

**An optimization model for electricity usage
in smart homes**

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An optimization model for electricity usage in smart homes

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Abstract: Smart homes have the potential to achieve optimal energy consumption with appropriate scheduling. It is expected that 35% of households in North America and 20% in Europe will be smart homes by 2020. However, current optimization models have limitations on the number of appliances considered and/or their reliability. This paper presents a new appliance-oriented integrated linear optimization model that finds an optimal trade-off between cost and comfort. We consider the use of energy in equipment, appliances, and electric vehicles and take into account renewable local generation, batteries, and demand-response. We use selected models from the literature and analyze them in detail. Our model can be used to find an optimal consumption pattern and the corresponding trade-off between cost and comfort. We present computational results to validate the model and indicate how it overcomes the limitations of previous models.

Keywords: Smart home, energy management system, power demand, residential loads, multi-class appliances

1 Introduction and related work

The European Technology Platform Smart Grid in 2006 defined a smart grid to be “an electricity network integrating users, consumers, and generators in order to produce and deliver economic, secure and sustainable electricity supplies.” Smart grids are used worldwide [1, 2], and many distribution companies use demand-response pricing mechanisms in the residential sector. The number of smart homes has increased considerably in recent years. In North America the number of smart homes is expected to reach 46.2M by 2020 [3], corresponding to 35% of households. In Europe, 44.9M are expected by 2020, corresponding to 20% of households. Governments are supportive because smart homes allow investment in the grid infrastructure to be postponed. Moreover, if the local generation comes from renewables, the environmental impact of coal/oil-based generation is reduced.

Smart homes require smart meters and an energy management system (EMS), so that they can avoid peak-period consumption and obtain financial incentives from demand-response and by injecting electricity into the grid. See [4] and [5] for surveys of modeling approaches and [6] and [7] for demand-response.

We assume that householders are interested in managing their electricity bills while ensuring a certain level of comfort. This is a scheduling problem. Existing models for the scheduling of appliances [8, 9] often do not account for the operational and energy-consumption characteristics of each device. Roh & Lee [9] developed detailed models for five classes of appliances, but they do not consider thermal machines, multiple trips for electric vehicles (EV), or combined heat and power (CHP).

Detailed EMS models give more accurate information and can better predict consumption, which is important for the evaluation of expansion strategies in the medium- and long-term [47]. Table 1 summarizes the EMS models in the literature: columns 2 to 16 indicate the features included in each model. The column *IBR constraints* indicates that the model uses inclining blocking rates (IBR), a pricing scheme where the prices increase for each incremental block of consumption. The column *Fraction of step* indicates that the appliances can be used for brief cycles. This occurs when, for example, a model uses a fixed interval of 10 min but allows the A/C to operate in cycles of 2.5 min. The column *Comfort* indicates the model used for the comfort function, and *Pricing* indicates the pricing policy, where TOU is time-of-use and RTP is real-time pricing. Finally, the column *Objective* indicates the optimization objective(s).

Yu et al. [10] use a two-phase approach to find the consumption of appliances. In the first phase they use hourly time intervals, and the results are used to initiate the second phase, which has a more precise time interval and thus gives more accurate results. They also consider the home layout, and they are the only authors in our survey to include a daily budget constraint. Zhuang et al. [23] use IBR pricing with blocks of one hour. Hourly IBR pricing leads to fewer coupling constraints, but in some markets the IBR blocks are days or even months. Arabali et al. [24] include an explicit formula for forecasting the photovoltaic (PV) production. They use the clearness index approach [48, Section 2.9] to calculate the solar radiation.

Table 1 shows that no papers use a detailed model for every appliance. Even the most comprehensive models [34, 35, 44, 45, 46] have limitations. Finding realistic optimal approaches to smart home scheduling requires accurate EMS models. We propose an optimization model that incorporates detailed submodels to optimize the trade-off between cost and comfort. We consider thermal appliances, EVs, an energy storage system (ESS), and CHP. We also use production and load profiles for the appliances and electronics. Our contributions are as follows:

1. We validate existing models for appliances.
2. We present a new model that optimizes electricity sales and purchases and off-grid local generation.
3. We propose an EV model that includes hybrid vehicles.

The rest of this paper is organized as follows. In Section 2, we introduce the models for the appliances, IBR pricing, thermal machines, and other components. Section 3 describes our instances, and Section 4 presents the results. Section 5 provides concluding remarks.

Table 1: Summary of features of EMS models.

Reference	Wind Generation	PV	Solar Collector	Energy Storage System	Water Heater	Shower	Refrigerator/Fridge	HVAC	Phase Machines	Noninterruptible Machines	EV	CHP	Lighting	IBR Constraints	Fraction of Step	Comfort	Pricing	Objective
Our	x	x	x	x	x	x	x	x	x	x	x	x		x	x	L	TOU	↓ D vs C
[10]								x							x	Q	NC	↓ TD
[11]													x			L		↓ IL
[12]				x					x	x							NC	↓ B
[13]				x					x	x						Q	NC	↓ C vs D vs B
[14]								x	x	x						Q	DA RTP	↓ D vs C
[15]										x							DA RTP	↓ C
[15]									x								DA RTP	↓ C
[16]					x											L	DA TOU	↓ B
[17]				x						x							NC	↓ C
[18]								x								L	DA NC	↓ B vs D
[19]									x							Q	DA NC	↓ D vs C
[20]									x	x						Q	NC	↓ D vs C
[21]									x	x						CF	NC RTP	↑ -D vs C
[22]				x				x	x	x		x				L	DA NC	↓ C
[23]									x	x				x		L	NC	↓ C vs WT
[24]	x	x		x												L		↓ C
[25]					x			x								L	NC	↑ -D vs C
[26]					x			x								L	NC	↑ -D vs C
[27]									x	x				x		L	RTP	↓ C vs WT
[28]									x	x							RTP, TOU	↓ C
[29]				x						x	x					L	TOU	↓ C / vs D
[30]				x										x			NC	↓ C
[8]									x	x						NL	TOU	↓ B
[31]										x							DA TOU	↓ C
[32]									x	x						L	NC	↓ C / vs PL
[33]				x					x	x							DA RTP	↓ C
[34]	x	x		x	x			x	x	x		x				NL	RTP	many
[35]				x	x			x	x	x		x				NL	RTP	many
[36]				x				x	x	x						L	NC	↓ C vs D
[37]					x			x	x	x	x						RTP	↓ B
[38]				x	x			x		x	x						RTP	↓ C
[39]					x					x							NC	↓ C
[40]									x	x						L	TOU	↓ C / vs PL
[41]			x		x			x	x	x	x					L	RTP	↓ C
[42]					x		x	x									TOU	↓ C
[43]								x	x	x						L	NC	↓ C
[9]					x		x	x		x						NL	TOU	↑ -D vs C
[44]	x	x		x	x		x	x		x		x	x			L	RTP	↓ C
[45]	x	x		x	x		x	x		x		x	x			L	RTP	↓ C / vs PL
[46]				x	x		x	x		x	x	x					TOU	↓ C

Legend: ↓: Minimize ↑: Maximize IL: Illumination level D: Discomfort -D: Comfort C: Cost L: Linear Q: Quadratic
 NC: Not constant TD: Thermal discomfort B: Electricity bill BL: Battery loss DA: Day ahead
 CF: concave function WT: Waiting time NL: Nonlinear PL: Peak load

2 Mathematical models

In this section, we describe the models for the appliances, EVs, and prices. The Appendix presents the variables, sets, and parameters as well as the full optimization model. Note that we consider only power consumption.

Objective Function: We minimize a weighted sum of comfort and cost (8), where the cost is given by (9) and the EV cost. The cost at each time t is given by the energy acquisition cost for block i minus the energy sold plus the price of CHP.

Flow Conservation: The coupling constraints (10) ensure energy conservation.

Financial Incentives: Inequality (11) limits the energy that can be sold. Variable E_v^t represents the case where the customer can inject electricity into the grid: see (10) and (9). In some markets, the customer receives in exchange an energy credit for future consumption. In this case, we set $E_v^t = 0$ in (9), rewrite (10) as (166), and replace constraint (11) by $E_{av}^0 = 0$ to initialize the power credit variable.

IBR constraints: Constraint (12) limits the energy that can be bought. Constraints (13) to (16) are the main IBR constraints. (13) ensures that the daily consumption is within the capacity. (14) controls the activation of the blocks, and (15) ensures that at most one block is used. (16) are the binary restrictions.

2.1 PV and solar collector

The PV and solar collector models calculate the values of the parameters E_{pv}^t and T_{in}^t . We first need to estimate T_o^t so we can use the formulas from [49, Example 9, Section 14.12]. We also need to know the quantity of solar radiation available at the surfaces of the panels and collectors. We used the equations from [48, Chapter 1,2] (without shading and “track moving” but with “clear sky”). We implemented four models: the isotropic diffuse model, HDRK model, Perez model, and the ASHRAE revised clear sky model (“Tau Model”) from [49, Chapter 14] and [50, Chapter 35]. From these we selected the Perez model. We also considered the absorptance effect.

We must convert the solar radiation into power for the PV and heat for the solar collector. For the PV, we use the approach from [51] with $V = V_{mp}$; we replace Equation 3 of [51] by Equation 9 of [52]. For the cell temperature, we use the approach from [48, Section 23.3]. For the solar collector, we use the collector-efficiency equations from [48, Chapters 3–6] with the factor $F' = 0.8$, the temperature of the plates equal to the outside temperature, the temperature difference between the plates and glass set to 20°C, all the layers of glass having the same temperature, and the inlet water coming from the street.

2.2 Wind turbines

The wind turbine (WT) model forecasts the value of E_{wt}^t . Villaneuva & Feijóo [53] propose a relationship between wind speed and WT power produced. They consider operation at maximum capacity and specify cut-in and cut-out inequalities. We have slightly modified their strict inequalities to include the end-points:

Wind speed interval (m/s)	Output power (W)	
$U < U_{CutIn}$	0	
$U_{CutIn} \leq U < U_{Pmax}$	$0.5A\rho U^3 C_{p2}$	(1)
$U_{Pmax} \leq U \leq U_{CutOut}$	P_{max}	
$U > U_{CutOut}$	0	

where U is the wind speed (m/s); U_{CutIn} is the lowest wind speed (m/s) at which it is possible to obtain power from the wind; U_{Pmax} is the wind turbine power speed rate (m/s); A is the area of the air-stream, measured in a perpendicular plane to the direction of the wind speed (m^2); ρ is the air density ($kg.m^{-3}$); C_{p2} is the power coefficient, which is the ratio between the power produced by the WT and the power carried by the free air-stream; U_{CutOut} is the upper limit (m/s) at which it is possible to get power from the wind; and P_{max} is the rated power (W).

The Weibull probability distribution function (PDF) of U at a specified location is

$$f_U(U) = \begin{cases} \frac{k}{C} \left(\frac{U}{C}\right)^{k-1} e^{-(U/C)^k}, & U \geq 0 \\ 0, & U < 0 \end{cases}, \text{ where } k \text{ is a shape parameter and } C \text{ is a scale parameter [53].}$$

We can find ρ using the ideal gas law for dry air, Equation 1.18 from [54]: $\rho = \frac{p}{RT}$, where p is the absolute pressure (101.325 kPa), T is the temperature (K), and R is the specific gas constant for dry air ($287.058 \frac{J}{kg.K}$).

2.3 Heating, Ventilation, and Air Conditioning (HVAC)

Constraints (17) to (27) represent the room temperature, and (28) and (29) permit deviation and count the discomfort from the target temperature during the intervals when the user needs the HVAC.

Shao et al. [55] propose a model with a 1.2% total daily energy difference. Given an initial temperature T_0 , their model (with slightly changed notation) is:

$$\begin{aligned} T_{i+1} &= T_i + \Delta_{t_2} \left(\frac{G_i}{\Delta_c} + \frac{C_{HVAC}}{\Delta_c} w_{AC,i} \right), \\ \Delta_c &= C_{air} V_{house2}, \\ G_i &= E_i (T_{out,i} - T_i) + (SHGC) \times A_{window_south} \times H_{solar} \times \frac{3.142 \frac{Btu}{Wh}}{10.76 \frac{ft^2}{m^2}} + H_p, \\ E_i &= \left(\frac{A_{ceiling2}}{R_{ceiling2}} + \frac{A_{wall2}}{R_{wall2}} + \frac{A_{window2}}{R_{window2}} + \frac{11.77 Btu}{^\circ F \times ft^3} n_{ac} V_{house2} \right) \end{aligned}$$

where T_i is the room temperature in interval i ($^\circ F$); Δ_{t_2} is the length of interval i (hours); G_i is the heat gain rate (Btu/h) during interval i ; C_{HVAC} is the heating capacity (if positive) and the cooling capacity (if negative) in Btu/h; and Δ_c is the energy (Btu) needed to change the air temperature by $1^\circ F$. Further, A_{wall2} , $A_{ceiling2}$, and $A_{window2}$ are the areas (ft^2) of the walls, ceiling, and windows; R_{wall2} , $R_{ceiling2}$, and $R_{window2}$ are the heat resistance ($^\circ F \cdot ft^2 \cdot h/Btu$) of the walls, ceiling, and windows; n_{ac} is the number of air changes in interval i (1/h); V_{house2} is the house volume (ft^3); $T_{out,i}$ is the external temperature in interval i ($^\circ F$); SHGC is the solar heat gain coefficient of the windows; H_{solar} is the solar radiation heat power (W/m^2); H_p is the heat gain from the occupants (Btu/h); and C_{air} is the specific heat capacity of air for a typical room condition (1.012 J/gK or 0.0195 Btu/ $ft^3^\circ F$).

Assumptions: A_{wall2} in [55] is derived from A_{floor2} assuming that the height of the house is 10 ft. If A_{wall} is not given by the user, we assume that the floor is square and calculate A_{wall2} accordingly. We set $A_{window_south} = A_{window2}$. We set $C_{HVAC} = P_{HVAC} y_{HVAC}$, where y_{HVAC} is the binary variable and P_{HVAC} is the rated power for HVAC (W).

H_p can be found ([34], [56, pp. 41–43]) via $H_p = P_{activity} A_{body}$, where $P_{activity}$ is the metabolic rate and $A_{body} = 0.202 m^{0.425} h^{0.725}$, where m is the mass of the body (kg), and h is the height (m). We define np^t to be the number of occupants at time t . H_p is then given by:

$$H_p^t = \sum_{i=1}^{np^t} P_{activity,i}^t A_{body,i}^t \quad \forall t \in T.$$

For SHCG, we used the average of the values of [49, p. 353]. H_{solar} is calculated from our PV model. We now discuss the calculation of n_{ac} . We first note that the infiltration loss $\frac{\Delta_c}{\Delta_c} \left(\frac{11.77 Btu}{^\circ F \times ft^3} n_{ac} V_{house} \right) (T_{out,i} - T_i)$ is large. Therefore, we assume that the term $\frac{11.77 Btu}{^\circ F \times ft^3}$ should be multiplied by 10^{-3} . If we consider the infiltration formula $\frac{\Delta_c}{\Delta_c} (n_{ac} V_{house} \rho C_{air}) (T_{out,i} - T_i)$ where ρ is the air density, we see that $\frac{11.77 Btu}{^\circ F \times ft^3}$ replaces ρC_{air} in the formula. For $\rho = 1.2$ and $C_{air} = \frac{1012 J}{kg^\circ C}$, we have $\rho C_{air} = \frac{1214.4 J}{^\circ C \times m^3} \approx \frac{0.018 Btu}{^\circ F \times ft^3}$. That value matches the formula from [49, Chapter 17]: $n_{ac} = ACH = \frac{3.6 Q_i}{V}$, $Q_i = A_L IDF$, $A_L = A_{es} A_{ul}$, where ACH is the air changes per hour; Q_i is the infiltration airflow rate (L/s); V is the building volume (m^3); A_L is the building effective leakage area (cm^2) (including the flue) at a reference pressure difference of 4 Pa, assuming a discharge coefficient CD of 1; IDF is the infiltration driving force ($L/(s \cdot cm^2)$); A_{es} is the building exposed surface area (m^2); and A_{ul} is the unit leakage area (cm^2/m^2). This gives $ACH = \frac{3.6 A_{es} A_{ul} IDF}{V}$. With the values given for IDF [49, Table 5 of Chapter 17] and A_{ul} [49, Table 3 of Chapter 17], n_{ac} has an acceptable value.

Our other assumptions are as follows: i) there is a single conditioned space; ii) no independent thermal storage is coupled to the main heating/cooling equipment; iii) the control of humidity is neglected; iv) internal heat sources of the equipment are neglected; v) the temperature is constant throughout the space.

We have rewritten the model from [55] with the considerations above to arrive at an on-off model composed of constraints (17) to (20), (23), (28), and (29) as well as

$$x_{HVAC}^t = P_{heating}y_{HVAC}^t + P_{cool}y_{air}^t \quad \forall t \in T.$$

Flexibility: (17) introduces the on-off feature, since y_{HVAC}^t and y_{air}^t are binary variables. Thus, we are working with fixed power in a fixed time interval. However, forcing a machine to operate for a full interval could generate an overcharge. We therefore introduce the variable z_a^t to allow appliances to operate for shorter periods. Let z_a^t be the fraction of Δ_t during which appliance a is on. We have $z_a^t y_a^t = z_a^t$. If $y_a^t = 1$ then $0 \leq z_a^t \leq 1$; if $y_a^t = 0$ then $z_a^t = 0$. We can model the implications with inequalities such as (24) and (25). Therefore, with the new variable, we can build a single model that is represented by constraints (17) to (29). With the objective function (8), we can, without any impact on the results, drop the variables y_{HVAC}^t and y_{air}^t and the restrictions (20) and (23) to (25). We refer to this as the *Fraction of step*.

The strategy above does not indicate when the machine is on/off in the fixed time interval. Given x_{HVAC}^t for each $t \in T$, we can find whether the machine is on or off at each second via the following optimization problem:

Parameters

S Number of seconds in Δ_t

x Value of x_{HVAC}^t (W)

P Nominal machine power (W)

Variables

$y_i \in \{0, 1\}$ 1 if machine is on in second i ; 0 otherwise.

$e \geq 0$ Error variable (W)

$$\min_{e, y} e \quad (2)$$

$$\sum_{i=1}^S P y_i = x \times S + e \quad (3)$$

$$y_i \leq y_{i+1} \quad i \in \{1, \dots, S-1\} \quad (4)$$

$$y_i \in \{0, 1\} \quad (5)$$

$$e \geq 0 \quad \forall t \in T \quad (6)$$

The objective function (2) minimizes the energy error. Constraint (3) assigns the energy consumed during the time interval.

The timing of the use of an appliance is important: the temperature at the end of the period depends on exactly when it is used. If it is used at the end of the period, the results are closer to the results for the complete optimization model. Constraint (4) ensures that it is turned on as late as possible in the interval. If instead of the assignment at the end of period, another period is wished, Constraint (4) has to be changed appropriately.

Model (2) to (6) gives a difference between $\sum_{i=1}^S P y_i$ and x that is less than $|P|$. With $P = 25$ kw, the difference in temperature will be around 0.12°C at some point in the time interval.

We note that the addition of z_a^t does not violate energy conservation. For example, suppose an appliance has a power of 5000 W and it operates for 50% of the time, giving a consumption of $5000 W \times 0.5\Delta_t$. It will contribute $2500 W \times \Delta_t$ to the conservation constraint.

2.4 Water heaters

Constraints (30) to (34) represent the water heating temperature, and (35) and (36) permit deviation and count the discomfort from the target temperature during the intervals when the user needs hot water. Constraint (37) places an upper bound on the temperature. If $E_{wh} = 0$, there is no water heater (WH) and $T_{out,wh}^t$ is at most the street water temperature; constraints (30) to (32) and (34) are then omitted.

The model was constructed from [48, Equation 8.3.3]. We rewrote it as follows:

$$T^{t+1} = T^t + \frac{\frac{\Delta t}{60}}{(v_{tank} C_p)} [(3.6 * 10^3 P_{wh} - 227.4 f_r^t C_p (T^t - T_{inlet}^t) - 3600 (UA)_{wh} (T^t - T_{room}^t)]$$

where Δt is the length of each time slot (min); V_{tank} is the WH volume (dcm³); C_p is the water thermal capacity ($J/kg^\circ C$); P_{wh} is the rated power for the WH (W); f_r^t is the hot water flow rate in interval t (gal/min); UA is the loss coefficient area product for the WH ($W/^\circ C$); T_{inlet}^t is the water temperature from the solar collector or street in interval t ($^\circ C$); T_{room}^t is the exterior temperature around the WH in interval t ($^\circ C$); and T^t is the mixed water temperature in the WH in interval t ($^\circ C$). Note that T_{room}^t comes from the HVAC model.

We validate our model by comparing it with the following model [55]:

$$T_{outlet,i+1} = \frac{T_{outlet,i}(V_{tank2} - f_{ri}\Delta_t)}{V_{tank2}} + \frac{T_{inlet}f_{ri}\Delta_t}{V_{tank2}} + \frac{1gal}{8.34lb} [p_{WH,i} \frac{3412Btu}{kWh} - \frac{A_{tank2}(T_{outlet,i} - T_a)}{R_{tank}}] \frac{\Delta_t}{60 \frac{min}{g}} \frac{1}{V_{tank2}}$$

where $T_{outlet,i}$ is the mixed water temperature in the WH in interval i ($^\circ F$); T_{inlet} is the temperature of the inlet water ($^\circ F$); T_a is the room temperature ($^\circ F$); f_{ri} is the hot water flow rate in interval i (gpm); A_{tank2} is the surface area of the WH (ft²); V_{tank2} is its volume (gallons); R_{tank} is its heat resistance ($^\circ F \cdot ft^2 \cdot h/Btu$); and Δ_t is the duration of the time intervals (min).

For a validation example, we convert units where necessary and use the data: room temperature given in Figure 1, $t_{outlet,0} = 77^\circ F$, $p_{WH,i} = 5 \text{ kW } \forall i$, $A_{tank2} = 5 \text{ ft}^2$, $R_{tank} = 25^\circ F \cdot ft^2 \cdot h/Btu$, $\Delta_t = 10 \text{ min}$, $t_{inlet} = 104^\circ F$, $V_{tank2} = 80 \text{ gallons}$, $f_{ri} = x$ (gallons/min) for $x \in \{0, 1, 5, 10\} \forall i$. If we assume that the WH is always on, we obtain the results of [55] when $f_r^t > 0$, as shown in Figures 2 and 3. When $f_r^t = 0$, the model deviates after about $150^\circ C$, but in practice there are mechanisms to avoid water evaporation. Thus, our model is equivalent to that of [55].

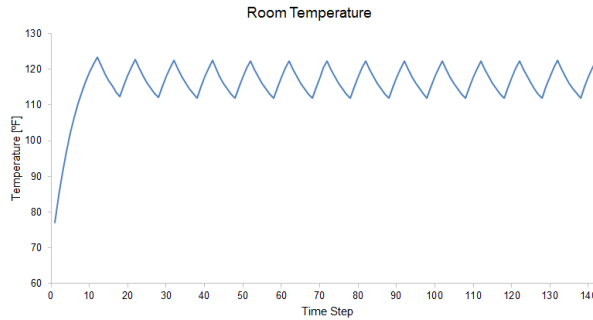


Figure 1: Room temperature for validation example.

We assume that all the water inside the WH is at the same temperature. The *Fraction of step* idea can be applied by relaxing y_{wt}^t .

2.5 Shower

Constraints (38) and (39) place bounds on the shower temperature without using the shower resistance. Constraints (40) to (42) represent the shower temperature, and (43) and (44) permit deviation and count the discomfort from the target temperature during the intervals when the user needs the shower. Constraints (45) are the binary restrictions.

We obtain model (38) to (45) from [56, Chapter 1] through the thermodynamics formula $\dot{Q} = \dot{m} C_p \Delta T$ where $\dot{m}$ is the mass flow rate of a fluid flowing in a pipe (kg/s); C_p is the specific heat of the fluid ($J/kg^\circ C$); ΔT is the temperature difference ($^\circ C$) and $\dot{Q}$ is the rate of net heat transfer into or out of the control

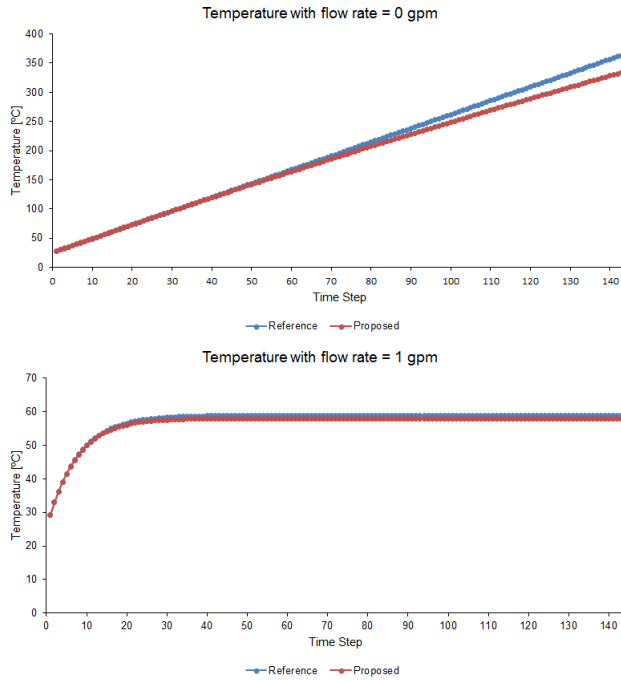


Figure 2: Comparison of our model and reference model from [55]: WH always on and low flow rate.

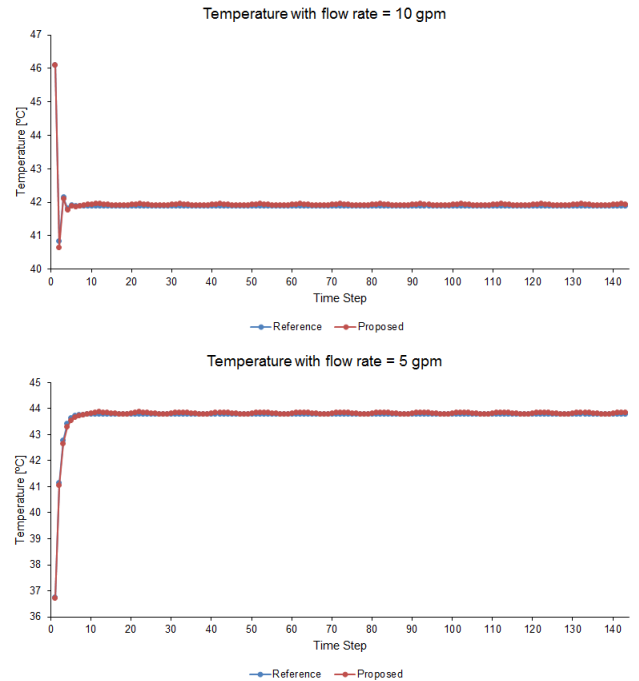


Figure 3: Comparison of our model and reference model from [55]: WH always on and high flow rate.

volume (j/s). Moreover, $\dot{m} = \rho V A_c$ where ρ is the fluid density; V is the average fluid velocity in the flow direction; and A_c is the cross-sectional area of the pipe or duct. This gives $\dot{Q} = \rho \dot{V} C_p \Delta T$, where $\dot{V}$ is the volumetric flow rate (m^3/s). We write $P = \rho \dot{V} C_p \Delta T$ where P is the power (W) lost or injected.

For each step $t \in D_{chu}$, $P_{chu} = P$, f_r is the equivalent measure of $\dot{V}$ (gpm) and $\Delta T = T_{out,chu} - T_{chu,hand}$. This gives (40) and (41). The *Fraction of step* idea can be applied by relaxing $y_{chu,hot}^t$.

2.6 Batteries

Constraints (46) establish the relationships between the state of charge (SOC), discharging and charging powers, and battery float losses. Constraints (47) place a lower bound on the SOC. Constraints (48) place an upper bound on the float losses that is activated when a threshold is reached. Constraints (49) set an initial value for the SOC. Constraints (50) to (53) set bounds on the discharging and charging power. Constraints (54) ensure that at each $t \in T$ the battery either charges or discharges. Constraints (55) to (57) are the binary restrictions.

We based our model on [34], [35], and [38]. The differences between these models will be explained using the following data: $|T| = 144$, $E_{bat} = 24 \text{ kWh}$, $SOC_{max} = 80\%$, $SOC_{min} = 20\%$, $P_{max}^{ch} = P_{max}^{dch} = 3.3 \text{ kW}$, $P_{min}^{ch} = P_{min}^{dch} = 0.3 \text{ kW}$, $\eta^{ch} = \eta^{dch} = 0.91$, and $\mu = 1$. We define the objective function $\max_{t \in T} 30 SOC^2 - \sum_{t \in T} SOC^t$. Thus, in interval 1, the battery will charge at a maximal power and in interval 2 it will discharge at a maximal power: see Figure 4. Energy is given by $\Delta_t \times \text{Power}$, so we show only power in the energy flow in Figures 4 and 5.

The solution to the models of [34] and [35] is $SOC^2 = 22.0854$, $SOC^t = 20 \forall t \in T \setminus \{2\}$, $P_{batt}^{ch,1} = P_{batt}^{dch,2} = 3.003$, and the other power variables are null. Thus, the charging consumption was 3.003 KW instead of 3.3 KW. For the discharging, $P_{batt}^{dch,2}$ takes the value of the power inside the battery instead of the power sent to the house. Thus, when we put these variables into the energy conservation constraints, they use different values than the values sent to or consumed in the battery model. Therefore, there is a energy construction if we take the home as the reference point.

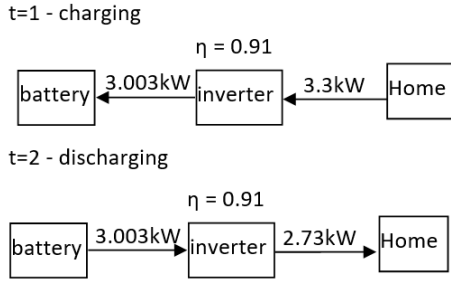


Figure 4: Solutions of battery model.

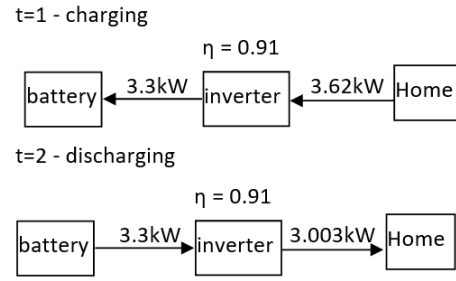


Figure 5: Solutions of battery model.

The model from [38] gives the same objective value, but it reproduces exactly the process shown in Figure 4. However, we could have maximal power charging in the battery, as shown in Figure 5.

Our model implements the energy flow shown in Figure 5, which gives a better objective value.

We also consider the battery float charge loss. According to [57], the global loss is given by $L_{Bat} = L_{Bat,eff} + E_{Float}$, where $L_{Bat,eff}$ is the energy efficiency loss, represented in our model by μ ; and E_{Float} is the battery float charge loss, which results from the energy used to maintain the battery charge when SOC $\approx 100\%$. Constraints (48) implement this.

2.7 Fridge

We consider a frost-free top mount refrigerator. Constraints (58) and (59) represent the freezer temperature and the fridge temperature, respectively. Constraint (60) establishes the initial freezer temperature, and Constraint (61) establishes the initial fridge temperature. Constraints (62) initialize the x_{refri}^t values for the link with the complete model. Constraints (63) set $x_{freezer}^t = 0$ because the freezer temperature depends on the fridge temperature. Constraints (64) to (67) permit deviation and count the discomfort from the target temperature, and Constraints (68) are the binary restrictions.

A transient model has been developed [58, 59] for the temperatures of refrigerated compartments; the predicted energy consumption has a maximum deviation of $\pm 2\%$. Our model is based on this, with some adjustments. We observe a correlation between the power consumption and the air temperature at the evaporator outlet [59, Figures 2.15 and 2.16]. We apply Newton's law of cooling to find the temperature at the evaporator outlet when the compressor is operating. When it is off, we have a choice of two methods. In the first, as suggested by the author via email correspondence, we assume that the temperature at the evaporator outlet is the same as the air temperature at the evaporator inlet:

$$T_{o,e}^t = T_{newton} y_{comp}^t + (r T_{freezer}^{t-1} + (1-r) T_{refri}^{t-1}) (1 - y_{comp}^t),$$

where $T_{o,e}^t$ is the temperature at the evaporator outlet; T_{newton} is the temperature obtained from Newton's law of cooling; and r is the air mass fraction supplied to the freezer. In the second method, we use Newton's law of cooling for the temperature increase around the evaporator outlet: $T_{o,e}^t = T_{newton} y_{comp}^t + T_{newton2} (1 - y_{comp}^t)$. Figure 6 compares these methods with the reference values calculated using the equations from [59] and experimental data provided by email by the authors. We selected method 2.

We can rewrite Equation 3.71 from [59] as $(T_{o,e}^t)' = T_{o,e}^t + \frac{3600 w_{fan}}{\dot{m}_a C_{p,a}} y_{comp}^t$, where $(T_{o,e}^t)'$ is the air temperature of the fan discharge ($^{\circ}C$); w_{fan} is the rated fan power (W); $\dot{m}_a$ is the total mass flow rate of air (kg/h); and $C_{p,a}$ is the specific heat at a constant pressure (J/kg $^{\circ}C$). The fan is on only when the compressor is on.

With $\kappa \in \{T_{newton2}, r T_{freezer}^{t-1} + (1-r) T_{refri}^{t-1}\}$ and following [59], we can write the above equations as (167) and (168) where $T_{eq,f}^t$ ($T_{eq,r}^t$) is associated with the steady-state temperature of the freezer (fridge) if constant conditions are maintained at time t ($^{\circ}C$); UA_f (UA_r) is the global freezer (fridge) thermal conductance (W/ $^{\circ}C$); $\dot{m}_{d,f}$ ($\dot{m}_{d,r}$) is the air flow leaving the freezer (fridge) when the door is open; T_a^t is the exterior temperature around the fridge at time t ($^{\circ}C$); UA_m is the global mullion thermal conductance (W/ $^{\circ}C$);

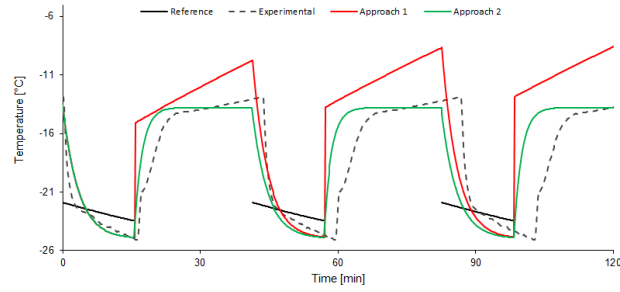


Figure 6: Comparison of ways to find $T_{o,e}^t$.

$T_f^t = T_{freezer}^t \forall t \in T$; and $T_r^t = T_{refri}^t \forall t \in T$. Note that T_a^t is T_{room}^t from the HVAC model. Using these equations, we obtain $\forall t \in \{1, \dots, T-1\}$, $\forall * \in \{r, f\}$:

$$T_*^{t+1} = T_{eq,*}^t - (T_{eq,*}^t - T_*^t) e^{\left[-\frac{60\Delta t(Ua_* + UAm + \frac{\dot{m}_* C_{air}}{3600} y_{comp}^t)}{C_*} \right]}.$$

Figures 7 and 8 compare the results from these equations with experimental results and the reference model [59, Condition 3], assuming standard masses in the enclosure thermocouples and a fixed air density.

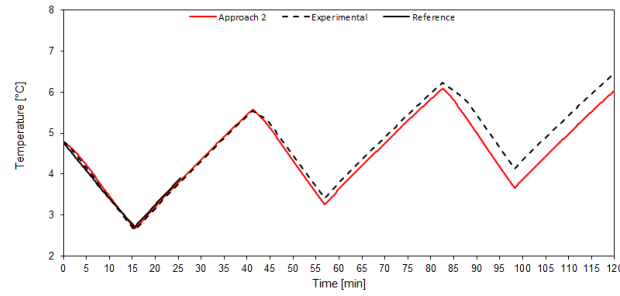


Figure 7: Our results compared to experimental results and reference model from [59] (fridge temperature).

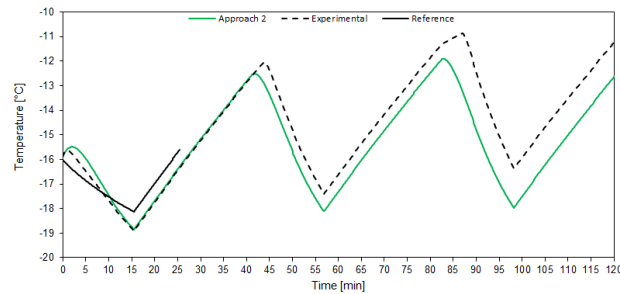


Figure 8: Our results compared to experimental results and reference model from [59] (freezer temperature).

This model is nonlinear and will increase the complexity of our optimization model. For simplicity, we perform a linear regression, obtaining:

$$\begin{aligned} T_f^t &= \begin{cases} -7/25t - 13, & y_{comp} = 1 \\ 15/67t - 15 \times 92/67 - 5, & y_{comp} = 0 \end{cases} \\ \Rightarrow T_f^{t+1} &= T_f^t - \frac{7}{25} \Delta t y_{comp} + \frac{15}{67} \Delta t (1 - y_{comp}) \end{aligned} \quad (7)$$

for the freezer and $T_r^{t+1} = T_r^t - 3.5/25 \Delta t y_{comp} + 7.25/67 \Delta t (1 - y_{comp})$ for the fridge. Figure 9 presents simulation results for the linearizations. They agree well with the reference model [59]. We thus obtain (68) to (68).

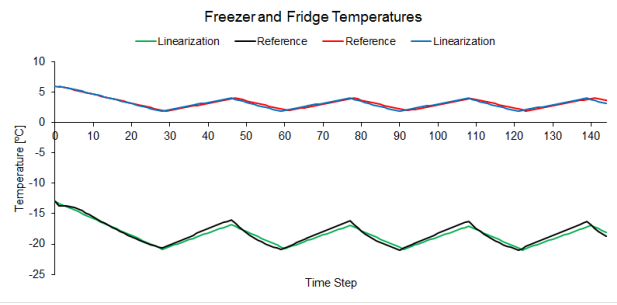


Figure 9: Comparison of simulation results for fridge (freezer) from linearization and reference model.

2.8 Plug-in hybrid electric vehicle

Constraints (94) and (95) establish the relationships between the state of charge, the discharging and charging powers, and the float loss. The vehicle could arrive without energy, so constraints (96) place a lower bound of zero on the SOC. Constraints (97) place an upper bound on the float losses that is activated when a threshold is reached. Constraints (98) and (99) establish an initial value for the SOC. The former is activated when the first event is a departure, and the latter is activated when the first event is an arrival. Constraints (100) and (101) create a link between consecutive trips to ensure energy conservation. Constraints (102) and (103) place a lower bound on the SOC (sufficient to reach the nearest gas station) in intervals when travel is necessary. Constraints (104) to (107) establish bounds on the discharging and charging power. Constraints (108) ensure that at each $t \in T$ the battery either charges or discharges. Constraints (109) to (128) ensure null values for the intervals when the EV battery cannot be used. Constraints (129) calculate the fuel cost for each trip s . Constraint (130) gives the total cost of the fuel consumption. Constraints (131) to (138) are the binary and nonnegative restrictions.

An EV model is essentially the same as a battery model [35], but there are more constraints, including minimal SOC constraints and trip-signal constraints to ensure that the EV charges and discharges only when at home. Sousa et al. [60] present an EV model that is similar to a battery model. Our model is based on [35] and [60] and considers multi-trips [61]. The EV can make multiple trips in a day. In addition, we consider hybrid vehicles.

2.9 CHP

Constraints (69) to (76) control the on/off status of the CHP. Constraints (77) and (78) initialize the status variable and the associated electricity production. Constraints (79) to (84) enforce ramp-up and ramp-down limits. Constraints (85) to (86) place lower and upper bounds on the electricity generated by the CHP. Constraints (87) to (88) relate to the CHP operation. Constraints (89) measure the CHP fuel cost. Constraints (90) to (93) impose the variable domains.

Our model is based on [62] and [63] where efficiency is a function of the electrical power generated. The image of that function has values that are close to each other, so we consider the average efficiency to be a parameter rather than a variable.

2.10 A_{IEUI} : Set of inelastic appliances with uninterruptible operation

Constraints (139) and (140) assign the load profile of task $a \in A_{IEUI}^t$ to time $t \in T$. Constraints (141) eliminate solutions where the appliance cannot start its operation. Constraints (142) ensure that the task is done at most once, between the beginning of the horizon and the last start time permitting the completion of the task. Constraints (143) and (144) prevent overlap between tasks for the same appliance. Constraints (145) to (147) count the discomfort from the deviation from the preferred time. Constraints (149) and (150) are the binary restrictions.

A_{IEUI} is a set of inelastic appliances with uninterruptible operation (see [9]). In practice, the operation of these appliances is represented by a load profile. Load profiles forecast the power variation over a period of time for an appliance, frequently as an average. They are a useful approximation. For example, Tsagarakis et al. [64, 65] use aggregated load profiles, while [66] uses load profiles for specific machines. The former use a time-use survey (TUS) to construct the load profiles. However, the consumption found through TUS diaries can be different from the load measurements obtained by submetering [67]. The latter approach uses controlled tests to measure consumption, and the resulting profiles provide a good approximation. UK-DALE [68] is an open-access data-set giving the consumption of domestic appliances in the UK. The data are based on measurements of real electricity consumption.

Our model is task oriented, an idea from [22] that proposes a model with parameters to indicate how many times each appliance will be used. In our case, we consider individual load profiles for each use of each appliance. Therefore, our model includes also appliances that are used intermittently, classified as A_{iei} by [9].

2.11 A_{phases}

Constraints (151) and (152) assign the load profile of each phase p of task $a \in A_{phases}^t$ to time $t \in T$. Constraints (153) eliminate solutions where the appliance cannot start its operation. Constraints (154) ensure that the task is done at most once, between the beginning of the horizon and the last start time permitting the completion of the task. Constraints (155), from [69], are precedence constraints between consecutive phases of the same task. Constraints (156) and (157) prevent overlap between tasks for the same appliance. Constraints (158) to (162) count the discomfort from the deviation from the preferred time. Constraints (164) and (165) are the binary restrictions.

Each load profile is separated in sequential phases, and the operation can be interrupted between phases. The appliances in A_{phases} include dishwashers [70], washing machines [71], and dryers [66].

The model has flexibility enough to accommodate other constraints if the client need it. For example, if there is a need of maximum time Max^t , between phases, we can add $\forall a \in A_{phases}^t, p = 1 \dots PH_a - 1$:

$$\sum_{tt=1}^{|T|} tty_{a,p+1}^{tt} - \sum_{t=1}^{|T|} ty_{a,p}^t \leq Max^t.$$

If, for instance, the dryer d are going to be used after the washing machine b , we can add:

$$\sum_{tt=1}^{|T|-LL} tty_{d,1}^{tt} \geq \sum_{t=1}^{|T|-(\lceil \frac{D_{b,PH_b}}{\Delta t_2} \rceil - 1)} \left(t + \left\lceil \frac{D_{b,PH_b}}{\Delta t_2} \right\rceil \right) y_{b,PH_b}^t \text{ where } LL = \left\lceil \sum_{p=1}^{PH_d} D_{d,p} \frac{1}{\Delta t_2} \right\rceil - 1.$$

3 Data and instances

In this section we discuss our data and how we create the instances.

3.1 Locations

Brazil is expected to become one of the largest smart-grid markets in the world by 2023 [2]. The country has a total installed capacity of 161 GW [72], which is expected to grow to 224 GW by 2030 [73, Table 6.26] via initiatives such as the Cities of the Future Project [74]. The climatic data was taken from [49, Chapter 14]. We calculated the temperature based on 5% dry-bulb design conditions. Wind speed data was taken from [75].

3.2 Prices

In Brazil, there are two pricing options: the conventional tariff and the white tariff. The conventional tariff is constant, whereas the white tariff is a TOU pricing with a single price for weekends and holidays and three weekday prices:

1. Peak hours: Three consecutive daily hours defined by the distributor.
2. Intermediate peak hours: The hour before and the hour after the peak hours.
3. Nonpeak hours: The remaining hours of the day.

We use the electricity prices published in April 2017 for the white tariff. We summed two tariffs, following [76, p. 57].

The IBR pricing defined by federal law [77] specifies certain discounts:

1. For the portion of consumption below 30 kWh/month, the discount is 65%.
2. For the portion between 31 and 100 kWh/month, the discount is 40%.
3. For the portion between 101 and 220 kWh/month, the discount is 10%.

We divided the monthly limits defined above by 30 days. This makes our model less realistic compared to the actual reality, but since smart meters track and report energy consumption in intervals of minutes, we suppose that a pricing scheme in the day will be more important than in the whole month. The law applies to a subgroup of the population, but we apply it to everyone.

Brazil has a net metering incentive. If the energy injected into the grid is greater than that consumed, the customer receives an energy credit for future consumption. In addition, we assume that, as in other markets, Brazil's consumers will in the future have the opportunity to sell electricity, i.e., a *feed-in tariff*. Thus, we consider $\nu^t = \frac{\lambda^t}{2} \forall t \in T$.

3.3 Capacity

We set the capacity E_{th}^t to 75 kW, which is the residential capacity in Brazil.

3.4 PV and solar collector

We take the data from data-sheets or use the estimates in [48]. We consider the PVs *Canadian Solar CS5P250M*, *Yingli Solar JS150*, and *Siemens SM110* and the solar collectors *CSi Sodramar*, *TERMOMAX*, *Soletrol*, and *Sunda Seido 10*. The ground reflectance was generated from the uniform distribution $U(0.13, 3)$ according to [49, Table 5, Chapter 14].

3.5 Wind turbines

The data comes from the data-sheets for the *Raum Energy 3.5 kW Wind Turbine System* and the *Raum Energy 1.3 kW Wind Turbine System*.

3.6 HVAC

For the house, we generated resistance values from a uniform distribution (see [55]), converting the units to $(\text{m}^2 \text{ } ^\circ\text{C.h/J})$. The house height is 3.048 m, with $A_{ceiling} = A_{floor} = 100 \text{ m}^2$. We set $P_{heating} = P_{cool} = 60 \frac{\text{W}}{\text{m}^2} \times A_{floor}$, an approximate value suggested by some manufacturers. We set A_{ul} between 0.7 and 2.8. We set IDF to 0.031 for cooling and 0.086 for heating. The other parameters are $T_f^t = 23^\circ\text{C} \forall t \in T$, $H_p^t = 97.57 \text{ W} \forall t \in T$.

3.7 Water heaters and shower

We use $T_f^{chu} = T_f^{wh} = 50^\circ\text{C}$ (see [78]). We set $P_{wh} \sim U(4000, 5000) \text{ W}$ (see [55]). The heat resistance is given by $U(12, 25) \frac{^\circ\text{F.ft}^2\text{h}}{\text{Btu}}$ converted to $(\text{m}^2 \text{ } ^\circ\text{C.h/J})$. We take the WH area and diameter from the data-sheets for Giant-142ETE, Giant-152ETE, Giant-172ETE, Rheem-PROPH50, Rheem-PROPH65, and Rheem-PROPH80. We use the diameter to calculate $(UA)_{wh}$ and V_{tank} . We set $T^{max} = 100^\circ\text{C}$ and $E_{wh} = 1$.

We took the temperature of the water from the street (T_{inlet}^t) from [79, Graphic 4]. We projected the missing intervals so that the last projected value is equal to the first collected value.

For fr_{wh}^t and fr_{chu}^t , we used the data available in [80]. For each client, we aggregate the daily consumption in intervals of Δ_t . Inter-period consumption is calculated proportionally. P_{chu} is obtained from the data-sheets for Lorenzetti's showers.

3.8 Batteries

We used the following values from [38]: $E_{bat} = 24 \text{ kWh}$, $SOC_{min} = 20\%$, $SOC_{max} = 100\%$, $p_{max}^{ch} = 4 \text{ kW}$, $p_{min}^{dch} = 4 \text{ kW}$, $p_{min}^{ch} = 0.3 \text{ kW}$, $p_{min}^{dch} = 0.3 \text{ kW}$, $\eta^{ch} = 0.91$, $\eta^{dch} = 0.91$, $\mu = 1$. According to [57], in a low-storage system, the float charge losses represent around 100 W , so $p_{loss} = 0.1 \text{ kW}$.

3.9 Fridge

The fridge is described in [58] and [59]. The other parameters are $T_{freezer}^{start} = -13^\circ\text{C}$, $T_{refri}^{start} = 6^\circ\text{C}$, $P_{comp} \sim U(150, 170) \text{ W}$ considering experiments on power in permanent regime from [81], $T_f^{freezer} = -18^\circ\text{C}$, and $T_f^{refri} = 2^\circ\text{C}$.

3.10 Electric vehicles

We used the Nissan Leaf's battery specifications [82]: $EV_{bat} = 24 \text{ kWh}$, $EV SOC_{max} = 90\%$, $EV SOC_{min} = 0\%$ and $EV P_{max}^{ch} = EV P_{max}^{dch} = 3.6 \text{ kW}$. The other parameters are based on the battery parameters: $EV P_{min}^{ch} = EV P_{min}^{dch} = 0.3 \text{ kW}$, $EV \mu = 1$, and $EV p_{loss} = 0.1 \text{ kW}$.

The Nissan Leaf has a battery capacity of 24 kWh and a range of 160 km [60]. We calculated $km^{100} = \frac{EV_{bat}}{\text{Electricity consumption}} = \frac{24 \text{ kWh}}{150 \text{ Wh/km}} = 160 \text{ km}$, as in [60].

We simulated the battery-charging model for 7 h. At time 0, the SOC was 0. With the specified rate of 3.6 kW/h , the SOC should be 25.2 kWh after 7 h. We therefore defined $\eta_{ev}^{dch} = 24/25.2 \approx 0.95$. Moreover, we set $\eta_{ev}^{dch} = \eta_{ev}^{ch}$.

The number of trips is set to $n_{trip} \sim U(1, 4)$ and t_{start}^s and t_{end}^s are generated randomly. The first trip has a minimum duration of 8 h, the second 4 h, the third 1 h, and the fourth 0.5 h.

The other parameters are $price_gas = 3 \text{ \$/l}$, $consumption_gas = 10 \text{ km/l}$, $EV SOC_{last_day} = 50\%$, $Km_{next}^s \sim U(0, km^{100}) \text{ km} \forall s \in \{1, \dots, n_{trip}\}$, $EV SOC_{min}^{end} \sim U(EV SOC_{retorno_car}, MM)\%$, where $MM = \text{maximum value of } EV SOC_{retorno_car}$, and finally, $EV SOC_{retorno_car} \sim U(0, 30)\%$.

3.11 CHP

We take the data from [63, Table 4] and the data-sheet for CHP CP5WN-SPB. For that CHP, $rate_{CHP}^{fuel} = \frac{0.72 \text{ gallon/h}}{17.8 \text{ kWh}} = \frac{3.79 \text{ l/gallon} \times 0.72 \text{ gallon/h}}{17.8 \text{ kWh}} = 0.153 \text{ l/kWh}$.

Thus, the values are: $d_{CHP}^{on} = d_{CHP}^{off} = 60 \text{ min}$, $nb_{CHP}^\mu = nb_{CHP}^\eta = 3$, $r_{CHP}^{down} = r_{CHP}^{up} = 0.05 \text{ kW/min}$, $y_{CHP}^{last_day} = y_{CHP}^{ini} = 0$, and $P_{CHPe}^{ini} = 0 \text{ kW}$. Fixing μ and $\eta \in \{1, 2, 3\}$ we set: $P_{CHP,min}^{k,\mu}$ and $P_{CHP,min}^{k,\eta} \in \{0, 0.2, 0.5\} \text{ kW}$, $P_{CHP,max}^{k,\mu}$ and $P_{CHP,max}^{k,\eta} \in \{0.2, 0.5, 1\} \text{ kW}$, $\eta_{CHP,p}^k \in \{0, 80, 85\}$, and $\mu_{CHP,p}^k \in \{0, 40, 35\}$.

We set the overall efficiency to the average of the efficiencies given by the piecewise linear function, discarding null efficiencies. Thus, $\bar{\eta} = 0.825$ and $\bar{\mu} = 0.375$.

3.12 A_{IEUI} and A_{phases}

We use the data from [68]. We start the analysis from the first day that has measured data at midnight. Missing data is assumed to indicate no consumption. We aggregated the power data into 10-min intervals based on the average values. We calculated probabilities for the time and duration of the use of each appliance. Using these probabilities, we selected a time t when that appliance can be started for S_a and its duration D_a . We obtain P_a^h from the aggregated power in the interval between t and $t + D_a$. Finally, we set $DS_a = D_a + U(0, 4)$.

For A_{phases} , we performed the steps above for each phase. Dishwashers have three phases: washing, draining, and drying [70]. We tried to find patterns to break the load profile into these phases: see Figure 10. Washing machines have three phases: water heating, washing, and spinning [71]. Dryers have two phases: with and without heat [66]. In [66], the duration of the dryer’s second phase is between 5 and 15 min, so we consider the final 10 min of the load profile to be the second phase.

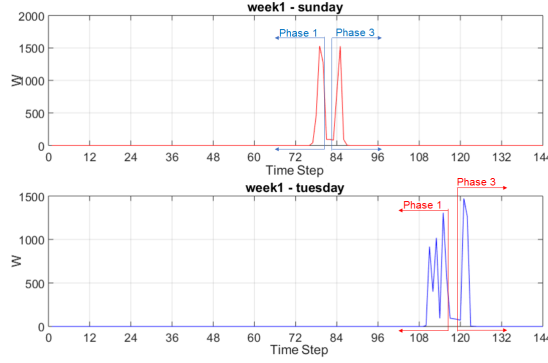


Figure 10: Defining phases in the dishwasher.

We used data from UK-DALE since Brazil does not have an equivalent database.

4 Results and discussion

In this section, we solve the scheduling problem using various models. We replaced the comfort constraints in every model by a fixed start time for the appliances based on load profiles, and fixed bounds for desired temperatures for the thermal appliances. Thus, objective function considers only the cost instead of the trade-off between cost and comfort. The parameters not previously defined are as follows: $H = 24$, $|T| = 2$, and $w_c = w_u = w_t = 1$.

4.1 Models from [45]

We consider the appliance models in [44, 45] with the following adjustments:

1. We assume that $S_t^{ESS} = S_t^{ess}$.
2. The available data corresponds to time windows and power for four appliances in [45, Table 1] and electricity prices in [45, Figure 3]. Since the devices based on load profiles have a fixed usage initialization, we use the data from Section 3 instead of that from [45].
3. For the freezer and fridge, the computation of β^{fr} , α^{fr} and γ^{fr} is not specified. We assumed that β^{fr} is related to the action of opening and closing the fridge doors, α^{fr} is related to the evaporator temperature, and γ^{fr} is related to the losses.
4. For HVAC, α^{ac} and α^{ht} are undefined. We assumed that $EP_t = EP_t^\Theta : \Theta \in \{ac, ht\}$. The parameters β^{ac} , β^{ht} , ρ^{ac} , and ρ^{ht} are computed as in our model, considering infiltration and losses based on the structure of the house.
5. For lighting, there is no constraint that associates illumination level with consumption. Therefore, we did not consider this.

We start our experiments with an “initial” combination composed by A_{IEUI} , fridges, WTs, PVs, IBR constraints, and selling options. We progressively add more appliances; see Table 2.

In Ins. 1, we assign consumption profiles over the horizon and respect the bounds on the freezer and fridge temperatures. Both models give the same cost.

In Ins. 2, we add a battery, a type of ESS. The charging and discharging efficiencies are set to 100%. The model of [45] allows the battery to be off during the entire horizon, but with charging and discharging.

Table 2: Costs for model from [45] and our model.

Ins.	Combination	[45] cost (\$)	Our cost (\$)
1	“initial”	4.02	4.02
2	Ins. 1 + ESS	1.61	1.49
3	Ins. 2 + HVAC	1.93	6.83
4	Ins. 3 + CHP, WH	1.93	6.83
5	Ins. 4 + A_{phases} , Shower	-	9.06
6	Ins. 5 + EV	-	30.03
7	Ins. 4 + Boiler, TSS	1.93	-

Their cost is 8% higher than ours. There are two reasons for this. First, our model considers the SOC at the beginning of an interval, whereas [45] considers it at the end. Thus, in the final interval our model records a discharge while the model from [45] does not. Second, we have lower bounds on the charge and discharge variables. If we remove these two differences, we find the same solution.

In Ins. 3, for the model of [45], HVAC adds 32 cents to the bill while our model adds around 440 cents. Note that in [45], the objective function value is optimal with respect to all machines and appliances. Like the ESS, the HVAC is off throughout the horizon, but it contributes to the indoor temperature. As we will see in Ins. 9, the main issue in this experiment is the solar gain.

In Ins. 4, both models adjust the use of fridge and batteries, achieving the same cost as in Ins. 3.

In Ins. 5, 6, and 7, we add appliances that are considered either by our model or by [45], obtaining results for just one of the models.

In Ins. 3, the HVAC and the battery cannot emit or absorb energy if they are not turned on. For example, a battery could have the constraint set $AC_b = \{\text{BatPower}_t \leq \text{BatCapacity} \times y_t^{\text{on,off}} \forall t \in T\}$, where $y_t^{\text{on,off}}$ is binary, and similar steps give the constraint set AC_h for HVAC. Table 3 shows the results when we add the sets AC_b and AC_h .

Table 3: Costs for model from [45] and our model, with additional constraints from [45].

Ins.	Combination	[45] cost (\$)	Our cost (\$)
1	“initial”	4.02	4.02
8	Ins. 1 + ESS, AC_b	2.13	1.49
9	Ins. 8 + HVAC	2.13	6.83
10	Ins. 9 + AC_h	infeasible	6.83
11	Ins. 8 + solar gain	infeasible	6.83
12	Ins. 10 + disjoint set	2.28	6.83

For Ins. 8, we note that the model from [45] has a cost around 43% higher than ours. The cost for [45], compared with Ins. 2, is increased because of the battery losses. For Ins. 9, HVAC works without consumption because the activation constraints are not considered. When we add these constraints in Ins. 10, [45] becomes infeasible. Moreover, in Ins. 9, our model consumes energy trying to keep the indoor temperature within the bounds. This does not happen in [45], as shown in Figure 11. The model from [45] does not consider solar gain. This has a considerable effect on the indoor temperature [34], so our model consumes more energy and is more expensive.

When we add solar gain in Ins. 11, [45] becomes infeasible. Ins. 12 assumes that the time windows (TWs) for the A/C and heating form a disjoint set. The model from [45] now gives a feasible solution. However, there is still no consideration of solar gain, and finding the optimal disjoint set of TWs is difficult since there are $2^{|T|}$ possible combinations. If we remove the solar gain from the analysis, our model decreases the cost by around 280%.

The thermal energy storage model in [45] follows the ESS model, so a similar analysis can be performed. The CHP in [45] comes from [46], and we present this analysis in Section 4.3.

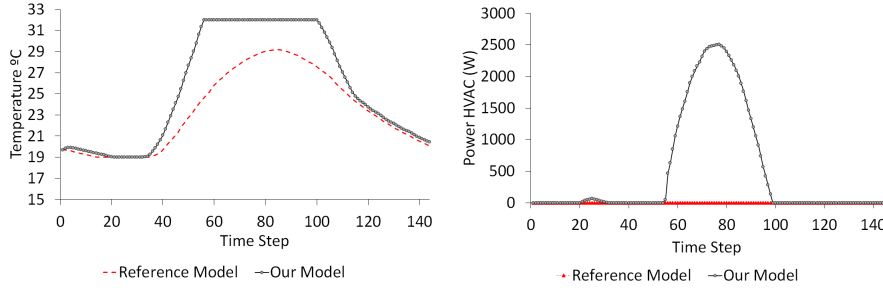


Figure 11: Results for temperature of Ins. 9.

4.2 Results from [34, 35]

The models from [34, 35] consider equivalent batteries, water heating, and underfloor HVAC models. Moreover, they have the same energy flow conservation constraints.

We discussed the limitations of the battery models in Section 2.6. The water heating model from [34, 35] is given below:

$$T_{st}(t+1) = T_{st}(t) + \frac{V_D^{th}(t)}{(V_{tot})} (T_{cw} - T_{st}(t)) + \frac{P_{aux}^{th}(t) + P_{CHP}^{th}(t)}{V_{tot}C_w} - \frac{A_{st}}{R_{st}} (T_{st}(t) - T_b(t)) \quad \forall t \in T/\{|T|\},$$

where V_{tot} is the total WH volume; $V_D^{th}(t)$ is the hourly hot water demand at time t ; $T_{cw}(t)$, $T_b(t)$, and $T_{st}(t)$ are the entering cold water, environment around the WH, and hot water temperatures at time t , respectively; A_{st} is the surface area of the WH; and R_{st} is the thermal resistance of the WH insulation material. We need to change the units given in [35]. Using our parameter Δ_{t_2} , we have:

$$T_{st}(t+1) = \frac{V_D^{th}(t)}{(V_{tot})} (T_{cw} - T_{st}(t)) + \Delta_{t_2} \frac{P_{aux}^{th}(t) + P_{CHP}^{th}(t)}{V_{tot}C_w} - \frac{\Delta_{t_2}}{V_{tot}C_w} \frac{A_{st}}{R_{st}} (T_{st}(t) - T_b(t)) + T_{st}(t) \quad \forall t \in T/\{|T|\}.$$

This modified model is close to our model. There are two differences. First, we use demand as the rate volume per time, but multiplying it by the time interval results in V_D^{th} . The second difference is related to the technology. We use an electrical resistance as the auxiliary power while [34, 35] use a boiler fed by gas. Since gas and electricity have different prices, for a fair comparison, we assume that the three hot water models ([35], [34], and ours) use electricity as auxiliary power. Note that the three models link the HVAC and CHP systems.

Our HVAC is directly in contact with the air that heats the principal floor, while the HVACs from [34, 35] are on the basement floor. Otherwise, the models are equivalent. For a fair comparison, we assume that all the models have the HVAC on the main floor.

The CHP model in [35] differs from that in [34] by a binary on/off variable and one constraint. We test both versions. The *schedulable tasks and residential load* model has more constraints in [35] than in [34], but as we set the starting time for the appliances based on the load profiles, the models are feasible.

Finally, since [34] considers WTs and PVs, we add these to [35]. Thus, the conservation constraints for both come from [34, Equation 37] with $\delta = 1$.

We start our experiments with an “initial” combination composed by the A_{IEUI} and A_{phases} appliances, WTs, PVs, IBR constraints, and selling options. We progressively add more appliances; see Table 4.

In Ins. 1, we assign consumption profiles over the horizon. All the models give the same cost.

In Ins. 2, we add a battery. The charging and discharging efficiencies are set to 100%. For [34, 35] we do not place an upper bound on the initial SOC. Their SOC starts at the maximal value allowed, which gives a result 296% cheaper than our model.

Table 4: Costs for models from [34, 35] and our model.

Ins.	Combination	[35] cost (\$)	[34] cost (\$)	Our cost (\$)
1	“initial”	3.9506	3.9506	3.9506
2	Ins. 1 + ESS	-5.2055	-5.2055	1.3134
3	Ins. 2 + SOC_{min}	1.3102	1.3102	1.3134
4	Ins. 3 + D_A	1.2716	1.2716	2.0013
5	Ins. 1 + HVAC	8.7767	8.7767	8.7767
6	Ins. 3 + HVAC	6.6601	6.6601	6.6629
7	Ins. 6 + D_A	6.5999	6.5999	7.3859
8	Ins. 5 + WH, CHP_{ini}^{off}	16.6304	infeasible	9.7961
9	Ins. 5 + WH, CHP_{ini}^{on}	infeasible	infeasible	10.5269
10	Ins. 9 + D_A	infeasible	infeasible	-4.4723
11	Ins. 8 + D_A	16.6304	infeasible	-2.4028

Suppose SOC_{min} is the bound on the initial SOC. We add SOC_{min} to the models from [34, 35] in Ins. 3. Then, their results are 0.24% cheaper than our result because they do not specify a minimal power for the charging and discharging variables. If we also add these constraints, we obtain the same cost as for our model.

In Ins. 4, we change the efficiencies to the values in [35]. D_A refers to the data from [35]. Their cost is 36.5% lower than our cost. The most important part of this difference is explained by the energy construction in the battery constraints linked with the energy conservation flow in [34, 35] (see Section 2.6).

In Ins. 5 and 6, we test HVAC and HVAC with ESS, respectively. The efficiencies in the battery model are 100%. The results are identical for Ins. 5, and they are slightly different for Ins. 6 for the same reason as in Ins. 3. However, for Ins. 7, with D_A , the [34, 35] results are 10.6% cheaper than ours because of the energy creation.

In Ins. 8, we test the CHP with the WH and HVAC. The CHP is off in the first interval, indicated by CHP_{ini}^{off} in Table 4. In the [35] results, CHP and the boiler were not used. The only way to increase the water temperature is by increasing the room temperature, but this solution is 69.8% more expensive than our results. In [34] the ramp-up constraints do not allow the start-up of the CHP. Moreover, the off status is not allowed. Therefore, the [34] model is infeasible.

In Ins. 9, we changed the CHP status at the first interval. It is on with a maximal production of electrical power, indicated by CHP_{ini}^{on} in Table 4. Constraint 17 from the [35] model makes the problem infeasible, and the upper bound on P_{CHPt}^t in the [34] model makes the problem infeasible.

Ins. 10 and 11 are identical to Ins. 8 and 9, but with D_A . The analysis for Ins. 8 and 9 applies. For Ins. 11, our model sells electricity while the [35] model buys it. Our solution is around 114% cheaper than the [35] solution.

4.3 Results from [46]

There are two models from [46] that we did not consider in the analysis: the fridge model (29–31 in [46]) and the HVAC model (32–33 in [46]). These models have a scheduled consumption solution as input. For each interval, the models postpone or advance the time when the appliance will be turned on in that input solution. Thus, [46] assumes that the cooling goods (food), for the fridge, and the living space (air), for the HVAC, have enough thermal inertia to act as a buffer. We do not know how to determine the initial buffers, needed as parameters. We assume we have a scheduled solution for the fridge and a null initial buffer for the cooling goods. We consider only a fridge, and we optimize the electricity cost using the models from [46] and our model. The solution found by [46], shown in Figure 12, is not feasible because the temperatures become too high.

We started our experiments with an “initial” combination composed by the A_{IEUI} and PV appliances, IBR constraints, and selling options. WTs are considered in all the instances except 1,2,3, and 6. We progressively add more appliances; see Table 5.

In Ins. 1, we assign consumption profiles in the horizon and we consider the production from the PV. Both models give the same cost.

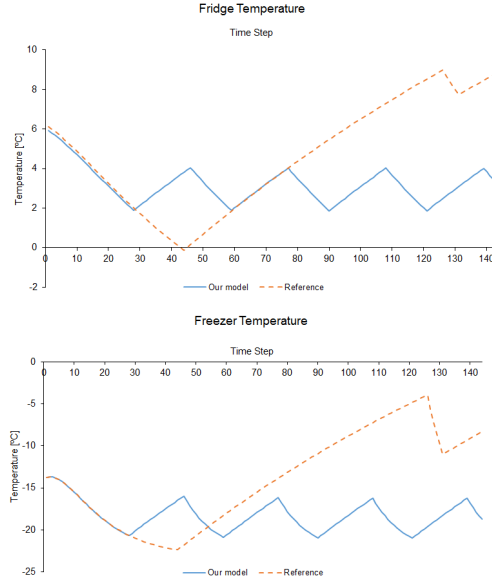


Figure 12: Results for temperature of reference fridge model from [46] and our model.

Table 5: Costs for model from [46] and our model.

Ins.	Combination	[46] cost (\$)	our cost (\$)
1	“initial”	12.6557	12.6557
2	Ins. 1 + ESS	11.6846	10.4669
3	Ins. 2 + ESS*	11.6463	10.4669
4	Ins. 1 + CHP	12.6557	12.3806
5	Ins. 1 + WT	3.5393	3.5393
6	Ins. 5 + ESS	4.5219	0.8055
7	Ins. 6 + ESS*	4.5219	0.8055
8	Ins. 6 + EV	infeasible	21.8308
9	Ins. 8 + CHP	infeasible	21.8308
10	Every appliance	infeasible	30.0261
11	Ins. 5 + CHP	3.5393	3.521
12	Ins. 11 + off grid, WS	infeasible	20.7146
13	Ins. 5 + EV	infeasible	23.3725
14	Ins. 5 + EV	infeasible	3.9683
15	Ins. 5 + EV*	11.0408	3.9683
16	Ins. 15 + ESS	10.6995	2.7601
17	Ins. 16 + CHP	10.6995	2.7601

In Ins. 2, we add the ESS. In contrast to [45], the ESS model in [46] is designed specifically for a flywheel. The [46] solution continuously charges and discharges at the same time to maintain feasibility. It is 1.12 times more expensive than our solution.

Given the continuous charging and discharging operation of Ins. 2, in Ins. 3 we assume that the ESS power variables in [46] ($p_{min}^{ess,r}$ and $p_{max}^{ess,r} : r \in \{eex, eim\}$) should be allowed to be zero, allowing the model to either charge or discharge. We change Constraints 10 from [46] to: $y_t p_{min}^{ess,r} \leq p_t^{ess,r} \leq y_t p_{max}^{ess,r} \forall r \in \{eex, eim\}, t \in T$, where $y_t \in \{0, 1\}$ is an on/off variable at time t . This is represented by ESS* in Table 5. In the [46] results, the battery does not always charge and discharge at the same time. This explains the cost decrease compared to Ins. 2.

Ins. 4 uses the appliances from Ins. 1 plus CHP. The CHP is off in the first interval. The costs for the two models are almost the same. In our model CHP is used between intervals 99 and 123, and in the [46] model CHP is not used. This will be discussed in our analysis of Ins. 11.

In Ins. 5, we add the production from WT. There is no WT in [46], but it could decrease the cost by a factor of 4 compared with the cost for Ins. 1. We therefore considered WT for the next experiments.

In Ins. 6, we add ESS to Ins. 5. The [46] solution continuously charges and discharges at the same time to maintain feasibility: it is 5.6 times more expensive than our solution. The difference between the [46] solution and our solution is larger in Ins. 6 than in Ins. 2. Moreover, we can compare the cost reduction after the addition of WT, using the results from Ins. 2 and 6. We reduced the cost by a factor of 13 while the [46] model reduced it by 2.6. This is because when there is energy coming from WT, the battery has to be used more to sell energy at high prices. If the battery model has limitations, increased use of the battery will have an impact on the cost. If batteries are not considered, the differences are reduced, as shown by the solutions for Ins. 4 and 11.

We use ESS* for Ins. 7, as in Ins. 3. Compared to Ins. 6, the results do not change because $y_t = 1 \forall t \in T$. In Ins. 8, 9, and 10, the addition of EVs makes the models from [46] infeasible. We will explain this below.

Ins. 11 has the same appliances as Ins. 5 plus CHP. The CHP is off in the first interval. Our model and the [46] model give similar costs. In our model CHP is used between intervals 113 and 118; in the [46] model it is not used. This is because the ramp-up and ramp-down constraints in [46] do not allow the CHP to change state. This can be problematic when there is more demand or in off-grid systems.

In Ins. 12, we force the use of CHP via an off-grid system. If a surplus of energy occurs, it is lost, i.e., we waste the surplus (WS). Since fuel is more expensive than electricity in our data, the cost of meeting the demand is higher, and it cannot be met by the [46] model because of the ramp-up and ramp-down constraints.

In Ins. 13, we consider the appliances from Ins. 5 plus EV. The infeasibility in the [46] model is because that model cannot consider the case where the vehicle departs at time k_0 , returns at time k_1 , and departs again at time k_2 ($k_2 > k_1 > k_0$). This implies noncyclic behavior since there is an odd number of departure-arrival events. The bounds $E_t^{evh,ub}$ and $E_t^{evh,lb}$ in the [46] model cannot handle this case.

In Ins. 14, we change the EV parameters to allow only one departure, where the vehicle is plugged in at interval 7 and the departure is at interval 110. Here, the scenario is appropriate for the [46] model, but there is no zero energy assignment when the EV is traveling because of Constraint 22 from [46]. In Ins. 15, we changed this constraint to: $e_t^{evh} = e_{t-1}^{evh} + (p_t^{evh,eev} \eta^{evh,eev} - \frac{p_t^{evh,eim}}{\eta^{evh,eim}}) \Delta_{t2} + k_t^{evh} (E_t^{evh,ub} - e_{t-1}^{evh}) \forall t \in T$. This change is indicated by EV* in Table 5. It corrects the problem of Ins. 14, but the inequalities 21 from [46] do not allow the power variables to be zero. This increases the cost, and our cost is around 278% cheaper.

Ins. 16 adds ESS to Ins. 15; Ins. 17 adds CHP to Ins. 16. For both models, ESS reduces the final cost, but CHP does not (it is not used). Our cost is 389% cheaper.

4.4 Application

We now present an application of our model to a house in Belo Horizonte, Brazil, during the summer. We used a PC with a 19 Ghz Intel Core i5-4300U and 8 GB of RAM. Figure 13 shows the forecast data, the electricity price, and the behavior of the appliances in the optimal solution.

In the Water Temperature and Water Temperature in Shower plots, a Temp. wish value of zero means that the user does not care about the temperature. The plots show that the temperature needs are almost met. For instance, the water temperature is acceptable for the shower and for general use after step 36. The temperature before step 12 is unacceptable because the WH power and the thermal energy provided by CHP cannot sufficiently increase the initial temperature in the interval before the first and second uses. This is confirmed by the plot *Fraction of time with power on*, which shows that WH is on from the start until near step 12. The CHP operates at a maximal speed until step 12. The air temperature deviates from the target around 12 a.m. because the A/C power is insufficient. Note that in *Fraction of time with power on* the A/C is on whenever the temperature is above the target. The fridge and freezer quickly adjust their internal temperatures.

Discharge mode is preferred for the batteries when the price is high. The customer does not buy energy between steps 96 and 125 because it is too expensive. Moreover, CHP is preferred around those times. The EV makes its planned trips and is used for storage when electricity is expensive, e.g., in the steps around 100. The appliances with load profiles are used during the permitted time windows.

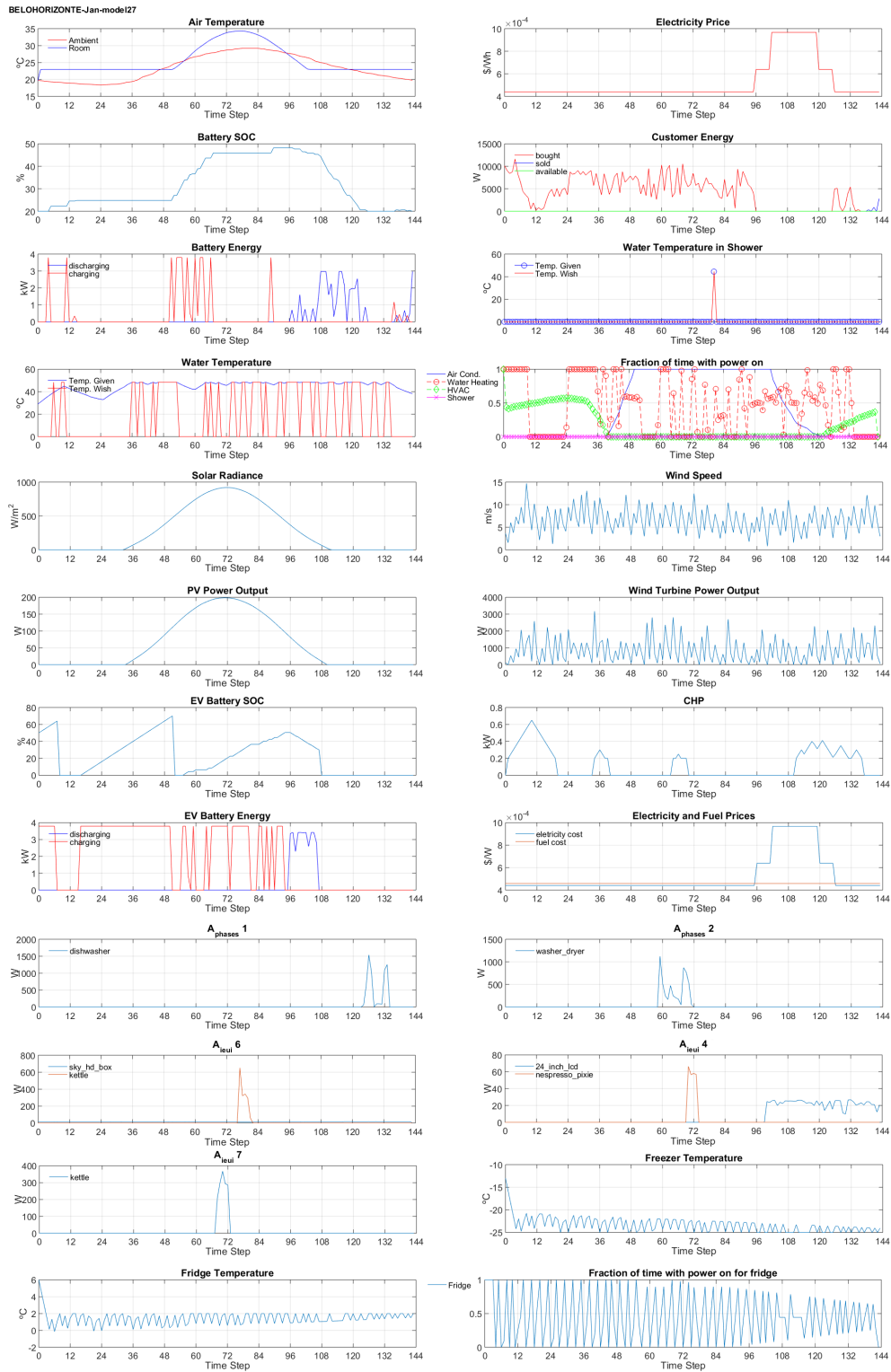


Figure 13: Results for Belo Horizonte in January.

There are 105391 constraints and 14886 variables. Around 30% of the variables are integers. CPLEX solved the problem in 36.64s.

5 Conclusions

The number of smart homes is expanding worldwide. At the same time, EMS research is exploring how to use energy in an optimal way. One approach is to model the appliances of smart homes, but current models have limitations. We define specific models for the various appliances and integrate them into a new optimization model. We compare our model with models from the literature, showing that it overcomes the limitations of other models. We also present an example of the use of our model. Future work will extend it to consider multiple smart homes and aggregators.

Appendix I Model

Sets

T	Set of all time sub-intervals in the entire scheduling time interval.
A	Set of electric appliances.
$A_{IEUI} \subseteq A$	Set of inelastic appliances with an uninterruptible operation.
T	Set of all time sub-intervals in the entire scheduling time interval.
A	Set of electric appliances.
$A_{IEUI} \subseteq A$	Set of inelastic appliances with an uninterruptible operation.
A_{IEUI}^t	Set of tasks from appliances $\in A_{IEUI}$.
$A_{IEUI}^{t,a} \subseteq A_{IEUI}^t$	Set of tasks from A_{IEUI}^t of the same appliance $a \in A_{IEUI}$.
$A_{Phases} \subseteq A$	Set of appliances with interruptible phases.
A_{Phases}^t	Set of tasks from appliances $a \in A_{Phases}$.
$A_{Phases}^{t,a} \subseteq A_{Phases}^t$	Set of tasks from A_{Phases}^t of the same appliance $a \in A_{Phases}$.
$A^t \in \{A_{Phases}^t \cup A_{IEUI}^t\}$	Set of tasks of appliances $a \subseteq A$.
D_{air}	set of durations for hot or cold air needs.
D_{wt}	set of durations for hot water needs.
D_{chu}	set of durations for shower needs.
Ξ	set of all variables in the problem.
B	set of blocks in IBR.

Variables

C^t	Monetary cost at sub-interval t (\$).
E_g^t	Amount of fixed power took from the grid at sub-interval t (W).
E_v^t	Amount of fixed power put back in the grid at sub-interval t (W).
x_a^t	Amount of fixed power consumption of appliance a at sub-interval t (W).
E_{av}^t	Amount of fixed power credit available at sub-interval t (W).
V_a^t	Discomfort related to temperature deviation from target temperature of appliance a at sub-interval t .
U_a^t	Discomfort related to time of use deviation from target time of use for appliance a at sub-interval t .
ζ_a	Discomfort of not doing the task $a \in A_{IEUI}^t$.
$\Psi_{a,p}$	Discomfort of not doing the operation of phase p of task $a \in A_{Phases}^t$.
$P_{bat}^{ch,t}$	Charging power of battery (kW).
$P_{bat}^{dch,t}$	Discharging power of battery (kW).
$y_{bat}^{ch,t}$	Integer that indicates whether the charging operation of battery is on or off at sub-interval t .
$y_{bat}^{dch,t}$	Integer that indicates whether the discharging operation of battery is on or off at sub-interval t .
SOC^t	Battery's state of charge at sub-interval t (%).
y_{float}^t	Binary that represent the activation of float losses at sub-interval t .
T_{room}^t	Room temperature at sub-interval t (°C).
y_{HVAC}^t	Integer that indicates whether the operation of heating is on or off at sub-interval t .
y_{air}^t	Integer that indicates whether the operation of air-conditioning is on or off at sub-interval t .
z_{air}^t	Fraction of Δ_t in time step t in which air-conditioning machine is on.
z_{HVAC}^t	Fraction of Δ_t in time step t in which heating machine is on.
$T_{out,wh}^t$	Mixed water temperature in the WH at sub-interval t (°C).
y_{wh}^t	Integer that indicates whether the operation of WH is on or off at sub-interval t .
$T_{chu,hand}^t$	Shower water temperature manually adjusted at sub-interval t (°C).
$T_{out,chu}^t$	Water temperature getting out of the shower at sub-interval t (°C).
$y_{chu,hot}^t$	Integer that indicates whether the operation of resistance shower is on or off at sub-interval t .

$T_{freezer}^t$	Freezer temperature at sub-interval t ($^{\circ}\text{C}$).
T_{refri}^t	Fridge temperature at sub-interval t ($^{\circ}\text{C}$).
y_{comp}^t	Integer that indicates whether the operation of refrigerator's compressor is on or off at sub-interval t .
y_a^t	Integer that indicates whether the operation of task $a \in A_{IEUI}^t$ starts at sub-interval t .
$y_{a,b}$	Binary that indicates if task $a \in A_{IEUI}^t$ has to be done after task $b \in A_{IEUI}^t$ be finished.
$y_{a,p}^t$	Integer that indicates whether the operation of phase p of task $a \in A_{Phases}^t$ starts at sub-interval t .
$y_{a,b}^p$	Binary that indicates if task $a \in A_{Phases}^t$ has to be done after task $b \in A_{Phases}^t$ be finished.
C_{ev}	Cost of the fuel used in the EV (\$).
$P_{EVbat}^{ch,t}$	Charging power of EV battery (kW).
$P_{EVbat}^{dch,t}$	Discharging power of EV battery (kW).
$y_{EVbat}^{ch,t}$	Integer that indicates whether the charging operation of EV battery is on or off at sub-interval t .
$y_{EVbat}^{dch,t}$	Integer that indicates whether the discharging operation of EV battery is on or off at sub-interval t .
$EVSOC^t$	EV battery's state of charge at sub-interval t (%).
$y_{EVfloat}^t$	Binary that represent the activation of float losses for each period t in EV battery.
Km_{fuel}^s	Amount of Km from trip s using fuel (km).
C_{ev}^s	Cost related to the fuel used in trip s (\$).
P_{CHPe}^t	Electrical power output from CHP at period of time t (KW).
P_{CHPt}^t	Thermal power output from CHP at period of time t (KW).
$y_{CHP}^t \in \{0,1\}$	Determine if CHP is on or off at sub-interval t .
$z_{CHP}^t \in \{0,1\}$	Determine if CHP is turned on at sub-interval t .
C_{CHP}^t	Cost related to CHP's fuel at sub-interval t (\$).
$P_{CHP,fuel}^t$	Power input for CHP from fuel conversion at sub-interval t (KW).
$zd_{CHP}^t \in \{0,1\}$	Determine if CHP is turned off at sub-interval t .
$y_{up}^t \in \{0,1\}$	Determine if CHP is increasing its power output at sub-interval t .
$y_{down}^t \in \{0,1\}$	Determine if CHP is decreasing its power output at sub-interval t .
w_{up}^t	Fraction of time period t in which CHP is increasing its power output.
w_{down}^t	Fraction of time period t in which CHP is decreasing its power output.
$E_g^{t,i}$	Amount of fixed power took from the grid at sub-interval t inside block $i \in B$ (W).
$y_g^i \in \{0,1\}$	Determine if block $i \in B$ is selected or not.

Constants

H	Number of hours in the entire scheduling time (hours). Suppose that t is an index to represent a sub-interval of time and we want to analyze $H = 24$ hours. If we cut this day in 2 sub-intervals, so, $\Delta t = 12$, $ T = 2$, $t \in T = \{1, 2\}$, where $t = 1$ represent the interval time between 00:00h to 11:59h and $t = 2$ represent the interval time between 12:00h to 23:59h.
$\Delta t_2 = H/ T $	Duration of each time sub-interval (hours).
$\Delta t = 60\Delta t_2$	Duration of each time slot (minutes).
w_u	Weight factor for the discomfort level related to preferred start time of appliances.
w_c	Weight factor for the cost.
w_t	Weight factor for the temperature discomfort level.
λ^t	Price for unit energy consumption from grid at sub-interval t (\$/Wh).
ν^t	Price for unit energy selling to grid at sub-interval t (\$/Wh).
E_{pv}^t	Fixed power forecasted from photo-voltaic panels at sub-interval t (W).
E_{wt}^t	Fixed power forecasted from wind turbines at sub-interval t (W).
E_{th}^t	Grid power capacity obtained at sub-interval t (W).
S_a	Preferable starting time for task $a \in A_{IEUI}^t$ (h).
DS_a	Preferable deadline to starting time for task $a \in A_{IEUI}^t$ (h).
F_a	Preferable finishing time for task $a \in A_{IEUI}^t$ (h).
DF_a	Preferable deadline to finish time for task $a \in A_{IEUI}^t$ (h).
D_a	Usage duration for task $a \in A_{IEUI}^t$ (h).
P_a^h	Fixed power of task $a \in A_{IEUI}^t$ at interval h of his load profile (W).
$P_{a,p}^h$	Fixed power for phase p of task $a \in A_{Phases}^t$ at interval h of his load profile (W).
$D_{a,p}$	Usage duration for phase p of task $a \in A_{Phases}^t$ (h).
PH_a	Number of phases of task $a \in A_{Phases}^t$.

T_o^t	Ambient temperature at sub-interval t ($^{\circ}\text{C}$).
T_f^t	Thermostat set point temperature for HVAC at sub-interval t ($^{\circ}\text{C}$).
$P_{heating}$	Rated power for heating (W).
P_{cool}	Rated power for cooling (W).
ρ_{air}	Density of air (kg/m^3).
V_{house}	Volume of the house (m^3).
C_{air}	Heat capacity ($\text{J}/\text{kg}^{\circ}\text{C}$).
A_{wall}	Area of the wall (m^2).
$A_{ceiling}$	Area of the ceiling (m^2).
A_{window}	Area of the window (m^2).
R_{wall}	Heat resistance of the wall ($\text{m}^2 \text{ }^{\circ}\text{C}.\text{h}/\text{J}$).
$R_{ceiling}$	Heat resistance of the ceiling ($\text{m}^2 \text{ }^{\circ}\text{C}.\text{h}/\text{J}$).
R_{window}	Heat resistance of the window ($\text{m}^2 \text{ }^{\circ}\text{C}.\text{h}/\text{J}$).
n_{ac}	Number of air changes (1/h).
$SHGC$	Solar Heat Gain Coefficient average.
H_{sun}^t	Solar radiation heat power at sub-interval t (W/m^2).
$P_{activity}^t$	Metabolic rates for a given activity at sub-interval t (W/m^2).
A_{body}	Person's body area inside the home (m^2).
V_{tank}	Volume of the WH (dcm^3).
P_{wh}	Rated power for WH (W).
fr_{wh}^t	Hot water flow rate for WH at sub-interval t (gpm).
T_{wh}^t	Preferable temperature for hot water ($^{\circ}\text{C}$).
T_{in}^t	Water temperature from solar collector or street at sub-interval t ($^{\circ}\text{C}$).
$(UA)_{wh}$	Loss coefficient area product for WH ($\text{W}/^{\circ}\text{C}$).
$C_p = (4190 \text{ J}/\text{kg}^{\circ}\text{C})$	Thermal capacity.
T_{max}	Maximal temperature for water.
E_{wh}	Binary constant to say if water heating machine exists or not.
P_{chu}	Rated power for shower resistance (W).
fr_{chu}^t	Hot water flow rate for shower at sub-interval t (gal/min).
$C_e = (4190 \text{ J}/\text{kg}^{\circ}\text{C})$	Water specific heat.
T_{inlet}^t	Water temperature coming from street at sub-interval t ($^{\circ}\text{C}$).
T_f^{chu}	Preferable temperature for hot water in shower ($^{\circ}\text{C}$).
E_{bat}	Battery capacity (kWh).
SOC_{max}	Upper bound of battery SOC (%) without float loss.
SOC_{min}	Lower bound of battery SOC (%).
P_{max}^{ch}	Battery maximum charging power (kW).
P_{max}^{dch}	Battery maximum discharging power (kW).
P_{min}^{ch}	Battery minimum charging power (kW).
P_{min}^{dch}	Battery minimum discharging power (kW).
η^{ch}	Inverter battery's charging efficiency.
η^{dch}	Inverter battery's discharging efficiency.
μ	Battery's charging/discharging efficiency.
p_{loss}	Battery's power float loss (kW).
$T_{freezer}^{start}$	Initial freezer temperature ($^{\circ}\text{C}$).
T_{refri}^{start}	Initial fridge temperature ($^{\circ}\text{C}$).
P_{comp}	Rated power for refrigerator's compressor machine (W).
$T_f^{freezer}$	Preferable temperature for freezer compartment ($^{\circ}\text{C}$).
T_f^{refri}	Preferable temperature for fridge compartment ($^{\circ}\text{C}$).
EV_{bat}	EV battery capacity (kWh).
$EV_{SOC_{max}}$	Upper bound of EV battery SOC (%).
$EV_{SOC_{min}}$	Lower bound of EV battery SOC (%).
$EV_{P_{max}^{ch}}$	EV battery maximum charging power (kW).
$EV_{P_{max}^{dch}}$	EV battery maximum discharging power (kW).
$EV_{P_{min}^{ch}}$	EV battery minimum charging power (kW).
$EV_{P_{min}^{dch}}$	EV battery minimum discharging power (kW).
$EV_{\eta^{ch}}$	Inverter EV battery's charging efficiency.
$EV_{\eta^{dch}}$	Inverter EV battery's discharging efficiency.
EV_{μ}	EV battery's charging/discharging efficiency.
$EV_{p_{loss}}$	EV battery's power float loss (kW).
n_{trip}	Number of complete trips in the day.
t_{start}^s	Step time in which car arrive at home at trip s . Car arrives at the end of period.

t_s^{end}	Step time in which car get out from home for trip s . Car leaves at the beginning of period.
km_{100}^{end}	Car autonomy with SOC equals to 100% (km).
$EV SOC_{min}^{end}$	Minimal SOC wish when car get out from home (%).
$consumption_gas$	Distance attained by one liter of fuel (km/l).
$price_gas$	Fuel price per liter (\$/l).
$P_{gas} = \frac{price_gas}{consumption_gas}$	Fuel price per km wheeled (\$/km).
$EV SOC_retorno_car$	Forecasted SOC when car arrive at home. (%)
$EV SOC_last_day$	EV SOC battery at the end of the previous day.(%)
Km_{next}^s	Distance forecated to next trip s (km).
$rate_{CHP}^{fuel}$	Fuel price per kWh for CHP (l/KWh).
$price_{CHP}^{on}$	CHP start up cost (\$).
d_{CHP}^{on}	Give the minimal duration that CHP has to be on after turned on (min).
d_{CHP}^{off}	Give the minimal duration that CHP has to be off after turned off (min).
nb_{CHP}^{η}	Number of piecewise functions for the overall efficiency function.
$P_{CHP,min}^{k,\eta} \forall k = \{1..nb_{CHP}^{\eta}\}$	Minimal power at breakpoint k in micro-CHP piecewise linear overall efficiency function (KW).
$P_{CHP,max}^{k,\eta} \forall k = \{1..nb_{CHP}^{\eta}\}$	Maximal power at breakpoint k in micro-CHP piecewise linear overall efficiency function (KW).
$\eta_{CHP,p}^k \forall k = \{1..nb_{CHP}^{\eta}\}$	Image at breakpoint k in micro-CHP piecewise linear overall efficiency function (%).
nb_{CHP}^{μ}	Number of piecewise for electrical generation efficiency function.
$P_{CHP,min}^{k,\mu} \forall k = \{1..nb_{CHP}^{\mu}\}$	Minimal power at breakpoint k in micro-CHP piecewise linear electrical generation efficiency function (KW).
$P_{CHP,max}^{k,\mu} \forall k = \{1..nb_{CHP}^{\mu}\}$	Maximal power at breakpoint k in micro-CHP piecewise linear electrical generation efficiency function (KW).
$\mu_{CHP,p}^k \forall k = \{1..nb_{CHP}^{\mu}\}$	Image at breakpoint k in micro-CHP piecewise linear electrical generation efficiency function (%).
r_{CHP}^{down}	Maximum ramp down for CHP (kW/min).
r_{CHP}^{up}	Maximum ramp up for CHP (kW/min).
$P_{KW} = rate_{CHP}^{fuel} \times price_gas$	Price for each KW generated (\$/KWh).
$y_{CHP}^{last_day}$	Determines if CHP is on or off at last time period in the day before.
$\bar{\eta}$	Overall efficiency average.
$\bar{\mu}$	Electrical generation efficiency average.
y_{CHP}^{ini}	Initial value for variable y_{CHP} .
P_{CHPe}^{ini}	Initial value for variable P_{CHPe} (kW).
$E.bl^i$	Represents the limit of energy to be considered inside of block $i \in B$ (Wh).
$discount^i$	Discount to stay in block $i \in B$ (%).

$$\min_{\Xi} w_c \left(\sum_{t \in T} C^t + C_{ev} \right) + w_t \left(\sum_{t \in T} \sum_{a \in A} V_a^t \right) + w_u \left(\sum_{a \in A_{phases}^t} \sum_{p=1}^{|PH_a|} \Psi_{a,p} + \sum_{a \in A_{IEUI}^t} \zeta_a + \sum_{t \in T} \sum_{a \in A^t} U_a^t \right) \quad (8)$$

$$C^t = \Delta_t \left[\lambda^t \left(\sum_{i \in B} (1 - discount^i) E_g^{t,i} \right) - \nu^t E_v^t \right] + C_{CHP}^t \quad \forall t \in T \quad (9)$$

$$\sum_{a \in A} x_a^t = 10^3 (P_{EVbatt}^{ch,t} + P_{bat}^{ch,t} - P_{bat}^{dch,t} - P_{EVbatt}^{dch,t} - P_{CHPe}^t) + E_{pv}^t - E_v^t + E_{wt}^t + \sum_{i \in B} E_g^{t,i} \quad \forall t \in T \quad (10)$$

$$E_v^t \leq E_{th}^t \quad \forall t \in T \quad (11)$$

$$E_g^{t,i} \leq E_{th}^t \quad \forall t \in T, i \in B \quad (12)$$

$$\Delta_t \sum_{t \in T} E_g^{t,i} \leq E.bl^i \quad \forall i \in B \quad (13)$$

$$\Delta_t E_g^{t,i} \leq E.bl^i y_g^i \quad \forall t \in T, i \in B \quad (14)$$

$$\sum_{i \in B} y_g^i \leq 1 \quad (15)$$

$$y_g^i \in \{0, 1\} \quad \forall i \in B \quad (16)$$

$$T_{room}^{t+1} = T_{room}^t + \frac{\Delta t_2}{V_{house} \rho_{air} c_{air}} (G^t + 3600 (P_{heating} z_{HVAC}^t - P_{cool} z_{air}^t)) \quad \forall t \in T / \{|T|\} \quad (17)$$

$$G^t = \frac{\frac{A_{ceiling}}{R_{ceiling}} + \frac{A_{wall}}{R_{wall}} + \frac{A_{window}}{R_{window}} + n_{ac} V_{house} \rho_{air} c_{air}}{+3600(SHGC)A_{window}H_{sun}^t + 3600H_p^t} (T_o^t - T_{room}^t) \quad \forall t \in T/\{|T|\} \quad (18)$$

$$T_{room}^1 = T_o^1 \quad (19)$$

$$y_{HVAC}^t \in \{0, 1\} \quad \forall t \in T \quad (20)$$

$$y_{air}^t \in \{0, 1\} \quad \forall t \in T \quad (21)$$

$$x_{HVAC}^t = P_{heating} z_{HVAC}^t + P_{cool} z_{air}^t \quad \forall t \in T \quad (22)$$

$$1 \geq y_{HVAC}^t + y_{air}^t \quad \forall t \in T \quad (23)$$

$$z_{HVAC}^t \leq y_{HVAC}^t \quad \forall t \in T \quad (24)$$

$$z_{air}^t \leq y_{air}^t \quad \forall t \in T \quad (25)$$

$$z_{HVAC}^t \geq 0 \quad \forall t \in T \quad (26)$$

$$z_{air}^t \geq 0 \quad \forall t \in T \quad (27)$$

$$V_{HVAC}^t \geq T_{room}^t - T_f^t \quad \forall t \in D_{air} \quad (28)$$

$$V_{HVAC}^t \geq -T_{room}^t + T_f^t \quad \forall t \in D_{air} \quad (29)$$

$$T_{out,wh}^{t+1} = \frac{T_{out,wh}^t + \frac{\Delta t_2}{(v_{tank} C_p)} [-227.4 f_{r_{wh}}^t C_p (T_{out,wh}^t - T_{in}^t)]}{+3.6 \times 10^3 (x_{wt}^t + 1000 P_{CHP}^t) - 3600(UA)_{wh} (T_{out,wh}^t - T_{room}^t)} \quad \forall t \in T/\{|T|\} \quad (30)$$

$$T_{out,wh}^1 = T_{in}^1 \quad (31)$$

$$x_{wt}^t = P_{wt} y_{wt}^t \quad \forall t \in T \quad (32)$$

$$x_{wt}^t = 0 \quad \forall t \notin D_{wt} \quad (33)$$

$$y_{wt}^t \in \{0, 1\} \quad \forall t \in T \quad (34)$$

$$V_{wh}^t \geq T_f^{wh} - T_{out,wh}^t \quad \forall t \in D_{wh} \quad (35)$$

$$V_{wh}^t \geq -T_f^{wh} + T_{out,wh}^t \quad \forall t \in D_{wh} \quad (36)$$

$$T_{out,wh}^t \leq T_{in}^{max} E_{wh} + T_{in}^t (1 - E_{wh}) \quad \forall t \in T \quad (37)$$

$$T_{chu,hand}^t \leq T_{out,wh}^t \quad \forall t \in D_{chu} \quad (38)$$

$$T_{chu,hand}^t \geq T_{inlet}^t \quad \forall t \in D_{chu} \quad (39)$$

$$T_{out,chu}^t = T_{chu,hand}^t + \frac{60 x_{chu}^t}{3.79 \times C_e f_{r_{chu}}^t} \quad \forall t \in D_{chu} \quad (40)$$

$$x_{chu}^t = P_{chu} y_{chu,hot}^t \quad \forall t \in D_{chu} \quad (41)$$

$$x_{chu}^t = 0 \quad \forall t \notin D_{chu} \quad (42)$$

$$V_{chu}^t \geq T_f^{chu} - T_{out,chu}^t \quad \forall t \in D_{chu} \quad (43)$$

$$V_{chu}^t \geq -T_f^{chu} + T_{out,chu}^t \quad \forall t \in D_{chu} \quad (44)$$

$$y_{chu,hot}^t \in \{0, 1\} \quad \forall t \in D_{chu} \quad (45)$$

$$SOC^{t+1} = SOC^t + \frac{100 \Delta t_2}{E_{bat}} \left(P_{bat}^{ch,t} \eta^{ch} \mu - \frac{P_{bat}^{dch,t}}{\eta^{dch} \mu} - p_{loss} y_{float}^t \right) \quad \forall t \in T \setminus \{|T|\} \quad (46)$$

$$SOC^t \geq SOC_{min} \quad \forall t \in T \quad (47)$$

$$SOC^t \leq SOC_{max} + (100 - SOC_{max}) y_{float}^t \quad \forall t \in T \quad (48)$$

$$SOC^1 = SOC_{min} \quad (49)$$

$$P_{bat}^{ch,t} \leq \frac{P_{max}^{ch}}{\eta^{ch} \mu} y_{bat}^{ch,t} \quad \forall t \in T \quad (50)$$

$$P_{bat}^{ch,t} \geq \frac{P_{min}^{ch}}{\eta^{ch} \mu} y_{bat}^{ch,t} \quad \forall t \in T \quad (51)$$

$$P_{bat}^{dch,t} \leq P_{max}^{dch} \eta^{dch} \mu y_{bat}^{dch,t} \quad \forall t \in T \quad (52)$$

$$P_{bat}^{dch,t} \geq P_{min}^{dch} \eta^{dch} \mu y_{bat}^{dch,t} \quad \forall t \in T \quad (53)$$

$$1 \geq y_{bat}^{dch,t} + y_{bat}^{ch,t} \quad \forall t \in T \quad (54)$$

$$y_{bat}^{dch,t} \in \{0, 1\} \quad \forall t \in T \quad (55)$$

$$y_{bat}^{ch,t} \in \{0, 1\} \quad \forall t \in T \quad (56)$$

$$y_{float}^t \in \{0, 1\} \quad \forall t \in T \quad (57)$$

$$T_{freezer}^{t+1} = T_{freezer}^t - \frac{7\Delta t}{25} y_{comp}^t + \frac{15\Delta t}{67} (1 - y_{comp}^t) \quad \forall t \in T \setminus \{|T|\} \quad (58)$$

$$T_{refri}^{t+1} = T_{refri}^t - 0.1467\Delta t y_{comp}^t + 0.1196\Delta t (1 - y_{comp}^t) \quad \forall t \in T \setminus \{|T|\} \quad (59)$$

$$T_{freezer}^1 = T_{freezer}^{start} \quad (60)$$

$$T_{refri}^1 = T_{refri}^{start} \quad (61)$$

$$x_{refri}^t = P_{comp} y_{comp}^t \quad \forall t \in T \quad (62)$$

$$x_{freezer}^t = 0 \quad \forall t \in T \quad (63)$$

$$V_{freezer}^t \geq T_f^{freezer} - T_{freezer}^t \quad \forall t \in T \quad (64)$$

$$V_{freezer}^t \geq 0 \quad \forall t \in T \quad (65)$$

$$V_{refri}^t \geq T_f^{refri} - T_{refri}^t \quad \forall t \in T \quad (66)$$

$$V_{refri}^t \geq 0 \quad \forall t \in T \quad (67)$$

$$y_{comp}^t \in \{0, 1\} \quad \forall t \in T \quad (68)$$

$$y_{CHP}^t \geq z_{CHP}^t \quad \forall t \in T, \quad \forall \tau \in \{t..min(t + \lceil \frac{d_{CHP}^{on}}{\Delta t} \rceil - 1, |T|)\} \quad (69)$$

$$y_{CHP}^t \leq 1 - z_{CHP}^t \quad \forall t \in T, \quad \forall \tau \in \{t..min(t + \lceil \frac{d_{CHP}^{off}}{\Delta t} \rceil - 1, |T|)\} \quad (70)$$

$$z_{CHP}^{t+1} \leq 1 - y_{CHP}^t \quad \forall t \in \{1..|T| - 1\} \quad (71)$$

$$z_{CHP}^{t+1} \leq y_{CHP}^t \quad \forall t \in \{1..|T| - 1\} \quad (72)$$

$$z_{CHP}^t \geq y_{CHP}^t - y_{CHP}^{t-1} \quad \forall t \in \{2..|T|\} \quad (73)$$

$$z_{CHP}^t \geq -y_{CHP}^t + y_{CHP}^{t-1} \quad \forall t \in \{2..|T|\} \quad (74)$$

$$1 \geq z_{CHP}^t + z_{CHP}^t \quad \forall t \in T \quad (75)$$

$$z_{CHP}^1 \geq y_{CHP}^1 - y_{CHP}^{last.day} \quad (76)$$

$$y_{CHP}^1 = y_{CHP}^{ini} \quad (77)$$

$$P_{CHPe}^1 = P_{CHPe}^{ini} \quad (78)$$

$$P_{CHPe}^{t+1} \geq P_{CHPe}^t - 60\Delta t_2 r_{CHP}^{down} w_{down}^t - max(P_{CHP,max}^{1,\mu}, P_{CHP,max}^{1,\eta}) z_{CHP}^{t+1} \quad \forall t \in \{1..|T| - 1\} \quad (79)$$

$$P_{CHPe}^{t+1} \leq P_{CHPe}^t + 60\Delta t_2 r_{CHP}^{up} w_{up}^t + max(P_{CHP,max}^{1,\mu}, P_{CHP,max}^{1,\eta}) z_{CHP}^{t+1} \quad \forall t \in \{1..|T| - 1\} \quad (80)$$

$$1 \geq w_{up}^t + z_{CHP}^{t+1} \quad \forall t \in \{1..|T| - 1\} \quad (81)$$

$$1 \geq w_{down}^t + z_{CHP}^{t+1} \quad \forall t \in \{1..|T| - 1\} \quad (82)$$

$$w_{up}^t \leq y_{CHP}^t \quad \forall t \in T \quad (83)$$

$$w_{down}^t \leq y_{CHP}^t \quad \forall t \in T \quad (84)$$

$$P_{CHPe}^t \geq max(P_{CHP,max}^{1,\mu}, P_{CHP,max}^{1,\eta}) y_{CHP}^t \quad \forall t \in T \quad (85)$$

$$P_{CHPe}^t \leq min(P_{CHP,max}^{nb,\mu}, P_{CHP,max}^{nb,\eta}) y_{CHP}^t \quad \forall t \in T \quad (86)$$

$$P_{CHPe}^t + P_{CHPt}^t = P_{CHP,fuel}^t \bar{\eta} \quad \forall t \in T \quad (87)$$

$$P_{CHPt}^t = P_{CHPe}^t \bar{\mu} \quad \forall t \in T \quad (88)$$

$$C_{CHP}^t \geq P_{CHP,fuel}^t P_{KW} \Delta t_2 \quad \forall t \in T \quad (89)$$

$$z_{CHP}^t, z_{dCHP}^t \in \{0, 1\} \quad \forall t \in T \quad (90)$$

$$y_{CHP}^t \in \{0, 1\} \quad \forall t \in T \quad (91)$$

$$w_{up}^t, w_{down}^t \leq 1 \quad \forall t \in T \quad (92)$$

$$w_{up}^t, w_{down}^t \geq 0 \quad \forall t \in T \quad (93)$$

$$EV SOC^{t+1} = \frac{EV SOC^t + \frac{100\Delta t}{EEV_{bat}} (P_{EVbat}^{ch,t} \eta_{Ev}^{ch} \mu_{Ev} - \frac{P_{EVbat}^{dch,t}}{\eta_{Ev}^{dch} \mu_{Ev}} - EV P_{loss} y_{EVfloat}^t)}{\quad} \quad \forall t \in T \setminus \{|T|\}, s \in \{1..n_{trips}\}: \quad (94)$$

$$t_{start}^s \leq i < t_{end}^s, t_{start}^1 \leq t < t_{end}^1$$

$$EV SOC^{t+1} = \frac{EV SOC^t + \frac{100\Delta t}{EEV_{bat}} (P_{EVbat}^{ch,t} \eta_{Ev}^{ch} \mu_{Ev} - \frac{P_{EVbat}^{dch,t}}{\eta_{Ev}^{dch} \mu_{Ev}} - EV P_{loss} y_{EVfloat}^t)}{\quad} \quad \forall s \in \{1..n_{trips}\}, \quad (95)$$

$$t \in \{1..t_{end}^1 - 1\} \cup \{t_{start}^s..t_{end}^{s+1} - 1\}: t_{start}^1 > t_{end}^1$$

$$EV SOC^t \geq 0 \quad \forall t \in T \quad (96)$$

$$EV SOC^t \leq \frac{EV SOC_{max} + (100 - EV SOC_{max}) y_{EVfloat}^t}{\quad} \quad \forall t \in T \quad (97)$$

$$EV SOC^1 = EV SOC_{lat_day} \quad : t_{start}^1 > t_{end}^1 \quad (98)$$

$$EV SOC^{t_{start}^1} = EV SOC_{retorno_car} \quad : t_{start}^1 < t_{end}^1 \quad (99)$$

$$EV SOC^{t_{start}^s} = EV SOC^{t_{end}^s} - 100 \frac{Km_{next}^s - Km_{fuel}^s}{Km^{100}} \quad \forall s \in \{1..n_{trips}\}: \quad (100)$$

$$t_{start}^s > t_{end}^s$$

$$EV SOC^{t_{start}^{s+1}} = EV SOC^{t_{end}^s} - 100 \frac{Km_{next}^s - Km_{fuel}^s}{Km^{100}} \quad \forall s \in \{1..n_{trips} - 1\}: \quad (101)$$

$$t_{start}^1 < t_{end}^1$$

$$EV SOC^{t_{end}^s} \geq EV SOC_{min}^{end} \quad \forall s \in \{1..n_{trips}\} \quad (102)$$

$$EV SOC^{t_{end}^{n_{trips}+1}} \geq EV SOC_{min}^{end} \quad : t_{start}^1 > t_{end}^1 \quad (103)$$

$$P_{EVbat}^{ch,t} \leq \frac{EV P_{max}^{ch}}{\eta_{Ev}^{ch} \mu_{Ev}} y_{EVbat}^{ch,t} \quad \forall t \in T \quad (104)$$

$$P_{EVbat}^{ch,t} \geq \frac{EV P_{min}^{ch}}{\eta_{Ev}^{ch} \mu_{Ev}} y_{EVbat}^{ch,t} \quad \forall t \in T \quad (105)$$

$$P_{EVbat}^{dch,t} \leq EV P_{max}^{dch} \eta_{Ev}^{dch} \mu_{Ev} y_{EVbat}^{dch,t} \quad \forall t \in T \quad (106)$$

$$P_{EVbat}^{dch,t} \geq EV P_{min}^{dch} \eta_{Ev}^{dch} \mu_{Ev} y_{EVbat}^{dch,t} \quad \forall t \in T \quad (107)$$

$$y_{EVbat}^{dch,t} + y_{EVbat}^{ch,t} \leq 1 \quad \forall t \in T \quad (108)$$

$$y_{EVbat}^{dch,t} = 0 \quad \forall s \in \{1..n_{trips}\}, t \in \{t_{end}^s \leq t < t_{start}^s\}: \quad (109)$$

$$t_{start}^1 > t_{end}^1$$

$$y_{EVbat}^{ch,t} = 0 \quad \forall s \in \{1..n_{trips}\}, t \in \{t_{end}^s \leq t < t_{start}^s\}: \quad (110)$$

$$t_{start}^1 > t_{end}^1$$

$$y_{EVfloat}^t = 0 \quad \forall s \in \{1..n_{trips}\}, t \in \{t_{end}^s \leq t < t_{start}^s\}: \quad (111)$$

$$t_{start}^1 > t_{end}^1$$

$$EV SOC^t = 0 \quad \forall s \in \{1..n_{trips}\}, t \in \{t_{end}^s < t < t_{start}^s\}: \quad (112)$$

$$t_{start}^1 > t_{end}^1$$

$$y_{EVbat}^{dch,t} = 0 \quad \forall i \in \{t_{end}^{n_{trips}+1}..|T|\} : \quad (113)$$

$$t_{start}^1 > t_{end}^1$$

$$y_{EVbat}^{ch,t} = 0 \quad \forall i \in \{t_{end}^{n_{trips}+1}..|T|\} : \quad (114)$$

$$t_{start}^1 > t_{end}^1$$

$$y_{EVfloat}^t = 0 \quad \forall i \in \{t_{end}^{n_{trips}+1}..|T|\} : \quad (115)$$

$$t_{start}^1 > t_{end}^1$$

$$EV SOC^t = 0 \quad \forall i \in \{t_{end}^{n_{trips}+1} + 1..|T|\} : \quad (116)$$

$$t_{start}^1 > t_{end}^1$$

$$y_{EVbat}^{dch,t} = 0 \quad \forall t \in \{1..t_{start}^1 - 1\}: \quad (117)$$

$$t_{start}^1 < t_{end}^1$$

$$y_{EVbat}^{ch,t} = 0 \quad \forall t \in \{1..t_{start}^1 - 1\}: \quad (118)$$

$$t_{start}^1 < t_{end}^1$$

$$y_{EVfloat}^t = 0 \quad \forall t \in \{1..t_{start}^1 - 1\}: \quad (119)$$

$$t_{start}^1 < t_{end}^1$$

$$EV SOC^t = 0 \quad \forall t \in \{1..t_{start}^1 - 1\}: \quad (120)$$

$$t_{start}^1 < t_{end}^1$$

$$y_{EVbat}^{dch,t} = 0 \quad \forall s \in \{1..n_{trips} - 1\}, t \in \{t_{end}^s..t_{start}^{s+1} - 1\}: \quad (121)$$

$$t_{start}^1 < t_{end}^1$$

$$y_{EVbat}^{ch,t} = 0 \quad \forall s \in \{1..n_{trips} - 1\}, t \in \{t_{end}^s..t_{start}^{s+1} - 1\}: \quad (122)$$

$$t_{start}^1 < t_{end}^1$$

$$y_{EVfloat}^t = 0 \quad \forall s \in \{1..n_{trips}-1\}, t \in \{t_{end}^s..t_{start}^{s+1}-1\}: \quad (123)$$

$$EVSOC^t = 0 \quad \forall s \in \{1..n_{trips}-1\}, t \in \{t_{end}^s+1..t_{start}^{s+1}-1\}: \quad (124)$$

$$y_{EVbat}^{dch,t} = 0 \quad \forall t \in \{t_{end}^{n_{trips}}..|T|\}: \quad (125)$$

$$y_{EVbat}^{ch,t} = 0 \quad \forall t \in \{t_{end}^{n_{trips}}..|T|\}: \quad (126)$$

$$y_{EVfloat}^t = 0 \quad \forall t \in \{t_{end}^{n_{trips}}..|T|\}: \quad (127)$$

$$EVSOC^t = 0 \quad \forall t \in \{t_{end}^{n_{trips}}+1..|T|\}: \quad (128)$$

$$C_{ev}^s \geq Km_{fuel}^s P_{gas} \quad \forall s \in \{1..n_{trips}\} \quad (129)$$

$$C_{ev} \geq \sum_{s=1}^{n_{trips}} C_{ev}^s \quad (130)$$

$$y_{EVbat}^{dch,t} \in \{0,1\} \quad \forall t \in T \quad (131)$$

$$y_{EVbat}^{ch,t} \in \{0,1\} \quad \forall t \in T \quad (132)$$

$$y_{EVfloat}^t \in \{0,1\} \quad \forall t \in T \quad (133)$$

$$y_{EVfloat}^t \in \{0,1\} \quad \forall t \in T \quad (134)$$

$$y_{EVfloat}^t \in \{0,1\} \quad \forall t \in T \quad (135)$$

$$C_{ev}^s \geq 0 \quad \forall s \in \{1..n_{trips}\} \quad (136)$$

$$Km_{fuel}^s \geq 0 \quad \forall s \in \{1..n_{trips}\} \quad (137)$$

$$C_{ev} \geq 0 \quad (138)$$

$$x_a^{t+h} \geq P_a^h y_a^t \quad \forall a \in A_{IEUI}^t, \forall t \in T, \quad (139)$$

$$\forall h \in \{0, \dots, \lceil \frac{D_a}{\Delta t_2} \rceil - 1\}: t+h \leq |T|$$

$$x_a^t \leq \sum_{i=\max(1, t - (\lceil \frac{D_a}{\Delta t_2} \rceil - 1) + 1)}^t P_a^{t-i} y_a^i \quad \forall a \in A_{IEUI}^t, \forall t \in T \quad (140)$$

$$y_a^t = 0 \quad \forall a \in A_{IEUI}^t, \quad (141)$$

$$t \in T: t > |T| - (\lceil \frac{D_a}{\Delta t_2} \rceil - 1)$$

$$\sum_{t=1}^{|T| - (\lceil \frac{D_a}{\Delta t_2} \rceil - 1)} y_a^t + \zeta_a = 1 \quad \forall a \in A_{IEUI}^t \quad (142)$$

$$\sum_{tt=1}^{|T| - (\lceil \frac{D_a}{\Delta t_2} \rceil - 1)} t t y_a^{tt} \geq \sum_{t=1}^{|T| - (\lceil \frac{D_b}{\Delta t_2} \rceil - 1)} t y_b^t \quad (143)$$

$$\forall a \in A_{IEUI}^t, \forall b \in A_{IEUI}^t, \forall k \in A_{IEUI}: a \in A_{IEUI}^{t,k}, b \in A_{IEUI}^{t,k}$$

$$\sum_{tt=1}^{|T| - (\lceil \frac{D_b}{\Delta t_2} \rceil - 1)} t t y_b^{tt} \geq \sum_{t=1}^{|T| - (\lceil \frac{D_a}{\Delta t_2} \rceil - 1)} t y_a^t \quad (144)$$

$$\forall a \in A_{IEUI}^t, \forall b \in A_{IEUI}^t, \forall k \in A_{IEUI}: a \in A_{IEUI}^{t,k}, b \in A_{IEUI}^{t,k}$$

$$U_a^t \geq 0 \quad \forall a \in A_{IEUI}^t, \forall t \in T \quad (145)$$

$$U_a^t \geq S_a y_a^t - t y_a^t \quad \forall a \in A_{IEUI}^t, \forall t \in T \quad (146)$$

$$U_a^t \geq -D S_a y_a^t + t y_a^t \quad \forall a \in A_{IEUI}^t, \forall t \in T \quad (147)$$

$$\zeta_a \geq 0 \quad \forall a \in A_{IEUI}^t \quad (148)$$

$$y_a^t \in \{0,1\} \quad \forall a \in A_{IEUI}^t, \forall t \in T \quad (149)$$

$$y_{b,a} \in \{0,1\} \quad \forall a \in A_{IEUI}^t, \forall b \in A_{IEUI}^t, \forall k \in A_{IEUI}: a \in A_{IEUI}^{t,k}, b \in A_{IEUI}^{t,k} \quad (150)$$

$$\forall k \in A_{IEUI}: a \in A_{IEUI}^{t,k}, b \in A_{IEUI}^{t,k}$$

$$x_a^{t+h} \geq P_{a,p}^h y_{a,p}^t \quad \forall a \in A_{Phases}^t, \forall t \in T, \forall p=1..PH_a, \forall h \in \{0, \dots, \lceil \frac{D_{a,p}}{\Delta t_2} \rceil - 1\}: t+h \leq |T| \quad (151)$$

$$x_a^t \leq \sum_{p=1}^{PH_a} \sum_{i=\max(1, t - (\lceil \frac{D_{a,p}}{\Delta t_2} \rceil - 1) + 1)}^t P_{a,p}^{t-i} y_{a,p}^i \quad \forall a \in A_{Phases}^t, \forall t \in T \quad (152)$$

$$y_{a,p}^t = 0 \quad \forall a \in A_{Phases}^t, p=1..PH_a, t \in T: t > |T| - (\lceil \frac{D_{a,p}}{\Delta t_2} \rceil - 1) \quad (153)$$

$$\sum_{t=1}^{|T| - (\lceil \frac{D_{a,p}}{\Delta t_2} \rceil - 1)} y_{a,p}^t + \Psi_{a,p} = 1 \quad \forall a \in A_{Phases}^t, p=1..PH_a \quad (154)$$

$$\sum_{tt=t}^{|T|} y_{a,p}^{tt} + \sum_{tt=1}^{t-1 + \lceil \frac{D_{a,p}}{\Delta t_2} \rceil} y_{a,p+1}^{tt} \leq 1 \quad \forall a \in A_{Phases}^t, \forall p=1..PH_a-1, \forall t \in T: t < |T| + 1 - \lceil \frac{D_{a,p}}{\Delta t_2} \rceil \quad (155)$$

$$\sum_{tt=1}^{|T| - (\lceil \frac{\sum_{p=1}^{PH_a} D_{a,p}}{\Delta t_2} \rceil - 1)} tt y_{a,1}^{tt} \geq \sum_{t=1}^{|T| - (\lceil \frac{D_{b,PH_b}}{\Delta t_2} \rceil - 1)} t y_{b,PH_b}^t + (\lceil \frac{D_{b,PH_b}}{\Delta t_2} \rceil)(1 - \Psi_{b,1}) - |T| y_{b,a}^p \quad \forall a \in A_{Phases}^t, \forall b \in A_{Phases}^t, \forall k \in A_{Phases}: a \in A_{Phases}^{t,k}, b \in A_{Phases}^{t,k} \quad (156)$$

$$\sum_{tt=1}^{|T| - (\lceil \frac{\sum_{p=1}^{PH_b} D_{b,p}}{\Delta t_2} \rceil - 1)} tt y_{b,1}^{tt} \geq \sum_{t=1}^{|T| - (\lceil \frac{D_{a,PH_a}}{\Delta t_2} \rceil - 1)} t y_{a,PH_a}^t + (\lceil \frac{D_{a,PH_a}}{\Delta t_2} \rceil)(1 - \Psi_{a,1}) - |T|(1 - y_{b,a}^p) \quad \forall a \in A_{Phases}^t, \forall b \in A_{Phases}^t, \forall k \in A_{Phases}: a \in A_{Phases}^{t,k}, b \in A_{Phases}^{t,k} \quad (157)$$

$$U_a^t \geq 0 \quad \forall a \in A_{Phases}^t, \forall t \in T \quad (158)$$

$$U_a^t \geq S_a y_{a,1}^t - t y_{a,1}^t \quad \forall a \in A_{Phases}^t, \forall t \in T \quad (159)$$

$$U_a^t \geq -D S_a y_{a,1}^t + t y_{a,1}^t \quad \forall a \in A_{Phases}^t, \forall t \in T \quad (160)$$

$$U_a^t \geq F_a y_{a,PH_a}^t - t y_{a,PH_a}^t \quad \forall a \in A_{Phases}^t, \forall t \in T \quad (161)$$

$$U_a^t \geq -D F_a y_{a,PH_a}^t + t y_{a,PH_a}^t \quad \forall a \in A_{Phases}^t, \forall t \in T \quad (162)$$

$$\Psi_{a,p} \geq 0 \quad \forall a \in A_{Phases}^t, \forall p=1..PH_a \quad (163)$$

$$y_{a,p}^t \in \{0, 1\} \quad \forall a \in A_{Phases}^t, \forall t \in T, \forall p=1..PH_a \quad (164)$$

$$y_{b,a}^p \in \{0, 1\} \quad \forall a \in A_{Phases}^t, \forall b \in A_{Phases}^t, \forall k \in A_{Phases}: a \in A_{Phases}^{t,k}, b \in A_{Phases}^{t,k} \quad (165)$$

Appendix II Auxiliary equations

$$1000(p_{EVbatt}^{ch,t} + P_{bat}^{ch,t}) + \sum_{a \in A} x_a^t + E_{av}^t = E_{av}^{t-1} + E_{wt}^t + E_{pv}^t + \sum_{i \in B} E_g^{t,i} + 1000(p_{EVbatt}^{dch,t} + P_{bat}^{dch,t} + P_{CHPe}^t) \quad \forall t \in T \quad (166)$$

Appendix III Temperature refrigerator equations

$$T_{eq,f}^t = \frac{(U A_f + \dot{m}_d c_{pa}) T_a^t + U A_m T_r^t + \frac{r \dot{m}_a c_{pa}}{3600} (T_{newton} y_{comp}^t + (\kappa)(1 - y_{comp}^t) + \frac{3600 w_{fan}}{\dot{m}_a C_{p,a}} y_{comp}^t) y_{comp}^t}{U A_f + U A_m + \dot{m}_d C_{p,a} + \frac{\dot{m}_f C_{p,a}}{3600} y_{comp}^t} \quad (167)$$

$$= \frac{(U A_f + \dot{m}_d c_{pa}) T_a^t + U A_m T_r^t + \frac{r \dot{m}_a c_{pa}}{3600} (T_{newton} y_{comp}^t + \frac{3600 w_{fan}}{\dot{m}_a C_{p,a}} y_{comp}^t)}{U A_f + U A_m + \dot{m}_d C_{p,a} + \frac{\dot{m}_f C_{p,a}}{3600} y_{comp}^t} \quad \text{because } (y_{comp}^t)^2 = y_{comp}^t$$

$$\begin{aligned}
T_{eq,r} &= \frac{(UA_r + \dot{m}_{d,r} c_{pa}) T_a^t + UA_m T_f^t + \frac{(1-r)\dot{m}_a c_{pa}}{3600} (T_{newton} y_{comp}^t + (\kappa)(1-y_{comp}^t) + \frac{3600wfan}{\dot{m}_a C_{p,a}} y_{comp}^t) y_{comp}^t}{UA_r + UA_m + \dot{m}_d C_{p,a} + \frac{\dot{m}_r C_{p,a}}{3600} y_{comp}^t} \\
&= \frac{(UA_r + \dot{m}_{d,r} c_{pa}) T_a^t + UA_m T_f^t + \frac{(1-r)\dot{m}_a c_{pa}}{3600} (T_{newton} y_{comp}^t + \frac{3600wfan}{\dot{m}_a C_{p,a}} y_{comp}^t)}{UA_r + UA_m + \dot{m}_d C_{p,a} + \frac{\dot{m}_r C_{p,a}}{3600} y_{comp}^t} \text{ because } (y_{comp}^t)^2 = y_{comp}^t
\end{aligned} \tag{168}$$

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