Downlink Erlang Capacity Estimation in CDMA Cellular Networks

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Abstract

This paper presents an analysis of the forward link capacity of a cellular network, based on IS-95 CDMA technology. The forward link, or downlink, refers to the base station-to-mobile direction. This study deals with power limitations at the base station, and aims at evaluating the probability that a power shortage occurs.

Keywords: CDMA, capacity, cellular network, forward link, power shortage.

Résumé

Ce rapport de recherche présente une analyse de la capacité sur le lien descendant dans un réseau cellulaire, basée sur la technologie IS-95 CDMA. Le lien descendant, encore appelé lien avant, se rapporte à la direction de la station vers le mobile. L'étude discute les limitations de la puissance à la station de base, et a pour but d'évaluer la probabilité qu'un manque de puissance survienne.

1 Introduction

The capacity of a cellular CDMA network has systematically been studied according to the reverse link limitations [1]-[2]-[3], mainly because it has been (and rightfully so) considered to be the critical link. The situation could drastically change with the introduction of wireless multiple (non voice) services, including web browsing, as this adds up to an asymetric traffic with the balance heavily tilted forwards downlink traffic. In turn this could result in cell capacity being limited by antenna power availability at the base station. We are aware of only one paper by Hiltunen and Bernardi [4] which is a contribution towards that question when two types of traffic are considered. Our paper deals with the simpler issue of downlink capacity estimation for a single (voice) type of traffic. However, unlike [4] which relies on simulations to compute the influence of the random spatial distribution of mobiles on downlink capacity, the latter is computed using appropriate probabilistic averaging. Also a more general study of the influence of neighboring cells on downlink capacity is carried out.

Note that each time a mobile is to generate a call, at least one base station has to create a link with it. This implies that the base station must allocate a certain amount of power to the user. The more users there are, the more power is needed by the base station. Base stations are often assumed to have infinite power or equivalently enough power to satisfy all incoming calls on the forward link, as long as the reverse link capacity limit is not reached. This is not always true in a real network. Indeed, for an unusually spread out cell traffic, the associated base station could lack power. This could then become the effective limiting factor on the overall capacity of the cell.

The forward link capacity is estimated here by calculating the probability that the power needed by a station exceeds a certain threshold. This threshold could be set at the total power available at the base station. In our study, we chose to consider a safety margin so as te reduce the probability that the antenna actually reaches the state where it has no power left.

Throughout our analysis, we assume that each base station sends to the mobiles it is actively linked with, the minimal amount of power sufficient to maintain the required signal to noise ratio (SNR=energy per bit/total interference power spectral density). This is tantamount to assuming perfect power control (power control imperfections, usually modeled as log-normal variables, are thus not considered).

The paper is organized as follows. In section 2, we consider the case of an isolated cell. In section 3, we consider the same estimation problem first in the case of two interacting cells and subsequently, of multiple interacting cells. A system interaction matrix is defined and while in principle, configurations involving widely varying traffic intensities from cell to cell could be treated, we carry out calculations only for cell traffics which are assumed to be perfectly identical at all times. Section 4 is our conclusion.

2 Capacity Estimation for an Isolated Cell

An isolated cell is considered here. The antenna of the base station is assumed to be isotropic. Under such a hypothesis, the analysis could be easily modified to apply to a sector (for a sectorized cell), rather than a non sectorized cell.Thus, while in the following, the word “cell” will be used, it could be replaced by the word “sector”, whenever one deals with a sectorized antenna.

2.1 Expression of The Total Requirements at Base Station

The theory is based on the Carrier-to-Interference ratio (CIR), which should be the same for any user. The CIR for one user i is given by

$$CIR = \gamma = \frac{R}{W}.SNR \tag{1}$$

$$= \frac{\textit{Power of signal received by user } i}{\textit{Power of interferences received by user } i}$$

$$= \frac{P_i/\alpha_{i0}}{(I_i)/\alpha_{i0} + \mathcal{N}_{therm}} \tag{2}$$

where

- P_{pilot} = power dedicated to the pilot channel.
- P_i = power allocated to mobile i,
- P_T = total base station power emission requirement,
- I_i = total interfering power for mobile i,
  P_{pilot}, P_i, P_T, *represent the different powers as emitted at the base station antenna.*
- α_{i0} = path loss between base station and mobile i,
- γ = CIR required for every user,
- W/R = processing gain, (R = bit rate, W = bandwidth),
- SNR = Signal-to-Noise Ratio,
- $\mathcal{N}_{therm}$ = thermal noise.

The pilot channel is a signal constantly sent by the base station. This signal is used by mobiles whenever they wish to make a call. By tracking pilot channels, a user can locate the nearest stations and attempt to create a link with one of them.

For i, signals sent by mobiles other than i do not create interferences on the forward link, because frequencies on the uplink and downlink directions are non overlapping. Furthermore, under ideal conditions there should exist no interferences between signals sent by the base station to different users in the same cell. This is because at the base station antenna, signals sent on the forward link are perfectly orthogonal to each other [12]. However, propagation through space distorts the signals (reflections, diffractions,. . .) so that mutual orthogonality of the latter no longer holds when signals reach a given mobile in the cell. We shall consider a worst case analysis whereby all of the base station power allocated to mobile j is considered to be interfering with the signal sent the mobile i, if $i \neq j$.

Usually, propagation loss α_i (also called path loss) is modelled as a fourth power of the distance r_i, multiplied by a log-normal random variable (representing the shadowing effects): $\alpha_{i0} = r_i^4 \cdot 10^{\zeta/10}$ where ζ is a Gaussian random variable. Within our modeling framework, shadowing effects are neglected. This is because despite the reality of imperfect power control, in the downlink direction, the major source of power variability is mobile random spatial spread.

In this case, the power allocated for user i can be expressed as

$$P_i = \frac{\gamma}{1+\gamma}(P_T + \alpha_{i0}\mathcal{N}_{therm}) \tag{3}$$

Considering a lossless base station antenna:

$$P_T = P_{pilot} + \sum_i P_i \tag{4}$$

P_T can then be expressed as follows:

$$P_T = \frac{1}{1 - N\frac{\gamma}{1+\gamma}} \left(P_{pilot} + \frac{\gamma}{1+\gamma}\mathcal{N}_{therm} \sum_{i=1}^{N} \alpha_{i0} \right) \tag{5}$$

where in(5), N is the current numbers of cell users.

2.2 Maximal Number of Users

In expression (5), the total power emitted can become negative, whenever N becomes too high. This of course, has no physical meaning in a real system. It only indicates that the system cannot sustain more than a certain number of users at the same time if it is to provide the required SNR for each user. Let us denote by N_{max} the maximal number of allowable users connected to the base station at any one time. From 10, N_{max} is given by

$$N_{max} = \left\lfloor \frac{1+\gamma}{\gamma} \right\rfloor \tag{6}$$

When the number of users approaches this value, the system approaches instability, and the powers allotted to users can increase without limit. Eventually, several calls will be dropped before the system returns to a stable state. N_{max} can be viewed as a "pole capacity", a notion developed in [4].

2.3 Voice Activity

One of the main features of IS-95 CDMA is that less power is emitted when there is no communication going on during a call. Thus whenever the user is not talking, it is not necessary for the mobile to send as much power as when it is. Likewise for the base station. From that point of view, the state of the downlink signal will be called downlink signal activity. It will be accounted for in power calculations by modelling signalling state as a binary random process; more specifically, it will be modeled as a Bernoulli random variable with parameter ν, between 0.4 and 0.5. This exactly corresponds to the usual uplink voice activity model which should be symmetric.

If the downlink signal activity random variable is included in the analysis, equations are affected as follows:

- Total power emitted P_T is now given by

$$P_T = P_{pilot} + \sum_{i}^{N} \nu_i P_i \tag{7}$$

  where ν_i is the downkink signal dedicated to user i activity state.

- The total power of the interferences perceived by the mobile i is

$$I_i = P_T - \sum_{j=1,j\neq i}^{N} \nu_j P_j \tag{8}$$

The revised expression for the power allotted to each user i reads as follows:

$$P_i = \frac{\gamma}{1+\nu_i\gamma}(P_T + \alpha_{i0}\mathcal{N}_{therm}) \tag{9}$$

Finally:

$$P_T = \frac{1}{1-\sum_{i=1}^{N}\frac{\nu_i\gamma}{1+\nu_i\gamma}}\left(P_{pilot} + \mathcal{N}_{therm}\sum_{i=1}^{N}\frac{\nu_i\gamma}{1+\nu_i\gamma}\alpha_{i0}\right) \tag{10}$$

If there are only k users listening out of N users connected at a certain time (ν_i equals 1 for k users, and 0 for the others), the total power emitted is given by

$$P_T = \frac{1}{1-k\sum_{i=1}^{k}\frac{\gamma}{1+\gamma}}\left(P_{pilot} + \mathcal{N}_{therm}\sum_{i=1}^{k}\frac{\gamma}{1+\gamma}\alpha_{i0}\right) \tag{11}$$

which is the same as (5), with k now replacing N.

2.4 Computation of Base Station Antenna Power Shortage Probabilities

Base station power shortage occurs whenever the total power emitted by the base station exceeds a predefined threshold P_{th}. In order to estimate the probability that P_T exceeds this threshold, expression (10) containing three random variables of which two, N,$\nu_i$are time dependent and the third, α_{i0}, is spatially dependent has to be evaluated. Recall that N = number of users engaged in a call, ν_i = downlink signal activity random process for user i, and α_{i0} = propagation loss between the base station and mobile i.

The above three variables can be assumed to be independent. The following nested conditioning of base station antenna power shortage probability ca be written:

$$\begin{aligned} Prob\,[P_T > P_{th}] \;\; &= \textstyle\sum_{N=1}^{N_{max}} \sum_{k=1}^{N} Prob\,[P_T > P_{th}|k, N] \\ &\qquad .Prob\,[\text{k active users}|N] \\ &\qquad .Prob\,[\text{N users connected}] \end{aligned} \tag{12}$$

where $Prob[A|B]$ refers to the probability of event if conditioned on event B.

2.4.1 Expression for $Prob\,[$k active users$|N]$

Given that ν_i is a Bernoulli random variable with parameter ν, it is possible to write:

$$Prob\,[\text{k active users}|N] = \sum_{k=0}^{N} C_k^N \nu^k (1-\nu)^{N-k} \tag{13}$$

where in 2.4.1 C_k^N indicates the number of distincts element combinations chosen among N elements.

2.4.2 Expression for $Prob\,[$N users connected$]$

Users are modelled by a M/M/m queueing model system with no queueing space. The maximum number of servers m is given by the pole capacity found earlier (6). Therefore, for $N < N_{max}$,

$$Prob[N] = \frac{\rho^N/N!}{\sum_{j=0}^{N_{max}} \rho^j/j!} \tag{14}$$

where $\rho = \frac{\lambda}{\mu}$ represents the trafic in Erlang, in the cell. λ is the user arrival rate, μ is the inverse of the mean conversation length.

2.4.3 Expression for $Prob\,[P_T > P_{th}|k, N]$

This probability is equivalent to the probability that $\sum_{i=1}^{N} \alpha_{i0}$ exceeds a certain threshold. Conditional on N users being connected with k users active, using (11) it is possible to write:

$$Prob[P_T > P_{th}|k, N] = Prob[\sum_{i=1}^{k} \alpha_{i0} > \Omega(k)|k, N] \tag{15}$$

where

$$\Omega(k) = \frac{1+\gamma}{\gamma \mathcal{N}_{therm}} \left[P_{th}(1 - \frac{k\gamma}{1+\gamma}) - P_{pilot} \right] \tag{16}$$

Given that α_{i0} depends on the distance between the base station and the user, assumptions as to the spatial distribution of users within the cell must be made. In this study, two user spatial distributions have been considered: A uniform distribution along cell radius, and a uniform distribution on the surface of the cell.

The central limit theorem is used to approximate the distribution $\sum \alpha_{i0}$ via a Gaussian. Thus, only the mean and variance of α_{i0} are required so as to express the different probabilities of interest. Up to a normalizing constant corresponding to attenuation for a unit distance, the results for non sectorized cells are as follows:

- for a uniform distribution along radius,

$$\begin{array}{rcccl} mean(\alpha_{i0}) & = & & & \frac{1}{5}R^4 \\ Var(\alpha_{i0}) & = & mean\left(\alpha_{i0}^2\right) - mean\left(\alpha_{i0}\right) & = & \frac{16}{225}R^8 \end{array} \tag{17}$$

- for a uniform distribution over all the area,

$$\begin{array}{rcccl} mean(\alpha_{i0}) & = & & & \frac{1}{3}R^4 \\ Var(\alpha_{i0}) & = & mean\left(\alpha_{i0}^2\right) - mean\left(\alpha_{i0}\right) & = & \frac{4}{45}R^8 \end{array} \tag{18}$$

2.5 Results for an Isolated Base Station

Power Shortage Probability calculations have been carried out based on the following data: P_{pilot} has been set to 2W for a 10W total power available at the base station (20% of the total power available), γ=2.45E-2 (for a standard required SNR of 7dB), $\mathcal{N}_{therm}$=3.14E-11 (corresponding to a noise density of -166dBm/Hz), ν=0.4.

The results for each distribution considered are given in Fig 1 and 2. According to Fig 1, for a 1% blocking rate, a 1 km cell could meet a traffic of 55 Erl. As one could expect because of higher power requirements with distance, capacity vanishes as cell size grows. For a 10 km cell, only a few Erlangs of traffic can be sustained if the blocking rate has to be lower than 1%. However, designing a cell of that size would have to mean that there is not a large amount of traffic within the region so that a single base station covering all the area would have to be sufficient.

For a distribution of users uniform on cell surface, it is expected that capacity should be less than for a distribution uniform over radius. This is because for an identical number of active users, the former will imply a higher mean distance from base station and therefore higher path loss and power requirements.

Such behavior is confirmed in the computations summarized in Fig. of the influence of the two spatial distributions 2 where a comparison is carried out. While there are no significant differences for small cells (1 km radius), for a 4 km radius cell, the loss in capacity for uniform over cell surface mobile distribution becomes more visible. The difference between the two distributions is about 4 Erl.

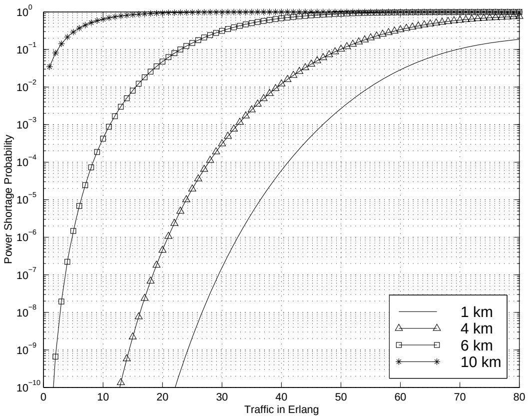

Figure 1: Base station antenna power shortage probability for an isolated cell - users are uniformly distributed along cell radius - Cell radius is specified in white box.

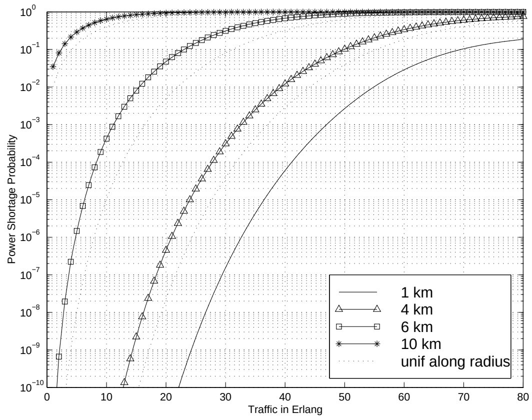

Figure 2: Base station antenna power shortage probability for an isolated cell - Full lines are associated with a mobile distribution uniform over cell area. Dotted lines are associated with a mobile distribution uniform over cell radius - Cell radius is specified in white box.

3 Multiple Interacting Cells

3.1 Interaction between two cells

When there several neighboring cells in a network, signals sent by one tend to create interferences for the other cells. They are called intercell interferences. In order to study how the power sent by one cell affects another one, let us first consider two cells of a network respectively called the main cell and the interfering cell.

Let P_{T_0} and P_{T_1} respectively denote the total power emitted by the main base station antenna and the interfering base station antenna. User i must receive from the main cell sufficient power level to sustain the required CIR. More specifically:

$$CIR = \frac{P_i/\alpha_{i0}}{(P_{T_0} - P_i)/\alpha_{i0} + P_{T_1}/\alpha_{i1} + \mathcal{N}_{therm}} = \gamma \tag{19}$$

where α_{i0} and α_{i1} represent propagation loss between the mobile and, respectively, the base station of the main cell and that of the interfering cell.

Solving for P_i in (19), and as in (11) yields:

$$P_T = P_{T_0}(K_0) + \Delta(K_0)\, P_{T_1} \tag{20}$$

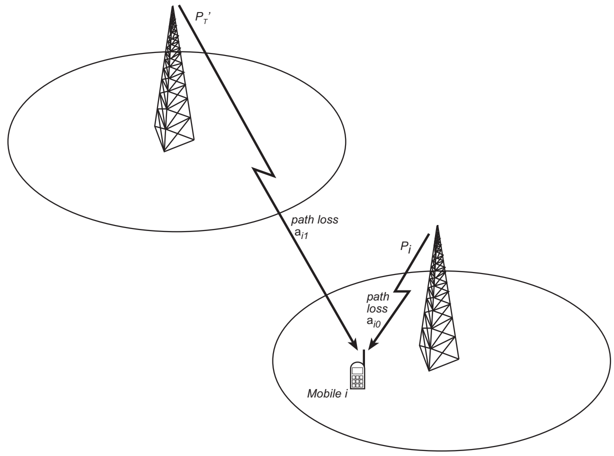

Figure 3: Interaction between two cells

where in (20), K_0 is the number of active users in the main cell, and:

$$P_{T_0}(K_0) \quad = \quad \frac{1}{1 - \frac{K_0 \gamma}{1+\gamma}} \left(P_{pilot} + \frac{\gamma}{1+\gamma} \mathcal{N}_{therm} \sum_{i=1}^{K_0} \alpha_{i0} \right) \tag{21}$$

$$\Delta_{01}(K_0) \quad = \quad \frac{\gamma}{1 - (K_0 - 1)\gamma} \sum_{i=1}^{K_0} \frac{\alpha_{i0}}{\alpha_{i1}} \tag{22}$$

The expression for the power emitted by the main cell can be divided into two components as can be seen from eq. (20).

- one component, $P_{T_0}(K_0)$, corresponds to the isolated cell expression of eq. (5). Equivalently, it is the self interference power contribution.
- a second component, $\Delta_{01}(K_0).P_{T_1}$, represents the extra power requirement caused by external cell interference. Notice that it is linear in the external cell power with a coefficient of influence $\Delta(K_0)$

3.2 Interaction Matrix

In general, however there will be more than just two interacting cells, so that a system of equations, linear in the unknown total power P_{T_j}, $j = 1,\ldots,M$, where M is the number of interacting cells:

$$[\Delta] \, . \, [P_T] = \left[P_T^S \right] \tag{23}$$

where $K_1,\ldots,K_M$ respectively represent the number of listening users in cells $i,\ldots,M$, and:

$$[\Delta] = \begin{bmatrix} 1 & -\Delta_{01}(K_0) & \cdots & -\Delta_{0M}(K_0) \\ -\Delta_{10}(K_1) & 1 & \cdots & -\Delta_{1M}(K_1) \\ \vdots & & \ddots & \vdots \\ -\Delta_{M0}(K_M) & \cdots & \cdots & 1 \end{bmatrix} \tag{24}$$

$$[P_T] = \begin{bmatrix} P_{T_0} \\ P_{T_1} \\ \vdots \\ \vdots \\ P_{T_M} \end{bmatrix} \tag{25}$$

$$[P_T^S] = \begin{bmatrix} P_{T_0}^S \\ P_{T_1}^S \\ \vdots \\ \vdots \\ P_{T_M}^S \end{bmatrix} \tag{26}$$

Each $\Delta_{ij}(K_i)$, $i \neq j$, represents the coefficient of influence of external base station power in cell j, on the base station power in cell i. The $[\Delta]$ matrix will be called the interaction matrix. It is a random matrix over cell space and time. It is often admitted that only the first three rings of surrounding cells have a significant influence on the main cell. Thus,while the above system of equations must theoretically be of infinite dimension, as far main cell calculations are concerned, one needs only to consider a matrix corresponding to 19 cells (Figure 4).

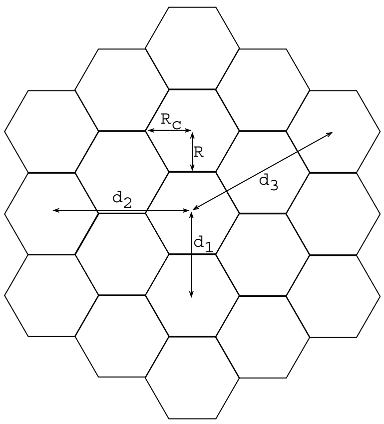

Figure 4: First three rings of surrounding cells

3.3 Base station antenna power shortage probability estimation

In order to develop estimates of antenna power shortage probabilities in the multi cell case, we shall proceed following steps (i) and (ii):

- step(i): develop the estimates conditional on knowledge of individual number of users in the main cell as well as other relevant surrounding cells, specifically N_0, N_1,..., N_M.
- step(ii): using an appropriately defined joint number of users in cells distribution, use the results in step(i) to compute the unconditional power shortage probability estimates.

Furthermore, and in order to keep computations tractable, we shall make the following (important) simplification: "Conditional on the number of users in the main cell as well as relevant surrounding cells, $\Delta_{ij}(K_i)$, $j = 0,\ldots,M$ will be treated as constants, essentially identified with their conditional means $E\left[\Delta_{ij}(K_i)|K_i\right]$". Note that this simplification is of the same order as that utilized in evaluating extra cell interference in the uplink direction.

Now, denoting by $P^S_{T_c}$, P_{T_c} respectively the multi-cell and isolated cell base station antenna random power vectors conditional on the number of users, it is possible to write from (23):

$$\left[P_{T_c}\right] = \left[\overline{\Delta_c}\right]^{-1} . \left[P^S_{T_c}\right] \tag{27}$$

where in (27), $\left[\overline{\Delta_c}\right]^{-1} = \left[E\left[\Delta_{ij}(K_i)|K_i\right]\right]$. Note that through the rest of the paper $\overline{a}$ will denote the mean of some quantity a.

Under the assumption of independence of spatial distributions of users from cell to cell, and based on the single isolated cell analysis of Section 2, $P^S_{T_c}$, and thus P_{T_c} can be considered to be Gaussian vectors, probabilistically characterized by their mean and covariance matrix. in particular:

$$\begin{aligned} E\left[P_{T_c}\right] &= \left[\overline{\Delta_c}\right]^{-1} .E\left[P^S_{T_c}\right] && (28)\\ Cov\left[P_{T_c}\right] &= \left[\overline{\Delta_c}\right]^{-1} .Cov\left[P^S_{T_c}\right] . \left[\overline{\Delta'_c}\right]^{-1} && (29)\end{aligned}$$

where in (29), $\overline{\Delta'_c}$ is the transpose of matrix $\overline{\Delta_c}$:

$$\begin{aligned} Cov\left[P_{T_c}\right] &= E\left[(P_{T_c} - E\left[P_{T_c}\right]).(P_{T_c} - E\left[P_{T_c}\right])'\right]\\ Cov\left[P^S_{T_c}\right] &= E\left[(P^S_{T_c} - E\left[P^S_{T_c}\right]).(P^S_{T_c} - E\left[P^S_{T_c}\right])'\right]\end{aligned}$$

Based on the analysis in Section 2, it is possible to write:

$$E\left[P_{T_{ci}}\right] = \frac{P_{pilot} + \mathcal{N}_{therm}\frac{\gamma}{1+\gamma}K_i\overline{\alpha_{j0}}}{1 - K_i^2\frac{\gamma}{1+\gamma}} \tag{30}$$

where i=0, 1,..., M and j=1,..., K_i

Furthermore, under the assumptions of independence of mobile cell spatial distribution from cell to cell, it is possible to write:

$$Cov\left[P^S_{T_c}\right] = Diag\left[Var\left(P^S_{T_ci}\right)\right] \tag{31}$$

where

$$Var\left(P^S_{T_ci}\right) = \frac{\mathcal{N}^2_{therm}\left(\frac{\gamma}{1+\gamma}\right)^2 K_i^2\overline{\alpha_{j0}}}{\left(1 - K_i^2\frac{\gamma}{1+\gamma}\right)^2}.Var\left(\alpha_{j0}\right) \tag{32}$$

where i=0, 1,..., M and j=1,..., K_i

It remains to compute matrix $\overline{\Delta_c}$ in equation (29). Recall that:

$$E\left[\Delta_{ij}(K_i)|K_i\right] = \frac{\gamma}{1-(K_i-1)\gamma}K_i\overline{\frac{\alpha_{li}}{\alpha_{lj}}} \tag{33}$$

where i, j=0,..., M and l=1,..., K_i, and the averaging in (33) has to be carried out over all mobiles within cell i. Averaging calculations will be detailed in subsection D. Notice that conditional random vector P_{T_c} is Gaussian, and based on (28), (29), (30) and (31), the following conditional probabilities can be evaluated:

$$Pr\left[P_{T_i} > P_{th}|K_0, K_1, \ldots, K_M\right] \tag{34}$$

From that knowledge, and the knowledge of the joint probability distribution function of the number of users in cells i=0, 1,..., M, one can obtain the unconditional probability of antenna power exceeding the allowable threshold P_{th} for a given traffic level in Erlangs. Let $P(N_0, N_1, \ldots, N_M)$ be that joint distribution function. In general, one can write:

$$\begin{aligned} Pr\left[P_{T_0} > P_{th}\right] = \sum_{N_0=0}^{N_0=N_{max}} \cdots \sum_{N_M=0}^{N_M=N_{max}} P(N_0, N_1, \ldots, N_M). \\ \sum_{K_0=0}^{K_0=N_0} \cdots \sum_{K_M=0}^{K_M=N_M}\left[\prod_{i=1}^{M} C^{N_i}_{K_i}\nu^{K_i}(1-\nu)^{N_i-K_i}\right]. \\ Pr\left[P_{T_0} > P_{th}|K_0, K_1, \ldots, K_M\right] \end{aligned} \tag{35}$$

Note that in (35), we have considered that given N_0, N_1,...,N_M, the numbers of listening users in each cell are independent random variables. On the other hand, if for simplicity we assume that at all times, the number of total and listening users in each cell are respectively equal, using (14), one can write:

$$\begin{aligned} Pr\left[P_{T_0} > P_{th}\right] = \sum_{N=0}^{N_{max}} \frac{\rho^N/N!}{\sum_{j=0}^{N_{max}}\rho^j/j!}. \\ \left(\sum_{K=0}^{N} C^N_K\nu^K(1-\nu)^K.\right. \\ \left. Pr\left[P_{T_0} > P_{th}|K_0 = \cdots = K_M = K\right]\ \right) \end{aligned} \tag{36}$$

Furthermore, under such conditions of perfect symmetry, eq. (23) yields:

$$P_{T_{0c}}\left(1-\sum_{j=1}^{M}E\left[\Delta_{0j}(K)|K\right]\right)=P_{T_{0c}}^{S} \tag{37}$$

where in turn, using (32) and (33) yields:

$$E\left[P_{T_{0c}}\right] = \frac{P_{pilot}+\mathcal{N}_{therm}\frac{\gamma}{1+\gamma}K\overline{\alpha_{l0}}}{(1-K^2\frac{\gamma}{1+\gamma})(1-\sum_{j=1}^{M}E[\Delta_{0j}(K)|K])} \tag{38}$$

$$Var\left[P_{T_{0c}}\right] = \left[\frac{\mathcal{N}_{therm}\frac{\gamma}{1+\gamma}K\overline{\alpha_{l0}}}{(1-K^2\frac{\gamma}{1+\gamma})(1-\sum_{j=1}^{M}E[\Delta_{0j}(K)|K])}\right]^2 Var\left(\alpha_{l0}\right) \tag{39}$$

Note that (38) and (39) greatly simplify the computation of $Pr[P_{T_0} > P_{th}|K_0 = \ldots = K_M = K]$ in eq. (36).

3.4 Averaging Calculations

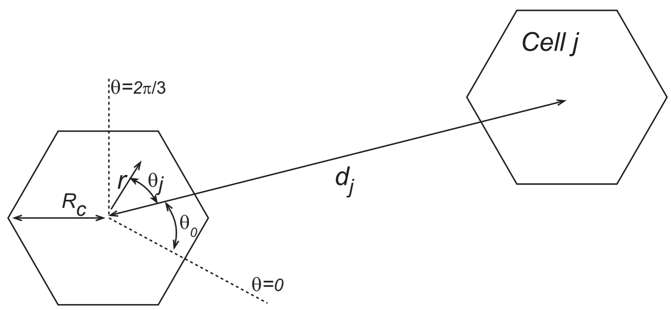

Figure 5: Notations

In the following, we discuss the calculation of $\frac{\overline{\alpha_{li}}}{\alpha_{lj}}$, l=1,…, K_i, i=0,…, M, j=0,…, N, $j \neq i$.

Integration of a ratio of two path losses has to be carried out; one path loss corresponds to the distance between the base station antenna in the main cell and the mobile, and the other one represents the distance between the base station antenna of a surrounding cell and the mobile. For notations, see Figure 5; $p(r,\theta)$ is the spatial probability density function of mobiles within a cell.

$$\frac{\overline{\alpha_{li}}}{\alpha_{lj}}\int_0^{2\pi}\int_{=0}^{R_c}\frac{r^4}{r^2+d_i^2-2rd_i\cos(\theta_j)}p(r,\theta)drd\theta \tag{40}$$

Equation (40) corresponds to a double integral in the r and θ variables. Note that $\theta_j = \theta - \theta_0$. We shall consider a maximal external interference situation for which $\theta_0 = \pi/3$. It is possible to show that:

- for a radially uniform mobile spatial distriution with A,B,Z defined below:

$$\begin{aligned} \int_0^{R_c} & \frac{r^4}{r^2 + d_i^2 - 2rd_i cos\theta_j} p(r,\theta)dr \\ = & \frac{2}{R_c}\mathcal{R}e\left[R_c - A\left(\frac{1}{R-z} + \frac{1}{z}\right) + Bln(\frac{z-R_c}{z})\right] \end{aligned} \tag{41}$$

- for a spatially uniform mobile spatial with A,B,Z defined below:

$$\begin{aligned} \int_0^{R_c} & \frac{r^4}{r^2 + d_i^2 - 2rd_i cos\theta_j} p(r,\theta)dr \\ = & \frac{2}{R_c}\mathcal{R}e\left[\frac{R_c^2}{2} + B.R_c - A.z\left(\frac{1}{R-z} + \frac{1}{z}\right)\right. \\ & \left. +(A + B.z)ln(\frac{z-R_c}{z})\right] \end{aligned} \tag{42}$$

where:

$$\begin{aligned} z &= d_i e^{j\theta} \\ A &= \frac{z^4}{z - z^*)^2} \\ B &= \frac{2z^4 - 4z^3z^*}{(z-z^*)^3} \end{aligned}$$

Following (41) and (42), the integration with respect to θ has to be carried out numerically.

3.5 Numerical Results

In order to limit required calculations, we have relied on expression (36) based on an identical number of users in every cell at all times. However the theory developed in subsection 3.3 can be used under more general hypotheses. In particular each cell could have its own number of users varying independently from the others.

The probability of base station antenna power exceeding threshold power P_{th} is plotted in Fig. 6 and 7.

A blocking probability of 1% is still considered. In Fig. 6 The capacity of a 1 km cell is about 38 Erl, thus indicating a capacity loss of 16 Erl as compared to the results obtained earlier for an isolated cell of identical radius. The loss is due to the interferences introduced by surrounding cells. Indeed, the inter-cell interferences increase the overall power that needs to be emitted by the base station and thus increase the risk for the base station to be in power shortage.

For the second distribution (uniform in space), the same remarks can be made. Capacity has fallen from 55 Erl for a single cell to 30 Erl, for a multiple cell configuration. This time, there is a difference in capacity between the two distributions even for small cells (about 8 Erl for 1 km cells), whereas this was not the case with isolated cells. This can be explained by the fact that users at cell borders tend to be more sensitive to interferences coming from surrounding cells, because they are closer to the surrounding base stations. Since such users tend to be more numerous in the case of a spatially uniform mobile distribution as compared to a radially uniform mobile distribution, the former exhibits a lower Erlang capacity.

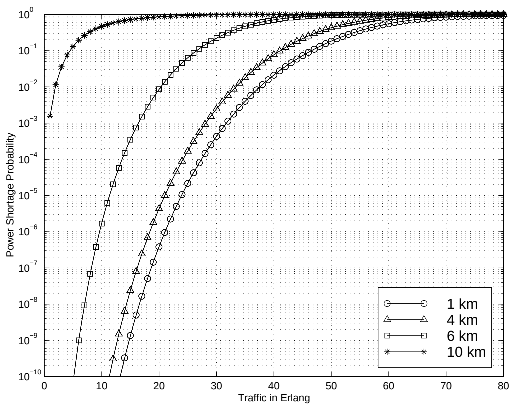

Figure 6: Base station antenna power shortage probability when accounting for extra cell interference and under a radially uniform mobile distribution

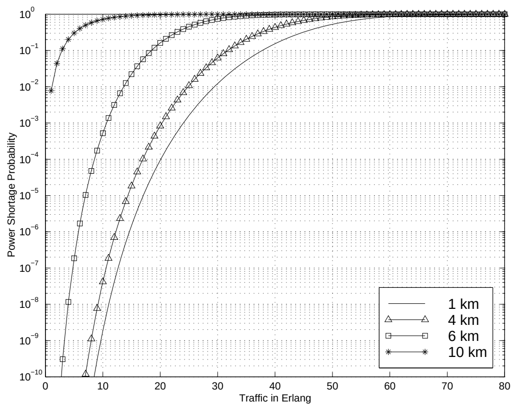

Figure 7: Base station antenna power shortage probability when accounting for extra cell interference and under a spatially uniform mobile distribution

4 Conclusion

A new method to evaluate downlink capacity in CDMA cellular networks has been developed. The method can account for various mobile spatial distributions. Downlink capacity is expected to become an increasingly critical factor with upcoming third generation mobile phone systems, which allow applications such as web-surfing or e-mail. The new services mainly affect downlink cell capacity, because the new kind of traffic introduced flows mostly from the base station towards the mobiles. Indeed, when surfing or reading e-mail, little data is sent by the user (web-page requests,. . .). On the other hand, downloading web-pages, audio or video files significantly increases traffic on the forward link, and therefore, more base station antenna power is used.

The theory presented here can be extended to a multi-service context, by requiring different signal-to-noise ratios for each kind of applicatiob (voice data, internet surfing, video data) and accounting for different bit rates to each. Such an extension will be the object of future work.

In addition, several improvements can be made to the current theory. In particular, one could account for the shadowing effects in the formula for the propagation loss. Also, some significant approximations had to be made on the statistical variables to simplify the calculation of different probabilities. If possible, carrying out the calculations without considering any simplifications should lead to more accurate results. Finally, the interaction matrix has not been evaluated in the case of sectorized cells.

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