xp_H + (1-x)\,p_L\right]B + m_B\right) - \delta_H\right) \\ -U\left(W_0 - \left(\left[xp_H + (1-x)\,p_L\right]B + m_B\right) - \delta_L\right) \\ +p_H\pi\int_0^\Lambda\left[U\left(W+B-(1-d_H)\,\lambda\right) - U\left(W+B-(1-d_L)\,\lambda\right)\right]g\left(\lambda\right)d\lambda\end{pmatrix}.\end{aligned}$$

This problem is even less tractable analytically than the first. Nevertheless, it is solvable numerically so that we can compare the welfare of agents under the different situations. This is what we do in the following section.

5 Example and empirical analysis

As it is quite clear from the modeling section of the paper, comparing a retirement plan with long-term care insurance is not a trivial matter. The timing of information vis-à-vis the decision to acquire a retirement product complicates the analysis. In particular, if the purchase of the retirement product occurs before agents learn their risk types, which occurs before agents choose their long-term care insurance product, then not only are we unable to find a closed-form solution to the problem without specifying an explicit utility function, we are also unable to find a general structure for the contact. This is in sharp contrast to the situation where agents had information before purchasing a retirement vehicle and a long-term care insurance contract. In such a case, we showed that both the high risk and the low risk agents were purchasing contracts that offered full protection against the loss that can befall them if they indeed need long-term care.

What we present in this section of the paper are simulations of contract structures based on parameter values in the traditional Rothschild-Stiglitz framework with no subsidization across agents or products. From those parameter values, we will be able to infer more general results on the impact of each parameter on the agents' preference for the pre-information retirement vehicle (DB plan) versus the post-information retirement vehicle (DC plan). We present simulations that also incorporate the risk of needing of long-term care once old-age is reached (π), the difference between the two types of agents' probability of needing retirement income ($p_H - p_L$), and on the proportion of high risks in the economy (x).

We will be using the following starting (initial) values for our exercise:

Initial (starting) values for the parameters				
$U(C) = \ln(C)$;	$W_0 = 25$;	$W = 5$;	$p_L = 0.55$;	$p_H = 0.95$;
$m_A = m_B = 1$;	$\mu = 1$;	$\int_0^\Lambda \lambda g(\lambda)\, d\lambda = 5$;	$\pi = 0.2$;	$x = 0.3$

The final wealth for the high and low risk individuals, as a function of each state of the world, as well as a summary of the contract choice are given in the following table.

Consumption in each state of the world as a function of agent type: The case where the defined contribution pension plan is preferred						
Defined contribution situation; $EU(\bullet) = 4.64$						
	$EU(C)$	LTC	Retirement	*Period* 0	*Period* 1 Healthy	*Period* 1 Sick
High risk (0.3)	5.11	$d_H = 1.00$	$A_H = 8.74$	13.74	13.74	13.74
Low risk (0.7)	4.45	$d_L = 1.00$	$A_L = 4.19$	20.15	9.19	9.19
Defined benefit situation; $EU(\bullet) = 4.56$						
	$EU(C)$	LTC	Retirement	*Period* 0	*Period* 1 Healthy	*Period* 1 Sick
High risk (0.3)	5.32	$d_H = 0.84$	$B = 10.77$	14.98	15.77	14.98
Low risk (0.7)	4.24	$d_L = 0.22$	$B = 10.77$	15.66	15.77	11.86
Low risk (*exit*)	4.30	$d_L = 0$	$B = 10.77$	16.78	15.77	10.77
Values used for the example: $U(C) = \ln(C); W_0 = 25; W = 5; p_L = \underline{0.55}; p_H = 0.95;$ $m_A = m_B = \mu = 1; \int_0^\Lambda \lambda g(\lambda)\, d\lambda = 5; \pi = 0.2; x = \underline{0.3}.$						

In such a situation, agents prefer to have a defined contribution contract since their expected utility under a DB contract is 4.56 (calculated as $0.3 \times 5.32 + 0.7 \times 4.24 = 4.56$), which is less than their expected utility under a DC contract, 4.64 (calculated as $0.3 \times 5.11 + 0.7 \times 4.45 = 4.64$). From this information, we are able to find that the optimal level of long-term care insurance under the defined contribution plan is such that both the high risk and the low risk agents receive full insurance (i.e., $d_L = d_H = 1$). In terms of the retirement contract, the high risk chooses $A_H = 8.74$ whereas the low risk chooses $A_L = 4.19$. In a defined benefit plan, the long-term care insurance choices are $d_H = 0.84$ and $d_L = 0.22$ so that neither agent chooses to have full long-term care insurance (and the low-risk even less so). In the DB case, the retirement contract is of course the same for the low and high risk agents (i.e., $B = 10.77$) given that they purchased the retirement contract before any one has any information about their risk type.

One aspect of the defined benefit contract we did not explicitly examine, but is always a possibility, is for the agents (the low risk most likely) to decide not to buy any long-term care insurance contract after entering a defined benefit retirement arrangement. This is what is meant in the line titled "*Low risk* (*exit*)". Given the parameter values, we see that the low risk would be better off by exiting the long-term care insurance market and rely only on the wealth transfer associated with the defined benefit retirement vehicle to pay for the potential cost of long-term care. The reason for preferring to remain uninsured vis-à-vis the long-term care risk is that the fixed loading factor and the cost of signalling eliminate all benefits of having long-term care insurance.

Changing the parameter values (say letting $x = 0.5$ instead of $x = 0.3$, and $p_L = 0.65$ instead of $p_L = 0.55$), we find that agents would prefer to enter a DB contract instead since their expected utility under a DB contract is now 4.82, which is more than the expected utility of 4.75 they anticipate under a DC contract.

Consumption in each state of the world as a function of agent type: The case where the defined benefit pension plan is preferred						
Defined contribution situation; $EU(\bullet) = 4.75$						
	$EU(C)$	LTC	Retirement	*Period* 0	*Period* 1 Healthy	*Period* 1 Sick
High risk (0.5)	5.11	$d_H = 1.00$	$A_H = 8.74$	13.74	13.74	13.74
Low risk (0.5)	4.39	$d_L = 1.00$	$A_L = 4.57$	19.38	9.57	9.57
Defined benefit situation; $EU(\bullet) = 4.82$						
	$EU(C)$	LTC	Retirement	*Period* 0	*Period* 1 Healthy	*Period* 1 Sick
High risk (0.5)	5.21	$d_H = 0.91$	$B = 9.78$	14.31	14.78	14.31
Low risk (0.5)	4.42	$d_L = 0.29$	$B = 9.78$	14.98	14.78	11.25
Low risk (*exit*)	4.48	$d_L = 0$	$B = 9.78$	16.17	14.78	9.78
Values used for the example: $U(C) = \ln(C); W_0 = 25; W = 5; p_L = \underline{0.65}; p_H = 0.95;$ $m_A = m_B = \mu = 1; \int_0^\Lambda \lambda g(\lambda)\, d\lambda = 5; \pi = 0.2; x = \underline{0.5}.$						

By changing each value one at the time, we are able to find the tipping point where agents prefer to have a retirement scheme chosen before to after the revelation of information. Although we do not provide all possible examples for ease of presentation, we are able to show that the greater is the probability of needing long-term care (i.e., the greater is π), the more likely individuals will opt for a pre-information investment vehicle (that is, a defined benefit pension plan). The greater is the proportion of high risks (x), the greater is an agent's likelihood of choosing a DB plan. The greater is the difference between p_H and p_L, the greater is the likelihood that an agent will choose the defined contribution pension plan. Also, the greater is the expected possible loss associated with long-term care (i.e., the greater is $\int_0^\Lambda \lambda g(\lambda)\, d\lambda$), the more likely to prefer the DB plan. The greater is the agent's initial wealth, the greater his preference for the pre-information investment vehicle. Finally, more risk averse agents will choose the defined benefit pension plan. In summary we find the following:

The greater is	The more likely
π	DB
x	DB
$p_H - p_L$	DC
$\int_0^\Lambda \lambda g(\lambda)\, d\lambda$	DB
W_0, W	DC
$CRRA$	DB

How does that match the observed empirical evidence? The last two lines are related to individual wealth and risk aversion. To that end, there is ample evidence that richer individuals prefer DC plans (Gerrans and Clark 2013, Brown and Weisbenner 2014). So the (W_0, W) line seems to be respected in reality. With respect to risk aversion, Gerrans and Clark (2013) find that younger individuals are more likely to choose a defined contribution plan to a defined benefit plan. If age is correlated with risk aversion, then we have evidence that older more risk averse individuals are more likely to opt for a DB plan. Finally, it is commonly accepted that women are more risk averse then men. In Drolet and Morissette (2014), it is found that 33% of employed Canadian women and 24% of employed Canadian men aged 25 to 54 were covered by DB plans. So the $CRRA$ prediction seems to hold in reality.

The other four predictions are harder to assess.

- If healthy individuals become healthier in general compared to less healthy individuals (i.e., $p_H - p_L$ becomes larger) then DC plans should be preferred. The problem is that we observe an increase in longevity for all; it is not obvious that $p_H - p_L$ is increasing.

- Is the proportion of high risks (x) decreasing? Again, the problem is that it is not obvious how we can measure the proportion of high risks in the economy, let alone how their numbers are evolving over time.

- Is the probability of needing LTC conditional on being alive at an advanced age (π) increasing? If so, then a DB plan would be preferable. But as individuals are healthier, it may just be that long-term care is less likely so that more sudden death events become more likely. Again, it is not obvious to us what is the actual directionality here.

- Is the real cost of LTC ($\int_0^\Lambda \lambda g(\lambda)\, d\lambda$) increasing? If so DB plan should be more interesting.

These are empirical questions about the model that will need to be taken to the data. Note that we have not taken into account measures of education in the model. Drolet and Morissette (2014) find that education is correlated with the choice of DB over DC plans, but Brown and Weisbenner (2014) find the opposite. The difference between the two studies is that former is conducted using Canadian data whereas the second uses American data. Could it be that the differences between Canadian and American results are driven only by differences across the two countries with respect to $p_H - p_L$, π, x, and $\int_0^\Lambda \lambda g(\lambda)\, d\lambda$? The apparent empirical regularity is that DB plans is more popular in Canada than in the United States. Our model would predict such a result provided that $p_H - p_L$ is greater in the U.S. than in Canada, that π and x are greater in Canada than in the U.S.. Is that the case? Anecdotal evidence seems to indicate that the wedge between long-lived and short-lived individuals is higher in the U.S., at least amongst the most educated. Finally, with respect to the cost of long term care, one could imagine that this is a lowly-skilled labor intensive industry in which the U.S. market should have a cost advantage given the lower minimum wage in place in the United States compared to Canada.

6 Robustness and other model approaches

6.1 What if LTC losses are type-specific?

One of the assumptions we made early in the model was that the conditional expected long-term care loss (i.e., $\pi \int_0^{\Lambda_H} \lambda g_H(\lambda)\, d\lambda$) was the same for all agent types. In other words, agents only differed with respect to the probability of reaching an advanced age, p_i. What if we were now to assume that $\int_0^{\Lambda_H} \lambda g_H(\lambda)\, d\lambda > \int_0^{\Lambda_L} \lambda g_L(\lambda)\, d\lambda$? In other words, suppose there is a positive correlation between the probability of living to an old age and the potential loss (in finance, this would be similar to saying that there is a positive correlation between the probability of default and the loss given default). From a modelling perspective, we can show that in the DC case, the high risk individual still obtains full income smoothing so that $d_H = 1$ and $A_H = \frac{W_0 - W - m_A - \mu - p_H \pi \left(\int_0^{\Lambda_H} \lambda g_H(\lambda) d\lambda \right)}{1 + p_H}$. We also can show that the low risk individual now receives less than full insurance on the LTC market ($d_L < 1$) and must bear some wealth risk on the retirement market ($A_H < \frac{W_0 - W - m_A - \mu - p_L \pi \left(\int_0^{\Lambda_L} \lambda g_L(\lambda) d\lambda \right)}{1 + p_L}$). The question is: what impact does it have on the decision to enter the retirement contract before or after one's risk type is known?

Interestingly, as the high risk's conditional expected long-term care loss increases in comparison to the low risk's, we can show that the value of waiting for more information about one's risk type increases. This is true even if, ultimately, the low risk individuals obtain less LTC insurance protection after the risk types are known.

We run here a simulation similar to the one we did before, using the same initial value for the utility function ($U(C) = \ln(C)$), for the wealth in each period ($W_0 = 25$; $W = 5$), the probability of getting to the second period and the probability of needing long-term care ($p_L = 0.55$; $p_H = 0.95$; $\pi = 0.2$), proportion of high risk individuals ($x = 0.7$), and the same loading factors ($m_A = m_B = \mu = 1$). The only difference we make is to use different potential losses to the agents that need long-term care: $\int_0^{\Lambda_L} \lambda g_L(\lambda)\, d\lambda = 5$ and $\int_0^{\Lambda_H} \lambda g_H(\lambda)\, d\lambda = 5.4$.

Solving empirically this problem gives us an interesting result that if agents choose their retirement contract before they have any information, and then they acquire the long-term care insurance, then the agents' choice with respect to the long-term care protection is such that $d_H = 94\%$ and $d_L = 27\%$. Clearly, this is not full insurance for either. Looking now at the case where agents wait until they have private information before choosing their retirement, we find $d_H = 1$ (so full insurance) and $d_L = 5\%$. With respect to the wealth in each state of the world, we find that the low risk individual must accept a much more volatile wealth when agents decide to wait for the information to be transmitted before making their retirement choices than when they can contract before such information is revealed.

Despite the fact that ex ante agents know that the patient low-longevity risk individuals will buy very little insurance for their potential long-term care needs (less than 5%) and that they will have a much more volatile wealth distribution across states of the world than the impatient low-longevity risk individuals, they still prefer to wait to know what their risk type is before making those decisions.

When high risk individuals face a lower loss when entering long-term care (so that $\int_0^{\Lambda_H} \lambda g_H(\lambda)\, d\lambda = 4.6$), the results are similar to those we have already presented with respect to the choice of contracts with less information (i.e., $d_H = 94\%$ and $d_L = 27\%$). When agents know their risk type, however, the choice of the low-longevity risk individual changes such that $d_L = 142\%$. In other words, the low risk individual chooses a long-term care contract that over-compensates him in the event that such a state of the world occurs. But of course, such a result is only possible if the indemnity given by the insurer is monetary and not in nature such as the reimbursement of actual expenses or the direct provision of help.

7 Discussion and Conclusion

In this paper we analyze an economy where risk averse agents face two levels of uncertainty: longevity and long-term care. We posit that agents learn over time what their risks of needing longevity hedging products and of long-term care insurance are. In addition to that, agents are able to commit to purchasing the retirement product before or after they learn about their risk type. Apart from the long-term care issue, the problem is similar to that of the choice between a defined benefit and a defined contribution investment product. In a defined benefit system, individuals are typically grouped together with their fellow workers in such a way that each individual's specific risk type is irrelevant. In a defined contribution scheme, however, agents get to convert their retirement savings into an annuity at the time of their retirement. Surely at that point in time they have acquired more information about their health risk type than when they had the opportunity of having a defined benefit pension plan.

In a world where there is no long-term care risk, it is well known that risk averse agents will strictly prefer the defined benefit pension plan. The reason is that, in a competitive market, waiting to know if we are high risk or low risk before purchasing insurance (or planning retirement) is tantamount to choosing a lottery with mean zero (i.e., a mean-preserving spread). At the same time, economic theory tells us that one should smooth consumption over time so that the purchasing annuities should be welfare increasing (Yaari, 1965). Yet, this is not what we observe in reality as individuals rarely purchase annuities (see Mitchell *et al.* 1999 *inter alia*).

When we introduce long-term care insurance that can be purchased only after the risk type is known, it become possible that individuals prefer to wait to buy their retirement product. In other words, defined contribution pension plans can become preferred when combined with the purchase of long-term care insurance. The reason is that when agents choose to wait before acquiring their retirement product to have information about their risk type, they are able to have full long-term care insurance, no matter what their risk type is. That full insurance result is not possible when agents purchase their retirement product before knowing what their type is. In fact, we find in our model that full long-term care insurance is not even chosen by the high risk individual. To our knowledge, this is the first time that such a result has been found.

Standard insurance theory tells us that the costly nature of long-term care makes such the management of such a risk extremely valuable for risk averse individuals. Surprisingly, however, only 4% of long-term care expenditures are paid for by private insurance (see Brown and Finkelstein 2007). The limited insurance coverage of long-term care costs has important implications for the welfare of the elderly, and for their adult children as well is they are forced to quit their job (or cut down on their hours worked) to take care of their elderly parents.

Although we do not directly address the question of the thinness of the long-term care insurance market or of the quasi-absence of the bundling of annuities with long-term care insurance (or life-care annuities as defined in Murtaugh *et al.* 2001, Warshawsky 2007, and Zhou-Richter and Gründl 2011),[15] we are able to offer some possible explanation based on the characteristics of the equilibria we find. Another possibility, which is outside of this model, is that individuals have access to informal care so that they do not need to pay for long-term care insurance. A third possibility is that the annuity market is so unattractive (see all the papers on the thinness of the annuity market) as to make the bundling of the contracts not attractive enough. In other words, the omnipresence of a DB-like social security system is such that it is crowding out the long-term care risk market.

[15] As a response to the shortcomings of both the long-term care insurance and the annuity market, it has been suggested that a product bundling both product lines would help mitigate the problems encountered in each individual segment. Warshawsky (2007) coined the term "Life Care Annuity" (LCA) for this bundled product.

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9 Appendix: Proofs

9.1 Post-information choice with zero-profit for each contract & type

By substituting the four zero-profit constraints where appropriate, the problem in its Langrangian form takes the following form:

$$\begin{aligned}
\max_{A_L,d_L,A_H,d_H,\gamma} \Omega_L^{after} &= U\left(W_0-(p_LA_L+m_A)-\left(p_L\pi\left(\int_0^{\Lambda} d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right) \\
&+p_L\left[(1-\pi)U(W+A_L)+\pi\int_0^{\Lambda}U(W+A_L-(1-d_L)\lambda)g(\lambda)\,d\lambda\right] \\
&+\gamma\begin{pmatrix} U\left(W_0-(p_HA_H+m_A)-\left(p_H\pi\left(\int_0^{\Lambda} d_H\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right) \\ +p_H\left[(1-\pi)U(A_H)+\pi\int_0^{\Lambda}U(W+A_H-(1-d_H)\lambda)g(\lambda)\,d\lambda\right] \\ -U\left(W_0-(p_LA_L+m_A)-\left(p_L\pi\left(\int_0^{\Lambda} d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right) \\ -p_H\left[(1-\pi)U(W+A_L)+\pi\int_0^{\Lambda}U(W+A_L-(1-d_L)\lambda)g(\lambda)\,d\lambda\right]\end{pmatrix}
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
\frac{\partial}{\partial A_L} &: 0=-U'\left(W_0-(p_LA_L+m_A)-\left(p_L\pi\left(\int_0^{\Lambda} d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)p_L \\
&+p_L\left[(1-\pi)U'(W+A_L)+\pi\int_0^{\Lambda}U'(W+A_L-(1-d_L)\lambda)g(\lambda)\,d\lambda\right] \\
&+\gamma\begin{pmatrix} U'\left(W_0-(p_LA_L+m_A)-\left(p_L\pi\left(\int_0^{\Lambda} d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)p_L \\ -p_H\left[(1-\pi)U'(W+A_L)+\pi\int_0^{\Lambda}U'(W+A_L-(1-d_L)\lambda)g(\lambda)\,d\lambda\right]\end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial d_L} &: 0=-U'\left(W_0-(p_LA_L+m_A)-\left(p_L\pi\left(\int_0^{\Lambda} d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)p_L\pi\left(\int_0^{\Lambda}\lambda g(\lambda)\,d\lambda\right) \\
&+p_L\pi\int_0^{\Lambda}\lambda U'(W+A_L-(1-d_L)\lambda)g(\lambda)\,d\lambda \\
&+\gamma\begin{pmatrix} U'\left(W_0-(p_LA_L+m_A)-\left(p_L\pi\left(\int_0^{\Lambda} d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)p_L\pi\left(\int_0^{\Lambda}\lambda g(\lambda)\,d\lambda\right) \\ -p_H\pi\int_0^{\Lambda}\lambda U'(W+A_L-(1-d_L)\lambda)g(\lambda)\,d\lambda\end{pmatrix}
\end{aligned}$$

$$\frac{\partial}{\partial A_H}: 0=\gamma\begin{pmatrix} -p_HU'\left(W_0-(p_HA_H+m_A)-\left(p_H\pi\left(\int_0^{\Lambda} d_H\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right) \\ +p_H\left[(1-\pi)U'(W+A_H)+\pi\int_0^{\Lambda}U'(W+A_H-(1-d_H)\lambda)g(\lambda)\,d\lambda\right]\end{pmatrix}$$

$$\frac{\partial}{\partial d_H}: 0=\gamma\begin{pmatrix} -p_H\pi\left(\int_0^{\Lambda}\lambda g(\lambda)\,d\lambda\right)U'\left(W_0-(p_HA_H+m_A)-\left(p_H\pi\left(\int_0^{\Lambda} d_H\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right) \\ +p_H\pi\int_0^{\Lambda}\lambda U'(W+A_H-(1-d_H)\lambda)g(\lambda)\,d\lambda\end{pmatrix}$$

$$\frac{\partial}{\partial \gamma}:0=\left(\begin{array}{c} U\left(W_0-\left(p_H A_H+m_A\right)-\left(p_H \pi\left(\int_0^{\Lambda} d_H \lambda g(\lambda)\, d\lambda\right)+\mu\right)\right) \\ +p_H\left[(1-\pi) U\left(W+A_H\right)+\pi \int_0^{\Lambda} U\left(W+A_H-\left(1-d_H\right) \lambda\right) g(\lambda)\, d\lambda\right] \\ -U\left(W_0-\left(p_L A_L+m_A\right)-\left(p_L \pi\left(\int_0^{\Lambda} d_L \lambda g(\lambda)\, d\lambda\right)+\mu\right)\right) \\ -p_H\left[(1-\pi) U\left(W+A_L\right)+\pi \int_0^{\Lambda} U\left(W+A_L-\left(1-d_L\right) \lambda\right) g(\lambda)\, d\lambda\right] \end{array}\right)$$

The high risk allocation is easy to find using $\frac{\partial}{\partial A_H}$ and $\frac{\partial}{\partial d_H}$. These first order conditions are exactly the same as those that we had when types were known. This means that the high risk individual is able to obtain his first best allocation. As we recall, the first best allocation for the high risks means that $d_H = 1$ and $A_H = \frac{W_0 - W - m_A - \mu - p_H \pi\left(\int_0^{\Lambda} \lambda g(\lambda) d\lambda\right)}{1+p_H}$.

Finding the low risk's allocation requires more work. From $\frac{\partial}{\partial A_L}$ and $\frac{\partial}{\partial d_L}$, we find

$$\gamma=\frac{\begin{array}{c} U'\left(W_0-\left(p_L A_L+m_A\right)-\left(p_L \pi\left(\int_0^{\Lambda} d_L \lambda g(\lambda)\, d\lambda\right)+\mu\right)\right) p_L \\ -p_L\left[(1-\pi) U'\left(W+A_L\right)+\pi \int_0^{\Lambda} U'\left(W+A_L-\left(1-d_L\right) \lambda\right) g(\lambda)\, d\lambda\right] \end{array}}{\begin{array}{c} U'\left(W_0-\left(p_L A_L+m_A\right)-\left(p_L \pi\left(\int_0^{\Lambda} d_L \lambda g(\lambda)\, d\lambda\right)+\mu\right)\right) p_L \\ -p_H\left[(1-\pi) U'\left(W+A_L\right)+\pi \int_0^{\Lambda} U'\left(W+A_L-\left(1-d_L\right) \lambda\right) g(\lambda)\, d\lambda\right] \end{array}}$$

and

$$\gamma=\frac{\begin{array}{c} U'\left(W_0-\left(p_L A_L+m_A\right)-\left(p_L \pi\left(\int_0^{\Lambda} d_L \lambda g(\lambda)\, d\lambda\right)+\mu\right)\right) p_L \pi\left(\int_0^{\Lambda} \lambda g(\lambda)\, d\lambda\right) \\ -p_L \pi \int_0^{\Lambda} \lambda U'\left(W+A_L-\left(1-d_L\right) \lambda\right) g(\lambda)\, d\lambda \end{array}}{\begin{array}{c} U'\left(W_0-\left(p_L A_L+m_A\right)-\left(p_L \pi\left(\int_0^{\Lambda} d_L \lambda g(\lambda)\, d\lambda\right)+\mu\right)\right) p_L \pi\left(\int_0^{\Lambda} \lambda g(\lambda)\, d\lambda\right) \\ -p_H \pi \int_0^{\Lambda} \lambda U'\left(W+A_L-\left(1-d_L\right) \lambda\right) g(\lambda)\, d\lambda \end{array}}$$

so

$$\begin{aligned} 0 \quad = \quad & \frac{\begin{array}{c} U'\left(W_0-\left(p_L A_L+m_A\right)-\left(p_L \pi\left(\int_0^{\Lambda} d_L \lambda g(\lambda)\, d\lambda\right)+\mu\right)\right) p_L \\ -p_L\left[(1-\pi) U'\left(W+A_L\right)+\pi \int_0^{\Lambda}\left(W+A_L-\left(1-d_L\right) \lambda\right) g(\lambda)\, d\lambda\right] \end{array}}{\begin{array}{c} U'\left(W_0-\left(p_L A_L+m_A\right)-\left(p_L \pi\left(\int_0^{\Lambda} d_L \lambda g(\lambda)\, d\lambda\right)+\mu\right)\right) p_L \\ -p_H\left[(1-\pi) U'\left(W+A_L\right)+\pi \int_0^{\Lambda}\left(W+A_L-\left(1-d_L\right) \lambda\right) g(\lambda)\, d\lambda\right] \end{array}} \\ & -\frac{\begin{array}{c} U'\left(W_0-\left(p_L A_L+m_A\right)-\left(p_L \pi\left(\int_0^{\Lambda} d_L \lambda g(\lambda)\, d\lambda\right)+\mu\right)\right) p_L \pi\left(\int_0^{\Lambda} \lambda g(\lambda)\, d\lambda\right) \\ -p_L \pi \int_0^{\Lambda} \lambda U'\left(W+A_L-\left(1-d_L\right) \lambda\right) g(\lambda)\, d\lambda \end{array}}{\begin{array}{c} U'\left(W_0-\left(p_L A_L+m_A\right)-\left(p_L \pi\left(\int_0^{\Lambda} d_L \lambda g(\lambda)\, d\lambda\right)+\mu\right)\right) p_L \pi\left(\int_0^{\Lambda} \lambda g(\lambda)\, d\lambda\right) \\ -p_H \pi \int_0^{\Lambda} \lambda U'\left(W+A_L-\left(1-d_L\right) \lambda\right) g(\lambda)\, d\lambda \end{array}} \end{aligned}$$

and

$$
\begin{aligned}
0 \;=\; & U'\begin{pmatrix} W_0-(p_LA_L+m_A) \\ -\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\end{pmatrix} p_LU'\begin{pmatrix} W_0-(p_LA_L+m_A) \\ -\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\end{pmatrix} p_L\pi\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right) \\
& -U'\begin{pmatrix} W_0-(p_LA_L+m_A) \\ -\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\end{pmatrix} p_Lp_H\pi\int_0^\Lambda \lambda U'(W+A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda \\
& -p_L\begin{bmatrix} (1-\pi)U'(W+A_L) \\ +\pi\int_0^\Lambda U'(W+A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda\end{bmatrix} U'\begin{pmatrix} W_0-(p_LA_L+m_A) \\ -\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\end{pmatrix} p_L\pi\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right) \\
& +p_L\begin{bmatrix} (1-\pi)U'(W+A_L) \\ +\pi\int_0^\Lambda U'(W+A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda\end{bmatrix} p_H\pi\int_0^\Lambda \lambda U'(W+A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda \\
& -U'\begin{pmatrix} W_0-(p_LA_L+m_A) \\ -\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\end{pmatrix} p_LU'\begin{pmatrix} W_0-(p_LA_L+m_A) \\ -\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\end{pmatrix} p_L\pi\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right) \\
& +U'\begin{pmatrix} W_0-(p_LA_L+m_A) \\ -\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\end{pmatrix} p_Lp_L\pi\int_0^\Lambda \lambda U'(W+A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda \\
& +p_H\begin{bmatrix} (1-\pi)U'(W+A_L) \\ +\pi\int_0^\Lambda U'(W+A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda\end{bmatrix} U'\begin{pmatrix} W_0-(p_LA_L+m_A) \\ -\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\end{pmatrix} p_L\pi\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right) \\
& -p_H\begin{bmatrix} (1-\pi)U'(W+A_L) \\ +\pi\int_0^\Lambda (A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda\end{bmatrix} p_L\pi\int_0^\Lambda \lambda U'(W+A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda
\end{aligned}
$$

Simplifying all the terms, we find the following condition

$$
\begin{aligned}
0 \;=\; & \int_0^\Lambda \lambda U'(W+A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda \\
& -\left[(1-\pi)U'(W+A_L)+\pi\int_0^\Lambda U'(W+A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda\right]\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right)
\end{aligned}
$$

The only solution to this equality is to have $d_L=1$. To see why, note that when $d_L=1$, we find

$$
U'(W+A_L)\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right)=U'(W+A_L)\left[(1-\pi)+\pi\int_0^\Lambda g(\lambda)\,d\lambda\right]\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right)
$$

Which is obviously true since $\left[(1-\pi)+\pi\int_0^\Lambda g(\lambda)\,d\lambda\right]=1$. It therefore means that $d_H=d_L=1$.

Finally, we only have the retirement vehicle choice for the low risk agent to find. Concentrating on the first order condition with respect to the Lagrangian multiplier, $\frac{\partial}{\partial\gamma}$, and substituting for $d_H=1$ we find

$$
\begin{aligned}
0 \;=\; & U\left(W_0-(p_HA_H+m_A)-\left(p_H\pi\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right)+\mu\right)\right) \\
& -U\left(W_0-(p_LA_L+m_A)-\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right) \\
& +p_HU(W+A_H)-p_H\left[(1-\pi)U(W+A_L)+\pi\int_0^\Lambda U(W+A_L-(1-d_L)\lambda)\,g(\lambda)\,d\lambda\right]
\end{aligned}
$$

Now substituting in for $A_H = \frac{W_0 - W - m_A - \mu - p_H \pi \left(\int_0^\Lambda \lambda g(\lambda) d\lambda\right)}{1+p_H}$, the previous equation writes as

$$\begin{aligned}
(1+p_H) U (W + A_H) &= U \left(W_0 - (p_L A_L + m_A) - \left(p_L \pi \left(\int_0^\Lambda d_L \lambda g(\lambda) \, d\lambda \right) + \mu \right) \right) \\
&\quad + p_H \left[(1-\pi) U (W + A_L) + \pi \int_0^\Lambda U (W + A_L - (1 - d_L) \lambda) g(\lambda) \, d\lambda \right]
\end{aligned}$$

Substituting now for $d_L = 1$ yields

$$(1+p_H) U (W + A_H) = U \left(W_0 - (p_L A_L + m_A) - \left(p_L \pi \left(\int_0^\Lambda \lambda g(\lambda) \, d\lambda \right) + \mu \right) \right) + p_H U (W + A_L)$$

We remember that the first best is given by $A_L = \frac{W_0 - W - m_A - \mu - p_L \pi \left(\int_0^\Lambda \lambda g(\lambda) d\lambda\right)}{1+p_L}$. Substituting this value in the previous equation gives us

$$(1+p_H) U \left(\frac{W_0 - W - m_A - \mu - p_L \pi \left(\int_0^\Lambda \lambda g(\lambda) \, d\lambda \right)}{1+p_L} \right) = (1+p_H) U \left(\frac{W_0 - W - m_A - \mu - p_H \pi \left(\int_0^\Lambda \lambda g(\lambda) \, d\lambda \right)}{1+p_H} \right)$$

which cannot be a solution because $\frac{W_0 - W - m_A - \mu - p_L \pi \left(\int_0^\Lambda \lambda g(\lambda) d\lambda\right)}{1+p_L} > \frac{W_0 - W - m_A - \mu - p_H \pi \left(\int_0^\Lambda \lambda g(\lambda) d\lambda\right)}{1+p_H}$. So this means that $A_L^* < \frac{W_0 - W - m_A - \mu - p_L \pi \left(\int_0^\Lambda \lambda g(\lambda) d\lambda\right)}{1+p_L}$. Consequently, the low risk agents is accepting an allocation that does not give him his first best allocation through perfect income smoothing.•

9.2 Post-information choice with zero-profit in the industry

From the Lagrangian

$$\begin{aligned}
\max_{\substack{A_H, A_L, \alpha_L, \alpha_H, \\ \xi_H, \xi_L}} \Omega_L^{after} &= U (W_0 - \xi_L) + p_L \left[(1-\pi) U (W + A_L) + \pi \int_0^\Lambda U (W + A_L - (1 - d_L) \lambda) g(\lambda) \, d\lambda \right] \\
&\quad + \eta \left[\begin{array}{c} x \xi_H + (1-x) \xi_L - x p_H \left[A_H + \pi d_H \int_0^\Lambda \lambda g(\lambda) \, d\lambda \right] - \mu - m_A \\ - (1-x) p_L \left[A_L + \pi d_L \int_0^\Lambda \lambda g(\lambda) \, d\lambda \right] \end{array} \right] \\
&\quad + \gamma \left[\begin{array}{l} U (W_0 - \xi_H) + p_H \left[(1-\pi) U (W + A_H) + \pi \int_0^\Lambda U (W + A_H - (1 - d_H) \lambda) g(\lambda) \, d\lambda \right] \\ -U (W_0 - \xi_L) - p_H \left[(1-\pi) U (W + A_L) + \pi \int_0^\Lambda U (W + A_L - (1 - d_L) \lambda) g(\lambda) \, d\lambda \right] \end{array} \right]
\end{aligned}$$

we find the following first order conditions:

$$\begin{aligned}
\frac{\partial}{\partial A_L} \quad : \quad & 0 = p_L \left[(1-\pi) U' (W + A_L) + \pi \int_0^\Lambda U' (W + A_L - (1 - d_L) \lambda) g(\lambda) \, d\lambda \right] \\
& + \eta \left[- (1-x) p_L \right] \\
& + \gamma \left[-p_H \left[(1-\pi) U' (W + A_L) + \pi \int_0^{\Lambda_i} U' (W + A_L - (1 - d_L) \lambda) g(\lambda) \, d\lambda \right] \right]
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial}{\partial d_L} \quad : \quad 0 &= p_L \left[\pi \int_0^{\Lambda} \lambda U' \left(W + A_L - (1 - d_L) \lambda \right) g(\lambda) \, d\lambda \right] \\
&+ \eta \left[- (1 - x) p_L \pi \left(\int_0^{\Lambda} \lambda g(\lambda) \, d\lambda \right) \right] \\
&+ \gamma \left[- p_H \pi \int_0^{\Lambda_i} \lambda U' \left(W + A_L - (1 - d_L) \lambda \right) g(\lambda) \, d\lambda \right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial A_H} \quad : \quad 0 &= \eta \left[- x p_H \right] \\
&+ \gamma \left[p_H \left[(1 - \pi) U' \left(W + A_H \right) + \pi \int_0^{\Lambda} U' \left(W + A_H - (1 - d_H) \lambda \right) g(\lambda) \, d\lambda \right] \right]
\end{aligned}
$$

$$
\frac{\partial}{\partial d_H} : 0 = \eta \left[- x p_H \pi \left(\int_0^{\Lambda} \lambda g(\lambda) \, d\lambda \right) \right] + \gamma \left[p_H \pi \int_0^{\Lambda} \lambda U' \left(W + A_H - (1 - d_H) \lambda \right) g(\lambda) \, d\lambda \right]
$$

$$
\frac{\partial}{\partial \xi_L} : 0 = - U' \left(W_0 - \xi_L \right) + \eta \left[1 - x \right] + \gamma \left[U' \left(W_0 - \xi_L \right) \right]
$$

$$
\frac{\partial}{\partial \xi_H} : 0 = \eta \left[x \right] + \gamma \left[- U' \left(W_0 - \xi_H \right) \right]
$$

$$
\begin{aligned}
\frac{\partial}{\partial \eta} \quad : \quad 0 &= x \xi_H + (1 - x) \xi_L - x p_H \left[A_H + \pi d_H \left(\int_0^{\Lambda} \lambda g(\lambda) \, d\lambda \right) \right] \\
&- (1 - x) p_L \left[A_L + \pi d_L \left(\int_0^{\Lambda} \lambda g(\lambda) \, d\lambda \right) \right] - \mu - m_A
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \gamma} \quad : \quad 0 &= U \left(W_0 - \xi_H \right) + p_H \left[(1 - \pi) U \left(W + A_H \right) + \pi \int_0^{\Lambda} \lambda U' \left(W + A_H - (1 - d_H) \lambda \right) g(\lambda) \, d\lambda \right] \\
&- U \left(W_0 - \xi_L \right) - p_H \left[(1 - \pi) U \left(W + A_L \right) + \pi \int_0^{\Lambda} \lambda U' \left(W + A_L - (1 - d_L) \lambda \right) g(\lambda) \, d\lambda \right]
\end{aligned}
$$

First, let us find the optimal long-term insurance contract for the high risk. From $\frac{\partial}{\partial A_H}$ and $\frac{\partial}{\partial d_H}$, we find

$$
\begin{aligned}
\frac{\eta x}{\gamma} &= (1 - \pi) U' \left(W + A_H \right) + \pi \int_0^{\Lambda} U' \left(W + A_H - (1 - d_H) \lambda \right) g(\lambda) \, d\lambda \\
\frac{\eta x}{\gamma} &= \frac{1}{\left(\int_0^{\Lambda} \lambda g(\lambda) \, d\lambda \right)} \int_0^{\Lambda} \lambda U' \left(W + A_H - (1 - d_H) \lambda \right) g(\lambda) \, d\lambda
\end{aligned}
$$

so that

$$
\frac{\int_0^{\Lambda} \lambda U' \left(W + A_H - (1 - d_H) \lambda \right) g(\lambda) \, d\lambda}{\int_0^{\Lambda} \lambda g(\lambda) \, d\lambda} = (1 - \pi) U' \left(W + A_H \right) + \pi \int_0^{\Lambda} U' \left(W + A_H - (1 - d_H) \lambda \right) g(\lambda) \, d\lambda
$$

The solution to this is to have $d_H = 1$.

Let us now find the retirement contract for the high risk given that result that $d_H = 1$, we find

$$\frac{\eta x}{\gamma} = U'(W + A_H)$$

From $\frac{\partial}{\partial \xi_H}$,

$$\frac{\eta x}{\gamma} = U'(W_0 - \eta_H) = U'(W + A_H)$$

so that

$$A_H = W_0 - W - \eta_H$$

since the high risk agent will have an income of A_H in every state of the world. So we have perfect income smoothing.

The third aspect of the proof is to examine the long-term care insurance contract for the low risk. From $\frac{\partial}{\partial \xi_L}$,

$$U'(W_0 - \xi_L)(1 - \gamma) = \eta [1 - x]$$

Substituting for $\eta = U'(W_0 - \xi_L)\left(\frac{1-\gamma}{1-x}\right)$ in $\frac{\partial}{\partial A_L}$ and $\frac{\partial}{\partial d_L}$ gives, respectively,

$$\begin{aligned} 0 \;=\;& p_L \left[(1-\pi)U'(W + A_L) + \pi \int_0^{\Lambda} U'(W + A_L - (1-d_L)\lambda)\, g(\lambda)\, d\lambda\right] \\ & - U'(W_0 - \xi_L)(1-\gamma)p_L - \gamma p_H \left[(1-\pi)U'(W + A_L) + \pi \int_0^{\Lambda_i} U'(W + A_L - (1-d_L)\lambda)\, g(\lambda)\, d\lambda\right] \end{aligned}$$

and

$$\begin{aligned} 0 \;=\;& p_L \pi \int_0^{\Lambda} \lambda U'(W + A_L - (1-d_L)\lambda)\, g(\lambda)\, d\lambda \\ & - U'(W_0 - \xi_L)(1-\gamma)p_L \pi \left(\int_0^{\Lambda} \lambda g(\lambda)\, d\lambda\right) - \gamma p_H \pi \int_0^{\Lambda_i} \lambda U'(W + A_L - (1-d_L)\lambda)\, g(\lambda)\, d\lambda \end{aligned}$$

Isolating γ gives us

$$\gamma = \frac{p_L \left[(1-\pi)U'(W + A_L) + \pi \int_0^{\Lambda} U'(W + A_L - (1-d_L)\lambda)\, g(\lambda)\, d\lambda\right] - U'(W_0 - \xi_L)p_L}{p_H \left[(1-\pi)U'(W + A_L) + \pi \int_0^{\Lambda_i} U'(W + A_L - (1-d_L)\lambda)\, g(\lambda)\, d\lambda\right] - U'(W_0 - \xi_L)p_L}$$

and

$$\gamma = \frac{p_L \pi \int_0^{\Lambda} \lambda U'(W + A_L - (1-d_L)\lambda)\, g(\lambda)\, d\lambda - U'(W_0 - \xi_L)p_L \pi \left(\int_0^{\Lambda} \lambda g(\lambda)\, d\lambda\right)}{p_H \pi \int_0^{\Lambda_i} \lambda U'(W + A_L - (1-d_L)\lambda)\, g(\lambda)\, d\lambda - U'(W_0 - \xi_L)\gamma p_L \pi \left(\int_0^{\Lambda} \lambda g(\lambda)\, d\lambda\right)}$$

Putting the two together we have that $d_L = 1$ is the only solution. To see why, substitute for $d_L = 1$ in the two previous equalities yields

$$\gamma = \frac{p_L U'(W+A_L) - U'(W_0-\xi_L)\, p_L}{p_H U'(W+A_L) - U'(W_0-\xi_L)\, p_L}$$

and

$$\gamma = \frac{p_L \pi U'(W+A_L)\left(\int_0^\Lambda \lambda g(\lambda)\, d\lambda\right) - U'(W_0-\xi_L)\, p_L \pi \left(\int_0^\Lambda \lambda g(\lambda)\, d\lambda\right)}{p_H \pi U'(W+A_L)\left(\int_0^{\Lambda_i} \lambda g(\lambda)\, d\lambda\right) - U'(W_0-\xi_L)\, \gamma p_L \pi \left(\int_0^\Lambda \lambda g(\lambda)\, d\lambda\right)}$$

Simplifying the $\int_0^\Lambda \lambda g(\lambda)\, d\lambda$ completes the proof that the low risk agent chooses a long-term insurance contract that provides him with full insurance ($d_L = 1$) despite the presence of adverse selection.

The only part of the proposition that remains to be shown is that the low risk individual chooses an allocation in which his income is not perfectly equal in every state. To do so, substitute for $d_L = d_H = 1$ and that $A_H = W_0 - W - \xi_H$ in $\frac{\partial}{\partial \gamma}$ and find

$$(1+p_H)\, U(W-\xi_H) = U(W_0-\xi_L) + p_H U(W+A_L)$$

Clearly it cannot be that $A_L = W_0 - W - \xi_L$. This means that $A_L < W_0 - W - \xi_L$ is not the first best allocation.•

9.3 Choice of Long-term care insurance given previous choice of retirement vehicle

In this situation, agents get to choose their long-term care insurance product after they have already chosen their retirement vehicle (B^*, β^*). This means that agents are maximizing only with respect to the insurance product. The adverse selection problem can then be written as

$$\max_{\substack{d_L,\delta_L \\ d_H,\delta_H}} \Omega_L^{before} = U(W_0-\beta^*-\delta_L) + p_L\left[(1-\pi)\, U(W+B^*) + \pi \int_0^\Lambda U(W+B^*-(1-d_L)\lambda)\, g(\lambda)\, d\lambda\right]$$

subject to the zero profit conditions

$$\delta_L = p_L \pi \left(\int_0^\Lambda d_L \lambda g(\lambda)\, d\lambda\right) + \mu$$

$$\delta_H = p_H \pi \left(\int_0^\Lambda d_H \lambda g(\lambda)\, d\lambda\right) + \mu$$

and an incentive compatibility constraint that guarantees that the high risk individual is better off purchasing the contract that is designed for him than purchasing the contract designed for the low risk

$$\begin{aligned} 0 &< U(W_0-\beta^*-\delta_H) + p_H\left[(1-\pi)\, U(W+B^*) + \pi \int_0^\Lambda U(W+B^*-(1-d_H)\lambda)\, g(\lambda)\, d\lambda\right] \\ &\quad - U(W_0-\beta^*-\delta_L) - p_H\left[(1-\pi)\, U(W+B^*) + \pi \int_0^\Lambda U(W+B^*-(1-d_L)\lambda)\, g(\lambda)\, d\lambda\right]. \end{aligned}$$

By substituting the two relevant zero-profit constraints where appropriate, the problem in its Langrangian form is the following:

$$
\begin{aligned}
\max_{d_L,d_H,\gamma} \Omega_L^{before} &= U\left(W_0-\beta^*-\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)\\
&+p_L\left[(1-\pi)U(W+B^*)+\pi\int_0^\Lambda U(W+B^*-(1-d_L)\lambda)g(\lambda)\,d\lambda\right]\\
&+\gamma\left(\begin{array}{c}
U\left(W_0-\beta^*-\left(p_H\pi\left(\int_0^\Lambda d_H\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)\\
+p_H\left[(1-\pi)U(W+B^*)+\pi\int_0^\Lambda U(W+B^*-(1-d_H)\lambda)g(\lambda)\,d\lambda\right]\\
-U\left(W_0-\beta^*-\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)\\
-p_H\left[(1-\pi)U(W+B^*)+\pi\int_0^\Lambda U(W+B^*-(1-d_L)\lambda)g(\lambda)\,d\lambda\right]
\end{array}\right).
\end{aligned}
$$

The first order conditions are

$$
\begin{aligned}
\frac{\partial}{\partial d_L} &: 0=-U'\left(W_0-\beta^*-\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)p_L\pi\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right)\\
&+p_L\pi\int_0^\Lambda \lambda U'(W+B^*-(1-d_L)\lambda)g(\lambda)\,d\lambda\\
&+\gamma\left(\begin{array}{c}
U'\left(W_0-\beta^*-\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)p_L\pi\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right)\\
-p_H\pi\int_0^\Lambda \lambda U'(W+B^*-(1-d_L)\lambda)g(\lambda)\,d\lambda
\end{array}\right)\\
\frac{\partial}{\partial d_H} &: 0=\gamma\left(\begin{array}{c}
-p_H\pi\left(\int_0^\Lambda \lambda g(\lambda)\,d\lambda\right)U'\left(W_0-\beta^*-\left(p_H\pi\left(\int_0^\Lambda d_H\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)\\
+p_H\pi\int_0^\Lambda \lambda U'(W+B^*-(1-d_H)\lambda)g(\lambda)\,d\lambda
\end{array}\right)\\
\frac{\partial}{\partial\gamma} &: 0=\left(\begin{array}{c}
U\left(W_0-\beta^*-\left(p_H\pi\left(\int_0^\Lambda d_H\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)\\
+p_H\left[(1-\pi)U(W+B^*)+\pi\int_0^\Lambda U(W+B^*-(1-d_H)\lambda)g(\lambda)\,d\lambda\right]\\
-U\left(W_0-\beta^*-\left(p_L\pi\left(\int_0^\Lambda d_L\lambda g(\lambda)\,d\lambda\right)+\mu\right)\right)\\
-p_H\left[(1-\pi)U(W+B^*)+\pi\int_0^\Lambda U(W+B^*-(1-d_L)\lambda)g(\lambda)\,d\lambda\right]
\end{array}\right).
\end{aligned}
$$

These first order conditions will be anticipated by the agent at the time he chooses his optimal retirement contract before learning any information about his type. •

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