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# Incorporating geological and equipment performance uncertainty while optimizing short-term mine production schedules

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**Abstract:** Short-term production scheduling in open pit mining consists of defining the extraction sequence and process allocation of mineralized material over time-scales of either months, weeks or days. An effective short-term production schedule will ensure compliance with the production targets and restrictions imposed by the long-term plan. The method proposed herein outlines a new approach to simultaneously optimize the short-term production sequence with the mobile equipment allocation plan while incorporating both material type, grade and equipment performance uncertainty. A new simulation methodology is introduced to generate more realistic equipment performance scenarios, as well as a new concept of including ramp positions in the formulation to facilitate minable extraction patterns. This short-term model is benchmarked against the conventional design at one of the largest copper mines in the world, and the results show improved production target compliance by delivering more consistent material quantity and quality to each processing destination, and a physical extraction sequence that has a greater likelihood of being executed according to plan in the face of equipment performance and truck cycle time uncertainty.

# 1 Introduction

Production scheduling of open pit mines consists of defining the extraction sequence and process allocation of mineralized material. These decisions are made at different time scales which consider different objectives subject to different technical and operational constraints. Short-term mine production scheduling (STMPS) entails developing an extraction sequence at short time-scales of either months, weeks or days. The objectives in STMPS are significantly different than those used in long-term scheduling since the optimizer is no longer motivated by time-valued cash flows. Given that the long-term plan forecasts the value that will be recovered within one fiscal year, the motivation to target high metal grades in earlier periods no longer exists. The goals of short-term production scheduling are to maximize compliance with the long-term plan by: i) delivering consistent tonnage and grades to each processing destination; ii) minimizing the number of shovel moves necessary to extract new material; and iii) minimizing the non-productive time of the fleet by ensuring that adequate trucking resource is allocated to each shovel.

STMPS optimization gained considerable interest since the 1970's given the substantial value that can be added by minimizing operating costs. Wilke and Reimer (1977) develop a linear programming (LP) model for an STMPS that maximizes profit while delivering material that satisfies mining and milling capacities and blending requirements. Fytas (1985) also develops an LP formulation to optimize an STMPS constrained by the amount of ore and waste that can be produced each period, along with multi-element blending requirements. As LP approaches may lead to infeasible mining progression, integer programming has since been deemed a more appropriate method for optimizing mine production schedules. Smith (1998) develops a mixed-integer programming (MIP) formulation to maximize ore production complying with grade blending requirements. The author concludes that the major drawback of the approach is the computational burden required to solve for many binary variables given large, multi-period applications. Alternatively, Kumral and Dowd (2002) develop a multi-objective simulated annealing heuristic to optimize an STMPS problem. In this approach, sub-optimal solutions derived from Lagrangian parametrization are improved considering ore quantity and ore quality.

The LP and MIP methods described above are what Alarie and Gamache (2002) define as the upper-stage of the common multi-stage STMPS optimization approach. In this framework, the upper-stage generates the extraction schedule satisfying blending and capacity constraints; this schedule then imposes the targets for the lower-stage which entails optimizing the dispatching of mobile equipment throughout the mine. The separation of these highly integrated problems inevitably leads lost value in the form of under-utilized equipment, higher operating cost, and extraction sequences that cannot be achieved given the fleets true productivity. The optimization of the production schedule and the fleet allocation plan should be done simultaneously to allow for complete communication between these variables to create more valuable and more attainable plans. L'Heureux et al. (2013) incorporate the sequencing of production activities such as drilling, blasting, and extraction into their MIP formulation along with a shovel movement optimization. A small example where production is scheduled on large aggregates of blocks is used, and the proposed model lacks the ability to be scaled to real problem sizes. Eivazy and Askari-Nasab (2012) formulate an MIP that incorporates fixed-grade stockpile reclamation, variable destinations, and variable truck haulage routing. The model is solved using CPLEX by aggregating blocks into clusters to reduce the size of the problem. Both optimization models reduce the number of decision variables by relying on aggregated scheduling units. Mousavi et al. (2016) highlight three reasons why aggregation should be avoided. First, it reduces the granularity of the solution, which misrepresents the selectivity of the mining equipment. Second, averaging block grades through aggregation intensifies the grade smoothing effect which will mislead the optimizer. Lastly, when schedules optimized using aggregated units are evaluated at the block level, the solutions are often infeasible in the original context of the problem due to the re-exposure of grade variability. The authors develop a block-level optimization model that includes stockpile reclamation and more detailed block access requirements. The problem is solved using a hybrid heuristic involving simulated annealing, branch and bound and large neighbourhood search.

The major drawback of these short-term optimization models is that they ignore the variability and uncertainty in both the geological characteristics and the fleet performance. The shortcomings of conventional grade estimation techniques and the detrimental impacts they impose on production schedules have been studied extensively in the past (Ravenscroft, 1992; Vallée, 2000; Dimitrakopoulos et al., 2002; Godoy, 2003; Ramazan and Dimitrakopoulos, 2013; others). Dimitrakopoulos and Jewbali (2013) incorporate grade uncertainty in the schedule optimization of a large gold mine. The short-term grade uncertainty is modelled by updating the stochastic simulations with simulated future grade control data using conditional simulation by successive residuals (CSSR). A stochastic integer programming (SIP) formulation (Dimitrakopoulos and Ramazan, 2008) is used to maximize NPV and minimize the ore and grade target deviations, but this method ignores the positioning and productivity of the mobile equipment fleet. Ignoring uncertainty pertaining to equipment performance can also have substantial negative ramifications on short-term production scheduling.

Random fluctuations in weather, operator habits, road conditions and maintenance trends are all factors that lead to mobile equipment performance variability. The failure to extract the scheduled material will result in deviations from the plan that might cause rippling effects throughout the life of mine. There has been very little investigation as to how equipment performance uncertainty, jointly with material and grade uncertainty, can be incorporated into STMPS considering that all mining operations observe these effects in practice. Topal and Ramazan (2010) propose an MIP formulation to minimize the maintenance cost of using mining equipment under the assumption that the cost varies with equipment age. This formulation is extended to incorporate stochastic maintenance cost as well (Topal and Ramazan, 2012). Burt et al. (2016) adopt a similar concept for the problem of purchasing and salvaging equipment for long-term mine production, given that equipment availability and maintenance costs vary with age. Unfortunately, these approaches consider equipment performance on a long-term scale and therefore cannot be adopted to STMPS.

Matamoros and Dimitrakopoulos (2014) introduce a model that incorporates both geological (grades and material types) and equipment performance uncertainty into an STMPS optimization. Geostatistical simulations are coupled with historical equipment performance data to inform an optimization model with both sources of uncertainty. The optimizer aims to meet the monthly grade blending requirements while generating minable shapes that respect a pre-defined direction of mining. This work has four drawbacks. First, the model only allows the monthly production to deviate below the uncertain shovel capacity. The production schedule is then ultimately bounded by the worst-case shovel scenario, negating the value of stochastic information. Second, the authors treat equipment utilization as a result of the optimized schedule rather than an uncertain input to scheduling opimization. Equipment utilization reflects unplanned non-productive time, however, the authors define utilization as the ratio between scheduled production time and the available equipment time. It is not realistic to assume that once a piece of equipment has completed its planned production, it will remain idle until the end of the period. Third, the equipment scenarios are generated independently. Fleet performance parameters of different units of equipment tend to show strong correlations to one another and the simulation approach should respect such relationships. Lastly, the approach used to ensure mineable shapes is inefficient and is limited to models with few decision variables.

A new formulation to optimize STMPS, proposed herein, builds on the model introduced by Matamoros and Dimitrakopoulos (2014). The model optimizes short-term schedules by minimizing the production deviations from targets provided by the long-term plan, and by generating extraction patterns that can be achieved with far greater confidence given the fleet's uncertain levels of productivity (availability). These objectives are optimized while accounting for uncertainty in both material types, grades, equipment performance ) and truck cycle times. Below are the contributions and model improvements that will be incorporated in this stochastic short-term mine production scheduling (SSTMPS) optimization approach:

1. Adapt the short-term model to accommodate mining complexes; which are large mining operations comprised of multiple mines, multiple material types and multiple processing destinations.
2. Include location-dependent shovel movement in the formulation.
3. Develop a new simulation approach to generate realistic equipment performance scenarios that reproduce the intrinsic correlations between different equipment performance parameters.
4. Introduce the concept of horizontal block access constraints to facilitate smoother mining patterns that can be incorporated without increasing solving time

The following section outlines the main concepts, as well as the full formulation of the SSTMPS optimization model. The model is then applied to a very large copper mining complex to assess the effectiveness of the stochastic optimization when compared to a more conventional approach. Finally, conclusions and venues of future work are discussed.

# 2 Stochastic short-term mine production scheduling

## 2.1 Short-term production scheduling concepts

The time-step for STMPS can vary from months, weeks, days or shifts, however, the final extent of the time horizon should coincide with the amount of available material taken from the long-term plan. The three-dimensional space of a mining operation is partitioned into two distinct domains, as can be seen in Figure 1. Selective mining units (SMU), or *blocks*, are the smallest discretization of space in mine production scheduling. Each block has a known position, metal grade(s), tonnage, and can be extracted in one period. An *area* represents a pre-defined region of the mine comprised of a large grouping of blocks. In the optimization model, areas are used to describe equipment positions, different truck cycle times, shovel movements and the associated movement cost.

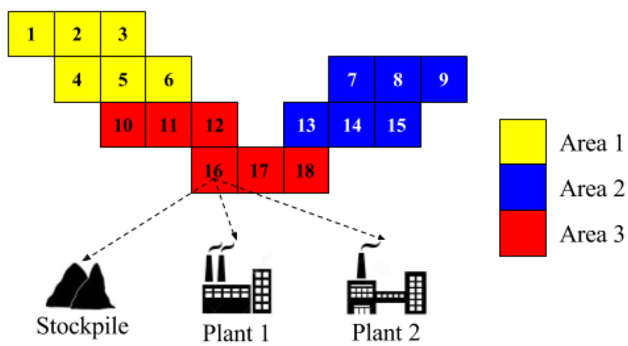


**Figure 1: Configuration of blocks within areas and possible destinations of a block.**

## 2.2 Mineable schedules with horizontal precedence

An STMPS must be implementable because it usually directly precedes operational planning. Shovel movement, shovel proximities, equipment access and face progression must be incorporated to make this planning transition as seamless. Aside from traditional vertical precedence constraints, encouraging spatially connected blocks to be extracted in the same period will yield extraction patterns that are easily minable and will limit unnecessary shovel movement. The direction of mining in an implementable schedule is also important; this is typically enforced by adding some of the immediate in-bench neighbours to each block's precedence set. This precedence direction is consistent for every block in the mine and must be pre-defined as one of the four directions: access from the north, south, east or west. There are two major drawbacks to this approach: it tends to produce square, unnatural-looking schedules, and the direction of mining should be allowed to vary by mine and by region, all of which will have different access points.

A new approach is proposed to ensure reasonable mining shapes and progression of mining. Specifically, a modified pre-processing technique that allows for each block in the mine to have a unique horizontal precedence set along a continuum of access directions. The underlying assumption is that a significant amount of planning has already been established prior to the STMPS optimization. For each bench, in each area being considered in the short-term time horizon, a permanent or temporary ramp should have already been designed. Knowing the locations of these ramp entrance points, a unique direction of access for each block can be established. Figure 2 shows an example of how a ramp's positioning (blue) is used to establish the horizontal precedence set (cyan) of certain blocks (red).

The pre-processing technique to determine the horizontal precedence of each block can be summarized as follows:

1. Identify the bench and area that a block belongs to.
2. Establish the direction to the appropriate ramp access point.
3. Allow for a tolerance angle $\beta$ about the access direction. The magnitude of this angle can be varied to impose equipment access constraints.
4. Add each block within the tolerance range to the horizontal precedence set. The transitivity property of the precedence constraints is exploited; only the minimal set of preceding blocks is needed.

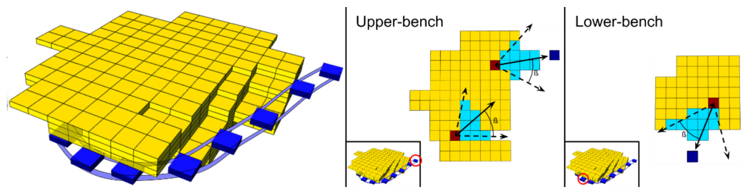


**Figure 2: An example area of a mine and the associated ramp design (left), horizontal precedence sets on the upper-bench (middle), and a horizontal precedence set on the lower-bench (right).**

## 2.3 Uncertain model parameters

This optimization model is formulated as a stochastic framework considering three sources of uncertainty: 1) metal grades and associated material types, 2) equipment performance and 3) truck cycle times. Each of these types of uncertainty is included in the formulation using simulated scenarios. The mapping of unknown correlated metal grades at the block-level is generated through a joint conditional simulation approach known as DBMAFSIM ( Boucher and Dimitrakopoulos, 2009). To quantify the uncertain equipment performance, three different parameters are simulated: i) the availability; ii) the utilization of all mobile equipment; and (iii) the hourly production of the shovels. The overall tonnage production for each piece of equipment can then be described using these parameters. Since the underlying factors that cause this variability impact the entire fleet simultaneously, these performance indicators tend to be strongly correlated, and the simulations must reproduce these relationships. The joint simulation of equipment performance parameters is done following a projection into an orthogonal space using principal component analysis (PCA) (Hotelling, 1933). Following the independent simulation of the uncorrelated components, the final simulation is generated using a back-rotation into the original space. Finally, since truck cycle times are also highly variable, although not necessarily directly related to availability and utilization, including them in a stochastic framework might allow for more robust equipment matching decisions in short-term mine planning. For each truck type, each area and each destination, a distribution for cycle times is sampled calculated from historical data.

## 2.4 Model formulation

The model is formulated as an SIP (Birge and Louveaux, 1997). The model yields two primary deliverables: i) a complete mobile equipment plan, consisting of shovel placements and truck allocation; and ii) a block-level extraction sequence that guides the progression of mining activity. The objectives are to minimize tonnage and grade target deviations at each destination, minimize equipment idle times, and generate a schedule that can be achieved with greater confidence given equipment uncertainty. The formulation is introduced below, followed by the solution approach.

### 2.4.1 Indices

| **Indices** | |
|---|---|
| $t$ | is a time period, $t = 1, \ \ldots, \ T$ |
| $l$ | is an individual shovel (loader), $l = 1, \ \ldots, L$ |
| $m$ | is a model of haulage truck, $m = 1, \ \ldots, \ M$ |
| $j$ | is an area of the mine, $j = 1, \ \ldots, \ J$ |
| $i$ | is a block within area $j$, $i \in i(j)$ |
| $p$ | is a destination (crusher, waste dump, leach-pad, etc.), $p = 1, \ \ldots, P$ |
| $k$ | is a material type, $k = 1, \ \ldots, K$ |
| $\varepsilon$ | is a metal element, $\varepsilon = 1, \ \ldots, \ E$ |
| $s$ | is a geological scenario, $s = 1, \ \ldots, S$ |
| $\alpha$ | is an equipment performance scenario, $\alpha = 1, \ \ldots, A$ |

### 2.4.2 Model parameters

| **Equipment parameters** | |
|---|---|
| $H_t$ | is the total number of hours in period $t$ |
| $A^L_{tlj\alpha}$ | is the availability of shovel $l$ in period $t$ in area $j$ for equipment scenario $\alpha$ |
| $U^L_{tlj\alpha}$ | is the utility of shovel $l$ in period $t$ in area $j$ for equipment scenario $\alpha$ |
| $Q^L_{tlj\alpha}$ | is the production rate (t/h) of shovel $l$ in period $t$ in area $j$ for equipment scenario $\alpha$ |
| $A^M_{tmj\alpha}$ | is the availability of truck type $m$ in period $t$ in area $j$ for equipment scenario $\alpha$ |
| $U^M_{tmj\alpha}$ | is the utility of truck type $m$ in period $t$ in area $j$ for equipment scenario $\alpha$ |
| $Q^M_m$ | is the tonnage capacity of truck type $m$ |
| $R_{tmjp\alpha}$ | is the cycle time for truck type $m$ delivering material from area $j$ to destination $p$ in period $t$ for equipment scenario $\alpha$ |
| $\Lambda^{start}_{0l}$ | is the area location of shovel $l$ at time $t = 0$ |
| $\Lambda^{max}_j$ | is the maximum allowable shovels in area $j$ |
| $M^{max}_m$ | is the maximum number of trucks of type $m$ available in the fleet |
| **Operational parameters** | |
| $Ton_{ji}$ | is the tonnage of block $i$ in area $j$ |
| $\theta_{jis}$ | is the material type of block $i$ in area $j$ for geological scenario $s$ |
| $G_{ji\varepsilon s}$ | is the grade of element $\varepsilon$ of block $i$ in area $j$ for geological scenario $s$ |
| $D^{min}_{tp}, \ D^{max}_{tp}$ | are the minimum/maximum tonnage targets (resp.) at destination $p$ in period $t$ |
| $G^{min}_{tp\varepsilon}, \ G^{max}_{tp\varepsilon}$ | are the min/max grade targets (resp.) for element $\varepsilon$ at destination $p$ in period $t$ |
| $C^{Lmove}_{lj'j}$ | is the cost to move shovel $l$ from area $j'$ to area $j$ |
| $C^{L-}_j, \ C^{L+}_j$ | are the lower/upper deviation costs (resp.) for shovel tonnage production in area $j$ |
| $C^{M-}_j, C^{M+}_j$ | are the lower/upper deviation costs (resp.) for truck hours in area $j$ |
| $C^{D-}_p, \ C^{D+}_p$ | are the lower/upper tonnage target deviation costs (resp.) at destination $p$ |
| $C^{G-}_{p\varepsilon}, \ C^{G+}_{p\varepsilon}$ | are the lower/upper grade target deviation costs (resp.) for element $\varepsilon$ at destination $p$ |
| $F_{jips}$ | dictates whether block $i$ in area $j$ can be sent to destination $p$ in geological scenario $s$ |
| $F_{jips}=$ | $\begin{cases} 1 & \text{If the material type of block } i \text{ in scenario } s \text{ can be sent to destination } p \\ 0 & \text{Otherwise} \end{cases}$ |
| $\rho_v(i(j))$ | is the set of blocks that precede block $i$ in the vertical direction |
| $\rho_h(i(j))$ | is the set of blocks that precede block $i$ in the horizontal direction (Figure 2) |
| $\rho(i(j))$ | is the complete set of blocks that precede block $i$, $\rho\,(i(j)) = \rho_v(i(j)) \cup \rho_h(i(j))$ |

### 2.4.3 Decision variables

| **Integer decision variables** | |
|---|---|
| $x_{tji}=$ | $\begin{cases} 1 & \text{If block } i \text{ in area } j \text{ is extracted } by \text{ period } t \\ 0 & \text{Otherwise} \end{cases}$ |
| $\lambda_{tlj}=$ | $\begin{cases} 1 & \text{If shovel } l \text{ is located in area } j \text{ in period } t \\ 0 & \text{Otherwise} \end{cases}$ |
| $\Omega_{tlj'j}=$ | $\begin{cases} 1 & \text{If shovel } l \text{ moves from area } j' \text{ to area } j \text{ in period } t \\ 0 & \text{Otherwise} \end{cases}$ |
| $\tau_{tmj}$ | is the number of trucks of model $m$ allocated to area $j$ in period $t$ |
| **Continuous decision variables** | |
| $n_{tmjps}$ | is the number of trips that trucks of model $m$ allocated to area $j$ make to destination $p$ in period $t$ for geological scenario |
| $h_{tmjps\alpha}$ | is the hours that trucks of model $m$ allocated to area $j$ require to deliver material to destination $p$ in period $t$ for geological scenario $s$ and equipment performance scenario $\alpha$ |
| $d^{D-}_{tps}$, $d^{D+}_{tps}$ | are the lower/upper tonnage deviations (resp.) at destination $p$ in period $t$ for geological scenario $s$ |
| $d^{G-}_{tp\varepsilon s}$, $d^{G+}_{tp\varepsilon s}$ | are the lower/upper grade deviations (resp.) of element $\varepsilon$ at destination $p$ in period $t$ for geological scenario $s$ |
| $d^{L-}_{tj\alpha}$, $d^{L+}_{tj\alpha}$ | are the lower/upper shovel production deviations (resp.) in area $j$ in period $t$ for equipment performance scenario $\alpha$ |
| $d^{M-}_{tjs\alpha}, d^{M+}_{tjs\alpha}$ | are the lower/upper truck hour deviations (resp.) in area $j$ in period $t$ for geological scenario $s$ and equipment performance scenario $\alpha$ |

### 2.4.4 Objective function

$$
\begin{aligned}
\min &\sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{j'=1}^{J}\sum_{j=1}^{J} C^{Lmove}_{lj'j}\cdot\Omega_{tlj'j} && \text{Part 1} \\
&+\frac{1}{S}\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S} C^{D-}_{p}\cdot d^{D-}_{tps} + C^{D+}_{p}\cdot d^{D+}_{tps} && \text{Part 2} \\
&+\frac{1}{S}\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{\varepsilon=1}^{E}\sum_{s=1}^{S} C^{G-}_{p\varepsilon}\cdot d^{G-}_{tp\varepsilon s} + C^{G+}_{p\varepsilon}\cdot d^{G+}_{tp\varepsilon s} && \text{Part 3} \\
&+\frac{1}{A}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{\alpha=1}^{A} C^{L-}_{j}\cdot d^{L-}_{tj\alpha} + C^{L+}_{j}\cdot d^{L+}_{tj\alpha} && \text{Part 4} \\
&+\frac{1}{S\cdot A}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{\alpha=1}^{A} C^{M-}_{j}\cdot d^{M-}_{tjs\alpha} + C^{M+}_{j}\cdot d^{M+}_{tjs\alpha} && \text{Part 5}
\end{aligned}
\tag{1}
$$

Part 1 of the objective function minimizes the total shovel movement costs. The cost of moving a shovel between two areas is a function of the lateral and vertical distance between the two areas. Parts 2 and 3 minimize the expected costs of deviating from tonnage and grade targets at each destination. For each area in the mine, Part 4 minimizes the costs associated with the discrepancy between the scheduled extraction tonnage and the production provided from the shovels across all equipment scenarios. Part 5 minimizes the deviations between truck hours required to deliver the scheduled material in an area, and the total truck hours assigned to that area. If trucking hours deviate above the required hours, trucks will be idle and under-utilized, and if the hours deviate below, shovels will endure waiting times instead. The optimal number of trucks is assigned to the shovels in an area to improve the equipment productivity in the face of performance and cycle time uncertainty.

#### 2.4.5 Model constraints

**Scheduling constraints**

$$x_{tji} \geq x_{(t-1)ji} \qquad \forall\, t, j, i\,(j) \tag{2}$$

$$x_{tji} \leq x_{tn} \qquad \forall\, t, j, i \in i(j), n \in p(i(j)) \tag{3}$$

$$\begin{aligned} &\sum_{j=1}^{J}\sum_{i=1}^{i(j)} Ton_{ji} \cdot F_{jips} \cdot (x_{tji} - x_{(t-1)ji}) + d_{tps}^{D-} \geq D_{tp}^{min} \quad \forall\, t, p, s \\ &\sum_{j=1}^{J}\sum_{i=1}^{i(j)} Ton_{ji} \cdot F_{jips} \cdot (x_{tji} - x_{(t-1)ji}) - d_{tps}^{D+} \leq D_{tp}^{max} \quad \forall\, t, p, s \end{aligned} \tag{4}$$

$$\begin{aligned} &\sum_{j=1}^{J}\sum_{i=1}^{i(j)} Ton_{ji} \cdot \left(G_{ji\varepsilon s} - G_{tp\varepsilon}^{min}\right) \cdot F_{jips} \cdot (x_{tji} - x_{(t-1)ji}) + d_{tp\varepsilon s}^{G-} \geq 0 \quad \forall\, t, p, \varepsilon, s \\ &\sum_{j=1}^{J}\sum_{i=1}^{i(j)} Ton_{ji} \cdot \left(G_{ji\varepsilon s} - G_{tp\varepsilon}^{max}\right) \cdot F_{jips} \cdot (x_{tji} - x_{(t-1)ji}) - d_{tp\varepsilon s}^{G+} \leq 0 \quad \forall\, t, p,\ \varepsilon, s \end{aligned} \tag{5}$$

Constraints (2) ensure that each block cannot be extracted more than once, and block precedence is enforced by constraints (3). Constraints (4) and (5) set the tonnage and average grade deviations at each destination for each geological scenario respectively.

**Shovel constraints**

$$\sum_{l=1}^{L} \lambda_{tlj} \leq L_j^{max} \qquad \forall\, t, j \tag{6}$$

$$\sum_{j=1}^{J} \lambda_{tlj} = 1 \qquad \forall\, t, l \tag{7}$$

Constraints (6) limit the number of shovels that can operate in a given area in one period. The size of the area will dictate the maximum number of shovels permitted to ensure all units have enough space to operate safely. Constraints (7) enforce the rule that each shovel must be assigned to exactly one area in each period.

$$\lambda_{tlj} + \lambda_{(t-1)lj'} - \Omega_{tlj'j} \leq 1 \qquad \forall\, t, l, j, j' \neq j \tag{8}$$

$$-\lambda_{tlj} + \Omega_{tlj'j} \leq 0 \qquad \forall\, t, l, j, j' \neq j \tag{9}$$

$$-\lambda_{(t-1)lj'} + \Omega_{tlj'j} \leq 0 \qquad \forall\, t, l, j, j' \neq j \tag{10}$$

Constraints (8) to (10) are used to set the shovel movement variables. All three are necessary to ensure that if a shovel is present in two different areas in consecutive periods, the movement variable, $_{tlj'j}$, is activated and the associated movement cost is incurred in the objective function.

$$x_{tji} - x_{(t-1)ji} \leq \sum_{l=1}^{L} \lambda_{tlj} \quad \forall\, t, j, i(j) \tag{11}$$

$$\sum_{i=1}^{i(j)} Ton_{ji} \cdot \left(x_{tji} - x_{(t-1)ji}\right) - \sum_{l=1}^{L} \left(Q_{tlj\alpha}^{L} \cdot H_t \cdot A_{tlj\alpha}^{L} \cdot U_{tlj\alpha}^{L}\right) \cdot \lambda_{tlj} + d_{tj\alpha}^{L-} - d_{tj\alpha}^{L+} = 0 \qquad \forall\, t, j, \alpha \tag{12}$$

Constraints (11) ensure that a block can be extracted only if at least one shovel is present in the area where this block is located. Constraints (12) link the tonnage extracted in each area to the production provided by the shovels present in that area. Since shovel production is uncertain, violations are permitted but are penalized in the objective function.

**Truck constraints**

$$\sum_{j=1}^{J} \tau_{tmj} \leq M_m^{max} \qquad \forall\, t, m \tag{13}$$

$$\sum_{i=1}^{i(j)} Ton_{ji} \cdot F_{jips} \cdot (x_{tji} - x_{(t-1)ji}) - \sum_{m=1}^{M} \left(Q_m^M \cdot n_{tmjps}\right) = 0 \qquad \forall\, t, j, p, s \tag{14}$$

$$h_{tmjps\alpha} - n_{tmjps} \cdot R_{mjp\alpha} = 0 \qquad \forall\, t, m, j, p, s, \alpha \tag{15}$$

$$\sum_{m=1}^{M} \tau_{tmj} \cdot \left(H_t \cdot A_{tmj\alpha}^M \cdot U_{tmj\alpha}^M\right) - \sum_{m=1}^{M} \sum_{p=1}^{P} h_{tmjps\alpha} + d_{tjs\alpha}^{M-} - \ d_{tjs\alpha}^{M+} = 0 \qquad \forall\, t, j, s, \alpha \tag{16}$$

Constraints (13) limit the number of trucks assigned across all areas in each period by the fleet size of each model. Constraints (14) set the number of trips that each truck model in each area requires to deliver the scheduled tonnage to each destination. Unlike shovel production, truck production does not depend on the *equipment performance* scenario since truck production only consists of constant truck payloads and the number of trips; however, the number of trips to each destination does vary by *geological* scenario since a block is sent to different destinations depending on its uncertain material type. Furthermore, the *time* required to deliver material to its destination, and the trucking *time* that will be provided by assigning a truck to an area both depend on the equipment performance scenario. Constraints (15) set the trucking hours required to deliver material in each geological and equipment performance scenario. Constraints (16) link the number of trucks that are assigned to an area to achieve the planned production schedule with as much certainty as possible. The hour deviations set in constraints (16) are minimized in the objective function to maximize equipment productivity.

## 2.5 Solution approach

The SSTMPS model described in Section 2.4 is solved using the branch and cut algorithm implemented in CPLEX v.12.6.1.0 in a Visual Studio 13 (C++) environment. In contrast to long-term planning, there are fewer binary decisions variables in short-term planning since there are fewer blocks. However, this reduction in extraction variables is negated by the additional equipment plan variables which substantially increase the size of the problem. To provide an efficient solution method for the SSTMPS model, two heuristic algorithms are proposed to reduce the solving time.

### 2.5.1 Sliding window heuristic

The goal of the sliding time window heuristic (STWH) (Pochet and Wolsey, 2006) is to reduce the solving time of large integer programs. This is done by iteratively solving relaxed sub-problems that have fewer binary variables. Cullenbine et al. (2011) extend the STWH to the mine production scheduling problem; the block extraction variables within a pre-defined number of periods, or *window*, are kept as binary, whereas, those associated with the remaining periods are relaxed to continuous. The sub-problem is solved, the binary variables are fixed to the solved values, the window is shifted forward by one period, and the process is repeated until all binary variables in the original problem have been fixed. This heuristic approach has shown to substantially reduce the solving time for realistic mine scheduling problem sizes and still achieve an optimality gap of less than 2.5% (Cullenbine et al., 2011). It is noted, however, that these gap results are dependent on the geometry and distribution of the grades in the mineral deposit. The STWH applied to the SSTMPS is outlined as follows:

1. A time window of one period is selected.
2. Model all variables and constraints in period $t$ consistent with the formulation described in Section 2.4. Model all binary decisions as continuous in periods $t'$, where $t' > t$.
3. Solve the relaxed sub-problem and fix the binary variables associated with period $t$.
4. Move the window to period $t + 1$.
5. Repeat until all periods have been considered.

### 2.5.2 Early start heuristic

The solving time of each STWH iteration can be reduced by fixing a number of binary variables using the earliest start heuristic (ESH) (Topal, 2008). This heuristic eliminates binary block extraction decisions based on the logic that it is impossible to mine some blocks in certain periods given the capacities for shovel production in active mining areas. The binary variable associated with extracting a block in a given period can be fixed to zero if that block is preceded by an amount of material greater than the extraction capacity of that period. The earliest period that a block can be extracted can be defined, and all earlier extraction decisions can be eliminated. Three variants of this ESH are used at the beginning of each sliding window iteration where the extraction variables are only binary in period $t$:

1. **Shovel production**. For each block $i$ that has yet to be mined by period $t$, a recursive algorithm is used to determine the complete set of blocks that precede block $i$ that also remain to be extracted by period $t$. The total tonnage contained within this complete precedence set is compared to the total shovel production capacity of period $t$. Since shovel production is uncertain, the performance scenarios with the highest production rates for each shovel are combined to form an upper-bound. If the total tonnage of blocks requiring extraction to access block $i$ exceeds the best case shovel performance, block $i$ is deemed unreachable and the decision variable $x_{tji}$, is fixed to zero.
2. **Shovel presence**. This variant exploits the structure of the SSTMPS model; blocks in area $j$ can only be extracted in period $t$ if area $j$ is *active*, meaning at least one shovel is present in area $j$ during period $t$ (constraints (11)) and shovels must be present in exactly one area for the entire duration of one period (constraints (7)). Therefore, given the complete precedence set of block $i$ and knowing which area $j$ each of these preceding blocks belongs to, the total number of active areas required to access block $i$ can be determined. If the active areas required exceeds the total number of shovels in the fleet, the block is deemed unreachable and the decision variable $x_{tji}$, is fixed to zero. Figure 3 helps demonstrate this variant of the EHS. In a mine with two shovels, the extraction of block 2 in period $t$ could be eliminated since three shovels would have to be present in areas 1, 2 and 3 in period $t$. On the other hand, block 1 could be extracted in period $t$ since only two shovels would need to be present in areas 2 and 3.
3. **Area elimination**. Following the previous two EHS variants, another pass is made to determine if an entire area of the mine can be eliminated. If the extraction variables of all of the remaining blocks contained in area $j$ have been fixed to zero in period $t$, then all of the shovel presence variables $\lambda_{tlj}$, and truck allocation variables $\tau_{tmj}$, are fixed to zero in period $t$. These variable eliminations are far less likely to be realized, but experiments have demonstrated that these variable eliminations have a significant impact on the solving time because the equipment presence variables tend to complicate the problem the most.

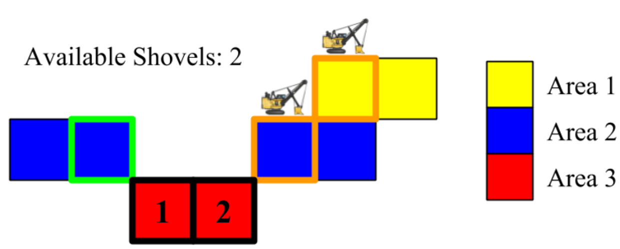


**Figure 3: The precedence set of block 1 (green) and block 2 (orange).**

# 3 Stochastic short-term mine production scheduling of a copper mining complex

In this section, the proposed SSTMPS formulation is applied to a very large copper mining complex to demonstrate its effectiveness in generating short-term production schedules and equipment plans. The outline of the operation and the details of the input parameters are presented below.

### 3.1 Copper operation outline

This copper mining complex uses conventional truck and shovel methods to extract material from two different mines: the Large Pit and the Small Pit. Material is extracted from each of these sources and is then trucked to a processing destination depending on the material's type. A block's material type is defined as sulphide, oxide or waste depending on its total copper and soluble copper content. Material flow through the various processing streams of this mining complex converts raw material into one of two saleable products: copper concentrate or copper cathodes. However, the only components in the system that are relevant to this short-term optimization for this simplified case study are those where mobile equipment is directly involved: the points of shovel extraction and the truck dumping locations. Figure 4 outlines the processing destinations within the mining complex and the types of material each one accepts. Waste and oxide blocks from both mines are sent to the waste dump and oxide leach pad respectively. Sulphides, on the other hand, are allocated depending on the block's location and its total copper grade. Sulphide blocks from either pit that are below a threshold total copper value are sent to the bio-leach pad; blocks above this threshold are send to each mines designated sulphide crusher.

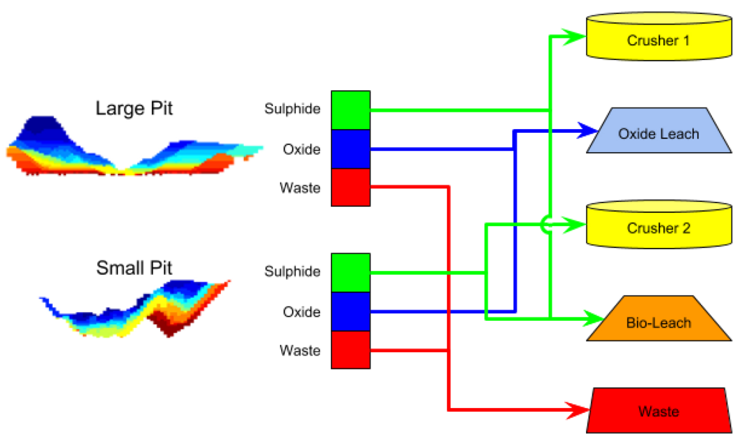


**Figure 4: Flow of material types from each mine to their processing destinations.**

The chosen period length is one month, and the extent of the material to be scheduled is one years' worth of production taken from period nine of the long-term plan (Figure 5). This particular year of production can be easily separated into seven mining areas; two areas in the Small Pit (0 and 1) and the remaining five in the large pit (2, 3, 4, 5 and 6). The long-term plan also defines the number of mobile equipment units that can be allocated, and the desired tonnage targets for each destination. The annual processing targets are equally divided into 12 months to ensure that the destinations receive uniform tonnages throughout the year. Metal grade targets are not relevant in this particular study since material type definitions already reflect minimum copper grade thresholds; grade requirements at each destination will always be respected as a result. The vertical slope angle is 45° and the horizontal precedence set for each block is determined using the method outlined in Section 2.2. The black blocks in Figure 5 represent the ramp positions for each bench in each area, which are used to define the horizontal precedence direction with a tolerance angle $\beta$, of 46°.

The tonnage deviation (penalty) costs at each destination can be seen in Table 1. The sulphide crushers have the highest priority and waste production is penalized the least. Amongst most destinations, shortages are penalized more heavily than surpluses to reflect this operation's emphasis on satisfying minimum tonnage requirements to maximize processor utility. The complete mobile equipment fleet consists of 215 haul trucks and 16 shovels (Table 2). The shovel movement costs between each pair of areas can be seen in Table 3. It should be noted that the cost of moving a shovel from one mine to the other is substantially higher than movements within the same mine. This is to heavily discourage unnecessarily long shovel moves to keep the equipment plan realistic.

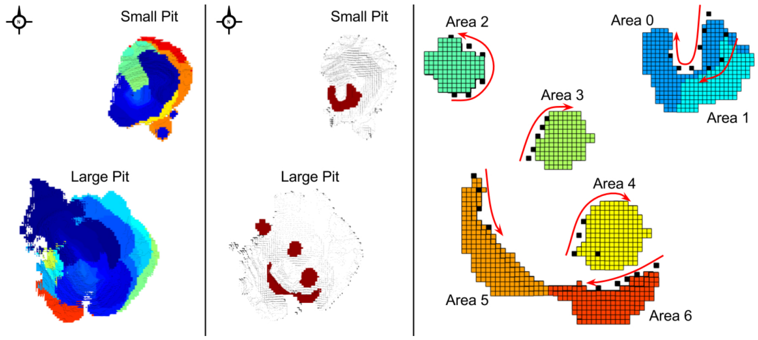


**Figure 5: Mine's long-term schedule (left). Year 9 of the long-term schedule (middle). The 7 areas within year 9, and the descending (arrows) direction of the ramps (right).**

**Table 1: Tonnage deviation costs for each processing destination.**

| Processing Destination | Shortage deviation costs ($/tonne) | Surplus deviation costs ($/tonne) |
|---|---|---|
| Large Pit Crusher | 3.15 | 2.95 |
| Small Pit Crusher | 2.85 | 2.65 |
| Bio-Leach | 1.00 | 1.30 |
| Oxide Leach | 3 | 2 |
| Waste Dump | 2 | 2 |

**Table 2: Mobile equipment fleet size.**

| Trucks | | | Shovels | |
|---|---|---|---|---|
| Model type | Payload (tonnes/trip) | Number of units | Model type | Number of units |
| 1 | 350 | 142 | 1 | 8 |
| 2 | 320 | 54 | 2 | 3 |
| 3 | 220 | 19 | 3 | 5 |
| | | **215** | | **16** |

A total of 20 orebody simulations are used to quantify the variability in the material types and the copper grade for this stochastic optimization. The joint conditional simulations of total copper and soluble copper for the deposit are generated using direct block simulation combined with min/max autocorrelation factors (Boucher and Dimitrakopoulos, 2009). The simulated grades are then filtered through the 4,437 blocks in year 9 for this short-term optimization study. The material type of each block, in each geological scenario, is computed given the simulated copper values. The Small Pit tends to display far more copper grade variability than the Large Pit, and this phenomena will become apparent in the results of the optimization. To model the equipment performance uncertainty, the PCA approach from Section 2.3 is used to jointly simulate 10 equipment performance scenarios consisting of the correlated availabilities, utilities for all equipment and hourly productions for shovels. A brief description and validation of these equipment scenarios and cycle time simulations are presented below.

**Table 3: Equipment movement and production deviation costs.**

| Area | Shovel movement cost (000' $/move) | | | | | | | Shovel deviation cost ($/tonne) | | Truck deviation cost ($/hour) | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | |
| 0 | - | 34 | 2,816 | 2,938 | 3,382 | 3,653 | 3,725 | 3 | 2 | 60 | 60 |
| 1 | 34 | - | 2,830 | 2,915 | 3,337 | 3,636 | 3,679 | 3 | 2 | 60 | 60 |
| 2 | 2,816 | 2,830 | - | 144 | 254 | 212 | 304 | 3 | 2 | 60 | 60 |
| 3 | 2,938 | 2,915 | 144 | - | 118 | 146 | 180 | 3 | 2 | 60 | 60 |
| 4 | 3,382 | 3,337 | 254 | 118 | - | 130 | 69 | 3 | 2 | 60 | 60 |
| 5 | 3,653 | 3,636 | 212 | 146 | 130 | - | 131 | 3 | 2 | 60 | 60 |
| 6 | 3,725 | 3,679 | 304 | 180 | 69 | 131 | - | 3 | 2 | 60 | 60 |

## 3.2 Equipment simulations

### 3.2.1 Equipment performance

The equipment simulations are generated using historical data that measures the monthly availability and utility for shovels and trucks, and the hourly shovel production at the mine. This data was collected and aggregated on an equipment model basis (3 truck types and 3 shovel types) for each pit. However, since shovels are modelled independently in this SSTMPS formulation, the cumulative distribution functions that are constructed in the PCA space are sampled enough times to populate the simulations for each shovel independently. Table 4 and Figure 6 show how the simulated Large Pit equipment performance scenarios reproduce the univariate statistics and bivariate correlations, respectively.

There are clear relationships shown in Figure 6, specifically, the strong utility correlations between the different shovel models, and similarly, between the different truck models. This is logical since equipment will tend to share similar levels of productivity when affected by the same unplanned events, such as mine shutdowns, inclement weather, etc. Most of the availabilities share these strong relationships as well since maintenance performance tends to impact the entire fleet in a similar fashion.

**Table 4: Mean and standard deviations for both original and simulated data (Large Pit).**

| | | Average Value | | Standard Deviation | |
|---|---|---|---|---|---|
| | | Original Data | Simulated Data | Original Data | Simulated Data |
| Shovel 1 | Avail. (%) | 0.836 | 0.836 | 0.0496 | 0.0449 |
| | Utility (%) | 0.679 | 0.685 | 0.0610 | 0.0562 |
| | Prod. (t/h) | 4521.0 | 4528.8 | 313.44 | 298.04 |
| Shovel 2 | Avail. (%) | 0.906 | 0.907 | 0.0246 | 0.0223 |
| | Utility (%) | 0.714 | 0.716 | 0.0434 | 0.0399 |
| | Prod. (t/h) | 4810.9 | 4823.6 | 468.68 | 479.96 |
| Shovel 3 | Avail. (%) | 0.806 | 0.810 | 0.0517 | 0.0481 |
| | Utility (%) | 0.760 | 0.765 | 0.0484 | 0.0452 |
| | Prod. (t/h) | 4767.0 | 4831.1 | 811.23 | 784.73 |
| Truck 1 | Avail. (%) | 0.751 | 0.749 | 0.0503 | 0.0460 |
| | Utility (%) | 0.830 | 0.830 | 0.0123 | 0.0107 |
| Truck 2 | Avail. (%) | 0.812 | 0.811 | 0.0621 | 0.0538 |
| | Utility (%) | 0.830 | 0.832 | 0.0232 | 0.0206 |
| Truck 3 | Avail. (%) | 0.811 | 0.809 | 0.0584 | 0.0515 |
| | Utility (%) | 0.830 | 0.831 | 0.0224 | 0.0201 |

### 3.2.2 Truck cycle times

For this particular application, the only information available pertaining to truck cycle times is the average length of time it takes to reach each destination from each bench in each mine, which is independent of the truck model. A route cycle time in this formulation is defined as the amount of time it takes a truck to complete one cycle from a given area to a destination. The route times are calculated by taking the average cycle time to each destination across all of the blocks contained in an area. A Gaussian distribution is then created, considering a variance of 10% of the mean route time. The mean and variances for each of these route times can be seen in Table 5. Each of these route time distributions are independently sampled 10 times and randomly linked to an equipment performance scenario to yield 10 complete equipment performance simulations.

## 3.3 Optimization results

The SSTMPS formulation in Section 2.4 is solved using the approach described in Section 2.5 and CPLEX is used to solve the sub-problems of each STWH iteration. The final stochastic solution is generated in approximately 9 hours on an Intel Xeon E5-2697 (2.60 GHz, 128 GB RAM). The physical short-term production

| | | Loader 1 | | | Loader 2 | | | Loader 3 | | | Truck 1 | | Truck 2 | | Truck 3 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | A | U | P | A | U | P | A | U | P | A | U | A | U | A | U |
| Loader 1 | A | - | 0.226 | 0.294 | 0.120 | 0.241 | 0.111 | 0.141 | 0.277 | 0.066 | 0.155 | 0.055 | 0.224 | 0.244 | 0.078 | 0.269 |
| | U | 0.226 | - | 0.182 | 0.058 | 0.643 | 0.155 | 0.544 | 0.837 | 0.497 | 0.149 | 0.003 | 0.272 | 0.143 | 0.398 | 0.373 |
| | P | 0.294 | 0.182 | - | 0.030 | 0.305 | 0.615 | 0.425 | 0.068 | 0.083 | 0.254 | 0.122 | 0.022 | 0.304 | 0.323 | 0.082 |
| Loader 2 | A | 0.120 | 0.058 | 0.030 | - | 0.291 | 0.073 | 0.152 | 0.175 | 0.224 | 0.062 | 0.275 | 0.435 | 0.389 | 0.187 | 0.167 |
| | U | 0.241 | 0.643 | 0.305 | 0.291 | - | 0.181 | 0.170 | 0.677 | 0.222 | 0.288 | 0.054 | 0.099 | 0.079 | 0.465 | 0.041 |
| | P | 0.111 | 0.155 | 0.615 | 0.073 | 0.181 | - | 0.172 | 0.200 | 0.448 | 0.211 | 0.010 | 0.058 | 0.218 | 0.195 | 0.119 |
| Loader 3 | A | 0.141 | 0.544 | 0.425 | 0.152 | 0.170 | 0.172 | - | 0.393 | 0.271 | 0.167 | 0.114 | 0.022 | 0.029 | 0.084 | 0.399 |
| | U | 0.277 | 0.837 | 0.068 | 0.175 | 0.677 | 0.200 | 0.393 | - | 0.649 | 0.319 | 0.168 | 0.065 | 0.310 | 0.474 | 0.452 |
| | P | 0.066 | 0.497 | 0.083 | 0.224 | 0.222 | 0.448 | 0.271 | 0.649 | - | 0.081 | 0.144 | 0.062 | 0.260 | 0.015 | 0.379 |
| Truck 1 | A | 0.155 | 0.149 | 0.254 | 0.062 | 0.288 | 0.211 | 0.167 | 0.319 | 0.081 | - | 0.156 | 0.175 | 0.044 | 0.592 | 0.246 |
| | U | 0.055 | 0.003 | 0.122 | 0.275 | 0.054 | 0.010 | 0.114 | 0.168 | 0.144 | 0.156 | - | 0.185 | 0.888 | 0.024 | 0.691 |
| Truck 2 | A | 0.224 | 0.272 | 0.022 | 0.435 | 0.099 | 0.058 | 0.022 | 0.065 | 0.062 | 0.175 | 0.185 | - | 0.150 | 0.047 | 0.137 |
| | U | 0.244 | 0.143 | 0.304 | 0.389 | 0.079 | 0.218 | 0.029 | 0.310 | 0.260 | 0.044 | 0.888 | 0.150 | - | 0.057 | 0.723 |
| Truck 3 | A | 0.078 | 0.398 | 0.323 | 0.187 | 0.465 | 0.195 | 0.084 | 0.474 | 0.015 | 0.592 | 0.024 | 0.047 | 0.057 | - | 0.155 |
| | U | 0.269 | 0.373 | 0.082 | 0.167 | 0.041 | 0.119 | 0.399 | 0.452 | 0.379 | 0.246 | 0.691 | 0.137 | 0.723 | 0.155 | - |

| | | Loader 1 | | | Loader 2 | | | Loader 3 | | | Truck 1 | | Truck 2 | | Truck 3 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | A | U | P | A | U | P | A | U | P | A | U | A | U | A | U |
| Loader 1 | A | - | 0.248 | 0.303 | 0.189 | 0.311 | 0.143 | 0.174 | 0.295 | 0.102 | 0.089 | 0.028 | 0.203 | 0.268 | 0.118 | 0.233 |
| | U | 0.248 | - | 0.133 | 0.034 | 0.650 | 0.217 | 0.595 | 0.864 | 0.498 | 0.191 | 0.035 | 0.212 | 0.217 | 0.340 | 0.435 |
| | P | 0.303 | 0.133 | - | 0.024 | 0.265 | 0.641 | 0.326 | 0.004 | 0.167 | 0.289 | 0.141 | 0.051 | 0.391 | 0.343 | 0.066 |
| Loader 2 | A | 0.189 | 0.034 | 0.024 | - | 0.303 | 0.044 | 0.145 | 0.175 | 0.209 | 0.079 | 0.234 | 0.428 | 0.335 | 0.172 | 0.133 |
| | U | 0.311 | 0.650 | 0.265 | 0.303 | - | 0.136 | 0.224 | 0.682 | 0.199 | 0.307 | 0.045 | 0.077 | 0.108 | 0.459 | 0.086 |
| | P | 0.143 | 0.217 | 0.641 | 0.044 | 0.136 | - | 0.285 | 0.277 | 0.501 | 0.191 | 0.012 | 0.084 | 0.302 | 0.226 | 0.162 |
| Loader 3 | A | 0.174 | 0.595 | 0.326 | 0.145 | 0.224 | 0.285 | - | 0.432 | 0.267 | 0.135 | 0.162 | 0.039 | 0.004 | 0.033 | 0.393 |
| | U | 0.295 | 0.864 | 0.004 | 0.175 | 0.682 | 0.277 | 0.432 | - | 0.641 | 0.337 | 0.174 | 0.056 | 0.346 | 0.429 | 0.468 |
| | P | 0.102 | 0.498 | 0.167 | 0.209 | 0.199 | 0.501 | 0.267 | 0.641 | - | 0.056 | 0.152 | 0.112 | 0.271 | 0.020 | 0.391 |
| Truck 1 | A | 0.089 | 0.191 | 0.289 | 0.079 | 0.307 | 0.191 | 0.135 | 0.337 | 0.056 | - | 0.093 | 0.166 | 0.028 | 0.636 | 0.211 |
| | U | 0.028 | 0.035 | 0.141 | 0.234 | 0.045 | 0.012 | 0.162 | 0.174 | 0.152 | 0.093 | - | 0.190 | 0.847 | 0.046 | 0.656 |
| Truck 2 | A | 0.203 | 0.212 | 0.051 | 0.428 | 0.077 | 0.084 | 0.039 | 0.056 | 0.112 | 0.166 | 0.190 | - | 0.144 | 0.050 | 0.137 |
| | U | 0.268 | 0.217 | 0.391 | 0.335 | 0.108 | 0.302 | 0.004 | 0.346 | 0.271 | 0.028 | 0.847 | 0.144 | - | 0.105 | 0.691 |
| Truck 3 | A | 0.118 | 0.340 | 0.343 | 0.172 | 0.459 | 0.226 | 0.033 | 0.429 | 0.020 | 0.636 | 0.046 | 0.050 | 0.105 | - | 0.115 |
| | U | 0.233 | 0.435 | 0.066 | 0.133 | 0.086 | 0.162 | 0.393 | 0.468 | 0.391 | 0.211 | 0.656 | 0.137 | 0.691 | 0.115 | - |

**Figure 6: Absolute coefficient of correlation for each equipment performance parameter. Original historical data (top) and simulated data (bottom). Stronger relationships are highlighted in darker green.**

**Table 5: Mean and variances for route cycle times.**

| Processing Destinations | | From area 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| Crusher 1 | Mean | - | - | 62.10 | 69.79 | 55.96 | 41.91 | 41.71 |
| | Var. | - | - | 6.21 | 6.98 | 5.60 | 4.19 | 4.17 |
| Crusher 2 | Mean | 45.52 | 42.20 | - | - | - | - | - |
| | Var. | 4.55 | 4.22 | - | - | - | - | - |
| Bio-Leach | Mean | 89.08 | 84.90 | 90.49 | 97.08 | 85.22 | 73.16 | 72.99 |
| | Var. | 8.91 | 8.49 | 9.05 | 9.71 | 8.52 | 7.32 | 7.30 |
| Oxide Leach | Mean | 119.52 | 116.47 | 79.67 | 90.10 | 71.34 | 52.27 | 51.99 |
| | Var. | 11.95 | 11.65 | 7.97 | 9.01 | 7.13 | 5.23 | 5.20 |
| Waste Dump | Mean | 62.73 | 59.10 | 74.59 | 81.77 | 68.86 | 55.72 | 55.54 |
| | Var. | 6.27 | 5.91 | 7.46 | 8.18 | 6.89 | 5.57 | 5.55 |

schedule is displayed in Figure 7. The monthly mining patterns tend to be well connected and obey the directions of mining imposed by the ramp locations.

Figure 8 demonstrates how the optimizer assigns equipment to certain areas of the mine to move material. Allocating more shovels to area 4 near the end of the year allows for an increased extraction rate. More trucks are also required in these later months to deliver the extracted material. However, there is some variation as to how many trucking units are assigned in each of these months. Since different truck models have different

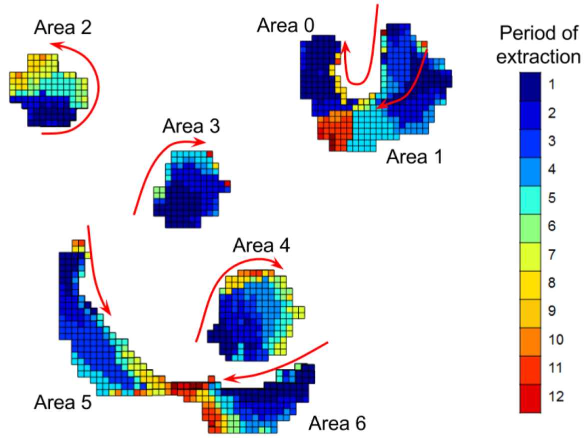


**Figure 7: Stochastic short-term production schedule with descending ramp directions.**

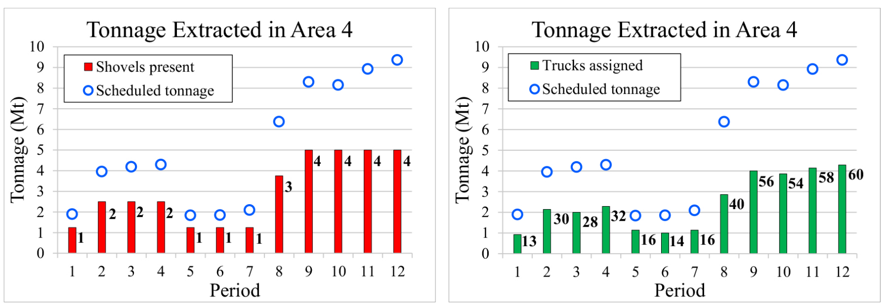


**Figure 8: Example of tonnage extracted from area 4. Shovels present (left) and trucks allocated (right) necessary to extract the scheduled material.**

payloads and since cycle times depend on the destinations of blocks, similarly sized groupings of blocks in the same area can require significantly different trucking times. Figure 9 shows a best-case and worst-case example of the shovel movement schedule. Shovel 11 only makes one very short move in the small pit throughout the entire year, whereas Shovel 15 makes four moves within the large pit. Only three shovel moves between the two pits occurred in this optimized equipment plan. These inter-pit moves could be discouraged even further using higher penalty costs, but this sensitivity was not explored in this particular study.

| Shovel #11 | Period | Start | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Area | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| Shovel #15 | Period | Start | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Area | 6 | 6 | 4 | 4 | 4 | 6 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |

**Figure 9: Movement schedules for two different shovels. Areas are colour-coded.**

To demonstrate the benefits that incorporating grade and equipment uncertainty have in short-term production scheduling, the stochastic solution is benchmarked against a more conventional approach. A deterministic version of the proposed model is solved, and comparisons are drawn in terms of destination production target satisfaction, and equipment performance deviations. This conventional optimization uses an estimated orebody and average equipment performances. Specifically, the orebody model is generated using ordinary kriging and the identical material type definitions are applied. The availabilities and utilities are considered to be the average across all simulated scenarios, and the mean values for all route cycle times are used.

### 3.3.1 Impact of geological uncertainty

The $10^{th}$, $50^{th}$ and $90^{th}$ percentiles of the monthly tonnages sent to each destination are shown in Figure 10. Since block destinations depend on the blocks uncertain material type, a distribution of possible tonnages is expected. The blue risk profiles in the left column are the tonnages from the stochastic solution, and the orange profiles on the right are from the conventional design. All tonnages presented below are expressed as a percentage of the destination target for confidentiality purposes. A copper grade comparison at each destination is irrelevant in this study since destination decisions are based solely on material types; grade targets are always respected in both the stochastic and conventional solutions because of the a priori definition of the material types. This comparison shows how the stochastic solution is able to better-manage the geological risk. At each destination in this mining complex, the expected tonnages in the stochastic solution are consistently within the targets, and the tonnage profile is far less variable when compared to the conventional results. It can be seen that the satisfaction of these tonnage targets tends to deteriorate more in the later period for both the stochastic and conventional solution; this is an artifact of the sequential nature of the STWH solution approach.

Through an improved understanding of material type and grade variability in the stochastic model, the optimization process can better blend geological risk amongst different periods to facilitate a mine plan that can be implemented with greater confidence. The conventional plan is expected to frequently violate the tonnage targets, which will inevitably lead to increased re-handling costs as well as decreased utilization of the processors. The greater copper grade variability present in the small pit can be observed from the results in Figure 10. This pit provides the majority of the oxide-leach material and all of the small pit crusher material; both of which display the highest degree of tonnage variability in the stochastic and conventional solutions alike. Regardless, the stochastic solution tends to better manage this variability and deliver more certain monthly tonnages to the related destinations.

### 3.3.2 Impact of equipment uncertainty

Figure 11 shows the production risk profiles of both solutions with respect to the equipment uncertainty. The inclusion of the equipment variability tends to allow the stochastic optimizer to yield a production schedule that is far more likely to be achieved. The stochastic production is comfortably within the expected operating ranges throughout the year (Figure 11; left). In contrast, the conventional design often chooses to extract groupings of blocks that are highly unlikely to be produced by the mobile fleet (Figure 11; right). The overall risk profile of the stochastic schedule is only slightly tighter than the conventional one, measured by the distance between the P-10 and P-90 profiles in each month. This is because the historical hourly productions are very similar amongst all loading units in this study. There is little to be gained by grouping different shovels together in the same areas, whose production profiles would complement each other's to minimize these variabilities by significant amounts. Also, the total extraction graphs can be misleading since lower and upper equipment production deviations from different areas may appear to balance out. The production in Area 0 (Figure 11; bottom) and the absolute shovel deviations in Figure 12 are more reliable in demonstrating how the stochastic solution yields production schedules that are far more attainable.

Figure 13 shows the cumulative absolute truck hour deviations averaged across all cycle time scenarios. This graph compares the truck hour deviations between both solutions. Although more consistent, the deviations in the conventional case are equal to, if not less than, that of the stochastic solution. It can be inferred that there is little benefit of including cycle time uncertainty in the manner proposed herein. There are two major factors that may contribute to this conclusion. First, the conventional optimizer decides to extract less tonnage than in the stochastic solution, specifically in waste and oxide material (Figure 10). Having relatively long cycle times, under-producing these material types places less strain on truck production which results in less truck hour shortages. This can be further supported by the fact that 70% of the total truck hour deviations are shortages in the stochastic solution compared to 60% in the conventional case. Second, and more importantly, the chosen cycle time simulation approach adds very little value to the stochastic solution. The symmetric nature of Gaussian distributions, coupled with equivalent shortage and surplus truck hour deviation penalties (Table 3), allows for cycle time uncertainty to be fully characterized by its mean value, just as proposed in the conventional approach. For there to be value gained from optimal

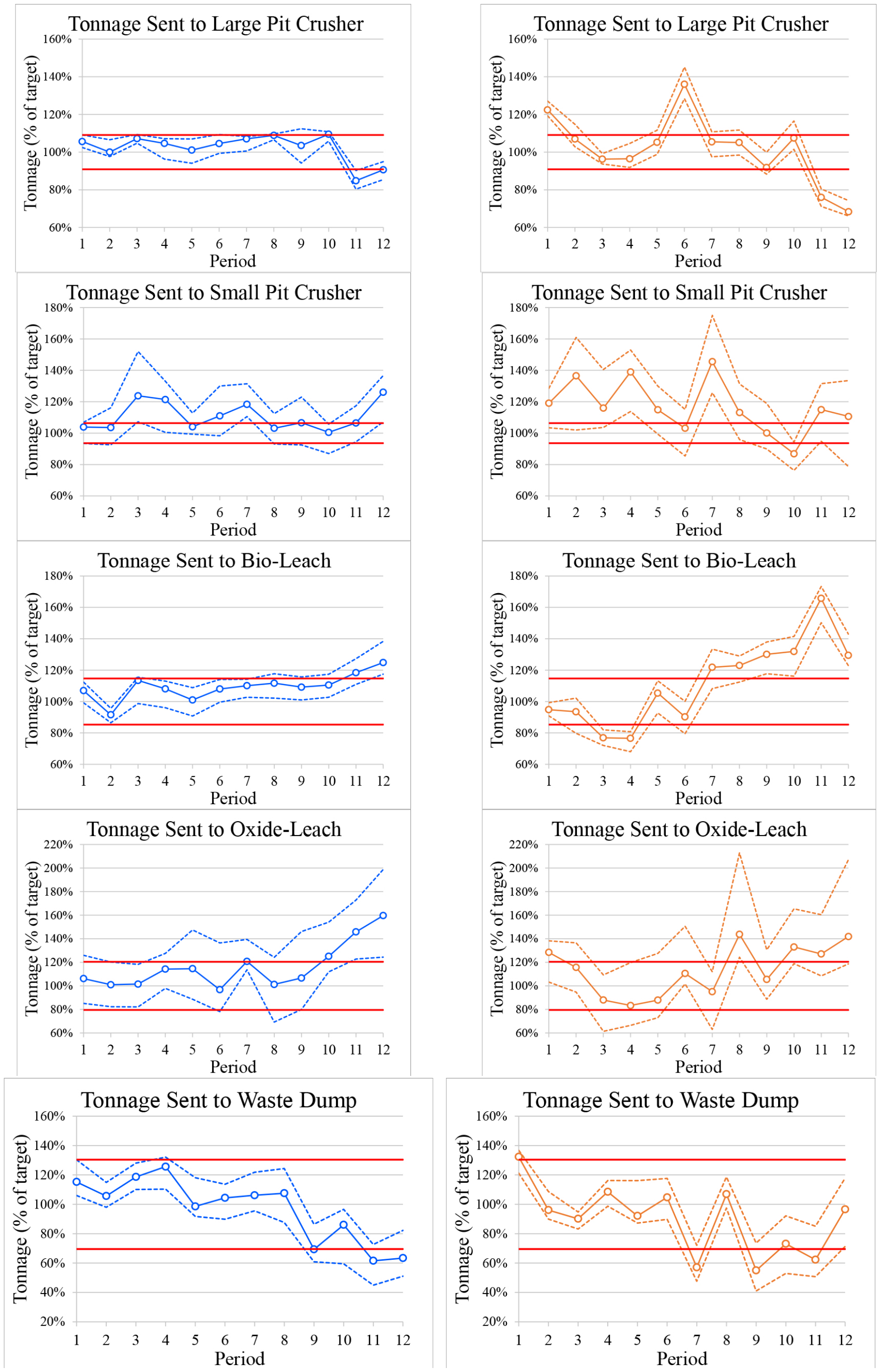


**Figure 10: Tonnage risk profiles for the stochastic (left) and conventional (right) production schedules. The dotted lines are the $10^{th}$ and $90^{th}$ percentiles, the solid lines are the $50^{th}$ percentiles, and the red lines are the destination's upper and lower targets.**

truck assignments through incorporating cycle time uncertainty, more insights would need to be drawn from a better understanding of the factors that result in cycle time uncertainty. Some potential factors could be: truck payload densities for different material types, road quality variations by season, ramp grade, etc.

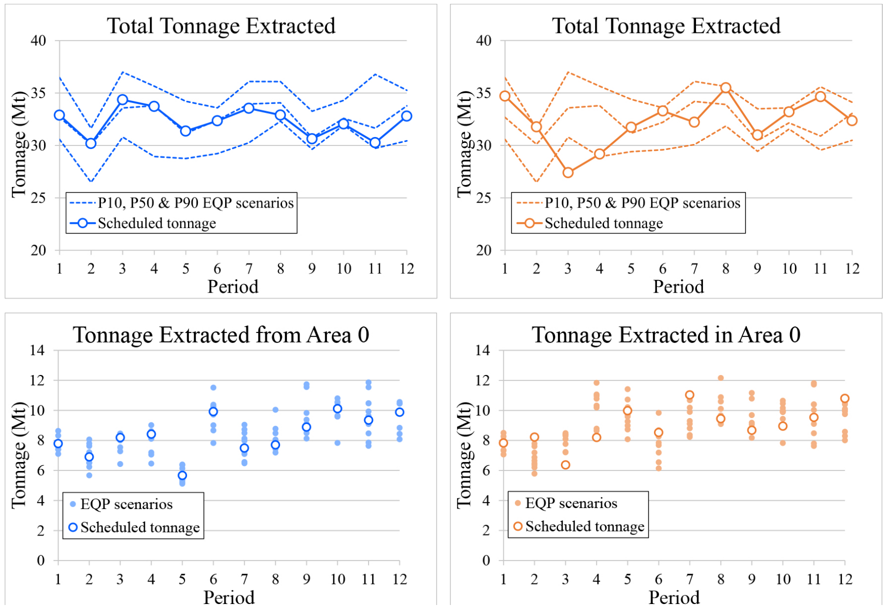


**Figure 11: Scheduled production each month compared to the uncertain equipment production for the stochastic solution (left) and conventional solution (right). Total tonnage extracted (top) and an example of tonnage extracted from one area (bottom).**

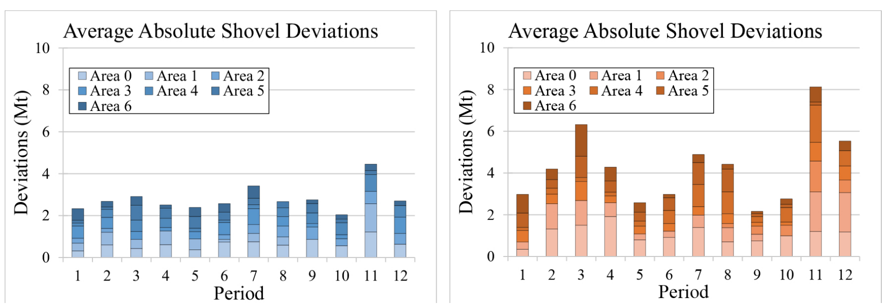


**Figure 12: Absolute shovel production deviations in each area averaged across all equipment performance scenarios. Stochastic (left) and conventional (right) solutions.**

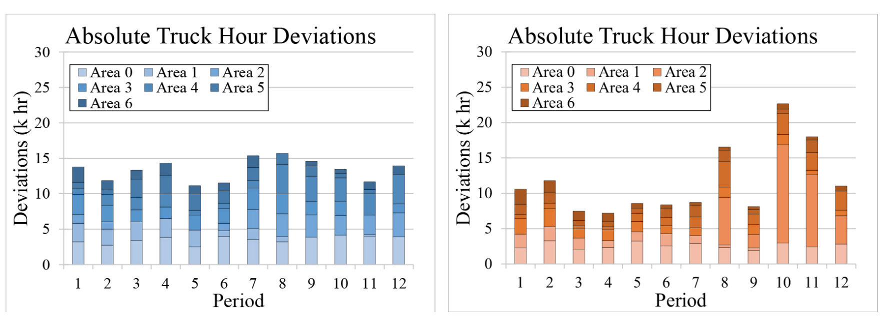


**Figure 13: Absolute trucking hour deviations in each area averaged across all cycle time scenarios. Stochastic (left) and conventional (right) solutions.**

# 4 Conclusions and future work

This work proposes a new stochastic short-term model to jointly optimize the equipment plan and production schedule of mining complexes while incorporating both geological and equipment performance uncertainty. This formulation introduces: a new approach to generate more realistic equipment performance scenarios, a more accurate shovel movement formulation, and an efficient means to encourage connected extraction patterns with the notion of horizontal precedence. The stochastic short-term mine production scheduling formulation is applied to a very large copper mining complex and the results are compared to conventional methods to show the benefits of including geological and equipment uncertainties in the problem. The stochastic formulation generates implementable equipment plans and extraction patterns while successfully controlling the operational risk in terms of delivering more reliable tonnages to the processors, and defining an extraction sequence that is far more likely to be achieved.

There are three main areas that may be improved and should be the focus for avenues of future work. 1) The complexity and size of the case study presented in this work approaches the limits that commercial solvers can handle efficiently. To accommodate models with more decision variables, future work should focus on developing fast and computationally efficient heuristic solvers. 2) More comprehensive cycle time distributions should be investigated. There is significant value to be gained through allowing new stochastic formulations to make well-informed decisions considering complex equipment variability factors in short-term production scheduling. 3) To expand into different areas of mine plan optimization, other aspects could be jointly optimized, such as: optimal equipment maintenance scheduling, optimal temporary ramp placement, and optimal equipment usage in terms of loading factors and trucking speeds.

# 5 References

Alarie, S., and Gamache, M. (2002). Overview of Solution Strategies Used in Truck Dispatching Systems for Open Pit Mines. International Journal of Surface Mining, Reclamation and Environment, 16(1), 59–76.

Birge, J. R., and Louveaux, F. (1997). Introduction to stochastic programming (2 Ed.): Springer.

Boucher, A., and Dimitrakopoulos, R. (2009). Block Simulation of Multiple Correlated Variables. Mathematical Geosciences, 41(2), 215–237.

Burt, C., Caccetta, L., Fouché, L., and Welgama, P. (2016). An MILP approach to Multi-location, Multi-Period Equipment Selection for Surface Mining with Case Studies. Journal of industrial and management optimization, 12(2), 403–430.

Cullenbine, C., Wood, R. K., and Newman, A. (2011). A sliding time window heuristic for open pit mine block sequencing. Optimization Letters, 5(3), 365–377.

Dimitrakopoulos, R., Farrelly, C.T. and Godoy, M. (2002) Moving forward from traditional optimization: grade uncertainty and risk effects in open pit design. Transactions of Institute of Mining and Metallurgy (Section A: Mining Technology), 111, A82–A88.

Dimitrakopoulos, R., and Jewbali, A. (2013). Joint stochastic optimisation of short and long term mine production planning: method and application in a large operating gold mine. Mining Technology, 122(2), 110–123.

Dimitrakopoulos, R., and Ramazan, S. (2008). Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions. Mining Technology, 117(4), 155–160.

Eivazy, H., and Askari-Nasab, H. (2012). A mixed integer linear programming model for short-term open pit mine production scheduling. Mining Technology, 121(2), 97–108.

Fytas, K. G. (1985). A computerized model of open pit long and short term production scheduling. (PhD), Queen's University, Kingston, Ontario, Canada.

Godoy, M. (2003). The effective management of geological risk in long-term production scheduling of open pit mines. (Ph.D. Dissertation), University of Queensland, Brisbane, Australia. Goovaerts, P. (1997). Geostatistics for natural resources evaluation. New York: Oxford University Press.

Hotelling, H. (1933). Analysis of a complex of statistical variables into principal components. Journal of Educational Psychology, 24(6), 417–441.

Kumral, M., and Dowd, P. A. (2002). Short-term mine production scheduling for industrial minerals using multi-objective simulated annealing. Paper presented at the Proceedings 30th APCOM Symposium, Fairbanks, Alaska, USA.

L'Heureux, G., Gamache, M., and Soumis, F. (2013). Mixed integer programming model for short term planning in open-pit mines. Mining Technology, 122(2), 101–109.

Matamoros, M., and Dimitrakopoulos, R. (2014). Stochastic short-term production scheduling accounting for fleet allocation, operational considerations, belnding restrictions and future multi-element ore control data. (MEng), McGill University.

Mousavi, A., Kozan, E., and Liu, S. Q. (2016) Open-pit block sequence optimization: a mathematical model and solution technique. Engineering Optimization, 48(11), 1932–1950.

Pochet, Y., and Wolsey, L. A. (2006). Production planning by mixed integer programming: Springer Science and Business Media.

Ramazan, S., and Dimitrakopoulos, R. (2013). Production scheduling with uncertain supply: a new solution to the open pit mining problem. Optimization and Engineering, 14(2), 361–380.

Ravenscroft, P. J. (1992). Risk analysis for mine scheduling by conditional simulation. Transactions Australian Intitute of Mining and Metallurgy, pp 104–108.

Smith, M. L. (1998). Optimizing short-term production schedules in surface mining: Integrating mine modeling software with AMPL/CPLEX. International Journal of Surface Mining, Reclamation and Environment, 12(4), 149–155.

Stanford, California. (1984). Min/max autocorrelation factors for multivariate spatial imagery. Standford University, Department of Statistics(6).

Topal, E. (2008). Early start and late start algorithms to improve the solution time for long-term underground mine production scheduling. Journal of the Southern African Institute of Mining and Metallurgy, 108, 99–107.

Topal, E., and Ramazan, S. (2010). A new MIP model for mine equipment scheduling by minimizing maintenance cost. European Journal of Operational Research, 207(2), 1065–1071.

Topal, E., and Ramazan, S. (2012). Mining truck scheduling with stochastic maintenance cost. Journal of Coal Science and Engineering (China), 18(3), 313–319.

Vallée, M. (2000). Mineral resource + engineering, economic and legal feasibility = ore reserve. CIM Bulletin, 93(1038), 53–61.

Wilke, F. L., and Reimer, T. H. (1977). Optimizing the short-term production schedule for an open-pit iron ore mining operation. Paper presented at the Proceedings 15th International APCOM Symposium, Brisbane, Australia.